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COMPREHENSIVE PHYSICS PRACTICALS

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Comprehensive Physics Practicals

Ngaiza Series of Advanced Physics

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TABLE OF CONTENTS

Preface ..... **i**

PART ONE GROUNDWORK **1**

1 Inside the Practical Paper 2
2 Instruments and Readings 6
3 Tables and Graphs 25
4 Knowing What Your Results Should Be 39
5 Error Analysis 51
6 Report Writing and the Unseen Question 62

PART TWO MECHANICS **73**

7 The Simple Pendulum Family 74
8 Compound Pendulums and Laminas 106
9 Bifilar Suspension and Torsion 126
10 Springs and Hooke's Law 146
11 Elasticity: Young's Modulus 165
12 Moments, Equilibrium and Applications 181

PART THREE HEAT **199**

13 Cooling Curves and Newton's Law of Cooling 200
14 Comparative Cooling 217
15 Specific Heat Capacity 242
16 Heating Curves and Change of State 262

PART FOUR CURRENT ELECTRICITY **277**

17 Ohm's Law and Direct Resistance 278
18 E.M.F. and Internal Resistance 298
19 The Metre Bridge Family 324
20 Potentiometer 352

PART FIVE GUIDE-LISTED BUT RARELY EXAMINED EXPERIMENTS **368**

21 Vibrations and Waves: Resonance 369
22 Light: Reflection and Refraction 386
23 Fluid Dynamics: Viscosity 405
24 Surface Tension 416

Solutions to Practice Experiments **427**

Index of Experiments **487**

Index of Apparatus **491**

PREFACE

There is a morning in every physics course when the reading stops. You are handed a thread, a bob, a metre rule and a stopwatch, told to find the acceleration due to gravity, and left alone with them for the better part of an afternoon. What you do in those hours is examined, and the marks are not given for knowing physics. They are given for a table with a unit in every heading, a graph with a title and two labelled axes, a slope taken from a triangle a marker can see, a value with its unit, and a sentence setting that value beside the one that was expected.

Those are the easiest fifty marks in the subject to earn and the most regularly thrown away, because the practical is the paper that is examined and never taught. You are told to be careful. Nobody tells you what careful looks like.

That is what this book is for.

Part One, Groundwork, is the craft of it: what the paper contains and where its marks actually lie; how to read every instrument on the bench, and how many figures the reading entitles you to write; how to build a table and draw a graph that earn their marks before the physics is looked at; how to know what your results should be before you take them; what an uncertainty is, and what it is not; and how to write the answer, including the answer to a question you have never seen.

Parts Two to Five are the physics: seventy-two experiments worked in full and seventy-five practice questions, through Mechanics, Heat and Current Electricity, and the vibrations, light, fluids and surfaces that close the course.

No paper sets the same experiment twice in the same words, so the book is not built as a list of experiments. It is built in families. Each chapter takes one method and turns it over: the same apparatus with a different unknown, the same readings plotted on a different pair of axes, the same relation with one term added that the question expects you to notice. By the fourth variety the family is familiar, and a fifth one you have never met is recognisable.

Every table of readings printed here is a specimen, and the readings in it are of the kind careful hands actually record. They scatter. They round to what the instrument allows and no further. They do not sit flat on the line, because real readings do not. Learning to see at a glance that a table could not have come off a bench is most of the way to producing one that could.

Three people recur in the openings: Mr. Akilikubwa, who teaches, and two students, Kipute and Kipanga. They are there because a question is easier to want to answer when somebody in the room wants to know.

How this book is put together

Every chapter opens with a table of its varieties, saying what each experiment asks you to do and what comes out of it. Read that table first. It is the map of the family.

A worked experiment then runs in one order, the same every time: the opening, the Aim, the Apparatus, the Setup with its drawing, the Procedure, the Questions, the Table of results, Making the measurement count, the Answers to the questions and Examiner's eye. Work the Questions from the Table of results before you turn to the Answers. Read the other way round, the section is worth very little.

A practice question carries the opening, the Procedure and the Questions, and nothing under them. Its answers, and the Examiner's eye that belongs to it, are gathered at the back of the book under Solutions to Practice Experiments, so that you can work the question first and mark it afterwards.

Two of the sections are written for different moments. Making the measurement count is for the bench: what to do with your hands, why the apparatus is arranged as it is, and where the relation printed in the question came from. Examiner's eye is for the night before: what gets thrown away on a question of this kind, and how not to throw it away.

At the very back stand two indexes. The Index of Experiments lists what the book determines and by what method, so that a reader who wants the specific heat capacity of a solid can find every set that yields one. The Index of Apparatus lists every experiment under the apparatus it calls for, so that a school can see at once what it is equipped to set up.

There are three ways through the book. If you are revising one topic, take its chapter, work the practice questions and mark them at the back. If you have one evening left, read Part One, and then the Examiner's eye of the

families your paper is likely to draw on. If you are teaching, the varieties tables and the Index of Apparatus between them will build a mock practical out of whatever is in the cupboard.

The team behind this book

Comprehensive Physics Practicals is the product of the Ngaiza Education Hub. It was built by a team of experienced physics educators who share a common conviction: that Tanzanian students deserve world-class teaching materials written for their context, their syllabus, and their aspirations. For this volume they added one more: that a student left alone with a thread and a stopwatch should have been shown, long before that morning, exactly what is being asked of him.

Writers: Mr. Amir Juma, Mr. Kelvin Mashu, Mr. Endrew Mariki, and Mr. Morris Mulokozi.

Editors: Dr. Aloyce Julian and Dr. Alex Muganyizi.

Designer: Mr. Frank Mwita.

I am grateful to every member of this team for their dedication, their expertise, and their refusal to settle for "good enough". I also thank the students whose hard questions and honest frustrations shaped every chapter. They reminded us, constantly, that a textbook which cannot be understood by the student it was written for has failed, no matter how correct its physics may be.

Ngaiza Lusima
Series Creator and Director

PART ONE

GROUNDWORK

Chapter 1

INSIDE THE PRACTICAL PAPER

On the first morning of the practical course **Mr. Akilikubwa** wrote a single number on the board, 200, and then sat down without explaining it.

The class waited. **Kipute**, who had already worked out that waiting was usually the wrong strategy, asked what it meant.

"Minutes," he said. *"That is the whole of it. Three hours and twenty minutes, once, at the end of two years. Fifty marks. Three questions, and no choice about which of them you answer."* He looked round the room. *"Most candidates walk into that laboratory having never been told what is inside the paper, which is a strange way to sit an examination. You will not be among them."*

He picked up the chalk again. *"By the end of this lesson you will know how the fifty marks are divided, what the first page hands you free, which topics the paper is allowed to ask about, and where the marks are actually lost. None of it is secret. It is simply that almost nobody reads it."*

What is in front of you

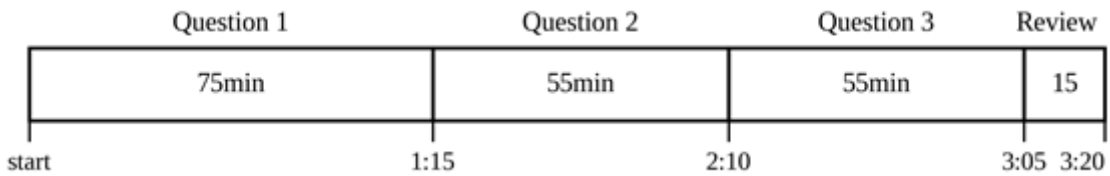
The practical paper is a single sitting of three hours and twenty minutes. It carries three questions, all of them compulsory, and you answer them at a bench with the apparatus in front of you. There is no choice to make and nothing to leave out.

The paper exists in three alternatives, and for a practical reason. A laboratory seats only so many candidates at once, so a school with more candidates than benches sends them in two or three sittings, each taking a different alternative. The friend you revised with may sit a different one after you. All three are built to the same plan and test the same skills, so the alternative you receive changes the apparatus in front of you, not the physics you need to know. This is the single most important thing to understand about preparing: you cannot revise one experiment and hope. You revise a family of experiments, and you learn what they have in common.

How the fifty marks are divided

Question	What it is drawn from	Marks	Minutes to spend
1	Mechanics	20	75
2	Heat	15	55
3	Current Electricity	15	55
	Reading back through what you have written		15
	Total	50	200

Question 1 is worth a third more than either of the others, and it has always been the Mechanics experiment. Questions 2 and 3 carry fifteen marks each, and have always been Heat and Current Electricity, in that order. You therefore know, before you open the paper, which three parts of the syllabus you are about to be asked about.



The arithmetic in that strip is worth doing yourself. Fifty marks in two hundred minutes comes to

$$\frac{200 \text{ min}}{50 \text{ marks}} = 4 \text{ min per mark}$$

so a twenty-mark question is worth about eighty minutes and a fifteen-mark question about sixty. The budget above trims each of those a little in order to leave a quarter of an hour at the end. That quarter of an hour is the most profitable fifteen minutes in the whole paper, and the first thing a candidate in trouble abandons.

What the first page gives you

Before the first question there is a page of instructions, and candidates turn past it. Read it. It tells you, every year, that there are three questions and all are to be answered; that Question 1 carries twenty marks and the other two fifteen each; that mathematical tables and non-programmable calculators may be used; that all writing must be in blue or black ink, except drawings, which must be in pencil; and that your examination number goes on every page.

Then comes a line that repays attention: *the following information may be useful*, and beneath it the constants. The value of π is given every year as 3.14. The specific heat capacity of water appears whenever the Heat question needs it, usually as 4.2J/gK, and that of copper when the calorimeter you are handed is a copper one. What is never given is a quantity the paper intends you to measure: you will not be handed a value for the acceleration due to gravity, because finding it is frequently the point.

Practical tip 1: *Use the value of π the paper gives you, not the one stored in your calculator.*

The difference looks negligible and is not quite. With the given value,

$$4\pi^2 = 4 \times (3.14)^2 = 39.4384$$

whereas a calculator, left to itself, returns 39.4784. Carried through a pendulum experiment, that shifts your value of the acceleration due to gravity by about 0.1%. It is small, but it is avoidable, and it is the kind of quiet inconsistency that turns a clean piece of work into an argument. The paper hands you a value so that everybody's arithmetic agrees. Use it. Every worked answer in this book does.

Where the paper is allowed to go

Ten areas of the syllabus can be set as a practical question at this level. The list is short enough to learn, and knowing it tells you what you can safely stop worrying about.

Area the paper may set	Worked in this book at
Structure and properties of matter	Chapters 10, 11 and 24
Forces in equilibrium	Chapter 12
Motion in a straight line	Chapter 7
Simple harmonic motion	Chapters 7 to 10
Rotation of rigid bodies	Chapters 8 and 9
Fluid dynamics	Chapter 23
Light	Chapter 22
Heat	Chapters 13 to 16
Vibrations and waves	Chapter 21
Current electricity	Chapters 17 to 20

Where the marks actually are

A practical question is not marked on whether your final number is right. It is marked on the trail you leave. Six things are looked for on a graph, and they are worth committing to memory, because they are the same six every time:

1. the title,
2. the scales,
3. the axes,
4. the transfer of your points onto the grid,
5. the best straight line or curve,
6. the indication of the slope.

Not one of those six is physics. Every one of them is free to a candidate who knows the list and expensive to one who does not. The model answers in this book set out all six every time, in the same order, until the order is a habit.

The same principle governs the rest of the answer. Your table earns marks for being a table: headed columns, a unit in each heading and nowhere else, a consistent number of decimal places down each column, and derived columns actually worked out. Your conclusion earns marks for being a conclusion: a value, its unit, and a sentence comparing it with what was expected.

Which question to take first

Take them in order unless you have a reason not to. Question 1 carries the most marks and you are freshest at the start, which is the whole argument. Two cautions are worth holding, though.

The Heat question usually runs on a clock of its own. Once you have poured hot water and started the stopwatch, the readings arrive every minute or two for the next twenty, and you cannot hurry them or leave them. Look at Question 2 early enough that you are not still waiting for a kettle when the invigilator calls the last half hour.

The Electricity question is the one that goes wrong. A circuit that will not balance eats time in a way a swinging pendulum never does, and it eats it at the end of the paper when you have least to spare. Never leave Question 3 to the final thirty minutes if you can help it.

Examiner's eye

- The commonest single loss in the whole paper is not physics. It is a graph with no title, or axes with no units on them, thrown away by a candidate who could do the experiment perfectly well.
- You are marked on the table you present, not on the figures you scribbled at the bench. Rule the table before you take the first reading, and write into it.
- Answer the question that is asked. If it says state the physical meaning of the slope, a number is not an answer; if it says determine, an estimate read off by eye is not an answer.
- A value with no unit is not a value. This costs marks somewhere in almost every script.
- The question is not finished when the number appears. Compare it with what was expected, and say in one sentence why the two differ. That sentence is often worth as much as the calculation that produced the number.
- Keep the last fifteen minutes. Check that every graph has a title, every column a unit, every answer a unit, and every question a response. Candidates lose more marks to blank parts than to wrong ones.

Drills

Work each one before reading the answer under it.

1. You have finished Question 1 and the clock shows that one hour and thirty-five minutes have gone. Are you ahead or behind, and by how much? What should you do about it?

   Answer: Behind by twenty minutes: the budget allows seventy-five. Do not try to win them back on Question 2, whose cooling readings cannot be hurried. Win them back on writing: tabulate as you measure, and keep the comment short. Give up five review minutes, but not all fifteen.

2. The paper gives $\pi = 3.14$. Your calculator holds a longer value. Which do you use, and what practical difference does it make?

   Answer: Use 3.14, the value you were given. In a pendulum experiment the two values differ in $4\pi^2$ by about one part in a thousand, so the acceleration due to gravity comes out about 0.1% apart. The difference is small; the reason to prefer the given value is that your arithmetic then agrees with everyone else's.

3. Name the six things looked for on a graph.

   Answer: Title, scales, axes, transfer of points, best line or curve, and indication of the slope. If you could not list all six from memory, learn them now. They are the cheapest marks in the paper.

4. Two candidates obtain the same slope and the same final value. One writes "g = 9.78". The other writes "g = 9.78ms^{-2}, which is 0.3% below the accepted value, because the swing was a few degrees wide rather than vanishingly small." Which earns more, and why?

   Answer: The second, and by more than most candidates expect. The first is a number with no unit, which is not a value at all, and it stops before the question is finished. The second gives a value, its unit, a comparison and a physical reason, for perhaps forty seconds of extra writing.

5. Your school is too big to sit the practical in one session, so you take one alternative and the group after you takes another. What does that mean for the way you revise?

   Answer: That revising a single experiment is useless. The alternatives use different apparatus to test the same skill, so what transfers is the family: what is plotted against what, and what the slope and intercept mean. Learn one variety and the rest become recognisable, not new.

6. You are told to write in blue or black ink. What must not be in ink, and why does the instruction exist?

   Answer: Drawings, which must be in pencil, and that includes your graph. A line drawn in ink cannot be adjusted once you see where the points really lie, and a best line almost always needs adjusting. Plot and draw in pencil, then letter the title and axes in ink if you wish.

Chapter 2

INSTRUMENTS AND READINGS

Mr. Akilikubwa opened the lesson by cutting a length of constantan wire into exactly one piece and holding it above his head.

"One wire," he said. *"One diameter. Thirty-one of you. Measure it."*

The micrometers went round the class, and the answers came back to the board in a column. Eleven students had 0.42mm. Nine had 0.45mm. Six had 4.2mm. Three had 21mm, which would have made the wire thicker than a pencil. Two, from the back row, had 0.042mm, which would have made it thinner than the smoke coming off the Bunsen burner.

"Interesting," he said. *"The wire has been in this room the whole time. It has not grown, it has not shrunk, and it has not divided itself into five wires of different sizes. So either twenty of you are wrong, or I am holding a miracle."*

Kipute, who had written 0.45mm, said that her micrometer was faulty.

"Show me."

She closed it on nothing at all, and it read three hundredths of a millimetre. It had been reading three hundredths of a millimetre too much all morning, on every wire, in every hand that had held it.

"Then it is not faulty," said **Mr. Akilikubwa.** *"It is honest. It told you exactly what it believes zero to be, and not one of you asked it. The rest of you did not even get that far. You read the wrong scale, or you read the right scale in the wrong units, and you wrote the answer down without once stopping to ask whether a wire that size could exist."*

He gave the wire back to the front bench. *"From today you will ask. Every instrument on that bench will mislead you, politely and predictably, in a way you can correct. Marks in the practical paper go to the candidates who know how each one does it."*

What the paper is really testing

When the paper hands you an instrument it is testing three separate things, and you can lose marks on any of them without touching the physics.

- **Reading**: putting the right number to what the instrument shows, including the part you have to estimate.
- **Correcting**: knowing what the instrument does when it is measuring nothing, and taking that off.
- **Recording**: writing the number down with the right number of figures, in a column whose heading carries the unit.

Everything in this chapter is one of those three. Nothing in it is difficult. All of it is examined.

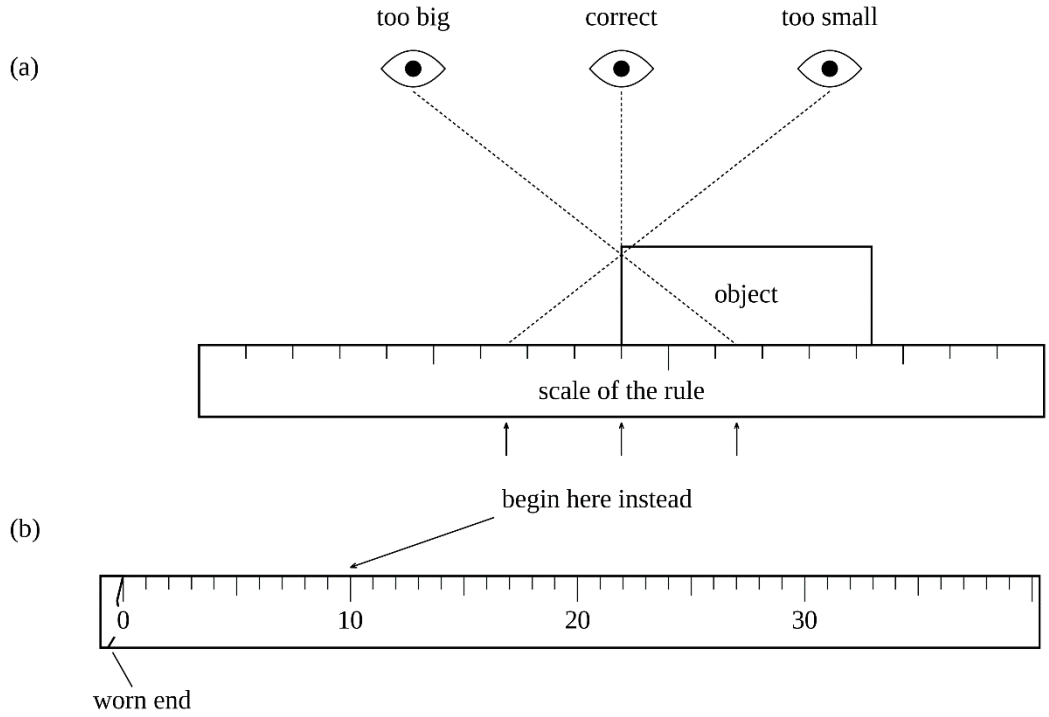
One idea runs through the whole chapter, so meet it now. Every instrument has a **least count:** *the smallest change it can show you.* For a metre rule that is 1mm, for a vernier calliper 0.01cm, for a micrometer 0.01mm, for a stopwatch 0.01s. The least count decides how many figures you are entitled to write, and writing more of them does not make you look careful. It makes you look as though you do not know what your instrument can do.

The metre rule

The humblest instrument on the bench is also the one that carries the most marks, because almost every experiment in this book measures a length with it. It is graduated in millimetres, so you record a length to the nearest millimetre: 24.7cm, not 24.5cm and not 24.70cm.

Two faults cost marks, and both are avoidable.

Parallax: The scale of the rule lies on the wood, but the thing you are measuring almost never does. There is a gap of a millimetre or two between the mark and the scale, and if your eye is off to one side your line of sight crosses that gap at an angle and lands on the wrong graduation. Look straight down on the mark, so that your eye, the mark and the graduation are in one vertical line. Where you can, lay the rule on its graduated edge so that the scale touches the object and the gap disappears altogether.

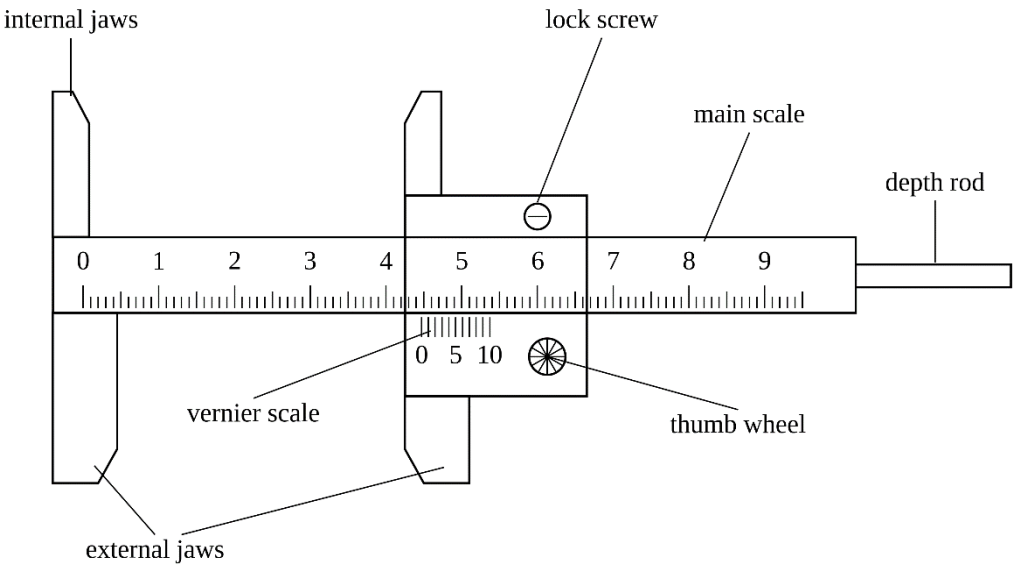


End wear: The zero of a metre rule is the end of the wood, and the end of the wood is the part that has been dropped, dragged along benches and used to point at blackboards for twenty years. Never measure from it. Lay the object against a round number well inside the scale. The 10.0cm mark is the usual choice. Read the far end and subtract. You have then used two engraved marks and no worn edge, and the two readings are of equal quality.

When a length has to be read from a rule held vertically, as when you measure the length of a pendulum, clamp the rule rather than hold it, keep it truly vertical, and read it with your eye level with the mark. A rule leaning by 5° reads a vertical length 0.4% too long, which is 0.4cm in a metre.

The vernier calliper

A vernier calliper measures external diameters between its lower jaws, internal diameters between its upper jaws, and depths with the rod that slides out of the far end. Its least count is 0.01cm, ten times finer than a metre rule.



The trick that buys the extra figure is worth understanding rather than memorising. Ten divisions of the vernier scale are made exactly as long as nine divisions of the main scale, so one vernier division is 0.9mm and falls short of a main-scale division by 0.1mm. Slide the vernier along by a tenth of a millimetre and a different pair

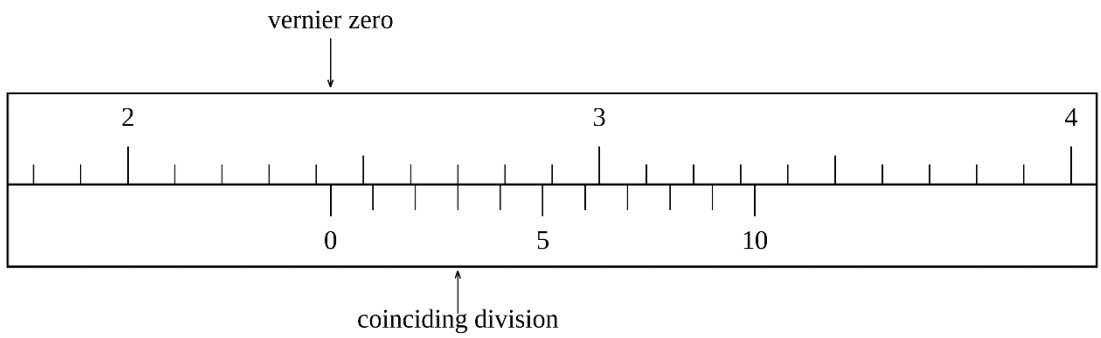
of marks comes into line. Whichever vernier division lines up with a main-scale division tells you how many tenths of a millimetre the vernier zero has moved past the last whole millimetre.

$$\text{least count}=\frac{\text{smallest main-scale division}}{\text{number of vernier divisions}}=\frac{1\text{mm}}{10}=0.1\text{mm}=0.01\text{cm}$$

To take a reading, do these two things in this order.

(i) Read the main scale at the vernier zero, taking the last mark the vernier zero has passed. This gives you whole millimetres.

(ii) Run your eye along the vernier until you find the division that lines up exactly with a main-scale division. Its number is the number of hundredths of a centimetre.

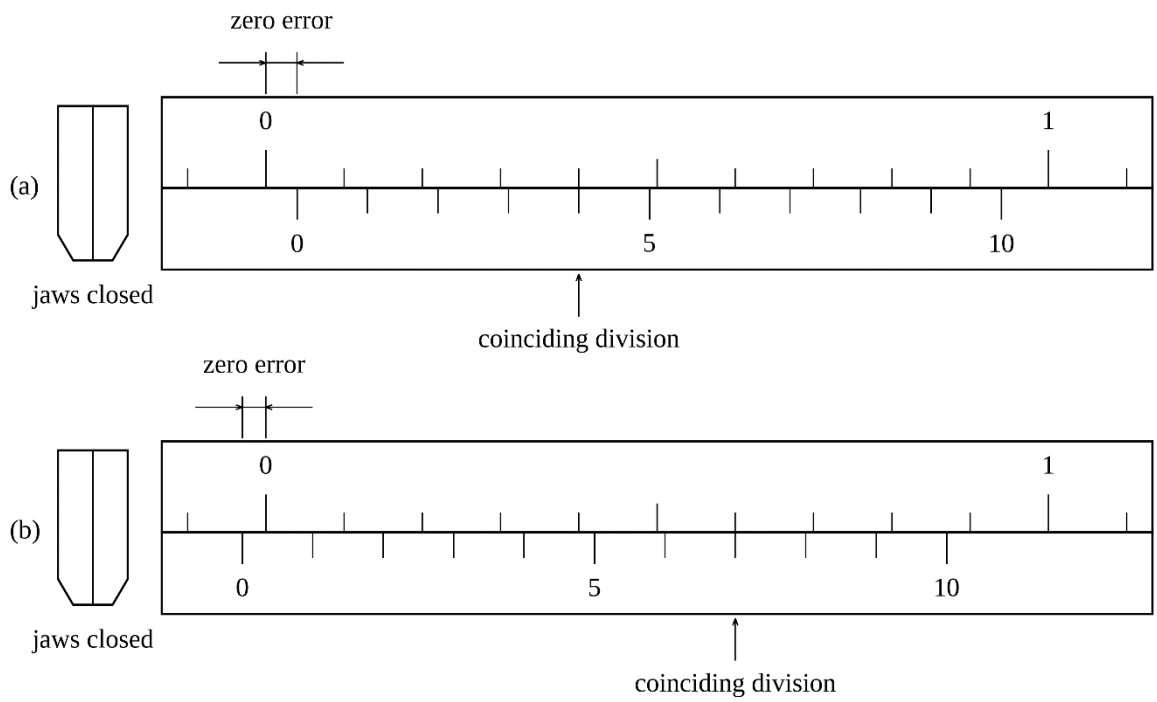


In the drawing the vernier zero has passed the 2.4cm mark and has not reached 2.5cm, so the main-scale reading is 2.4cm. Division 3 of the vernier is the one in line with a main-scale mark. The reading is therefore

$$2.4\text{cm}+(3\times0.01\text{cm})=2.43\text{cm}$$

Write it as 2.43cm even when the last figure is zero. A reading of 2.40cm and a reading of 2.4cm are different claims about how carefully you worked, and only one of them is worth the mark.

Zero error. Before you measure anything, close the jaws and look. If the two zeros are exactly in line the instrument has no zero error and you may forget about it. If they are not, the offset is there in every reading you will take that day.



In (a) the vernier zero has moved to the right of the main-scale zero, and division 4 is the one in line. The instrument reads 0.04cm when it is holding nothing, so the zero error is $+0.04$cm and every reading is 0.04cm too big.

In (b) the vernier zero lies to the left of the main-scale zero. Count the coinciding division, here 7, and take it from ten:

$$\text{zero error}=-(10-7)\times0.01\text{cm}=-0.03\text{cm}$$

so the instrument reads 0.03cm too little. In both cases the same sentence puts it right, and it is worth learning as a sentence:

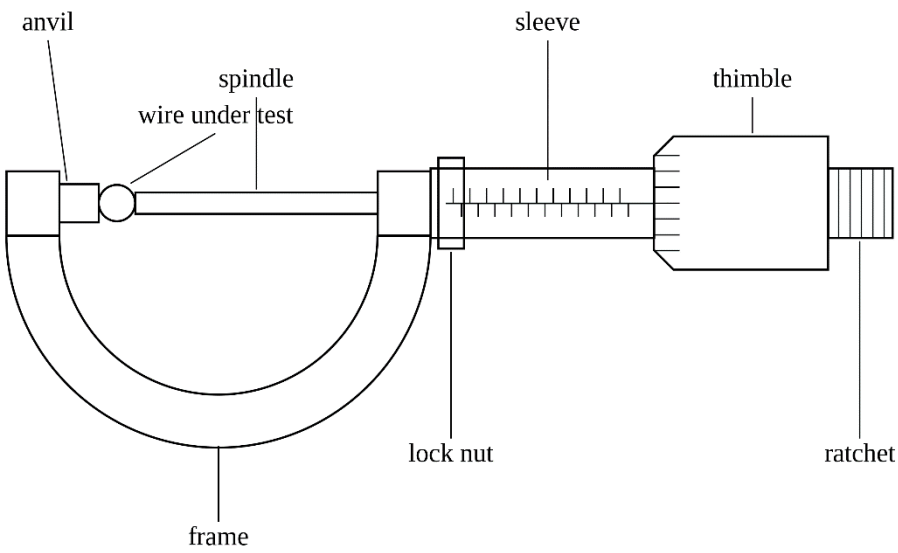
$$\text{corrected reading} = \text{observed reading} - \text{zero error}$$

Subtract, with the sign. A calliper with a zero error of $+0.04$cm that reads 1.68cm on a rod gives a corrected 1.64cm, and one with a zero error of -0.03cm that reads 1.68cm gives 1.71cm. Add when you should subtract and you lose the mark twice, once in the table and once in the final answer that comes out of it.

Two smaller habits. Tighten the jaws with the thumb wheel, not by pulling on the slider, and stop as soon as the jaws touch: a calliper squeezed hard on a soft object measures the squeeze. And measure with the flat of the jaws, never with their tips, because the tips are on the end of a long lever and bend.

The micrometer screw gauge

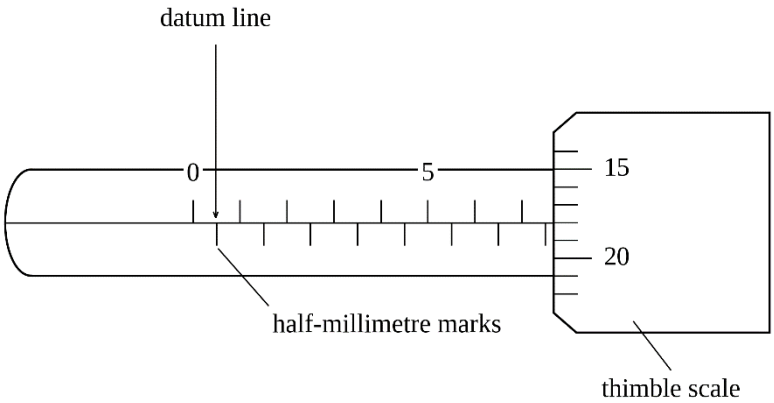
The micrometer measures the small dimensions the calliper cannot: the diameter of a wire, the thickness of a metre rule, the diameter of a ball bearing. Its least count is 0.01mm, another ten times finer, and it is the instrument on which most marks are thrown away.



The object goes between the fixed **anvil** and the moving **spindle**. The spindle is driven by a screw of pitch 0.50mm, so one full turn of the **thimble** advances it by half a millimetre. The rim of the thimble carries fifty divisions, so

$$\text{least count} = \frac{\text{pitch}}{\text{divisions on the thimble}} = \frac{0.50\text{mm}}{50} = 0.01\text{mm}$$

The **sleeve** carries a long horizontal datum line with whole millimetres marked above it and half-millimetres marked below it. That second row is where the marks are lost. Read the sleeve first, counting the half-millimetre mark if the edge of the thimble has passed it, then read the thimble division that lies on the datum line.



Here the thimble edge has passed the 7mm mark and also the half-millimetre mark beyond it, so the sleeve reads 7.5mm. The thimble division on the datum line is 18. The reading is

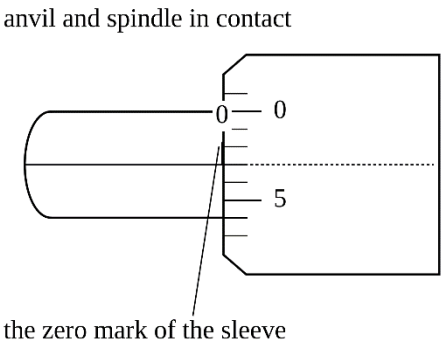
$$7.5\text{mm}+(18\times0.01\text{mm})=7.68\text{mm}$$

Miss the half-millimetre mark and you get 7.18mm: an answer that is wrong by exactly 0.50mm. If you ever compare two results and find they differ by half a millimetre, you have found this mistake and not a faulty instrument.

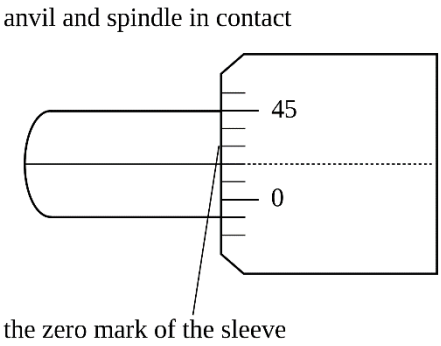
Always close on the ratchet. The small knob at the end is a clutch. Turn the thimble until the anvil and spindle are nearly touching the object, then finish with three clicks of the ratchet and stop. The ratchet slips at a fixed force, so every reading is taken with the same grip, and yours will agree with the next candidate's. Closing the jaws with the thimble instead is how a micrometer acquires a zero error in the first place, and how a soft wire acquires a flat side.

Zero error. Wipe the faces, close them gently on nothing with the ratchet, and read.

(a)



(b)



In (a) the thimble has gone three divisions past zero, so the instrument reads 0.03mm when the faces are touching and the zero error is $+0.03\text{mm}$. In (b) the zero of the thimble has not yet reached the datum line; the division on the line is 48, which is two short of fifty, so the zero error is -0.02mm. The correction is the same sentence as before, so a wire that reads 0.42mm on the instrument in (a) is really

$$0.42\text{mm}-(+0.03\text{mm})=0.39\text{mm}$$

and the same wire on the instrument in (b) would read 0.37mm and be corrected to 0.39mm as well. That is the point of the correction: two different instruments, one wire, one answer.

Take readings at three places along a wire, and at each place take a second reading with the wire turned through a right angle. Average the six. Wire is not perfectly round and it is not the same thickness along its length, and the examiner expects to see that you knew.

Examiner's eye

- Write every reading to the full resolution of the instrument, trailing zeros included. On a vernier that means two decimal places in centimetres, on a micrometer two decimal places in millimetres, on a metre rule one decimal place in centimetres.
- The number of decimal places must not wander down a column. A column reading 2.43, 2.4, 2.40 is marked down even though every entry is a fair reading of the instrument, and an entry of 2.435 is worse still, because a vernier cannot give that third decimal at all.
- Units go in the column heading, never beside every entry.

- Subtract a zero error, with its sign. Half of those who notice a zero error at all still add it.
- State that you checked for a zero error even when there was none. It is a precaution, and precautions carry marks.
- An answer that is wrong by exactly 0.50mm on a micrometer, or by exactly 0.10cm on a vernier, is a misread scale and not a mistake in the physics. Look there first.

The stopwatch

A digital stopwatch reads to 0.01s, and that figure is a fiction. Your reaction time is about 0.2s, twenty times larger, so the instrument is nowhere near the weakest part of the measurement. You are.

This is why you are told to time twenty or thirty oscillations rather than one. The periodic time comes from

$$T=\frac{t}{N}$$

and the same division that turns t into T turns your timing error into a twentieth or a thirtieth of itself:

$$\Delta T=\frac{\Delta t}{N}=\frac{0.20s}{20}=0.010s$$

On a period of about 2s that is half a percent. Since g is found from T^2, the error it leaves in g is twice as large:

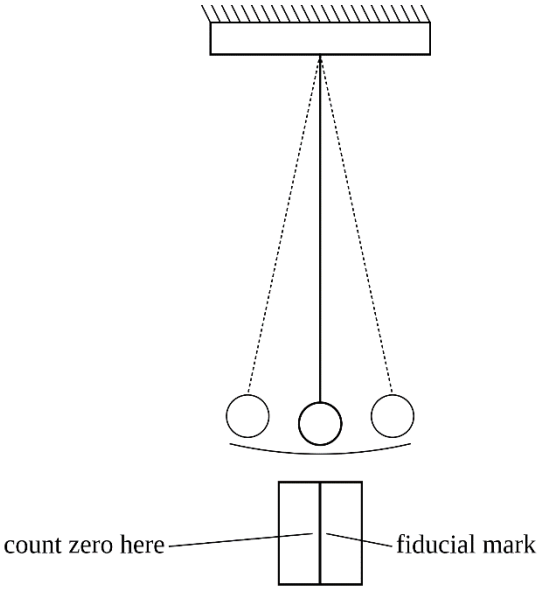
$$\frac{\Delta g}{g}=2\times\frac{\Delta T}{T}=2\times\frac{0.010s}{2.00s}=1.0\%$$

Time a single swing instead and the same 0.20s becomes a 10% error in the period and a 20% error in the value of g. That is the difference between a good result and a meaningless one. Timing thirty oscillations rather than twenty improves it again, by a third.

There is better news hidden in that arithmetic. Most of your reaction time is not random at all: you are late starting by about as much as you are late stopping, and the two delays cancel in the subtraction the watch performs. What survives is only the *variation* in your reaction, perhaps 0.05s, provided you start and stop in the same way. The 1.0% worked out above is therefore the worst case, for a candidate whose starting and stopping are unrelated; work consistently and you will do several times better than it. So the rule is not simply to be quick. It is to be consistent.

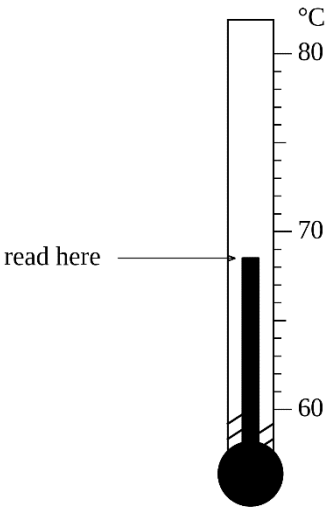
Being consistent means all of this:

- Count at the lowest point of the swing, never at the extremes. The bob is moving fastest at the bottom, so it spends the least time near the mark and your eye can place it most sharply. At the extremes it is almost stationary and you cannot tell the instant it turns.
- Use a fiducial mark: a vertical line on a card, or the edge of the clamp stand, placed behind the bob at its rest position. Judge every passage against that line and not against your memory of where the bob was.
- Count the passages in one direction only. The bob crosses the mark twice per oscillation, and counting both is how a period comes out exactly half of what it should be.
- Say “zero” as you start the watch, not “one”. Counting the release as one means you stop after nineteen oscillations while writing twenty, so the period comes out five percent too small and the value of g about eleven percent too large.
- Let the pendulum settle into a steady swing before you start counting, and take each set of readings twice.



The thermometer

The thermometer supplied for the practical paper reads from 0°C to 100°C and is graduated in degrees. You can read it to the nearest half degree by judging where the top of the thread sits between two marks, so temperatures are recorded as 68.0°C or 68.5°C and never as 68.3°C.

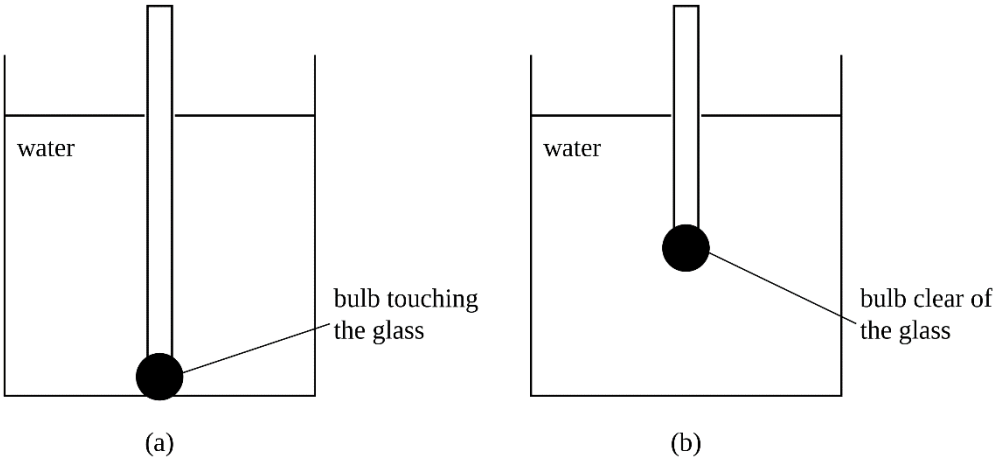


In the thermometer drawn above the top of the thread stands halfway between the 68°C and 69°C marks, so the temperature is 68.5°C. Enter it in the table with that decimal place, and give every other entry in the column one decimal place too.

The thermometer measures its own bulb and nothing else, which is the source of every mistake made with it.

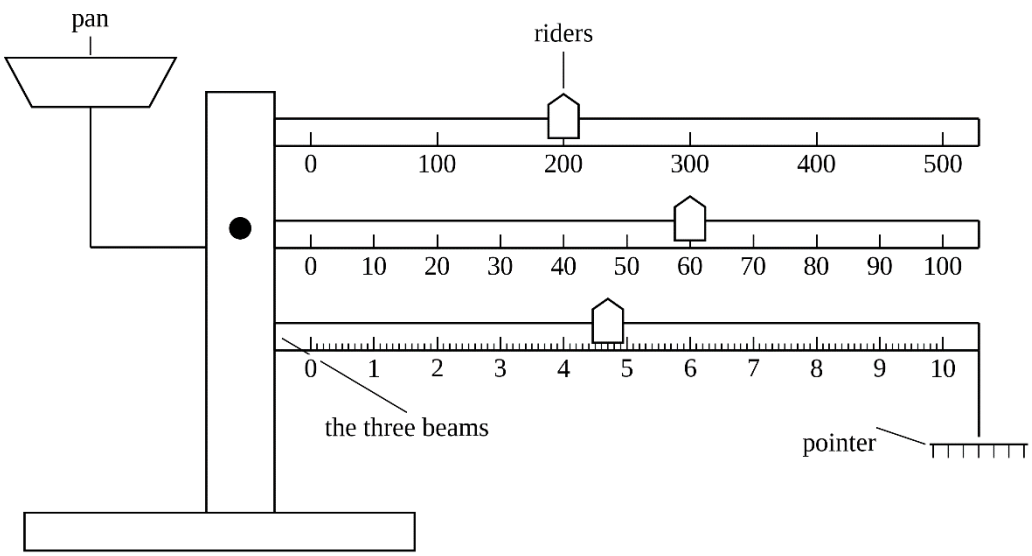
- The bulb must be well inside the liquid, not touching the bottom or the wall of the vessel. Glass at the bottom of a beaker over a flame is hotter than the water in it, and a bulb resting there reads the glass.
- Stir before you read, and stir with the stirrer rather than with the thermometer. A beaker being heated is several degrees hotter at the bottom than at the top, and until you mix it there is no single temperature to record.
- Wait for the thread to stop moving. The bulb takes a few seconds to reach the temperature of what surrounds it, and in a cooling experiment, where you read at fixed times, you must read on the move and accept the small lag rather than pause the experiment.

- Never lift the thermometer out to read it. The moment it leaves the liquid the thread begins to fall, and the number you write down belongs to the air.
- Read with your eye level with the top of the thread. The stem is a thick glass cylinder and reading it from above or below is parallax again, magnified by the lens the glass makes.



The balance

Masses are found on a beam balance, and the one you are most likely to meet has three beams: a rear beam in steps of 100g, a middle beam in steps of 10g, and a front beam graduated to 0.1g. The total mass is the sum of the three rider positions.



The riders read 200g, 60g and 4.7g, so the mass on the pan is

$$200g+60g+4.7g=264.7g$$

Work in this order and it will come out right every time. Level the balance and check that the pointer swings evenly about its centre mark with the pan empty and all three riders at zero; if it does not, adjust the screw under the pan until it does. Put the load on. Move the *rear* rider notch by notch until the pointer just drops, then come back one notch. Do the same with the middle rider. Then slide the front rider, which is not notched, until the pointer swings evenly again.

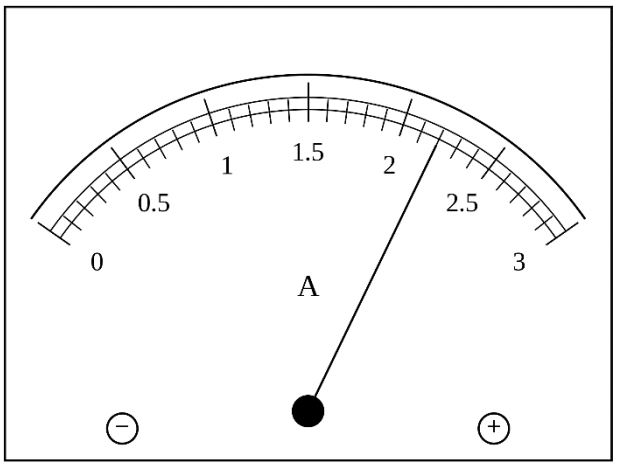
Then two habits. Read the balance while the pointer is still swinging gently and symmetrically about the centre, rather than waiting for it to come to rest; a balance that has stopped has stopped on friction. And when you want

the mass of a liquid, weigh the empty vessel, then the vessel with the liquid in it, and subtract. Never pour a liquid into the pan.

A digital balance asks less of you but still asks something. Let it settle and read zero before you load it, use the tare so that you weigh contents rather than container and contents, keep it out of a draught, and record every mass to the number of decimal places the display gives, including the trailing zeros.

The ammeter and the voltmeter

An ammeter measures the current through a component, so it goes in series with it. A voltmeter measures the potential difference across a component, so it goes in parallel with it. Both are moving-coil instruments with a definite polarity: current must enter at the terminal marked + and leave at the one marked -, and a meter connected the other way round drives its pointer against the stop on the wrong side of zero. Connect the whole circuit, check it, and only then close the key.



Work out the least count before you take a reading, not after. The meter above runs from 0A to 3A. Between 0 and 0.5 there are five spaces, so one small division is

$$\frac{0.5\text{A}}{5}=0.1\text{A}$$

and the pointer, which stands exactly on the second small division past 2, reads 2.20A. Where the pointer falls between two divisions, estimate the half and record 2.25A, keeping the same two decimal places all the way down the column. Do the same arithmetic on the voltmeter you are given before you record anything, and do it again if you change the range.

Choose the range so that the pointer sits in the upper part of the scale. A moving-coil meter is guaranteed to a percentage of its *full-scale* reading, not of the reading you happen to take. That guarantee is printed on the dial as a class number: a meter marked class 2.0 is accurate to two percent of its full-scale value. On the 0A to 3A range that is

$$2.0\%\times 3\text{A}=0.06\text{A}$$

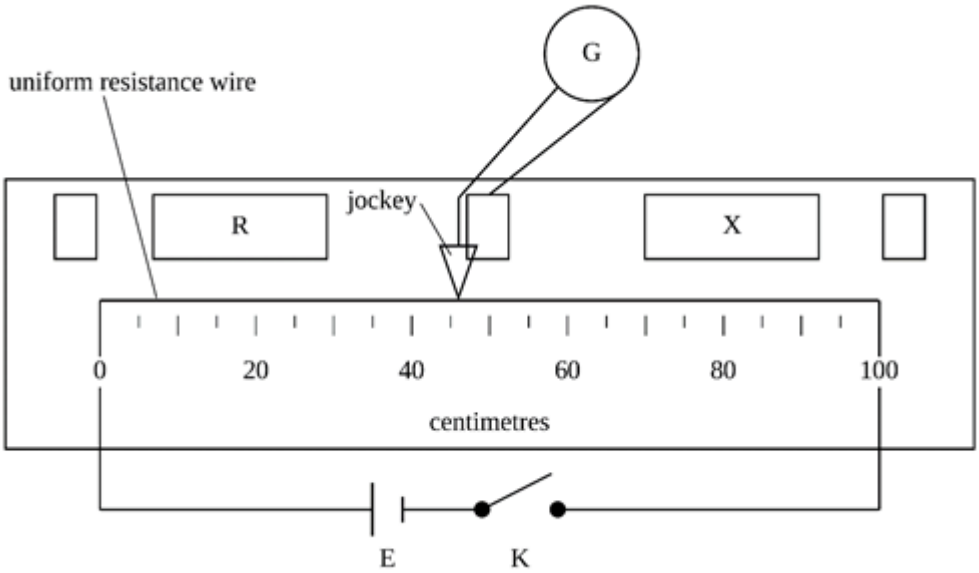
anywhere on the dial. Used at 2.70A that is an uncertainty of 2.2%. Used at 0.30A the same 0.06A is 20%, and no amount of care in the rest of the experiment will recover what that costs. If your readings all huddle near the bottom of the scale, change the range or change the circuit.

Kill parallax. A good meter has a strip of mirror set into the scale. Move your head until the pointer hides its own reflection, and read at that position. Where there is no mirror, put your eye directly above the pointer.

Two last habits. Tap the case gently with a fingernail before reading, so that a pointer held by friction is freed. And open the key between readings: current left flowing warms the coil under test, its resistance rises, and a set of readings taken over ten minutes will drift in one direction and bend a graph that ought to be straight.

The galvanometer and the jockey

A galvanometer is not there to be read. In the bridge and potentiometer experiments it is there to tell you when a current is zero, and a null reading is the most accurate reading in physics because it does not depend on the instrument being calibrated at all. It only has to be honest about nothing.



The jockey is the sliding contact you press on the bridge wire. There is one rule about it and it matters more than anything else in the chapter.

Tap, do not drag. Press the jockey down, look at the galvanometer, lift it, move it, press again. Dragging it along wears a groove in the wire. A worn wire is thinner where it has been dragged, so its resistance per centimetre is no longer the same all the way along, and the uniformity of that wire is the one assumption the whole method rests on. One careless candidate can ruin the apparatus for every candidate who uses the bench afterwards.

Find the balance by bracketing rather than by hunting. Tap near one end and note which way the needle kicks; tap near the other end and it will kick the other way; the balance point is between them, and half a dozen halvings will place it to the nearest millimetre. While you are still far from balance, keep a protective resistance in series with the galvanometer so that a large out-of-balance current cannot damage it, and short the protection out only for the final, sensitive adjustment.

Keep the balance point between 40cm and 60cm. For a metre bridge with the known resistance R in the left-hand gap and the unknown X in the right, and a balance point L centimetres from the left-hand end,

$$X=R\left(\frac{100\text{-}L}{L}\right)$$

and the fractional error that an uncertainty ΔL in the balance point leaves in the answer is

$$\frac{\Delta X}{X}=\frac{100\Delta L}{L(100\text{-}L)}$$

That denominator is largest at the middle of the wire. Balancing at 50cm with the jockey placed to the nearest millimetre gives

$$\frac{100\text{cm}\times 0.1\text{cm}}{50\text{cm}\times 50\text{cm}}=0.4\%$$

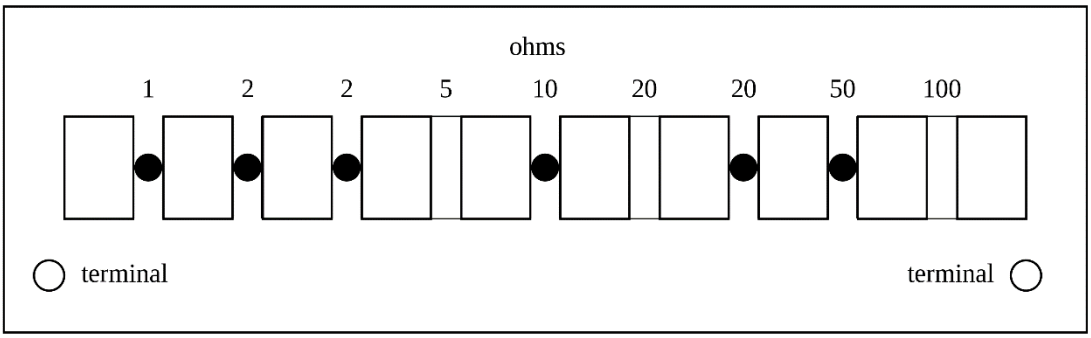
while the same millimetre at 5cm gives

$$\frac{100\text{cm}\times 0.1\text{cm}}{5\text{cm}\times 95\text{cm}}=2.1\%$$

five times worse, before you even consider that the end connections matter most where the balance point is near an end. If the balance comes out near an end, do not accept it. Change R until it moves towards the middle. That single decision is worth more than any care you can take elsewhere in the experiment.

The resistance box

A plug-type resistance box is a row of brass blocks with a coil between each neighbouring pair, and a tapered plug that short-circuits the coil when it is pushed home. So the rule is the opposite of what students expect on first meeting it.



The resistance in circuit is the sum of the values whose plugs have been taken out. In the box drawn above the plugs are out of the 5, the 20 and the 100, so

$$R=5\Omega+20\Omega+100\Omega=125\Omega$$

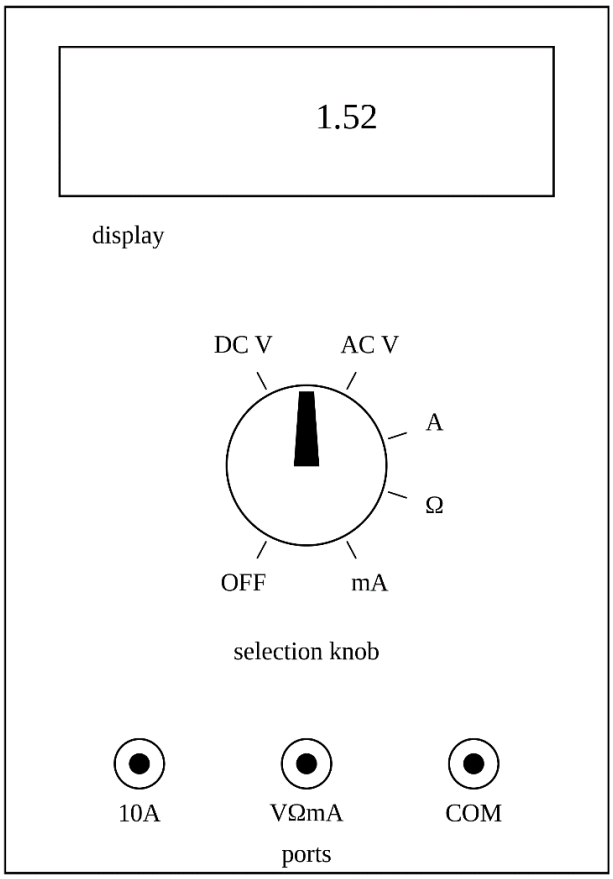
and every other coil is short-circuited by its plug and contributes nothing. With every plug in place the box reads zero, which is why a box left fully plugged in a circuit is a piece of wire.

Press each plug home and give it a small twist as you seat it. A plug that is dirty or loose adds a fraction of an ohm you have not counted, and in a bridge experiment where the box may be set to 2Ω, a tenth of an ohm is five percent of your known resistance. Keep the spare plugs in the holes provided rather than on the bench, where they collect dust and roll onto the floor.

Two things more. The values on a box are the *nominal* values of the coils, good to about half a percent, so a box is a known resistance and not a perfect one. And a box carrying current warms up: choose settings that keep the current modest, and open the key between readings, or the resistance you are treating as known will quietly climb while you work.

The multimeter

A digital multimeter measures voltage, current, resistance and continuity with one instrument. It has three parts you must know: the display, the selection knob and the ports.



The black lead always goes into the port marked COM. The red lead goes into the port marked VΩmA for voltage, resistance and small currents, and only into the port marked 10A when you expect a current larger than the milliampere port will take. Moving the red lead is part of changing the measurement, and forgetting to move it back is how a multimeter is destroyed.

On the knob, the direct-voltage setting is the one marked DC V, or on many meters a V with a straight line beside it. Direct voltage is what cells and batteries give, and what every circuit in this book uses. The alternating-voltage setting, marked AC V or a V with a wavy line, is for what comes out of the wall, and you have no reason to touch it in a school laboratory.

Then four habits, in the order you will need them.

- Start on the highest range and come down until the display gives you the most figures it can. A display showing 1 or OL on its own is telling you the quantity is bigger than the range, not that it is infinite.
- To measure a voltage, put the leads across the component with the circuit working. To measure a current, the meter has to be in the circuit, so break the circuit and put it in the gap. Putting a meter set to read current straight across a cell short-circuits the cell through the meter, and the fuse is the cheapest thing that then fails.
- To measure a resistance, take the component out of the circuit first. A multimeter measures resistance by passing its own small current, and anything else connected to the component offers that current another path and gives a low answer.
- Do not hold both probe tips between your fingers while measuring a large resistance. Your body is a resistance of a few hundred thousand ohms in parallel with the one you are trying to measure.

If the display shows a negative sign, nothing is wrong. You have the leads the other way round, and the meter is reporting that faithfully. Swap them, or simply write the size and note the direction.

Examiner's eye

- Say what you did, not what you would normally do. "The time for 20 complete oscillations was taken, and the periodic time obtained by dividing by 20" earns a mark that "I used a stopwatch" does not.
- Never offer "human error", "instrumental error" or "parallax" on its own as a source of error. Name the instrument, say which way the reading is pushed and pair it with the precaution: "variation in reaction time at starting and stopping the stopwatch, which may push a recorded time either way; reduced by timing 20 oscillations instead of one and by starting and stopping in the same way."
- A reading taken with the pointer in the bottom fifth of a meter scale is a weak reading however carefully it was taken, and an examiner can see it in your table.
- A metre bridge balance quoted outside 40cm to 60cm invites the question of why you did not change the known resistance, and the answer costs marks in the comment.
- Record temperatures to the half degree, currents and voltages to the least count you worked out, masses to the decimal places the balance gives. A table whose figures are inconsistent with the instruments named in the apparatus list is marked down before the graph is even looked at.
- All working is done in ink; all drawings and graphs in pencil. A graph drawn in ink cannot be corrected, and a candidate who has to correct it in ink loses more than the mark for the line.

Making the measurement count

Practical tip 1: *Know the size of the answer before you read the instrument. A reading that could not possibly be right is the easiest of all mistakes to catch, and the only one you can catch without repeating the experiment.*

Every quantity you will be asked to measure lives inside a narrow range, fixed by the apparatus the laboratory is asked to provide. Learn the ranges. Then, when the micrometer says 4.2mm for a piece of wire, you will know at once that the decimal point is in the wrong place, because no laboratory wire is four millimetres thick.

Quantity	**What the apparatus makes possible**
Diameter of a resistance wire	0.27mm to 0.46mm
Diameter of a pendulum bob	20mm to 30mm
Thickness of a metre rule	5mm to 8mm
Room temperature	20°C to 30°C
Boiling water	close to 100°C
Electromotive force of a fresh dry cell	1.5V
Two such cells in series	3.0V
Current in the circuits of this book	0.1A to 1.5A
Balance point on a metre bridge, box well chosen	40cm to 60cm
Periodic time of a pendulum of about 1m	close to 2s

Practical tip 2: *The least count of the instrument fixes how many figures you may write, in the table and in everything computed from it.*

A micrometer whose least count is 0.01mm gives 4.35mm: three significant figures, and the last of them already uncertain. Average three such readings and a calculator will offer you 4.353333mm. You may not write that. The mean of three readings from an instrument that cannot resolve better than 0.01mm is 4.35mm, and quoting more figures claims a precision no instrument in the room possesses.

The same discipline runs into the derived columns. A time of 36.42s for 20 oscillations gives a period of 1.821s, and squaring it gives 3.316s², which you would round to 3.32s² because the graph you are about to plot cannot be read to better than three figures anyway. Chapter 3 takes this up in detail; for now, let the instrument decide and not the calculator.

Practical tip 3: *A zero error shifts an intercept but leaves a slope untouched, provided the quantity it affects goes onto the graph just as it was measured. This is why so many questions ask you for a slope.*

Suppose the true length is d and your instrument has a zero error e, so the length you actually record, call it d_r, is $d_r=d+e$ every time. If the theory says $y=md+c$, then written in terms of the number you put in the table,

$$y=m(d_r-e)+c=md_r+(c-me)$$

which is still a straight line, with exactly the same slope m and an intercept changed from c to $c-me$. Take your answer from the slope and an unnoticed zero error costs you nothing at all. Take it from the intercept and it costs you everything.

The protection is not unconditional, and the condition is worth knowing. It holds only while the affected quantity enters the graph linearly. If the theory is $y=md^2+c$ and you plot y against d_r^2, what you have really plotted is

$$y=m(d_r-e)^2+c=md_r^2-2med_r+me^2+c$$

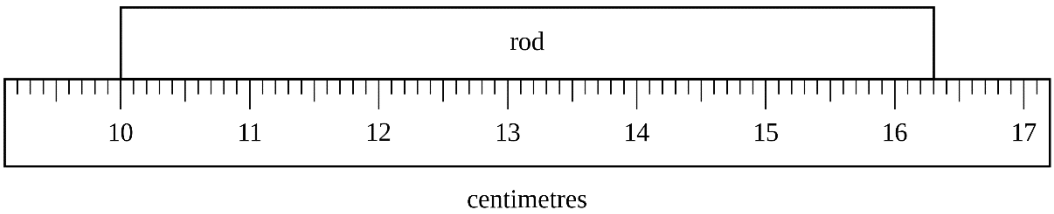
which is not a straight line in d_r^2 at all. The graph bends very slightly, and the slope you read off is wrong. So: check the zero, always; and when you are told which pair of axes to use, notice whether your measured quantity is being squared, inverted or logged before you decide how much a small constant error matters.

Drills

Work each one on paper before you read the answer printed beneath it. The scales are drawn to be read, not to be guessed at.

Drill 1

The rod below is being measured against a metre rule. Write down the length of the rod, and say why it was not laid against the end of the rule.



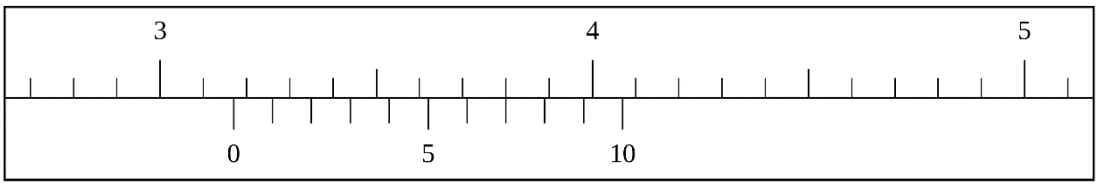
Answer

The left-hand end of the rod is at the 10.0cm mark and the right-hand end is at the 16.3cm mark, so the length is 16.3cm-10.0cm=6.3cm.

The end of a metre rule is worn, so its zero is not where the engraving says it is. Laying the rod against an engraved mark well inside the scale means both readings are taken from engraved marks of equal quality.

Drill 2

The jaws of this vernier calliper coincided exactly when they were closed. Read it.



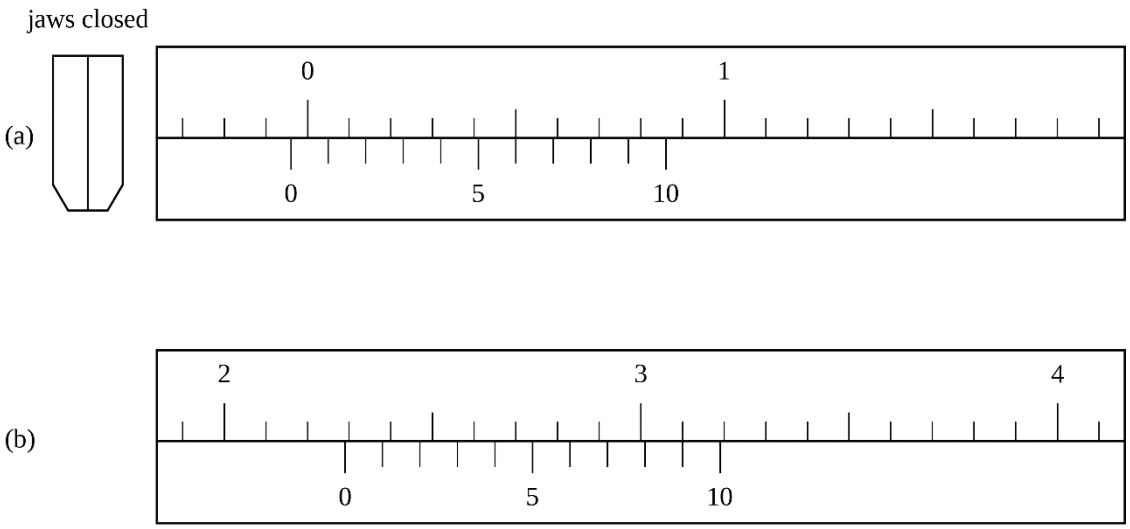
Answer

The vernier zero has passed the 3.1cm mark and has not reached 3.2cm, so the main-scale reading is 3.1cm. Division 7 of the vernier is in line with a main-scale division.

$$3.1\text{cm}+(7\times0.01\text{cm})=3.17\text{cm}$$

Drill 3

Diagram (a) shows the scales of a vernier calliper with its jaws closed, and diagram (b) shows the same scales with a metal rod between the jaws. State the zero error and give the corrected diameter of the rod.



Answer

In (a) the vernier zero lies to the left of the main-scale zero and division 6 is the one in line, so the zero error is negative:

$$\text{zero error}=-(10-6)\times0.01\text{cm}=-0.04\text{cm}$$

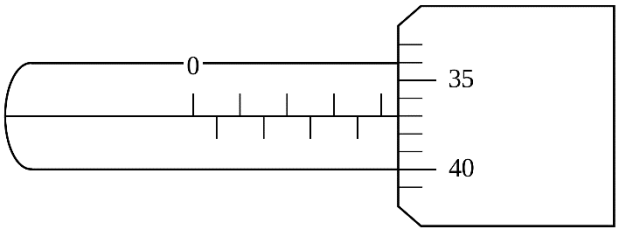
In (b) the vernier zero has passed 2.2cm and division 9 coincides, so the observed reading is 2.29cm. Correcting it,

$$2.29\text{cm}-(-0.04\text{cm})=2.33\text{cm}$$

Accept: 2.33cm only. Adding the zero error instead of subtracting it gives 2.25cm, which is the commonest wrong answer in this family.

Drill 4

Read this micrometer screw gauge, which has no zero error.



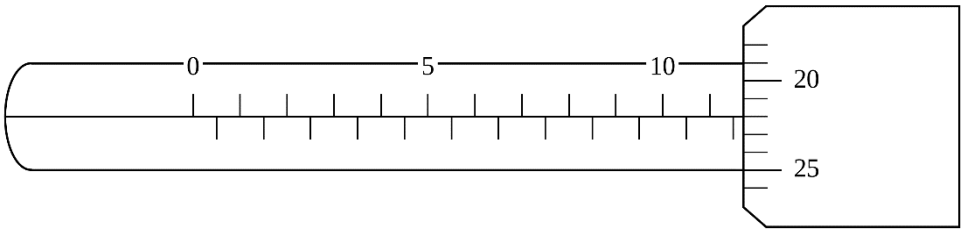
Answer

The thimble edge has passed the 4mm mark on the sleeve, and the half-millimetre mark beyond it is still covered, so the sleeve reads 4.0mm. The thimble division on the datum line is 37, two beyond the 35 that is printed.

$$4.0\text{mm}+(37\times0.01\text{mm})=4.37\text{mm}$$

Drill 5

Read this micrometer screw gauge, which has no zero error.



Answer

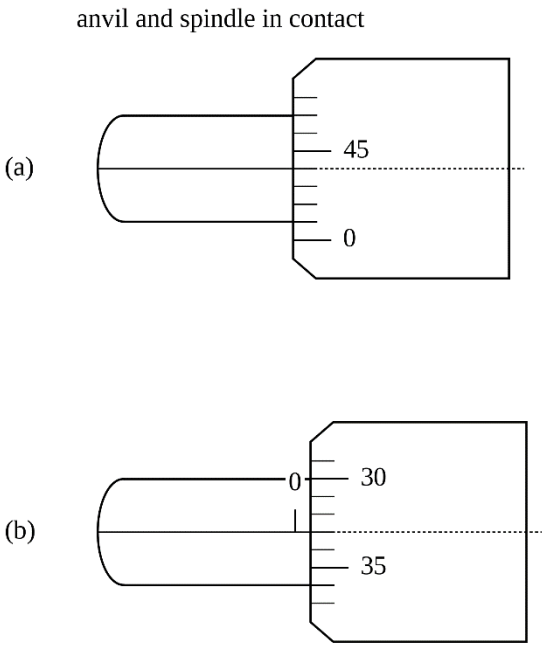
The thimble edge has passed the 11mm mark and the half-millimetre mark beyond it is showing, so the sleeve reads 11.5mm. The thimble division on the datum line is 22.

$$11.5\text{mm}+(22\times0.01\text{mm})=11.72\text{mm}$$

Accept: 11.72mm only. An answer of 11.22mm means the half-millimetre mark was missed, and the error is exactly 0.50mm.

Drill 6

Diagram (a) shows a micrometer closed on nothing, and diagram (b) shows it closed on a wire. Give the zero error and the true diameter of the wire.



Answer

In (a) the zero of the thimble has not yet reached the datum line. The division on the line is 46, four short of fifty, so the zero error is -0.04mm.

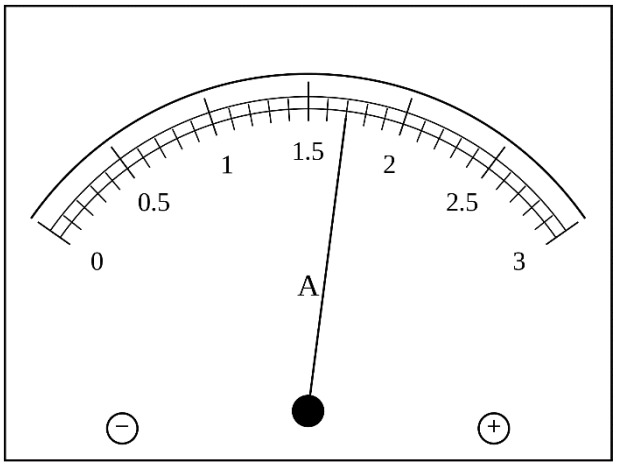
In (b) the sleeve shows only the 0mm mark and the thimble reads 33, so the observed diameter is 0.33mm. Correcting it,

$$0.33\text{mm}-(-0.04\text{mm})=0.37\text{mm}$$

Accept: 0.37mm only. Notice the size of what you have just put right: four hundredths of a millimetre is more than a tenth of the diameter being measured, so on wire this thin the zero check is not a formality.

Drill 7

Work out the least count of this ammeter, then read it.



Answer

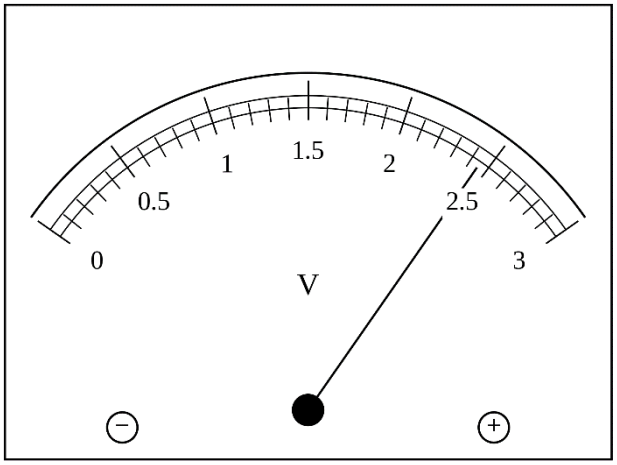
There are five spaces between 0 and 0.5, so one small division is $0.5\text{A}\div5=0.1\text{A}$.

The pointer stands on the second small division past 1.5, so the current is 1.70A.

Accept: 1.70A, recorded to the same two decimal places as every other entry in the column.

Drill 8

Read this voltmeter, and say what the last figure of your answer depends on.



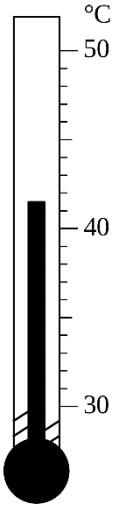
Answer

One small division is 0.5V÷5=0.1V. The pointer lies halfway between 2.4 and 2.5, so the reading is 2.45V. The last figure is estimated by eye, not read off a graduation. It is the doubtful figure, and it is still worth writing: a reading given as 2.4V throws away information the instrument gave you.

Accept: 2.45V.

Drill 9

Read this thermometer, and give the reading in the form you would enter in a table.



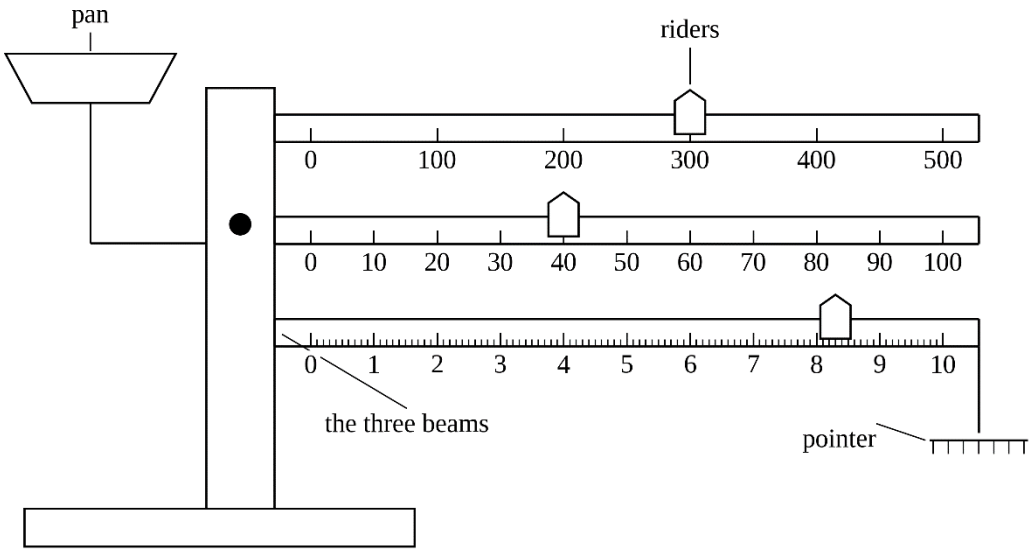
Answer

The thread stands halfway between the 41°C and 42°C marks, so the temperature is 41.5°C.

Every entry in the column must then carry one decimal place, and that decimal place can only ever be .0 or .5, because the thermometer is graduated in whole degrees.

Drill 10

Read the three beams of this balance and give the mass on the pan.



Answer

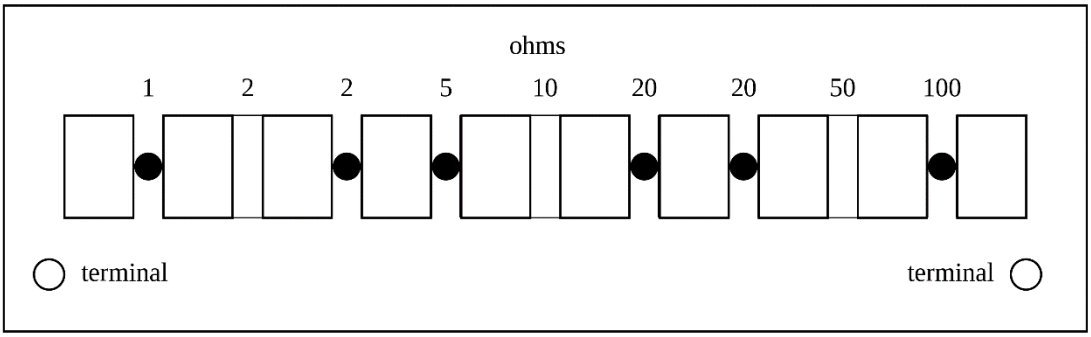
The rear rider is in the 300g notch, the middle rider in the 40g notch, and the front rider stands at 8.3g.

$$300g+40g+8.3g=348.3g$$

Accept: 348.3g. Record it to one decimal place, since that is what the front beam resolves.

Drill 11

Give the resistance this box has in circuit, and state what it would read if every plug were pushed home.



Answer

The plugs are out of the 2Ω, the 10Ω and the 50Ω coils, and those are the only coils in circuit:

$$R=2\Omega+10\Omega+50\Omega=62\Omega$$

With every plug pushed home every coil is short-circuited and the box reads zero. A resistance box left fully plugged is a piece of wire.

Drill 12

A pendulum is timed for 20 complete oscillations and the stopwatch reads 36.42s. (a) Find the periodic time. (b) A second candidate counted the release as "one". How many oscillations did that candidate actually time, and what is the true periodic time? (c) By what percentage would that candidate's value of g be wrong, and in which direction?

Answer

(a) $T=\frac{36.42s}{20}=1.821s$.

(b) Counting the release as "one" and stopping on "twenty" means only 19 complete oscillations were timed, so

$$T=\frac{36.42s}{19}=1.917s$$

(c) Since g is proportional to $1/T^2$, and the recorded period is too small by the factor 19/20,

$$\frac{g_{wrong}}{g_{true}}=\left(\frac{20}{19}\right)^2=1.108$$

so the value of g comes out about 11% too large. This is the single commonest way of turning a well-performed pendulum experiment into a wrong answer.

Drill 13

A metre bridge is set up with a resistance box in the left-hand gap and a coil of about 20Ω in the right-hand gap. (a) Which single plug should be taken out of the box, and where will the balance point then lie? (b) A candidate instead left only the 1Ω plug out and found the balance at 4.8cm. Find the resistance of the coil, and say what should be done next and why.

Answer

(a) Take out the 20Ω plug. With the known resistance equal to the unknown, the two halves of the wire must be equal, so the balance lies at the middle of the wire, close to 50cm.

(b) With R=1Ω and L=4.8cm,

$$X=R\left(\frac{100\text{-}L}{L}\right)=1\times\frac{95.2}{4.8}=19.8\Omega$$

The answer is right, but it has been obtained badly. At 4.8cm a jockey placed to the nearest millimetre carries a fractional error of $\frac{100\text{cm}\times0.1\text{cm}}{4.8\text{cm}\times95.2\text{cm}}=2.2\%$, against 0.4% at the middle of the wire. Take out the 20Ω plug instead and repeat the balance.

Accept: 19.8Ω, or any value from 19.5Ω to 20.1Ω.

Drill 14

You have to find the resistance of a coil that is joined into a circuit, and you expect a few hundred ohms. State (a) which port each lead goes into, (b) what the selection knob is set to, (c) the one thing you must do before the probes touch the coil, and (d) why pinching a probe tip in each hand is a bad habit even though it would make no measurable difference here.

Answer

(a) The black lead goes into COM and the red lead into VΩmA. The 10A port is only for large currents.

(b) The ohms setting, on the lowest range that is still above a few hundred ohms, so that the display gives the most figures it can.

(c) Free at least one end of the coil from the rest of the circuit. The meter measures resistance by driving its own small current through the coil, and anything still connected in parallel offers that current a second path, giving a reading below the true value.

(d) Your body is a resistance of a few hundred thousand ohms, and holding both tips puts it in parallel with whatever you are measuring. Against a few hundred ohms the effect is nothing; against a resistance of a megaohm it is most of the answer. The habit you form on the easy measurement is the one you will use on the hard one.

Chapter 3

TABLES AND GRAPHS

The argument had been running since the mock scripts came back, and it had reached the stage where both sides were appealing to a higher authority.

"Twice the marks you got," said **Kipanga**, waving his paper. *"Twice. And my answer was nowhere near yours."*

"Your value of g was 11.2," said **Kipute.**

"And yet."

Mr Akilikubwa took both scripts, laid them side by side on the bench, and began with neither of the two answers.

"Kipanga. A title. Both axes labelled with the quantity and the unit. Scales a person could read. Every reading transferred to the paper where it belongs. A line that balances the points. A slope triangle I can see from the back of the room. Six things, all of them settled before you arrived anywhere near the physics, and you lost none of them."

He turned the other script over. *"Kipute. Your value of g is the better one, and I am glad of it. But you gave your graph no title, left the units off both axes, chose a scale of two centimetres to three units and drew it the size of a postage stamp, so that a millimetre of paper stood for a tenth of everything you had measured. You joined your points one to the next, and you took your slope from two of them a centimetre apart. Six things. You gave away five of them, and you managed it before you had made a single mistake in physics."*

Kipute looked at the two scripts for a while. *"So the drawing is worth more than the answer."*

"On a question like this one the drawing carries more than the answer," said **Mr Akilikubwa**, *"and it is the most dependable part of the whole examination, because not one of those six things asks you to understand anything. They ask you to be careful. This week you will learn to be careful, and I will stop having this conversation."*

The six things the examiner looks for

When your graph is marked, the examiner looks for six things, and they are the same six every time:

- the **title**;
- the **scales**;
- the **axes**;
- the **best line or curve**;
- the **slope indication**;
- the **transfer of points** from your table onto the paper.

Read that list again and notice what is missing from it. Not one of the six asks you to know any physics. You can earn every one of them on a graph whose final answer turns out to be wrong, and you can lose every one of them on a graph whose answer is perfect. They are marks for care, and care can be practised.

The rest of this chapter takes them in the order you meet them at the bench, together with the two things that travel with them, the table and the intercept: the table you build the graph from, the scales, the axes and title, the points, the line, and the slope and intercept you take off it. A worked example runs through the whole of it, using a set of pendulum readings, and the six are shown together on one finished sheet.

The table

The graph is only as good as the table it comes from, and the table is marked in its own right. Four rules cover almost everything.

The unit goes in the heading, once. Write the heading as the quantity, by its symbol rather than its name, followed by its unit in brackets, and then write nothing but numbers underneath. A column headed L (m) with entries reading 1.000, 0.850, 0.700 is right. A column headed Length with entries reading 1.000m, 0.850m, 0.700m is wrong three times over: the heading gives the name of the quantity instead of its symbol, it does not say what unit you worked in, and the column is now a column of labels rather than a column of numbers.

The number of decimal places must not wander. Every entry in a column comes from the same instrument, so every entry carries the same number of decimal places, trailing zeros included. A stopwatch column reading 40.22, 36.90, 33.7, 29.68 is marked down for that third entry, which should be 33.68. The lengths in the table below carry three decimal places for the same reason: a metre rule is read to the nearest millimetre, so under a heading of L (m) a thread set at one metre is entered as 1.000 and never as 1.00.

A derived column is computed, not copied. If you square a period of 2.011s you write 4.044s², not 4.04s² and not 4.044121s². Chapter 2 gave the rule: the instrument decides how many figures you may carry, and the calculator does not get a vote. The times here were read to four significant figures, so the periods and their squares carry four as well. Work each derived column from the printed column beside it, so that a marker can check any row with a calculator, and drop to three figures only at the end, when you quote the slope you have taken off the graph.

Every derived column gets its own heading, with its own unit. A column of squared periods is headed T^2 (s^2). A column of reciprocals is headed 1/I (A^{-1}). Getting the unit of a derived column right is half of getting the unit of the slope right, and the unit of the slope is where most of the arithmetic marks in the practical paper live.

Here is the table the worked example uses. A thread was shortened in steps and the time for 20 complete oscillations recorded at each length.

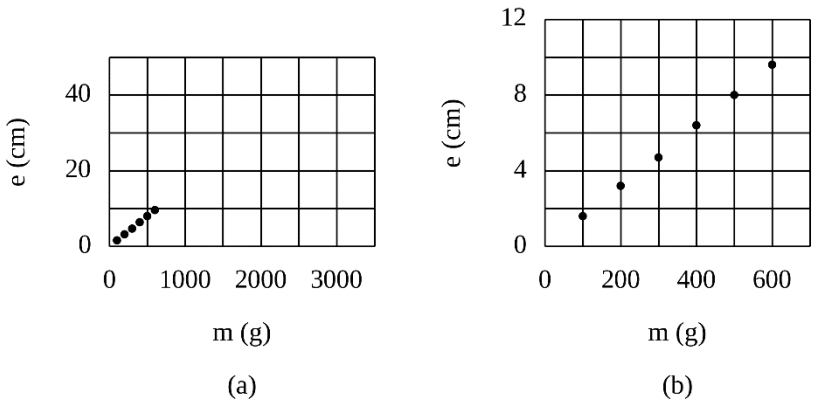
L (m)	t for 20 oscillations (s)	T (s)	T^2 (s^2)
1.000	40.22	2.011	4.044
0.850	36.90	1.845	3.404
0.700	33.68	1.684	2.836
0.550	29.68	1.484	2.202
0.400	25.48	1.274	1.623
0.250	20.02	1.001	1.002

Read down it and check the four rules for yourself. Each heading carries a unit; no entry carries one. The lengths are given to three decimal places, the times to two, the periods to three and the squares to three. Every period is its time divided by 20, and every square is that period multiplied by itself, so a marker can check any row in a few seconds. That is the point of a table: it should be checkable without being explained.

Choosing the scales

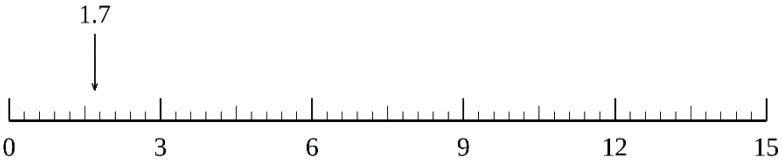
More of the six are thrown away here than anywhere else, and the scales are settled before you plot anything. First, the sheet itself. The graph paper you are given is ruled in large squares of two centimetres, each divided into ten small squares of two millimetres, and every scale in this book is quoted as two centimetres, one large square, to so many units. Then two rules, in this order.

Your points must fill more than half of each axis, and your axes must use the sheet. More than half of the width and more than half of the height. Filling one axis well and leaving the points bunched at one end of the other is not enough, and neither is a pair of axes drawn small in a corner of a large sheet. The one excuse for leaving part of the sheet empty is an axis that has to reach zero.

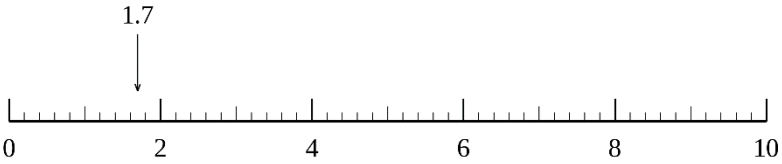


Both sheets carry the same six readings, of the extension e of a spring against the mass m hung on it. In (a) the scales were chosen without looking at the readings, so the whole experiment sits in the bottom corner and every point is within a few millimetres of its neighbours. Nothing can be read off that graph, and no care taken later can rescue it. In (b) the scales were chosen after looking at the largest reading in each column, and the same six points now use the sheet they were given.

The scale must be one you can read. A scale is a number of units to a large square, and it must be 1, 2 or 5 units, or those multiplied or divided by any power of ten: 0.1, 0.2, 0.5, 1, 2, 5, 10, 20, 50, 100 and so on. Never anything else: not 3, not 4, not 6, not 7, not 8, not 9.



2cm to 3 units: one small square is 0.3



2cm to 2 units: one small square is 0.2

The reason is arithmetic, not taste. A large square is ten small squares across. On the upper scale above, at two centimetres to three units, each small square is 0.3 of a unit, so a reading of 1.7 sits five and two thirds of a small square along: you will not place it correctly, you will not read it back correctly, and you will do that six times over. On the lower scale, at two centimetres to two units, each small square is 0.2 of a unit, the same reading sits exactly eight and a half small squares along, and your eye finds it at once.

So work in this order. Look at the largest and the smallest value in each column. Decide whether an axis has to start at zero, and which one, which the question settles whenever it turns on the line passing through the origin or on an intercept. Then choose the 1, 2 or 5 scale that spreads the range over as much of the paper as it will reach, and check that your points will use more than half of the axis you have drawn.

For the pendulum table above, the lengths run from 0.250m to 1.000m and the squares of the periods from $1.002s^2$ to $4.044s^2$. The graph asks whether T^2 is proportional to L, so both axes must start at zero. On a sheet 18cm wide and 24cm high that gives nine large squares across and twelve up. Two centimetres to 0.20m across gives an axis from 0 to 1.20m, six large squares of the nine, and two centimetres to $0.50s^2$ up gives an axis from 0 to $4.50s^2$, nine large squares of the twelve. The points then occupy five eighths of the horizontal axis and two thirds of the vertical one. Both axes reach zero, so the question about proportionality can be answered, and both stop short of the edge of the sheet only because the zero has to be on them.

Labelling the axes, and the title

Label each axis with the same heading you used in the table: the quantity, then its unit in brackets. L (m) along the bottom, T^2 (s^2) up the side. An axis labelled only "length", or only "m", or not at all, loses the mark for axes even when the numbers beside it are perfect.

The title has a fixed form, and you should use it every time without thinking about it:

A graph of (the vertical quantity and its unit) against (the horizontal quantity and its unit).

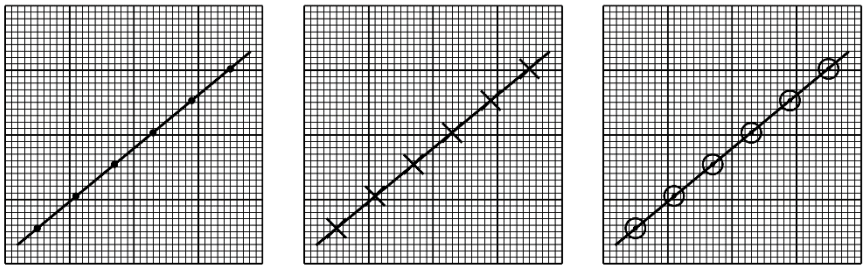
So: *A graph of T^2 (s^2) against L (m)*. The word **against** is what the marker is looking for, and the order matters: what you plotted up the page comes first. A title of "Graph of pendulum experiment" earns nothing. Neither does a title written on the answer booklet instead of on the graph paper, so write it at the top of the sheet itself.

One more thing belongs on the sheet and is forgotten every year: your examination number. Graph paper is loose. A sheet that comes adrift from the booklet with nothing on it to say whose it is cannot be marked at all.

Plotting the points

Use a sharp pencil, and put every reading in your table onto the paper. Six readings means six points; a candidate who plots five has lost the transfer mark whatever the sixth would have shown.

Place each point to the nearest half of a small square. That is what the scale was chosen for, and it is what the marker checks: a point more than half a small square from where your own table says it belongs is a wrong transfer.



(a) (b) (c)

Mark each point with a small, sharp dot, as in (a). If you find your dots disappearing under the line, use a fine cross as in (b), or ring each one as in (c). Examiners accept all three. What they will not accept is a blob: a dot the size of a small square cannot be located to half a small square, and it hides the very thing the line is being judged against.

Do not join the points as you go, and do not write the coordinates beside them; the graph is a drawing, not a list. The only coordinates that belong on the sheet are those of the two points you take your slope from, and they are dealt with under The slope, below.

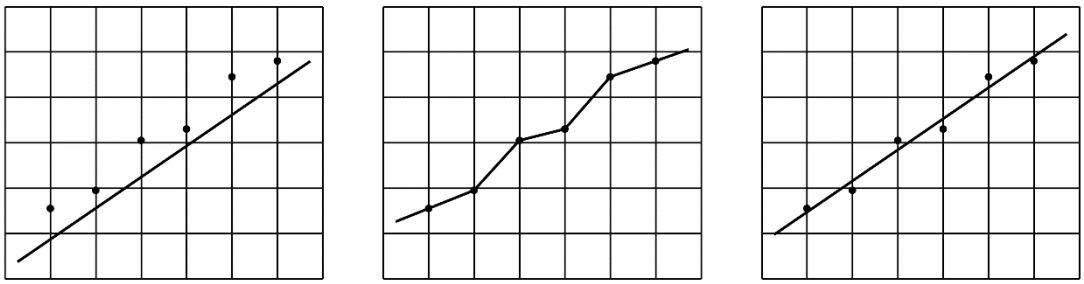
Examiner's eye

- The title, the axis labels and the scales can all be written down before you plot a single point. Do them first, while you are calm, and they cannot be forgotten in the rush at the end.
- Check the scale by plotting your largest reading first. If that reading falls off the paper, or only reaches halfway up it, you have found out before you have plotted the other five.
- A scale that is not 1, 2 or 5 units to a large square is marked wrong even if every point is correctly placed on it.
- Everything drawn on the graph sheet, the points and the line and the triangle, goes in pencil, because a line drawn in ink cannot be rubbed out and redrawn when you see it is wrong. Your written working in the booklet is in ink as usual.
- Write your examination number on the graph sheet.

The best straight line

Your six points will not lie on a line. They lie near one, and your job is to find the line they are near.

Use a transparent ruler, so that you can see the points on both sides of the edge at once. Lay it across them and move it until the points are as evenly divided as you can make them, some above the edge and some below, with the ones nearest the ends given the same weight as the ones in the middle. Then draw the line right across your scales, not merely from the first point to the last, because a long line gives a more accurate slope.



(a) (b) (c)

Three attempts at the same six points. In (a) the line is straight and neat, and every one of the six points lies above it; whoever drew it never looked at the points at all. In (b) the points have been joined one to the next, which produces a shape that is not a line at all and that has a different slope on every segment, so no single slope can be read from it. In (c) the line balances: three points above it, three below, and none of them far away. Only (c) earns the mark.

Three further habits are worth having. Carry the line as far as an intercept, if the question wants one, since an intercept can only be read from a line that reaches the axis. Do not force the line through the origin unless the theory says it must pass through it, and if the theory does say so, let the points tell you whether it really does. And if one point lies far off the line while the other five sit neatly, draw the line through the five, mark the odd one with an arrow from the margin, and write a sentence saying that you have treated it as a doubtful reading. Do not rely on a ring for this if you have already ringed every point. That sentence turns a mistake into evidence of judgement.

The slope

Choose two points **on the line**, not two points from your table. The line is your best estimate of the truth; the table entries are the raw measurements, and using them throws away the averaging the line has just done for you. If a table entry happens to sit exactly on the line you may use it, but say so.

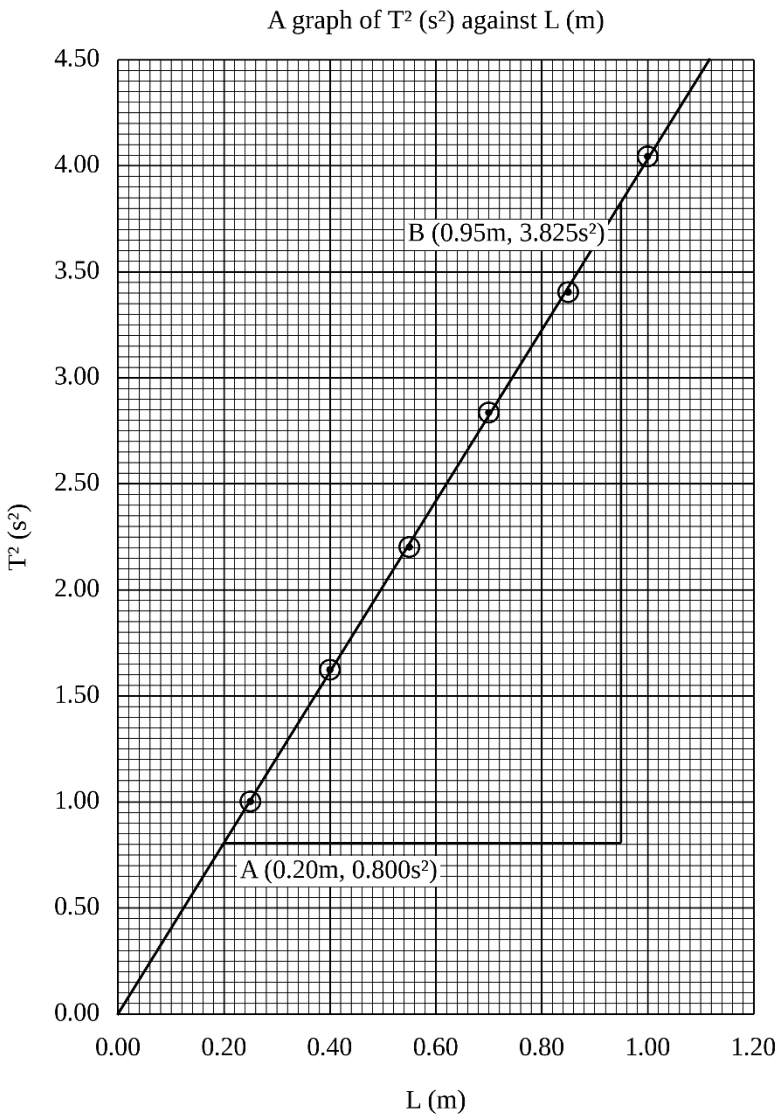
Take the two points far apart. Drop a horizontal and a vertical from them so that the line itself becomes the hypotenuse of a right-angled triangle, and make that triangle span more than half the drawn line. A triangle a centimetre across gives a slope with no significant figures worth having, and it also loses the mark for slope indication, which is awarded for the triangle being visible on the sheet at all.

Write the coordinates of the two corners beside them, then divide:

$$\text{slope}=\frac{y_2-y_1}{x_2-x_1}$$

and give the answer to three significant figures with its unit. The unit of a slope is the unit of the vertical axis divided by the unit of the horizontal axis, and it is worth writing that division out the first few times you do it. A slope taken from a graph of T^2 (s^2) against L (m) is in s^2m^{-1}, and quoting it as a bare number loses the mark however right the number is. The one exception is a graph whose two axes carry the same unit, such as 1/v (m^{-1}) against 1/u (m^{-1}), where u and v are the object and image distances of a lens: there the units cancel, the slope is a bare number, and you should say in a few words that it is.

Here is the finished graph of the table above, with all six things on it. The points are ringed, because the line passes so close to every one of them that a plain dot would disappear underneath it. Only the examination number is missing, and that is to keep the drawing clear; on your own sheet it goes at the top.



The line is drawn from the origin right up to the top rule, which is as far as this scale will take it, and it balances the points. The triangle runs from A to B, about two thirds of the length of the line. Neither A nor B is a reading from the table; both were read off the line itself, at positions the ruling lets the eye fix to half a small square. Taking their coordinates from the sheet,

$$\text{slope}=\frac{3.825\text{s}^2\text{-}0.800\text{s}^2}{0.95\text{m-}0.20\text{m}}=\frac{3.025\text{s}^2}{0.75\text{m}}=4.03\text{s}^2\text{m}^{-1}$$

To turn that into a value of g, go back to the equation the graph was built from. The readings came from $T = 2\pi\sqrt{(L/g)}$. Squaring both sides and making T^2 the subject gives

$$T^2=\frac{4\pi^2}{g}L$$

which is the form y = mx with T^2 up the page and L across it, so the slope is $4\pi^2$ divided by g, and

$$g=\frac{4\pi^2}{\text{slope}}=\frac{4\times(3.14)^2}{4.03\text{s}^2\text{m}^{-1}}=9.79\text{ms}^{-2}$$

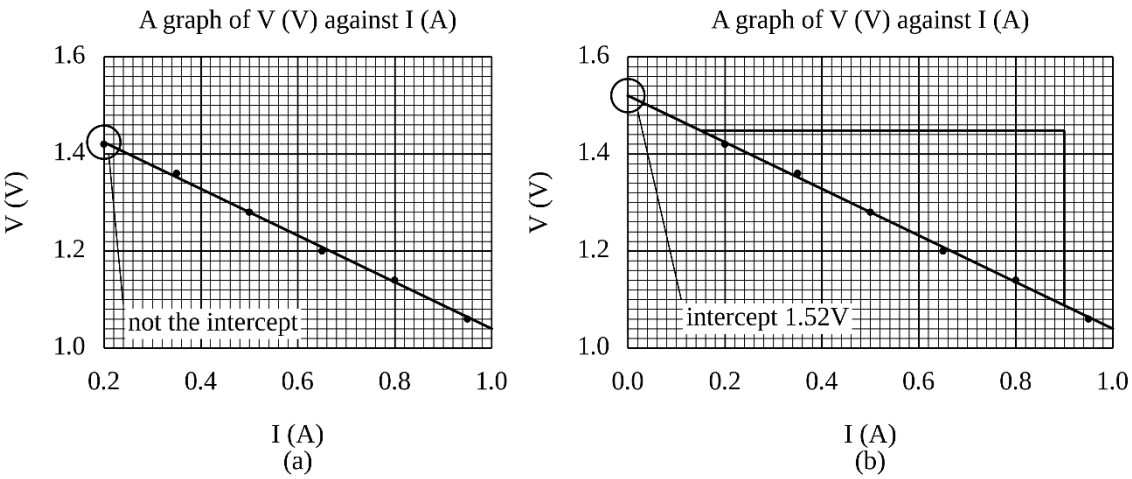
Compare that with the accepted value, 9.81ms^{-2}. The result is low by 0.02ms^{-2}, which is 0.2%, and a comment saying so in one sentence is worth writing. About half of that difference is not the experiment at all: it comes from using 3.14 for π rather than the fuller value on your calculator. The rest belongs to the experiment. Where a value of π is printed at the front of your paper, use that one and no other, so that a marker can follow your arithmetic, and say which value you used. Notice also that the line passes through the origin, which is the graph's

own way of saying that T^2 is directly proportional to L, with no correction needed to the length that was measured.

The intercept, and where it must be read

An intercept on the vertical axis is the value the line reaches when the horizontal quantity is zero, and an intercept on the horizontal axis is the reverse. It usually carries a physical meaning worth marks on its own. It is also the single easiest thing in this chapter to get wrong, and the way it goes wrong is always the same.

The intercept is read where the line crosses the axis drawn through zero, not where it runs off the edge of your paper. If your horizontal axis begins at 0.2 rather than at 0, then the value at the left-hand edge of the grid is simply the value at 0.2. It is not the intercept, and writing it down as one is worth no marks at all.



Both sheets carry the same six readings of the potential difference V across a cell and the current I taken from it. In (a) the horizontal axis starts at 0.2A, so the line meets the left-hand edge at 1.42V, and a candidate in a hurry writes down 1.42V as the electromotive force. In (b) the axis has been drawn from zero, the line reaches it, and the intercept is 1.52V. The difference between those two numbers is a tenth of a volt, and it came from the choice of where the axis started.

So: **decide before you choose your scales whether the question needs an intercept.** If it does, the axis carrying the quantity you are not taking the intercept from must reach zero, even when that wastes part of the sheet. For an intercept on the vertical axis it is the horizontal axis that has to start at zero; the vertical axis may start where it likes, and in the sheets above it starts at 1.0V. You may extend the line beyond your last point as a dashed continuation to reach that zero, but only if the zero is on the sheet at all. If you find afterwards that your axis does not reach zero, do not guess. Take the slope from your triangle, take the coordinates of any point on the line, and work the intercept out from $c = y - mx$, saying in words that you obtained it that way. The safer course is to redraw the graph with the axis starting at zero; a calculated intercept keeps the mark only if you show that working.

The graph in (b) also has a negative slope, and negative slopes are worth a sentence of their own. Reading the line itself at two currents well apart, neither of them a reading from the table,

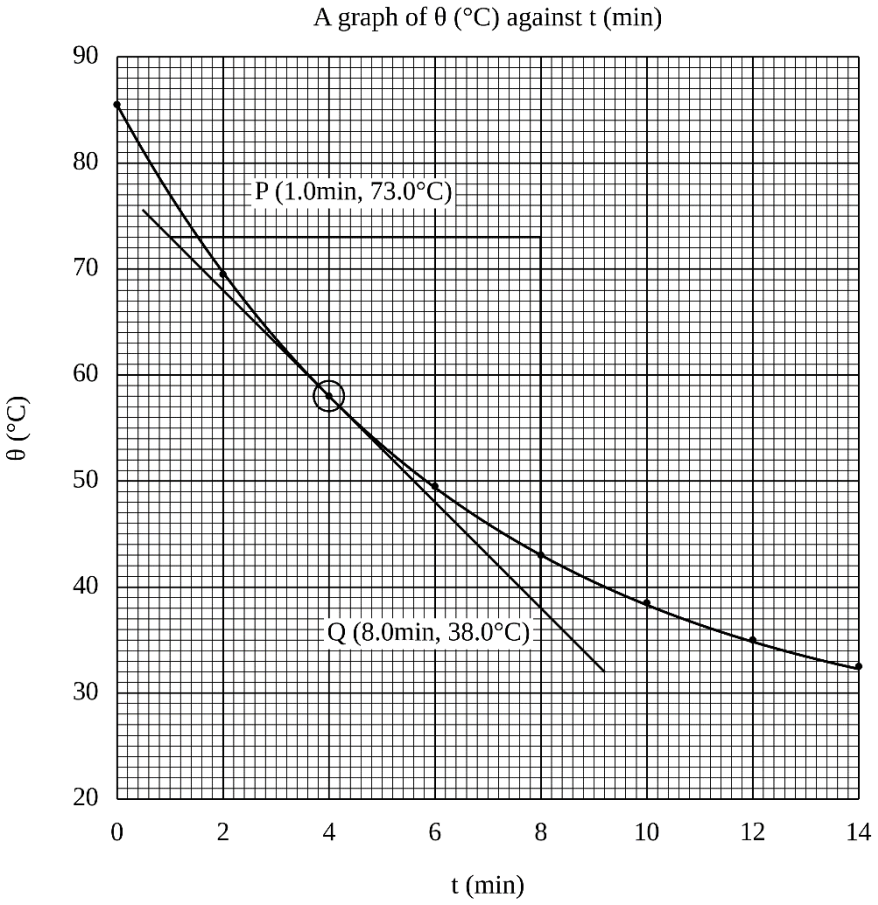
$$\text{slope}=\frac{1.09\text{V}-1.45\text{V}}{0.90\text{A}-0.15\text{A}}=\frac{-0.36\text{V}}{0.75\text{A}}=-0.480\text{VA}^{-1}$$

Keep the minus sign all the way through. It is not untidy; it is part of the answer. Comparing with the equation the graph was built from, $V = E - Ir$, the slope is $-r$ and the intercept is E, so this cell has an electromotive force of 1.52V and an internal resistance of 0.480Ω. A candidate who drops the sign reports a negative internal resistance, which does not exist.

Curves, and the tangent

Not every graph is a straight line. When the question asks for a curve, draw a smooth curve: one continuous freehand sweep that passes through or near every point, with no corners and no sudden changes of direction. Turn the paper so that your hand moves comfortably along the curve rather than across it, and draw it in one movement rather than in short strokes.

What a curve is usually for is its slope at a particular place, and that means a tangent. A tangent is a straight line that touches the curve at one point and runs in the same direction as the curve does there. Its slope is the slope of the curve at that point.



The curve is that of a body cooling towards room temperature, and the tangent has been drawn at the point where t = 4.0min. Two methods will place it for you.

(i) **The mirror method.** Stand a small plane mirror on the paper across the curve at the point, and turn it until the curve and its reflection run into one another without a kink. The mirror is then at right angles to the curve, so draw a line along the foot of the mirror and then the tangent at right angles to it, through the point.

(ii) **The chord method.** Mark the curve at two points the same short distance to the left and to the right of yours, measured along the bottom axis, join those two marks with a faint straight line, and then draw the tangent parallel to it through the point itself. The closer together you take the two marks, the better the tangent, and the harder it is to draw.

Then treat the tangent exactly as you would treat a best straight line: extend it well beyond the point, take a triangle spanning more than half of it, and read the two corners off the sheet. For the tangent drawn above, using P and Q,

$$\text{slope}=\frac{38.0°\text{C}-73.0°\text{C}}{8.0\text{min}-1.0\text{min}}=\frac{-35.0°\text{C}}{7.0\text{min}}=-5.00°\text{Cmin}^{-1}$$

The temperature your answer belongs to is the one where the tangent touches the curve, not one of the corners you took the triangle from. Here the curve stands at 58.0°C when t = 4.0min. So the slope of the curve there is $-5.00°\text{Cmin}^{-1}$, and the body is cooling at 5.00°C per minute at the moment when its temperature is 58.0°C. The minus sign says that the temperature is falling, and if you drop it from your final line you must let the word cooling carry it instead and say so. The answer means nothing without the temperature at which it was taken: a rate read from a curve belongs to one point on that curve and to no other.

Examiner's eye

- The slope triangle must be drawn on the graph, not left to arithmetic in the answer booklet. The mark is for showing it.

- Take the slope from the line. Marks are lost every year by candidates who quietly use two rows of their own table instead, and it shows, because the slope comes out to be exactly what two rows of the table give, which a hand-drawn line almost never does.
- Give the slope its unit: the unit of the vertical axis over that of the horizontal one. A slope from a graph of V (V) against I (A) is in VA^{-1}, which is an ohm.
- Keep a negative slope negative all the way to the final line, unless you replace the sign with a word such as falling or cooling and say that you have done so.
- An intercept quoted from a graph whose axis does not begin at zero earns nothing unless you say how you obtained it and show the working.
- A curve drawn as a series of straight segments between points is not a curve, and a tangent drawn without the curve being smooth first is not a tangent.
- Where the quantity has an accepted value, g being the common one, finish with a sentence comparing yours with it as a percentage. It costs one line and it is expected.

Making the measurement count

Practical tip 1: *You can work out the slope you ought to get before you plot anything, and a slope that comes out upside down, or wrong by a power of ten, is then caught in seconds.*

The graph in this chapter should have a slope of $4\pi^2$ divided by g. Putting the accepted value in,

$$\text{expected slope}=\frac{4\times(3.14)^2}{9.81\text{ms}^{-2}}=4.02\text{s}^2\text{m}^{-1}$$

so before a single point was plotted you knew the answer had to come out near $4s^2m^{-1}$. Any slope that is out by a plain factor, whether it is a hundred, a four hundred or a simple turning upside down, has a cause just as simple, and Drill 10 chases three of them down. All of them are invisible unless you know what to expect.

Practical tip 2: *Almost every relation in this book can be bent into the form y = mx + c, and the whole skill is deciding what to plot against what.*

The method has three steps, and it never varies. Write the relation down. Rearrange it until one measured quantity, or a simple function of one, stands alone on the left, and another stands multiplied by a constant on the right. Whatever multiplies the second is the slope; whatever is left over is the intercept. The table below is the catalogue for this book; you will meet every line of it again.

Relation	Plot	Slope	Intercept
$T = 2\pi\sqrt{(L/g)}$	T^2 against L	$4\pi^2/g$	zero
$T = 2\pi\sqrt{((L+d)/g)}$	T^2 against L	$4\pi^2/g$	$4\pi^2d/g$
$T = aL^n$	$\log_{10}T$ against $\log_{10}L$	n	$\log_{10}a$
$T^2h = (4\pi^2/g)(h^2 + k^2)$	T^2h against h^2	$4\pi^2/g$	$4\pi^2k^2/g$
$V = E - Ir$	V against I	$-r$	E
$E = I(R + r)$	1/I against R	1/E	r/E
$R = \rho L/S$	R against L	ρ/S	zero
$1/u + 1/v = 1/f$	1/v against 1/u	−1	1/f
$L = v/(4f) - e$	L against 1/f	v/4	−e
$\log_{10}(\theta - \theta_0) = C - Kt$	$\log_{10}(\theta - \theta_0)$ against t	−K	C

A symbol means what its own line says it means, and one of them does double duty. In the eighth line u, v and f are the object distance, the image distance and the focal length of a lens; in the ninth, v is the speed of sound, f is the frequency and L is the length of the air column. Everywhere else L is a length, T a period, V a potential difference, I a current, R a resistance and θ a temperature. The letters you never measure are these: a, n, C and K are constants the graph is there to find; d is the error in the measured length; h is the distance from the pivot to the centre of mass and k the radius of gyration; ρ is resistivity and S a cross-section; e is the end correction of the tube; and θ_0 is the temperature of the room. Each is met properly in the chapter that uses it.

Two remarks about the table. The third line is the one to reach for whenever a question says "show that the relation is of the form" rather than giving you the formula: taking logarithms turns any power law into a straight

line whose slope is the power itself. And the last line is the one to read carefully. State the base of every logarithm you plot. It decides what the intercept of the third line means, since a is ten to the power of that intercept if you plotted logarithms to base ten, and the exponential of that intercept if you plotted natural ones. It decides the slope of the last line too: to base ten the slope is −K as written, and the constant in the natural-logarithm form of the same law is 2.303 times larger.

Practical tip 3: *A graph is a calculation you can check by looking at it, so look at it before you leave the bench.*

Three checks take about a minute between them and catch most of what goes wrong. Does the line have roughly the slope you predicted? Do the points lie along it rather than curving away at one end, and if they curve, is the relation you assumed actually the right one? And does the intercept mean anything sensible: an end correction that is a few millimetres rather than a few metres, an electromotive force near 1.5V rather than near 15V?

A graph that fails any of those three is telling you something while you can still do something about it. The same graph, checked an hour later at home, tells you the same thing and there is nothing left to do.

Drills

Work each one on paper before you read the answer printed beneath it.

Drill 1

A candidate submits the table below. Find four separate faults in it, and say what each one should have been.

Length	time (s)	T	T squared
100.0cm	40.22	2.011	4.04
85.0cm	36.90	1.845	3.40
70.0cm	33.7	1.684	2.8359
55.0cm	29.68	1.484	2.20

Answer

The unit is written beside every entry in the first column instead of once in its heading. The unit belongs in the heading, written once, with the entries reading 100.0, 85.0, 70.0, 55.0.

The headings of the third and fourth columns carry no unit at all. They should read T (s) and T^2 (s^2).

The times do not carry a constant number of decimal places: 33.7 should be 33.68, since the stopwatch gives hundredths.

The squared column does not carry a constant number of figures. The times were read to four significant figures, so every entry in it should carry four: 4.044, 3.404, 2.836 and 2.202. As printed, three entries have been rounded to three figures and one carries five.

A fifth fault, if you found it: the first two headings give the name of the quantity instead of its symbol, and the second does not say what was timed. They should read L (cm) and t for 20 oscillations (s).

Accept: any four of these five.

Drill 2

A set of readings runs from 0.15m to 0.85m in one column and from 12.4s^2 to 61.8s^2 in the other. You have a sheet of graph paper 18cm wide and 24cm high. The lengths go across the sheet and the squared periods up it, and neither axis has to start at zero. Choose a scale for each axis, and say what fraction of each axis your points will fill.

Answer

Across the sheet you have 18cm, which is nine large squares. The range to be covered is 0.85 − 0.15 = 0.70m. A scale of 2cm to 0.1m gives an axis from 0.10m to 0.90m, which is eight large squares and fits.

Up the sheet you have 24cm, which is twelve large squares. The range is 61.8 − 12.4 = 49.4s^2. A scale of 2cm to 5s^2 gives an axis from 10s^2 to 65s^2, which is eleven large squares and fits.

The points then span 0.70m of an axis 0.80m long, and 49.4s^2 of an axis 55s^2 long: about 88% across and 90% up.

Accept: 2cm to 0.1m and 2cm to 5s². A vertical scale of 2cm to 10s² is not acceptable here: nothing in the question forces the axis to reach zero, so there is no excuse for one that spreads the whole range over five large squares of the twelve you were given and leaves the rest of the sheet empty.

Drill 3

Explain, with a number, why a scale of 2cm to 4 units should never be used, and state the two permitted scales nearest to it.

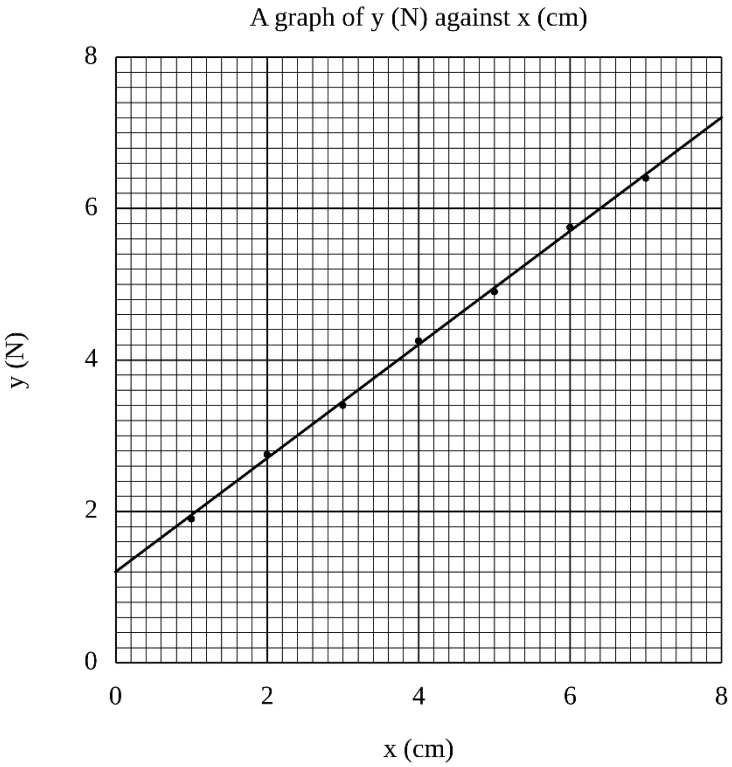
Answer

A large square is ten small squares across, so 2cm to 4 units makes each small square 0.4 of a unit. A reading of 1.3 then falls at 1.3 ÷ 0.4 = 3.25 small squares, a quarter of a square past the third ruling. The eye can judge a whole small square and a half of one, and nothing finer, so that reading can be neither placed nor read back.

The permitted scales are 1, 2 and 5 units to a large square, and the same multiplied or divided by any power of ten. The two nearest to 4 are 2 units and 5 units to a large square, which make each small square 0.2 and 0.5 of a unit.

Drill 4

Read the slope of this graph. Give it to three significant figures with its unit, and state the intercept on the vertical axis.



Answer

Take two points on the line and well apart, choosing values you can read to half a small square, and make the triangle span most of the line rather than half of it. At x = 1.2cm the line is at 2.10N and at x = 7.2cm it is at 6.60N:

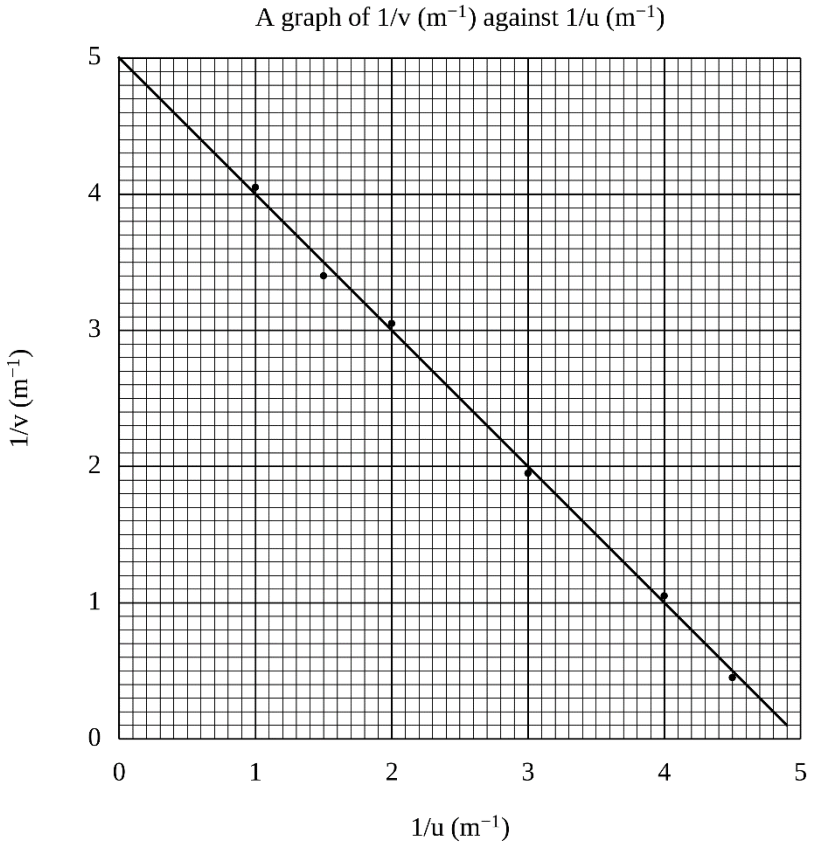
$$\text{slope}=\frac{6.60\text{N}-2.10\text{N}}{7.2\text{cm}-1.2\text{cm}}=\frac{4.50\text{N}}{6.0\text{cm}}=0.750\text{Ncm}^{-1}$$

The line meets the vertical axis at 1.20N, and since that axis is drawn through x = 0 the value may be read straight off.

Accept: 0.735 to 0.765Ncm⁻¹, and 1.15 to 1.25N.

Drill 5

This graph was drawn from measurements on a converging lens, where u is the object distance and v the image distance. Find the slope and the intercept, and hence the focal length of the lens.



Answer

Take two points on the line and well apart, reading them from the line itself and not from the plotted points. At $1/u = 0.50\text{m}^{-1}$ the line is at 4.50m^{-1}, and at $1/u = 3.50\text{m}^{-1}$ it is at 1.50m^{-1}:

$$\text{slope}=\frac{1.50\text{m}^{-1}-4.50\text{m}^{-1}}{3.50\text{m}^{-1}-0.50\text{m}^{-1}}=\frac{-3.00\text{m}^{-1}}{3.00\text{m}^{-1}}=-1.00$$

The slope of this graph has no unit, because the two axes carry the same one. The intercept on the vertical axis is 5.00m^{-1}.

The lens equation rearranges to $1/v=-1/u+1/f$, so the intercept is 1/f and

$$f=\frac{1}{5.00\text{m}^{-1}}=0.200\text{m}=20.0\text{cm}$$

Accept: slope −1.00 with no unit, intercept 4.90 to 5.10m^{-1}, f from 19.6cm to 20.4cm.

Drill 6

A candidate plots a graph whose horizontal axis runs from 20°C to 100°C. The line meets the left-hand edge of the grid at 1.02m, and the candidate writes "intercept = 1.02m". State what is wrong, and what should have been done instead.

Answer

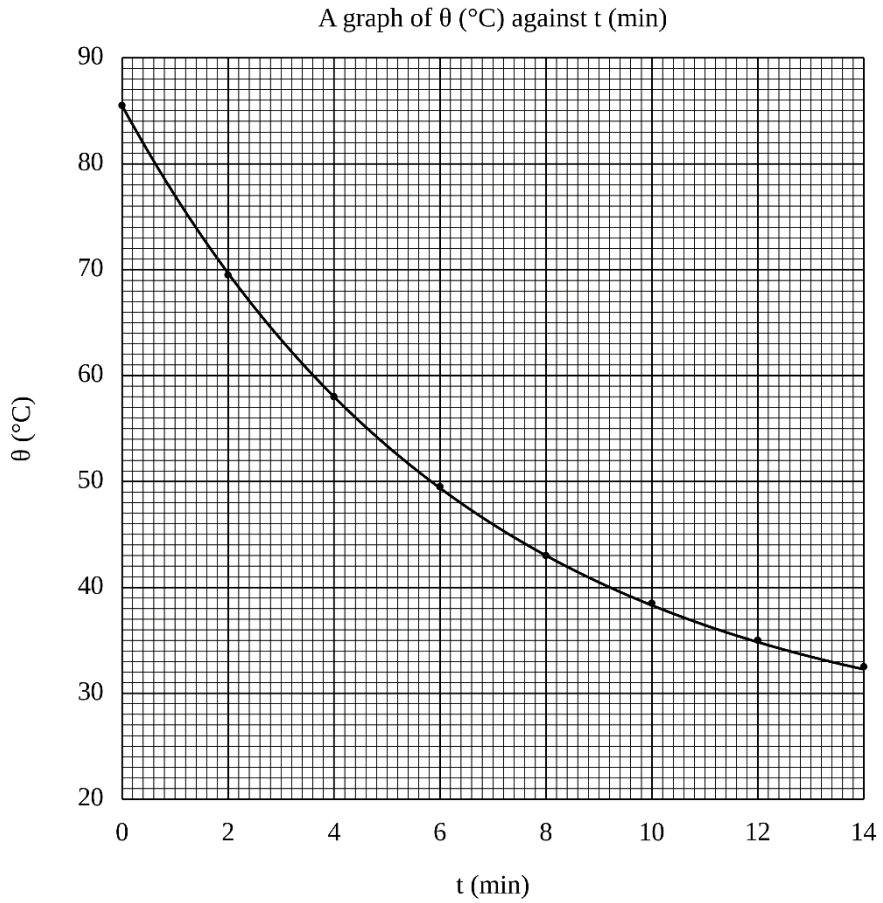
The intercept is the value the line reaches when the horizontal quantity is zero, and this axis never reaches zero. The number 1.02m is simply the length at 20°C.

Do what *The intercept, and where it must be read* said: redraw the graph with the horizontal axis starting at 0°C, or work the intercept out from c = y − mx and say in words that you calculated it. A dashed continuation cannot help, because 0°C is not on the sheet at all.

Redrawing is the safer of the two, because a calculated intercept carries every error the slope carries and then a second one from the point you chose, while a read intercept carries neither.

Drill 7

On the cooling curve below, draw the tangent at t = 8.0min and use it to find the rate of cooling at that instant. State the temperature to which your answer belongs.



Answer

At t = 8.0min the curve is at 43.0°C. Draw a tangent there and take a triangle across it, for example from t = 4.0min, where the tangent stands at about 54.0°C, to t = 12.0min, where it stands at about 32.0°C:

$$\text{slope}=\frac{32.0°\text{C}-54.0°\text{C}}{12.0\text{min}-4.0\text{min}}=\frac{-22.0°\text{C}}{8.0\text{min}}=-2.75°\text{Cmin}^{-1}$$

So the slope is $-2.75°\text{Cmin}^{-1}$, and the body is cooling at about 2.75°C per minute when its temperature is 43.0°C. Quoting the rate without the temperature is worth nothing, because a different point on the same curve gives a different rate.

Accept: -3.0 to $-2.4°\text{Cmin}^{-1}$, or the same figure written as a rate of cooling, with the temperature stated as 43.0°C. A tangent drawn by eye cannot do better than that, which is why the tolerance is wide.

Drill 8

None of these three relations is in the catalogue above. For each one, state what should be plotted against what to give a straight line, and what the slope and the intercept of that line are. (a) A trolley given the same push down a runway each time, where $v^2 = u^2 + 2as$, so that its speed u at the first gate is the same at every trial and the distance s to the second gate is varied. (b) A fixed mass of gas at constant temperature, where pV = k. (c) A radioactive source, where the count rate N falls as $N = N_0e^{-\lambda t}$.

Answer

(a) The relation is already linear in s. Plot v^2 against s; the slope is 2a, so the acceleration is half the slope, and the intercept is u^2.

(b) Divide by V: $p=k\left(\frac{1}{V}\right)$. Plot p against 1/V; the slope is k and the intercept is zero, which is the check that the temperature really did stay constant.

(c) Take logarithms to base ten: $\log_{10}N=\log_{10}N_0-\frac{\lambda}{2.303}t$. Plot $\log_{10}N$ against t; the slope is $-\lambda/2.303$ and the intercept is $\log_{10}N_0$. Plotting natural logarithms instead gives a slope of $-\lambda$, which is why the base must be stated.

Drill 9

A candidate takes a slope from a graph of T^2 (s^2) against L (m) and writes "slope = 4.0271". Give two separate reasons why that line loses marks, and write what should have been written.

Answer

The slope carries no unit. The vertical axis is in s^2 and the horizontal in m, so the slope is in s^2m^{-1}.

Five significant figures claim a precision the graph cannot possibly carry. A point on a graph can be read to about half a small square, which here is about one part in a hundred, and the two readings that make a slope each carry that. Three significant figures is the most that can be justified, and the third of them is already the doubtful one.

It should read: slope = $4.03s^2m^{-1}$.

Drill 10

A graph of T^2 (s^2) against L (m) is expected to have a slope of about $4s^2m^{-1}$. One candidate obtains $0.248ms^{-2}$, a second obtains $0.0403s^2m^{-1}$, and a third obtains $1610s^2m^{-1}$. Say what each has most probably done.

Answer

A slope of $0.248ms^{-2}$ is the reciprocal of $4.03s^2m^{-1}$, unit and all, so that candidate has plotted the axes the other way round: L up the page against T^2 across it. The graph is not wrong, but the slope is now $g/4\pi^2$ and must be used that way, so g is $4\pi^2$ multiplied by the slope rather than divided by it.

A slope of $0.0403s^2m^{-1}$ is a hundred times too small, and a hundred is the number of centimetres in a metre, so that candidate has plotted the lengths as centimetres, 100.0 down to 25.0, on an axis labelled in metres. Every horizontal distance is a hundred times too large, so the slope is a hundred times too small. Written honestly it is $0.0403s^2cm^{-1}$, and a hundred times that is $4.03s^2m^{-1}$.

A slope of $1610s^2m^{-1}$ is 400 times too large, and 400 is 20 squared. That candidate has squared the time for twenty oscillations instead of the period. The cure is to divide by 20 first and square afterwards; this candidate never divided at all.

All three are caught in seconds by working out the expected slope before plotting, which is the whole point of Practical tip 1.

Chapter 4

KNOWING WHAT YOUR RESULTS SHOULD BE

The pendulum bench was the last one free, and **Kipute** had taken it because nobody else wanted it.

She timed her six lengths, wrote them into her table, and then, instead of going straight to the graph paper, she turned her page over and did something that took her rather less than two minutes.

"You are working," said **Mr Akilikubwa**, stopping behind her. *"The readings are taken. What is there left to work at?"*

"I am checking them, sir. The period of a pendulum is two pi root L over g. I know L. I know g near enough. So I know what the watch ought to have said, and I am seeing whether it said it."

"And?"

Kipute pointed at the fourth row. *"Five of them agree to a fifth of one percent. That one is out by five percent."*

"So it is wrong."

"So it is worth a second look." She went back to the bench, set the thread again at 0.700m, and timed it once more. The watch said 33.74s, not the 32.09s she had written. She had counted the swing she released the bob on, so she had timed nineteen oscillations and called them twenty.

Two benches away, **Kipanga** was already ruling his axes. He plotted six points, one of which was a great deal lower than the other five, drew a line that satisfied nobody, and got a value of g that no marker could accept. He was not careless. He simply did not know, until the paper came back, that anything had gone wrong.

That is the whole difference, and this chapter is about it. Every experiment you will meet is built on an equation. If you know the equation and roughly what the apparatus is, you can say what the readings will be before you take them, to within a fraction of one percent. That single habit will earn you more marks than any other in this book, because it turns the two hours at the bench from a gamble into a check.

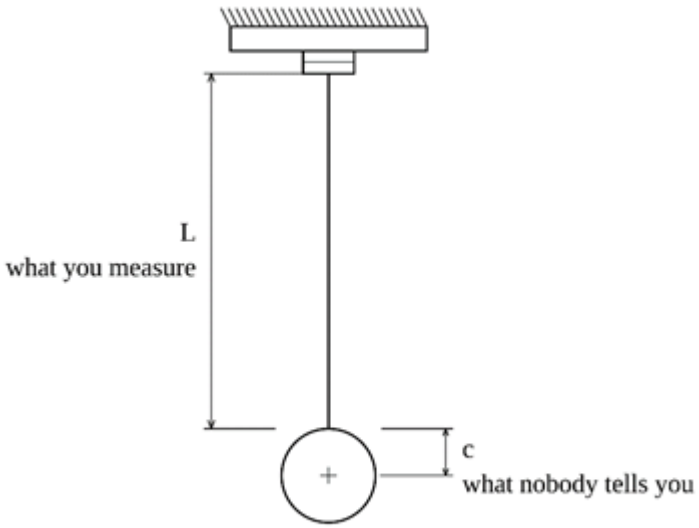
There is an equation behind every table

A practical question hands you apparatus and a procedure. It does not usually hand you the relation, because finding the relation, or using it, is part of what is being marked. But there is always one, and it is always a relation you have met in the theory course. The catalogue of relations in Chapter 3 has most of them.

Take the pendulum. The relation is

$$T=2\pi\sqrt{\frac{L_e}{g}}$$

in which L_e is the **effective length:** *the distance from the point of suspension to the centre of gravity of the bob.* That is the sentence to read twice. It is not the length of the thread, and it is not the length you can most easily measure. From here on it is written L + c, because that is how you will have to build it.



the pendulum swings on L + c, not on L

What you measure with the metre rule is L, from the cork pads down to the point where the thread is tied to the bob. What the pendulum actually swings on is L + c, where c is the distance from that point to the centre of the bob, which for a 25mm brass bob is 1.25cm, half its width. Nobody will tell you c. It is not printed in the

question and it is not in the apparatus list. It is the first of the constants you have to supply for yourself, and this chapter is largely about where such constants come from and what they are worth. In Chapter 3 the lengths had already been measured to the centre of the bob, which is why that graph went through the origin and this one will not.

The constants nobody gives you

School apparatus is not infinitely various. The bob is a 25mm brass sphere; the calorimeter is a copper can of between fifty and a hundred and twenty grams; the cell is a size D dry cell; the wire is constantan of one of a few standard gauges. Each of those has a value, or a narrow range of values, that you can carry in your head. Those numbers are a small bank of constants, and here is enough of it to work with; the rest arrives in the chapter that needs it.

Apparatus	The numbers you are not given
Pendulum bob, 25mm brass	radius 1.25cm, and 1.0 to 1.5cm across the sizes stocked; the mass makes no difference to the period
Helical spring, school pattern	spring constant 25 to 30Nm^{-1}, but batches differ, so treat it as a guide and not a value; about a third of the spring's own mass adds to the load it carries
Metre rule as a cantilever	Young's modulus of wood 9 to 12GPa; breadth about 2.5cm; thickness 5 to 8mm, as Chapter 2 measured it
Copper calorimeter with stirrer	mass 55 to 120g; specific heat capacity of copper 380 to 400Jkg^{-1}K^{-1}; with 150 to 200g of water the cooling constant is 0.028 to 0.038min^{-1}
Dry cell, size D	fresh: 1.50 to 1.60V with an internal resistance of 0.3 to 0.6Ω; worked hard: 1.40V and 0.8 to 1.5Ω
Constantan wire, SWG 28	diameter 0.376mm; resistivity 4.7 to 4.9 × 10^{-7}Ωm; about 4.3Ω to the metre
Tuning forks, kit set	341.3, 384, 426.6, 480 and 512Hz; sound travels at 346 to 350ms^{-1} in a warm room; the end correction of a tube is about 0.6 of its radius

You are not being asked to memorise a table of physical constants. You are being asked to know, roughly, what the thing in front of you is like, so that when your arithmetic produces a spring constant of 2700Nm^{-1} or a cell of 15V you notice before the marker does.

Predicting the table before you start

Here is the pendulum question as it is usually set: attach the thread so that the length from the point of suspension to the top of the bob is 1.000m, time 20 complete oscillations, shorten by 0.100m and repeat down to 0.500m.

Before touching the apparatus, put the numbers in. Take c as 0.0125m, take g as 9.81ms^{-2}, and take π as 3.14, because that is the value printed at the front of the paper. For the first length,

$$T=2\times3.14\times\sqrt{\frac{1.0125\text{m}}{9.81\text{ms}^{-2}}}=2.018\text{s}$$

and the watch, which will be stopped after twenty oscillations, ought to read twenty times that, 40.36s. Work from the rounded period each time, so that anybody following you reaches the same figure. Five more lines of the same arithmetic give the whole column, and the whole thing takes about three minutes.

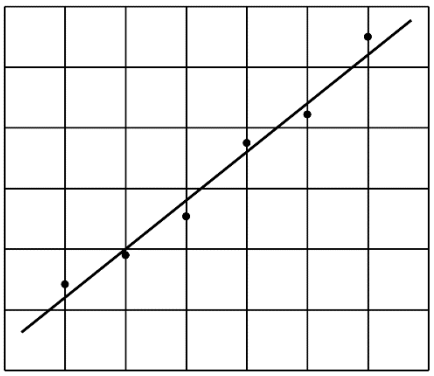
L (m)	L + c (m)	T predicted (s)	t predicted (s)
1.000	1.0125	2.018	40.36
0.900	0.9125	1.915	38.30
0.800	0.8125	1.807	36.14
0.700	0.7125	1.692	33.84
0.600	0.6125	1.569	31.38
0.500	0.5125	1.435	28.70

Three minutes have now bought you a great deal. You know that every reading will lie between 28 and 41 seconds, so you know a reading of 4s or of 400s is a blunder. You know the readings fall by two seconds for each 0.100m at the long end and by nearly three at the short end, so you know the order they must come in. And you have a number to compare each reading against the moment you have written it down.

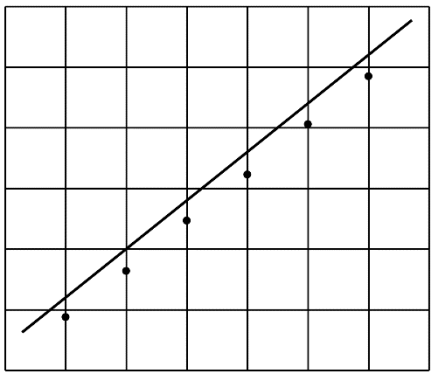
What careful hands actually record

Your readings will not equal the prediction, and they should not. Two quite different things push them away from it, and telling them apart is the skill.

Random scatter is the wobble of a hand and an eye. It is as likely to be up as down, it is different in every row, and it is small. **A systematic bias** is a push that goes the same way every time: a watch stopped a little early in every row, a pointer whose zero was never checked, a thermometer read from above every time. Scatter makes your points sit on both sides of the line. A bias puts them all on one side.



(a)



(b)

These two are sketches of a pattern, not finished sheets; a sheet you hand in carries a title and both axis labels. In both of them the line drawn is the one the theory predicts, not a line drawn through the points. In (a) the six points sit on both sides of it: that is scatter, it is honest, and the best line through those points is the theory line. In (b) all six sit below it by about the same amount. No amount of scatter does that.

It matters which kind of bias you have. A push that is the same number of seconds in every row, such as a watch stopped early, lowers the whole line without tilting it, so your slope survives and your intercept does not. A push that is the same fraction in every row, such as a metre rule whose divisions are wrong, tilts the line as well, and then the slope carries it too. Chapter 2 made the same point about a zero error, which shifts an intercept and leaves a slope alone. Until you have found out which you have, you do not know which of your two answers to trust.

Each family of experiments has its own signature, and it is worth knowing them:

Family	Its signature
Timed oscillations	every time a shade short of the prediction, by a tenth to half of one percent, with a few hundredths of scatter either way, written to 0.01s
Cooling and heating runs	temperatures at whole and half degrees only, the room's temperature written down and unchanged, most readings within half a degree of the curve and none of them rising
Method of mixtures	a final temperature 0.5 to 1.5°C below the ideal one, because heat escapes during the transfer
Circuit readings	current to 0.05A and potential difference to 0.05V, a digit of jitter either way, and a cell that loses about one percent of its electromotive force across a long run
Bridge balance lengths	to the nearest millimetre, with a millimetre or two of scatter, and every balance inside the 40cm to 60cm band Chapter 2 asked for

The timing bias is the one you will meet most. You start the watch on a swing and stop it on a swing, so most of your reaction time cancels, as Chapter 2 explained. What is left divides in two. The part that varies from row to row is the scatter Chapter 2 called reaction time, and it pushes a reading either way. The part that does not vary is a small anticipation, because a hand that knows the count is coming moves a little early, and that one always shortens the reading. It is worth a few hundredths of a second, which on these timings is a tenth to three tenths of one percent.

Checking a table, row by row

Write the prediction beside the reading and take the difference, both as a plain number and as a percentage of the prediction. Three extra columns, and you can fill them at the bench in the time it takes to reset the thread.

L (m)	t predicted (s)	t recorded (s)	Δt (s)	Δt/t (%)
1.000	40.36	40.28	−0.08	−0.2
0.900	38.30	38.22	−0.08	−0.2
0.800	36.14	36.10	−0.04	−0.1
0.700	33.84	33.74	−0.10	−0.3
0.600	31.38	31.34	−0.04	−0.1
0.500	28.70	28.64	−0.06	−0.2

Read the last column downwards. Every entry is negative, and every entry is smaller than half of one percent. They are not all the same size either, which is the scatter riding on the bias. One sentence is worth writing, and it can be as plain as this:

Each recorded time is between one and three tenths of one percent short of the value the relation predicts, and every one is short by a similar amount, which points to a small bias in the timing rather than to careless reading.

It is worth knowing where the bias came from. The watch was stopped a shade early, which takes the same few hundredths off every reading; and g at your bench is not exactly the value the book quotes, which lengthens or shortens them all together. Neither is a mistake. Both are why the check is made against a band and not against an exact number.

So what counts as agreement? Use these until a chapter gives you a narrower rule:

- a timed reading, within 1% of the prediction;
- a temperature, within one division of the thermometer, which is usually 1.0°C;
- a length on a metre rule, within 2mm;
- a current or a potential difference, within 5%, because meters are the coarsest instruments on the bench;
- a balance length on a metre bridge, within 5mm.

Anything outside its band is not necessarily wrong. It is a reading worth taking again, and taking a reading again costs a minute. The fourth row of the table above reads 33.74s because it was timed twice. On the first attempt it read 32.09s against a prediction of 33.84s, which is 5.2% out, more than five times the band. The first figure was not a bad measurement; it was nineteen oscillations counted as twenty, and 32.09 multiplied by 20 and divided by 19 is 33.78s, within four hundredths of a second of what the watch gave the second time. That is the arithmetic that identifies the fault.

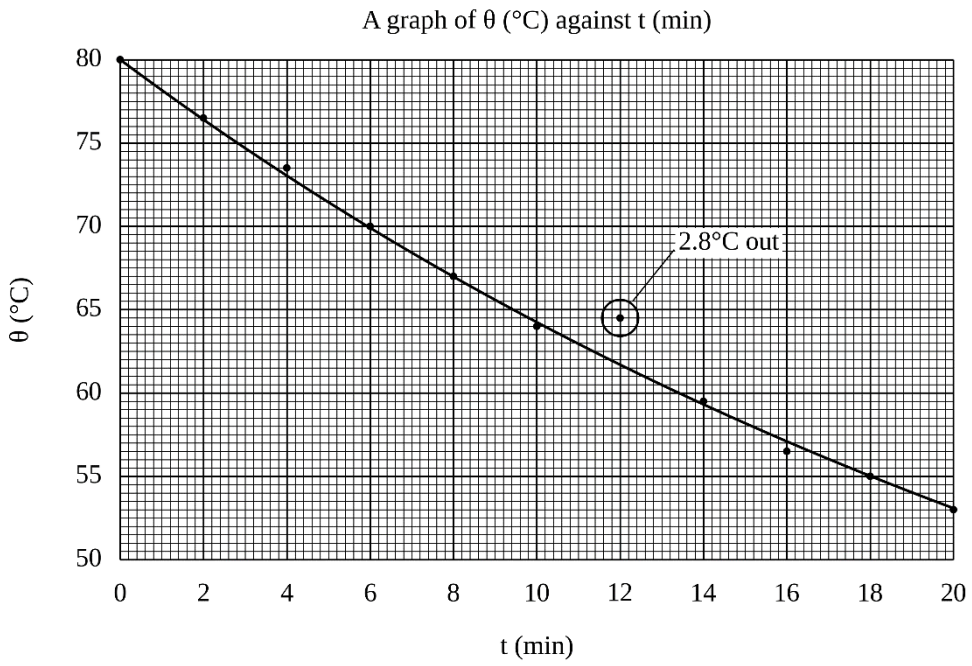
Never rub out a reading you have decided against. Rule one line through it, write the new one beside it, and say in a sentence why. A crossed-out figure with a reason beside it is evidence that you were watching. An erasure is evidence of nothing at all.

A cooling run is checked the same way, against the relation the chapter on heat will derive:

$$\theta=\theta_0+(\theta_i-\theta_0)e^{-kt}$$

where θ_0 is the temperature of the room, θ_i the temperature you started from, and k the cooling constant from the bank. This k belongs to the exponential form. Chapter 3 plots $\log_{10}(\theta - \theta_0)$ against t, whose slope is −K, and K is this k divided by 2.303; say which of the two you are quoting. With a room at 26.0°C, a start at 80.0°C and k of 0.0345min^{-1}, the predicted column and the recorded column look like this.

t (min)	θ predicted (°C)	θ recorded (°C)	Δθ (°C)
0	80.0	80.0	0.0
2	76.4	76.5	+0.1
4	73.0	73.5	+0.5
6	69.9	70.0	+0.1
8	67.0	67.0	0.0
10	64.2	64.0	−0.2
12	61.7	64.5	+2.8
14	59.3	59.5	+0.2
16	57.1	56.5	−0.6
18	55.0	55.0	0.0
20	53.1	53.0	−0.1

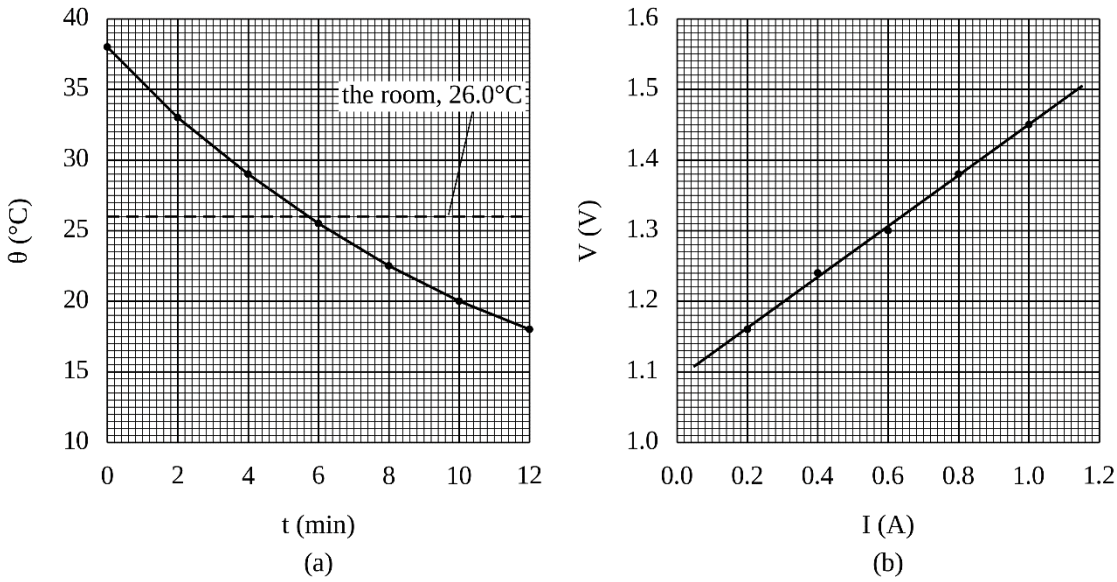


Every recorded temperature falls on a whole or a half degree, because that is all a thermometer graduated in degrees can give. Every difference is under half a degree, which is what rounding and a little lag between them do, except one. At twelve minutes the reading is 2.8°C above the prediction, which is nearly three divisions, and it is higher than the reading two minutes before it. Water cooling towards a cool room does not warm up. On the graph it is the one point that stands off the curve.

Take it again is not open to you here, because twelve minutes does not come round twice. Ring the point as you plot it, draw the curve through the other ten, and write one line saying that you treated it as doubtful. That is what a marker wants to see, and it is worth more than a tidy curve with a bad reading buried in it.

Readings that cannot be right

Some readings are not merely unlikely; they are impossible, and they can be recognised at a glance, without going back through the working. Learn the short list and run down it before you leave the bench.

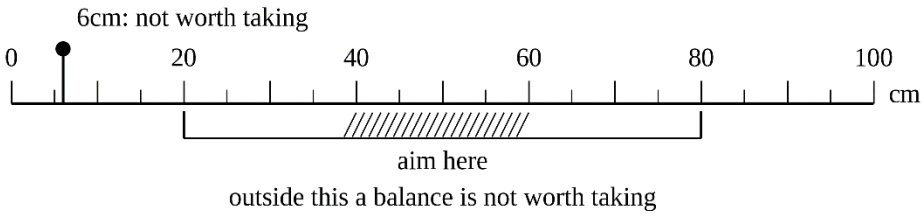


In (a) a body cooling in a room at 26.0°C has been recorded at 18.0°C. By conduction, convection and radiation nothing cools below its surroundings; only evaporation can take a liquid lower, and never by eight degrees. Either the room temperature was written down wrong, or the thermometer was read wrong, or the last readings were written down after the apparatus had been put away. In (b) the potential difference across a cell rises as more current is drawn from it. It falls, always, because the current has to be driven through the cell's own resistance. One of the two columns has been written down the page in the wrong order, so the largest potential difference now stands beside the largest current instead of the smallest.

Here is the rest of the list.

- A value of g outside 9.5 to 10.1ms^{-2}. Anything further out is a fault in the counting, the length or the graph, not a property of your bench.
- A refractive index below 1, or above about 2 for glass.
- A balance point on a metre bridge outside 20cm to 80cm. Near either end the two lengths are so unequal that a millimetre of error costs several times what the same millimetre costs at the middle, which is why Chapter 2 asks you to bring the balance back to between 40cm and 60cm.
- A period that does not lengthen when the pendulum is lengthened, or a resistance that does not rise when the wire is lengthened.
- A specific heat capacity larger than that of water, or a density smaller than that of air, for any solid or liquid you will be handed on a bench.
- A final temperature in a mixing experiment above the hotter of the two starting temperatures.
- A temperature that rises during a cooling run.

The drawing below shows where a balance point can be trusted and where it cannot. The dot stands at 6cm, which is not worth taking at all.



If a reading on your page is on that list, it is not a result. Go back and find out why, and if there is no time to find out, say in your report that the reading is impossible and what you think went wrong. A candidate who

writes that sentence has shown the examiner something a bare table cannot: that they were reading their own results.

Finishing an exercise the apparatus cut short

Take the discipline first, because it matters more than the method that follows it. In an examination a reading you did not take is not a reading. If an instrument fails, ask for another and take it again. If there is genuinely no time, write down what happened, in one line, beside the gap. Nothing in this section belongs in a script.

What it is for is your own practice: finishing an exercise the apparatus cut short, so that the graph still gets drawn and the analysis still gets done, and finding out whether the readings you did take hang together at all. A stopwatch is passed to the next bench and does not come back; a thermometer is knocked off the stand at the eighth minute. Two readings will carry you to the end of the exercise.

The method has one idea in it. Your readings differ from the prediction in a way that hardly changes down the table, because the bias that makes the difference is the same in every row. Measure that difference where you have readings, and carry it to the rows where you have none. Take the two readings at opposite ends of the range, on purpose, because that is where they hold the rest steady.

Here the timing bias is a fixed number of seconds, so what is carried is the difference in seconds and not a ratio. Suppose only the longest and the shortest were timed: 40.28s against a prediction of 40.36s, and 28.64s against a prediction of 28.70s. The two differences are

$$40.28\text{s}-40.36\text{s}=-0.08\text{s} \quad \text{and} \quad 28.64\text{s}-28.70\text{s}=-0.06\text{s}$$

whose mean is −0.07s. Take that off each remaining prediction.

L (m)	t predicted (s)	t estimated for practice (s)
0.900	38.30	38.23
0.800	36.14	36.07
0.700	33.84	33.77
0.600	31.38	31.31

The bench, when the watch came back, gave 38.22s, 36.10s, 33.74s and 31.34s. The estimates are within three hundredths of a second of every one of them, and that tells you one thing only: the readings you did take hang together. It tells you nothing whatever about the readings you did not.

One anchor removes the bias; a second catches any drift in it, which is why two are better than one. Where the bias is a fixed fraction rather than a fixed amount, a mis-scaled rule or a meter reading a steady percentage high, take the ratio of reading to prediction instead of the difference and multiply. Look at your two anchors and let them tell you which of the two is the steadier.

Practising when there is no laboratory

This is where the habit pays best of all, and it needs nothing but paper. A whole practical question, from table to final comment, can be rehearsed at a desk.

(i) Choose a relation from the catalogue and a set of values for the quantity you would vary. Six values, evenly spaced, covering a range of at least two to one.

(ii) Take the constants from the bank, and choose them near the middle of their range so that the numbers behave.

(iii) Work out the column the instrument would show, and then let the instrument have its say: apply the family's bias, add a few hundredths of scatter, and round to what the instrument can actually read. These are invented numbers and you know it. They exist so that the plotting and the analysis get practised; not one of them ever goes near a script.

(iv) Now set aside where the numbers came from. Tabulate them, choose your scales, plot, draw the best line, take the slope, and answer the questions the paper would ask.

(v) Only then go back to the constants and see whether your slope returns them. If it does, every step in between was sound.

That last step is what a laboratory cannot give you. At the bench you can check your slope against what the theory expects, but never against the truth, because nobody at the bench knows this spring's constant or this bob's radius. At the desk you chose them yourself, so every rehearsal marks itself.

Examiner's eye

- Do the prediction before you touch the apparatus, not after. Afterwards it is only arithmetic; beforehand it is a plan.
- A prediction is not an answer. Keep it on the back of the page, or rule its column off from the recorded one and head it predicted, so that nobody can mistake which is which.
- The differences between prediction and reading should be small and should mostly fall the same way. If they are large, or if they change sign wildly, the readings are the problem. If they are all one way and all much the same size, look at the apparatus and not at your hands, and say in one line what you think is biasing every row.
- Write the room temperature down at the start of every heat experiment, even when the question does not ask for it. Half the checks in the heat chapters need it.
- Cross out, do not erase, and give the reason in a few words.
- **Never adjust a reading to agree with a prediction.** The prediction tells you which reading to take again; the bench, and only the bench, tells you what it is.

Making the measurement count

Practical tip 1: *The graph catches a blunder; only the percentage difference catches a bias. A push of two or three tenths of a percent is invisible on graph paper and plain in a column of arithmetic.*

Look again at the pendulum table. Every reading is low, by between one and three tenths of one percent. On the sheet Chapter 3 chose for that graph, two centimetres to $0.50\,s^2$, that shifts each point by about a third of a small square, which no eye will ever see. Yet it moves the slope, and the slope is the answer. Taking the six recorded rows, squaring the periods and drawing the best line gives a slope of $4.01\,s^2m^{-1}$.

L (m)	t for 20 oscillations (s)	T (s)	T^2 (s^2)
1.000	40.28	2.014	4.056
0.900	38.22	1.911	3.652
0.800	36.10	1.805	3.258
0.700	33.74	1.687	2.846
0.600	31.34	1.567	2.455
0.500	28.64	1.432	2.051

From that slope,

$$g=\frac{4\pi^2}{\text{slope}}=\frac{4\times(3.14)^2}{4.01s^2m^{-1}}=9.84ms^{-2}$$

which is 0.3% above the accepted $9.81ms^{-2}$. Times that are all a little short make periods that are all a little short, squares that are shorter still, a slope that is too small, and therefore a g that is too big. The graph never told you. The column of percentages told you before you drew it.

Practical tip 2: *An intercept is where the constant nobody gave you comes back out.*

Squaring the pendulum relation and putting the effective length in full gives

$$T^2=\frac{4\pi^2}{g}L+\frac{4\pi^2}{g}c$$

so a graph of T^2 against L is a straight line whose slope is $4\pi^2$ divided by g and whose intercept on the vertical axis is that same slope multiplied by c. The line of the table above meets the axis at about $0.05s^2$, and

$$c=\frac{0.05s^2}{4.01s^2m^{-1}}=0.012m$$

which is 1.2cm. The bob is 25mm across, so its radius is 1.25cm, and the graph has handed that back to within a millimetre. Do not ask more of it than that: the intercept here is one small square above the origin, and a value read to half a small square is good to no better than half itself. In this table we had put c in ourselves, so all the check shows is that the arithmetic closes; at the bench the intercept is genuinely new. That is worth remembering the next time a question asks what an intercept means. It is not decoration. It is the constant the question left out, handed back to you.

Practical tip 3: *If you cannot predict a reading, you have not yet understood the experiment, and making the prediction is the quickest way to find that out.*

The rearrangement you would need for the prediction is the same rearrangement the question will ask for early on, and the constants you would need are the ones it will ask you to find at the end. Predicting a table is not extra work laid on top of the question. It is the question, done early and roughly, while there is still time to profit from it.

Drills

Work each one on paper before you read the answer printed beneath it.

Drill 1

A pendulum is set up with the thread 0.400m long from the suspension to the top of a 25mm brass bob. Predict the time for 20 complete oscillations, taking g as $9.81ms^{-2}$ and π as 3.14, and state the range within which a recorded value would be acceptable.

Answer

The bob's radius is 0.0125m, so the effective length is 0.400 + 0.0125 = 0.4125m.

$$T=2\times3.14\times\sqrt{\frac{0.4125m}{9.81ms^{-2}}}=1.288s$$

so the predicted time for 20 oscillations is 20 × 1.288 = 25.76s.

A timed reading is acceptable within 1%, which is 0.26s, so anything from 25.50s to 26.02s. A reading of 24.4s or of 27.1s should be taken again.

Accept: 25.7 to 25.8s, with a band of about ±0.3s.

Drill 2

A candidate times 20 oscillations of a pendulum and records 38.30, 36.14, 33.84, 31.38, 28.70 and 40.36 seconds for lengths of 0.900, 0.800, 0.700, 0.600, 0.500 and 1.000m. Without doing any arithmetic at all, give two reasons for looking at this table again.

Answer

The rows are not in the order of the lengths, with the 1.000m row written last. Nothing is wrong with the pairing, so the graph would survive it, but a table read downwards should march in one direction, and a row written out of place is one slip away from being paired with the wrong length.

Every reading agrees with the prediction exactly, to the hundredth of a second, in all six rows. Six hand-timed readings never do that. A table with no scatter in it has not been measured.

Accept: both points, each with a reason. One alone is half an answer.

Drill 3

A room is at 27.0°C. A student records the temperature of cooling water at two-minute intervals as 74.0, 71.0, 68.0, 65.0, 62.5, 60.0, 58.0, 56.0, 54.0 and 52.3°C. One entry cannot have come from the thermometer described in Chapter 2. Which, and why?

Answer

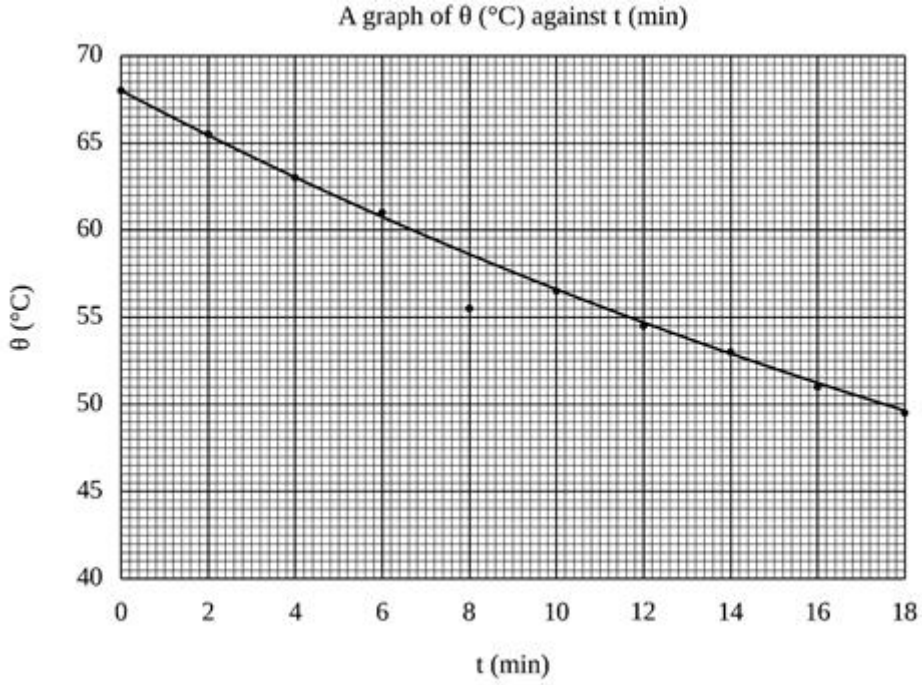
52.3°C. A thermometer graduated in whole degrees is read to the nearest half division, so every entry must end in .0 or .5. A tenth of a degree is a precision the instrument does not have.

It should be recorded as 52.0°C or 52.5°C, whichever the mercury was nearer.

Note that none of the readings is impossible in the other sense: all of them are above the room's 27.0°C, as they must be.

Drill 4

The graph below was drawn from a cooling run. One reading is badly out. Say which it is, estimate by how much, and say what should have been done about it.



Answer

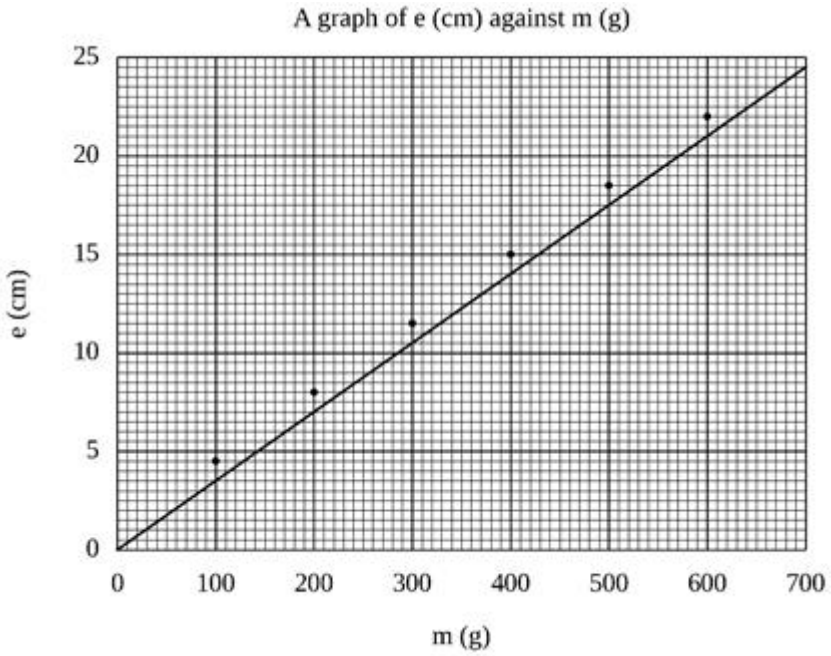
The reading at t = 8.0min. Every other point lies on the smooth curve; that one sits about 3°C below it, at 55.5°C where the curve gives about 58.5°C.

Three degrees is three divisions of the thermometer, well outside the one-division band. It could not have been taken again, because a cooling run is read at fixed times and eight minutes does not come round twice. It should have been ringed as it was plotted, the curve drawn through the other nine, and a sentence written to say that it was treated as doubtful.

Accept: the eight-minute reading, out by 2.5 to 3.5°C.

Drill 5

A spring is loaded in steps of 100g and the extension e is measured. The line drawn on the sheet below is the one the spring constant in the bank predicts. Describe the pattern of the six points, name the fault, and say how the readings should be used.



Answer

Every point lies about 1.0cm above the predicted line, and by the same amount at every load. That is a systematic bias, not scatter: scatter would put some points above and some below.

The fault is a zero error. The pointer was not read against the scale with the spring unloaded, so every extension carries the same extra centimetre.

The slope is unaffected, so the spring constant taken from the slope is still right. The extensions themselves should each have 1.0cm subtracted, or better, the unloaded reading should be taken and every extension worked out as a difference from it.

Accept: a bias of 0.9 to 1.1cm, identified as a zero error.

Drill 6

A candidate obtains a spring constant of 0.27Nm^{-1} from a school helical spring, an internal resistance of 12Ω from a fresh size D cell, and a balance point at 92cm on a metre bridge. Say what is wrong with each, without recomputing anything.

Answer

A school spring is 25 to 30Nm^{-1}. A value of 0.27Nm^{-1} is a hundred times too small, which is exactly what happens when an extension measured in centimetres is divided into the force as though the centimetres were metres.

A fresh size D cell has an internal resistance of 0.3 to 0.6Ω, so 12Ω is twenty times anything the cell can offer. Internal resistance comes from the slope of V against I, not from the intercept, and a slope twenty times too large usually means the triangle was taken over a centimetre of line, or that the two axes were read against each other's scales. Take the triangle again over more than half the drawn line.

A balance at 92cm leaves only 8cm in the other arm, so a millimetre of error in the jockey changes the ratio by more than one percent. The resistance in the box should be changed until the balance comes back to the 40cm to 60cm band Chapter 2 asked for, and the reading taken again.

Drill 7

Knowing that the stopwatch would be taken away, a student times the two extreme lengths first: 37.18s at L = 0.850m and 25.72s at L = 0.400m. The predictions for those two lengths are 37.24s and 25.76s. Estimate the reading at L = 0.650m, whose prediction is 32.64s, and say what you would write in your report.

Answer

The two differences are 37.18 − 37.24 = −0.06s and 25.72 − 25.76 = −0.04s, whose mean is −0.05s. The bias is a fixed number of seconds, so it is the difference and not the ratio that is carried.

32.64 − 0.05 = 32.59s.

In your own practice that is the figure to carry into the table so that the graph can still be drawn, and it should be marked as an estimate. In an examination you would not write it in the table at all: you would ask for another watch, and failing that write one line saying that the stopwatch was taken away after the second reading and that no further timings were taken.

Accept: 32.55 to 32.65s, with the report sentence.

Drill 8

Explain why a prediction is made with g taken as 9.81ms^{-2} even though the whole purpose of the experiment is to measure g.

Answer

The prediction is not an answer and is never quoted as one. It is a rough expectation, used to decide whether a reading is worth keeping.

The local value of g differs from 9.81ms^{-2} by a few parts in a thousand, which changes a predicted time by a few hundredths of a second. That is far inside the 1% band the check uses, so the prediction is good enough for its purpose while being **useless** as an answer.

The measured value must come from the slope of your own graph and from nothing else. Using 9.81 in a prediction and then quoting 9.81 as your result is the one way this method can be misused, and it is obvious to a marker, because your slope will not support it.

Drill 9

A cooling curve is to be predicted for a copper calorimeter of 100g holding 180g of water, starting at 78.0°C in a room at 25.0°C. Taking the cooling constant as 0.035min^{-1}, predict the temperature after 10.0min and after 30.0min, and say which of the two predictions you would trust less.

Answer

The excess over the room is 78.0 − 25.0 = 53.0°C at the start, and it falls by a factor of e^{-kt}.

$$\theta = 25.0°\text{C} + 53.0°\text{C} \times e^{-0.35} = 62.3°\text{C}$$

$$\theta = 25.0°\text{C} + 53.0°\text{C} \times e^{-1.05} = 43.5°\text{C}$$

The thirty-minute prediction is the weaker of the two. The cooling constant is known to only about 15%, and the error that causes grows in step with kt: at 10min it shifts the predicted excess over the room by about 5%, at 30min by about 16%. A prediction reaching far beyond the readings that fixed the constant is always the loosest.

Accept: 62 to 63°C and 42 to 45°C, with the second named as the less reliable.

Drill 10

Design a home rehearsal of an experiment that measures the resistance per metre of a constantan wire of SWG 28. Give the relation, the constants you would take, one row of the table you would generate, and the check you would make at the end.

Answer

The relation is $R = \rho l/A$, the form Chapter 3 catalogues, so a graph of R against l is a straight line through the origin whose slope is ρ/A, the resistance per metre.

From the bank, SWG 28 constantan has a diameter of 0.376mm and a resistivity of about $4.8 \times 10^{-7}\Omega\text{m}$. Its cross-section is $\pi(0.188 \times 10^{-3}\text{m})^2 = 1.11 \times 10^{-7}\text{m}^2$, so the resistance per metre is $4.8 \times 10^{-7} \div 1.11 \times 10^{-7} = 4.3\Omega\text{m}^{-1}$, which is what the bank quotes.

At l = 0.600m the relation gives $R = 4.3 \times 0.600 = 2.6\Omega$, two figures because the resistance per metre carries two. Then let the bench have its say: a metre bridge delivers that resistance through a balance length read to the nearest millimetre, with a millimetre or two of scatter.

The check at the end closes the loop: the slope of your graph should come back to $4.3\Omega\text{m}^{-1}$, and multiplying it by the cross-section should return the resistivity you started from. If it does not, the error is in your analysis, not in your hands.

Accept: 4.2 to $4.5\Omega\text{m}^{-1}$, with the closing check stated.

Chapter 5

ERROR ANALYSIS

Kipanga had finished early, which for **Kipanga** was a warning sign, and he was reading the last question over **Kipute's** shoulder.

"State two sources of error in this experiment," he said. *"That is two free marks. Human error and parallax error. Done."*

Kipute did not look up from her graph. *"Two free marks for what? You could have written that on the bus."*

Mr Akilikubwa did not look up either. *"Whose hand?"*

"Sir?"

"Human error. Which human, doing what, and which way did it push the reading? Parallax. Looking at which scale, from which side, and how much did it cost you? You have written down two words that could be attached to any experiment in this building. An examiner who cannot tell from your answer which experiment you did will not give you anything for it, and he is right not to."

He came over and read the rest of the page. *"You have written g equals 9.84282 metres per second squared. You measured a length with a wooden rule and a time with your thumb, and you are offering me six figures. Two of those figures are yours. The other four are the calculator's opinion, and the calculator was not at the bench."*

This chapter is about the difference between those two kinds of figure. It will show you how to put a number on the doubt in a single reading, how the doubt travels through the arithmetic into your final answer, how to tell which measurement is doing the damage, how to close with a sentence that sets your answer against the accepted one, and how to name a source of error so that it earns the marks Kipanga threw away.

Mistakes, errors and uncertainties

In conversation, mistake and error mean much the same thing. In a laboratory they do not, and the marks go to the laboratory meaning.

A mistake is something you did wrong. You read the wrong scale, you counted nineteen swings as twenty, you wrote 33.6 when the watch said 33.68. The laboratory name for it is a **gross error**, and it has no size and no direction worth quoting, because the right response to it is not to estimate it but to put it right. Chapter 4 was about catching gross errors before you leave the bench. Never offer one as a source of error.

An error is what is left when nothing has gone wrong. The metre rule is engraved to the millimetre and no further. Your thumb is not the watch. The thermometer takes a moment to catch up with the water. None of that is carelessness, none of it can be removed, and all of it can be estimated. That estimate is the **uncertainty**. The error is the imperfection in the measurement; the uncertainty is the number you put on it, written after the reading with a plus-or-minus sign.

Errors come in two kinds, and you met them in Chapter 4 as scatter and bias. A **random error** is as likely to push a reading up as down. It shows itself when you repeat a measurement and get a slightly different answer, and repeating is what reduces its effect on the mean, because the pushes cancel. A **systematic error** goes the same way every time: an unchecked zero, a thermometer always read from above, a watch always stopped a fraction early. Repeating a measurement a hundred times does not touch a systematic error. Only finding it does.

Two more words belong with this pair. **Precision** is how closely repeated readings agree, so random error and the resolution of the instrument are what limit it. **Accuracy** is how close those readings lie to the true value, so systematic error is what limits it. The two are independent, which is why a set of readings can be precise and still be wrong, and why a small scatter is never on its own proof that the answer is right.

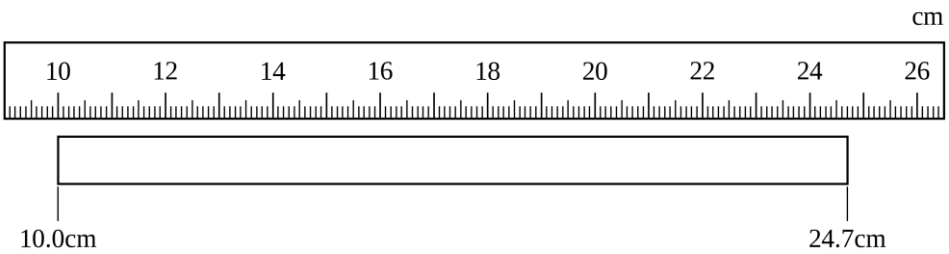
The uncertainty in a single reading

Start with the instrument. Chapter 2 gave every instrument its least count, the smallest change it can show, and the uncertainty in one reading follows from it in one of two ways. On some instruments the scale is engraved and your eye takes the last step: a metre rule, a thermometer, a moving-coil meter. You can see which of two marks the reading falls nearer, and the uncertainty is half a division. Where the instrument itself hands you the last figure, as a vernier calliper or a micrometer or a beam balance does, you cannot do better than the least count, and you quote the whole of it.

Instrument	Least count	Uncertainty to quote for one reading
Metre rule	1mm	±0.05cm on each end, so ±0.1cm on a length
Vernier calliper	0.01cm	±0.01cm
Micrometer screw gauge	0.01mm	±0.01mm
Stopwatch, read by hand	0.01s	±0.2s, because the reaction time is coarser than the least count
Thermometer, 1°C divisions	1°C	±0.5°C, and ±1°C on a temperature change
Moving-coil meter	one small division	half a division, or the accuracy class printed on the meter face if that is larger
Triple-beam balance	0.1g	±0.1g
Metre bridge, balance length	1mm	±0.1cm

Two of those rows you should know by heart, because they are the two you are most likely to get wrong. The first is the stopwatch. Its least count is 0.01s, but nothing about your hand is good to a hundredth of a second, so the uncertainty is the one Chapter 2 gave for a reaction time, about 0.2s. That 0.2s covers the whole interval, the starting and the stopping together, so it is not doubled. Quote the hand, not the watch.

The second row is the metre rule. A length taken from one is the difference between two readings, and each of them carries its own uncertainty. The rod above is laid against the 10.0cm mark and its far end read at 24.7cm. Each reading is good to about half a millimetre. The length is 14.7cm, and the uncertainty in it is the two uncertainties added: ±0.1cm, not ±0.05cm. Whenever a quantity is the difference of two readings, its uncertainty is the sum of theirs.



10.0cm and 24.7cm, each read to ±0.05cm

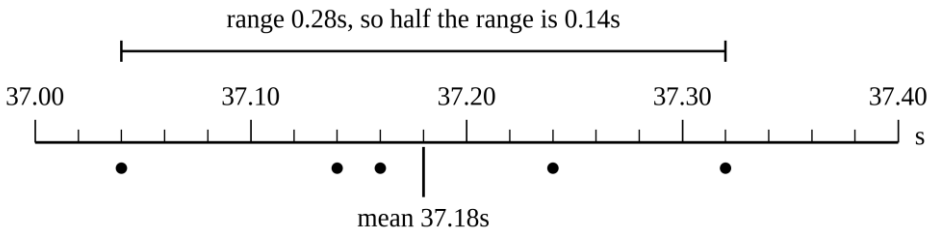
Repeated readings and the half-range

Where a reading can be taken again cheaply, take it again. Five timings of the same pendulum, over 20 oscillations each:

Attempt	t for 20 oscillations (s)
1	37.16
2	37.04
3	37.32
4	37.14
5	37.24

Two numbers come out of that. The **mean** is the value you carry forward. The **half-range**, which is half the distance from the smallest reading to the largest, is a fair estimate of the uncertainty you quote with that mean, and a deliberately generous one. Here the mean is 37.18s and the half-range is 0.14s, which sits comfortably

beside the 0.2s Chapter 2 allows for a reaction time. Where the two disagree, take the larger. Here that is the ±0.2s of reaction time, and the half-range of 0.14s is only a rough check on it.



What the five readings bought is not a smaller uncertainty but the right to trust the one you are quoting. Had they scattered over a whole second, the ±0.2s would have been a fiction, and the half-range would have been the honest estimate.

Now divide. The 20 oscillations took 37.18s, so the period is 1.859s, and the 0.2s of uncertainty is divided by 20 along with everything else:

$$\Delta T = \frac{0.2s}{20} = 0.01s$$

That single division is why the question tells you to time 20 oscillations and not one, and it is worth saying so in your report. The ±0.2s is the same whether you time one swing or 20: set against a single swing of 1.859s it is 11% of the reading, and set against 37.18s it is 0.54%. Dividing by 20 then carries that 0.54% into the period unchanged, because dividing by an exact number leaves a percentage alone. Repeating a reading reduces a random error; timing many oscillations is what makes the 0.2s of reaction time small.

The propagation of uncertainty

Your final result is rarely something you measured. It is something you computed from two or three things you measured, and each of them brings its own uncertainty into the answer. An uncertainty quoted in the unit of the measurement is an **absolute uncertainty**; the same uncertainty expressed as a fraction of the reading and multiplied by a hundred is a **percentage uncertainty**. Carrying them through a calculation is what propagation means, and five rules cover every calculation in this book.

When you	Do this with the uncertainties	Because
add or subtract	add the absolute uncertainties	two ends of a rule, each ±0.05cm, give a length ±0.1cm
multiply or divide	add the percentage uncertainties	an area from two lengths, each ±1%, is ±2%
raise to a power	multiply the percentage uncertainty by the power	a diameter ±2.63% makes an area ±5.26%
multiply by a constant	the percentage uncertainty does not change	halving a number halves its uncertainty too
take a mean of n readings	take half the range, or the instrument's own uncertainty if that is larger	the spread is what you saw

The second and third rules are the ones that matter, and they work in percentages rather than in units, so convert first. A resistance recorded as 4.30Ω ± 0.05Ω has a percentage uncertainty of

$$\frac{0.05\Omega}{4.30\Omega} \times 100\% = 1.16\%$$

and it is that 1.16% that gets added to the others. Notice what the third rule does to a squared quantity. A diameter known to 2.63% gives an area known to 5.26%, because the area carries the diameter twice. A quantity that is cubed carries it three times. That is what tells you, in advance, which measurement will dominate the uncertainty in the answer.

A worked answer with its uncertainty

The pendulum above had an effective length of 0.8625m: 85.0cm of thread read on a metre rule, and 2.50cm for the diameter of the bob read on a vernier calliper, so 1.25cm for its radius. The rule contributes ±0.1cm and the calliper ±0.005cm on that radius, so the length is uncertain by ±0.105cm, which is ±0.1cm to one significant figure, or 0.001m. Its period was 1.859s, uncertain by ±0.01s. The relation, rearranged for g, is

$$g=\frac{4\pi^2 L}{T^2}$$

so, putting the numbers in with π as 3.14,

$$g=\frac{4\times(3.14)^2\times 0.8625\text{m}}{(1.859\text{s})^2}=9.84\text{ms}^{-2}$$

Now the uncertainty. The length enters to the first power, so its percentage goes in once; the period is squared, so its percentage goes in twice:

$$\frac{\Delta L}{L}=\frac{0.001\text{m}}{0.8625\text{m}}\times 100\%=0.12\%$$

$$\frac{\Delta T}{T}=\frac{0.01\text{s}}{1.859\text{s}}\times 100\%=0.54\%$$

$$\frac{\Delta g}{g}=0.12\%+2\times 0.54\%=1.2\%$$

and 1.2% of 9.84 is 0.12, so

$$g=(9.8\pm 0.1)\text{ms}^{-2}$$

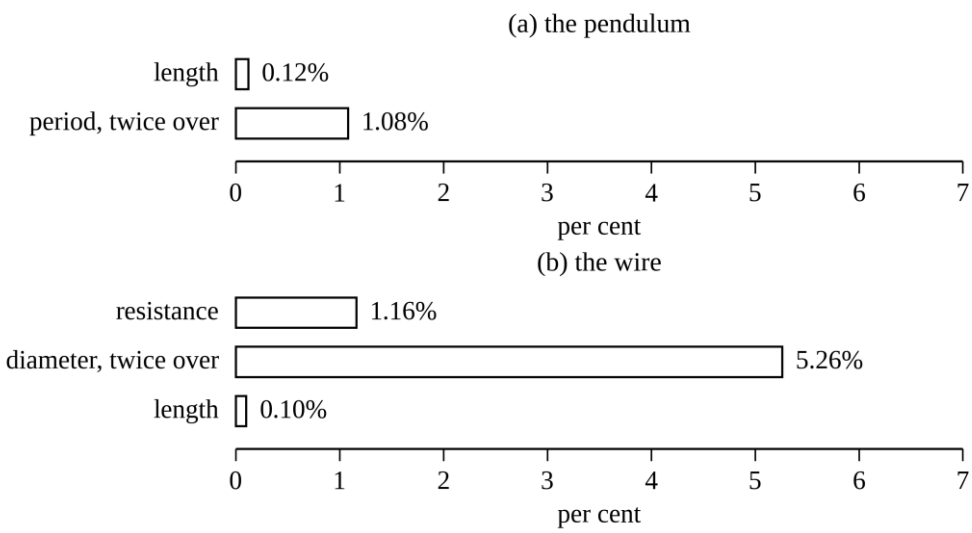
Three things about that last line. The uncertainty is given to one significant figure, because it is an estimate and an estimate will not support two. The result is then rounded to the same decimal place as the uncertainty, which is what turns 9.84 into 9.8: it would be absurd to write a hundredths figure you have just admitted you cannot trust. And the unit is written once, outside the bracket, because it belongs to both numbers.

One more thing about the arithmetic that produced it. The period went in as 1.859s, with all three of its figures, and the rounding was done once, at the end. Round in the middle of a calculation and you lose in the last figure exactly what the rounding was meant to protect.

That is also the answer to Kipanga. A calculator will offer you 9.84282. The uncertainty tells you where to stop, and it stops you at the second figure.

Which measurement dominates the uncertainty

Look at the two contributions in that sum, and at the three in the wire experiment below it, where the resistance of a metre of wire was 4.30Ω ± 0.05Ω on a metre bridge, its diameter 0.38mm ± 0.01mm on a micrometer, and its length 1.000m ± 0.001m.



In (a) the timing contributes nine times what the length does. Measuring that length to the nearest tenth of a millimetre with a travelling microscope would improve your value of g by nothing you could notice. Timing 40 oscillations instead of 20 would halve the largest term in the sum. In (b) the diameter contributes more than four

times the other two together, because it is small, because the micrometer is working at the limit of its resolution, and above all because it is squared.

So the resistivity of the wire comes out as

$$\rho=\frac{RA}{l}=\frac{R\pi d^2}{4l}=4.87\times10^{-7}\Omega m$$

with an uncertainty of 1.16% + 2 × 2.63% + 0.10% = 6.5%, which is 0.32 × $10^{-7}\Omega$m, so

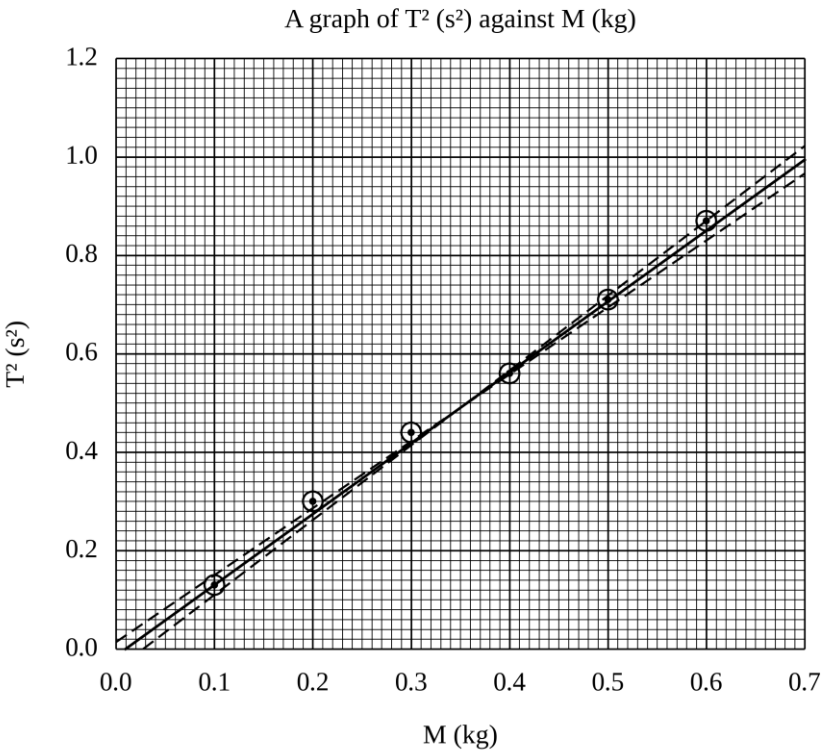
$$\rho=(4.9\pm0.3)\times10^{-7}\Omega m$$

Improve the dominant term, and leave the others alone. Spend ten minutes on the length and one minute on the diameter and you have spent the ten minutes for nothing. Work out the percentages first, and let them tell you where to be careful.

The uncertainty in a slope

Most results in the practical paper come from a slope, so the uncertainty in the slope is the uncertainty in the result. You do not need statistics for it. You need a ruler and two more lines.

Draw your best line as Chapter 3 taught. Then draw the steepest and the shallowest lines you could reasonably defend, the maximum and minimum slope lines, letting each end of the line move by about one small square, which is as closely as an eye can fix a line against a row of points. Take the slope of each, and half the difference between the two extremes is the uncertainty in the best one.



A mass hanging on a spring was set oscillating, and T^2 plotted against M. The best line has a slope of $1.44s^2kg^{-1}$. The steepest and shallowest lines drawn through the same points have slopes of $1.52s^2kg^{-1}$ and $1.36s^2kg^{-1}$, so

$$\Delta_{slope}=\frac{1.52-1.36}{2}=0.08s^2kg^{-1}$$

which is 5.6% of 1.44. The theory gives $T^2 = 4\pi^2M/k$, so the spring constant is $4\pi^2$ divided by the slope:

$$k=\frac{4\pi^2}{1.44s^2kg^{-1}}=27.4Nm^{-1}$$

and because k is inversely proportional to the slope, its percentage uncertainty is the slope's: 5.6% of 27.4 is 1.5, which as an uncertainty rounds up to 2, so k = (27 ± 2)Nm^{-1}. That is a spring constant of twenty-seven newtons per metre, known to within two, which is an honest description of what a sheet of graph paper and a stopwatch can tell you about a spring.

Comparing your result with the accepted value

Do not confuse two percentages that sound alike.

Your uncertainty is what your own apparatus allows: 1.2% for the pendulum above. It says how far your result could move even if you did everything right. **The percentage difference** is how far your result actually sits from the accepted value:

$$\text{percentage difference} = \frac{|\text{your value-accepted value}|}{\text{accepted value}} \times 100\%$$

For g = 9.84ms⁻² against the accepted 9.81ms⁻² that is 0.3%. Use the unrounded 9.84 here and not the 9.8 you quoted: the rounding belongs to the final statement, not to the arithmetic that leads to it. The sentence a marker wants sets the two side by side:

The value obtained differs from the accepted value by 0.3%, which is well inside the 1.2% uncertainty of the measurement, so the two agree.

That is the strongest closing sentence available to you, and it is worth more than a value that happens to be nearer. If the difference is larger than your uncertainty, say so, and name the systematic error you think is responsible. Write that the difference is three times your uncertainty and name an unchecked zero, and you have shown more than the candidate who landed on 9.81 by luck.

Sources of error and precautions

Almost every practical question ends by asking for sources of error or precautions, and almost every candidate throws the marks away. The fault is always the same: the answer could have been written without doing the experiment.

A source of error earns its mark when it has three parts. **Name the measurement**, not the person. **Say which way it pushes the reading**, or say that it pushes either way. **Pair it with the precaution** that reduces it. In one line:

Variation in reaction time at starting and stopping the stopwatch, which may push a recorded time either way; reduced by timing 20 oscillations instead of one and by starting and stopping as the bob passes the centre of its swing.

Compare that with Kipanga's "human error". The first names a measurement, a direction and a cure; the second names no measurement, no direction and no cure. Here is a starting bank, one line for each family you will meet.

Experiment	The error worth naming	The precaution that answers it
Simple pendulum	reaction time at starting and stopping the watch, which may push a time either way	time 20 oscillations rather than one, and start and stop on a swing through the middle
Cooling curve	the thermometer lags behind the water, so a falling temperature reads high	stir continuously and read on the move, at the instant the clock reaches the mark
Metre bridge	contact resistance at the crocodile clips and the terminals, which adds to whichever resistance it sits beside and shifts the balance point	clean the ends of the wire and tighten every terminal, then interchange the two resistances, balance again and take the mean of the two balance lengths
Method of mixtures	heat lost on the way across and to the room during the run, both of which make the final temperature too low	carry the solid across quickly, lag the calorimeter and start the water a little below room temperature
Young's modulus by cantilever	the thickness enters as a cube, so its percentage uncertainty is tripled and swamps everything else	measure the thickness with a micrometer at five places along the rule and take the mean
Resonance tube	the end correction, which the simple formula ignores, so that every resonance length comes out short	plot the resonance length against 1/f and read the end correction from the intercept rather than guessing it

Use that table with care. The errors in it are the ones worth naming, which means the ones that are large enough to matter for that experiment: do not offer the wind from an open window when the thickness of a rule enters as a cube. And a precaution is something you did, not an attitude you held. The table above uses the bare imperative for shortness; in your own report write what was done: "the water was stirred continuously", not "be careful with the thermometer". "Be careful" is not a precaution.

Examiner's eye

- Quote an uncertainty to one significant figure, and round the result to match it. An answer of 9.84282 ± 0.12 is wrong twice in one line.
- Percentages add when you multiply or divide, and are multiplied by the power when you raise to a power. Absolute uncertainties add when you add or subtract. Convert to percentages only for the first kind, and convert back at the end.
- A quantity that is squared carries its percentage twice, and one that is cubed carries it three times. Most of the marks lost in this part of the paper are lost by forgetting that factor.
- Never write "human error", "instrumental error" or "parallax" on its own. Name the measurement, the direction and the cure.
- A precaution must answer the error you named. A pair that do not match earns nothing, however sensible each half sounds.
- If the question asks for two sources of error, give two, not four. Four sources of error, none of them properly named, read as guessing, and the marker takes the first two.

Making the measurement count

Practical tip 1: *Work out the percentage uncertainties before you take a single reading, and you will know which measurement to spend your time on.*

You can do it from the apparatus list alone. Take the cantilever, where the thickness of the rule enters as a cube. A micrometer on a rule 4.00mm thick gives 0.25%, tripled to 0.75%, and you may forget it; a metre rule on that same thickness is a difference of two readings, so ±0.1cm, which is 25%, tripled to 75%, and the experiment is dead before it starts. The depression is the other trap. It is also a difference of two readings on a millimetre scale, so it carries ±0.1cm, and a depression of only half a centimetre is uncertain by 20%. Load the rule until the depression is large enough to be worth reading. Before you sit down you know which instrument has to come out of the drawer, and where your first five minutes go.

Practical tip 2: *An uncertainty is an estimate, so treat it like one. Two significant figures in an uncertainty is a claim nobody can support.*

The percentages in the middle of a calculation may carry two or three figures, because dropping them early makes the sum wrong. The final uncertainty carries one, rounded in the ordinary way: 0.118 becomes 0.1, and 1.53 becomes 2, not 1.5. Where an uncertainty falls exactly between two figures, round it up, because an understated uncertainty is the one that misleads.

Practical tip 3: *A systematic error will not show in your scatter, so it will not show in your uncertainty either. Only something from outside your own readings can find it.*

Five timings that agree to a few hundredths tell you the random error is small. They tell you nothing whatever about a watch that starts late every time. Three things will catch one. A comparison with the accepted value, where a difference much larger than your own uncertainty is the fingerprint. An intercept that theory says should be zero and is not, which is how the end correction of a resonance tube is found. And a reversal: read the rule from the other end, or swap the two resistances in the bridge and balance again. Name the systematic error you suspect. It is worth more than a page that pretends there is none.

Drills

Work each one on paper before you read the answer printed beneath it.

Drill 1

A rod is laid on a metre rule with one end against the 15.0cm mark and the other end read at 41.6cm. Give the length of the rod and its uncertainty, and say why the uncertainty is not ±0.05cm.

Answer

The length is 41.6 − 15.0 = 26.6cm.

Each end is read against an engraved millimetre mark, so each reading is good to about half a millimetre, ±0.05cm.

A length is the difference of two readings, and the rule for a difference is that the absolute uncertainties add: 0.05 + 0.05 = 0.1cm.

Accept: 26.6cm ± 0.1cm, with the reason that both ends were read.

Drill 2

A candidate writes "sources of error: human error, parallax error, and the apparatus was not accurate". Replace the first of the three with a source of error worth naming in a cooling experiment, with its precaution, and say what is wrong with the other two.

Answer

The thermometer lags behind the water it is in, so a falling temperature is read a little high; reduced by stirring continuously and by reading at the instant the clock reaches the mark rather than afterwards.

"Parallax error" names no scale and no direction. The scale of a thermometer is engraved on the front of the stem and the thread lies behind it, so an eye held above the thread sights it against a higher division and every temperature reads a little high; the precaution is to read with the eye level with the top of the thread.

"The apparatus was not accurate" is not a source of error at all. It names nothing, it cannot be reduced, and it would fit any experiment ever done.

Accept: an error that is genuinely large in a cooling experiment, named with a direction and paired with a precaution that answers it, and both of the other two criticised.

Drill 3

A quantity is found from $Q = ab^2/c$. The percentage uncertainties are 1.0% in a, 3.0% in b and 2.0% in c. Find the percentage uncertainty in Q, and say which measurement is worth improving.

Answer

Multiplying and dividing means the percentages add, and b is squared, so it goes in twice:

$$\frac{\Delta Q}{Q} = 1.0\% + 2 \times 3.0\% + 2.0\% = 9.0\%$$

The b term contributes 6.0 of the 9.0, so b is the one to improve. Halving the uncertainty in b takes the total to 6.0%; halving the uncertainty in a takes it only to 8.5%.

Accept: 9.0%, with b named.

Drill 4

A stopwatch reading to 0.01s is used to time 20 oscillations, and the reading is 31.46s. Give the period and its uncertainty, and state the percentage uncertainty in the period.

Answer

The period is 31.46 ÷ 20 = 1.573s.

The uncertainty is the reaction time, not the least count of the watch: about 0.2s on the total, which the division by 20 turns into 0.2 ÷ 20 = 0.01s.

$$\frac{0.01\text{s}}{1.573\text{s}} \times 100\% = 0.64\%$$

The uncertainty sits in the second decimal place, so the calculator's third figure has to go: the period is quoted as 1.57s.

Accept: 1.57s ± 0.01s, and 0.6%. Note that ±0.01s is also the least count of the watch, so the right figure can be reached by the wrong route; the working, 0.2s ÷ 20, is what shows you did not take it.

Drill 5

Five readings of a resonance length are 24.6, 24.9, 24.5, 24.8 and 24.7cm. Give the mean and the uncertainty you would quote, and explain why five readings are better than one.

Answer

The mean is (24.6 + 24.9 + 24.5 + 24.8 + 24.7) ÷ 5 = 123.5 ÷ 5 = 24.7cm.

The range is 24.9 − 24.5 = 0.4cm, so half the range is 0.2cm, and that is the uncertainty to quote. It is larger than the ±0.1cm the rule alone would give, which tells you that finding the resonance is harder than reading the scale.

Repeating reduces a random error, because the pushes that make one reading high and another low tend to cancel in the mean. It does nothing at all to a systematic error, such as a tube whose zero is not where you think it is.

Accept: 24.7cm ± 0.2cm, with repetition named as the cure for random error only.

Drill 6

The diameter of a wire is measured with a micrometer as 0.42mm, and the least count is 0.01mm. The wire's cross-sectional area is computed from $A = \pi d^2/4$. Find the percentage uncertainty in the area, and say what you should do about it.

Answer

The uncertainty in the diameter is one least count, 0.01mm, so

$$\frac{\Delta d}{d} = \frac{0.01\text{mm}}{0.42\text{mm}} \times 100\% = 2.4\%$$

The diameter is squared, so the area carries it twice: 2 × 2.4% = 4.8%.

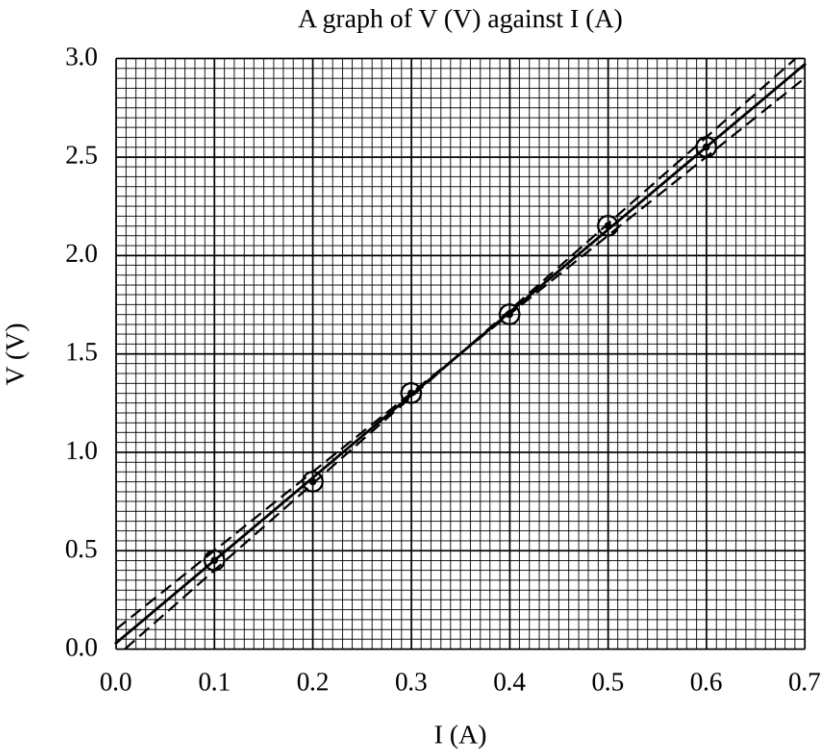
Nearly 5% from one reading of one place on the wire. Measure the diameter at five places along it, in two directions at each place because the wire is rarely round, and take the mean.

Be clear about what that buys. It cannot reduce the uncertainty below the micrometer's own ±0.01mm, which is a lower limit; what it buys is a fair figure for an uneven wire instead of one reading of one place, and a half-range that tells you at once whether the wire is uniform. If that half-range comes out larger than 0.01mm, quote the half-range. The 4.8% is the real message: the diameter is the experiment.

Accept: 4.8%, with the diameter named as the dominant term in the uncertainty, and repeated measurement along the wire as the response to it.

Drill 7

Using the graph below, read the slope of the best line, then the slopes of the steepest and shallowest lines drawn through the same points, and hence the resistance of the wire in that circuit with its uncertainty.



Answer

Reading the best line at I = 0.10A and I = 0.60A gives 0.45V and 2.55V, so

$$\text{slope}=\frac{2.55\text{V}-0.45\text{V}}{0.60\text{A}-0.10\text{A}}=4.20\text{VA}^{-1}$$

The two dashed lines are the steepest and the shallowest that can be defended, each end having been moved one small square. Read them at the same two currents. The steeper gives 0.40V and 2.60V, so its slope is 4.40VA^{-1}; the shallower gives 0.50V and 2.50V, so its slope is 4.00VA^{-1}.

The uncertainty in the slope is half the difference between them: $(4.40 - 4.00) \div 2 = 0.20\text{VA}^{-1}$.

A slope of V against I is a resistance, so $R = (4.2 \pm 0.2)\Omega$, which is 4.8%.

Accept: 4.2Ω with an uncertainty between 0.15Ω and 0.25Ω.

Drill 8

You obtain $g = 9.62\text{ms}^{-2}$ with an uncertainty of 1.5%. Write the closing comparison, and say what it tells you.

Answer

The percentage difference from the accepted value is

$$\frac{9.81-9.62}{9.81}\times 100\%=1.9\%$$

The value obtained differs from the accepted value by 1.9%, which is larger than the 1.5% uncertainty of the measurement, so the two do not agree, and a systematic error is likely.

The difference being larger than the uncertainty is the point. Random errors are already inside the 1.5%, so something is pushing every reading the same way: a length measured from the worn end of the rule, or a watch stopped late, would both do it.

Accept: 1.9%, named as larger than the uncertainty, with a systematic error suspected.

Drill 9

The period of a pendulum, found by timing 10 oscillations with a hand-operated stopwatch, is 2.00s ± 0.02s, and its length is 1.000m ± 0.005m. Without computing g, state the percentage uncertainty in g, say which of the two measurements you should improve first, and state what timing twice as many oscillations would do to the total.

Answer

The period is squared in $g = 4\pi^2L/T^2$, so its percentage goes in twice:

$$\frac{\Delta T}{T}=1.0\% \quad \text{and} \quad \frac{\Delta L}{L}=0.5\%$$

$$\frac{\Delta g}{g}=0.5\%+2\times1.0\%=2.5\%$$

The timing contributes 2.0 of the 2.5, so the period is the one to improve. Timing twice as many oscillations halves its contribution and takes the total from 2.5% to 1.5%. Halving the uncertainty in the length would take it only to 2.25%.

Accept: 2.5%, with the period named.

Drill 10

A calculator returns 0.0004736m for a length whose uncertainty works out at 3.4%. Write the answer as it should appear.

Answer

3.4% of 0.0004736 is 0.0000161, which to one significant figure is 0.00002.

The answer is then rounded to the same place as the uncertainty, so it becomes 0.00047.

Written in standard form, which is what a quantity this size asks for,

$$(4.7\pm0.2)\times10^{-4}\text{m}$$

Accept: $(4.7 \pm 0.2) \times 10^{-4}$m, or 0.00047m ± 0.00002m.

Chapter 6

REPORT WRITING AND THE UNSEEN QUESTION

The invigilator finished reading the instructions, and the room went quiet. One bench at a time, it stopped being quiet.

On every bench stood a retort stand. A plastic ruler was clamped in each one, and from the end of the ruler hung a thread with a mass on it. Nobody had seen this arrangement before. Somewhere behind Kipute, **Kipanga** said, to nobody in particular, *"We did not do this one."*

Kipute read the question twice. Then she turned the paper over and wrote the following lines on the back:

> *Something oscillates: the mass. Two things pull it back: the thread and the ruler. So T^2 against L, and it will not go through the origin. The intercept is the ruler.*

She had not touched the apparatus, and the thinking was already finished. Everything after that was arithmetic.

Mr Akilikubwa was not in the room. He had said the useful thing three weeks earlier, in a lesson that had annoyed everybody. *"I am not going to teach you every experiment. There are more experiments than there are days. I am going to teach you four questions, and after that it will not matter what they put on the bench."*

This chapter is those four questions, and before them the sentences that frame every answer you write: the aim, the account of what was done, the meaning of a slope, the meaning of an intercept, and the sentence that closes the answer. Those sentences carry marks in every question of every paper, they take twenty seconds each, and they are thrown away more often than any graph.

The five sentences the paper keeps asking for

A table and a graph are the visible part of an answer. Around them sits a frame of prose, and the frame is marked. Five sentences do nearly all of it, and each has a fixed shape. Learn the shape and you will never sit looking at a blank line.

- stating the aim: what the experiment determines;
- describing what was done: what was set up, varied and measured;
- what the slope means;
- what the intercept means;
- the closing comment: how your result stands against the value it should have.

None of the five needs a fresh idea. All five can be written before you know a single reading, and three of them can be written before you touch the apparatus.

Stating the aim

The aim is the quantity you finish with. It is not the thing you did on the way to it. Three answers to the same question. You could have written any of the three after a good experiment:

- "To plot a graph of T^2 against L." That is a step. A step is not a purpose.
- "To investigate the simple pendulum." Investigate what about it? Nothing is named.
- *"To determine the acceleration due to gravity, g, using a simple pendulum."* A quantity, its symbol, and the method. Full marks.

The shape is always the same: to determine the quantity, with its symbol, by the method named. Where the question opens "You are required to determine …", the aim is that phrase turned round, and you may copy it. Where the question ends "Suggest the aim of the experiment", look at the last thing it asked you to calculate. That is the aim, every time.

Describing what was done

The question is written in the imperative, because it is telling you what to do: tie the thread, displace the bob, record the time. Your answer is not. Your answer is the record of an experiment that has already happened, so it goes into the past tense, and into the passive, and it never says I or we.

Three sentences are usually enough: what was set up, what was measured and with what, and what was repeated.

The pendulum was suspended from a split cork held in a clamp, and its length was measured with a metre rule from the underside of the cork to the centre of the bob. The bob was displaced through a small angle and released, and the time for twenty complete oscillations was recorded with a stopwatch. The procedure was repeated for six lengths between 0.250m and 1.000m.

Notice what is not there. No apparatus list, because the question printed one. No account of the reading that went wrong and was taken again, because a report records what was done, not what was suffered. And no instruction to the reader: "be careful to start the watch as the bob passes the centre" belongs among the precautions, not here.

What a slope means

"State the physical meaning of the slope" is the most reliable question in the paper, and the one you are most likely to leave blank when the graph is unfamiliar. It should never be blank. Work out the unit of the slope first, and the unit will tell you what the slope is.

The unit of a slope is the unit of the vertical axis divided by the unit of the horizontal axis. A slope of V against I is measured in volts per ampere, which is an ohm, and the only resistance in that circuit that the graph could be showing you is the internal resistance of the cell. You have the answer without touching the theory.

What is plotted	The slope is	So the slope means
T^2 against L, simple pendulum	$4\pi^2/g$, in s^2m^{-1}	$4\pi^2$ divided by the acceleration due to gravity
T^2 against M, mass on a spring	$4\pi^2/k$, in s^2kg^{-1}	$4\pi^2$ divided by the spring constant of the spring
V against I, cell on load	$-r$, in Ω	the internal resistance of the cell, negative because V falls as I rises
R against L, uniform wire	ρ/A, in Ωm^{-1}	the resistance of one metre of the wire
$\log(\theta - \theta r)$ against t, cooling body	$-k/2.303$, in min^{-1}	the cooling constant divided by 2.303, negative because the body cools
L against 1/f, resonance tube	$v/4$, in ms^{-1}	a quarter of the speed of sound in the air of the tube
1/v against 1/u, converging lens	-1, no unit	nothing at all: it is -1 for every lens. Here the intercept carries the physics
a^2 against T^2, torsion rod	$c/8\pi^2m$, in m^2s^{-2}	the torsion constant of the wire divided by $8\pi^2m$, where m is the mass of one rider

The last row but one is worth a moment. A slope that is the same number for every apparatus in the world carries no information, and saying so is the correct answer, not a failure to find one.

What an intercept means

An intercept is what the vertical quantity would be if the horizontal one were zero. Every intercept you meet is one of three things, and naming which one is most of the answer.

- **Required, and zero.** The theory says the line goes through the origin, and it does. That confirms the law, so write that it does.
- **Required, and not zero.** The intercept is a physical quantity in its own right, and the question will want it: an end correction, an emf, an effective length, the period of the other half of a hybrid oscillator.
- **Not allowed, and there anyway.** That is a systematic error, and it is measurable. Chapter 2 put it in one line: a zero error moves an intercept but leaves a slope alone. Name it, and if you can, say how big it was.

What is plotted	The intercept is	So the intercept means
T^2 against L, simple pendulum	zero, if all is well	the law confirmed. Write that the line passes through the origin, as the theory requires
1/v against 1/u, converging lens	1/f, in m^{-1}	the reciprocal of the focal length of the lens
V against I, cell on load	E, in V	the emf of the cell
L against 1/f, resonance tube	−e, in m	minus the end correction of the tube
T^2 against L, mass on a thread on a springy support	$4\pi^2M/k$, in s^2	the square of the period the mass would have on the support alone
a^2 against T^2, torsion rod	$-I_0/2m$, in m^2	minus I_0 divided by 2m. I_0 is the moment of inertia of the rod and riders about the wire with the riders at the axis, and m is one rider's mass

The closing comment

Chapter 5 gave the sentence and the arithmetic behind it, so this is only the reminder. Print the value with its unit and to a defensible number of figures. Work out the percentage difference from the accepted value. Set the two side by side, and give a verdict. If the difference is larger than your own uncertainty, say so and name the systematic error you suspect.

A separate question sometimes asks you to comment on the reliability of the results, which is not the same thing. Reliability is about the precision of the readings, not the accuracy of the result: how closely repeated timings agreed, and how well the points sat on the line. A set of points that lie on a straight line to within half a small square is a reliable set, whatever the final value turns out to be.

Examiner's eye

- The aim names a quantity. If your aim sentence contains the word graph, it is a method, not an aim.
- Past tense, passive, no I and no we. A report is a record, not a set of instructions.
- Never leave a physical meaning blank. Write the unit of the slope. The unit alone is worth something, and it usually hands you the rest.
- Do not describe apparatus the question has already described. The marks are for what you did with it.
- If the question gives you the relation, use the relation it gives. Marks are for applying the printed equation, never for producing a better one.
- An intercept that the theory says should be zero and is not is a finding, not an embarrassment. Report it.

Four questions for a question you have never seen

Now the four questions. Recent papers set apparatus that is in no textbook: a ruler used as a spring, a rod hung on a wire, a hotel with a hundred rooms and a heating problem. They are not testing whether you have met the apparatus. They are testing whether you can build the physics on the spot.

Four questions do it, in this order, and none of them needs the apparatus in your hands.

- **One.** What is oscillating, or what is the circuit? Name the thing that moves and the thing that carries the current. Everything else is scenery.
- **Two.** What pulls it back? Write the governing equation from the restoring mechanism, or from dimensions if the mechanism will not come.
- **Three.** What do you plot to get a straight line? Rearrange until the equation reads y = mx + c, and say at once what the slope and the intercept will mean.
- **Four.** What should the numbers be? Put the apparatus values in and predict the table before you measure it, exactly as in Chapter 4. Then you will know within a minute whether the bench is behaving.

Every oscillator is an inertia and a stiffness

Question two sounds like the hard one. It is not, because every oscillator in this paper, without exception, is the same oscillator wearing different clothes. Something has inertia and resists being accelerated. Something else is stiff and pulls it back. The period is

$$T=2\pi\sqrt{\frac{\text{inertia}}{\text{stiffness}}}$$

and the whole of question two is deciding what plays each part. Six families cover the paper.

The oscillator	The inertia is	The stiffness is	So the period is
simple pendulum	the bob, mass M	gravity, Mg/L	$2\pi\sqrt{(L/g)}$
mass on a spring	the mass M	the spring, k	$2\pi\sqrt{(M/k)}$
loaded cantilever	the mass M at the free end	the bending of the bar, $Ebd^3/4p^3$	$2\pi\sqrt{(4Mp^3/Ebd^3)}$
compound pendulum	the body, I about the pivot	gravity, Mgh	$2\pi\sqrt{(I/Mgh)}$
torsion pendulum	the body, I about the wire	the twist of the wire, $\pi\eta r^4/2L$	$2\pi\sqrt{(I/c)}$
bifilar suspension	the bar, I about the vertical	the threads, $mgd^2/4L$	$2\pi\sqrt{(4IL/mgd^2)}$

Two symbols in that table need saying out loud, because the formula changes if you take them to mean something else. In the compound pendulum row h is the distance from the pivot to the centre of mass. In the bifilar row d is the full separation of the two threads, with the centre of mass midway between them, and not the distance from the axis to one thread.

Read down the third column and you have the answer to almost any hybrid the paper can invent, because a hybrid is two of those stiffnesses acting on one inertia. When that happens, the two do not simply add. The mass has to move if either one gives way, so it is the compliances, the reciprocals of the stiffnesses, that add:

$$\frac{1}{k}=\frac{1}{k_1}+\frac{1}{k_2}$$

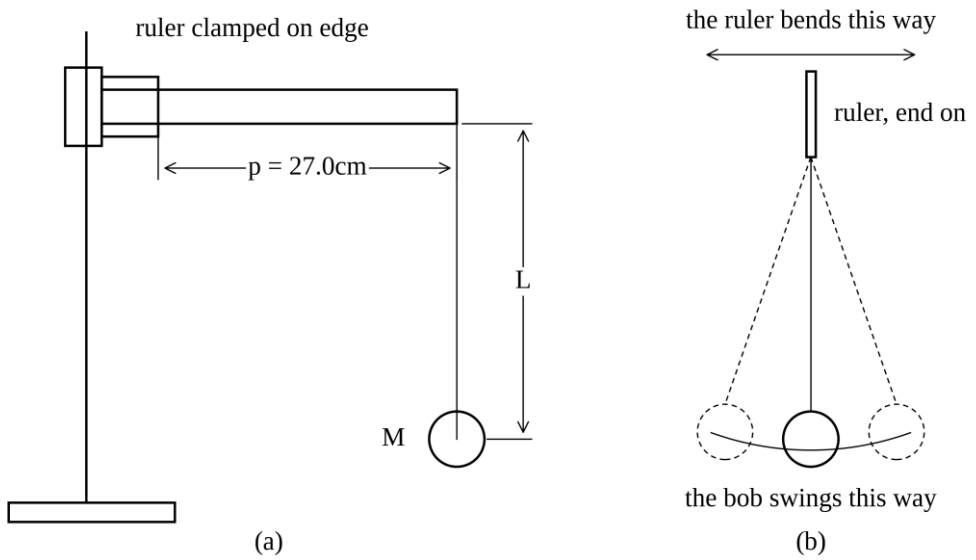
Multiply through by $4\pi^2M$ and the result comes out in a form worth remembering, because it turns a hard problem into an easy one:

$$T^2=T_1^2+T_2^2$$

Two restoring mechanisms in series add their periods in squares. That single line solves the question on Kipute's bench, and it is why she knew there would be an intercept before she had touched anything.

A ruler, a thread and a mass: two stiffnesses in series

A 30cm plastic ruler is clamped on edge in a retort stand so that a length p of it projects horizontally. A thread is taped to the free end, a mass M hangs from the thread, and the mass is set swinging in the vertical plane at right angles to the ruler, so that the ruler bends as the mass swings. The length L is measured from the underside of the ruler to the centre of the mass. The time for twenty oscillations is taken for five thread lengths.



Now the four questions.

One. The mass M is what oscillates. The ruler and the thread are the scenery, and the retort stand is not in the physics at all.

Two. Two things pull the mass back to the middle. The thread does, through gravity, with a stiffness Mg/L. The ruler does, by bending, with a stiffness k that belongs to the ruler and not to the mass. They are in series, because the mass cannot move sideways unless the thread swings or the ruler bends or both. So the periods add in squares, and since the pendulum contributes $4\pi^2L/g$ and the ruler $4\pi^2M/k$,

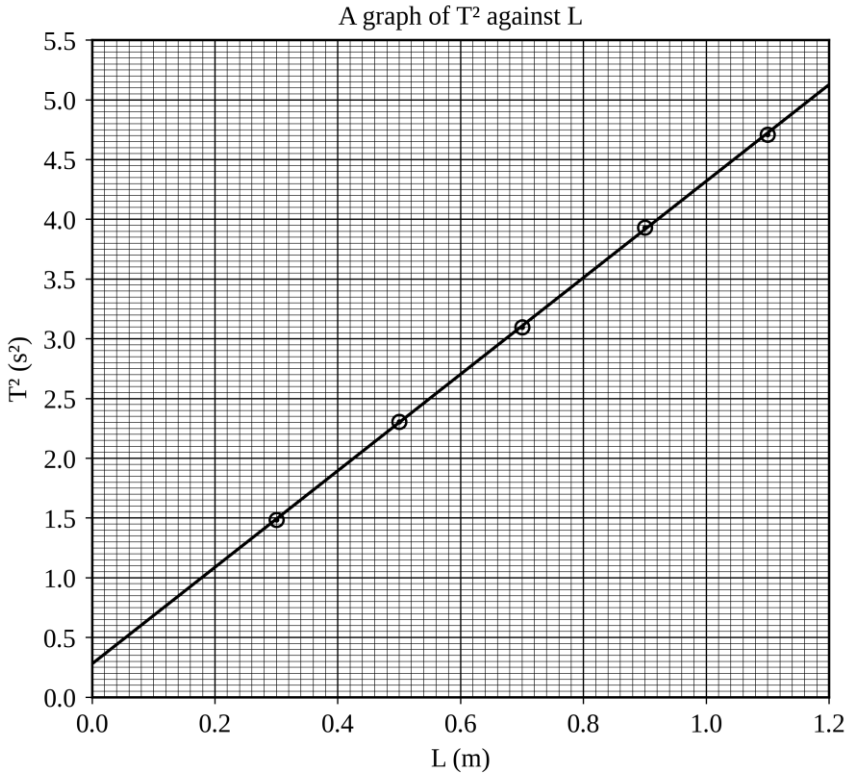
$$T^2 = \frac{4\pi^2}{g}L + \frac{4\pi^2 M}{k}$$

Three. That is already y = mx + c. Plot T^2 against L. The slope is $4\pi^2$ divided by g, and it has nothing to do with the ruler. The intercept is $4\pi^2M/k$, which is the square of the period the mass would have if the thread were shortened to nothing, leaving it swinging sideways on the ruler alone. Call that period Tc.

Four. With L about a metre the pendulum term alone gives T near 2s, so twenty oscillations will take about 40s. If the watch reads 4s or 400s, something is wrong with the apparatus and not with the theory.

The readings:

L (m)	t for 20 oscillations (s)	T (s)	T^2 (s^2)
1.100	43.40	2.170	4.709
0.900	39.66	1.983	3.932
0.700	35.20	1.760	3.098
0.500	30.38	1.519	2.307
0.300	24.36	1.218	1.484



The points lie on a good straight line which plainly does not pass through the origin. Taking the slope as Chapter 3 sets out, from a triangle spanning more than half the plotted range, between L = 0.400m and L = 0.900m, and reading both corners off the line and not off a point:

$$\text{slope}=\frac{3.92\text{s}^2\text{-}1.90\text{s}^2}{0.900\text{m-}0.400\text{m}}=4.04\text{s}^2\text{m}^{-1}$$

and since the slope is $4\pi^2/g$,

$$g=\frac{4\pi^2}{4.04\text{s}^2\text{m}^{-1}}=9.76\text{ms}^{-2}$$

which differs from the accepted 9.81ms^{-2} by 0.5%. The line meets the vertical axis at 0.280s^2, so Tc is 0.529s: left to itself on the ruler, with no thread at all, the mass would oscillate about twice a second.

That intercept is not a nuisance. It is the second half of the experiment. The stiffness of a bar of breadth b and thickness d, clamped so that a length p projects, is

$$k=\frac{\text{Ebd}^3}{4\text{p}^3}$$

and putting that into the intercept $4\pi^2\text{M/k}$ and rearranging for the Young modulus of the plastic gives

$$E=\frac{16\pi^2\text{Mp}^3}{\text{bd}^3\text{T}_c^2}$$

The ruler measures 2.80cm across on a vernier calliper and 3.00mm thick on a micrometer, which cannot span the breadth. The mass is 200g and p is 27.0cm, so

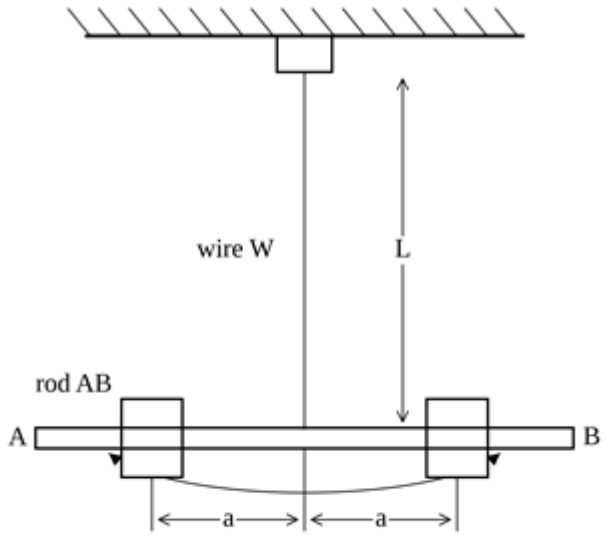
$$E=\frac{16\times3.14^2\times0.200\text{kg}\times(0.270\text{m})^3}{0.0280\text{m}\times(0.00300\text{m})^3\times0.280\text{s}^2}=2.93\times10^9\text{Pa}$$

About three thousand million pascals, which is right for the plastic a ruler is moulded from. One graph, one intercept, and a quantity nobody thought to ask for until the intercept refused to be zero.

A rod on a wire: an intercept worth having

A metal rod AB is hung at its centre from a wire W of length L, and two equal masses are clamped on the rod at a distance a either side of the wire. The rod is twisted a small angle in the horizontal plane and released, and

the time for ten oscillations is taken for five values of a. Only ten are timed here, and not twenty, because the period is long: ten of them already run to a minute, and the reaction time you met in Chapter 5 is about a third of one percent of that.



the rod is twisted in the horizontal plane

One. What oscillates is the rod together with its two riders, turning about the wire. Its inertia is not a mass but a moment of inertia, and it is not fixed: moving the riders outwards increases it.

Two. What pulls it back is the twist in the wire, whose torsional stiffness for a wire of radius r and length L is

$$c=\frac{\pi\eta r^4}{2L}$$

The moment of inertia is $I_0 + 2ma^2$, where I_0 covers the rod and the riders about their own centres and $2ma^2$ is the two riders treated as points at distance a. From $T = 2\pi\sqrt{(I/c)}$,

$$T^2=\frac{4\pi^2\left(I_0+2ma^2\right)}{c}$$

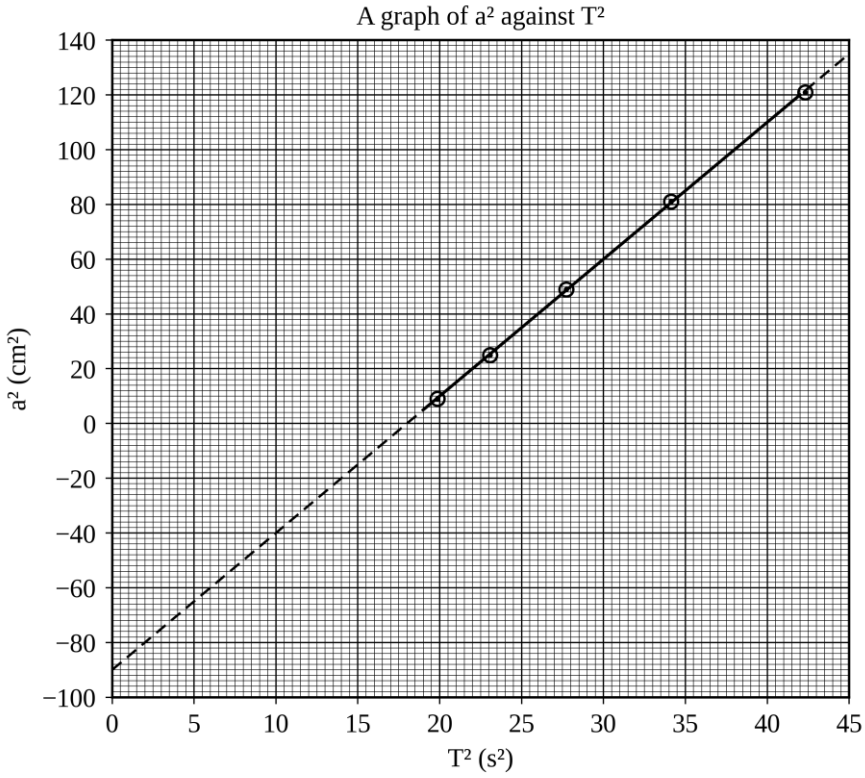
Three. Rearranging for a^2, which is the quantity that was set rather than measured:

$$a^2=\frac{c}{8\pi^2 m}T^2-\frac{I_0}{2m}$$

so a graph of a^2 against T^2 is a straight line of slope $c/8\pi^2m$ and intercept $-I_0/2m$. It looks the wrong way round, with the controlled quantity up the vertical axis. It is not wrong: a straight line is straight whichever way it is plotted, and this way the slope hands you the torsion constant and the intercept hands you the moment of inertia, both without a further rearrangement. What matters is that you say what your slope means.

Four. A steel wire of this thickness and a rod of this size give a period of a few seconds, so ten oscillations should take 45s to 70s.

a (cm)	a^2 (cm^2)	t for 10 oscillations (s)	T (s)	T^2 (s^2)
3.0	9.0	44.58	4.458	19.87
5.0	25.0	48.04	4.804	23.08
7.0	49.0	52.65	5.265	27.72
9.0	81.0	58.41	5.841	34.12
11.0	121.0	65.05	6.505	42.32



The horizontal axis has to start at zero here, because the intercept is wanted and the intercept lies at $T^2 = 0$. The points occupy the upper half of the sheet, and the line is carried back to the axis as a broken line: solid where you have evidence, broken beyond the last point, as Chapter 3 sets out.

$$\text{slope}=\frac{110.0\text{cm}^2\text{-}10.0\text{cm}^2}{40.0\text{s}^2\text{-}20.0\text{s}^2}=5.00\text{cm}^2\text{s}^{-2}$$

In SI that slope is $5.00\times10^{-4}\text{m}^2\text{s}^{-2}$, and each rider has a mass of 50.0g, so

$$c=8\pi^2\text{m}\times\text{slope}=8\times3.14^2\times0.0500\text{kg}\times5.00\times10^{-4}\text{m}^2\text{s}^{-2}=1.972\times10^{-3}\text{Nmrad}^{-1}$$

carrying four figures because the number is going straight into another calculation. The wire is 0.500m long and measures 0.600mm across on a micrometer, so its radius is 3.00×10^{-4}m and

$$\eta=\frac{2Lc}{\pi r^4}=\frac{2\times0.500\text{m}\times1.972\times10^{-3}\text{Nmrad}^{-1}}{3.14\times(3.00\times10^{-4}\text{m})^4}=7.75\times10^{10}\text{Pa}$$

The accepted shear modulus of steel is 8.0×10^{10}Pa, so the result is 3.1% low. A micrometer reading raised to the fourth power carries about 7% into the answer on its own, so 3.1% is comfortably inside the uncertainty of the measurement. The intercept is worth collecting too: it lies at -90.0cm^2, so

$$I_0=\text{-}2\text{m}\times\text{intercept}=\text{-}2\times0.0500\text{kg}\times(\text{-}90.0\times10^{-4}\text{m}^2)=9.00\times10^{-4}\text{kgm}^2$$

which is the moment of inertia the rod and riders would have with both riders pushed right up against the wire. Nobody measured it, and nobody could have measured it directly with the apparatus on the bench.

When the equation will not come: powers and logarithms

Sometimes the paper does not tell you the relation and you cannot derive it. It will then usually tell you the shape: T and L are related by $T = kL^n$, find k and n. Take logarithms of both sides:

$$\log T=n\log L+\log k$$

which is y = mx + c with log T up the vertical axis and log L along the horizontal. The slope is the power and the intercept is the logarithm of the constant. Nothing else is needed, and the method works whatever the power turns out to be.

Dimensions will usually tell you the answer before you plot anything. If T depends only on a length and on g, then seconds have to come out of metres and ms^{-2}, and the only way that happens is as the square root of a length divided by g. The power must be a half. A slope of 0.50 on log paper is then a confirmation rather than a discovery, and a slope of 0.80 is a warning that something in the apparatus is not what you assumed.

The intercept is worth having here too. If log k comes out at 0.303, then k is 2.01, in seconds per square root of a metre. Comparing $T = k\sqrt{L}$ with $T = 2\pi\sqrt{(L/g)}$ gives $k = 2\pi/\sqrt{g}$, so

$$g=\left(\frac{2\pi}{k}\right)^2=\left(\frac{2\times3.14}{2.01}\right)^2=9.76\text{ms}^{-2}$$

the same value the ruler and thread gave, from the same bench, by a different route.

Making the measurement count

Practical tip 1: *Two restoring mechanisms acting on one mass add the squares of their periods, never the periods themselves.*

The rule also tells you in advance which term will dominate. A stiff support contributes almost nothing and can be ignored; a floppy one can swamp the pendulum entirely. So read the intercept as a fraction of your readings: if it is a large fraction, the apparatus is telling you that the support, and not the thread, is doing most of the work, and the experiment is really a measurement of the support.

Practical tip 2: *Write the unit of a slope before you write its meaning. The unit is very nearly the answer.*

Doing this first also catches a mistake in the plotting, which is the reason to do it before anything else. If the unit of your slope turns out not to be the unit of anything, you have plotted the wrong pair of columns, and you have found that out while there is still time to plot them again.

Practical tip 3: *A power law becomes a straight line on logarithms, and dimensions tell you the power before you plot.*

Two cautions go with it. The logarithm of a quantity carrying a unit only means anything because the unit is understood to have been divided out first, so state the unit you used: a constant of 2.01 is a different number if L was in centimetres. And a logarithmic scale punishes a small reading badly, one small square covering a wider range of L at the top of the sheet than at the bottom, so keep every reading inside the same decade if the apparatus will let you.

Drills

Work each one on paper before you read the answer printed beneath it.

Drill 1

A question ends "(vi) Evaluate the shear modulus η of the wire W." A candidate writes as the aim: "The aim was to plot a graph of a^2 against T^2." Write a proper aim, and say what is wrong with the one given.

Answer

To determine the shear modulus, η, of the wire W by timing the torsional oscillations of a loaded rod suspended from it.

Plotting a graph is a step on the way. The aim is the quantity the experiment ends with, and the last thing the question asks for is that quantity.

Accept: an aim naming the shear modulus η of the wire as the quantity determined, with the criticism that a graph is a step and not a purpose.

Drill 2

Rewrite the following in the conventions of a report, and name the three faults in it. "First I tie the thread to the bob. Then take the metre rule and measure it. Displace the bob and start the stopwatch, and do not forget to count twenty swings."

Answer

The thread was tied to the bob and clamped between two cork pads. Its length was measured with a metre rule from the underside of the cork to the centre of the bob. The bob was displaced through a small angle and released, and the time for twenty complete oscillations was recorded with a stopwatch.

The faults: the present tense where the past is wanted; the first person and the imperative, neither of which belongs in a record; and an instruction to the reader, which records nothing that was done.

Accept: a past-tense, passive rewrite with no first person, and the three faults named.

Drill 3

A graph of R (Ω) against L (m) for a uniform wire is a straight line through the origin of slope $1.85\Omega m^{-1}$. State the physical meaning of the slope, and find the resistivity of the wire if its cross-sectional area is $2.65\times10^{-7}m^2$.

Answer

$R = \rho L/A$, so the slope is ρ/A, the resistance of one metre of the wire.

$\rho = \text{slope} \times A = 1.85\Omega m^{-1} \times 2.65\times10^{-7}m^2 = 4.90\times10^{-7}\Omega m$.

Accept: "the resistance of one metre of the wire", and $4.90\times10^{-7}\Omega m$.

Drill 4

Three graphs, three intercepts. For each, say whether the theory requires an intercept, and what this one means. (a) T^2 against L for a simple pendulum, intercept $+0.04s^2$. (b) L against 1/f for a resonance tube, intercept −0.008m. (c) V against I for a cell on load, intercept 1.52V.

Answer

(a) Not required. $T^2 = 4\pi^2L/g$ must pass through the origin, so every length was short by the same fixed amount Δ:

$$\Delta=\frac{g\times0.04s^2}{4\pi^2}=0.010m$$

which is close to the radius of the bob. The length was almost certainly read to the top of the bob and not to its centre.

(b) Required. $L + e = v/4f$, so the intercept is −e and the end correction of the tube is 0.008m.

(c) Required. $V = E - Ir$, so the intercept is the emf of the cell, 1.52V.

Accept: (a) not required, a systematic error of about 0.010m in the length, identified as reading to the top of the bob; (b) required, end correction 0.008m; (c) required, emf 1.52V.

Drill 5

A metal strip is clamped horizontally at one end, and a mass is fixed to the free end so that the strip vibrates up and down. Name the inertia, name the stiffness, and write the period.

Answer

The inertia is the mass M at the free end; the strip itself is light by comparison.

The stiffness is the bending of the strip, $k = Ebd^3/4p^3$ for a strip of breadth b and thickness d with a length p projecting.

$T = 2\pi\sqrt{(M/k)} = 2\pi\sqrt{(4Mp^3/Ebd^3)}$.

Accept: M named as the inertia, the bending of the strip named as the stiffness, and $T = 2\pi\sqrt{(4Mp^3/Ebd^3)}$.

Drill 6

You are told that $T = 2\pi\sqrt{((I + mr^2)/c)}$, where I, m and c are constants of the apparatus and r can be varied. What should you plot to obtain a straight line, and what will its slope and intercept give you?

Answer

Squaring,

$$T^2=\frac{4\pi^2m}{c}r^2+\frac{4\pi^2I}{c}$$

so plot T^2 against r^2. The slope is $4\pi^2m/c$, from which $c = 4\pi^2m \div \text{slope}$. The intercept is $4\pi^2I/c$, from which $I = c \times \text{intercept} \div 4\pi^2$.

Accept: T^2 against r^2, with c from the slope and I from the intercept.

Drill 7

A mass hangs on a thread from the end of a springy rod. On the rod alone, with no thread, the mass would oscillate with a period of 0.480s. On the thread alone, from a rigid support, it would oscillate with a period of 1.62s. Find the period of the two together.

Answer

The rod and the thread are two stiffnesses in series on one mass, so the periods add in squares:

$$T=\sqrt{(0.480s)^2+(1.62s)^2}=1.69s$$

Accept: 1.69s, obtained from $T^2 = T_1^2 + T_2^2$ and not from adding the periods.

Drill 8

Readings of T and L for an oscillator are plotted as log T against log L. The line has slope 0.50 and intercept 0.303. Write the relation between T and L, and hence find g.

Answer

log T = n log L + log k, so n = 0.50 and log k = 0.303, giving k = 2.01 and $T = 2.01\sqrt{L}$ with L in metres.

Dimensions require the power to be a half if T depends only on a length and on g, so the slope confirms the relation rather than discovering it.

Comparing $T = k\sqrt{L}$ with $T = 2\pi\sqrt{(L/g)}$ gives $k = 2\pi/\sqrt{g}$, so

$$g=\left(\frac{2\pi}{k}\right)^2=\left(\frac{2\times3.14}{2.01}\right)^2=9.76ms^{-2}$$

Accept: $T = 2.01\sqrt{L}$, and $g = 9.76ms^{-2}$.

Drill 9

A determination gives $\eta = 7.75\times10^{10}$Pa for a steel wire, and the uncertainty of the measurement works out at 7%. The accepted shear modulus of steel is 8.0×10^{10}Pa. Write the closing comment.

Answer

The percentage difference is (8.0 − 7.75) ÷ 8.0 × 100% = 3.1%.

The value obtained differs from the accepted value by 3.1%, which is inside the 7% uncertainty of the measurement, so the two agree.

Accept: 3.1%, named as smaller than the uncertainty, with the verdict that the two agree.

Drill 10

A U-tube of uniform bore contains a column of liquid of total length L. The liquid is displaced and released, and it oscillates. Work through the four questions, and say what you would plot to find g.

Answer

One. What oscillates is the whole column of liquid, of mass ρAL.

Two. If the liquid on one side rises by x, the other side falls by x, so the excess weight pulling it back is ρAg(2x) and the stiffness is 2ρAg. Hence

$$T=2\pi\sqrt{\frac{\rho AL}{2\rho Ag}}=2\pi\sqrt{\frac{L}{2g}}$$

and both the density and the bore have cancelled, which is the reason the experiment works with any liquid in any tube.

Three. Squaring,

$$T^2=\frac{2\pi^2}{g}L$$

so plot T^2 against L; the slope is $2\pi^2/g$ and $g = 2\pi^2 \div$ slope.

Four. With L about 0.400m the period is near 0.90s. A liquid column damps out quickly against the walls of the tube, so time ten oscillations rather than twenty and expect about 9s.

Accept: T^2 against L, slope $2\pi^2/g$, with the cancellation of ρ and A noted.

PART TWO

MECHANICS

Chapter 7

THE SIMPLE PENDULUM FAMILY

A bob on a thread is the cheapest apparatus in the laboratory, and it is also the one you are most likely to meet in the practical paper. Close to half of all Mechanics questions have been built on it. The reason is not sentiment. A pendulum takes a quantity you cannot weigh, the acceleration due to gravity, and turns it into two quantities you can measure with a metre rule and a stopwatch.

The family has eight varieties. On the page they look different; underneath they are almost the same experiment. Only two things change from one variety to the next: where the length is measured from, and what you plot against what. Learn the first two thoroughly and you will recognise the other six on sight.

The eight varieties at a glance

	What is different about it	What the graph gives you
7.1	Length measured to the centre of gravity of the bob	g, from a line through the origin
7.2	Length measured only to the point of attachment	g and the bob correction c
7.3	One thread, shortened by a measured amount each run	g, and the original length from the intercept
7.4	Length measured to a knot, with an unknown offset below it	g and the offset b
7.5	The law $T=kL^m$ tested, instead of the formula assumed	the exponent m and the constant k, and hence g
7.6	Length measured from a ring at the suspension, so the pivot is above the point measured from	g and the internal diameter of the ring
7.7	Point of suspension out of reach, so the height is measured from the bob	g and the height of the suspension
7.8	Mass varied, then swing angle, and the length relation tested instead of assumed	evidence that T depends on neither, and g from a line through the origin

Experiment 7.1 is worked below in full. Learn this one first. Every other variety in the chapter is this same experiment with a different definition of length, or a different pair of axes.

Experiment 7.1

Mr. Akilikubwa walked into the laboratory carrying a wall clock under one arm and wearing the face of a man who has been personally insulted.

"This clock," he said, setting it down on the bench, *"has lost eleven minutes since Monday. Eleven! At this rate it will arrive at Christmas sometime in January."*

Kipute leaned over and watched the little pendulum swinging behind the glass door. *"Sir, shall I carry it to the repair stall behind the bus stand?"*

"And pay two thousand shillings for something you are about to learn for free?" He opened the door, took hold of the small brass bob at the bottom of the pendulum, and slid it upwards by roughly the width of his thumbnail. *"There. The clock is cured. Do not clap yet, Kipute, because you cannot tell me why."*

Kipute opened her mouth. Then she closed it again.

"Exactly. So today you will find out. Tie this bob to a thread, count its swings, and before the lesson ends you will know why my thumbnail can repair a clock. You will also have weighed the pull of the entire Earth using a piece of string, which is not a bad morning's work."

Aim

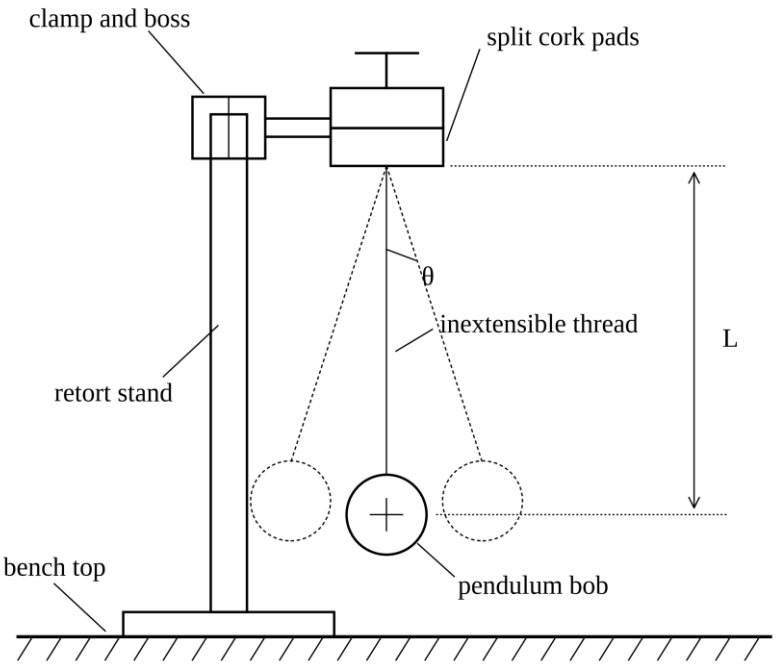
To determine the acceleration due to gravity, g, using a simple pendulum.

The same apparatus and the same readings will also answer two other aims that may be set from this rig: to show the relationship between the periodic time and the length of the pendulum, and to find the length of a pendulum whose periodic time is exactly two seconds.

Apparatus

Retort stand with clamp and boss, two split cork pads, a pendulum bob of brass about 25mm in diameter, inextensible thread about 110cm long, a metre rule, and a stopwatch reading to 0.01s.

Setup



Procedure

Proceed as follows:

(a) Tie the thread to the pendulum bob and grip its free end between the two cork pads in the clamp, so that the distance from the point of suspension to the centre of gravity of the bob is L = 1.00m.

(b) Displace the bob through a small angle, not more than about 5°, and release it so that it swings in one vertical plane.

(c) Find the time, t, for 20 complete oscillations. Start the stopwatch as the bob passes through its lowest point, and count from zero there.

(d) Draw the thread up through the cork pads so as to shorten the pendulum by 0.10m, then repeat (b) and (c).

(e) Continue until you have six readings, for L = 1.00, 0.90, 0.80, 0.70, 0.60 and 0.50m.

Questions

(i) Tabulate your results, including the values of T and T^2.

(ii) Plot a graph of L (m) against $T^2(s^2)$.

(iii) Determine the slope of your graph.

(iv) Use your slope to determine the value of the acceleration due to gravity, g.

(v) State the relationship between L and T^2 shown by your graph, and explain what the position of the line relative to the origin tells you about the way L was measured.

(vi) Use your graph to determine the length of a simple pendulum whose periodic time is exactly 2.00s.

(vii) A wall clock regulated by a pendulum is found to be gaining time. State, with a reason, whether its pendulum should be lengthened or shortened.

(viii) Give two sources of error in this experiment and two precautions taken against them.

Table of results

L **(m)**	t **for 20 oscillations (s)**	T=t/20 **(s)**	T^2 **(s^2)**
1.00	40.13	2.007	4.03
0.90	37.94	1.897	3.60
0.80	35.83	1.791	3.21
0.70	33.57	1.679	2.82
0.60	30.99	1.549	2.40
0.50	28.36	1.418	2.01

Making the measurement count

Practical tip 1: *The periodic time* T *of a simple pendulum, the length* L *measured to the centre of gravity of the bob, and the acceleration due to gravity* g *are connected by a single equation. Everything else in this experiment follows from it.*

$$T=2\pi\sqrt{\frac{L}{g}}$$

Your stopwatch never reads T directly. It reads t, the time for n complete oscillations, so the quantity you actually record is

$$t=nT=2\pi n\sqrt{\frac{L}{g}}$$

Put n = 20, π = 3.14 and the accepted value g = $9.81ms^{-2}$ into that, and you have one arithmetic recipe that tells you what each timing ought to come to before you take it:

$$t=40\pi\sqrt{\frac{L}{g}}=125.6\sqrt{\frac{L}{9.81ms^{-2}}}$$

Try it on the first length. When L = 1.00m,

$$t=125.6\sqrt{\frac{1.00m}{9.81ms^{-2}}}=125.6\times0.3193s=40.10s$$

and the stopwatch gave 40.13s. Repeat that substitution for the other five lengths, and you can set the expected times beside the times you recorded:

L **(m)**	t **expected (s)**	t **recorded (s)**	**difference (s)**	**difference (%)**
1.00	40.10	40.13	+0.03	+0.07
0.90	38.04	37.94	−0.10	−0.26
0.80	35.87	35.83	−0.04	−0.11
0.70	33.55	33.57	+0.02	+0.06
0.60	31.06	30.99	−0.07	−0.23
0.50	28.36	28.36	0.00	0.00

To conclude: every recorded time sits within 0.26% of the value the theory predicts, and the differences fall on both sides rather than all one way. Agreement of that order is well inside what a hand-held stopwatch can resolve, so the results are reliable, and the experiment is valid and successful.

Practical tip 2: *Once you have the recipe, one genuine reading can rebuild the rest of the table. This is what lets you rehearse the whole question at home, where there is no laboratory.*

Suppose the stopwatch failed after the first run. Divide the one reading you did get by the value the recipe expects:

$$\frac{40.13\text{s}}{40.10\text{s}}=1.0007$$

Now multiply every remaining expected time by that single ratio. The whole t column comes back to within 0.13s, which is closer than two careful hands agree on a repeat. One anchor reading corrects the systematic part of your timing. The two-anchor version, which also corrects an unknown apparatus constant, is drilled in Chapter 4.

Practical tip 3: *In this experiment the metre rule is read to the centre of gravity of the bob, so the radius of the bob is already inside your measurement. Read it only to the top of the bob and the effective length becomes* $L + c$*, where* c *is the radius of the bob, and your graph picks up an intercept.*

That is the whole difference between this experiment and Experiment 7.2, so settle it now. Quoting an intercept that your measurement never contained is one of the easiest marks in this family to throw away.

Answers to the questions

(i) See the table above. Notice the discipline of the columns: t to 0.01s, T to 0.001s, T^2 to 0.01s^2, and the units written in the headings only, never beside every entry.

(ii) Square the governing equation and make L the subject:

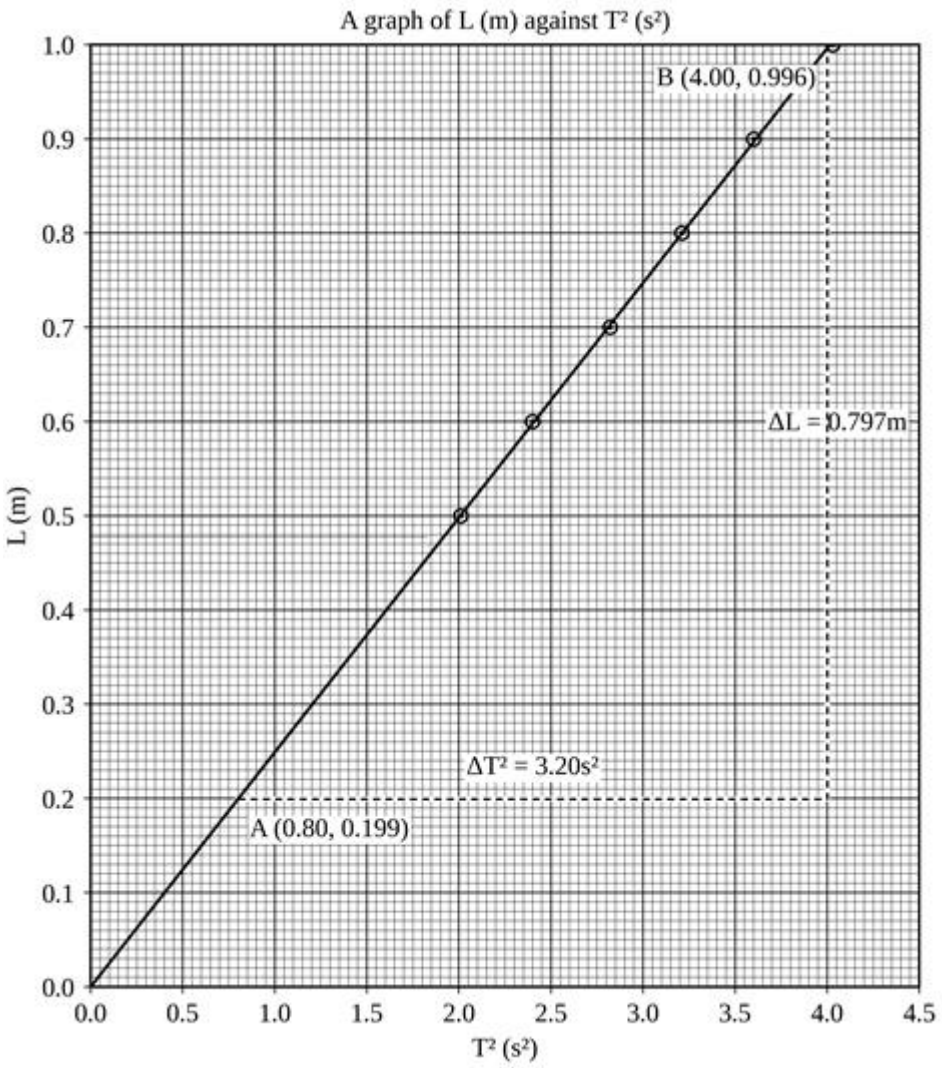
$$T^2=\frac{4\pi^2 L}{g}$$

$$L=\left(\frac{g}{4\pi^2}\right)T^2$$

That is the form $y=mx$, with L on the vertical axis and T^2 on the horizontal axis, and with no constant term. Your graph is therefore a straight line through the origin whose slope is

$$\text{slope}=\frac{g}{4\pi^2}$$

Give it the title *A graph of* $L(\text{m})$ *against* $T^2(\text{s}^2)$. Put L on the vertical axis at 2cm to 0.1m, running from 0 to 1.00m, and T^2 on the horizontal axis at 2cm to 0.5s^2, running from 0 to 4.50s^2. Both axes must start at zero here, because part (v) asks you what happens at the origin. Plot the six points, then draw one best straight line with the points balanced about it.



(iii) Choose two points that lie on your line and are well apart. Taking A ($0.80s^2$, 0.199m) and B ($4.00s^2$, 0.996m):

$$\text{slope}=\frac{0.996\text{m}-0.199\text{m}}{4.00\text{s}^2-0.80\text{s}^2}=\frac{0.797\text{m}}{3.20\text{s}^2}=0.249\text{ms}^{-2}$$

(iv) The slope is equal to g divided by $4\pi^2$, so

$$g=4\pi^2\times\text{slope}=4\times(3.14)^2\times0.249\text{ms}^{-2}=9.82\text{ms}^{-2}$$

Compare that with the accepted value, 9.81ms^{-2}. Your result is high by 0.01ms^{-2}, which is 0.1%, so it is a good result and your comment should say so plainly.

(v) The graph is a straight line passing through the origin, so L is directly proportional to T^2.

Because there is no intercept, no correction had to be added to the length you measured: L was read to the centre of gravity of the bob, and is therefore already the effective length of the pendulum. Had you read it only to the point of attachment, the line would have cut the L axis below the origin, at minus the radius of the bob.

(vi) A periodic time of T = 2.00s means $T^2 = 4.00s^2$. Read your best line at that value, or use the equation of the line:

$$L=0.249\text{ms}^{-2}\times4.00\text{s}^2=0.996\text{m}\approx1.00\text{m}$$

A pendulum that beats seconds is therefore almost exactly one metre long. That is why the cases of grandfather clocks are built to the height they are.

(vii) It should be **lengthened**. A clock that gains is running fast, so its pendulum is completing each swing in too short a time. Since T is proportional to the square root of L, you increase the period by increasing the length, and you do that by lowering the bob on its rod. Mr. Akilikubwa's clock had the opposite fault, which is why he raised his bob instead.

(viii) Take one precaution with each source of error, and name both specifically. A vague answer such as "human error" earns nothing.

- The reaction time of your hand in starting and stopping the watch, which shortens every recorded time. Timing 20 complete oscillations instead of one divides its effect on T by twenty, and starting the count as the bob passes its lowest point, where it is moving fastest, makes the instant of crossing easiest to judge.
- Parallax in reading the metre rule against the thread. Hold the rule vertical and close beside the thread, with your eye level with the mark you are reading.

Two further pairs held in reserve: air resistance gradually damping the swing, so keep the swing small and time the earliest oscillations; and the bob drifting into a conical swing instead of a plane one, so release it without a sideways push and start again if the path opens out.

Examiner's eye

- Units belong in the column headings, never beside every entry, and the number of decimal places must not wander down a column. Inconsistent decimals in the t column are the commonest table fault of all.
- When a question turns on where your line meets an axis, that axis must start at zero. An intercept claimed from a graph with a suppressed zero, and no explicit extrapolation, earns nothing. Neither does a graph so cramped that the line crosses only one corner of the sheet. Choose scales that satisfy both demands at once.
- The slope is not a bare number. Quote it to three significant figures with its units, read it from a triangle spanning more than half your line, and show on the graph the two coordinates you used.
- Part (v) wants a sentence, not a symbol. Name the relationship, which is direct proportionality, then say what the missing intercept tells you about the measurement. "The line passes through the origin" on its own is a description, not an answer.
- Check the size of your answer before you write the final line. You multiply the slope by $4\pi^2$; you do not divide by it. An answer near 0.006ms^{-2} or near 390ms^{-2} is not a value of g, and saying so in your comment costs nothing.

Practice 7.1

On Saturday **Kipute** found her small cousin **Neema** at the well, hauling the bucket half way up and then letting it swing on its rope, purely because that was more entertaining than carrying water home.

"You are wasting the whole morning," Kipute said.

"I am not wasting it. I am timing it," said **Neema.** "Full bucket, empty bucket, the swing always takes the same time. I have counted twice."

Kipute opened her mouth to argue, remembered **Mr. Akilikubwa's** thumbnail, and went to fetch a stopwatch instead. Her cousin had stumbled onto something worth measuring properly, although the water still had to be carried home afterwards.

Procedure

Proceed as follows:

(a) Suspend the bob from the clamp so that the length from the point of suspension to the centre of the bob is L = 90cm, measuring L in centimetres.

(b) Set the bob swinging through a small angle and find the time, t, for 30 complete oscillations.

(c) Reduce L by 10cm and repeat, continuing until you have six readings, the last of them at L = 40cm.

Questions

(i) Tabulate your results, including a column for t^2.

(ii) Plot a graph of $t^2(s^2)$ against L (cm).

(iii) Determine the slope of your graph and state its units.

(iv) Show that the slope of such a graph is equal to $4\pi^2n^2$ divided by g, where n is the number of oscillations timed, and hence determine the acceleration due to gravity in ms^{-2}.

(v) Use your graph to determine the time that 30 complete oscillations would take for a pendulum of length L = 55cm.

(vi) Neema repeats the whole experiment with a bucket twice as heavy. State, with a reason, how her graph would compare with yours.

(vii) Give one source of error in this experiment and state the precaution you would take against it.

Experiment 7.2

Kipanga finished before anybody else, which was usual, and put his hand up, which was also usual.

"Sir, my value of g is 9.4. The experiment is wrong."

Mr. Akilikubwa came and looked at the graph without saying anything for a while. It was a good graph. The points sat on the line, the line was ruled, the axes were labelled. It simply did not go through the origin, and **Kipanga** had dragged it down until it did.

"Your experiment is not wrong. Your line has been bullied. Where did you put the end of your metre rule?"

"At the cork, sir. And the other end at the bob."

"At the bob, or at the middle of the bob?"

Kipanga looked at his bob, which was a large one, the size of a small plum, and said nothing at all for some time. Then he said a word that **Mr. Akilikubwa** pretended not to hear.

That missing half-a-bob is the whole of this experiment. Measured to the top of the bob, every length is short by the same amount, and a fixed shortfall cannot bend a straight line; it can only slide it. So the graph keeps its slope and gains an intercept, and the intercept hands you back the very length you failed to measure.

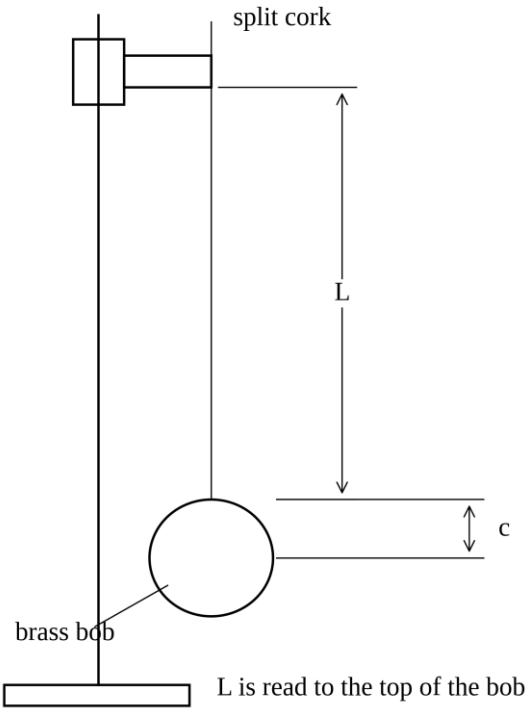
Aim

To determine the acceleration due to gravity, g, and the radius of the pendulum bob, by timing a simple pendulum whose length is measured to the top of the bob.

Apparatus

A large brass bob of about 4cm diameter, thread, a split cork, a retort stand with a boss and clamp, a metre rule, a vernier calliper and a stopwatch.

Setup



The thread is clamped between the two halves of a split cork so that it cannot slip, and the bob hangs free of the bench. The length L is read with the metre rule from the underside of the cork to the top of the bob, and deliberately not to its centre.

Procedure

Proceed as follows:

(a) Clamp the thread in the split cork so that the length L, measured from the underside of the cork to the top of the bob, is 0.900m.

(b) Displace the bob through a small angle, not more than about five degrees, and release it so that it swings in one vertical plane.

(c) Allow two or three swings to settle, then start the stopwatch as the bob passes through the centre of its swing and record the time t for 20 complete oscillations.

(d) Repeat the procedure in 7.2 (a) to (c) for L=0.800m, 0.700m, 0.600m, 0.500m and 0.400m.

(e) Measure the diameter of the bob with the vernier calliper.

Questions

(i) Tabulate your results, including the values of L(m), t(s), T(s) and $T^2(s^2)$.

(ii) Plot a graph of L(cm) against $T^2(s^2)$. Start the T^2 axis at zero, and carry the L axis below zero, from about −10cm, so that the intercept can be read.

(iii) Determine the slope of your graph and state its units.

(iv) Given that the swinging length of the pendulum is L+c, where c is a constant, show that $L=(g/4\pi^2)T^2$-c, and hence determine g.

(v) Read the intercept on the L axis and state what the constant c represents. Compare it with half the diameter you measured in 7.2 (e).

(vi) A student proposes to repeat the experiment with a bob 1.0cm across, to save time. Explain, using your graph, why that would make part (v) much harder to answer.

Table of results

L (m)	t for 20 oscillations (s)	T (s)	T^2 (s^2)
0.900	38.43	1.922	3.694
0.800	36.28	1.814	3.291
0.700	34.17	1.709	2.921
0.600	31.71	1.586	2.515
0.500	28.87	1.444	2.085
0.400	25.94	1.297	1.682

The diameter of the bob, on the vernier calliper, is 4.00cm.

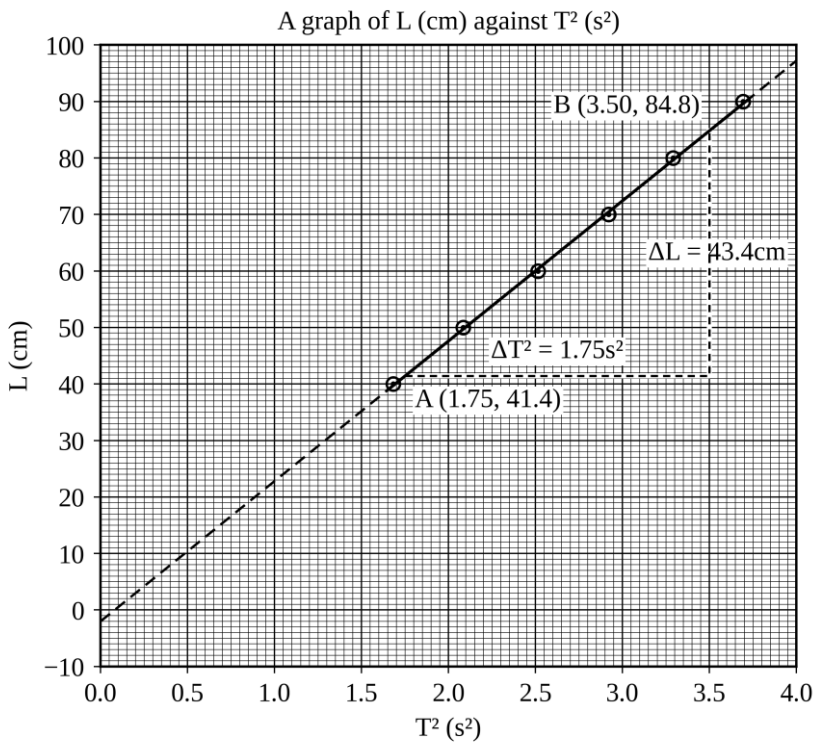
Making the measurement count

Practical tip 1: *When the question wants a correction, use the largest bob on the bench. An intercept is the least precise thing on a graph, so make the length you are reading off it as large as the apparatus will allow.*

The slope of a straight line is fixed by points that are far apart, so it is known well. The intercept is fixed by where the line arrives at an axis, which depends on the line's direction as well as its position, and a small twist of the ruler moves it. With a 4cm bob the intercept is about 2cm, or two whole small squares on the scale used here, and can be read. With a 1.0cm bob it would be 5mm, half a small square, which is the thickness of the line itself. The physics is the same; only the answer is unreadable.

Answers to the questions

(i) and (ii) The table is printed above. The T^2 axis begins at zero because the intercept is wanted and it lies at $T^2 = 0$, and the L axis is carried below zero because that is where the intercept turns out to be.



(iii) Taking a triangle spanning most of the readings, from $T^2=1.75s^2$ to $T^2=3.50s^2$, and reading both corners off the line and not off a point:

$$\text{slope}=\frac{84.8\text{cm}-41.4\text{cm}}{3.50\text{s}^2-1.75\text{s}^2}=24.8\text{cms}^{-2}$$

(iv) The swinging length is L+c, so

$$T=2\pi\sqrt{\frac{L+c}{g}}$$

Squaring both sides and making L the subject,

$$L=\frac{g}{4\pi^2}T^2-c$$

which is y=mx+c with L up the vertical axis, T^2 along the horizontal, a slope of $g/4\pi^2$ and an intercept of −c. The slope is 24.8cms^{-2}, which is 0.248ms^{-2}, so

$$g=4\pi^2\times\text{slope}=4\times3.14^2\times0.248\text{ms}^{-2}=9.78\text{ms}^{-2}$$

Against the accepted value 9.81ms^{-2} that is low by

$$\frac{9.81\text{ms}^{-2}-9.78\text{ms}^{-2}}{9.81\text{ms}^{-2}}\times100\%=0.3\%$$

a difference far inside anything a stopwatch and a metre rule could settle, and low in the direction a finite swing angle always pushes it.

(v) The line meets the L axis at −2.0cm, so c=2.0cm. It is the distance from the top of the bob to its centre, that is, the radius of the bob, which is the piece of the swinging length the metre rule never measured. Half the measured diameter is 2.00cm, so the graph has recovered a length nobody put into it. Do not expect this agreement every time: the intercept is the least certain part of a graph, and a centimetre either way here would still be a good result.

(vi) Nothing about the slope would change, because c does not appear in it, so g would come out just as well. But the intercept would fall from 2.0cm to about 0.5cm, which on the scale of this graph is half a small square and no larger than the width of the drawn line. The correction would be buried in the line and could not be read.

Examiner's eye

- A line that plainly misses the origin has not failed. Draw it where the points put it, and read the intercept. Forcing a line through the origin because the theory in your head says it should go there throws away the second half of the marks.
- Say which axis your intercept is on. "The intercept is -1.8" earns nothing; "the intercept on the L axis is -1.8cm" earns the mark and the unit that goes with it.
- c is a length and comes out in centimetres because L was plotted in centimetres. If you plot L in metres, the intercept is -0.018m. Both are right; a bare number is not.
- The slope here is $g/4\pi^2$ and not $4\pi^2/g$. Work out the unit of your slope before you decide which, and the unit will tell you: centimetres per second squared is an acceleration, so g is a multiple of the slope, not a divisor.

Practice 7.2

The carpenter at the end of the road hangs his plumb bob from a nail in the door frame when he wants a true vertical, and **Kipanga**, who had been sent to collect a stool, stood watching it swing for longer than the errand strictly required.

"It always takes the same time," he said, when he finally got home. *"Whether you push it hard or softly."*

"Then you have found out something worth measuring properly," said **Kipute**, *"and you have also lost my stool."*

Procedure

Proceed as follows:

(a) Suspend a pendulum bob from a thread held in a split cork, and measure the length L from the underside of the cork to the top of the bob. Begin with $L=0.850\text{m}$.

(b) Set the bob swinging through a small angle and record the time t for 20 complete oscillations.

(c) Repeat for $L=0.750\text{m}$, 0.650m, 0.550m, 0.450m and 0.350m.

Questions

(i) Tabulate L, t, T and T^2.

(ii) Plot a graph of $T^2(s^2)$ against L(m), and let the L axis begin at zero.

(iii) Determine the slope of the graph and state its units.

(iv) The swinging length is $L+c$. Show that $T^2=(4\pi^2/g)L+(4\pi^2/g)c$, and use the slope to find g.

(v) Use the intercept together with the slope to find c, and say why c cannot be read straight off the axis in this graph as it could when L was plotted against T^2.

(vi) Your value of g is above the accepted value. Name a systematic error that pushes g up rather than down, and say how you would test for it.

Experiment 7.3

There was one metre rule left in the cupboard and eleven benches wanting it.

"Then do not measure the length," said **Mr. Akilikubwa.** *"Measure how much you take away from it."*

He tied a bob to a long thread, clamped the thread in a cork, and made a pencil mark on the thread where it entered the cork. *"The length below the cork I do not know and will not measure. Now watch."* He drew the thread up through the cork until the mark had risen ten centimetres, and clamped it again. *"I have not measured the pendulum. I have measured what I removed from it, with the rule you all have to share."*

"But sir, we still do not know the length," said **Kipanga.**

"You will by lunchtime, and you will not have measured it. The graph will tell you what the pendulum was before you started shortening it."

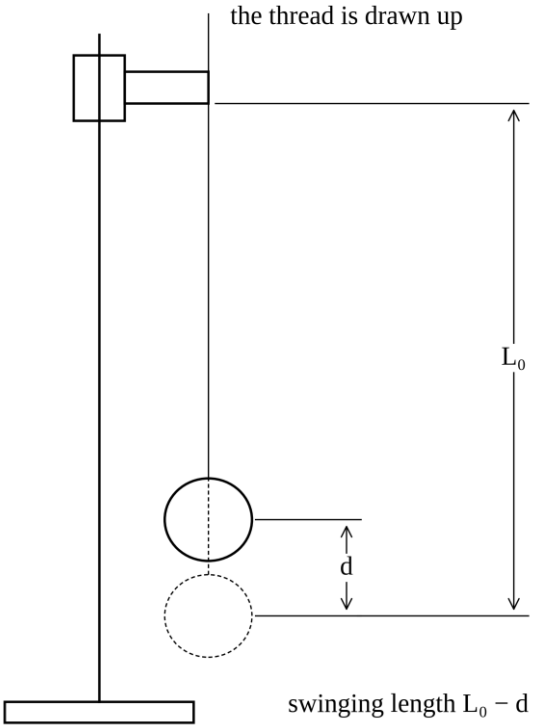
Aim

To determine the acceleration due to gravity, g, and the original length of a pendulum, by shortening it through measured distances.

Apparatus

A pendulum bob, a long thread, a split cork, a retort stand with a boss and clamp, a metre rule and a stopwatch.

Setup



The thread is clamped in the split cork with a good length hanging free, and a pencil mark is made on the thread level with the underside of the cork. Drawing the thread up through the cork by a distance d raises the mark by d and shortens the pendulum by d. Call the original length of thread below the cork, from the cork down to the point where the thread is tied to the bob, L_0. It is never measured. The swinging length also includes c, the short distance from that knot down to the centre of the bob, which is not measured either.

Procedure

Proceed as follows:

(a) Clamp the thread in the split cork so that the bob hangs about a metre below it, and mark the thread level with the underside of the cork.

(b) Draw the thread up through the cork until the mark stands 0.100m above the cork, and clamp it there. The pendulum has been shortened by d=0.100m.

(c) Set the bob swinging through a small angle and record the time t for 30 complete oscillations.

(d) Repeat the procedure in 7.3 (b) and (c) for d=0.200m, 0.300m, 0.400m, 0.500m and 0.600m.

Questions

(i) Tabulate your results, including the values of d(cm), t(s) and $t^2(s^2)$.

(ii) Plot a graph of d(cm) against $t^2(s^2)$.

(iii) Determine the slope of your graph and state its units.

(iv) The swinging length is L_0-d+c, where L_0 is the original length of thread below the cork and c is the distance from the knot at the bob to the centre of the bob. Show that $d=(L_0+c)-(g/4\pi^2n^2)t^2$, where n is the number of oscillations timed, and hence determine g.

(v) Read the intercept on the d axis and state what it represents.

(vi) Use your graph to predict the time for 30 oscillations when the pendulum has been shortened by 0.250m.

Table of results

d (cm)	t for 30 oscillations (s)	t^2 (s^2)
10.0	57.51	3307.4
20.0	54.27	2945.2
30.0	50.99	2600.0
40.0	47.28	2235.4
50.0	43.08	1855.9
60.0	38.64	1493.0

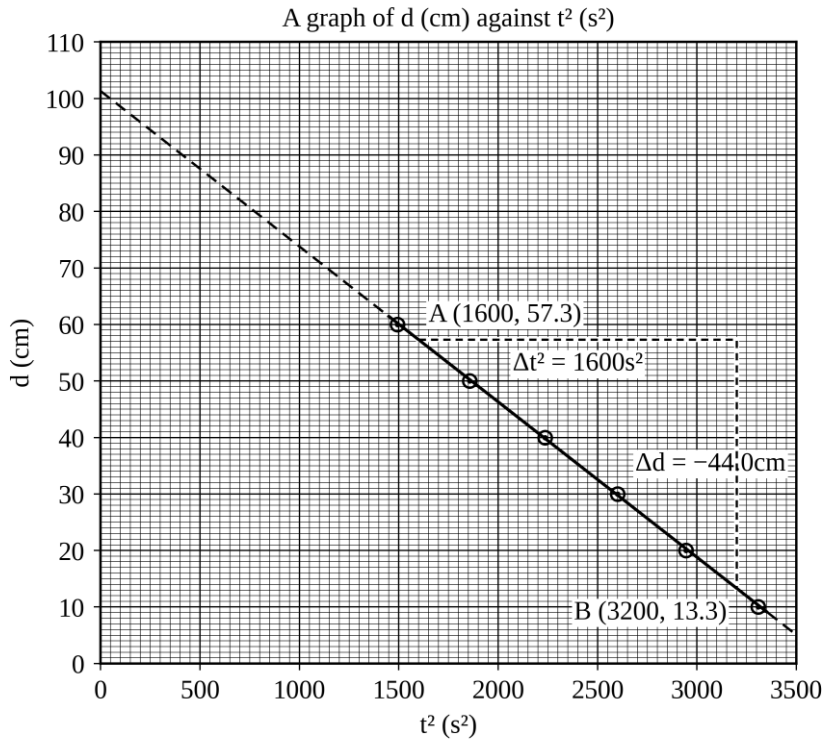
Making the measurement count

Practical tip 1: *When the question plots t^2 and not T^2, the slope carries the number of oscillations squared as well. Check the unit of your slope before you trust it.*

t is n times T, so t^2 is n^2 times T^2, and every slope taken against t^2 is smaller than the slope against T^2 by a factor of n^2. Here n is 30, so the factor is 900, and a candidate who forgets it reports a g of about $0.011 ms^{-2}$ and does not notice. The unit is the guard: the slope of a length against a time squared is an acceleration, so ask whether the number in front of it is a possible acceleration before writing it down. $0.0275 cms^{-2}$ is not. Nor is 900 times it, which is only $24.8 cms^{-2}$, the slope of Experiment 7.2. It is $4\pi^2 n^2$ times it, $976 cms^{-2}$, that is g, and multiplying by n^2 while forgetting the $4\pi^2$ is exactly the half-done conversion this check exists to catch.

Answers to the questions

(i) and (ii). The table is printed above. The d axis has to run well beyond the readings, because the intercept lies above every value of d that was measured, and it is the intercept the question wants.



(iii) Taking a triangle from $t^2=1600s^2$ to $t^2=3200s^2$, reading both corners off the line:

$$\text{slope}=\frac{13.3\text{cm}-57.3\text{cm}}{3200s^2-1600s^2}=-0.0275\text{cms}^{-2}$$

The slope is negative, as it must be: shortening the pendulum shortens the time, so d rises as t^2 falls.

(iv) The swinging length is L_0-d+c, so

$$T=2\pi\sqrt{\frac{L_0-d+c}{g}}$$

Squaring, replacing T by t/n, and making d the subject,

$$d=(L_0+c)-\frac{g}{4\pi^2n^2}t^2$$

which is a straight line of slope-$g/4\pi^2n^2$ and intercept L_0+c. Taking the size of the slope, 0.0275cms^{-2}, which is 0.000275ms^{-2},

$$g=4\pi^2n^2\times|\text{slope}|=4\times3.14^2\times30^2\times0.000275\text{ms}^{-2}=9.76\text{ms}^{-2}$$

low by 0.5% against the accepted 9.81ms^{-2}.

(v) The line meets the d axis at 101.3cm. That is L_0+c: the length the pendulum had before any thread was drawn up, measured from the cork to the centre of the bob. It is the distance you would have to shorten the pendulum by to reduce its period to zero, which is another way of saying it is the whole pendulum. Notice that it was obtained without ever laying the metre rule along the thread.

(vi) At d=25.0cm the line gives $t^2=2775s^2$, so t is the square root of that, about 52.7s. Reading the graph the other way round like this is worth practising: the line is a statement about every value of d, not only the six that were timed.

Examiner's eye

- A negative slope keeps its minus sign until you take its size deliberately and say that you are doing so. Writing $g=-9.76\text{ms}^{-2}$ and leaving it is a lost mark for a candidate who did everything else right.
- An intercept far outside the plotted points is normal here and is not a mistake. Draw the line broken where it leaves the readings, so the marker can see you know the difference between measuring and extrapolating.
- The quantity the intercept gives you is L_0+c, not L_0. If the question asks for L_0 alone, you must measure c or say that it is included.
- Read the number of oscillations off the question every time. Thirty here, twenty in the experiment before it. The n^2 in the slope is the commonest place in this chapter to throw a whole question away.

Practice 7.3

The rope swing over the river had been tied by somebody's uncle and was too long for the smaller children, so **Kipanga's** solution was to take three turns of the rope around the branch, which shortened it by an arm span and made him briefly popular.

"And now it swings faster," said **Kipute**, who had watched the whole operation. *"By exactly the amount you took off. You could work out how long it was to begin with, you know, without climbing up there again."*

Procedure

Proceed as follows:

(a) Clamp a thread in a split cork with the bob hanging about 0.9m below it, and mark the thread level with the underside of the cork.

(b) Draw the thread up until the mark stands 0.050m above the cork, and record the time t for 20 complete oscillations.

(c) Repeat for d=0.150m, 0.250m, 0.350m, 0.450m and 0.550m.

Questions

(i) Tabulate d(cm), t(s) and $t^2(s^2)$.

(ii) Plot a graph of d(cm) against $t^2(s^2)$.

(iii) Determine the slope and hence g.

(iv) State what the intercept on the d axis represents, and give its value.

(v) Comment on your value of g, and name the systematic error you think is responsible for the direction of the difference.

Experiment 7.4

The thread on Kipute's bench had been knotted in the middle by somebody the term before, and the knot would not come out.

"Then measure to the knot," said **Mr. Akilikubwa.**

"But the bob is below the knot, sir."

"It is. And you do not know how far below, and you are not going to find out, and it will not matter." He wrote a single line on the board and underlined it twice:

what you cannot measure, let the graph measure for you.

"Every length you write down will be wrong by the same amount. A wrongness that never changes is not a wrongness at all. It is a constant, and a constant is exactly the thing a straight line is best at handing back to you."

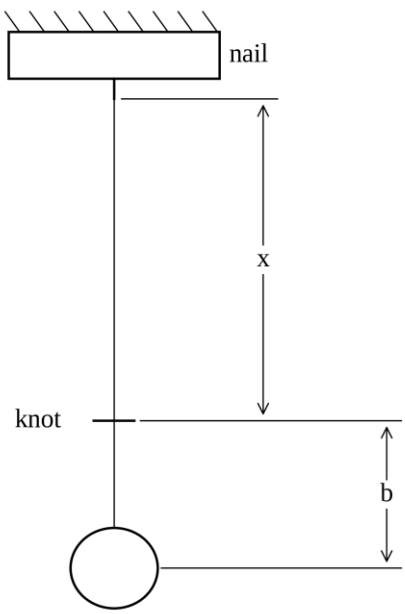
Aim

To determine the acceleration due to gravity, g, and an unmeasured offset in the length of a pendulum, from a graph of T^2 against the measured length.

Apparatus

A pendulum bob, a thread with a fixed knot in it, a nail fixed in a clamped wooden block, a metre rule and a stopwatch.

Setup



only x is measured

The thread hangs from the nail and the bob hangs below the knot. The distance x is read from the nail to the knot. The distance b, from the knot down to the centre of the bob, is fixed by the way the bob is tied on and is never measured, so the swinging length is x+b.

Procedure

Proceed as follows:

(a) Hang the thread over the nail so that the distance x from the nail to the knot is 0.800m, and hold it there with masking tape.

(b) Set the bob swinging through a small angle and record the time t for 20 complete oscillations.

(c) Repeat the procedure in 7.4 (a) and (b) for $x=0.700\text{m}$, 0.600m, 0.500m, 0.400m and 0.300m.

Questions

(i) Tabulate your results, including the values of $x(\text{m})$, $t(\text{s})$, $T(\text{s})$ and $T^2(\text{s}^2)$.

(ii) Plot a graph of $T^2(\text{s}^2)$ against $x(\text{m})$, extending the x axis far enough to the left of the origin to show where the line crosses it.

(iii) Determine the slope of your graph and state its units.

(iv) Given that the swinging length is $x+b$, show that $T^2=(4\pi^2/g)(x+b)$, and hence determine g.

(v) Determine b, and state what it is a distance between.

(vi) Explain why the value of g obtained from this graph does not depend on b at all.

Table of results

x (m)	t for 20 oscillations (s)	T (s)	T^2 (s^2)
0.800	37.68	1.884	3.549
0.700	35.66	1.783	3.179
0.600	33.32	1.666	2.776
0.500	30.62	1.531	2.344
0.400	28.05	1.403	1.968
0.300	24.82	1.241	1.540

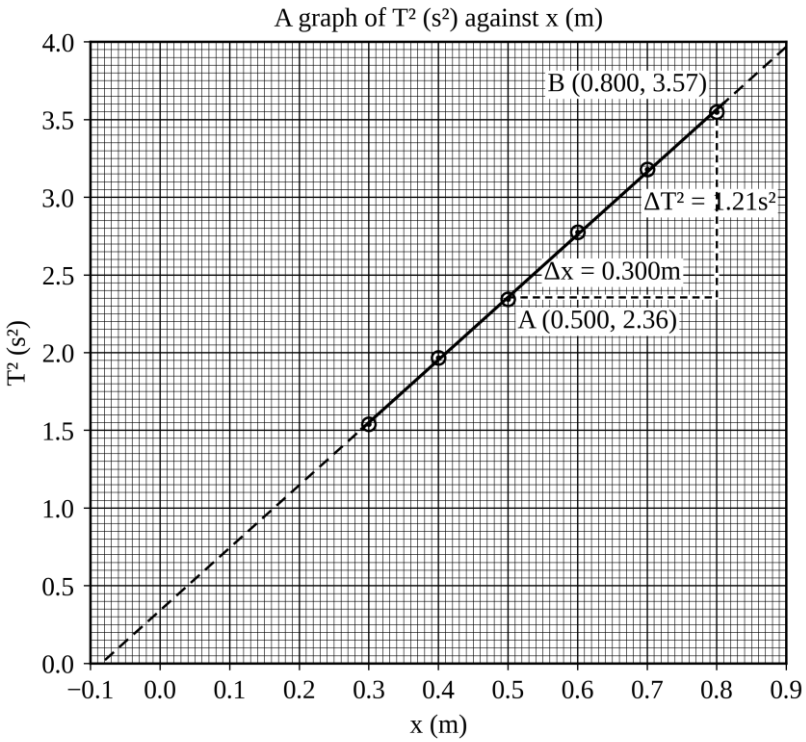
Making the measurement count

Practical tip 1: *A slope and an intercept from the same graph will give you a length you never measured, because dividing one by the other cancels* g.

The slope here is $4\pi^2/g$ and the intercept is $4\pi^2 b/g$. Neither can be turned into b on its own, because both contain g and g is not yet known. Their ratio contains nothing but b. That is a general move and it is worth recognising: whenever the slope and the intercept share a factor, dividing kills the factor and leaves the quantity behind it. It is how the end correction of a resonance tube is found, and how the effective mass of a spring is found, and it is why an intercept is worth reading even when the question has not asked for it.

Answers to the questions

(i) and (ii). The table is printed above. The x axis is carried back past zero, because the line crosses it at a negative value of x and that crossing is the answer to part (v).



(iii) Taking a triangle from x=0.500m to x=0.800m and reading both corners off the line:

$$\text{slope}=\frac{3.57\text{s}^2\text{-}2.36\text{s}^2}{0.800\text{m-}0.500\text{m}}=4.03\text{s}^2\text{m}^{-1}$$

(iv) The swinging length is x+b, so

$$T^2=\frac{4\pi^2}{g}(x+b)$$

which is a straight line of T^2 against x with slope$4\pi^2/g$, so

$$g=\frac{4\pi^2}{4.03\text{s}^2\text{m}^{-1}}=9.79\text{ms}^{-2}$$

which is 0.2% below the accepted 9.81ms^{-2}, about as close as a stopwatch and a metre rule will normally get.

(v) The intercept on the T^2 axis is 0.342s^2, and the intercept is $(4\pi^2/g)b$, which is the slope times b. So

$$b=\frac{\text{intercept}}{\text{slope}}=\frac{0.342\text{s}^2}{4.03\text{s}^2\text{m}^{-1}}=0.085\text{m}$$

and b is 8.5cm, the distance from the knot down to the centre of the bob. The same figure can be read straight off the graph as the point where the line crosses the x axis, at x=-0.085m, which says that a pendulum with the knot at the nail would still have 8.5cm of swinging length left in it.

(vi) Because b shifts the line sideways and nothing else. Adding a fixed length to every x moves every point the same distance to the right, and a set of points moved sideways as a body keeps the direction it had. The slope is unchanged, and the slope is the only thing g is taken from. This is the same argument as Experiment 7.2, where a fixed shortfall could not change the slope either.

Examiner's eye

- An axis may begin below zero when the question needs the crossing point, and that is one of the two situations where a scale not starting at zero is rewarded rather than penalised. Label the negative part of the axis properly. You will meet the other in Practice 7.8, where the whole variation is far too small to show on an axis that begins at zero.
- The intercept on the vertical axis and the intercept on the horizontal axis are different numbers with different units, and both are on this graph. Say which one you are quoting.

- b is a length, so the arithmetic that produces it must end in metres. Seconds squared divided by seconds squared per metre is metres; if your units do not do that, you have divided the wrong way round.
- Do not measure b with the rule to "check". The question has hidden it on purpose, and an answer that produces b from the ruler instead of from the graph answers a question nobody asked.

Practice 7.4

"I have done it exactly as we did in class," said **Kipanga**, *"and my offset has come out negative. A length cannot be negative. Therefore the experiment is wrong again."*

Kipute looked at where he had put the end of the metre rule, which was on the underneath of the bob, and decided to let him find it himself.

Procedure

Proceed as follows:

(a) Suspend a bob 4.00cm in diameter from a thread clamped in a split cork, and measure the distance x from the underside of the cork to the bottom of the bob. Begin with x=0.900m.

(b) Set the bob swinging through a small angle and record the time t for 20 complete oscillations.

(c) Repeat for x=0.800m, 0.700m, 0.600m, 0.500m and 0.400m.

Questions

(i) Tabulate x(cm), t(s), T(s) and $T^2(s^2)$.

(ii) Plot a graph of x(cm) against $T^2(s^2)$.

(iii) Determine the slope and hence g.

(iv) The swinging length is x+b. Read the intercept on the x axis and hence find b.

(v) Your value of b is negative. Explain what a negative b means here, and say what it would have to be numerically if the bob were a sphere.

(vi) State one change to the procedure that would make b come out positive.

Experiment 7.5

Mr. Akilikubwa wrote six characters on the board and then sat down, which he never did.

$$T=kL^m$$

"That is everything I am going to tell you today. There is a k and there is an m, and I know neither of them, and neither do you. Find them."

"Sir, the formula is two pi root L over g," said **Kipanga**, wounded.

"Today it is not. Today somebody has handed you an oscillator and told you only that the period goes as some power of the length. Perhaps it is a pendulum. Perhaps it is a wire, or a floating stick, or something nobody in this room has met. You are not being asked to remember. You are being asked to find out."

Kipute was already writing. She had seen it before, in Chapter 6: a power law becomes a straight line the moment you take logarithms of both sides, and a straight line gives up its two numbers without a fight.

Aim

To determine the power m and the constant k in the relation $T=kL^m$ for a simple pendulum, and hence to determine the acceleration due to gravity, g.

Apparatus

A pendulum bob, thread, a split cork, a retort stand with a boss and clamp, a metre rule, a stopwatch and mathematical tables or a calculator.

Setup

The arrangement is the plain one of Experiment 7.1: the thread is clamped between the halves of a split cork and the bob hangs free of the bench. The length L is measured from the underside of the cork, where the thread pivots, to the centre of the bob, so that no correction of the kind Experiments 7.2 to 7.4 recovered is hiding in these readings. That matters here more than usual: a constant added to every length would bend a logarithmic graph rather than shift it, and the slope would no longer be a clean half.

Procedure

Proceed as follows:

(a) Suspend the bob from a thread clamped in a split cork, and adjust the length L, from the underside of the cork to the centre of the bob, to 1.000m.

(b) Set the bob swinging through a small angle and record the time t for 20 complete oscillations.

(c) Repeat the procedure in 7.5 (a) and (b) for L=0.800m, 0.600m, 0.450m, 0.320m and 0.250m. Keep every length inside the same decade, that is, between 0.100m and 1.000m.

Questions

(i) Tabulate your results, including the values of L(m), t(s), T(s), logL and logT.

(ii) Plot a graph of logT against logL.

(iii) Determine the slope of your graph.

(iv) Given that $T=kL^m$, show that logT=mlogL+logk, and hence state the value of m.

(v) Read the intercept on the logT axis and determine k, giving its unit.

(vi) Use dimensions to show that if T depends only on L and on g then m must be a half, and comment on the agreement between that and your slope.

(vii) Given that $k=2\pi/\sqrt{g}$, determine g from your value of k.

Table of results

L (m)	t for 20 oscillations (s)	T (s)	log L	log T
1.000	40.09	2.005	0.000	0.302
0.800	35.85	1.793	-0.097	0.254
0.600	31.21	1.561	-0.222	0.193
0.450	26.86	1.343	-0.347	0.128
0.320	22.82	1.141	-0.495	0.057
0.250	20.00	1.000	-0.602	0.000

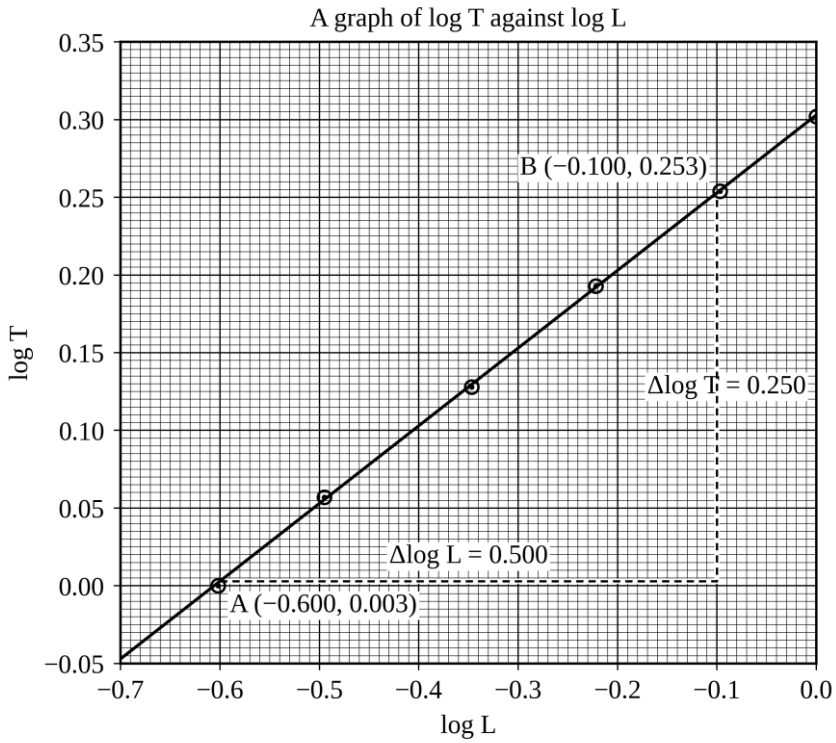
Making the measurement count

Practical tip 1: *Work out the power from dimensions before you plot anything. Then the slope confirms the law instead of discovering it, and a slope that misses is telling you the apparatus is not what you assumed.*

If T depends only on a length and on g, then seconds must be built out of metres and ms^{-2}. The only combination that gives a time is the square root of a length divided by g, so the power must be a half and nothing else is available. You know the answer to part (iv) before you take a single reading. What the graph is really for is the constant, and for the warning: a slope of 0.50 says the pendulum is behaving, and a slope of 0.80 says something in the apparatus is not a simple pendulum at all, which is a far more interesting result than a tidy one.

Answers to the questions

(i) and (ii). The table is printed above. Every value of logL is negative or zero, because every length is a metre or less, while logT stays positive because every period is a second or more. A negative logarithm is not a mistake.



(iii) Taking a triangle from logL=-0.600 to logL=-0.100:

$$m=\frac{0.253-0.003}{-0.100-(-0.600)}=\frac{0.250}{0.500}=0.500$$

(iv) Taking logarithms of $T=kL^m$, and using log(ab) = log a + logb and $\log(L^m)=m\log L$,

$$\log T=m\log L+\log k$$

which is y=mx+c with logT up the vertical axis and logL along the horizontal. The slope is therefore the power itself, and m = 0.500.

(v) The line meets the logT axis at 0.303, and that intercept is logk, so

$$k=\log^{-1}0.303=2.009$$

The unit is awkward and is worth stating properly. T is in seconds and L in metres, so k is in seconds per square root of a metre, $sm^{-1/2}$. Taking the logarithm of a quantity that carries a unit only means anything because the unit has been divided out first, which is why part (c) of the procedure insisted that every length be measured in metres.

(vi) Take dimensions, writing T for a time and L for a length. From $T=kL^m$,

$$[k]=\frac{[T]}{[L]^m}=TL^{-m}$$

and if k is built from g alone, its dimensions are those of a power of g, since a pure number in front of it carries none:

$$[k]=[g]^p=\left(LT^{-2}\right)^p=L^pT^{-2p}$$

The two expressions must agree index by index. Equating the index of T,

$$1=-2p,\text{ so } p=-\frac{1}{2}$$

and then the index of L,

$$-m=p,\text{ so } m=-p=\frac{1}{2}$$

Dimensions therefore fix the power at a half, and the slope at 0.500, before a single reading has been taken. The graph gives 0.500, so the pendulum is behaving as a simple pendulum should. What dimensions cannot fix is the pure number in front, because it has none of its own; that is why part (vii) has to be told that $k=2\pi/\sqrt{g}$.

(vii) Since $k=2\pi/\sqrt{g}$, squaring and rearranging gives

$$g=\left(\frac{2\pi}{k}\right)^2=\left(\frac{2\times3.14}{2.009}\right)^2=9.77ms^{-2}$$

which is 0.4% below the accepted $9.81 ms^{-2}$, and low in the direction a finite swing angle always pushes it.

Examiner's eye

- A logarithm has no unit, so neither axis on this graph carries one. Writing "logL(m)" is wrong. Say instead, in a note beside the table, that L was measured in metres.
- Keep every reading inside one decade if the apparatus will let you. Lengths from 0.100m to 1.000m span exactly one decade of logL, which fills the sheet; lengths from 0.900m to 1.000m span 0.046 and cannot be plotted usefully at all.
- The slope of this graph is a pure number and must be quoted as one. "m=0.500s" loses the mark that "m = 0.500" earns.
- If your slope comes out near 0.5 but not exactly, say so and quote it. Rounding 0.483 to "a half" because the theory says a half is the one thing this experiment is designed to stop you doing.

Practice 7.5

The tailor two doors down keeps a length of elastic hanging by his door with a weight on the end, and swings it when he is thinking. **Kipanga** timed it for a while and announced that the period went as the square root of the length, the same as everything else.

"You have timed one length," said **Kipute.** *"One length cannot tell you what anything goes as."*

Procedure

Proceed as follows:

(a) Suspend a bob from a thread and measure the length L from the point of suspension to the centre of the bob. Begin with L=0.900m.

(b) Record the time t for 20 complete oscillations.

(c) Repeat for L=0.700m, 0.550m, 0.420m, 0.320m and 0.240m.

Questions

(i) Tabulate L, t, T, logL and logT.

(ii) Plot a graph of logT against logL and determine its slope.

(iii) The period is believed to obey $T=kL^m$. State the values of m and k that your graph gives.

(iv) Determine g, given that $k=2\pi/\sqrt{g}$.

(v) A student claims that timing one length and using $T=2\pi\sqrt{L/g}$ would have given g just as well, and in a tenth of the time. Say what that method cannot do that this one can.

Experiment 7.6

The threads had all been cut too short, so **Mr. Akilikubwa** produced a box of steel rings and told them to hang the rings from the clamps and loop the threads through the rings.

"Now measure your length from the ring."

"From where on the ring, sir?" said **Kipute.**

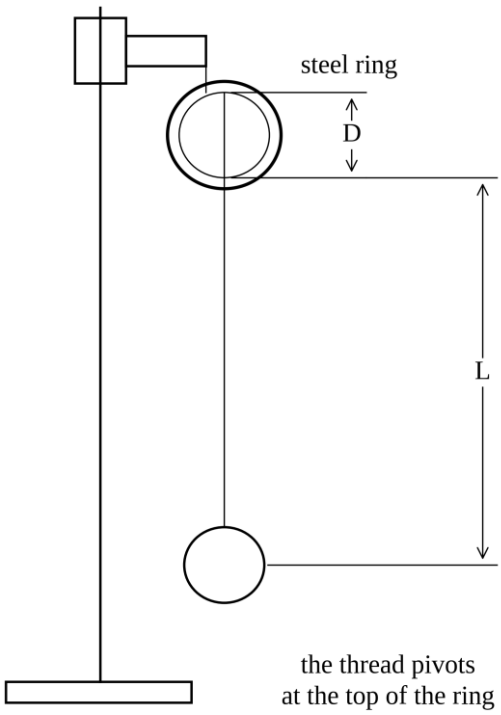
He looked genuinely delighted, which was always a bad sign. *"That is the whole question, and I am not going to answer it. Measure from the inside bottom of the ring, all of you, the same way, and let the graph tell us where the pendulum thinks it is hanging from."*

Kipanga measured from the inside bottom of the ring, wrote down his lengths, and was the first to notice that his line missed the origin by a long way, and that the miss was almost exactly the internal diameter of the ring.

Apparatus

A steel ring of about 5cm internal diameter, a pendulum bob, thread, a retort stand with a boss and clamp, a metre rule, a vernier calliper and a stopwatch.

Setup



The ring hangs from the clamp and the thread is looped through it, so the thread bears on the inside top of the ring and turns about that line. The length L is measured from the inside bottom of the ring to the centre of the bob, which is a point anybody can find with a rule, and which is not where the pendulum actually pivots.

Procedure

Proceed as follows:

(a) Hang the ring from the clamp and loop the thread through it so that the length L, from the inside bottom of the ring to the centre of the bob, is 0.850m.

(b) Set the bob swinging through a small angle in a plane at right angles to the ring, and record the time t for 20 complete oscillations.

(c) Repeat the procedure in 7.6 (a) and (b) for $L=0.750\text{m}$, 0.650m, 0.550m, 0.450m and 0.350m.

(d) Measure the internal diameter of the ring with the vernier calliper.

Questions

(i) Tabulate your results, including the values of $L(\text{m})$, $t(\text{s})$, $T(\text{s})$ and $T^2(\text{s}^2)$.

(ii) Plot a graph of $L(\text{cm})$ against $T^2(\text{s}^2)$, carrying the L axis below zero.

(iii) Determine the slope of your graph and state its units.

(iv) The swinging length is $L+D$, where D is a constant. Show that $L=(g/4\pi^2)T^2-D$, and hence determine g.

(v) Read the intercept on the L axis, state the value of D, and say what point on the apparatus the pendulum is actually turning about.

(vi) Suggest the aim of this experiment.

Table of results

L (m)	t for 20 oscillations (s)	T (s)	T^2 (s^2)
0.850	38.01	1.901	3.614
0.750	36.01	1.801	3.244
0.650	33.51	1.676	2.809
0.550	31.02	1.551	2.406
0.450	28.49	1.425	2.031
0.350	25.31	1.266	1.603

The internal diameter of the ring, on the vernier calliper, is 5.00cm.

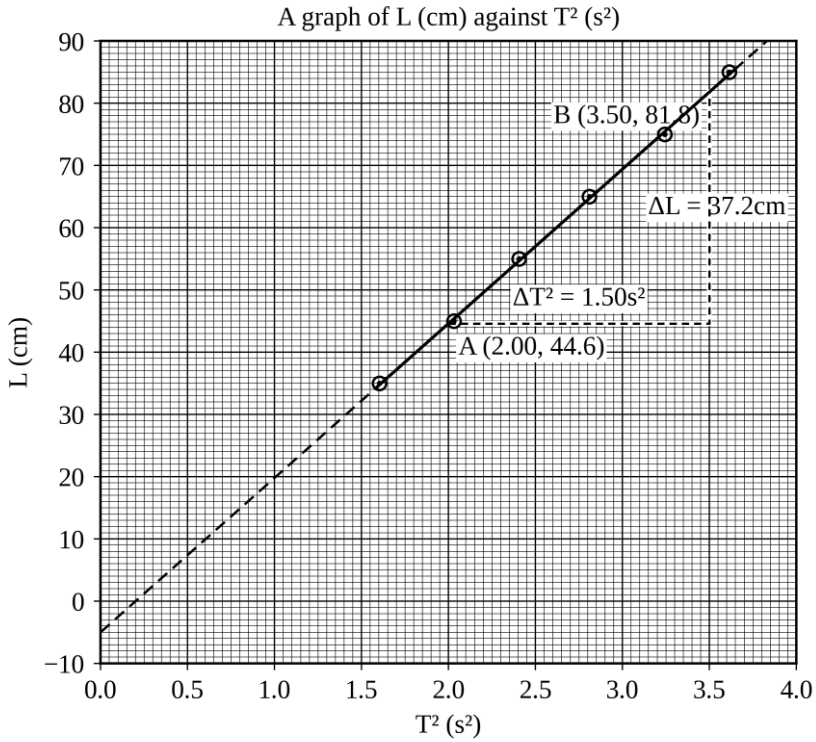
Making the measurement count

Practical tip 1: *A pendulum pivots where the thread last touches something solid, which is often not where the apparatus appears to end.*

A thread clamped in a split cork pivots at the underside of the cork. A thread looped through a ring pivots at the inside top of the ring. A thread hung over a nail pivots at the underside of the nail, and one wound twice round a rod pivots wherever it leaves the rod, which moves as you wind. In every case the point is fixed for the whole run, so it produces an intercept and not a change of slope, and the intercept measures the distance from wherever you did measure to wherever the pendulum really turns.

Answers to the questions

(i) and (ii). The table is printed above, and the L axis runs below zero because the intercept is negative.



(iii) Taking a triangle from $T^2=2.00s^2$ to $T^2=3.50s^2$:

$$\text{slope}=\frac{81.8\text{cm}-44.6\text{cm}}{3.50s^2-2.00s^2}=24.8\text{cms}^{-2}$$

(iv) The swinging length is L+D, so

$$T=2\pi\sqrt{\frac{L+D}{g}}$$

Squaring and making L the subject,

$$L=\frac{g}{4\pi^2}T^2-D$$

a straight line of slope $g/4\pi^2$ and intercept $-D$. The slope is 24.8cms^{-2}, which is 0.248ms^{-2}, so

$$g=4\pi^2\times0.248\text{ms}^{-2}=9.78\text{ms}^{-2}$$

0.3% below the accepted 9.81ms^{-2}.

(v) The line meets the L axis at -5.0cm, so $D=5.0\text{cm}$. That is the internal diameter of the ring, which the calliper gives as 5.00cm. The pendulum is turning about the inside top of the ring, one whole internal diameter above the point every length was measured from. Nobody put that distance into the calculation, and the graph found it anyway.

(vi) To determine the acceleration due to gravity and the position of the effective point of suspension of a pendulum hung through a ring. Two quantities are determined, so both belong in the aim, and neither of them is "to plot a graph of L against T^2".

Examiner's eye

- When the question asks you to suggest the aim, look at the last thing it asked you to calculate, and at anything else it asked you to determine along the way. Every determined quantity goes into the aim sentence.
- Swing the bob at right angles to the plane of the ring. Swung in the plane of the ring, the thread rolls along the inside of it as the bob moves, the pivot shifts during the swing, and the intercept stops meaning anything.
- D came out as the internal diameter and not the radius, which surprises most candidates. Say which distance your intercept is, and check it against something you measured.
- A ring is not the only offender. Any suspension with size to it, a hook, a chuck, a knot over a rod, hides the same kind of length. Meeting it once here should stop it costing you marks anywhere else.

Practice 7.6

The cork in the clamp was a fat one, and **Kipanga** had measured his lengths from the top of it, on the grounds that the top was easier to see.

"It will not matter," he said. *"It is the same cork every time."*

"It is the same cork every time," **Kipute** agreed, *"which is exactly why it will show."*

Procedure

Proceed as follows:

(a) Clamp a thread in a split cork and measure the length L from the TOP of the cork to the centre of the bob. Begin with $L=0.900\text{m}$.

(b) Record the time t for 20 complete oscillations.

(c) Repeat for $L=0.800\text{m}$, 0.700m, 0.600m, 0.500m and 0.400m.

Questions

(i) Tabulate L(cm), t(s) and $t^2(s^2)$.

(ii) Plot a graph of L(cm) against $t^2(s^2)$, taking the t^2 axis to zero.

(iii) Determine the slope, and hence g, remembering that the time is for 20 oscillations.

(iv) Read the intercept and state what it measures.

(v) Kipanga says measuring from the top of the cork rather than the bottom cannot matter, because it is the same cork every time. Say what is right and what is wrong in that.

Experiment 7.7

The pendulum for this one was already hanging when they came in: a bob on a long thread tied to a beam in the roof, the best part of three metres up, with no ladder in the building.

"Measure its length," said **Mr. Akilikubwa.**

Nobody moved.

"Good. You cannot, and neither can I, and that is the point. But you can measure how high the bob is off the floor, and you can lengthen the thread and measure that again, and the floor is not going anywhere. So you will measure the room instead of the pendulum, and the pendulum will tell you how high the beam is. It will also tell you g, but by now that is almost an insult."

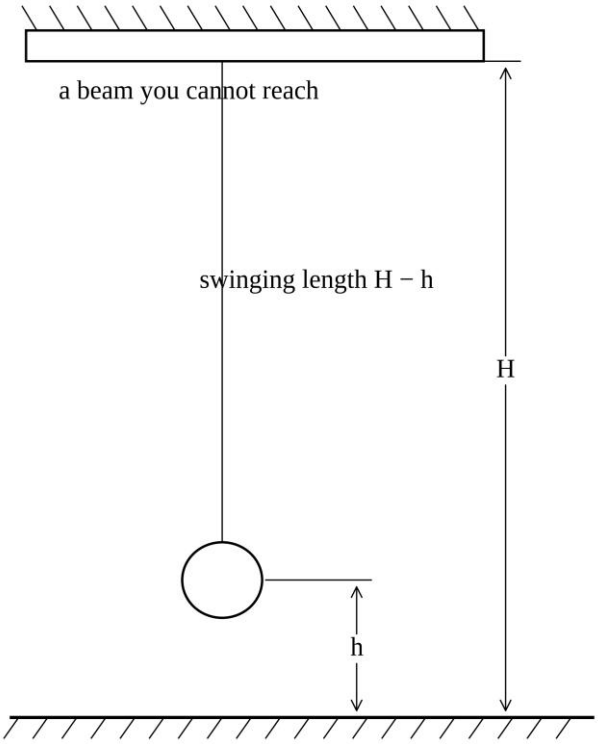
Aim

To determine the acceleration due to gravity, g, and the height H of an inaccessible point of suspension above the floor.

Apparatus

A bob on a long thread suspended from a high fixed point, a metre rule, masking tape and a stopwatch.

Setup



The height h of the centre of the bob above the floor is measured with the metre rule. The height H of the point of suspension above the same floor is never measured, and cannot be. Since both are measured from the floor, the swinging length is H-h, and lengthening the thread lowers the bob and makes h smaller.

Procedure

Proceed as follows:

(a) Adjust the thread so that the centre of the bob stands 0.300m above the floor, and measure h with the metre rule.

(b) Set the bob swinging through a small angle and record the time t for 20 complete oscillations.

(c) Shorten the thread so that h=0.600m and repeat, and continue for h=0.900m, 1.200m, 1.500m and 1.800m.

Questions

(i) Tabulate your results, including the values of h(cm), t(s) and $t^2(s^2)$.

(ii) Plot a graph of h(cm) against $t^2(s^2)$.

(iii) Determine the slope of your graph and state its units.

(iv) The swinging length is H-h. Show that $h=H-(g/4\pi^2n^2)t^2$, where n is the number of oscillations timed, and hence determine g.

(v) Read the intercept on the h axis and state what it represents.

(vi) Explain why the height of the beam can be found this way even though no rule was ever placed against it.

Table of results

h (cm)	t for 20 oscillations (s)	t^2 (s^2)
30.0	63.39	4018.3
60.0	59.29	3515.3
90.0	55.09	3034.9
120.0	50.73	2573.5
150.0	45.55	2074.8
180.0	40.12	1609.6

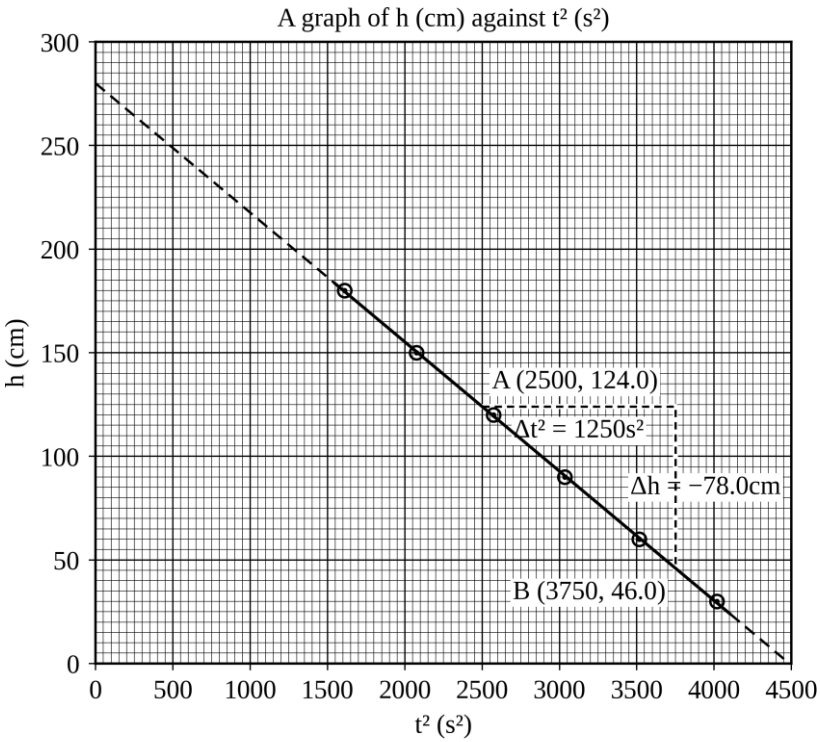
Making the measurement count

Practical tip 1: *When you cannot measure from the pivot, measure from any fixed datum you can reach. The intercept will find the pivot for you.*

The floor is not part of the pendulum and has nothing to do with the physics, and that is precisely why it works: it is fixed, and every reading is referred to the same fixed thing. The swinging length is the difference between two heights above that datum, one of which you know and one of which you do not, so the unknown one appears as a constant in the equation, and a constant in a straight line is an intercept. The same move measures the height of a lamp bracket, a tree branch or a ceiling fan, with a stopwatch and a piece of string.

Answers to the questions

(i) and (ii). The table is printed above. The h axis runs to 300cm and the t^2 axis to $4500s^2$, so that the intercept, which lies beyond every reading, is on the sheet.



(iii) Taking a triangle from $t^2=2500s^2$ to $t^2=3750s^2$:

$$\text{slope}=\frac{46.0\text{cm}-124.0\text{cm}}{3750s^2-2500s^2}=-0.0624\text{cms}^{-2}$$

The slope is negative because lengthening the thread lowers the bob and slows it: h falls as t^2 rises.

(iv) The swinging length is H-h, so

$$T=2\pi\sqrt{\frac{H-h}{g}}$$

Squaring, replacing T by t/n, and making h the subject,

$$h=H-\frac{g}{4\pi^2n^2}t^2$$

a straight line of slope-$g/4\pi^2n^2$ and intercept H. Taking the size of the slope, 0.0624cms^{-2}, which is 0.000624ms^{-2},

$$g=4\pi^2n^2\times|\text{slope}|=4\times3.14^2\times20^2\times0.000624\text{ms}^{-2}=9.84\text{ms}^{-2}$$

0.3% above the accepted 9.81ms^{-2}. A finite swing angle can only push g down, so it is not the explanation here; a watch stopped a fraction early shortens every time and pushes g up, and that fits.

(v) The line meets the h axis at 280.0cm, so H=2.80m. That is the height of the beam above the floor: the height the bob would have to be at for the swinging length, and therefore the period, to fall to zero.

(vi) Because H enters the equation as a constant and constants are what intercepts are made of. Every reading of h is a distance from the floor, and every swinging length is H minus one of them, so H is present in every row of the table even though it was never measured. Extending the line to t^2 = 0 asks what h would be if the pendulum took no time to swing at all, and the only bob that takes no time is one hanging at the beam itself.

Examiner's eye

- Measure h to the centre of the bob every time, not to its bottom. A consistent error there does not touch the slope, but it goes straight into H, and H is half the answer.
- The t^2 axis must reach zero or there is no intercept to read, even though no reading comes anywhere near it. Draw that part of the line broken.
- Check the intercept against the room. If your graph says the beam is 2.80m up, that is a believable ceiling; if it says 8m or 0.9m, you have made an arithmetic error, and it is better to find it yourself than to have it found for you.

- The slope is negative here and positive in most of this chapter. Take its size deliberately before putting it into the formula for g, and say that you are doing so.

Practice 7.7

The mango outside the classroom had one branch that hung lower than the others, and somebody's brother had tied a rope to it years ago.

"How high is that branch?" said **Kipanga**, who had been sent out for talking and had time on his hands.

He did not have a ladder either. He did have a stopwatch, because he had been sent out while holding one, and by the end of the lesson he had an answer and, for once, nobody argued with it.

Procedure

Proceed as follows:

(a) Suspend a bob from a long thread tied to a high fixed point, and adjust it so that the centre of the bob is 0.250m above the ground. Measure h with a metre rule.

(b) Record the time t for 20 complete oscillations.

(c) Repeat for $\text{h}=0.500\text{m}$, 0.750m, 1.000m, 1.250m and 1.500m.

Questions

(i) Tabulate h(cm), t(s) and $\text{t}^2(\text{s}^2)$.

(ii) Plot a graph of h(cm) against $\text{t}^2(\text{s}^2)$, taking the t^2 axis to zero.

(iii) Determine the slope and hence g.

(iv) Determine the height of the point of suspension above the ground.

(v) The bob is now to be set swinging with its centre 2.00m above the ground. Use your graph to predict the time for 20 oscillations, and say why the prediction is less trustworthy than the readings you took.

Experiment 7.8

There is a swing in the yard of the primary school next door, hung on two long chains from the arm of an old jacaranda, and at four o'clock there is always a queue for it. **Kipute** stopped on her way home because something about it was wrong.

A small girl was on it, going gently. Behind her, waiting, was her brother, who had perhaps three times her mass. When his turn came he pushed off hard and went far higher than she had. And the swing kept the same beat. **Kipute** counted it against her own pulse and it did not change.

She stood there long enough that people began to look at her.

In the morning she asked **Mr. Akilikubwa** about it. He did not answer. He went to the store cupboard and came back with a small aluminium bob, two brass bobs, a steel ball and a lead sinker, and laid them out on the bench in a row, heaviest last.

"You have told me twice this term that the mass does not matter," he said, "and both times you told me because the formula told you. Today the bench tells you."

"And the height, sir? He swung much higher than she did."

"You are right about the mass, and you will show it before lunch. About the height you are not quite right, and I am going to leave you wrong until tomorrow."

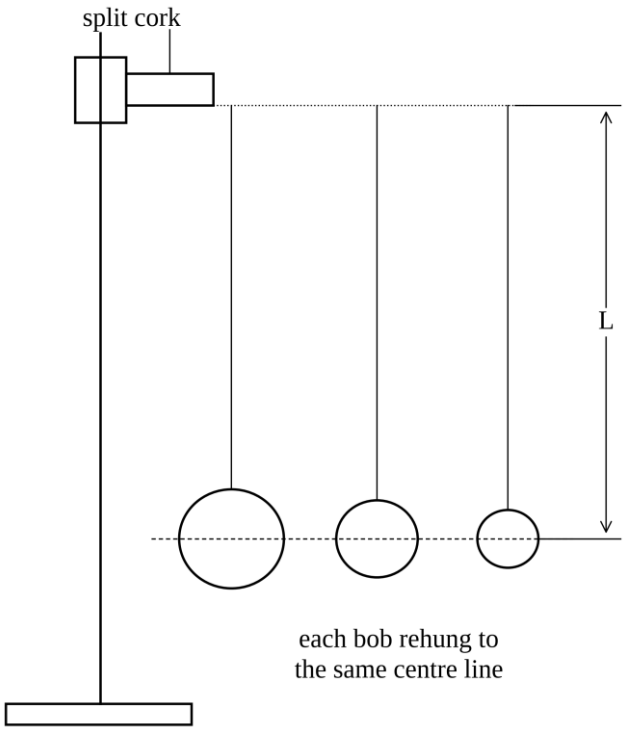
Aim

A question of this kind states none, and this one is harder to write than most, because one afternoon serves two ends and both of them belong in it. Answer (vii) sets it out.

Apparatus

Five bobs of different mass, all of them metal: one of aluminium, two of brass, one of steel and one of lead. Thread, a split cork, a retort stand with a boss and clamp, a metre rule, a vernier calliper, a balance reading to 0.1g and a stopwatch.

Setup



The rig is the plain one of Experiment 7.1, and this is the one variety in the family that changes neither where the length is measured from nor what is plotted against what. It changes what is varied at all. In the first part the length is held still and the bob is changed instead, and that is what the drawing insists on: the bobs are of different sizes, so each thread has to be shortened or lengthened until the centre of that bob reaches the same line. Measure each bob across with the vernier calliper, halve it, and set the thread so that L, from the underside of the cork to the centre of the bob, is 0.640m every time. Change the bob without resetting the thread and you have changed two quantities at once, which no experiment may do.

Procedure

Proceed as follows:

(a) Find the mass m of each of the five bobs on the balance, and measure the diameter of each with the vernier calliper.

(b) Suspend the first bob from a thread clamped in a split cork and adjust the thread until the length L, from the underside of the cork to the centre of the bob, is 0.640m.

(c) Set the bob swinging through a small angle and record the time t for 30 complete oscillations.

(d) Repeat the procedure in 7.8 (b) and (c) for each of the other four bobs, adjusting the thread each time so that L is again 0.640m.

(e) Keeping one bob, set $L=1.000\text{m}$ and record the time t for 30 complete oscillations.

(f) Repeat the procedure in 7.8 (e) for $L=0.810\text{m}$, 0.640m, 0.490m, 0.360m, 0.250m and 0.160m.

Questions

(i) Tabulate the results of the two parts separately. For the first part include $m(g)$, $t(s)$ and $T(s)$; for the second include $L(m)$, $\sqrt{L}(\sqrt{m})$, $t(s)$ and $T(s)$.

(ii) Plot a graph of $t(s)$ against $\sqrt{L}(\sqrt{m})$.

(iii) State whether your line passes through the origin, and say what that tells you about the way T depends on L.

(iv) Determine the slope of your graph and state its unit.

(v) Show that $t=(2\pi n/\sqrt{g})\sqrt{L}$, where n is the number of oscillations timed, and hence determine g.

(vi) Using the first table, state whether the period depends on the mass of the bob, and support your answer with a number.

(vii) Suggest the aim of this experiment.

Table of results

Five bobs at one length, L=0.640m:

m (g)	t for 30 oscillations (s)	T (s)
12.4	48.08	1.603
24.7	48.24	1.608
47.9	48.22	1.607
68.2	48.12	1.604
95.8	48.22	1.607

One bob at seven lengths:

L (m)	√L (√m)	t for 30 oscillations (s)	T (s)
1.000	1.000	60.09	2.003
0.810	0.900	54.09	1.803
0.640	0.800	48.27	1.609
0.490	0.700	42.25	1.408
0.360	0.600	36.14	1.205
0.250	0.500	30.04	1.001
0.160	0.400	24.02	0.801

Making the measurement count

Practical tip 1: *When you change the bob you change the length. Reset it before you time anything, or you will measure the length and call it the mass.*

Two bobs whose diameters differ by 1cm hang with their centres 0.5cm apart if the threads are cut to the same length. At L=0.640m that is 0.8% of the length, and because the period goes as the square root of the length it is 0.4% of the period, already larger than the whole spread of the first table. Reach for a wooden ball to get a very light bob and the difference becomes centimetres rather than millimetres, and it swamps everything.

Keep to bobs of metal for a second reason as well. A very light bob has to be a bulky one, and the air a bulky bob displaces lifts it enough to lengthen its period. That is a real effect and it has nothing to do with mass. A pendulum that appears to show the period depending on the mass of the bob is nearly always showing one of these two things instead.

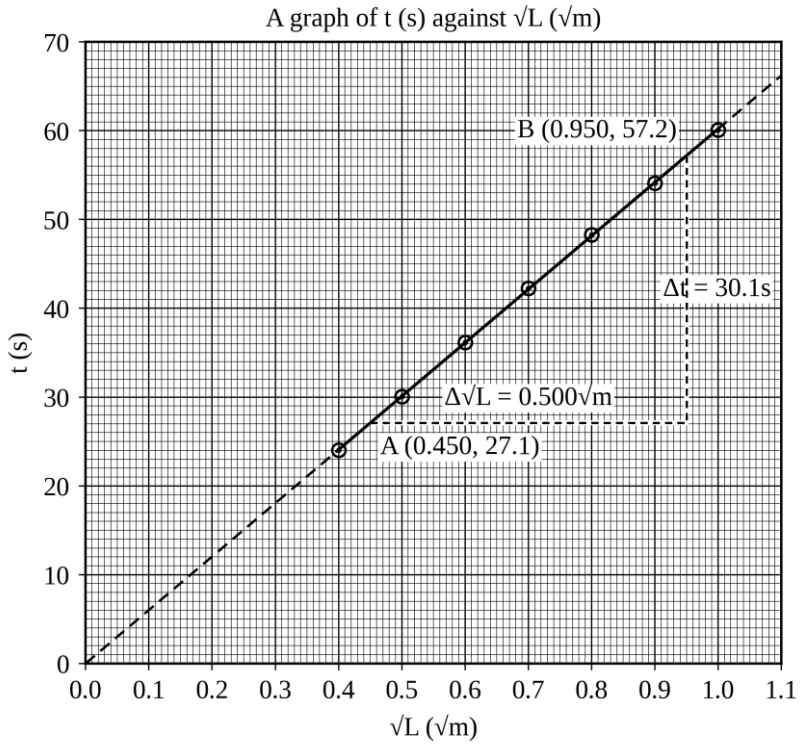
Practical tip 2: *A line that theory says must pass through the origin is still a line you draw through your points, not through the origin.*

Once you know where the line ought to go it is tempting to lay the ruler on the origin and swing it until it looks about right. Do not. Draw the best line through the readings first, and only then look at where it cuts the axis. If it misses the origin, that miss is a result and not a nuisance: it says that a constant length is hiding in your readings, or that your watch has a systematic start. A line forced through the origin can never tell you either, because you assumed the answer before you plotted.

Answers to the questions

(i) Both tables are printed above. They overlap at L=0.640m. The second part gives 48.27s there; the five bobs of the first table give times from 48.08s to 48.24s, so the extra run sits 0.03s above all of them. That is 0.06% of 48s, and well inside what repeated timings of one bob scatter by.

(ii) The graph is drawn below.



(iii) The line passes through the origin, which is what the relation demands: a pendulum of no length would take no time to swing. Since $t=nT$, the proportionality is between T and $\sqrt{L}$, and squaring both sides, T^2 is proportional to L, which is the relation Experiments 7.1 to 7.7 have each either assumed or corrected.

It also says something about the lengths themselves. Suppose a constant c were hidden in every one of them, as one was in Experiments 7.2, 7.3, 7.4 and 7.6. The recorded time would then be

$$t=60.2\sqrt{L+c}$$

which is not a straight line in $\sqrt{L}$ at all. Be careful about what you would actually see, though, because the bend is gentle over the range you plotted: with c as large as 2.0cm a ruler still lies across all seven points to within 0.12s, which is about the size of your own scatter. What the hidden constant really does is lift the line off the origin, to cut the t axis at about 1.9s, and pull the slope down from 60.2 to 58.8, which would hand you $g=10.26ms^{-2}$. So this graph does catch a hidden constant, but it catches it as a missed origin and a wrong answer rather than as something you can measure. On the T^2 against L axes of Experiment 7.2 the same constant gives a clean intercept that can be read straight off, which is why 7.2, 7.4 and 7.6 all recover their corrections there and this one does not try to.

(iv) Taking two points read off the line at $\sqrt{L}=0.450m^{1/2}$ and $\sqrt{L}=0.950m^{1/2}$, neither of which is one of your plotted readings:

$$\text{slope}=\frac{57.2s-27.1s}{0.950m^{1/2}-0.450m^{1/2}}=\frac{30.1s}{0.500m^{1/2}}=60.2sm^{-1/2}$$

(v) The period of a simple pendulum is

$$T=2\pi\sqrt{\frac{L}{g}}$$

and the recorded time is n periods, so

$$t=nT=2\pi n\sqrt{\frac{L}{g}}=\frac{2\pi n}{\sqrt{g}}\sqrt{L}$$

which is $y=mx$ with t up the vertical axis and $\sqrt{L}$ along the horizontal, and with no constant term, so the line must pass through the origin. The slope is $2\pi n/\sqrt{g}$, and rearranging it,

$$g=\left(\frac{2\pi n}{\text{slope}}\right)^2=\left(\frac{2\times3.14\times30}{60.2\text{sm}^{-1/2}}\right)^2=9.79\text{ms}^{-2}$$

This is below the accepted value by

$$\frac{9.81\text{ms}^{-2}-9.79\text{ms}^{-2}}{9.81\text{ms}^{-2}}\times100\%=0.2\%$$

(vi) No dependence can be detected. Across a mass range of nearly eight to one, from 12.4g to 95.8g, the largest difference between any two periods is 1.608s-1.603s=0.005s, and the periods do not order themselves by mass at all: the longest belongs to the second lightest bob, the shortest to the lightest, and the heaviest sits inside the range rather than at either end of it. Now set that spread against your own timing. The five recorded times scatter about their mean by about 0.07s, which over 30 oscillations is 0.07s÷30=0.002s on the period. Five readings whose scatter is 0.002s will normally spread over about two and a half times that, which is the 0.005s you have, so the table is what you would get by timing one bob five times over. The honest statement is not that the period is exactly independent of the mass but that this experiment can detect no dependence, and that any dependence there may be is smaller than 0.002s in 1.606s, or 0.1%.

(vii) Two quantities are established here, not one, so both belong in the aim, in the order the procedure takes them: to show that the period of a simple pendulum is independent of the mass of the bob, and to verify that the period is proportional to the square root of the length and hence to determine g. An aim that mentions only g throws the first table away, and the first table is half the work.

Examiner's eye

- A graph that has to be shown passing through the origin must have the origin on it, and that costs paper. Here the readings fill a little over half of each axis and the rest is empty. That is the price of the evidence, and it is worth paying. What you must not do is shrink the axes to fill the sheet and then claim the line goes through a corner you never plotted.
- $\sqrt{L}$ carries a unit and the axis must show one. "$\sqrt{L}$" on its own loses the mark that "$\sqrt{L}(\sqrt{m})$" earns, and $m^{1/2}$ is the same unit written the other way round.
- Writing "the period does not depend on the mass" because the formula says so earns nothing at all. The mark is in the number: quote the largest difference between your periods and set it against your own timing uncertainty.
- Plotting the recorded time rather than the period saves a column and keeps the reading you actually took, and it costs nothing, but n now lives inside the slope. Plot t and then use $g=4\pi^2/\text{slope}^2$ and you are out by a factor of n^2, which here is 900, and no amount of careful plotting will rescue it. Experiments 7.3 and 7.7 carry n in the same way.
- Five bobs are not five repeats of one bob, and the scatter is the evidence. Use the mean of the periods only as the base of a percentage; never replace the five with it and quote one number as the period.
- Set the angle, not the sideways pull. If you draw the bob aside by the same distance at every length, the short pendulums swing through the wider angles, and a bias that would have been the same on every row stops being the same and works on the slope instead. Practice 7.8 measures what that bias costs.

Practice 7.8

Mr. Akilikubwa kept his word. He came in the next morning, put a stopwatch down on the bench without a word, and waited.

"I went back to the swing," said **Kipute**. *"The wide swing is slower. Only just, but it is slower every time I do it."*

"By how much?"

She had not measured it. Nobody had. **Kipanga** observed that if it was only just, it was probably her counting.

"Then settle it," said **Mr. Akilikubwa**, and pushed the stopwatch across the bench. *"One length, one bob, and nothing changing but the angle. If Kipute is right the graph will say by how much, and it will also tell you what the period would have been had she pushed it no distance at all."*

Procedure

Proceed as follows:

(a) Suspend a bob from a thread so that the length L, from the point of suspension to the centre of the bob, is 1.000m.

(b) Draw the bob aside a horizontal distance of 8.7cm, so that the thread makes an angle $\theta=5°$ with the vertical, release it and record the time t for 20 complete oscillations.

(c) Repeat the procedure in 7.8 (b) for horizontal displacements of 17.4cm, 25.9cm, 34.2cm, 42.3cm and 50.0cm, which set θ to $10°$, $15°$, $20°$, $25°$ and $30°$.

Questions

(i) Tabulate your results, including $\theta(°)$, $\theta^2(°^2)$, $t(s)$ and $T(s)$.

(ii) For a swing of angle θ the period is given by $T=T_0(1+\theta^2/16)$, where θ is measured in radians and T_0 is the period the pendulum would have at a vanishingly small swing. Plot a graph of $T(s)$ against $\theta^2(°^2)$, beginning the T axis at 2.000s.

(iii) Read the intercept on the T axis and state what it represents.

(iv) Use the intercept to determine g.

(v) Determine the value of g that the $30°$ reading alone would have given, and comment on it.

(vi) Explain why the T axis has to be given a false origin, and say what precaution that makes necessary when the intercept is read.

Chapter 8

COMPOUND PENDULUMS AND LAMINAS

A simple pendulum has to pretend. It pretends that all its mass sits at one point, that the thread weighs nothing, and that the bob is small enough for its size not to matter. Chapter 7 spent eight experiments keeping that story honest, and a good share of its marks were for noticing where it broke down. This chapter drops the pretence altogether. Take any rigid body, drill a hole in it, hang it on a horizontal pivot and let it swing: a metre rule, a sheet of cardboard, a bicycle wheel, a signboard on one bolt. That is a compound pendulum, and it is what the paper sets when the pendulum question has to be about something other than a bob on a string.

Dropping the pretence costs you one new quantity and pays you back with a second answer. The new quantity is the **radius of gyration**, **k.** A rigid body resists being turned about a pivot, and how strongly it resists depends on how its mass is spread out, not only on how much of it there is. The radius of gyration is the single distance that describes the spreading: a body of mass m resists turning about an axis through its centre of gravity exactly as a single mass m placed a distance k from that axis would. For a uniform rule of length l the theory says k is $l/\sqrt{12}$, and in this chapter you measure it instead of taking it on trust.

Underneath, all four varieties are one equation. If h is the distance from the pivot to the centre of gravity, then

$$T=2\pi\sqrt{\frac{k^2+h^2}{gh}}$$

and only two things change from one variety to the next: which body you hang, and what you plot against what. There is no thread to stretch, no bob whose centre you have to guess at, and the mass of the body does not appear at all. Learn the first variety thoroughly and the other three are the same experiment in different clothes.

The four varieties at a glance

	What is different about it	What the graph gives you
8.1	A drilled metre rule swung about holes on one side of its centre of gravity	g, and the radius of gyration k of the rule
8.2	The same rule swung about holes on both sides, and a curve drawn instead of a line	k from the shape of the curve, and g
8.3	A circular cardboard lamina on a pin, holes along one radius	g, and the radius of gyration of the lamina
8.4	A rule pivoted at one fixed hole and loaded with a mass that is moved along it	g, and the radius of gyration of the rule

Experiment 8.1 is worked below in full. Learn this one first. The other three change only the body and the pair of axes; the last hangs two bodies on one pivot and asks a little more of you.

Experiment 8.1

The signboard at the school gate had hung from two bolts for as long as anyone could remember. In June the lower bolt gave up. For the rest of that term the board swung from the one bolt that was left, announcing the school's name to whichever direction the wind preferred, and the watchman had stopped apologising for it.

Mr. Akilikubwa arrived one Tuesday, stopped in front of the gate and did not go in. His class piled up behind him. He watched the board swing for a while, counting under his breath, and then remarked to nobody in particular that it was keeping better time than the clock in the staffroom.

Kipute asked, reasonably enough, how long its thread was.

There was no thread. There was no bob either. There was a painted plank hanging on a bolt, going back and forth with a period you could measure with a stopwatch and a certain amount of patience. Everything she had learned in the previous chapter began with a bob on the end of a string, and here was a pendulum with neither, keeping perfectly good time and refusing to explain itself.

By the end of the lesson she had a metre rule with a row of holes drilled along it, and the beginning of an explanation.

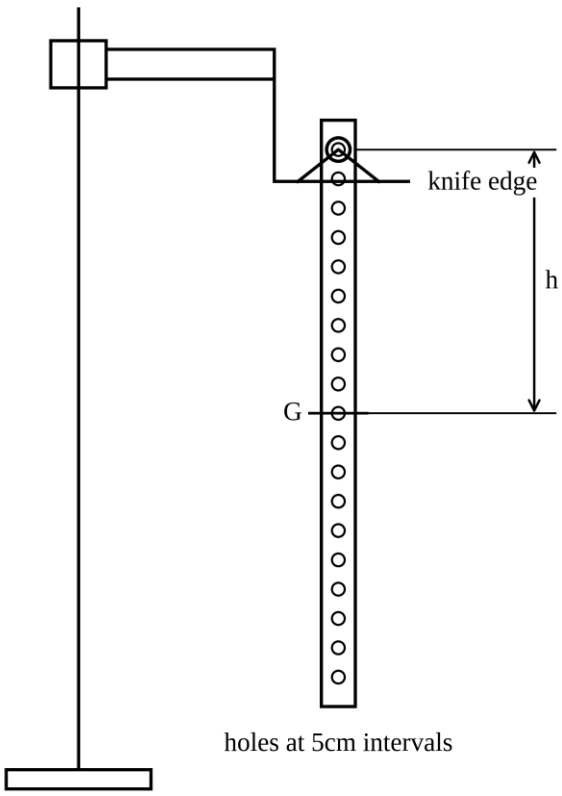
Aim

To determine the acceleration due to gravity, g, and the radius of gyration of a uniform metre rule about a horizontal axis through its centre of gravity.

Apparatus

A uniform metre rule drilled with small holes at 5cm intervals from the 5cm mark to the 95cm mark, a knife edge held horizontally in a clamp on a retort stand, and a stopwatch reading to 0.01s.

Setup



The rule hangs on the knife edge through one of its holes and swings in a vertical plane. The distance h is measured along the rule from the hole in use to the centre of gravity G.

Procedure

Proceed as follows:

(a) Balance the rule across the knife edge and note the graduation at which it balances. This is its centre of gravity, G.

(b) Hang the rule on the knife edge through the hole at the 5cm mark, so that h, the distance from the knife edge to G, is 0.450m.

(c) Displace the lower end of the rule about 2cm to one side and release it, so that it swings in a vertical plane.

(d) Allow two or three swings to settle, then record the time t for 20 complete oscillations.

(e) Repeat the procedure in 8.1 (b) to (d) using the holes at the 10, 15, 20, 25 and 30cm marks, for which h is 0.400m, 0.350m, 0.300m, 0.250m and 0.200m.

Questions

(i) Tabulate your results, including values of T, T^2, h^2 and T^2h.

(ii) Plot a graph of T^2h (s^2m) against h^2 (m^2).

(iii) Determine the slope of your graph and state its units.

(iv) The periodic time of a rigid body swinging about a horizontal axis a distance h from its centre of gravity is $T=2\pi\sqrt{(k^2+h^2)/(gh)}$, where k is the radius of gyration of the body about a parallel axis through its centre of gravity. Show that your graph should be a straight line, and use its slope to determine g.

(v) Read the intercept your line makes on the T^2h axis, and use it together with your slope to determine k.

(vi) The radius of gyration of a uniform rule of length l about an axis through its centre of gravity, at right angles to its length, is $l/\sqrt{12}$. Compare your value of k with the value this gives.

(vii) State one advantage this arrangement has over a bob on a thread, and one difficulty it introduces.

Table of results

h (m)	t for 20 oscillations (s)	T (s)	T^2 (s^2)	h^2 (m^2)	T^2h (s^2m)
0.450	31.89	1.595	2.544	0.2025	1.145
0.400	31.43	1.572	2.471	0.1600	0.988
0.350	30.90	1.545	2.387	0.1225	0.835
0.300	30.50	1.525	2.326	0.0900	0.698
0.250	30.69	1.535	2.356	0.0625	0.589
0.200	31.42	1.571	2.468	0.0400	0.494

Making the measurement count

Practical tip 1: *Where the governing equation comes from. Every question in this chapter is this derivation with different letters, so it is worth doing once, properly.*

Let the body have mass m and hang on a pivot O a distance h from its centre of gravity G. Displace it through a small angle θ. The weight mg acts vertically down through G, and its moment about O is $mgh\sin\theta$, which for a small angle is $mgh\theta$. That moment always turns the body back towards the vertical, so

$$I_O\frac{d^2\theta}{dt^2}=-mgh\theta$$

which is simple harmonic motion with $\omega^2=mgh/I_O$, and therefore

$$T=2\pi\sqrt{\frac{I_O}{mgh}}$$

The moment of inertia about the pivot is not the moment of inertia about the centre of gravity. The parallel axis theorem connects them: $I_O=I_G+mh^2$. Writing $I_G=mk^2$, which is what the radius of gyration means, gives $I_O=m(k^2+h^2)$, and the mass cancels out of the period completely:

$$\mathbf{T=2\pi\sqrt{\frac{k^2+h^2}{gh}}}$$

Notice what has and has not survived. The mass has gone, exactly as it goes from the simple pendulum, so a heavy rule and a light one of the same size swing alike. What has arrived is k, and it is the only thing in the equation that knows what shape the body is.

Practical tip 2: *One substitution tells you roughly what the first timing ought to come to. Do it before you take the other five, not after.*

Twenty oscillations take $t=20T$, so

$$t=40\pi\sqrt{\frac{k^2+h^2}{gh}}=125.6\sqrt{\frac{k^2+h^2}{gh}}$$

For a metre rule k is close to 0.289m. Putting that in, with the accepted value 9.81ms^{-2} and the first length h=0.450m,

$$t=125.6\sqrt{\frac{0.083521\text{m}^2+0.202500\text{m}^2}{9.81\text{ms}^{-2}\times0.450\text{m}}}=125.6\times0.2545\text{s}=31.97\text{s}$$

and the stopwatch gave 31.89s, which is short of that by 0.08s, or 0.25%. A hand-held stopwatch cannot do better than that, so nothing is wrong and the other five can be taken. Treat this as a check on your apparatus, not on your data. A first timing ten percent away from the recipe means you have mis-set h or lost count; a first timing within one percent you leave alone, whichever side of the recipe it falls.

Practical tip 3: *When the quantity you want is stuck in a denominator, multiply through by it. That single move turns almost every equation in this chapter into a straight line.*

Squaring the governing equation gives $T^2=\frac{4\pi^2(k^2+h^2)}{gh}$, and h is stuck underneath. Multiply both sides by h and it comes out:

$$T^2h=\frac{4\pi^2}{g}h^2+\frac{4\pi^2k^2}{g}$$

That is the equation of a straight line, with T^2h up the page and h^2 along it. There is a bonus hidden in it. The slope is $4\pi^2/g$ and the intercept is $4\pi^2k^2/g$, so their ratio is

$$\frac{\text{intercept}}{\text{slope}}=k^2$$

with g cancelled clean out of it. Your radius of gyration therefore does not depend on your value of g at all, and a poor g does not spoil it.

Practical tip 4: *Find the centre of gravity, do not assume it.*

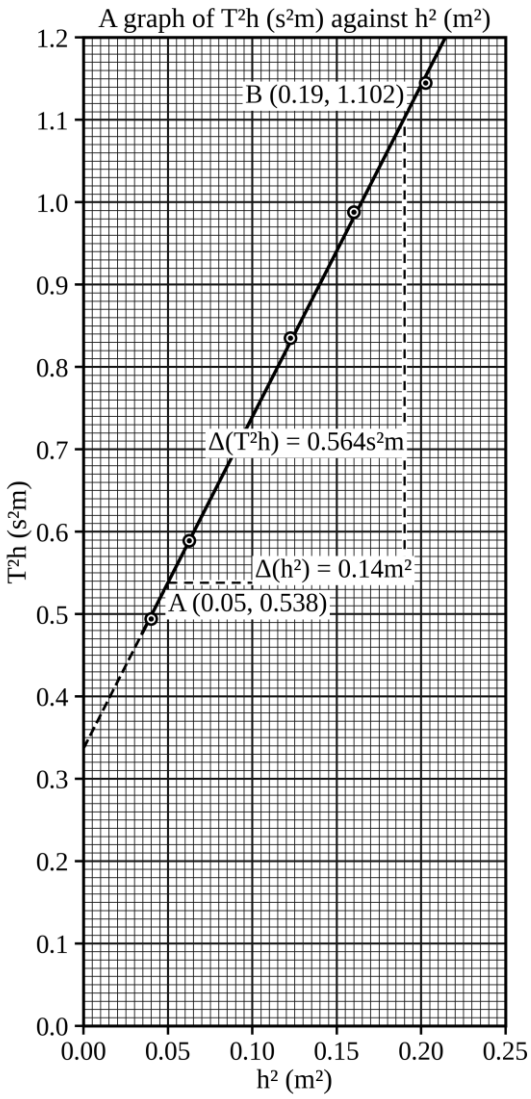
A metre rule is only uniform until somebody drops it. A chipped end, a brass edging strip, or nineteen holes drilled slightly off centre all move G, and if you take G to be the 50cm mark when it is really the 50.6cm mark then every one of your six values of h is wrong by 0.006m. That error does not slide the graph, because h appears on both axes. What it does instead is tilt the line, and it does so without spoiling its straightness: the six points still lie along a straight line to within a third of a millimetre of pencil. Six millimetres of error in G steepens the slope by 1.7%, which takes g from 9.81ms^{-2} down to 9.65ms^{-2} and k from 0.289m to 0.284m, and nothing on the graph will tell you. Balance the rule first. It takes ten seconds.

Answers to the questions

(i) and (ii). The table is printed above. Squaring the governing equation and multiplying by h, as in Practical tip 3, gives

$$T^2h=\frac{4\pi^2}{g}h^2+\frac{4\pi^2k^2}{g}$$

so T^2h goes on the vertical axis and h^2 on the horizontal one. Give it the title A graph of T^2h (s^2m) against h^2 (m^2). Put T^2h at 2cm to $0.10s^2m$, running from 0 to $1.20s^2m$, and h^2 at 2cm to $0.05m^2$, running from 0 to $0.25m^2$. That is 24cm by 10cm, which fits an A4 sheet upright with room to spare for the labels. Both axes must start at zero, because part (v) asks you to read the intercept. Plot the six points and draw one best straight line with the points balanced about it.



(iii) Take two points that lie on your line and are well apart. Taking A ($0.05m^2$, $0.538s^2m$) and B ($0.19m^2$, $1.102s^2m$):

$$\text{slope}=\frac{1.102s^2m-0.538s^2m}{0.19m^2-0.05m^2}=\frac{0.564s^2m}{0.14m^2}=4.03s^2m^{-1}$$

The units matter and they are not obvious: T^2h carries s^2m and h^2 carries m^2, so the slope carries s^2m^{-1}.

(iv) From the rearrangement above, the slope of the line is $4\pi^2/g$. Therefore

$$g=\frac{4\pi^2}{\text{slope}}=\frac{4\times3.14^2}{4.03s^2m^{-1}}=\frac{39.4384}{4.03}ms^{-2}=9.79ms^{-2}$$

Compare that with the accepted value $9.81ms^{-2}$. The result is low by $0.02ms^{-2}$, which is 0.2%, so the experiment has worked and your comment should say so plainly. Do not go hunting for a physical cause of a difference that small. Your six timings scatter about the line by something like a quarter of a percent, and a quarter of a percent in T is half a percent in T^2. A result inside that is as close as six hand timings can come, and which side of the accepted value it lands on is a matter of chance. The swing angle, which was the villain of Chapter 7, cannot be blamed here: drawing the far end of the rule aside by 2cm is an angle of about 1.3 degrees, and an angle that small lengthens the period by three parts in a hundred thousand.

(v) The line meets the T^2h axis at $0.337s^2m$. By Practical tip 3 the ratio of intercept to slope is k^2, with no g in it:

$$k^2=\frac{0.337s^2m}{4.03s^2m^{-1}}=0.0836m^2$$

$$k=\sqrt{0.0836m^2}=0.289m$$

(vi) For a uniform rule of length 1.000m,

$$\frac{l}{\sqrt{12}}=\frac{1.000m}{3.464}=0.289m$$

which is the value the graph gave. The agreement is worth pausing over. You did not measure the shape of the rule, you did not weigh it, and you never assumed a value for g: the intercept of a graph of timings handed you a property of the rule that a ruler alone could never have given.

(vii) One advantage: there is no thread and no bob, so the two worst faults of the simple pendulum both disappear. Nothing can stretch, and no length has to be measured to the invisible centre of a bob; every distance is read straight off the graduations of the swinging body itself. One difficulty: the period changes very little as h is changed. Look down your own t column, where the whole variation is little more than a second in thirty. Twenty oscillations are therefore the minimum worth timing, and a careless start costs you more here than it would with a bob on a string.

Examiner's eye

- Write the units of the slope on. Four of the eight graphs in this chapter have slopes in s^2m^{-1}, one has it in ms^{-2} and one in cms^{-2}, and it is the unit that tells you whether to multiply by $4\pi^2$ or to divide by it. If the units of your answer do not come out as ms^{-2} you have inverted something, and you can see it before you write the final line.
- h is measured from the pivot to the centre of gravity, not from the pivot to the end of the rule and not from the end of the rule to anywhere. Say in your report which mark you found G at. An examiner who cannot see that you located G assumes you did not.
- Both axes must run from zero, because the intercept is one of the answers. A graph whose horizontal axis begins at $0.04m^2$ gives a beautiful line and a worthless intercept.
- Do not quote k as a fraction of the length of the rule unless you have been asked for it that way. The question asks for a radius of gyration, which is a length, in metres. Twenty oscillations, every time. Time ten at one length and twenty at another, divide each by its own number, and your arithmetic is faultless while the scatter on half your points doubles. Those are the points the eye follows when it draws the line.

Practice 8.1

Two voices came out of the workshop behind the laboratory, neither of them lowered.

"It is heavier, so it must swing slower. Anybody can feel that," said **Kipanga**.

"Then swing it and time it," said **Kipute**.

"I do not need to time it. Look at the size of it."

What they were arguing about was a steel bar off the workshop bench, drilled along its length, which the fundi used for levering engine parts apart and which had many times the mass of any pendulum bob in the building. **Kipute** went and fetched the stopwatch anyway. She came back with it, and with the observation that if he was right about the mass he could tell her in advance what the times would be, and if he was wrong the graph would say so in about twenty minutes.

Procedure

Proceed as follows:

(a) Balance the bar to find its centre of gravity, G, and measure its length. It is 0.900m long and drilled at 5cm intervals.

(b) Hang the bar on a knife edge through the hole for which h, the distance from the knife edge to G, is 0.400m.

(c) Set it swinging through a small angle in a vertical plane and record the time t for 30 complete oscillations.

(d) Repeat the procedure in P8.1 (b) and (c) for h equal to 0.350m, 0.300m, 0.250m, 0.200m and 0.150m.

Questions

(i) Tabulate your results, including values of t^2, t^2h and h^2. Do not calculate the period.

(ii) Plot a graph of t^2h (s^2m) against h^2 (m^2).

(iii) Determine the slope and the intercept of your graph.

(iv) Given that $T=2\pi\sqrt{(k^2+h^2)/(gh)}$ and that $t=nT$, where n is the number of oscillations timed, show that the slope of such a graph is $4\pi^2n^2/g$, and hence determine g.

(v) Determine the radius of gyration k of the bar about an axis through G.

(vi) The radius of gyration of a uniform bar of length l about such an axis is $l/\sqrt{12}$. Use your value of k to calculate the length of the bar, and compare it with the length you measured.

(vii) **Kipanga** claims that a bar of twice the mass, but the same size and shape, would give a graph of twice the slope. State, with a reason, whether he is right.

Experiment 8.2

Kipute brought her table from Experiment 8.1 to the front of the room, and she was not happy with it.

"Sir, the times go down and then they come back up again at the short lengths. I have checked them twice."

Mr. Akilikubwa read the column without any sign of surprise. *"Are they wrong?"*

"No, sir. The graph is a perfect straight line."

"Then they are not wrong, and you have found something the straight line hides. Your graph knows about it; it simply is not asked. So do not plot a straight line this time. Plot the period itself against where you hung the rule, take five holes on each side of the middle and spread them as widely as the sheet will take, and draw the smooth curve those points want to lie on."

"And what am I looking for?"

"The place your own table has already pointed at. When you have found it, count fifty oscillations at a time, not twenty."

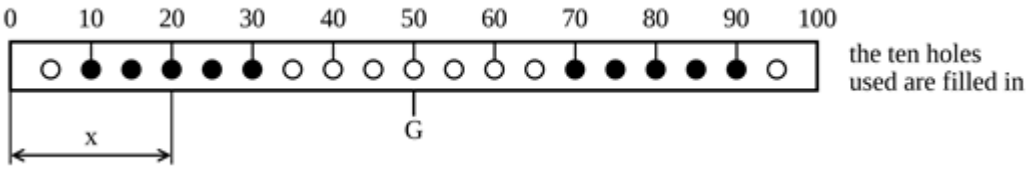
Aim

To investigate how the periodic time of a compound pendulum varies with the position of the pivot, and to use the result to determine the radius of gyration of a uniform metre rule and the acceleration due to gravity.

Apparatus

The drilled metre rule and knife edge of Experiment 8.1, and a stopwatch reading to 0.01s.

Setup



The same rule is used, but now it is hung from holes on both sides of its centre of gravity. The distance recorded is x, the graduation at which the hole in use is drilled, measured from the 0cm end of the rule, and not the distance from the centre of gravity. The ten holes used are filled in on the diagram.

Procedure

Proceed as follows:

(a) Balance the rule to find its centre of gravity, G.

(b) Hang the rule on the knife edge through the hole at the 10cm mark and record the time t for 50 complete oscillations, keeping the swing small.

(c) Repeat the procedure in 8.2 (b) for the holes at the 15, 20, 25 and 30cm marks.

(d) Turn the rule end for end and repeat for the holes at the 70, 75, 80, 85 and 90cm marks.

Questions

(i) Tabulate your results, including the periodic time T for each hole.

(ii) Plot a graph of T (s) against x (cm), using a false origin on the T axis, and draw a single smooth curve through the points. Do not join the two groups of points across the middle of the rule.

(iii) Describe the shape of your curve, and account for it using the equation $T=2\pi\sqrt{(k^2+h^2)/(gh)}$, where h is the distance from the pivot to the centre of gravity.

(iv) Show that the periodic time is least when h=k, and use your curve to determine k for this rule.

(v) Show that the least periodic time is $T=2\pi\sqrt{2k/g}$, and use it with your value of k to determine g.

(vi) State the length of the simple pendulum that would have the same periodic time as the rule at its least period.

(vii) Explain why 50 oscillations were timed in this experiment and only 20 in Experiment 8.1.

Table of results

x (cm)	h (cm)	t for 50 oscillations (s)	T (s)
10	40	78.28	1.5656
15	35	77.29	1.5458
20	30	76.60	1.5320
25	25	76.69	1.5338
30	20	79.13	1.5826
70	20	78.94	1.5788
75	25	76.62	1.5324
80	30	76.44	1.5288
85	35	77.00	1.5400
90	40	78.42	1.5684

Making the measurement count

Practical tip 1: *Why the curve has a lowest point, and where it is, without a line of calculus.*

Divide the top and bottom inside the square root by h:

$$T=2\pi\sqrt{\frac{k^2+h^2}{gh}}=2\pi\sqrt{\frac{1}{g}\left(\frac{k^2}{h}+h\right)}$$

Everything outside the bracket is fixed, so the period is least when $k^2/h+h$ is least. Now think about the two terms. Hang the rule far from its centre of gravity and h is large, so the bracket is large. Hang it close and k^2/h is large, so the bracket is large again. Between those two disasters there is a best place.

You can find it without calculus. The two terms multiply to give $(k^2/h)\times h=k^2$, which is fixed, and for two positive numbers whose product is fixed the sum is least when the two are equal. So the bracket is least when $k^2/h=h$, that is when h=k. Putting h=k back into the equation gives the least period itself:

$$T_{min}=2\pi\sqrt{\frac{k^2+k^2}{gk}}=2\pi\sqrt{\frac{2k}{g}}$$

Two results out of one line of algebra: the pivot that gives the quickest swing is one radius of gyration from the centre of gravity, and the period there depends on nothing but k and g. That is why this curve is worth drawing.

Practical tip 2: *Every compound pendulum has a simple pendulum that copies it exactly. Its length is the one number you can always compare across experiments.*

Set the two periods side by side, $T=2\pi\sqrt{L/g}$ for a simple pendulum and the compound formula for this one. They are the same equation if

$$L=\frac{k^2+h^2}{h}$$

and L is called the length of the **equivalent simple pendulum**. At the lowest point of your curve, where $h=k$, it is $L=2k$, and nowhere on the rule is it shorter. Keep this expression: Practice 8.2 turns it into a way of finding g without knowing where the centre of gravity is at all.

Practical tip 3: *Count fifty oscillations when the quantity you are hunting is a shallow minimum.*

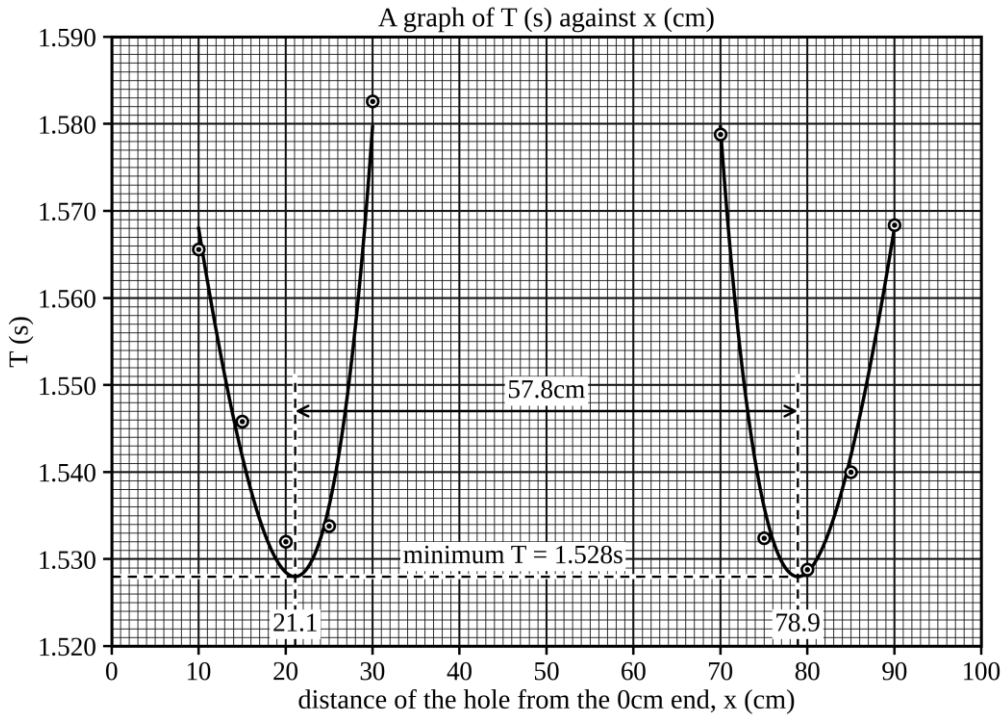
Look down your own T column. Between the largest and the smallest reading there is about $0.05s$, and the whole shape of the curve has to be drawn inside that. Suppose your reaction time is $0.2s$ at a start or a stop, which is ordinary. Divided by twenty oscillations that is $0.01s$ on T, a fifth of the entire range, and the bottom of the curve disappears into the scatter. Divided by fifty it is $0.004s$, and the minimum survives. When you are hunting a difference rather than a value, buy precision with patience.

Practical tip 4: *The two branches are a check on each other.*

The rule is uniform, so its two halves are identical and the curve must be symmetrical about the centre of gravity. The reading at the 10cm mark and the reading at the 90cm mark are the same experiment done twice, and so are the other four pairs. Compare them. A single pair that disagrees by more than the rest of your readings scatter has one bad timing in it; a whole branch sitting above the other means the rule is not uniform. Either way you know before you draw a single conclusion.

Answers to the questions

(i) and (ii). The table is printed above. Put T on the vertical axis at 2cm to $0.010s$, running from $1.520s$ to $1.590s$, and x on the horizontal axis at 2cm to 10cm, running from 0 to 100cm. That is 20cm by 14cm, and it is worth the paper: every answer here is read along x, and a cramped horizontal scale is exactly what stops you locating the lowest point. The T axis needs a false origin, because the whole variation is $0.05s$ and on an axis starting at zero every point would lie in one band. The x axis keeps its true origin. Draw two smooth curves, one through each group of five points, and do not join them across the middle: the period rises without limit as the pivot approaches the centre of gravity, and you have no readings there to justify a line.



(iii) Each branch falls to a lowest point and rises again, and the two branches are mirror images of each other in the centre of the rule. The reason is in Practical tip 1: the period depends on $k^2/h+h$, in which h pulls the period up when the pivot is far from the centre of gravity and k^2/h pulls it up when the pivot is near. The rule is uniform, so a hole h above the centre behaves exactly like a hole h below it, and that is the symmetry.

(iv) The bracket $k^2/h+h$ is a sum of two positive terms whose product is k^2, a constant. Such a sum is least when the two terms are equal, so the period is least when $h=k$. From the graph the two lowest points are at 21.1cm and 78.9cm, one on each side of the centre of gravity, so the distance between them is 2k:

$$k=\frac{78.9\text{cm}-21.1\text{cm}}{2}=28.9\text{cm}=0.289\text{m}$$

and for a uniform metre rule $l/\sqrt{12}=100\text{cm}/3.464=28.9\text{cm}$, which is what the curve gave. Reading the separation of the two minima, rather than the distance from one minimum to the centre, uses both branches, and it never uses the position of G at all.

(v) Read the least period off the lowest part of your curve. Both branches bottom out at 1.528s, as they must for a uniform rule. Putting $h=k$ into the governing equation gives $T=2\pi\sqrt{2k/g}$, so

$$g=\frac{8\pi^2k}{T_{min}^2}=\frac{8\times3.14^2\times0.289\text{m}}{(1.528\text{s})^2}=\frac{22.7954\text{m}}{2.3348\text{s}^2}=9.76\text{ms}^{-2}$$

which is 0.05ms^{-2} below the accepted 9.81ms^{-2}, or 0.5%. This route is less precise than the straight line of Experiment 8.1 and it should be: you are reading the bottom of a flat curve, where a millimetre of pencil is worth several tenths of a centimetre of k. Say so in your comment. An examiner would rather read an honest account of a method's limits than a claim of three-figure accuracy the graph cannot support.

(vi) By Practical tip 2 the equivalent simple pendulum has length $L=(k^2+h^2)/h$, and at the lowest point $h=k$, so

$$L=2k=2\times28.9\text{cm}=57.8\text{cm}$$

A simple pendulum 57.8cm long has a period of 1.528s, and that is the shortest period a metre rule can be made to copy, however you hang it.

(vii) Because the answers here are differences between periods, not periods. The entire useful range of T is about 0.05s, so a timing uncertainty that would be trivial in Experiment 8.1 is a fifth of everything you are trying to measure. Timing fifty oscillations instead of twenty divides that uncertainty by two and a half, at the cost of about a minute per reading.

Examiner's eye

- A false origin on one axis is allowed and often necessary. A false origin on the axis you read an answer along is not. Here T carries the false origin and x does not, which is the right way round, because every answer is read along x.
- Draw a curve, not a line, and draw it smoothly through the points rather than from point to point. A curve drawn as five straight segments puts its lowest point at whichever reading happens to be lowest, which throws away the whole advantage of having five of them.
- Do not carry the curve across the middle of the rule. There are no readings there, the period rises without limit as the pivot approaches the centre of gravity, and a confident line through empty paper is the easiest thing in a script for an examiner to notice.
- $h=k$ at the minimum is a result to quote, not to assume. One line, either the equal-terms argument or a derivative set to zero, earns the mark; writing "at the minimum $h=k$" without a reason does not.
- The two branches should give the same value of k. If they do not, say which you have used and why, rather than quietly averaging two numbers that disagree.

Practice 8.2

A sheet of paper appeared on the laboratory door on Monday morning, held up by four drawing pins because that corridor is draughty.

NOTICE. On the bench inside there is a steel bar 80cm long, drilled at 5cm intervals, and a stopwatch. Hang the bar from any hole and it will swing. Somewhere on that bar are two places, a long way apart and not the same distance from the middle, at which it swings with exactly the same period. Find them and you can measure the acceleration due to gravity without ever finding out where the centre of gravity of the bar is. The first correct answer keeps the bar. A. Akilikubwa.

By break somebody had crossed out the last sentence and written underneath it, in handwriting that everybody recognised as **Kipanga**'s, *"you still have to balance it, sir"*. The correction was wrong. It took the class most of the morning to see why, and by then two of them had already won the bar and were arguing about whose graph had been finished first.

Procedure

Proceed as follows:

(a) Hang the bar on a knife edge through the hole at the 5cm mark and record the time t for 50 complete oscillations, keeping the swing small.

(b) Repeat the procedure in P8.2 (a) for the holes at the 10, 15, 20 and 25cm marks.

(c) Turn the bar end for end and repeat for the holes at the 55, 60, 65, 70 and 75cm marks.

(d) Record for each hole the distance x of that hole from the 0cm end of the bar. At no point in this experiment do you need to know where the centre of gravity lies.

Questions

(i) Tabulate your results, including the periodic time T.

(ii) Plot a graph of T (s) against x (cm), using a false origin on the T axis, and draw a smooth curve through each group of points.

(iii) Draw a horizontal line across your graph at 1.400s and read the four values of x at which it cuts your curves.

(iv) A compound pendulum has the same periodic time as a simple pendulum of length $L=(k^2+h^2)/h$, where h is the distance from the pivot to the centre of gravity and k is the radius of gyration about it. Show that the two distances h_1 and h_2 which give the same periodic time satisfy $h_1+h_2=L$.

(v) Use two of your four readings, one from each curve, to obtain L, and hence determine g.

(vi) Explain why the position of the centre of gravity of the bar never enters your calculation.

(vii) Read from your curves the two positions at which the period is least, and hence determine the radius of gyration k of the bar. Compare it with $l/\sqrt{12}$ for an 80cm bar.

Experiment 8.3

There is a shop on the road down to the market that sells cooking oil, and above its door hangs a circular tin advertisement about the size of a bicycle wheel, nailed to the wall through one hole near its edge. On a still day it is an advertisement. On a windy day it is a pendulum, and that Saturday there was a wind.

Kipute stood in front of it long enough that the shopkeeper came out to ask what she wanted, and what she wanted was not for sale. A metre rule spreads its mass along a line, and she had already learned what to do with that. This thing spread its mass in two directions at once, over a whole circle of tin, and there was no obvious place to call its length. It swung anyway, steadily, a little slower than once a second, entirely indifferent to her difficulty.

She counted twenty swings against her watch, thanked the man, bought nothing, and went home to look for a carton big enough to cut a circle out of.

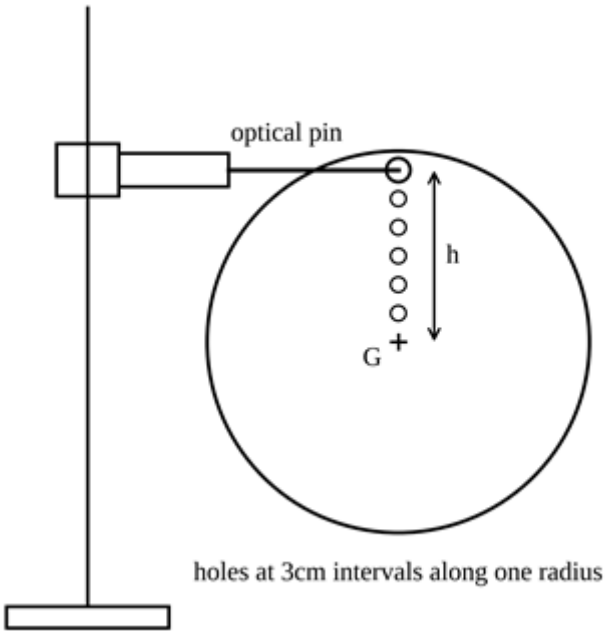
Aim

To determine the acceleration due to gravity, g, and the radius of gyration of a circular lamina about an axis through its centre at right angles to its plane.

Apparatus

A circular cardboard lamina of diameter 40.0cm, punched with small holes along one radius at 3cm intervals from 3cm to 18cm from the centre, an optical pin held horizontally in a clamp on a retort stand, a metre rule, and a stopwatch reading to 0.01s.

Setup



The lamina hangs on the pin through one of its holes and swings in its own plane. The distance h is measured from the centre G of the lamina to the hole in use. G is the point the compasses were set on when the circle was cut, and it can be confirmed by balancing the lamina on a pin point.

Procedure

Proceed as follows:

(a) Measure the diameter of the lamina with the metre rule, taking the mean of three diameters drawn in different directions.

(b) Hang the lamina on the pin through the hole 3.0cm from the centre, so that h=3.0cm.

(c) Displace the lamina through a small angle in its own plane and release it.

(d) Record the time t for 20 complete oscillations.

(e) Repeat the procedure in 8.3 (b) to (d) for the holes at h equal to 6.0cm, 9.0cm, 12.0cm, 15.0cm and 18.0cm.

Questions

(i) Tabulate your results, including values of T, T^2, T^2h and h^2, with h in centimetres.

(ii) Plot a graph of h^2 (cm^2) against T^2h (s^2cm).

(iii) Determine the slope of your graph and the intercept it makes on the h^2 axis, and state the units of each.

(iv) Given $T=2\pi\sqrt{(k^2+h^2)/(gh)}$, where k is the radius of gyration of the lamina about an axis through its centre at right angles to its plane, show that $h^2=\frac{g}{4\pi^2}T^2h-k^2$, and hence determine g in ms^{-2}.

(v) Use your intercept to determine k.

(vi) The radius of gyration of a uniform circular lamina of radius R about that axis is $R/\sqrt{2}$. Compare your value of k with the value this gives.

(vii) State why the h^2 axis must be carried well below zero, and what you would lose if it were not.

Table of results

h (cm)	t for 20 oscillations (s)	T (s)	T^2 (s^2)	T^2h (s^2cm)	h^2 (cm^2)
3.0	33.41	1.671	2.792	8.38	9
6.0	25.29	1.265	1.600	9.60	36
9.0	22.42	1.121	1.257	11.31	81
12.0	21.60	1.080	1.166	13.99	144
15.0	21.28	1.064	1.132	16.98	225
18.0	21.69	1.085	1.177	21.19	324

Making the measurement count

Practical tip 1: *The same equation, turned over. Learn to read a question for which pair of axes it wants, and then give it that pair and not the pair you are used to.*

Experiment 8.1 multiplied by h and stopped there, giving T^2h up the page against h^2 along it. This question asks for the same two quantities the other way round. Take the same line,

$$T^2h=\frac{4\pi^2}{g}h^2+\frac{4\pi^2k^2}{g}$$

and make h^2 the subject:

$$h^2=\frac{g}{4\pi^2}T^2h-k^2$$

Nothing physical has changed and everything on the page has. The slope is now $g/4\pi^2$ instead of $4\pi^2/g$, so you *multiply* by $4\pi^2$ to get g where before you divided. The intercept is now $-k^2$, so it is negative and the axis has to run below zero. Do not decide which arrangement you are using from memory. Read the question, write the rearranged equation down before you plot anything, and let it tell you.

Practical tip 2: *Centimetres are allowed, but they must be carried all the way through.*

This table keeps h in centimetres, because the lamina is a centimetres-sized object and converting six readings invites six mistakes. The cost is that the slope comes out in cms^{-2}, so $4\pi^2\times$slope gives g in cms^{-2} and not in ms^{-2}. An answer of about 978 is not wrong; it is unconverted. Divide by 100 at the very end, in one step you can point at, and give the unit the question asked for.

Practical tip 3: *Where the standard radii of gyration come from, and why they are worth memorising.*

The moment of inertia of a uniform disc of mass m and radius R about an axis through its centre at right angles to its plane is $\frac{1}{2}mR^2$. Since $I=mk^2$ defines the radius of gyration, $k^2=R^2/2$ and $k=R/\sqrt{2}$, which is about 0.707R. The two you will meet again and again in this paper are that one and the rule, $k=l/\sqrt{12}$, or about

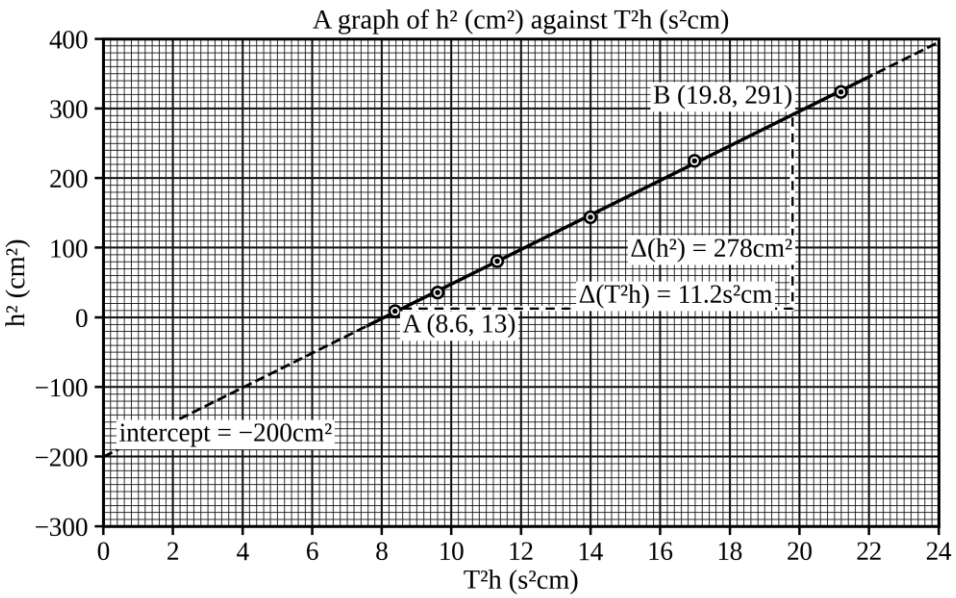
0.289l. For a rectangular lamina with sides a and b, swinging in its own plane, it is $\sqrt{(a^2+b^2)/12}$. Three lines, and between them they cover almost every body the paper hangs on a pin.

Practical tip 4: *Cardboard is light, and light bodies have their own troubles.*

A 40cm disc cut from a carton has a mass of only 30 to 70g, and it swings edge-on through the air, so the air itself takes very little out of it. What kills the swing is the hole rubbing on the pin, and what spoils the readings is the lamina drifting out of its own plane. Three habits deal with all of it. Punch the hole cleanly rather than tearing it, or the pin sits in a different place each time you rehang the lamina, and that error goes straight into h. Release the lamina by drawing one edge aside and letting go without any push, so that it starts in its plane and stays there. And time the first twenty oscillations rather than a later twenty, while the swing is still worth watching.

Answers to the questions

(i) and (ii). The table is printed above. Multiplying the squared governing equation by h and making h^2 the subject gives $h^2=\frac{g}{4\pi^2}T^2h\text{-}k^2$, so h^2 goes on the vertical axis and T^2h along the horizontal one. Put h^2 at 2cm to $100cm^2$, running from $-300cm^2$ to $400cm^2$, and T^2h at 2cm to $2s^2cm$, running from 0 to $24s^2cm$. That is 24cm by 14cm, so turn the sheet on its side. The T^2h axis keeps a true origin because the intercept is read there, and the h^2 axis must run well below zero because that intercept is negative.



(iii) Taking A ($8.6s^2cm$, $13cm^2$) and B ($19.8s^2cm$, $291cm^2$) on the line:

$$\text{slope}=\frac{291cm^2\text{-}13cm^2}{19.8s^2cm\text{-}8.6s^2cm}=\frac{278cm^2}{11.2s^2cm}=24.8cms^{-2}$$

and the line meets the h^2 axis at $-200cm^2$. The units of the slope follow from the axes: cm^2 divided by s^2cm leaves cms^{-2}, which is an acceleration, and that is the first sign that the rearrangement is the right way up.

(iv) Squaring $T=2\pi\sqrt{(k^2+h^2)/(gh)}$ and multiplying by h gives $T^2h=(4\pi^2/g)(k^2+h^2)$. Making h^2 the subject,

$$h^2=\frac{g}{4\pi^2}T^2h\text{-}k^2$$

which is a straight line of slope $g/4\pi^2$ and intercept $-k^2$. Therefore

$$g=4\pi^2\times\text{slope}=39.4384\times24.8cms^{-2}=978.1cms^{-2}$$

$$g=978.1cms^{-2}\times\frac{1m}{100cm}=9.78ms^{-2}$$

against the accepted $9.81ms^{-2}$, low by 0.3%. For a sheet of cardboard on a pin that is a good result, and three tenths of a percent is well inside what six hand timings and one read intercept can settle. Say that, and resist the

temptation to name a cause. A difference this small is the scatter of your own readings, and it could as easily have fallen on the other side.

(v) The intercept is $-k^2$, so

$$k^2=200\text{cm}^2, \qquad k=\sqrt{200\text{cm}^2}=14.1\text{cm}$$

(vi) With R=20.0cm,

$$\frac{R}{\sqrt{2}}=\frac{20.0\text{cm}}{1.414}=14.14\text{cm}$$

and the graph gave 14.1cm, which agrees to within the width of a pencil line on the intercept. Notice how little you had to know about the lamina. You never weighed it, you never worked out how its mass was distributed, and the graph still handed you a property of its shape.

(vii) Because the intercept is one of the two answers and it is negative. A graph whose vertical axis begins at zero would show you only the part of the line above the horizontal axis; the point where the line crosses the vertical axis would be off the paper, and k could not be read at all. Extending the line backwards by eye to a place the sheet does not show earns nothing.

Examiner's eye

- This graph has the measured quantity along the horizontal axis and the set quantity up the vertical one, which is the opposite of the usual convention. Plot what the question asks for. Swapping the axes back because it looks more natural turns the slope upside down and loses every mark that follows from it.
- A negative intercept is read where the line crosses the vertical axis, and the vertical axis is where the horizontal reading is zero. If your T^2h axis starts anywhere but zero, what you have read is not an intercept.
- k is a length, and about an axis through its own centre a disc cannot have one larger than its radius. A radius of gyration of 28cm for a disc of radius 20cm is arithmetically possible and physically impossible, and saying so costs one line and saves the question.
- Give g in the unit asked for. Working in centimetres throughout is sensible here, and leaving the answer as 978cms^{-2} when ms^{-2} was asked for is the commonest single loss of a mark in this chapter.
- The mean of three diameters, not one. A circle cut from a carton with scissors is not perfectly round, and R enters the comparison in part (vi) directly.

Practice 8.3

Kipanga cut his lamina out of the side of a carton, which was sound enough. Then he ruled the line for the holes by measuring in from the edge he had cut first, and that edge was not where he thought it was. The line of holes missed the centre of gravity by about two centimetres. A rectangle looks symmetrical from a distance, so nobody noticed, least of all **Kipanga**, and **Mr. Akilikubwa** let him finish the whole set before he asked him one question about it.

What follows is the experiment done with the holes on the line they should have been on: the line through the centre of gravity, parallel to the long side.

Procedure

Proceed as follows:

(a) Cut a rectangular cardboard lamina 60.0cm by 40.0cm. Find its centre of gravity G by ruling its two diagonals, and punch small holes along the line through G parallel to the long side, at 0.060m, 0.100m, 0.140m, 0.180m, 0.220m and 0.260m from G.

(b) Hang the lamina on a horizontal pin through the hole 0.060m from G, set it swinging in its own plane through a small angle, and record the time t for 20 complete oscillations.

(c) Repeat the procedure in P8.3 (b) for each of the other five holes.

Questions

(i) Tabulate your results, including T, T^2, T^2h and h^2, with h in metres.

(ii) Plot a graph of T^2h (s^2m) against h^2 (m^2).

(iii) Determine the slope and the intercept of your graph.

(iv) Determine g.

(v) Determine the radius of gyration k of the lamina about an axis through G at right angles to its plane.

(vi) For a rectangular lamina with sides a and b that radius of gyration is $\sqrt{(a^2+b^2)/12}$. Compare your value of k with the value this gives.

(vii) **Kipanga** punched his holes along a line 2cm to one side of the line through G, and measured h from the centre of the rectangle as though the line passed through it. State what this does to his graph, and why it is harder to notice than a mistake in timing.

Experiment 8.4

Down at the fish landing the buyers do not use a spring balance, because salt water eats springs. They use a steelyard: a horizontal iron bar hung from a hook near one end, with the fish in a basket at the short end and a single lump of iron that slides along the long end until the bar hangs level. One weight, one bar, and a scratched scale that has been read by four generations of the same family.

Mr. Akilikubwa grew up beside that market and he has never forgiven physics teachers for ignoring the steelyard, which he considers the most elegant instrument on the coast. What interests him about it here is not the balancing. It is what happens when the buyer, having finished, gives the bar a knock and lets it swing on its hook with the sliding mass still where he left it. The period of that swing depends on where the mass sits, and there is a whole experiment in that.

So: one rule, one pivot that never moves, one mass that does.

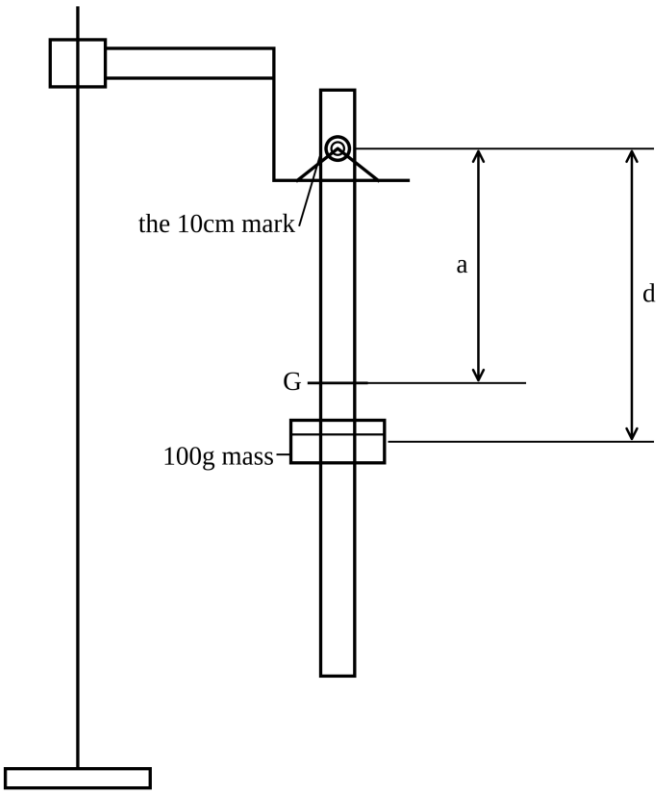
Aim

To determine the acceleration due to gravity, g, and the radius of gyration of a metre rule, using a rule pivoted at a fixed point and loaded with a mass equal to its own.

Apparatus

A uniform metre rule of mass 100g, drilled at the 10cm mark, a 100g mass in a clip that can be fixed at any graduation of the rule, a knife edge held horizontally in a clamp on a retort stand, a balance, and a stopwatch reading to 0.01s.

Setup



The rule hangs from the hole at the 10cm mark throughout, so the pivot never moves. The distance a is from the pivot to the centre of gravity of the rule alone, and the distance d is from the pivot to the added mass, which is moved to a new graduation between readings.

Procedure

Proceed as follows:

(a) Find the mass of the rule, and choose a mass equal to it. Balance alone to find its centre of gravity; it is at the 50cm mark, so $a=0.400\text{m}$ from the pivot at the 10cm mark.

(b) Hang the rule on the knife edge through the hole at the 10cm mark.

(c) Fix the 100g mass at the 30cm mark, so that $d=0.200\text{m}$.

(d) Set the rule swinging through a small angle in a vertical plane and record the time t for 20 complete oscillations.

(e) Repeat the procedure in 8.4 (c) and (d) with the mass at the 40, 50, 60, 70 and 80cm marks, for which d is 0.300m, 0.400m, 0.500m, 0.600m and 0.700m.

Questions

(i) Tabulate your results, including T, T^2, $a+d$, $T^2(a+d)$ and d^2.

(ii) Plot a graph of $T^2(a+d)$ (s^2m) against d^2 (m^2).

(iii) Determine the slope and the intercept of your graph.

(iv) Show that, when the added mass is equal to the mass of the rule,

$$T^2(a+d)=\frac{4\pi^2}{g}d^2+\frac{4\pi^2}{g}\left(k^2+a^2\right)$$

where k is the radius of gyration of the rule about its own centre of gravity, and hence determine g.

(v) Determine k_0, the radius of gyration of the rule alone about the pivot, and use $k_0^2=k^2+a^2$ to obtain the radius of gyration k of the rule about its own centre of gravity.

(vi) State what you would have to plot instead if the added mass were not equal to the mass of the rule.

(vii) Give one reason why the recorded times in this experiment do not fall steadily as d is increased from 0.200m.

Table of results

d (m)	t for 20 oscillations (s)	T (s)	T^2 (s^2)	a+d (m)	$T^2(a+d)$ (s^2m)	d^2 (m^2)
0.200	27.70	1.385	1.918	0.600	1.151	0.0400
0.300	27.73	1.387	1.924	0.700	1.347	0.0900
0.400	28.41	1.421	2.019	0.800	1.615	0.1600
0.500	29.64	1.482	2.196	0.900	1.976	0.2500
0.600	31.29	1.565	2.449	1.000	2.449	0.3600
0.700	32.77	1.639	2.686	1.100	2.955	0.4900

Making the measurement count

Practical tip 1: *Two bodies on one pivot. Add the moments of inertia, add the moments of the weights, and divide.*

The swinging body is now a rule of mass M with a mass m clipped to it. Both parts resist turning, so their moments of inertia about the pivot add:

$$I_0=M\left(k^2+a^2\right)+md^2$$

where k is the radius of gyration of the rule about its own centre of gravity, the bracket is the parallel axis theorem applied to the rule, and the last term treats the clipped mass as a small body at distance d. Both weights

also pull the system back towards the vertical, so the restoring moments add too, and for a small angle θ the total is $(Ma+md)g\theta$. Putting the two together,

$$T=2\pi\sqrt{\frac{M(l^2/12+a^2)+md^2}{g(Ma+md)}}$$

That is the general result, and it is not a straight line in any obvious pair of axes. Now make $m=M$. The mass cancels from top and bottom exactly as it did in Experiment 8.1, and what is left is

$$T^2(a+d)=\frac{4\pi^2}{g}d^2+\frac{4\pi^2}{g}\left(\frac{l^2}{12}+a^2\right)$$

Choosing the added mass to equal the mass of the rule is not a decoration. It is what turns an awkward equation into a straight line whose ordinate needs nothing but two distances.

Practical tip 2: *The intercept is a radius of gyration about the pivot, not about the centre of gravity. The parallel axis theorem gets you from one to the other, and you must use it in the right direction.*

The bracket in the intercept is $l^2/12+a^2$, and by the parallel axis theorem that is k_0^2, the square of the radius of gyration of the rule about the pivot. What the chapter has been calling k is about the centre of gravity, and the two are connected by

$$k_0^2=k^2+a^2$$

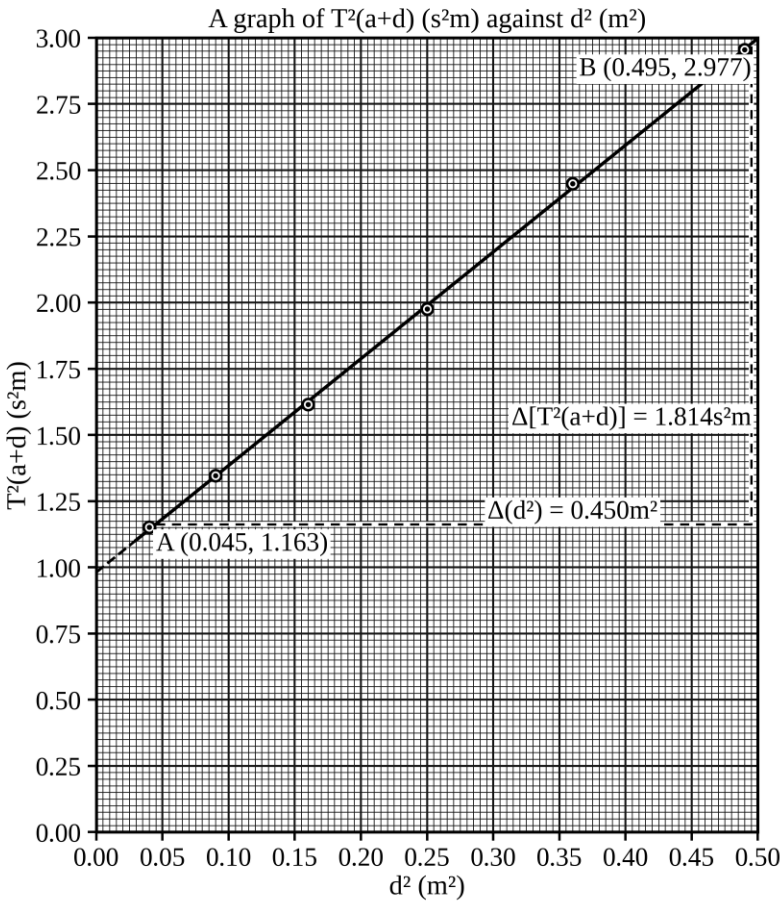
so you *subtract* a^2 to come back to k. Add when you should subtract and you get 0.635m for the radius of gyration of a rule that is only a metre long, which is impossible and therefore catchable. Check the size of every answer against the size of the apparatus.

Practical tip 3: *The times need not fall steadily, and a table that turns round is not a table with a mistake in it.*

Sliding the mass outwards does two opposite things at once. It increases the moment of inertia, which slows the swing, and it increases the restoring moment, which speeds it up. Which of them wins depends on where the mass already is. Near the pivot the restoring moment wins and the period falls; further out the moment of inertia wins and it rises. Between the two lies the least period, and in this experiment that place falls between your first two readings, at 0.235m from the pivot. That is why your first two times are all but identical while every reading after them is longer than the one before. The graph is untroubled by any of it: $T^2(a+d)$ against d^2 is a straight line through all six points whatever the raw times do.

Answers to the questions

(i) and (ii) The table is printed above. From Practical tip 1 the relation is $T^2(a+d)=(4\pi^2/g)d^2+(4\pi^2/g)k_0^2$, so $T^2(a+d)$ goes up the page and d^2 along it. Put $T^2(a+d)$ at 2cm to $0.25\text{s}^2\text{m}$, running from 0 to $3.00\text{s}^2\text{m}$, and d^2 at 2cm to 0.05m^2, running from 0 to 0.50m^2. That is 24cm by 20cm, which fills an A4 sheet upright. Both axes start at zero, because the intercept is part of the answer.



(iii) Taking A (0.045m^2, $1.163\text{s}^2\text{m}$) and B (0.495m^2, $2.977\text{s}^2\text{m}$):

$$\text{slope}=\frac{2.977\text{s}^2\text{m}-1.163\text{s}^2\text{m}}{0.495\text{m}^2-0.045\text{m}^2}=\frac{1.814\text{s}^2\text{m}}{0.450\text{m}^2}=4.03\text{s}^2\text{m}^{-1}$$

and the line meets the $T^2(a+d)$ axis at $0.981\text{s}^2\text{m}$.

(iv) The derivation is in Practical tip 1. The slope is $4\pi^2/g$, so

$$g=\frac{4\pi^2}{\text{slope}}=\frac{39.4384}{4.03}\text{ms}^{-2}=9.79\text{ms}^{-2}$$

which is 0.2% below the accepted 9.81ms^{-2}.

(v) The intercept is $(4\pi^2/g)k_0^2$ and the slope is $4\pi^2/g$, so their ratio is k_0^2:

$$k_0^2=\frac{0.981\text{s}^2\text{m}}{4.03\text{s}^2\text{m}^{-1}}=0.24342\text{m}^2,\qquad k_0=0.493\text{m}$$

Then, from $k_0^2=k^2+a^2$ with $a=0.400\text{m}$,

$$k^2=0.24342\text{m}^2-(0.400\text{m})^2=0.08342\text{m}^2,\qquad k=0.289\text{m}$$

and $l/\sqrt{12}$ for a metre rule is 0.289m, which is what the graph gave. It is worth seeing what has just happened. The rule was never hung from more than one hole; every reading was taken with the pivot in the same place. All the information came from moving a mass along it, and the graph still returned both g and a property of the rule that Experiment 8.1 needed six different pivots to find.

(vi) With m not equal to M the mass no longer cancels, and the general result of Practical tip 1 rearranges to

$$T^2(Ma+md)=\frac{4\pi^2}{g}md^2+\frac{4\pi^2}{g}M\left(k^2+a^2\right)$$

so you would plot $T^2(Ma+md)$ against d^2, which means weighing both the rule and the added mass and carrying them through every row. The line is still straight, but its slope is now $4\pi^2m/g$ rather than $4\pi^2/g$, so the added mass has to be divided out before g appears. Making the two masses equal buys you a simpler ordinate and a slope you can read directly.

(vii) Because moving the mass outwards increases the moment of inertia and the restoring moment at the same time, and the two pull the period in opposite directions. The least period falls at 0.235m from the pivot, which

lies between your first two values of d, so those two readings sit almost equally far up the two sides of one shallow dip: 27.70s and then 27.73s. Past the dip the moment of inertia wins outright, and every reading is longer than the one before it.

Examiner's eye

- The ordinate here is a product, $T^2(a+d)$, and it must be computed row by row and tabulated in a column of its own. A table that stops at T^2 and leaves the multiplication to be done on the graph paper loses the tabulation marks and usually the plotting marks too.
- The intercept gives the radius of gyration about the *pivot*. Quote it as though it were about the centre of gravity and you are out by a^2, which here is $0.160m^2$: not a small error and not one the arithmetic will reveal.
- The two masses must be equal for this pair of axes to work. If the question hands you a mass that is not equal to the mass of the rule, it is telling you to plot something else; go back to the general equation rather than forcing the simple one.
- Say in your report that the pivot was not moved. It is the whole design of the experiment, and it is also why the value of a appears once in the theory rather than six times in the table.

Practice 8.4

On the last afternoon of the topic **Mr. Akilikubwa** put a metre rule on the bench with a lump of plasticine squashed onto it, well off centre, and did not say how much plasticine.

He said that between them the four experiments of this chapter contained everything needed to work out the mass of that plasticine without ever putting it on a balance, and that anybody who could not see how should read their own answer to Experiment 8.4 again. Then he went to lunch.

Kipute got there by way of the balance point, which is where this question starts.

Procedure

Proceed as follows:

(a) Take a uniform metre rule of mass 100g, drilled at 5cm intervals, which balances at its 50.0cm mark. Clamp a load firmly to it at the 80.0cm mark.

(b) Balance the loaded rule on a knife edge and note the graduation at which it now balances. It is the 60.0cm mark.

(c) Hang the loaded rule on the knife edge through the hole at the 5cm mark, so that h, the distance from the pivot to the new balance point, is 0.550m.

(d) Set it swinging through a small angle and record the time t for 20 complete oscillations.

(e) Repeat the procedure in P8.4 (c) and (d) using the holes at the 10, 15, 20, 25 and 30cm marks, for which h is 0.500m, 0.450m, 0.400m, 0.350m and 0.300m.

Questions

(i) Tabulate your results, including T, T^2, T^2h and h^2.

(ii) Plot a graph of h^2 (m^2) against T^2h (s^2m).

(iii) Determine the slope of your graph and the intercept it makes on the h^2 axis.

(iv) Determine g.

(v) Determine the radius of gyration k of the loaded rule about its own centre of gravity.

(vi) Using the principle of moments and the two balance points, determine the mass of the load.

(vii) For the rule alone k is 0.289m. State, with a reason, whether you expect the loaded rule to have a larger or a smaller radius of gyration, and say whether your answer to (v) agrees.

Chapter 9

BIFILAR SUSPENSION AND TORSION

The last two chapters swung things. This one twists them, and the change of verb is the whole chapter. A pendulum leans out of the vertical and gravity pulls it back; a twisted body turns about a vertical axis and something twists it back. The mathematics of the two is identical, the apparatus is not, and two of the four varieties here measure something no pendulum can reach: how hard the material of a wire resists being sheared.

Everything that turns and returns obeys one equation. Put I for the moment of inertia of the twisted body about the axis it turns on, and C for the **torsional stiffness**, the restoring couple per radian of twist. Then

$$T=2\pi\sqrt{\frac{I}{C}}$$

which is the inertia-over-stiffness pattern of Chapter 8 with a couple in place of a force. I is a property of the body and C is a property of whatever holds it up, and the four varieties differ in that second half alone.

In the **bifilar suspension** a bar hangs level on two vertical threads. Twist it and the threads tilt, the bar rises very slightly, and gravity pulls it back. The stiffness works out at $C=Mgd^2/4L$, where d is the distance between the threads and L their length. Not one material property appears in it, and the mass cancels out of the period, so what the bifilar measures is g and the way the mass of the bar is spread along it.

In the **torsion pendulum** the same bar hangs from a single wire, and the restoring couple is the wire itself objecting to being twisted. Its stiffness is $C=\pi\eta r^4/2L$, where r is the radius of the wire, L its length and η the **shear modulus** of the material. Now the material is the whole point, and the period measures it.

Two names for one quantity. The shear modulus and the modulus of rigidity are the same thing: the stress needed to produce unit shear strain, in Nm^{-2}, the same units as the Young modulus. Papers write it η when a wire is being twisted and G when a spring is being loaded, and this chapter uses both so that neither symbol surprises you. Experiment 9.3 finds it dynamically, by timing a twist. Experiment 9.4 finds it statically, with no clock at all, from a spring you wind yourself on a test tube.

One symbol does double duty in this chapter, and it is better to meet that now than in the middle of a substitution. In the two bifilar experiments d is the distance between the threads. In the two spring experiments d is the diameter of the wire and D is the diameter of the coil. The two meanings have nothing to do with each other, so read d from the experiment you are in and never from memory.

One warning belongs here rather than anywhere later, because it governs how you spend your time. The stiffness of a wire goes as the **fourth power** of its radius. Measure the diameter one percent too large and your shear modulus is four percent too small, and no amount of careful timing will get it back. In this chapter the micrometer matters more than the stopwatch.

The four varieties at a glance

	What is different about it	What the graph gives you
9.1	A bar on two threads, twisted; the separation of the threads is changed	g, from a line through the origin
9.2	The same bar and threads, but the length of the threads is changed instead	g again, by a different road
9.3	A rod on a single wire carrying two masses, whose distance from the wire is changed	the shear modulus of the wire, and the moment of inertia of the rod
9.4	A spring wound by hand on a test tube, loaded and measured with a rule	the modulus of rigidity of the wire, with no timing at all

Experiment 9.1 is worked below in full. The second variety is the first one with a different quantity held constant, and the third and fourth change what supplies the stiffness.

Experiment 9.1

Mr. Akilikubwa had sent **Kipanga** to the back bench with a metre rule, two lengths of thread and instructions to hang the rule level. That part went well. What followed did not.

"Sir, it swings beautifully."

It did. He had pulled one end towards himself and let go, and the whole rule was swinging forwards and backwards on its two threads like a seat on a rope, which is a perfectly good motion and entirely the wrong one.

"Beautifully and uselessly," said **Mr. Akilikubwa**, without getting up. *"Stop it, take hold of both ends, and turn it. Not tip it. Turn it, the way you turn a key."*

Kipanga turned it through a few degrees and let go, and the rule wound itself back and forth about a vertical line through its middle, unhurried and regular, for a long time. Nothing about it looked like a pendulum. It was one.

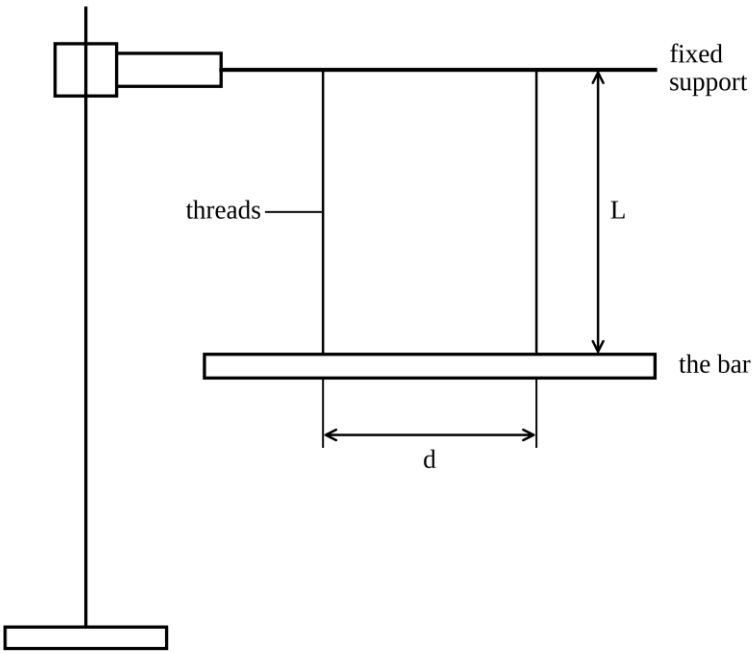
Aim

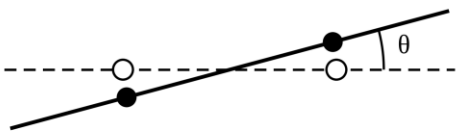
To determine the acceleration due to gravity, g, using a bifilar suspension.

Apparatus

A uniform metre rule, two lengths of inextensible thread, a fixed horizontal support, a metre rule for measuring, and a stopwatch reading to 0.01s.

Setup





seen from above, twisted through θ; the open circles mark where the threads hung before the twist

The bar hangs level on two vertical threads of equal length L, tied symmetrically about its centre a distance d apart. It is set oscillating by turning it in the horizontal plane through a small angle about the vertical axis through its centre and releasing it, so that it twists rather than swings. The lower drawing shows that twist seen from above.

Procedure

Proceed as follows:

(a) Suspend the metre rule horizontally by two threads, each of length L equal to 0.500m, tied at equal distances from the centre so that they are 0.800m apart.

(b) Check that the bar hangs level and that the two threads are vertical.

(c) Turn the bar in the horizontal plane through a small angle, not more than about five degrees, and release it so that it oscillates about the vertical axis through its centre.

(d) Allow two oscillations to settle, then record the time t for 20 complete oscillations.

(e) Repeat the procedure in 9.1 (a) to (d) with the threads 0.700m, 0.600m, 0.500m, 0.400m and 0.300m apart, keeping their length unchanged.

Questions

(i) Tabulate your results, including values of $1/d$ and the periodic time T.

(ii) Plot a graph of T (s) against $1/d$ (m^{-1}).

(iii) State the relationship between T and d shown by your graph, and say what the position of the line relative to the origin tells you.

(iv) Determine the slope of your graph and state its units.

(v) The periodic time of a bifilar suspension is $T=\frac{4\pi k}{d}\sqrt{L/g}$, where k is the radius of gyration of the bar about the vertical axis through its centre. Show that the slope of your graph is $4\pi k\sqrt{L/g}$.

(vi) Given that $k=l/\sqrt{12}$ for a uniform bar of length l, use your slope to determine g.

(vii) State two precautions you took, and say for each what would go wrong without it.

Table of results

d (m)	1/d (m^{-1})	t for 20 oscillations (s)	T (s)
0.800	1.25	20.51	1.026
0.700	1.43	23.41	1.171
0.600	1.67	27.43	1.372
0.500	2.00	32.91	1.646
0.400	2.50	40.89	2.045
0.300	3.33	54.57	2.729

Making the measurement count

Practical tip 1: *Where the bifilar formula comes from. It is three lines, and the third line is the one worth remembering.*

Let the bar have mass M and hang on two threads of length L tied d apart, so each thread is a distance $d/2$ from the axis. Turn the bar through a small angle θ. Each point of attachment moves sideways by $(d/2)\theta$, so each thread leans from the vertical by $(d/2)\theta/L$. Each thread carries half the weight, so the sideways pull it exerts is

$$\frac{Mg}{2}\times\frac{d\theta}{2L}$$

and the two of them, each acting at $d/2$ from the axis, give a restoring couple of

$$\text{couple}=2\times\frac{Mg}{2}\times\frac{d\theta}{2L}\times\frac{d}{2}=\frac{Mgd^2}{4L}\theta$$

so the torsional stiffness is $C=Mgd^2/4L$. Putting that and $I=Mk^2$ into $T=2\pi\sqrt{I/C}$,

$$T=2\pi\sqrt{\frac{Mk^2\times 4L}{Mgd^2}}=\frac{4\pi k}{d}\sqrt{\frac{L}{g}}$$

The mass has gone, exactly as it went from the simple pendulum and the compound pendulum, and for the same reason: it sits in the inertia and in the restoring couple and cancels between them. What is left is the shape of the bar, through k, and the geometry of the threads.

Practical tip 2: *Read the equation for which quantity you are about to change, and it tells you the graph before you take a single reading.*

In this experiment L and k are fixed and only d changes, so the whole of $4\pi k\sqrt{L/g}$ is a constant and

$$T=\left(4\pi k\sqrt{L/g}\right)\times\frac{1}{d}$$

which is T proportional to $1/d$: a straight line through the origin. In Experiment 9.2 the same equation has d fixed and L changing, and it gives a different straight line. One equation, two experiments, and the only skill is deciding which grouping of symbols is the constant.

Practical tip 3: *Turn it, do not tip it.*

A bar on two threads has two quite different motions with two quite different periods. Pull one end towards you and it swings like a seat on a rope, with a period set by the length of the threads alone. Turn it about the vertical and it twists, and that is the motion this experiment is about: the only one of the two whose period depends on the separation of the threads. Time the swing by mistake and every reading comes out much the same, because the swing does not know what d is: the points lie along a flat line nowhere near the origin, and the experiment has told you nothing.

You cannot separate the two by timing them, and it is worth knowing why. The twist and the swing have equal periods when $d=2k$, which for a metre rule is 0.58m, in the middle of the range of separations you are about to use. At 0.600m the two periods differ by four percent, which no stopwatch and no counting will resolve. You must separate them by looking. Start the bar by taking both ends between finger and thumb and rotating it, then watch two or three oscillations before you count: a swing shows itself as the whole bar travelling to and fro, which is unmistakable to the eye however close its period may be to the one you want.

Practical tip 4: *Keep the threads vertical and the same length as each other.*

The derivation assumed both threads hang vertically and are the same length. Tie one longer than the other and the bar hangs at a slope, and it is worth being exact about what that does. It does not unbalance the tensions: the bar is uniform and the threads are tied at equal distances from its centre, so taking moments about the centre gives each thread half the weight however the bar leans. What it does is give each thread its own tilt for a given twist, so that the restoring couple becomes

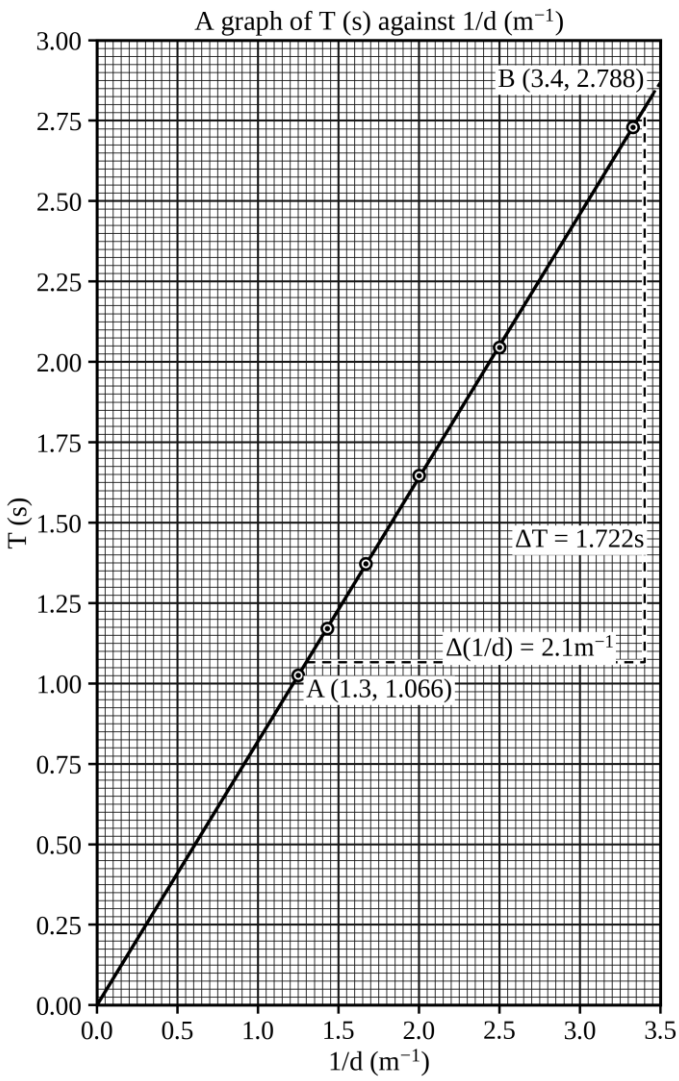
$$\text{couple}=\frac{Mgd^2\theta}{8}\left(\frac{1}{L_1}+\frac{1}{L_2}\right)$$

and the suspension behaves as though both threads had the length halfway between the two. In this experiment L is set once and never changed, so that mistake multiplies every period by the same factor: the line stays straight, stays through the origin, and is wrong. Nothing on the graph will show it and only the rule can catch it. Practice 9.2 takes the same fault into an experiment where L is the variable, and there the graph does give it away.

Tying the threads to a support narrower than d does something different and worse: they splay outwards, they are no longer vertical, and the geometry the derivation rests on has gone. Measure d between the threads at the bar, not at the support, and check by eye along the bar that both threads are upright.

Answers to the questions

(i) and (ii). The table is printed above. From the governing equation, with L and k both fixed, T is proportional to $1/d$, so T goes on the vertical axis and $1/d$ along the horizontal one. Give it the title A graph of T (s) against $1/d$ (m^{-1}). Put T at 2cm to 0.25s, running from 0 to 3.00s, and $1/d$ at 2cm to 0.5m^{-1}, running from 0 to 3.5m^{-1}. That is 24cm by 14cm and it fits an A4 sheet upright. Both axes must start at zero, because part (iii) asks about the origin.



(iii) The graph is a straight line passing through the origin, so T is inversely proportional to d: double the separation of the threads and you halve the period. The line goes through the origin because nothing in the equation adds a constant to T; every term that is not $1/d$ multiplies it. A line that missed the origin would mean either that a length had been measured from the wrong place or that the threads were splaying outwards instead of hanging vertically.

(iv) Take two points on the line, well apart. Taking A (1.3m^{-1}, 1.066s) and B (3.4m^{-1}, 2.788s):

$$\text{slope}=\frac{2.788\text{s}-1.066\text{s}}{3.4\text{m}^{-1}-1.3\text{m}^{-1}}=\frac{1.722\text{s}}{2.1\text{m}^{-1}}=0.820\text{sm}$$

The units are seconds divided by m^{-1}, which is sm. A slope in sm is unusual enough to be worth writing down: it is the signature of a graph plotted against a reciprocal.

(v) Squaring the governing equation is not needed here. Simply group the constants:

$$T=\frac{4\pi k}{d}\sqrt{\frac{L}{g}}=\left(4\pi k\sqrt{\frac{L}{g}}\right)\frac{1}{d}$$

and comparing with the equation of a straight line through the origin, the slope is $4\pi k\sqrt{L/g}$.

(vi) Square the slope and make g the subject. Since $k^2=l^2/12$,

$$\text{slope}^2=16\pi^2k^2\frac{L}{g}=\frac{16\pi^2l^2L}{12g}=\frac{4\pi^2l^2L}{3g}$$

$$g=\frac{4\pi^2 l^2 L}{3\times slope^2}=\frac{39.4384\times(1.000m)^2\times 0.500m}{3\times(0.820sm)^2}=\frac{19.7192}{2.0172}ms^{-2}=9.78ms^{-2}$$

against the accepted $9.81ms^{-2}$: a value low by 0.3%. That is inside the scatter of six hand timings and no cause need be named for it. Notice how little the bifilar asked of you: no knowledge of the mass of the bar, no bob, no knife edge, and the only lengths you had to measure were two you could reach with a rule.

(vii) Two that matter here, each with its consequence. First, start the bar by turning it and not by pushing one end, because the swinging motion has a period that does not depend on d at all, so timing it by mistake gives six nearly equal readings and a flat line. Second, tie the two threads to exactly the same length, because a suspension with unequal threads behaves as though both had the average of the two, and with L fixed that error multiplies every period alike: the graph stays straight, stays through the origin, and gives a value of g that is quietly wrong.

Examiner's eye

- The quantity plotted is $1/d$, not d. Work the reciprocals out in a column of the table before you go near the graph paper, and give that column a heading with its unit. A graph of T against d is a curve, and no amount of ruler will make it a line.
- Say which way round the proportionality goes. "T is proportional to $1/d$" and "T is inversely proportional to d" both earn the mark; "T depends on d" earns nothing.
- The slope carries sm, and that unit is your check on the substitution: $4\pi k\sqrt{L/g}$ is a length times a square root of a time squared, which is metres times seconds. A slope in sm^{-1} means you have plotted against d instead of $1/d$, and a graph of T against d is a curve in any case.
- Measure d at the bar. The threads hang vertically only if the two points they are tied to on the support are the same distance apart as the two points they are tied to on the bar, and the distance that appears in the formula is the one at the bar.
- Five degrees, not fifty. The derivation replaced the sine of the thread angle by the angle itself, and it also assumed the bar stays level. A violent twist lifts the bar noticeably and the period changes with amplitude.

Practice 9.1

The argument in the corridor had been going for some time before anybody thought to check the facts.

"You cannot find the mass of anything without a balance. That is what a balance is for," said **Kipanga**.

"You can find the mass of the bar hanging on the back bench," said **Kipute**, *"without taking it down and without putting it on anything."*

"With what? Your eyes?"

"With a stopwatch, a few slotted masses and a graph."

She was right, and the reason is worth following, because nothing in the method needs to know the acceleration due to gravity, the length of the threads, or how far apart they are. All of it cancels. What it does need is masses whose values you already know, and those are lying in the drawer with the figures stamped on them.

Procedure

Proceed as follows:

(a) Suspend a uniform bar of unknown mass M horizontally by two vertical threads, each 0.500m long and 0.600m apart.

(b) Fix a slotted mass m of 0.020kg to the middle of the bar with a little plasticine, directly over the axis about which the bar turns.

(c) Turn the bar through a small angle about the vertical axis through its centre, release it, and record the time t for 30 complete oscillations.

(d) Repeat the procedure in P9.1 (b) and (c) with m equal to 0.040kg, 0.060kg, 0.080kg, 0.100kg and 0.120kg, keeping every mass at the middle of the bar.

Questions

(i) Tabulate your results, including T^2 and $1/T^2$.

(ii) Plot a graph of $1/T^2$ (s^{-2}) against m (kg).

(iii) Determine the slope of your graph and the intercept it makes on the $1/T^2$ axis.

(iv) The lump sits over the axis, so it adds mass to the bar without adding to its moment of inertia. Show that $1/T^2$ is then proportional to $1+m/M$, and hence that the mass of the bar is given by intercept/slope.

(v) Determine the mass of the bar.

(vi) State why this method does not require you to know g, L or d.

(vii) Explain why the added mass must be at the middle of the bar, and say what would happen to your graph if the 0.120kg mass alone were fixed 0.10m from the centre.

Experiment 9.2

"Sir, why did we move the threads sideways? We could have made them longer."

Mr. Akilikubwa looked up. *"We could. What would that have cost you?"*

Kipute thought about it. *"Nothing, sir. It is the same equation."*

"It is the same equation and it is not the same graph, and the difference is worth an afternoon. Go and find out what you have to plot when it is the length that changes and the separation that stays put. And before you begin, tell me which of the two arrangements you would trust more. Not which is quicker. Which is harder to get wrong."

She did not have an answer for the second question until the end of the lesson, by which time she had a better one than he expected.

Aim

To determine the acceleration due to gravity, g, from a bifilar suspension in which the length of the threads is varied and their separation is kept constant.

Apparatus

As in Experiment 9.1: a uniform metre rule, two threads, a fixed horizontal support, a metre rule for measuring and a stopwatch reading to 0.01s.

Setup

The arrangement is the one drawn under Experiment 9.1. This time the threads stay 0.600m apart throughout and the quantity changed between readings is L, their length, measured from the support to the top surface of the bar.

Procedure

Proceed as follows:

(a) Suspend the metre rule horizontally by two vertical threads tied 0.600m apart, each of length L equal to 0.300m.

(b) Turn the bar through a small angle about the vertical axis through its centre, release it, and record the time t for 20 complete oscillations.

(c) Repeat the procedure in 9.2 (a) and (b) for L equal to 0.400m, 0.500m, 0.600m, 0.700m and 0.800m, keeping the separation of the threads unchanged.

Questions

(i) Tabulate your results, including T and T^2.

(ii) Plot a graph of T^2 (s^2) against L (m).

(iii) Determine the slope of your graph and state its units.

(iv) Using $T=\frac{4\pi k}{d}\sqrt{L/g}$, show that the slope of such a graph is $16\pi^2k^2/gd^2$.

(v) Given that $k=l/\sqrt{12}$ for a uniform bar, use your slope to determine g.

(vi) Explain why T^2 is plotted against L rather than T against L.

(vii) State which measurement in this experiment your value of g is most sensitive to, and justify your answer.

Table of results

L (m)	t for 20 oscillations (s)	T (s)	T^2 (s^2)
0.300	21.03	1.052	1.107
0.400	24.51	1.226	1.503
0.500	27.39	1.370	1.877
0.600	29.79	1.490	2.220
0.700	32.25	1.613	2.602
0.800	34.49	1.725	2.976

Making the measurement count

Practical tip 1: *The same equation gives a different straight line every time you change which symbol is the variable. Write the equation, then decide.*

Square the bifilar equation:

$$T^2 = \frac{16\pi^2 k^2}{gd^2} L$$

With d fixed, everything in front of L is a constant, so $T^2 \propto L$ and a graph of T^2 against L is a straight line through the origin. Compare that with Experiment 9.1, where L was fixed and the same equation gave T proportional to $1/d$. Nothing physical changed between the two; only the choice of which symbol to hold still.

Practical tip 2: *A slope that depends on the square of a length you measured once is worth more care than six timings.*

Look at where d sits: $16\pi^2 k^2/gd^2$ has it squared, and it is measured once, at the start, with a rule. An error of 2mm in a separation of 600mm is a third of a percent in d and therefore two thirds of a percent in g, and it repeats itself identically in every row of the table, so no amount of averaging will find it. The six timings, by contrast, scatter both ways and largely cancel. When you are deciding where to spend your care in an experiment, look for the quantity that is measured once and enters squared.

Practical tip 3: *Which arrangement to trust, and why the honest answer has two halves that point opposite ways.*

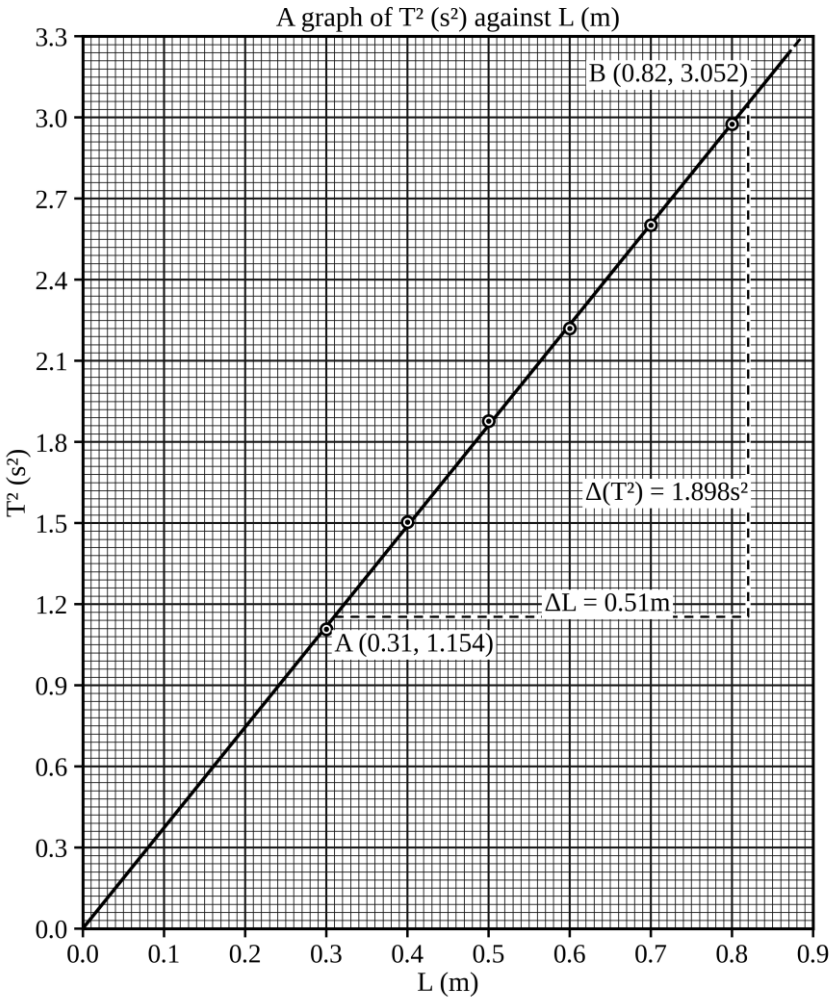
Experiment 9.1 changed d and left L alone; this one changes L and leaves d alone. The quantity you leave alone is measured once, so its error repeats identically in every row and no averaging will touch it. Ask where it lands. Here it is d, and g is $4\pi^2 l^2/3d^2$ divided by the slope, so d enters squared: half a percent in d is one percent in g. In Experiment 9.1 it is L, which enters to the first power only, so the same half a percent costs half as much. On that ground Experiment 9.1 is the safer.

Now the other half. Both versions ask you to re-tie the threads six times, and every re-tie is a chance to leave the two threads unequal. Practical tip 4 of Experiment 9.1 showed that such a pair behaves as though both threads had the average length, so the mistake amounts to an error in L. Here L is the horizontal variable and the graph is T^2 against L, so an error in L shifts the whole line sideways: the intercept moves off the origin and the slope, which is the only thing the answer uses, is untouched. In Experiment 9.1, with L fixed, the same mistake lands squarely on the slope and leaves no mark at all. On that ground this arrangement is very much the safer.

Which you trust more therefore depends on which mistake you are more likely to make: measuring one separation carelessly, or tying six pairs of threads by eye. Say so, and say why. A question that asks you to choose between two arrangements is not asking you to prefer the tidier one.

Answers to the questions

(i) and (ii). The table is printed above. Squaring the governing equation gives $T^2 \propto L$, so T^2 goes on the vertical axis and L along the horizontal one. Give it the title A graph of T^2 (s^2) against L (m). Put T^2 at 2cm to $0.3s^2$, running from 0 to $3.3s^2$, and L at 2cm to 0.1m, running from 0 to 0.9m. That is 22cm by 18cm. Both axes start at zero, because the line should pass through the origin and its doing so is evidence that nothing has been mismeasured.



(iii) Taking A ($0.31\text{m}, 1.154\text{s}^2$) and B ($0.82\text{m}, 3.052\text{s}^2$) on the line:

$$\text{slope}=\frac{3.052\text{s}^2\text{-}1.154\text{s}^2}{0.82\text{m-}0.31\text{m}}=\frac{1.898\text{s}^2}{0.51\text{m}}=3.72\text{s}^2\text{m}^{-1}$$

(iv) Squaring the governing equation,

$$T^2=\frac{16\pi^2k^2}{d^2}\times\frac{L}{g}=\left(\frac{16\pi^2k^2}{gd^2}\right)L$$

so the slope is $16\pi^2k^2/gd^2$, as required.

(v) Putting $k^2=l^2/12$ into the slope and making g the subject,

$$g=\frac{16\pi^2k^2}{d^2\times\text{slope}}=\frac{4\pi^2l^2}{3d^2\times\text{slope}}$$

$$g=\frac{39.4384\times(1.000\text{m})^2}{3\times(0.600\text{m})^2\times3.72\text{s}^2\text{m}^{-1}}=9.82\text{ms}^{-2}$$

against the accepted 9.81ms^{-2}. The two figures agree to better than a tenth of a percent, which is closer than this apparatus deserves and should be reported as luck rather than as skill.

(vi) Because T is proportional to the square root of L, so a graph of T against L is a curve and a curve has no slope you can quote. Squaring the period straightens it. The general move is the one this book keeps returning to: find the power of the variable that appears in the equation, and plot whichever combination makes that power one.

(vii) The separation of the threads. It enters the answer squared, so an error in it is doubled in g, and it is measured once rather than six times, so it cannot average away. The length of the bar enters squared too, through k, but a metre rule is a metre long by definition and there is nothing to get wrong. The timings, where most of the effort goes, are the least of the three.

Examiner's eye

- A line through the origin is a claim, and both axes must start at zero for you to be entitled to make it. Say in your comment that the line passes through the origin and what that confirms, rather than leaving the examiner to notice it.
- The slope here carries s^2m^{-1}. Check the unit before substituting: if you plotted T rather than T^2 your slope is in sm^{-1} and the formula will return nonsense with a straight face.
- Keep the separation fixed and say so. An examiner who reads a table with six lengths and no statement about d cannot tell whether you changed one quantity or two.

Practice 9.2

Kipanga tied his threads by eye. One of them turned out to be about two centimetres longer than the other, which does not sound like much until you look along the bar and see it sitting at a slope.

He got a straight line anyway. Experiment 9.1 would have let him keep it: there the same fault multiplies every reading alike and hides completely. Here it does not hide, and where it shows itself is the point of the question. That is why **Mr. Akilikubwa** made him do the whole thing again with the threads tied properly, and then work out afterwards what the first attempt would have looked like.

What follows is the experiment done with both threads the same length.

Procedure

Proceed as follows:

(a) Suspend a uniform metre rule horizontally by two vertical threads tied 0.500m apart, each of length L equal to 0.200m.

(b) Check by eye along the bar that it hangs level.

(c) Turn the bar through a small angle about the vertical axis through its centre and record the time t for 20 complete oscillations.

(d) Repeat the procedure in P9.2 (a) to (c) for L equal to 0.300m, 0.400m, 0.500m, 0.600m and 0.700m.

Questions

(i) Tabulate your results, including $\sqrt{L}$ and the periodic time T.

(ii) Plot a graph of T (s) against $\sqrt{L}$ ($\sqrt{m}$).

(iii) Determine the slope of your graph and state its units.

(iv) Show that the slope of such a graph is $4\pi k/(d\sqrt{g})$, and hence determine g, taking $k=l/\sqrt{12}$ for a uniform bar.

(v) Use your graph to find the periodic time when the threads are 0.450m long.

(vi) A second bar of the same length but twice the mass is hung on the same threads. State, with a reason, how its graph would compare with yours.

(vii) **Kipanga** ties one thread 0.020m longer than the other and repeats the whole experiment. State whether his points would still lie on a straight line, say what feature of his graph would betray the fault, and state what would be wrong with his value of g.

Experiment 9.3

The wire in the hardware stall at the top of the market comes off two reels. The *fundi* will tell you, at length and without being asked, that the one on the left is proper steel and the one on the right is something that was sold to him as steel, and that the difference is in the springiness. He is right about the difference. He has no way of putting a number on it, and neither has anybody else on that street.

Kipute bought a metre of each with her own money and carried them back to the laboratory, where there is a way. It takes a rod, two masses, a stopwatch and a micrometer, and it will tell you the shear modulus of a wire to within a few percent.

Of those four, the micrometer is the one that is not optional, and the reason is arithmetic rather than fussiness.

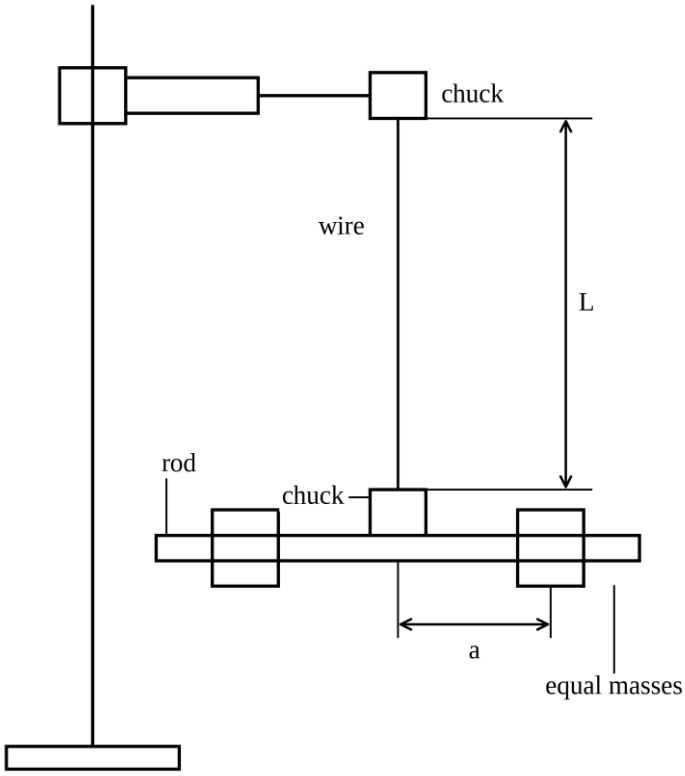
Aim

To determine the shear modulus of a steel wire, and the moment of inertia of a rod, using a torsion pendulum.

Apparatus

A steel wire of diameter about 1.00mm and length 0.500m, two chucks, a uniform rod, two equal masses of 0.0500kg that can be clamped to the rod, a micrometer screw gauge, a metre rule and a stopwatch reading to 0.01s.

Setup



The wire hangs from a chuck fixed in a clamp and carries a second chuck at its lower end, which grips the middle of a horizontal rod. The two masses are clamped to the rod at equal distances a from the wire. The system is set oscillating by turning the rod through a small angle about the wire and releasing it. L is the length of wire between the chucks.

Procedure

Proceed as follows:

(a) Measure the diameter of the wire with the micrometer screw gauge at six places along it, turning the wire between readings, and take the mean.

(b) Set the length of wire between the chucks to L equal to 0.500m.

(c) Clamp the two masses to the rod so that each is a distance a equal to 0.020m from the wire.

(d) Turn the rod through a small angle about the wire, release it, and record the time t for 20 complete oscillations.

(e) Repeat the procedure in 9.3 (c) and (d) for a equal to 0.040m, 0.060m, 0.080m, 0.100m and 0.120m.

Questions

(i) Tabulate your results, including a^2, T and T^2.

(ii) Plot a graph of T^2 (s^2) against a^2 (m^2).

(iii) Determine the slope of your graph and the intercept it makes on the T^2 axis.

(iv) The periodic time of a torsion pendulum is $T=2\pi\sqrt{I/C}$, where C is the torsional stiffness of the wire and I the moment of inertia of everything the wire carries. Taking $I=I_0+2ma^2$, show that the slope of your graph is $8\pi^2 m/C$, and hence determine C.

(v) For a wire of radius r and length L the torsional stiffness is $C=\pi\eta r^4/2L$, where η is the shear modulus of the material. Determine η.

(vi) Use the intercept to determine I_0, the moment of inertia of the rod and chuck alone.

(vii) The diameter of the wire was measured with a micrometer rather than a rule. Justify that choice with a calculation.

Table of results

a (m)	a^2 (m^2)	t for 20 oscillations (s)	T (s)	T^2 (s^2)
0.020	0.0004	21.05	1.053	1.109
0.040	0.0016	23.95	1.198	1.435
0.060	0.0036	27.71	1.386	1.921
0.080	0.0064	32.51	1.626	2.644
0.100	0.0100	37.65	1.883	3.546
0.120	0.0144	43.25	2.163	4.679

Making the measurement count

Practical tip 1: *Two masses on a rod. Add their moments of inertia to the rod's, and the graph separates the two for you.*

The wire carries a rod, a chuck and two clamped masses. The rod and the chuck keep the same moment of inertia throughout, call it I_0; each mass, treated as a small body at distance a from the axis, contributes ma^2. So

$$I=I_0+2ma^2$$

and putting that into $T=2\pi\sqrt{I/C}$ and squaring,

$$T^2=\frac{4\pi^2}{C}\left(I_0+2ma^2\right)=\frac{8\pi^2 m}{C}a^2+\frac{4\pi^2 I_0}{C}$$

The graph therefore does two jobs at once. Its slope carries only the known mass and the unknown stiffness, so it gives C without any knowledge of the rod; its intercept then gives the rod, using the C you have just found. A quantity you cannot weigh, cannot measure and would not know how to calculate falls out of the position of a line on a piece of paper.

Practical tip 2: *The fourth power. The chapter opened with it; here is what it costs, in numbers.*

The stiffness of a wire depends on r^4. Work out what that costs you. If your measured radius is one percent too large, then r^4 is about $4\times1\%$, four percent too large, and since η is proportional to $1/r^4$ your shear modulus comes out four percent too small. A wire of true diameter 1.00mm measured as 1.02mm, because nobody checked the micrometer for zero error, gives a shear modulus eight percent out, and there is nothing in the graph, the timings or the arithmetic to warn you.

That is why the micrometer, and why six readings rather than one. Drawn wire is not round to begin with and it is not uniform along its length, so take the diameter at several places and at several orientations and use the mean. Then halve it: the formula is in the radius, and forgetting to halve gives a shear modulus sixteen times too small, which at least has the merit of being obvious.

Practical tip 3: *Small twists, and let it settle.*

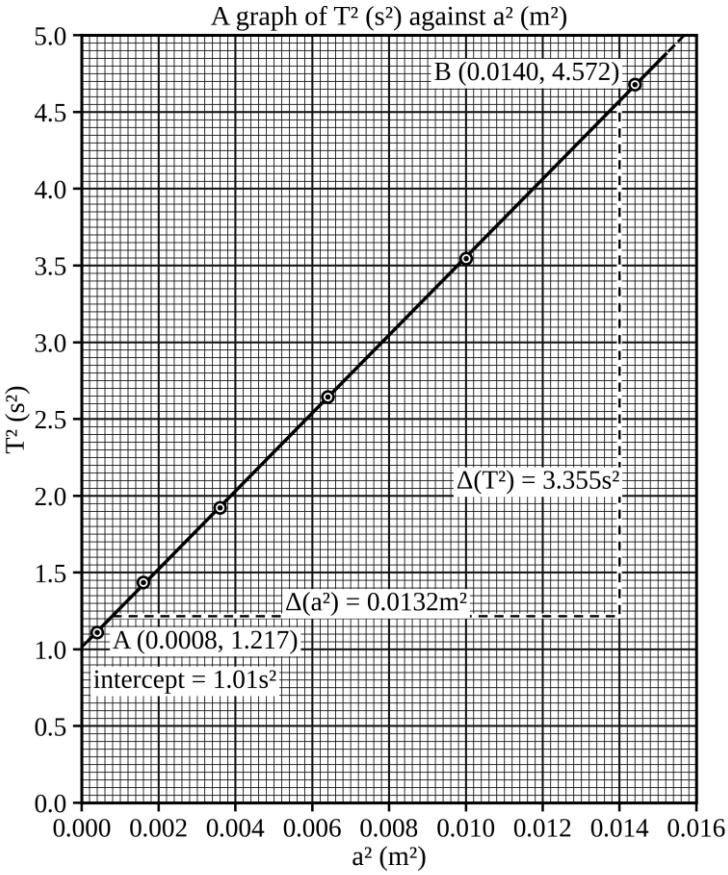
Turn the rod through a small angle only. A wire twisted too far does not come back to where it started; it takes a permanent set, and once it has, the readings taken before and after it happened do not belong to the same experiment. A quarter of a turn is already too much for a thin wire. Turn the rod through perhaps twenty degrees, watch two oscillations to check that the rod is twisting and not also swinging like a pendulum, and then start counting.

Practical tip 4: *Where to spend your care.*

Set the three measurements side by side. The timings are good to about a quarter of a percent and there are six of them, so they largely cancel. The length of the wire is good to a millimetre in five hundred, a fifth of a percent, and it enters to the first power. The diameter is good to perhaps a hundredth of a millimetre in one, one percent, and it enters to the fourth. The diameter is the experiment. Everything else is bookkeeping.

Answers to the questions

(i) and (ii). The table is printed above. From Practical tip 1 the relation is a straight line of T^2 against a^2, so T^2 goes on the vertical axis. Give it the title A graph of T^2 (s^2) against a^2 (m^2). Put T^2 at 2cm to $0.5s^2$, running from 0 to $5.0s^2$, and a^2 at 2cm to $0.002m^2$, running from 0 to $0.016m^2$. That is 20cm by 16cm, which fills an A4 sheet upright. Both axes must start at zero, because the intercept is one of the answers.



(iii) Taking A ($0.0008m^2$, $1.217s^2$) and B ($0.0140m^2$, $4.572s^2$):

$$\text{slope}=\frac{4.572s^2-1.217s^2}{0.0140m^2-0.0008m^2}=\frac{3.355s^2}{0.0132m^2}=254s^2m^{-2}$$

and the line meets the T^2 axis at $1.01s^2$.

(iv) The derivation is in Practical tip 1. The slope is $8\pi^2m/C$, so

$$C=\frac{8\pi^2m}{\text{slope}}=\frac{8\times3.14^2\times0.0500kg}{254s^2m^{-2}}=0.01553Nm$$

C is carried to four figures because it is used twice more, in (v) and in (vi). Round an intermediate that is going to be substituted again and you throw away accuracy you cannot get back; round at the end instead.

(v) Rearranging $C=\pi\eta r^4/2L$ for the shear modulus, with r half the measured diameter, so 0.500mm or $5.00\times10^{-4}m$:

$$\eta=\frac{2LC}{\pi r^4}=\frac{2\times0.500m\times0.01553Nm}{3.14\times(5.00\times10^{-4}m)^4}$$

$$\eta=\frac{0.01553}{1.963\times10^{-13}}Nm^{-2}=7.91\times10^{10}Nm^{-2}$$

The accepted shear modulus of steel is about $8.00\times10^{10}\text{Nm}^{-2}$, so this result is low by rather more than one percent. For a quantity that depends on the fourth power of a measured diameter that is a good result, and your comment should say which measurement it hangs on.

The right-hand reel, measured the same way the following afternoon, came out at less than half of that. The fundi was right about his two wires, and now there is a number for it.

(vi) The intercept is $4\pi^2 I_0/C$, so

$$I_0=\frac{\text{intercept}\times C}{4\pi^2}=\frac{1.01\text{s}^2\times0.01553\text{Nm}}{39.4384}=3.98\times10^{-4}\text{kgm}^2$$

(vii) Because the shear modulus depends on the fourth power of the radius. A metre rule reads to about 0.5mm, which on a wire of diameter 1.00mm is fifty percent, and fifty percent in the radius is a factor of five in r^4: the answer would be worthless. A micrometer reads to 0.01mm, which is one percent, and one percent in the radius is four percent in η. That is the whole justification, and it is worth writing out in full rather than saying that a micrometer is more accurate.

Examiner's eye

- The formula is in the radius and the micrometer gives you the diameter. Halve it, write the halving down, and check the size of your answer: a shear modulus of $5.00\times10^9\text{Nm}^{-2}$ for a steel wire is sixteen times too small and means exactly one thing.
- Both axes from zero. The intercept is the moment of inertia of the rod, and a graph whose a^2 axis starts at 0.002m^2 cannot give it.
- C is a couple per radian, and since the radian carries no unit it comes out simply in Nm. Quote it with that unit and carry it through to the next part rather than starting again from the slope: two substitutions are two chances to lose a power of ten.
- η and G are the same quantity, and a paper may use either. If the question calls it the modulus of rigidity, answer about the modulus of rigidity; do not introduce a symbol the question did not use.
- Take the mean of several diameters and say how many you took. One reading of a drawn wire tells you about one place on it.

Practice 9.3

A letter arrived at the school in the second week of term, addressed to **Mr. Akilikubwa** and written by a student who had left three years before and was now working at a workshop on the lake road.

Sir, we are making hinges from brass wire and the manager says the new wire is not as stiff as the old, but he will not say how he knows. I remember the experiment with the rod and the two weights. Here we have no way of changing where the weights sit, because the rod is welded. Can it still be done?

It can, and this question is the reply **Mr. Akilikubwa** sent back. Everything about the rod is left alone; what changes is the length of wire between the chucks.

Procedure

Proceed as follows:

(a) Hang a rod from a brass wire of diameter 1.20mm held in chucks, and clamp two masses to the rod. Do not move them again. The moment of inertia of the rod and its masses about the wire is $1.20\times10^{-3}\text{kgm}^2$.

(b) Set the length of wire between the chucks to L equal to 0.300m.

(c) Turn the rod through a small angle about the wire, release it, and record the time t for 20 complete oscillations.

(d) Repeat the procedure in P9.3 (b) and (c) for L equal to 0.400m, 0.500m, 0.600m, 0.700m and 0.800m.

Questions

(i) Tabulate your results, including T and T^2.

(ii) Plot a graph of T^2 (s^2) against L (m).

(iii) Determine the slope of your graph and state its units.

(iv) Using $T=2\pi\sqrt{I/C}$ and $C=\pi\eta r^4/2L$, show that the slope of such a graph is $8\pi I/\eta r^4$.

(v) Determine the shear modulus of the brass.

(vi) State what the line passing through the origin confirms about the apparatus.

(vii) The workshop manager's micrometer has never been checked and reads 0.05mm high, so he records the diameter as 1.25mm instead of 1.20mm. Calculate the shear modulus he would obtain from the same timings, and comment.

Experiment 9.4

There is one more way to measure the stiffness of a wire, and it uses no clock at all.

Kipute had gone back down to the market for two metres more of the left-hand reel, the one the *fundi* swore was proper steel and the one Experiment 9.3 had put a number to. **Mr. Akilikubwa** took the wire from her, wound a length of it round a test tube by hand, slid the coil off and let it hang from his finger.

"What is this?"

"A spring, sir."

"It is a straight wire that has been bent into a circle. Nothing in it bends when you pull it. It twists. Hang a load on it, measure how far it comes down, and you will have measured the same quantity you measured with the stopwatch last week, and if the two agree you will have done something worth doing."

That is the whole of this experiment. A spring is a torsion experiment wearing a disguise, and the disguise is thin enough to see through in five lines of algebra.

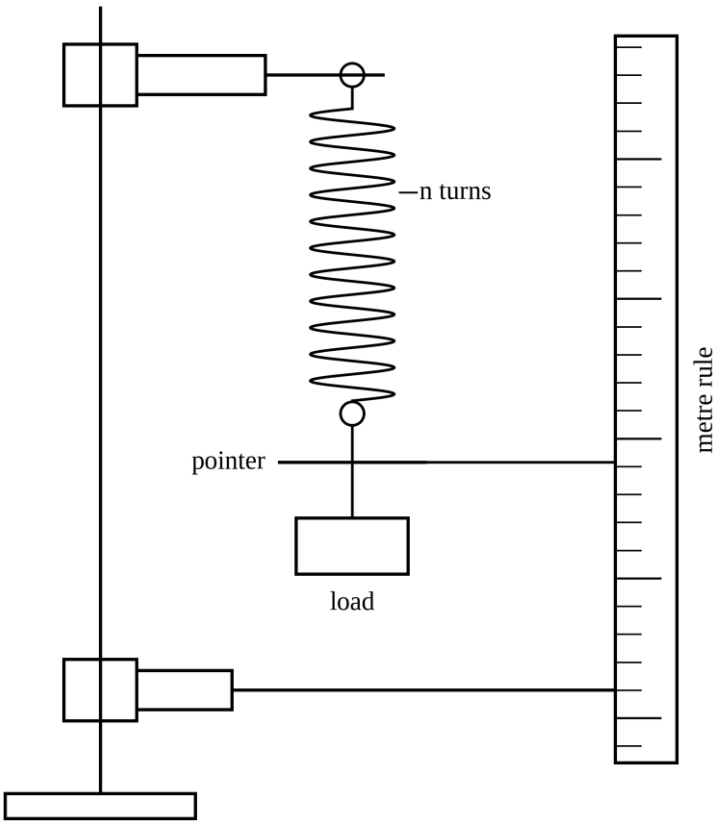
Aim

To determine the modulus of rigidity of steel using a helical spring.

Apparatus

About two metres of steel spring wire of diameter about 1.00mm, a test tube about 17mm in outside diameter to wind it on, a retort stand with clamp and boss, a metre rule clamped vertically, a light pointer, a mass hanger with slotted masses, a micrometer screw gauge and a pair of vernier callipers.

Setup



The spring hangs from the clamp with a light pointer fixed to its lower end, moving against a metre rule clamped vertically beside it. Loads are hung from the bottom of the spring and the pointer is read against the rule. The extension e for any load is the reading with that load less the reading with no load at all.

Procedure

Proceed as follows:

(a) Wind 30 close turns of the wire tightly on to the test tube, leaving a straight length at each end, slide the coil off and bend the two ends into hooks. Count the turns again on the finished spring.

(b) Measure the diameter d of the wire with the micrometer at six places along it, turning the wire between readings as in Experiment 9.3, and take the mean. Measure the outside diameter of the coil with the callipers, and subtract one wire diameter to obtain the mean coil diameter D.

(c) Hang the spring from the clamp, fix the pointer to its lower end, and record the reading of the pointer against the vertical rule with no load on the spring.

(d) Hang a mass m of 100g from the spring, allow it to come to rest, and record the new reading of the pointer.

(e) Repeat the procedure in 9.4 (d) for masses of 200g, 300g, 400g, 500g and 600g.

(f) Remove the masses one at a time and check that the pointer returns to its original no-load reading.

Questions

(i) Tabulate your results, including the extension e of the spring in millimetres.

(ii) Plot a graph of e (mm) against m (g).

(iii) Determine the slope of your graph and state its units.

(iv) The extension of a close-coiled helical spring of n turns of mean coil diameter D, wound from wire of diameter d, under a load F is $e=\frac{8FD^3n}{Gd^4}$, where G is the modulus of rigidity of the wire. Show that the slope of your graph is $8gD^3n/Gd^4$, and explain why a slope in mmg^{-1} may be used in that expression as it stands.

(v) Determine the modulus of rigidity of the steel.

(vi) State what the position of your line relative to the origin tells you, and what a line meeting the e axis above the origin would have meant.

(vii) The wire diameter was measured to 0.01mm and the coil diameter to 0.1mm. Compare the effect of these two uncertainties on your answer, and say which measurement deserved the greater care.

Table of results

The pointer read 328.0mm with no load on the spring.

m (g)	Pointer reading (mm)	e (mm)
100	348.6	20.6
200	371.1	43.1
300	391.5	63.5
400	414.5	86.5
500	435.0	107.0
600	455.8	127.8

Making the measurement count

Practical tip 1: *A spring is a twisted wire. Five lines take you from the torsion formula of Experiment 9.3 to the spring formula, and they are the same physics.*

Hang a load F on the spring and cut the wire anywhere you like. The load hangs a distance $D/2$ from the axis of the wire at the cut, so it applies a couple about that axis of

$$\text{couple}=F\times\frac{D}{2}$$

and the same couple acts at every point along the wire, because the load is the same distance from the axis wherever you cut. Now straighten the spring out in your mind. It is a single wire whose length L is

$$L=\pi Dn$$

n turns, each of circumference πD, twisted from one end to the other by that couple. Experiment 9.3 gives the twist, and in the same symbols: a wire of length L and radius r has torsional stiffness $\pi Gr^4/2L$, so the angle of twist is the couple divided by the stiffness,

$$\phi=\frac{FD}{2}\times\frac{2L}{\pi Gr^4}=\frac{FD\times\pi Dn}{\pi Gr^4}=\frac{FD^2n}{Gr^4}$$

The wire has radius $r=d/2$, so $r^4=d^4/16$ and

$$\phi=\frac{16FD^2n}{Gd^4}$$

Finally, the far end of that twisted wire carries the load at a distance $D/2$ from the axis, so as the wire turns through ϕ the load descends by

$$e=\phi\times\frac{D}{2}=\frac{8FD^3n}{Gd^4}$$

which is the formula the question gives you. It is worth doing once, because after it you will never again wonder why the modulus of rigidity and not the Young modulus governs a spring, and you will never write r where the question wants d.

Practical tip 2: *There is no stopwatch in this experiment. Every uncertainty you have is a length, and they do not all count the same.*

Three measured lengths go into the answer, and the exponent each one carries decides how much of your care it deserves.

- The wire diameter enters as d^4. The micrometer reads it to about 0.01mm in 1.00mm, one percent, and the fourth power turns that into four percent.
- The coil diameter enters as D^3. The callipers themselves resolve far better than 0.1mm, but a hand-wound coil is not perfectly round and that is about as well as its diameter can be known. In 19.0mm that is rather over half a percent, and the cube turns it into not quite two percent.
- The number of turns enters as n to the first power, and it is not measured at all: it is counted. Miscount one turn in thirty and you are three percent out, which is more than the coil diameter costs you. Counting is not free.

The extensions themselves are the least of your worries, because there are six of them and the graph averages them. Spend your time on the micrometer, then on the count, then on the callipers, and take the pointer readings briskly.

Practical tip 3: *Winding the spring, and what* D actually means.

Wind the turns tight against one another and keep the wire under tension against the former the whole way, or the coil will come off uneven and the turns at one end will carry more of the twist than the turns at the other. Leave a straight length at each end for the hooks; those straight ends are not turns and must not be counted as turns.

The coil springs open when you slide it off the tube, and by more than you would guess: this one was wound on a 17mm tube, which would have given a mean coil diameter of 18.0mm if it had held its shape, and measured a full millimetre wider than that once it was off. Measure the spring, never the former.

The callipers give you the outside diameter of the coil, which is greater than the diameter of the circle the wire itself travels round by one wire diameter, half a wire on each side. Hence

$$D=D_{outside}-d$$

and this spring read 20.0mm across the outside with 1.00mm wire, so D is 19.0mm. Forgetting to subtract puts D up by five percent and the answer up by sixteen.

Practical tip 4: *Take the load off again. If the pointer does not come back, the readings are worthless and so is the spring.*

A spring obeys $e=\frac{8FD^3n}{Gd^4}$ only while the wire stays elastic. Overload it and the wire takes a permanent set: the coils never close up again, the no-load reading has moved, and the extensions you measured after that moment were measured from a zero that no longer exists. The graph does not warn you, because the later points still lie on a line: a line with the wrong slope.

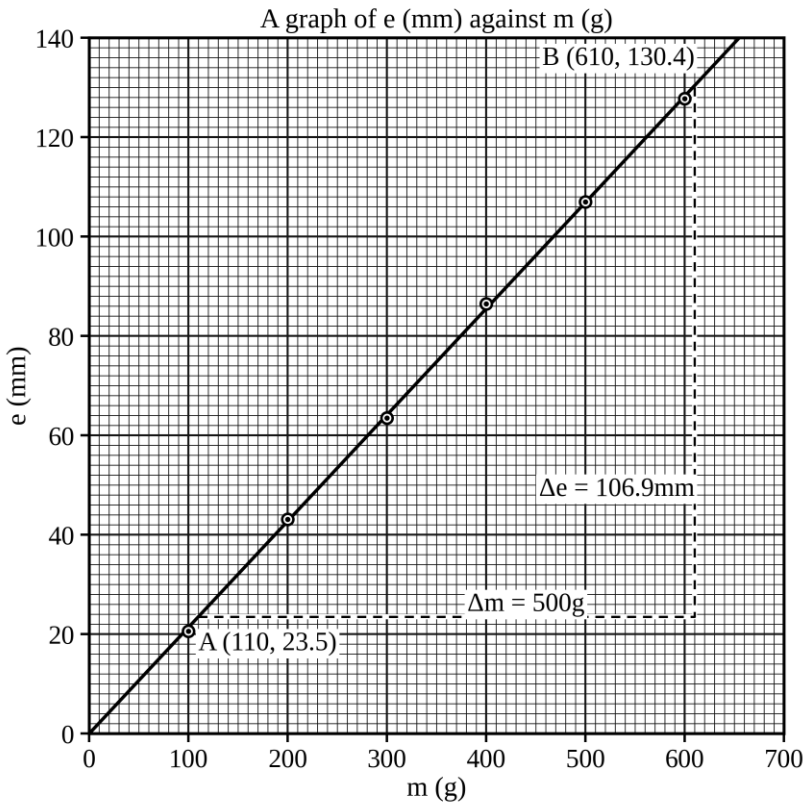
While you are about it, be clear about what the rule can tell you. Measuring the diameter of a wire with a rule means lining up two edges at once and is good to about half a millimetre, which is why Experiment 9.3 will not allow it. Reading a fine pointer against a millimetre scale is a different job: there is one edge, the scale is behind it, and with your eye level you can put the reading to a fifth of a division without embarrassment. That is where the tenths of a millimetre in the table come from, and they are honest only while the pointer stays fine and the eye stays level.

The check costs a minute. Unload the spring one mass at a time and read the pointer again with nothing on it. If it reads what it read at the start, every point in your table is trustworthy. If it does not, wind a new spring and stop one or two loads earlier. Read the pointer with your eye level with it as well, because a millimetre of parallax in an extension of twenty is worth more than anything the micrometer can do for you.

Answers to the questions

(i) The table is printed above. The extension is the pointer reading less the no-load reading of 328.0mm, and it belongs in the table as a column of its own, not worked out on the graph paper.

(ii) Put e on the vertical axis, since the load is what you chose and the extension is what followed. Give the graph the title A graph of e (mm) against m (g). Take e at 2cm to 20mm, running from 0 to 140mm, and m at 2cm to 100g, running from 0 to 700g. That is 14cm by 14cm; halving either scale to fill more of the paper would give 28cm, which will not fit, so this is the largest square available. Both axes start at zero, because the origin is part of the evidence here.



(iii) Taking A (110g, 23.5mm) and B (610g, 130.4mm):

$$\text{slope}=\frac{130.4\text{mm}-23.5\text{mm}}{610\text{g}-110\text{g}}=\frac{106.9\text{mm}}{500\text{g}}=0.214\text{mmg}^{-1}$$

(iv) The load is a weight, so F=mg, and

$$e=\frac{8FD^3n}{Gd^4}=\left(\frac{8gD^3n}{Gd^4}\right)m$$

which is e proportional to m: a straight line through the origin of slope $8gD^3n/Gd^4$.

As to the units, a slope of 0.214mmg^{-1} is a slope of 0.214mkg^{-1}, because dividing the top and the bottom of a fraction by a thousand leaves it unchanged. The number may therefore be used in the formula with metres and kilograms everywhere else and no conversion factor at all. Say this in your answer; it looks like an omission if you do not.

(v) Rearranging for the modulus of rigidity, with d equal to 1.00×10^{-3}m, D equal to 1.90×10^{-2}m and n equal to 30:

$$G=\frac{8gD^3n}{d^4\times\text{slope}}=\frac{8\times9.81\text{ms}^{-2}\times\left(1.90\times10^{-2}\text{m}\right)^3\times30}{(1.00\times10^{-3}\text{m})^4\times0.214\text{mkg}^{-1}}$$

$$G=\frac{1.615\times10^{-2}}{2.14\times10^{-13}}\text{Nm}^{-2}=7.55\times10^{10}\text{Nm}^{-2}$$

The accepted modulus of rigidity of steel is about $8.00\times10^{10}\text{Nm}^{-2}$, so this is low by not quite six percent, which is what Practical tip 2 said to expect. It is also worth putting beside the $7.91\times10^{10}\text{Nm}^{-2}$ that Experiment 9.3 obtained from the same reel of wire with a stopwatch and no spring at all. Two methods that share nothing but the wire, one of them dynamic and one of them static, agree to five percent. That agreement is a better argument for the physics than either number is on its own.

(vi) The line passes through the origin, which says that the extension is measured from the true unloaded position of the pointer and that the spring was obeying the formula from the very first load. A line meeting the e axis above the origin would have meant one of two things: that the no-load reading was taken wrongly, or that the turns of the spring were touching one another at the start, so that the first few grams were spent opening the coil rather than twisting the wire. Neither is fatal, because the slope is still the answer, but both must be reported.

(vii) The wire diameter enters as d^4 and the coil diameter as D^3, so

$$\frac{0.01\text{mm}}{1.00\text{mm}}=1\% \quad\rightarrow\quad 4\% \quad\text{in}\quad G$$

$$\frac{0.1\text{mm}}{19.0\text{mm}}=0.5\% \quad\rightarrow\quad 1.6\% \quad\text{in}\quad G$$

The wire diameter deserves the greater care, by a factor of rather more than two. Percentage uncertainties in a product are added, so the two diameters allow about six percent between them, and the answer is six percent from the accepted value: the discrepancy is inside the two measurements and there is no need to look for a cause beyond them. Note also where the coil diameter sits. Its two percent is beaten by a single miscounted turn, which costs three, and a miscount cannot be cured by buying a better instrument.

Examiner's eye

- D is the mean coil diameter and the callipers give you the outside diameter. Subtract one wire diameter, write the subtraction down, and do not measure the test tube.
- The slope is in mmg^{-1} and goes into the formula unchanged. A candidate who multiplies by a thousand somewhere gets an answer a thousand times out and usually writes it down without blinking.
- A modulus of rigidity comes out in Nm^{-2} and should be some tens of 10^9. Putting both diameters in as millimetres divides the answer by exactly a thousand, because G goes as D^3 over d^4, so an answer of $7.55\times10^7\text{Nm}^{-2}$ is that mistake and nothing else.
- Count the turns on the spring you loaded, not the turns you meant to wind. The two are different more often than anybody admits.
- There is no timing in this experiment, which tempts candidates to hurry it. The marks are in the micrometer, and the micrometer is what gets rushed.

Practice 9.4

The preparation room needed spring balances and had none. What it had was a reel of steel wire, a test tube and a store of slotted masses, so **Mr. Akilikubwa** set the class to make six springs on the same test tube from the same reel, differing in one respect only: the number of turns.

"Six springs is not a spring balance," said **Kipute**.

"No," said **Kipanga**, *"but six springs will tell you how many turns a spring balance wants, and then we only have to wind one more."*

"Make them," said **Mr. Akilikubwa**.

Procedure

Proceed as follows:

(a) Wind six springs on the same test tube from the same reel of steel wire, of 10, 20, 30, 40, 50 and 60 close turns. The wire has diameter 0.800mm and the mean coil diameter of every spring is 14.0mm.

(b) Hang the first spring from a clamp, fix a light pointer to its lower end, and record the reading of the pointer against a vertical metre rule with no load on the spring.

(c) Hang a mass of 200g from the spring and record the new reading of the pointer.

(d) Repeat the procedure in P9.4 (b) and (c) for each of the other five springs, using the same load throughout.

Questions

(i) Tabulate your results, including the extension e.

(ii) Plot a graph of e (mm) against n.

(iii) Determine the slope of your graph and state its units.

(iv) Using $e=\frac{8FD^3n}{Gd^4}$, show that the slope of such a graph is $8FD^3/Gd^4$.

(v) Determine the modulus of rigidity of the steel.

(vi) The hooks at the ends of the springs were all bent to the same pattern. State what the position of your line relative to the origin tells you about them.

(vii) A spring balance is wanted that extends by 100mm under a load of 200g. Use the slope of your graph to find how many turns it should have, and give one reason for treating that number with caution.

Chapter 10

SPRINGS AND HOOKE'S LAW

A pendulum borrows its stiffness from gravity, and gravity takes it back when you carry the apparatus to the moon. A twisted wire supplies its own, and Chapter 9 put a number on it. A spring supplies its own as well, and it is the one stiffness in this book you can see with your eyes: hang a load on a spring and it settles a certain distance lower, and that distance is the stiffness telling you what it is.

Two sentences carry the whole chapter. The first is that the extension is proportional to the force stretching the spring,

$$F=ke$$

where k is the spring constant, the force needed for unit extension, measured in Nm^{-1}. The second is that a mass M hung on the same spring and set bouncing keeps time,

$$T=2\pi\sqrt{\frac{M}{k}}$$

which is the inertia-over-stiffness pattern of Chapters 8 and 9 with the stiffness at its plainest. The k in the second equation is the k of the first. A rule and a stopwatch are measuring the same number two different ways, and everything in this chapter comes of putting them side by side.

The two equations do not need the same things to be known beforehand. The first turns masses into forces, so a value of g has to be carried into it before it can start; the second needs none. Working that difference is what the four varieties are for.

One warning belongs at the front, as it did in Chapter 9, because it governs where you spend your care. There the warning was a fourth power. Here it is a zero. Every extension in this chapter is a difference of two readings, the loaded one and the unloaded one, so the no-load reading enters every row that carries an extension; and a spring can be made to lie about where its own zero is. Nothing here is hard to measure. What is hard is being sure where zero is.

The four varieties at a glance

	What is different about it	What the graph gives you
10.1	The load is changed and the extension read with a rule	the spring constant, from a straight line
10.2	The same spring set bouncing, and the load changed	the spring constant again, and the mass of the spring
10.3	A calorimeter on the spring, and the volume of water in it changed	the mass of the empty calorimeter, without weighing it
10.4	The extension and the period of one spring, measured together	the acceleration due to gravity, with no pendulum

Experiment 10.1 is worked below in full. The second variety hangs the same spring from the same clamp and puts the rule away; the third changes what is hanging on it; the fourth changes nothing at all, and reads the rule and the stopwatch on the same load.

Experiment 10.1

Somebody had printed a card in capitals and tied it to the spring balance on the shelf in the science store:

DO NOT HANG MORE THAN FIVE KILOGRAMS ON THIS BALANCE. IT HAS ALREADY CARRIED TWENTY, ONCE, AND IT HAS NOT FORGIVEN.

The hand was the storekeeper's. **Mr. Akilikubwa** took the balance down and held it up so that **Kipute** could see the pointer, which was sitting well below the zero of the scale with nothing hanging on it at all.

"It thinks it is carrying something, sir."

"It does. The card says what was done to it. What the card does not say is what that has done to its readings, and neither can you, until you know what a spring does when nobody has abused it." He put it back on its nail. *"Take one off the shelf that is still behaving itself, and find out."*

Aim

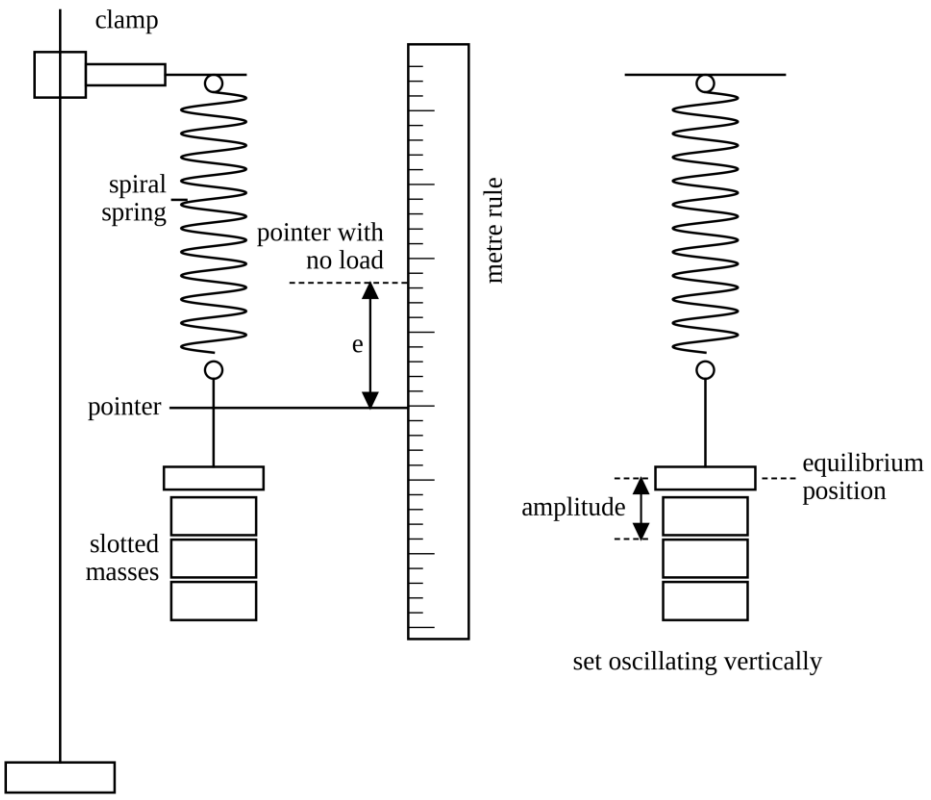
To determine the spring constant of a spiral spring.

Apparatus

A spiral spring of unstretched length about 0.25m, a retort stand with clamp and boss, a metre rule clamped vertically beside the spring, a light pointer, and a mass hanger with slotted masses, each of 20g.

Setup

The spring hangs from the clamp with a light pointer fixed to its lower end, moving against the metre rule. The loads hang from the lower end of the spring, below the pointer. The extension e produced by a load is the reading of the pointer with that load on the spring less its reading with no load at all. The right-hand drawing shows the same spring with its load set oscillating vertically, which is the arrangement of Experiment 10.2.



Procedure

Proceed as follows:

(a) Hang the spring from the clamp and fix the light pointer to its lower end so that it moves freely against the vertical metre rule.

(b) Record the reading of the pointer with no load on the spring.

(c) Hang a total mass M of 40g, the hanger included, from the lower end of the spring, allow it to come to rest, and record the reading of the pointer.

(d) Repeat the procedure in 10.1 (c) with masses of 80g, 120g, 160g, 200g and 240g.

(e) Remove the masses one at a time and record the reading of the pointer once more with no load on the spring.

Questions

(i) Tabulate your results, including the force F stretching the spring and the extension e of the spring.

(ii) Plot a graph of e (mm) against F (N).

(iii) State the relationship between e and F shown by your graph, and say what the position of your line relative to the origin confirms.

(iv) Determine the slope of your graph and state its units.

(v) Use your slope to determine the spring constant k of the spring.

(vi) At the end of the experiment the pointer read the same with no load as it had at the beginning. State what that tells you about the spring. Then state, with a reason, whether a spring balance whose pointer rests below the zero of its scale with nothing hanging on it reads too large or too small.

(vii) A second spring, wound from the same wire on the same former but with twice as many turns, is loaded in the same way. State, with a reason, how its graph would compare with yours.

Table of results

The pointer read 176.0mm with no load on the spring, both before the first load was hung on it and after the last had been taken off.

M (g)	F (N)	Pointer reading (mm)	e (mm)
40	0.392	208.0	32.0
80	0.785	240.4	64.4
120	1.177	270.9	94.9
160	1.570	302.8	126.8
200	1.962	335.5	159.5
240	2.354	367.8	191.8

Making the measurement count

Practical tip 1: *What Hooke's law says, and what the spring constant is a property of.*

The law is a statement about proportionality and nothing else: for a spring stretched within its limits, the extension is proportional to the force applied. The constant of proportionality, written the way F=ke writes it, is the force needed for unit extension, so a stiff spring has a large spring constant and a floppy one a small one, and the unit of both is the newton per metre.

Be clear about what k belongs to. It is a property of that spring, not of the steel it is made of: Chapter 9 showed that the same wire wound into more turns gives a spring that stretches further under the same load. The material property is the modulus of rigidity, which Chapter 9 measured and this chapter never mentions again.

Practical tip 2: *The newtons in your table were not measured. They were multiplied.*

You weighed nothing in this experiment. You took the mass stamped on each slotted mass, in grams, turned it into kilograms and multiplied by $9.81ms^{-2}$ to get a weight in newtons, because a force in newtons is a mass in kilograms times the acceleration due to gravity. So the first column of your table is a set of labels and the second is arithmetic done on them. Both carry the uncertainty of the labels, and for school slotted masses that is about one percent.

Count the hanger. It is part of what the spring is holding up and its own mass is stamped on it, so the mass in your table is the hanger together with the slotted masses riding on it. Leave the hanger out and every force in

the column is short by the same amount, which is the one mistake here that puts a false intercept on an otherwise faultless graph.

Practical tip 3: *A wrong zero spoils the origin and leaves the slope untouched, which is why the origin is worth arguing about.*

Take the no-load position of the pointer carelessly and you have not spoiled one reading; you have shifted all six by the same amount and in the same direction. Look at what that does to the graph. Every point moves up, or every point moves down, by the same distance, so the line keeps its slope exactly and moves bodily off the origin. The spring constant survives it completely; the evidence that the extension is proportional to the force does not.

Two habits follow. Take the no-load reading with the same care as the others, with your eye level with the pointer, because the origin of your graph is evidence and part (iii) asks you to use it. And keep the extension in a column of its own. The quantity you plot is the difference, not the reading. Do the subtraction over the top of the readings and the column ends up holding two kinds of number with nothing to tell them apart; do it in your head at the plotting stage and the table keeps no record of what you plotted.

Practical tip 4: *Two limits, and why it matters when the spring passed them.*

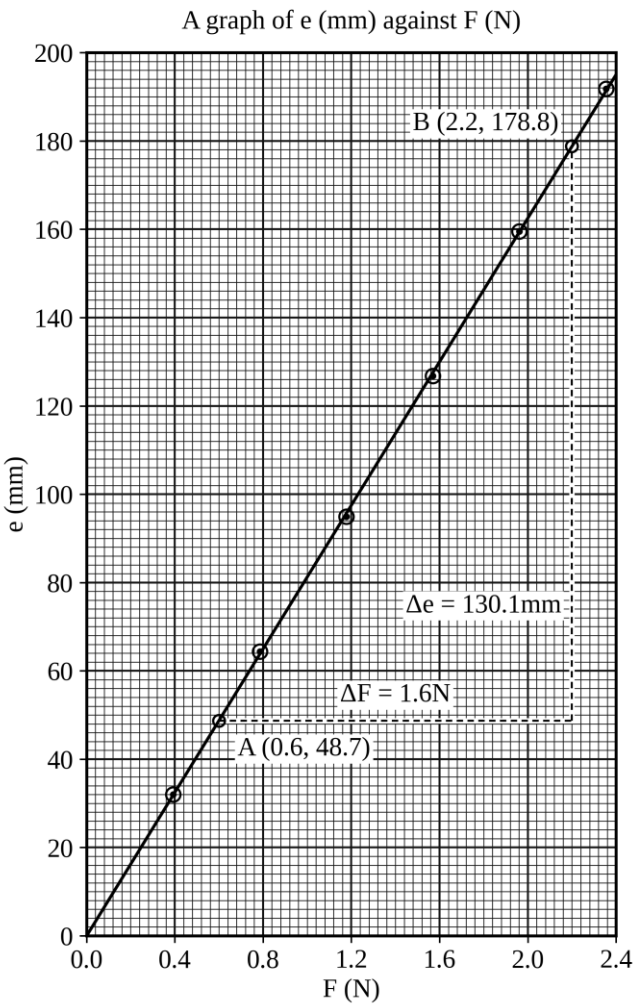
Load a spring far enough and two different things happen, one after the other. First the graph stops being straight: the extension is no longer proportional to the force, and the load at which that begins is the limit of proportionality. Then, a little further on, the spring stops coming back: unloaded, it is longer than it was, and that load is the elastic limit. Past it the wire has taken a permanent set, exactly as the wire in Chapter 9's torsion pendulum does when it is twisted too far.

The two limits come in that order and lie close together, so a graph that is straight from end to end is fair evidence that you stayed inside both. What a graph cannot show you is a set the spring took before you began. A spring that somebody overloaded last term is permanently longer than it was, and it will still give you a straight line through the origin and a perfectly good spring constant, because you took the no-load reading yourself, after the damage. What it will no longer do is agree with a scale engraved before the damage. Part (vi) asks which way that error runs.

The dangerous case is a set taken in the middle of your own readings. The rows after it are measured from a zero that has moved, so they sit too high, the line comes out too steep and the spring constant too small. Step (e) is the only check there is, and part (vi) asks you what it caught.

Answers to the questions

(i) and (ii). The table is printed above. F is the weight of the load, its mass in kilograms multiplied by 9.81ms^{-2}, so 40g gives 0.392N; e is the pointer reading less the no-load reading of 176.0mm. The extension is what followed from the load, so it belongs on the vertical axis. Give the graph the title *A graph of e (mm) against F (N)*. Take e at 2cm to 20mm, running from 0 to 200mm, and F at 2cm to 0.4N, running from 0 to 2.4N. That is 20cm by 12cm, which fits an A4 sheet upright. Both axes must start at zero, because part (iii) asks about the origin.



(iii) The points lie on a straight line through the origin, so the extension is directly proportional to the force: double the force and you double the extension, which is Hooke's law. The line passing through the origin confirms two things. The extensions were measured from the true unloaded position of the pointer, and the spring was obeying the law from the very first load, with no part of the first load spent opening turns that were touching one another at the start.

(iv) Take two points on the line, well apart. Taking A (0.6N, 48.7mm) and B (2.2N, 178.8mm):

$$\text{slope}=\frac{178.8\text{mm}-48.7\text{mm}}{2.2\text{N}-0.6\text{N}}=\frac{130.1\text{mm}}{1.6\text{N}}=81.3\text{mmN}^{-1}$$

The unit is millimetres per newton, a length divided by a force, and it is worth writing down because the next part depends on noticing that it is upside down.

(v) The slope of a graph of extension against force is e/F, which by $F=ke$ is $1/k$. The spring constant is therefore the reciprocal of the slope, and the slope must be in metres per newton before it is turned over:

$$k=\frac{1}{0.0813\text{mN}^{-1}}=12.3\text{Nm}^{-1}$$

(vi) That the spring took no permanent set while you were working: it returned to its original length, so the zero from which all six extensions were measured is still the zero and the whole table can be trusted. Had the final reading been the greater, the later extensions would have been measured from a position the pointer had already left.

A balance whose pointer rests below the zero of its scale with nothing hanging on it reads too large, and by a fixed amount. Its scale was engraved for a spring of a certain unloaded length; that spring is now permanently longer, so every load is read from a starting point further down the scale and the reading comes out as the true value plus whatever the permanent set amounts to in the units of the scale. Nothing is wrong with the spring itself, which obeys Hooke's law about its new length. What is wrong is that the scale no longer starts where the pointer does.

(vii) The graph would be a straight line through the origin of twice the slope, about 162.6mmN^{-1}, giving a spring constant of about 6.15Nm^{-1}, half of 12.3Nm^{-1}. Chapter 9 gives the reason: the extension of a helical spring is proportional to the number of turns, because each turn twists through the same angle and contributes the same share of the extension, so twice the turns is twice the extension for the same load and therefore half the spring constant.

Examiner's eye

- Plot the extension, not the pointer reading. A graph of the reading against F has exactly the same slope, so it will answer part (v); what it cannot do is pass through the origin, so it answers nothing in part (iii), where the whole point is that one quantity is proportional to another.
- The slope of this graph is not the spring constant. It is its reciprocal, and the units say so: the slope is a length over a force and a spring constant is a force over a length. Write both units down and the mistake becomes impossible.
- Convert the millimetres to metres before you turn the slope over. Convert afterwards, or not at all, and your answer is a thousand times out: a spring constant of 0.0123Nm^{-1} for a spring that visibly stretches under a slotted mass should stop the pen.
- Say in your comment that the line passes through the origin, and start both axes at zero so that you are entitled to say it. A graph whose axes begin where the readings begin answers part (iii) with a shrug.
- State the multiplication that turns a mass in grams into a force in newtons, and count the hanger in the mass. An examiner who sees an unexplained column of newtons cannot tell whether you divided by a thousand or forgot to.

Practice 10.1

The store had two springs left in the box and one clamp free. The first spring still had its paper label, with its spring constant on it in the storekeeper's handwriting. The label on the second had spent a rainy season under the hole in the roof and said nothing at all.

Kipanga hung the labelled spring from the clamp, hung the unlabelled one from the bottom of the labelled one, and hung the load from the bottom of that, because there was nowhere else to hang it. He also, without being told, clipped the pointer to the bottom of the lower spring instead of to the join between the two, and when **Kipute** asked him why, he had a reason ready.

One line on one sheet of graph paper now carries both springs, and the number the roof washed off the label is in it.

Procedure

Proceed as follows:

(a) Hang a spiral spring of spring constant 12.3Nm^{-1} from a clamp. Hang a second spring, whose spring constant is not known, from the lower end of the first, and fix a light pointer to the lower end of the second.

(b) Record the reading of the pointer against a vertical metre rule with no load on the springs.

(c) Hang a total mass M of 20g from the lower end of the second spring, allow it to come to rest, and record the reading of the pointer.

(d) Repeat the procedure in P10.1 (c) with masses of 40g, 60g, 80g, 100g and 120g.

Questions

(i) Tabulate your results, including the force F stretching the springs and the extension e of the pair.

(ii) Plot a graph of e (mm) against F (N).

(iii) Determine the slope of your graph and state its units.

(iv) Show that two springs of constants k_1 and k_2 joined end to end behave as a single spring of constant k given by $\frac{1}{k}=\frac{1}{k_1}+\frac{1}{k_2}$.

(v) Use your slope to determine the spring constant of the pair, and hence the spring constant of the second spring.

(vi) State which of the two springs provides the greater part of the extension, and by what factor.

(vii) A third spring, identical to the second, is joined below the pair and the loading is repeated. State the spring constant of the three together and the slope the new graph would have.

Experiment 10.2

The lorry that brings the maize down from the hill has a broken spring on the near side, and you can hear it long before you can see it. Empty, it takes the drain at the school gate in a single bounce and is finished with it. Loaded, it takes the same drain and then nods its way down the whole length of the road to the market, as though it were agreeing with something.

Mr. Akilikubwa stopped in the middle of a sentence to watch it go.

"Which of them bounces the more slowly, the empty one or the loaded one?"

Kipanga said the empty one, on the grounds that light things are quicker at everything. **Kipute** said the loaded one, on the grounds that you could hear it. Neither of them could say by how much, and the lorry was at the market by then, so the argument came indoors, where the load is known and the drain is a hand pulling gently downwards.

Aim

To determine the spring constant of a spiral spring from the vertical oscillations of a load hung from it, and to find the mass of the spring from the same graph.

Apparatus

As in Experiment 10.1: the spring, the retort stand with clamp and boss, a mass hanger and slotted masses of 20g, together with a stopwatch reading to 0.01s.

Setup

The arrangement is the right-hand drawing under Experiment 10.1. The pointer and the metre rule are no longer needed. The load is pulled down a short distance and released so that it oscillates vertically about its equilibrium position, and the amplitude is kept to about a centimetre throughout.

Procedure

Proceed as follows:

(a) Hang the spring from the clamp.

(b) Hang a mass M of 40g from its lower end.

(c) Pull the load down about a centimetre and release it, so that it oscillates vertically without swinging sideways. Allow two oscillations to settle, then record the time t for 30 complete oscillations.

(d) Repeat the procedure in 10.2 (b) and (c) with M equal to 80g, 120g, 160g, 200g and 240g.

Questions

(i) Tabulate your results, including T and T^2.

(ii) Plot a graph of T^2 (s^2) against M (kg).

(iii) The periodic time of a load M oscillating on a spiral spring of spring constant k and mass m_s is $T=2\pi\sqrt{\frac{M+m_s/3}{k}}$. Show that a graph of T^2 against M is a straight line, and write down expressions for its slope and for the intercept it makes on the T^2 axis.

(iv) Determine the slope of your graph and the intercept it makes on the T^2 axis, and state the units of the slope.

(v) Use your slope to determine the spring constant of the spring, and compare it with the value obtained in Experiment 10.1.

(vi) Use your intercept, together with your slope, to determine the mass of the spring. A balance gives the mass of this spring as 70g; comment on your value.

(vii) State, with a reason, whether the value of the spring constant obtained from this experiment depends on the acceleration due to gravity.

Table of results

M (kg)	t for 30 oscillations (s)	T (s)	T^2 (s^2)
0.040	13.52	0.451	0.203
0.080	17.18	0.573	0.328
0.120	20.42	0.681	0.464
0.160	22.83	0.761	0.579
0.200	25.46	0.849	0.721
0.240	27.39	0.913	0.834

Making the measurement count

Practical tip 1: *Why a load on a spring keeps time at all.*

Hang the load and let it settle. In that position the pull of the spring exactly balances the weight, which is why it settles there. Now pull it down a further distance x. The spring is stretched by x more than it was, so it pulls up harder than it did by kx, and since the weight was already balanced, that extra pull is the only unbalanced force acting on the load. Hence

$$Ma=-kx$$
$$a=-\frac{k}{M}x$$

The acceleration is proportional to the displacement and directed towards the equilibrium position, which is the definition of simple harmonic motion. Comparing with $a=-\omega^2x$ gives $\omega^2=k/M$ and therefore $T=2\pi\sqrt{\frac{M}{k}}$. Notice what has fallen out of the working: the weight of the load, which was used up in deciding where the load hangs and plays no part in what happens once it is hanging there.

Practical tip 2: *Why the mass of the spring gets a third of a vote.*

The load moves as one piece, every part of it with the same speed. The spring does not. Its top turn never moves at all, its bottom turn moves with the load, and the turns between move by amounts in between. So only part of the spring's mass is doing any moving, and adding up the shares for a uniform spring gives one third of it. That is the $m_s/3$ in the equation, and it is called the effective mass of the spring.

The one third is not exact. It is what you get by assuming that the spring stretches uniformly and that each turn keeps its share of the motion, which holds while the spring is light beside the load. Here it is not light: the lightest load in the table is lighter than the spring itself, and for that row the true period is about one and a half percent longer than the equation gives. It is the first point on the graph, it is the point the line is least entitled to, and it is one more reason not to make the loads any smaller than they are.

A third of 70g is 23.3g, which is a serious addition to a load of 40g and hardly worth mentioning beside one of 240g. That is exactly why a graph can find it. The same 23.3g is added to every load, and a constant added to the horizontal variable moves the whole line sideways, which shows itself as an intercept.

Practical tip 3: *How far to pull it down, and why there is a limit that has nothing to do with patience.*

At the top of each oscillation the spring is shorter than its equilibrium length by the amplitude. The load stretched the spring by e when it was put on, so an amplitude of exactly e brings the spring back to its unloaded length at the top of the swing. Go past that and the spring would have to push instead of pull, which a close-coiled spring cannot do: the coils come together, the force stops being proportional to anything, and the period stops meaning what the equation says it means. For the lightest load in this experiment e is only 32.0mm, read off the table of Experiment 10.1, so an amplitude of a centimetre is comfortably inside the limit and one of four centimetres is not.

There is a second limit, and it is worth knowing which of the two bites first. Whatever the spring is doing, gravity cannot pull the load down faster than g, so the motion cannot survive an amplitude for which $4\pi^2A/T^2$ exceeds g. For the lightest load here T^2 is $0.203s^2$ and that puts the limit at about 50mm. The spring reaches its unloaded length long before that, so in this experiment it is the spring that sets the limit; and it is the limit you can work out before you start, out of a column you have already written down.

Start it by pulling straight down, too. A load that swings sideways as well as bouncing is doing two things at once with two different periods, and the readings wander in a way no care with the stopwatch will mend.

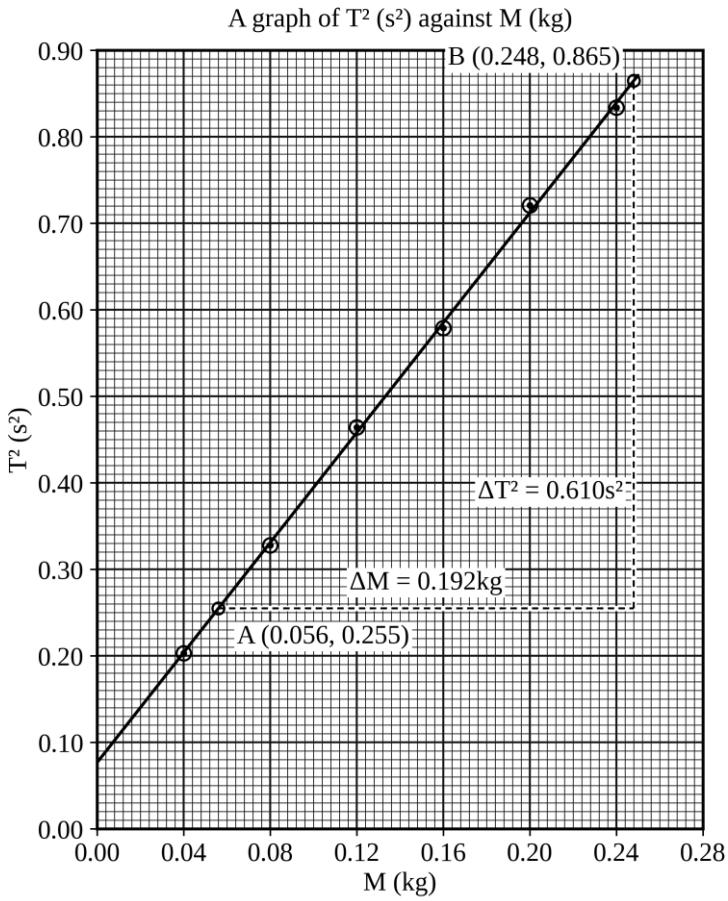
Practical tip 4: *The intercept is the weakest feature on the graph. Read it, and do not oversell it.*

On the scale used below, one small square is $0.01s^2$ and the intercept is $0.077s^2$, which is between seven and eight squares. You can read it to about half a square, so it carries a percentage uncertainty of something like seven percent, and the mass of the spring that comes out of it is worth two significant figures at the most. Say so when you quote it.

The intercept cannot be made larger, but it can be made easier to read. It is $4\pi^2 m_s/3k$, which holds no load term at all and is the same whatever you hang on the spring; what lighter loads do is shrink the vertical range, so that the same intercept covers a bigger fraction of it. What you pay is timing: lighter loads oscillate faster, so 30 oscillations take less time and the same reaction time costs a larger share of it. Practical tip 2 gives a second reason for keeping the loads up. The loads used here sit at the compromise, with the shortest run still over ten seconds.

Answers to the questions

(i) and (ii). The table is printed above, with T the recorded time divided by 30 and T^2 the square of the T you wrote down. From part (iii) the relation is a straight line of T^2 against M, so T^2 goes on the vertical axis. Give the graph the title *A graph of T² (s²) against M (kg)*. Take T^2 at 2cm to $0.1s^2$, running from 0 to $0.9s^2$, and M at 2cm to 0.04kg, running from 0 to 0.28kg. That is 18cm by 14cm, which fits an A4 sheet upright. Both axes must start at zero, because the intercept is one of the answers.



(iii) Squaring the equation given,

$$T^2 = \frac{4\pi^2}{k}M + \frac{4\pi^2 m_s}{3k}$$

which is of the form $y=mx+c$ with T^2 for y and M for x. The slope is $4\pi^2/k$ and the intercept on the T^2 axis is $4\pi^2 m_s/3k$.

(iv) Taking A (0.056kg, $0.255s^2$) and B (0.248kg, $0.865s^2$):

$$\text{slope} = \frac{0.865s^2 - 0.255s^2}{0.248\text{kg} - 0.056\text{kg}} = \frac{0.610s^2}{0.192\text{kg}} = 3.18s^2\text{kg}^{-1}$$

and the line meets the T^2 axis at $0.077s^2$.

(v) From part (iii) the slope is $4\pi^2/k$, so the spring constant is $4\pi^2$ divided by the slope. With $\pi=3.14$, $4\pi^2=39.4384$, and

$$k=\frac{39.4384}{3.18s^2kg^{-1}}=12.4Nm^{-1}$$

The unit deserves a line of its own: $1/s^2kg^{-1}$ is kgs^{-2}, and a kilogram per second squared is a newton per metre. Experiment 10.1 got $12.3Nm^{-1}$ from the same spring with a rule and a set of slotted masses. This experiment gets $12.4Nm^{-1}$ from a stopwatch. The two agree to about one percent, which is as close as two graphs read to three figures can be asked to come, and the agreement is the point of the pair: the number that decides how far a spring stretches is the number that decides how fast it bounces.

(vi) The intercept is $4\pi^2m_s/3k$ and the slope is $4\pi^2/k$, so their ratio is $m_s/3$, with the spring constant cancelling out of it. Hence

$$m_s=\frac{3\times0.077s^2}{3.18s^2kg^{-1}}=0.0726kg=72.6g$$

Call it 73g, since Practical tip 4 puts the mass that comes out of this intercept at two figures. The balance gives 70g, so the graph has found the mass of the spring to within about four percent without weighing it, and the one-third rule has survived a test it could have failed, which is the more interesting half of the result.

(vii) It does not. The period of an oscillation is decided by the inertia of what moves and the stiffness of what pulls it back, and neither of those is a weight. Gravity settled where the load hangs, and at that position its pull is exactly balanced by the spring, so it cancels out of the equation of motion, as Practical tip 1 shows. Every quantity used in this experiment is a mass in kilograms, a time in seconds or a length, and nowhere does a value of the acceleration due to gravity enter. Experiment 10.1 could not say that: the newtons in its table were masses multiplied by $9.81ms^{-2}$, so its spring constant rests on a value of g that the experiment itself never measured.

Examiner's eye

- Both axes from zero. A T^2 axis that starts at $0.2s^2$ gives a handsome line and throws away the mass of the spring, which is half the marks.
- The intercept is $4\pi^2m_s/3k$, not $4\pi^2m_s/k$. Multiply the ratio by three at the end and write the three down. Stop at the ratio and you report a third of the spring.
- The slope carries s^2 per kilogram, and $4\pi^2$ divided by it comes out in kgs^{-2}, which is Nm^{-1}. Show that conversion. A spring constant quoted in the wrong unit is a spring constant nobody can check.
- If your times drift one way and then back again, the load is swinging as well as bouncing: the sideways swing steals energy from the bouncing and gives it back. Stop it and start again rather than averaging it away.
- Two experiments on one spring should agree. Quote the other value, compare the two, and state what the comparison shows: that is the comment mark in part (v). Report a spring constant and never mention the first experiment and you have done half the work for half the credit.

Practice 10.2

Mr. Akilikubwa put two springs on the bench, wound from the same wire on the same former to the same number of turns, hung them side by side from one clamp, joined their lower ends with a light cross-piece and hung a load from the middle of it.

"Twice the spring, so twice as slow," said **Kipanga**.

"Twice the pull, so twice as fast," said **Kipute**.

"You cannot both be right," said **Mr. Akilikubwa**, *"and neither of you has the size of it. Time it, and come back with a number instead of an adverb."*

Procedure

Proceed as follows:

(a) Hang two identical spiral springs side by side from the same clamp and join their lower ends with a light rigid cross-piece. Use slotted masses of 50g for the loads.

(b) Hang a mass M of 100g from the middle of the cross-piece.

(c) Pull the load down about a centimetre and release it, so that the cross-piece stays horizontal and the load oscillates vertically without swinging sideways. Allow two oscillations to settle, then record the time t for 30 complete oscillations.

(d) Repeat the procedure in P10.2 (b) and (c) with M equal to 150g, 200g, 250g, 300g and 350g.

Questions

(i) Tabulate your results, including T and T^2.

(ii) Plot a graph of T^2 (s^2) against M (kg).

(iii) Each spring has spring constant k and mass m_s, and the cross-piece is light enough to be ignored. Show that the periodic time of the load is $T=2\pi\sqrt{\frac{M+2m_s/3}{2k}}$, and hence obtain expressions for the slope and the intercept of your graph.

(iv) Determine the slope of your graph and the intercept it makes on the T^2 axis.

(v) Determine the spring constant of one spring.

(vi) Determine the mass of one spring. A balance gives it as 83g; comment on your value.

(vii) The load of 200g is hung from one of the two springs alone. Calculate the periodic time it would then have, and state why it is not $\sqrt{2}$ times the value you measured with both.

Experiment 10.3

The balance in the chemistry laboratory has a pan the size of a saucer, and the copper calorimeter **Kipute** wanted to weigh had a base wider than that. It sat on the pan the way a hat sits on a nail. **Kipanga** steadied it with one finger, which the balance obligingly weighed as well, and a moment later the whole arrangement went over sideways and took the pan's little glass cover with it.

Mr. Akilikubwa collected the pieces without comment, poured a little water into the calorimeter and hung it from a spring by its own wire handle.

"You cannot weigh it on that," he said. *"So weigh the water instead, a little at a time, and let the calorimeter come out of the arithmetic."*

Aim

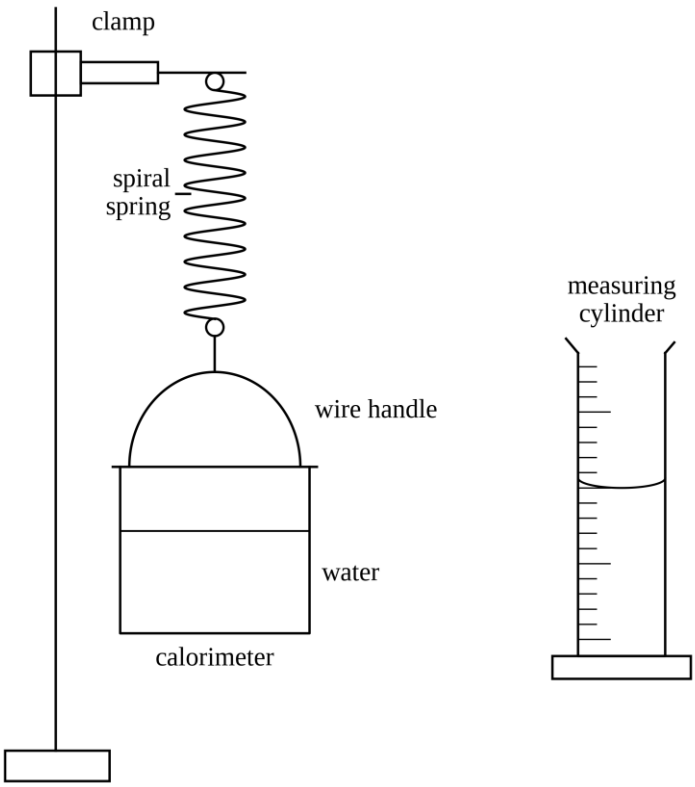
To determine the mass of a calorimeter, using a spiral spring and a measuring cylinder.

Apparatus

A stiff spiral spring, a retort stand with clamp and boss, a copper calorimeter of about $400cm^3$ with its own wire handle, a $100cm^3$ measuring cylinder, a supply of water, and a stopwatch reading to 0.01s.

Setup

The calorimeter hangs from the loop at the lower end of the spring by its own wire handle, so that nothing is on the spring but the calorimeter and what is inside it. Water is added to the calorimeter in measured amounts from the measuring cylinder, and after each addition the calorimeter is pulled down about a centimetre and released, so that the whole of it oscillates vertically as the load did in Experiment 10.2. V is the total volume of water in the calorimeter.



Procedure

Proceed as follows:

(a) Hang the spring from the clamp and hang the empty calorimeter from it by its handle.

(b) Measure $50cm^3$ of water in the measuring cylinder and pour it into the calorimeter.

(c) Pull the calorimeter down about a centimetre and release it, so that it oscillates vertically without swinging sideways. Allow two oscillations to settle, then record the time t for 30 complete oscillations.

(d) Repeat the procedure in 10.3 (b) and (c), adding $50cm^3$ of water each time, until the calorimeter holds $300cm^3$.

Questions

(i) Tabulate your results, including T and T^2.

(ii) Plot a graph of V (cm^3) against T^2 (s^2).

(iii) The periodic time of a load M on a spring of spring constant k and mass m_s is $T=2\pi\sqrt{\frac{M+m_s/3}{k}}$. Taking M to be the mass of the calorimeter together with the water in it, show that a graph of V against T^2 is a straight line, and obtain expressions for its slope and for the intercept it makes on the V axis.

(iv) Determine the slope of your graph and the intercept it makes on the V axis.

(v) Use your slope to determine the spring constant of the spring, taking the density of water as $1.00gcm^{-3}$.

(vi) The spring has a mass of 35g. Use your intercept to determine the mass of the calorimeter.

(vii) State, with a reason, how your value for the mass of the calorimeter and the slope of your graph would each be affected if the experiment were repeated with a heavier calorimeter of the same shape.

Table of results

V (cm^3)	t for 30 oscillations (s)	T (s)	T^2 (s^2)
50	15.54	0.518	0.268
100	17.80	0.593	0.352
150	19.54	0.651	0.424
200	21.39	0.713	0.508
250	22.90	0.763	0.582
300	24.64	0.821	0.674

Making the measurement count

Practical tip 1: *Why the spring had to be changed, and what is doing the weighing.*

The spring of Experiment 10.1 could not have carried this. A calorimeter with $300cm^3$ of water in it is over $400g$, and on a spring of $12.3Nm^{-1}$ that is more than $300mm$ of extension on top of the spring's own length: the calorimeter would be on the floor. A stiffer spring keeps the calorimeter clear of it, and keeps the periods in the same range as the rest of the chapter, so that 30 oscillations still take between fifteen and twenty-five seconds.

Notice what is being measured and what is not. Nothing that hangs on the spring is ever put on a balance. The masses come in through the water, whose density you know, and the volumes come from a measuring cylinder, and the calorimeter never leaves the loop. The one weighed number in the whole experiment is the mass of the spring, and the question gives you that: you weigh nothing yourself.

Practical tip 2: *Work in grams and cubic centimetres, and the intercept reads off as a mass with no arithmetic at all.*

The load on the spring is the calorimeter, plus the water, plus the third of the spring that Experiment 10.2 measured. Write m_c for the mass of the calorimeter and ρ for the density of the water. Putting that load into the equation and squaring,

$$T^2=\frac{4\pi^2}{k}(m_c+\rho V+m_s/3)$$

and making V the subject,

$$V=\frac{k}{4\pi^2\rho}T^2-\frac{m_c+m_s/3}{\rho}$$

Now choose the units. With the density of water at $1.00gcm^{-3}$, a volume in cubic centimetres and a mass in grams are the same number, so ρ disappears from the intercept and it reads off the axis directly as the mass of the calorimeter together with a third of the mass of the spring, in grams. There are no powers of ten to lose anywhere in this experiment, and that is worth arranging.

Practical tip 3: *The intercept is negative, and a negative volume of water is not nonsense.*

The line crosses the V axis below the origin, and what it says is this: to make the period fall to zero you would have to take water out of the calorimeter until nothing was left hanging on the spring at all, and the amount you would have to take out is the amount that weighs as much as the calorimeter and the effective mass of the spring. The intercept is a mass expressed as a volume of water, which is exactly the currency this experiment trades in.

It follows that both axes must be drawn out beyond the readings: the V axis below zero, far enough to reach the intercept, and the T^2 axis in to zero. The answer is off the end of the data in both directions, and a graph drawn only where the points are cannot give it.

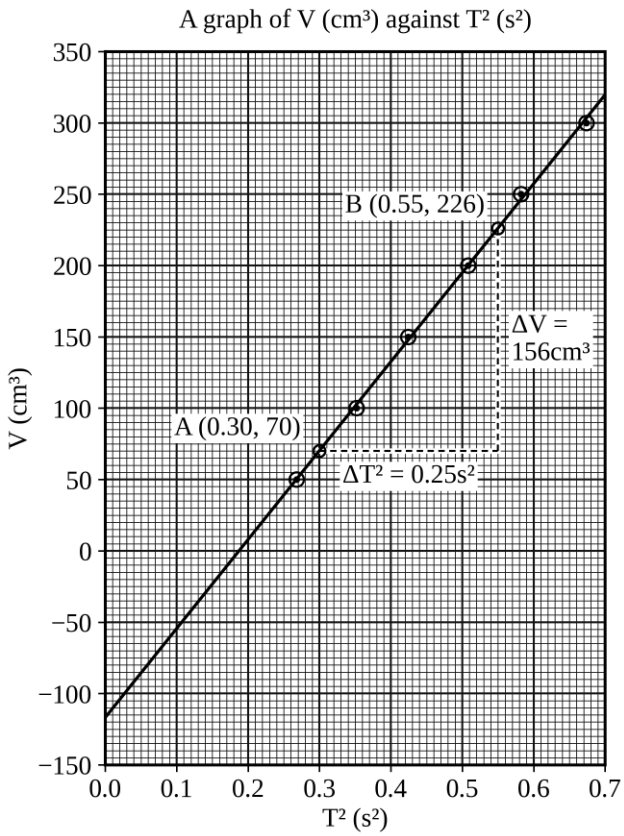
Practical tip 4: *Where the care goes: the lip of the measuring cylinder, and the surface of the water.*

The volume that matters is the total in the calorimeter, so every drop left behind on the lip of the cylinder is a drop missing from every row after it. Pour slowly and let the cylinder drain. Read the cylinder with your eye level with the bottom of the meniscus, and read it in the same way every time; a systematic misreading of the meniscus is worth more than the stopwatch will ever be.

Then start the oscillation gently. A calorimeter of water is not a rigid load: pull it down sharply and the surface slops from side to side, taking energy with it and giving it back, and the period wanders while it does. A displacement of a centimetre, straight down, and a moment to let the surface settle before you count, are the whole of the technique.

Answers to the questions

(i) and (ii). The table is printed above. From part (iii) the relation is a straight line of V against T^2. Give the graph the title *A graph of V (cm³) against T² (s²)*. Take V at 2cm to $50cm^3$, running from $-150cm^3$ to $350cm^3$, and T^2 at 2cm to $0.1s^2$, running from 0 to $0.7s^2$. That is 20cm by 14cm, which fits an A4 sheet upright. The V axis must run below zero, because the intercept is one of the answers and it lies there.



(iii) With m_c for the mass of the calorimeter and ρ for the density of the water, the load on the spring is $m_c+\rho V+m_s/3$, so

$$T^2=\frac{4\pi^2}{k}(m_c+\rho V+m_s/3)$$

and rearranging for V,

$$V=\frac{k}{4\pi^2\rho}T^2-\frac{m_c+m_s/3}{\rho}$$

which is a straight line of V against T^2 with slope $k/4\pi^2\rho$ and an intercept on the V axis of $-(m_c+m_s/3)/\rho$.

(iv) Taking A ($0.30s^2$, $70cm^3$) and B ($0.55s^2$, $226cm^3$) on the line:

$$\text{slope}=\frac{226cm^3-70cm^3}{0.55s^2-0.30s^2}=\frac{156cm^3}{0.25s^2}=624cm^3s^{-2}$$

and the line meets the V axis at $-117cm^3$.

(v) From part (iii) the slope is $k/4\pi^2\rho$, so

$$k=4\pi^2\rho\times\text{slope}=39.4384\times1.00gcm^{-3}\times624cm^3s^{-2}=24610gs^{-2}$$

and $24610gs^{-2}$ is $24.61kgs^{-2}$, which is $24.6Nm^{-1}$. The unit is worth following through: a kilogram per second squared is a newton per metre, and a spring constant is the only thing it can be.

(vi) The intercept is $-(m_c+m_s/3)/\rho$. With $\rho=1.00gcm^{-3}$ the intercept in cubic centimetres is that mass in grams, so the calorimeter and a third of the spring come to 117g between them. A third of 35g is 11.7g, so

$$m_c=117g-11.7g=105.3g$$

Call it 105g. The intercept was read to the nearest cubic centimetre, so the figure after the point is not one the graph can support.

(vii) The mass would come out larger, and the slope would not change at all. A heavier calorimeter adds the same extra mass to every reading, and a constant added to the load moves the line bodily down without tilting it: the intercept moves further below the origin and the slope, which holds only the spring constant and the density of the water, is untouched. That is worth knowing in both directions. It means the experiment measures the calorimeter through the intercept alone, and it means a mistake in the volumes, which tilts the line, cannot be mistaken for a heavier calorimeter.

Examiner's eye

- The quantity plotted is the total volume in the calorimeter, not the volume added each time. Write the running total in the table and label the column so that the reader can see which it is.
- The intercept is negative and it has to be read, not guessed. Draw the V axis down to -150cm^3 before you plot a single point; a graph that starts at the first reading throws away the answer to two of the seven parts.
- A slope in cm^3 per s^2 is not a spring constant. Multiply by $4\pi^2\rho$, and put the density in with its unit so that the grams and the cubic centimetres cancel where they should.
- Subtract a third of the mass of the spring from the intercept, not the whole of it. Only a third of the spring moves with the load, and none of it is riding in the calorimeter.
- Say that the calorimeter was pulled straight down and released gently, and that the water was not slopping. An examiner has no other way of knowing whether the load was behaving as a rigid one.

Practice 10.3

The kitchen's kerosene comes in a drum whose lid has not fitted properly since anybody can remember, and the drum stands where the rain can reach it. The cook brought a bottle of it to the laboratory, set it on the bench, and said that it was not burning as it used to and that in her opinion there was water in it.

"How much water?" asked **Kipute**.

The cook said that was what she had come to find out. **Kipanga** offered to settle it by smelling the bottle, and was not allowed to.

Mr. Akilikubwa pointed out that what the cook was asking about was a density: the same bottle of a wetter kerosene weighs a different amount. And a density is something a spring can measure.

Procedure

Proceed as follows:

(a) Hang a spiral spring of spring constant 28.5Nm^{-1} and mass 33g from a clamp, and hang an empty calorimeter from it by its handle.

(b) Measure 50cm^3 of the liquid from the bottle in a measuring cylinder and pour it into the calorimeter.

(c) Pull the calorimeter down about a centimetre and release it, so that it oscillates vertically without swinging sideways and the liquid is not slopping. Allow two oscillations to settle, then record the time t for 30 complete oscillations.

(d) Repeat the procedure in P10.3 (b) and (c), adding 50cm^3 each time, until the calorimeter holds 300cm^3.

Questions

(i) Tabulate your results, including T and T^2, with V the total volume of liquid in the calorimeter.

(ii) Plot a graph of V (cm^3) against T^2 (s^2).

(iii) Determine the slope of your graph and the intercept it makes on the V axis.

(iv) Show that the slope of such a graph is $k/4\pi^2\rho$, where ρ is the density of the liquid.

(v) Determine the density of the liquid, and state its relative density.

(vi) Determine the mass of the calorimeter, given that the intercept holds a third of the mass of the spring as well.

(vii) State how your graph would have differed had the calorimeter been filled with water instead of the liquid from the bottle.

Experiment 10.4

Kipanga had been quiet for most of the lesson, which was unusual, and then asked the question that stopped it.

"Sir, when we hang the load on the spring, the spring knows about gravity, because gravity is what pulls the load down. When we let it bounce, it does not: you said the weight cancels out. So the two experiments disagree about whether gravity is in the room."

"They do not disagree," said **Mr. Akilikubwa**. *"One of them has to know how strong gravity is and the other does not, which is a different thing and a more useful one. If you can see that far you can see the next step: gravity is the difference between them. Go and get it out."* He wrote nothing on the board. *"No pendulum. And you may not use the number on the label of any spring."*

Aim

To determine the acceleration due to gravity using a spiral spring and a metre rule, without a pendulum. No value of the spring constant enters the determination of g.

Apparatus

As in Experiment 10.1: the spring, the retort stand with clamp and boss, the light pointer, the metre rule clamped vertically, and a mass hanger with slotted masses, each of 20g; and a stopwatch reading to 0.01s.

Setup

Both drawings under Experiment 10.1 are in use at once. For each load the extension is read against the metre rule, and then the same load is set oscillating vertically and timed. The extension e is the pointer reading less the reading with no load, as before, and the load is not changed between the two measurements.

Procedure

Proceed as follows:

(a) Hang the spring from the clamp, fix the pointer to its lower end, and record the reading of the pointer with no load on the spring.

(b) Hang a total mass M of 40g, the hanger included, from the lower end of the spring, allow it to come to rest, and record the reading of the pointer.

(c) Pull the load down about a centimetre and release it, so that it oscillates vertically without swinging sideways. Allow two oscillations to settle, then record the time t for 30 complete oscillations.

(d) Repeat the procedure in 10.4 (b) and (c) with M equal to 80g, 120g, 160g, 200g and 240g.

Questions

(i) Tabulate your results, including the extension e, T and T^2.

(ii) Plot a graph of e (mm) against T^2 (s^2).

(iii) For a load M hanging at rest on the spring, $e=\frac{Mg}{k}$, and when it oscillates, $T=2\pi\sqrt{\frac{M+m_s/3}{k}}$. Use the first to eliminate M from the second, and hence show that $e=\frac{g}{4\pi^2}T^2-\frac{gm_s}{3k}$.

(iv) Determine the slope of your graph and the intercept it makes on the e axis.

(v) Use your slope to determine the acceleration due to gravity.

(vi) The spring constant of this spring was found in Experiment 10.1 to be $12.3Nm^{-1}$. Use your intercept to determine the mass of the spring, and compare it with the 70g a balance gives.

(vii) A student proposes to obtain g instead by dividing the spring constant of Experiment 10.1 by the spring constant of Experiment 10.2. Explain, with numbers, why the graph you have drawn is the better method.

Table of results

The pointer read 176.0mm with no load on the spring.

M (g)	Pointer reading (mm)	e (mm)	t for 30 oscillations (s)	T (s)	T^2 (s^2)
40	208.2	32.2	13.54	0.451	0.203
80	239.3	63.3	17.31	0.577	0.333
120	271.7	95.7	20.28	0.676	0.457
160	304.0	128.0	22.89	0.763	0.582
200	335.8	159.8	25.26	0.842	0.709
240	366.4	190.4	27.59	0.920	0.846

Making the measurement count

Practical tip 1: *Two equations, one unknown you do not want, and three lines of algebra.*

The load hanging at rest gives $e=\frac{Mg}{k}$, because the weight Mg is what the spring is holding up. The same load bouncing gives $T=2\pi\sqrt{\frac{M+m_s/3}{k}}$. Both contain k, which you are not allowed to know, and both contain M, which you do know but do not want. Make M the subject of the first, $M=ke/g$, and put it into the square of the second:

$$T^2=\frac{4\pi^2}{k}\left(\frac{ke}{g}+\frac{m_s}{3}\right)=\frac{4\pi^2 e}{g}+\frac{4\pi^2 m_s}{3k}$$

Multiply through by $g/4\pi^2$ and rearrange:

$$e=\frac{g}{4\pi^2}T^2-\frac{gm_s}{3k}$$

Look at what happened to k. It has gone from the first term altogether, which is why the slope of a graph of e against T^2 gives g and nothing else is needed for it. It survives in the second term, alongside the mass of the spring, which is why the intercept is the part of this graph that still needs help from somewhere else.

Practical tip 2: *What the two instruments are each contributing, and why they must not be separated.*

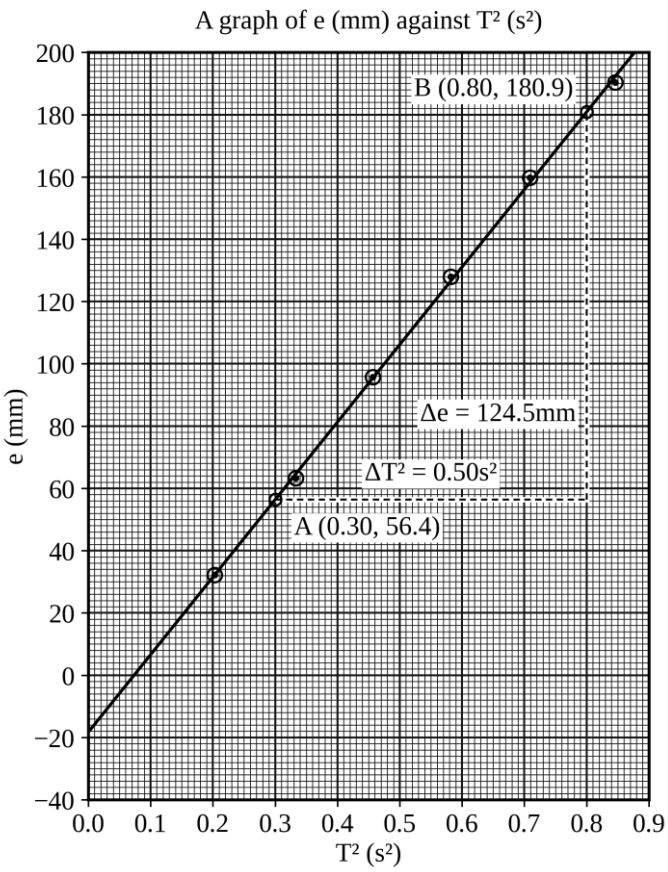
The rule measures Mg/k and the stopwatch measures $(M+m_s/3)/k$. Neither of them can tell you g on its own, because neither knows k; put them together and k divides out and g is left. That is the whole of the experiment, and it is why the pair of readings for each load must belong to the same load and the same clamp position. Read the pointer before you set the load bouncing, and do not touch the clamp between the two: a spring that has been nudged a millimetre down the clamp gives an extension from one arrangement and a period from another, and nothing on the graph will show that it happened.

Practical tip 3: *The chapter's zero, one last time.*

The no-load reading enters every extension in this table, and Practical tip 3 of Experiment 10.1 showed what a wrong one does: it moves the whole line without tilting it. Here the slope is the answer, so a wrong zero leaves g untouched and spoils only the intercept, and with it the mass of the spring. That is the lightest penalty the chapter has attached to a bad zero, and the reason is worth stating: the zero is subtracted from every pointer reading alike, so it cancels out of every difference the slope is made of.

Answers to the questions

(i) and (ii). The table is printed above, with e the pointer reading less 176.0mm and T^2 the square of the T you wrote down. From part (iii) a graph of e against T^2 is a straight line, so e goes on the vertical axis. Give the graph the title *A graph of e (mm) against T^2 (s^2)*. Take e at 2cm to 20mm, running from -40mm to 200mm, and T^2 at 2cm to $0.1s^2$, running from 0 to $0.9s^2$. That is 24cm by 18cm, which fits an A4 sheet upright. The e axis must run below zero, because the intercept lies there.



(iii) The working is Practical tip 1: from $e=\frac{Mg}{k}$, $M=ke/g$, and substituting into the square of the periodic time equation,

$$T^2=\frac{4\pi^2}{k}\left(\frac{ke}{g}+\frac{m_s}{3}\right)=\frac{4\pi^2 e}{g}+\frac{4\pi^2 m_s}{3k}$$

which rearranges to

$$e=\frac{g}{4\pi^2}T^2-\frac{gm_s}{3k}$$

a straight line of slope $g/4\pi^2$ and intercept $-gm_s/3k$ on the e axis.

(iv) Taking A ($0.30s^2$, 56.4mm) and B ($0.80s^2$, 180.9mm) on the line:

$$\text{slope}=\frac{180.9\text{mm}-56.4\text{mm}}{0.80s^2-0.30s^2}=\frac{124.5\text{mm}}{0.50s^2}=249\text{mms}^{-2}$$

and the line meets the e axis at -18.3mm.

(v) The slope is $g/4\pi^2$, so g is $4\pi^2$ times the slope, with the slope in metres per second squared:

$$g=39.4384\times0.249\text{ms}^{-2}=9.82\text{ms}^{-2}$$

against the accepted 9.81ms^{-2}. Claim no more for that than it will bear. One small square on this scale is 2mm of e, so each ordinate is read to about 1mm, and two of them over a rise of 124.5mm fix the slope to about one percent: this is 9.82ms^{-2} give or take 0.1ms^{-2}, and the lightest load carries the reservation Practical tip 2 of Experiment 10.2 attached to it. Within that band the agreement is complete, and it was got with a spring, a rule and a stopwatch, without a pendulum and without ever knowing how stiff the spring was.

(vi) The intercept is $-gm_s/3k$, and it is negative because m_s is positive. Rearranged for the mass, m_s is 3k times the size of the intercept, divided by g. The intercept measures 18.3mm, which is 0.0183m, so

$$m_s=\frac{3\times12.3\text{Nm}^{-1}\times0.0183\text{m}}{9.82\text{ms}^{-2}}=0.0688\text{kg}=69\text{g}$$

The balance gives 70g, and Experiment 10.2 obtained 73g from its own intercept. The chapter now has two graphical values for the mass of a spring neither experiment set out to weigh, and both are within a few percent of the balance. That is the one-third rule tested twice on the same spring: once through an intercept that needed no spring constant, and once through an intercept that borrowed one from Experiment 10.1.

(vii) The division does work in principle. The spring constant from the loading experiment was obtained from forces that were masses multiplied by 9.81ms^{-2}, so it carries the factor $9.81/g$, while the one from the timing experiment does not; their ratio is therefore $9.81/g$ and g follows. Put the numbers in and it gives

$$g=9.81\text{ms}^{-2}\times\frac{12.4\text{Nm}^{-1}}{12.3\text{Nm}^{-1}}=9.89\text{ms}^{-2}$$

which is a poor answer, and the fault is not in the physics. It is that 9.81ms^{-2} has to be put in at the beginning, to turn the slotted masses of Experiment 10.1 into forces, and it comes out again at the end multiplied by a ratio of two spring constants. That ratio is the whole of the measurement, and each constant is read off a graph to about one percent, so the ratio is good to about one and a half: moving either constant by one in its last figure moves this value of g by about 0.08ms^{-2}. What the method reports, then, is that g is 9.81ms^{-2} to within one and a half percent, which is not a determination of g but a check that the two experiments agree; and it is a coarse check, because the whole variation of g over the Earth's surface is about a third of that. The sign of the trouble is on the page: 9.89ms^{-2} lies further from the accepted value than that whole variation. The graph divides nothing and assumes nothing. Six pairs of readings fix one slope, and g comes out of that slope alone.

Examiner's eye

- Say in your answer that the slope needs neither the spring constant nor the mass of the spring. That is the point of the experiment, and the mark for part (iii) is for showing that k has gone from the first term, not for producing the equation.
- Draw the e axis down past zero before you plot anything. The intercept is negative, and an axis that begins at the smallest extension leaves nowhere for it to be.
- The slope comes off the paper in millimetres per second squared. Turn it into metres per second squared before multiplying by $4\pi^2$, or g comes out a thousand times too large. A value of g in the thousands is not an answer; it is a decimal point that has slipped.
- Read the pointer before you set the load bouncing, and record both readings for a load before you change the load. Two columns that do not belong to each other row by row cannot be plotted against each other.
- Prefer one slope drawn through six readings to a quantity assembled from two results measured separately. A slope carries the uncertainty of one graph; a quotient carries the uncertainty of both, and part (vii) is what that costs.

Practice 10.4

Kipanga's table for the next spring had six pointer readings in it and six times, and no reading at all for the spring with nothing on it, because he had hung the first mass before it occurred to him to look. He was ready to take the whole thing down and start again.

Kipute told him to plot it first. *"The spring does not know what you forgot to write down,"* he said.

Procedure

Proceed as follows:

(a) Hang a spiral spring of spring constant 18.0Nm^{-1} and mass 48g from a clamp, and fix a light pointer to its lower end so that it moves against a vertical metre rule.

(b) Hang a total mass M of 100g from the spring and record the reading x of the pointer against the rule.

(c) Pull the load down about a centimetre and release it, so that it oscillates vertically without swinging sideways. Allow two oscillations to settle, then record the time t for 25 complete oscillations.

(d) Repeat the procedure in P10.4 (b) and (c) with M equal to 200g, 300g, 400g, 500g and 600g.

Questions

(i) Tabulate your results, including T and T^2. No reading was taken with the spring unloaded.

(ii) Plot a graph of x (mm) against T^2 (s^2).

(iii) Taking x_0 for the reading the pointer would have given with no load, show that $x=x_0+\frac{g}{4\pi^2}T^2-\frac{gm_s}{3k}$, and hence that the slope of your graph does not depend on x_0 at all.

(iv) Determine the slope of your graph and the intercept it makes on the x axis.

(v) Determine the acceleration due to gravity.

(vi) Use your intercept to determine the reading the pointer would have given with no load on the spring.

(vii) State what has been lost by not recording that reading at the time, and name the parts of Experiment 10.1 that could not have been answered without it.

Chapter 11

ELASTICITY: YOUNG'S MODULUS

Everything in the physics store bends. Lean on the bench and it gives, a little; hang a stone on a metre rule and the rule dips; press your thumb against the flat of a hacksaw blade and it curves away from you and straightens when you let go. What is worth measuring is not that they bend but by how much, because the amount is a property of the material and is the same for every piece of it.

That last sentence is the one to hold on to, because the chapter turns on it. Chapter 10 measured a spring constant, and a spring constant belongs to a spring: wind the same wire into twice as many turns and the constant halves, though nothing has happened to the steel. A rule has a stiffness in the same sense, and it depends on how much of the rule is sticking out and on whether it is clamped flat or on edge. Young's modulus is what is left when all of that has been divided out. It belongs to the wood, or to the plastic the rule was moulded from, and to nothing else, and it is written E.

Two equations carry the three varieties below, and they are the same equation twice. A rule of breadth b and thickness d, clamped at one end with a length L of it projecting and a mass M hung at the free end, dips at that end by

$$s=\frac{4MgL^3}{Ebd^3}$$

and the same arrangement, set vibrating instead of left alone, keeps time with a period

$$T=2\pi\sqrt{\frac{4ML^3}{Ebd^3}}$$

The first is read off a scale and the second off a stopwatch, and both of them are E in disguise. Put the two side by side and the chapter has its subject.

One warning belongs at the front, as it has in each of the last two chapters, because it governs where you spend your care. In Chapter 9 the warning was a fourth power and in Chapter 10 it was a zero. Here it is a cube, and there are two of them. The projecting length is cubed, so one part in a hundred out in the length is three parts in a hundred out in the answer. The thickness is cubed as well, and the thickness of a rule is a few millimetres. Get that one reading wrong and no amount of good timing afterwards will recover what you have lost.

The three varieties at a glance

	What is different about it	What the graph gives you
11.1	The projecting length is changed and the loaded end is set vibrating	Young's modulus for the wood, from a straight line
11.2	A load hangs on a thread below the free end, and the thread is lengthened	the acceleration due to gravity, and Young's modulus for the plastic, from one line
11.3	The projecting length is changed again and nothing is allowed to move	Young's modulus for the wood again, from the sag alone

Experiment 11.1 is worked below in full. The second variety lets the load hang well below the rule on a long thread, so that the thread and the rule share the movement between them; the third puts the stopwatch away and reads a millimetre scale.

Experiment 11.1

The woodwork room had been quiet for a quarter of an hour, which was how **Mr. Akilikubwa** knew that something was going on in it. What was going on was a metre rule, clamped flat to the bench with a good third of it over the edge, and **Kipute** pulling the free end down and letting it go, over and over, while four boys listened.

"Longer," said **Kipute**, and slid the rule out. *"It only depends on how much of it is hanging out. Nothing else touches it."*

Kipanga clamped a second rule beside the first, half as far out again, and it flapped so much more slowly that the two of them would not stay in step for three beats together. **Kipute** slid his own rule out until they did, and looked pleased with himself, which was when he noticed who was standing in the doorway.

"Sir, how far out would it have to be to take a whole second to go down and come back?" That was **Kipanga**, who had been counting under his breath and had run out of rule.

"That depends on what you hang on the end of it," said **Mr. Akilikubwa**, *"which is the half of it neither of you has touched all afternoon. Clamp it properly, hang something on it you can trust, and find out."*

Aim

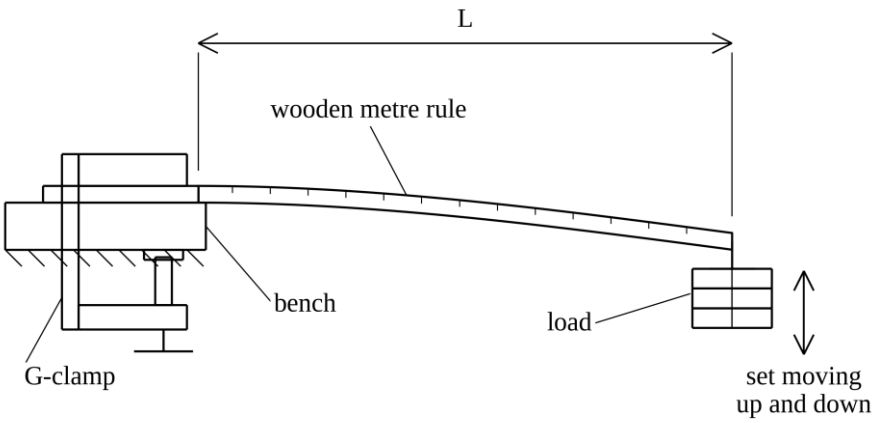
To determine Young's modulus for the wood of a metre rule from the vibrations of a loaded cantilever.

Apparatus

A wooden metre rule, a G-clamp, a bench with a straight edge, a mass hanger with slotted masses making up a total of 500g, a short loop of thread, a stopwatch reading to 0.01s, a micrometer screw gauge and a vernier calliper.

Setup

The rule is clamped flat to the bench at its own edge, so that a length L of it projects horizontally with nothing under it. The load hangs from the free end on a short loop of thread, and M is the mass of the hanger together with the slotted masses on it. Press the free end down and release it and the rule vibrates, carrying the load with it: the thread stays taut, so the load goes up and down with the rule. The projecting length is measured from the edge of the clamp to the free end.



Procedure

Proceed as follows:

(a) Measure the thickness of the rule with the micrometer screw gauge and its breadth with the vernier calliper.

(b) Clamp the rule flat to the bench with the G-clamp, at the edge of the bench, so that a length L of 32.0cm projects beyond the edge of the clamp, and tighten the clamp firmly.

(c) Hang the loop of thread over the free end and hang from it a total mass M of 500g, the hanger included.

(d) Press the free end down through about a centimetre and release it, so that it vibrates up and down without the load swinging. Allow two vibrations to settle, then record the time t for 50 complete vibrations.

(e) Slide the rule through the clamp and repeat the procedure in 11.1 (c) and (d) with L equal to 35.0cm, 38.0cm, 41.0cm, 44.0cm and 47.0cm.

Questions

(i) Tabulate your results, including L^3, T and T^2.

(ii) Plot a graph of T^2 (s^2) against L^3 (m^3).

(iii) State the relationship between T and L shown by your graph, and say what the position of your line relative to the origin tells you.

(iv) Determine the slope of your graph and state its units.

(v) The breadth of the rule is 24.8mm and its thickness 6.05mm. Use your slope to determine Young's modulus for the wood, given that the stiffness of a cantilever is $k=Ebd^3/4L^3$ and that $T^2=4\pi^2M/k$.

(vi) The projecting length was measured from the edge of the clamp. Suppose the rule in fact begins to bend a few millimetres inside the clamp. State, with a reason, whether the value of E you obtain would then be too large or too small.

(vii) A second rule, of the same wood and the same breadth but twice the thickness, is clamped in place of the first and the experiment is repeated with the same load and the same six lengths. State how the slope of the new graph and the value of E obtained from it would compare with yours, and give your reasoning.

Table of results

The breadth of the rule was 24.8mm and its thickness 6.05mm, and the mass hung at the free end was 500g throughout.

L (cm)	L^3 (m^3)	t for 50 vibrations (s)	T (s)	T^2 (s^2)
32.0	0.0328	10.77	0.215	0.0462
35.0	0.0429	12.58	0.252	0.0635
38.0	0.0549	14.09	0.282	0.0795
41.0	0.0689	15.93	0.319	0.1018
44.0	0.0852	17.68	0.354	0.1253
47.0	0.1038	19.36	0.387	0.1498

Making the measurement count

Practical tip 1: *One equation, and where each of your measurements enters it.*

The period of a loaded cantilever is $T^2=4\pi^2M/k$, the same inertia-over-stiffness pattern as every oscillator since Chapter 8, and the stiffness of a cantilever is

$$k=\frac{Ebd^3}{4L^3}$$

which is the stiffness of the rule at its free end, the force needed there for one metre of movement. Square the period, put that stiffness into it, and you have

$$T^2=\frac{16\pi^2M}{Ebd^3}L^3$$

so the slope of the graph part (ii) asks for carries E, the load and the cross-section, and nothing else. Look at what the stiffness is made of before you measure anything. The breadth appears once and the thickness three times, which is why the micrometer comes out for one of them and the calliper will do for the other. Part (vii) turns on the same three letters.

Practical tip 2: *Where the cantilever really begins.*

The length in the formula is the length that is free to bend, and that is not quite the length you can see. A clamp that is not tight lets the rule work loose inside it, and the rule then begins to bend somewhere under the jaw rather than at its edge. Tighten the clamp until the rule cannot be moved by hand, put a scrap of card between the jaw and the wood if the jaw is slipping, and measure from the edge of the clamp every time.

Two habits follow. Slide the rule and leave the clamp where it is, so that the point the length is measured from does not move between readings. And take the six lengths in one order and check two of them again at the end: an error that grows as you work down the bench is the hardest kind to see afterwards. Part (vi) asks which way an error of that sort runs.

Practical tip 3: *Timing fifty vibrations of something that will not hold still.*

A loaded rule at these lengths goes up and down between two and a half and five times a second, which is too fast to count by watching the rule. Count the load instead, or hold a pencil at the level the load reaches at the bottom of its travel and count the taps. Start the watch on a count of zero, not on a count of one, and stop it on fifty.

Press the end down about a centimetre and no more, and press it straight down. Do that and the load rides up and down on a taut thread and stays under the rule. Push it sideways at the start and the load begins to swing as well as bounce, the two motions beat against one another and the vibration you are timing dies away in a few seconds. If it starts to swing, stop, let it settle and start again.

Practical tip 4: *A cube magnifies every error in a length, and the smallest length worst of all.*

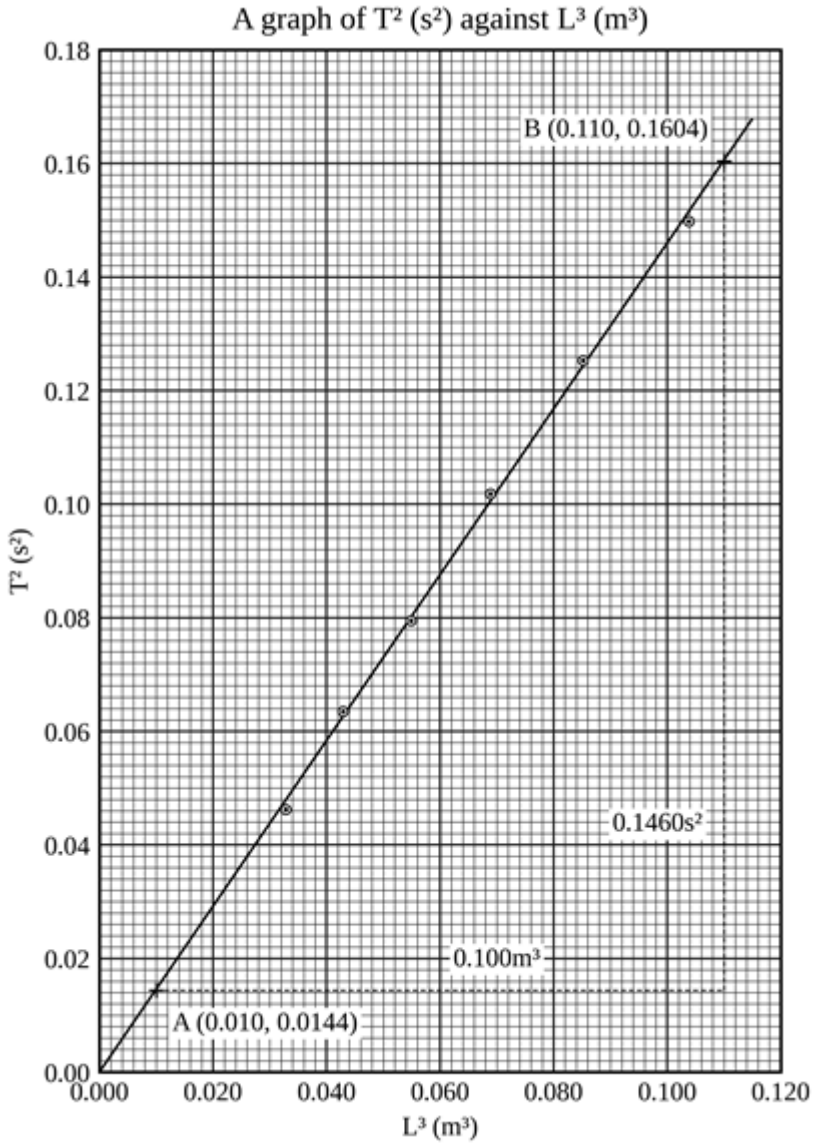
The abscissa of this graph is a length cubed, so an error in the length is trebled before it reaches the paper: a millimetre out in 400mm is a quarter of one percent in the length and three quarters of one percent in L^3. The thickness is cubed too, and it is the smallest quantity in the experiment, so it deserves the finest instrument you

have and three readings at different places along the rule. Take the mean of the three, and let the half-range of them tell you what that mean is worth.

Turn every length into metres before you cube it and keep it there. A length cubed in centimetres is a million times a length cubed in metres, and a Young's modulus is a large enough number already that a factor of a million in it does not look absurd until you compare it with something.

Answers to the questions

(i) and (ii). The table is printed above. L^3 is the projecting length in metres, cubed, and the period T is the recorded time divided by 50. Give the graph the title A graph of T^2 (s^2) against L^3 (m^3). Take L^3 at 2cm to $0.02m^3$, running from $0m^3$ to $0.12m^3$, and T^2 at 2cm to $0.02s^2$, running from $0s^2$ to $0.18s^2$. That is 12cm by 18cm, which fits an A4 sheet upright. Both axes must start at zero, because part (iii) asks about the origin.



(iii) The six points lie on a straight line through the origin, so $T^2 \propto L^3$ and therefore $T \propto L^{3/2}$: double the projecting length and the period is multiplied by 2.83, not by two. The line passing through the origin says something the slope cannot say. It says that no length has been added to the six you measured and none taken away: a rule bending from a point well inside the clamp makes every abscissa short by an amount that does not vanish as the rule is pushed in, and lifts the line off the origin on the high side. Be honest about how large that fault has to be before the line shows it. Three millimetres inside the jaw would raise the intercept by about a third of a small square, which is less than these six points already scatter, and the rule's own mass, which Practice 11.1 is about, pulls the intercept down by about as much again. The origin will catch a badly seated rule. It will not catch a slightly loose one.

(iv) Take two points on the line, well apart, and choose them where the ruling can be read. Taking A (0.010m^3, 0.0144s^2) and B (0.110m^3, 0.1604s^2):

$$\text{slope}=\frac{0.1604\text{s}^2\text{-}0.0144\text{s}^2}{0.110\text{m}^3\text{-}0.010\text{m}^3}=\frac{0.1460\text{s}^2}{0.100\text{m}^3}=1.46\text{s}^2\text{m}^{-3}$$

The unit is seconds squared per cubic metre, and it is worth writing down, because the next part is a substitution and a substitution with the wrong unit in it cannot be checked.

(v) From $T^2=16\pi^2ML^3/Ebd^3$, the slope of a graph of T^2 against L^3 is $16\pi^2M/Ebd^3$, so

$$E=\frac{16\pi^2M}{\text{slope}\times bd^3}$$

The two cross-section measurements go together first, in metres:

$$bd^3=0.0248\text{m}\times\left(6.05\times10^{-3}\text{m}\right)^3=5.492\times10^{-9}\text{m}^4$$

and then

$$E=\frac{16\times3.14^2\times0.500\text{kg}}{1.46\text{s}^2\text{m}^{-3}\times5.492\times10^{-9}\text{m}^4}=9.84\times10^9\text{Nm}^{-2}$$

The units come out as a kilogram divided by a second squared and a metre, which is a newton per square metre, so the arithmetic has kept its footing. A value near $1.0\times10^{10}\text{Nm}^{-2}$ is what seasoned hardwood gives; steel would be twenty times that and a plastic rule a third of it.

(vi) Too small. If the rule bends from a point inside the jaw, the length that is really bending is greater than the length you wrote down, so every period is longer than the measured length can account for. Plotted against the measured L^3 the points climb faster than they should, the slope comes out too large, and E is the slope turned over. A rule bending from 3mm inside the jaw at a mean length of about 0.40m would give a slope about one and a half percent high and a modulus one and a half percent low. That is a good deal more than it sounds: it is half the whole difference between this answer and the one Experiment 11.3 gets from the same rule, and that difference has a cause of its own.

(vii) The slope would be an eighth of yours, and the value of E would be the same. The stiffness holds bd^3, so doubling the thickness multiplies it by eight, and the slope, which is $16\pi^2M/Ebd^3$, is divided by eight:

$$\text{slope}=\frac{1.46\text{s}^2\text{m}^{-3}}{8}=0.183\text{s}^2\text{m}^{-3}$$

Now put that slope back through part (v). The eight that has gone into the denominator of the slope is the eight that bd^3 has gained, and the two cancel, so E comes out where it was, to the accuracy of a formula that takes no account of the rule's own mass. Stop on that for a moment: the rule has changed, the graph has changed and the answer has not, because what you are measuring is a property of the wood and the second rule is made of the same wood. A quantity that behaved otherwise would not deserve the name modulus.

Two things about that repeat are worth working out before you attempt it. The thicker rule carries twice as much of its own mass, so the small bias Experiment 11.3 uncovers is twice the size; and it droops far less, so the centimetre of press in the procedure is far too much. Press a stiff cantilever further down than its load makes it droop, and the thread goes slack at the top of every stroke.

Examiner's eye

- Cube the length before you plot it. A graph of T^2 against L is a curve, and a curve has no slope to hand to part (v).
- Write the slope with its units and put them through the substitution beside the numbers. A mass, divided by seconds squared per cubic metre and by a fourth power of length, leaves a newton per square metre; if your working does not end there, something has gone in upside down.
- Every gram hanging on the thread counts, the hanger included, and its own mass is stamped on it. Whether anything else counts as well is the question Practice 11.1 puts.

- Say what the origin is evidence for, and say how much it will carry. It is the only check the graph itself makes on where the bending began, and part (vi) shows how coarse a check that is.

Practice 11.1

Three streets from the school, at the end of the fish market, a woman carries her morning's trading home on a length of springy bamboo across her shoulder, a basket swinging at each end. The bamboo nods as she walks and she walks in step with it, so that the baskets come up as her shoulder comes up and the load never quite lands on her. **Mr. Akilikubwa** stopped to watch her go, which meant that the six boys behind him stopped too.

Kipanga said the nodding had nothing to do with the fish, and that an empty pole would nod at the same rate as a full one, because a pole is a pole. **Kipute** said that if that were so the woman would be walking in step with it on the way home and out of step with it on the way there, and she was not. The woman, who had been listening, handed **Kipanga** the pole with both baskets full and invited him to walk with it.

He did not get far. What settles the argument is a clamp, a rule and a set of small masses, and it settles rather more than the argument: this time the load is changed and the length is left alone, and the graph turns out to hold something the earlier one could not.

Procedure

Proceed as follows:

(a) Clamp a wooden metre rule flat to the bench with a G-clamp, at the edge of the bench, so that a length L of 55.0cm projects beyond the edge of the clamp, and tighten the clamp firmly.

(b) Hang a loop of thread over the free end and hang from it a total mass M of 60g, the hanger included.

(c) Press the free end down through about a centimetre and release it, so that it vibrates up and down without the load swinging. Allow two vibrations to settle, then record the time t for 50 complete vibrations.

(d) Repeat the procedure in P11.1 (b) and (c) with M equal to 100g, 140g, 180g, 220g and 260g, the projecting length being left untouched throughout.

Questions

(i) Tabulate your results, including T and T^2.

(ii) Plot a graph of T^2 (s^2) against M (kg).

(iii) Determine the slope of your graph and the intercept it makes on the T^2 axis.

(iv) The breadth of the rule is 25.6mm and its thickness 5.72mm. Use your slope to determine Young's modulus for the wood, given that the stiffness of a cantilever is $k=Ebd^3/4L^3$ and that $T^2=4\pi^2M/k$.

(v) Your line does not pass through the origin. State what is being set moving besides the load, and hence what quantity the intercept measures.

(vi) The projecting part of the rule has a mass of 61.6g. Use your intercept to find what fraction of that mass rides with the load, and compare it with 33/140.

(vii) State, with a reason, what would happen to the intercept if the projecting length were halved, the loads being unchanged.

Experiment 11.2

Mr. Akilikubwa had set the class one thing to do before the bell: build a pendulum that takes exactly one second to swing across and back, and be ready to prove it.

Kipute did the arithmetic before anybody else had found a stand, wrote the length on the board, cut the thread to it, tied a brass mass to the bottom and, because every stand in the room had already gone, tied the top of the thread to the end of a plastic rule clamped to the bench. He timed fifty swings, divided, and got a number that was not one second. He timed another fifty and got the same number again.

"The watch is slow, sir."

"It is not the watch," said **Kipanga** from two benches away, *"because mine is doing it as well, and mine is tied to a rule too."*

Mr. Akilikubwa looked at the two rules, both of them nodding gently at the far end, and did not say what was wrong. *"Lengthen the thread,"* he said. *"Then go on lengthening it until you have six lengths, and plot what you get. The answer is in the shape of the line, not in the watch."*

Aim

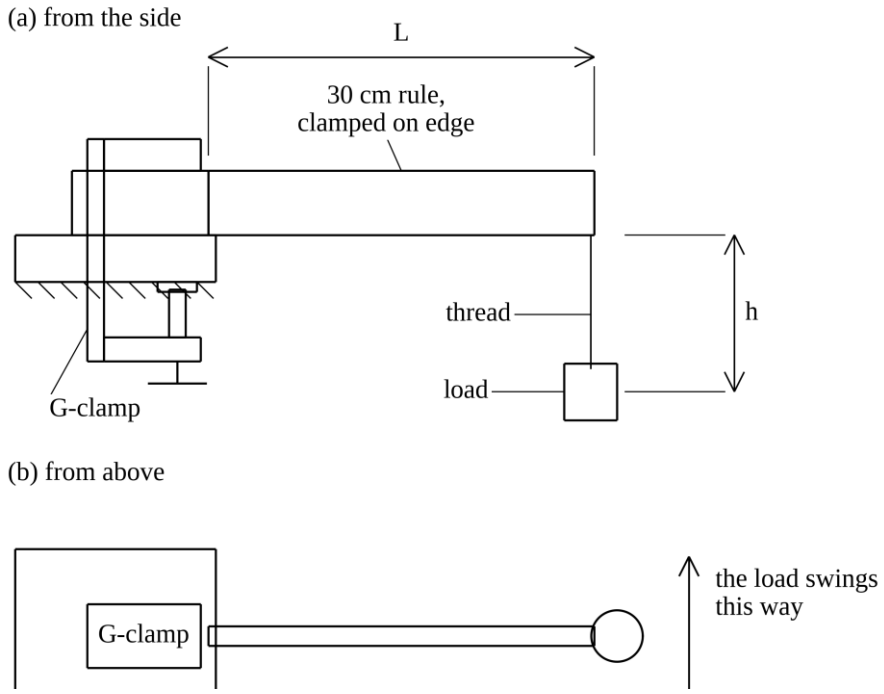
To determine the acceleration due to gravity and Young's modulus for the material of a rule, from the oscillations of a load hung by a thread below the free end of a clamped rule.

Apparatus

A plastic rule 30cm long, a G-clamp, a bench with a straight edge, a mass of 400g, a length of inextensible thread, a metre rule, a stopwatch reading to 0.01s, a micrometer screw gauge and a vernier calliper.

Setup

The plastic rule is clamped to the bench on edge, that is with its flat faces vertical, so that a length L of 22.0cm projects horizontally beyond the edge of the clamp. The thread is tied to the free end and the load hangs from it, and h is the distance from the free end of the rule to the centre of the load. The load is drawn a little to one side, in the direction at right angles to the rule, and released, so that it swings in a horizontal line while the free end of the rule moves with it. The projecting length is left untouched from the first reading to the last.



Procedure

Proceed as follows:

(a) Measure the thickness of the rule with the micrometer screw gauge and its breadth with the vernier calliper.

(b) Clamp the rule on edge at the edge of the bench with the G-clamp, so that a length of 22.0cm projects beyond the edge of the clamp, and tighten the clamp firmly.

(c) Tie the thread to the free end of the rule and hang the 400g mass from it so that h, the distance from the free end of the rule to the centre of the load, is 20.0cm.

(d) Draw the load about two centimetres to one side, in the direction at right angles to the rule, and release it. Allow two oscillations to settle, then record the time t for 25 complete oscillations.

(e) Repeat the procedure in 11.2 (c) and (d) with h equal to 30.0cm, 40.0cm, 50.0cm, 60.0cm and 70.0cm, the projecting length being left untouched throughout.

Questions

(i) Tabulate your results, including T and T^2.

(ii) Plot a graph of T^2 (s^2) against h (m).

(iii) Let k be the horizontal force needed to move the free end of the rule through one metre. Show that $T^2=(4\pi^2/g)h+4\pi^2M/k$.

(iv) Determine the slope of your graph and the intercept it makes on the T^2 axis.

(v) Use your slope to determine the acceleration due to gravity.

(vi) Use your intercept to determine k, and hence Young's modulus for the material of the rule, given that $k=Ebd^3/4L^3$, that the breadth b is 28.6mm and that the thickness d is 2.82mm.

(vii) State, with a reason, why the rule is clamped on edge and not flat.

Table of results

The projecting length was 22.0cm throughout and the load was 400g. The breadth of the rule was 28.6mm and its thickness 2.82mm.

h (cm)	h (m)	t for 25 oscillations (s)	T (s)	T^2 (s^2)
20.0	0.200	26.87	1.075	1.156
30.0	0.300	30.96	1.238	1.533
40.0	0.400	35.08	1.403	1.968
50.0	0.500	38.57	1.543	2.381
60.0	0.600	41.74	1.670	2.789
70.0	0.700	44.29	1.772	3.140

Making the measurement count

Practical tip 1: *Two springs in a row, and which of them you are looking at.*

Nothing in this arrangement is new. A load on a thread is the pendulum of Chapter 7 and a load on the end of a rule is the cantilever of Experiment 11.1, and here they are joined end to end, so that the load has to move both of them before it can go anywhere. Chapter 10 met the same joining in two springs hung one below the other, and what held there holds here: it is the compliances that add, not the stiffnesses. Part (iii) asks you to write that down for this arrangement.

What follows from it is worth knowing before you start. The softer of the two takes the larger share of the movement, so a short stiff rule leaves nearly all the work to the thread and a long floppy one takes most of it for itself. Neither extreme is any use here, because the graph needs both a slope worth measuring and an intercept worth reading.

Practical tip 2: *Measuring h, and keeping it what you measured.*

The length in the pendulum part of this experiment runs from the point of support to the centre of the load, so h is measured from the free end of the rule to the middle of the mass, not to the top of it and not to the knot. Measure it with the load hanging in place and the thread taut, holding the metre rule alongside without touching the thread.

A knot that slips takes the experiment with it, because every later reading is then measured from a support that has moved. Tie the thread to the rule so that it cannot ride along the end, retie it at each new length rather than paying it out, and measure h again after each oscillation run rather than before it.

Practical tip 3: *How far to pull it aside, and why two centimetres and not ten.*

Two things go wrong when the swing is large, and they pull in opposite directions, which is worse than either alone. The period of a pendulum grows with the angle of swing, by about one part in sixteen of the square of the angle in radians, so a swing of ten degrees lengthens the period by two parts in a thousand and the value of g that comes out of it by four. At two centimetres the angle here is under five degrees and the effect is smaller than anything you can read off the paper.

The other is the rule. Its free end moves with the load, and the cantilever formula holds while the bending is small compared with the projecting length. Pull the load half way across the bench and the free end travels far enough to leave that assumption behind, and the rule may take a permanent set, after which the stiffness you measure is not the stiffness the first three readings had.

Practical tip 4: *The slope and the intercept are not read to the same precision, and they do not deserve to be.*

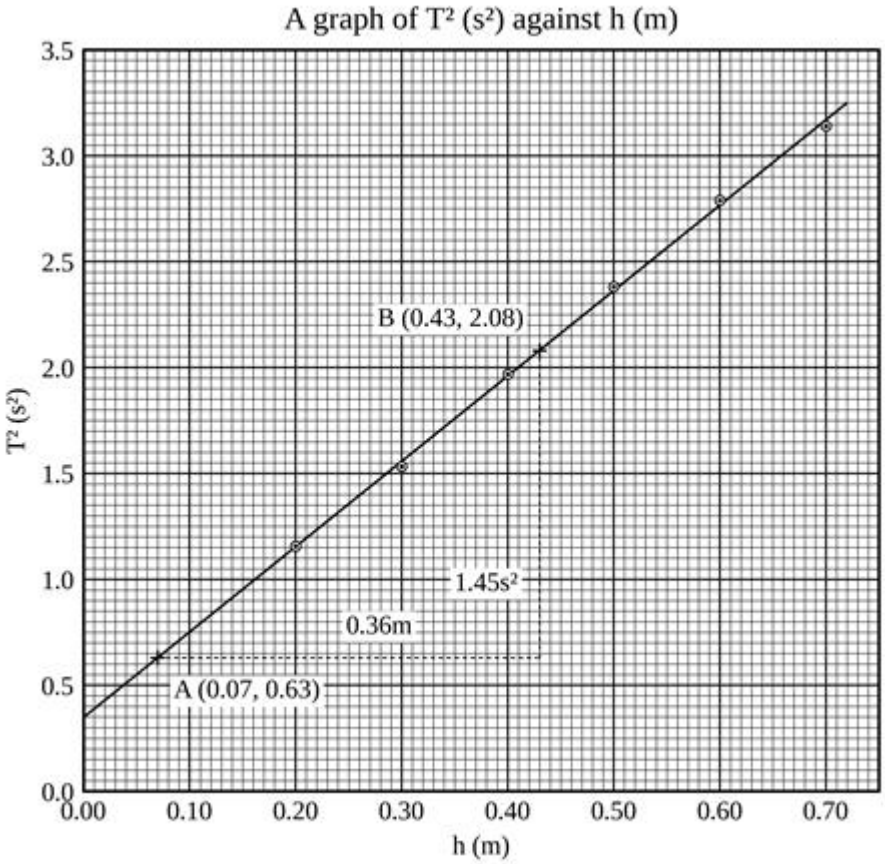
Look at what the graph offers. The slope is taken across the greater part of the plotted range, so a fifth of a small square at each end of the triangle is a small fraction of a large difference, and the percentage uncertainty in the

slope is about one. The intercept is one reading at one place, made where the line meets the axis, and a fifth of a small square there is nearer three percent of it.

Quote your answers accordingly. On this graph the last figure of anything that comes off the slope can be defended and the last figure of anything that comes off the intercept cannot, and part (vi) has two such answers one after the other, because Young's modulus is got from k and k comes from the intercept.

Answers to the questions

(i) and (ii). The table is printed above; T is the recorded time divided by 25. Give the graph the title A graph of T^2 (s^2) against h (m). Take h at 2cm to 0.10m, running from 0m to 0.75m, and T^2 at 2cm to $0.50s^2$, running from $0s^2$ to $3.50s^2$. That is 15cm by 14cm, which fits an A4 sheet. Both axes must start at zero, because part (iv) asks for the intercept.



(iii) Draw the load aside a small distance and hold it there. The thread pulls the free end of the rule sideways with a horizontal force F, so the free end has moved through $x_1=F/k$, by the definition of k. The load itself hangs at a small angle to the vertical through the free end, and for a mass M on a thread of length h the horizontal force needed to hold it a distance x_2 out of the vertical is $F=Mgx_2/h$, so $x_2=Fh/Mg$. The load has moved the two displacements added:

$$x=x_1+x_2=\frac{F}{k}+\frac{Fh}{Mg}=F\left(\frac{1}{k}+\frac{h}{Mg}\right)$$

so the arrangement behaves as one restoring system whose stiffness $K=F/x$ satisfies $1/K=1/k+h/Mg$. Its period is $T^2=4\pi^2M/K$, that is

$$T^2=4\pi^2\frac{M}{K}=4\pi^2M\left(\frac{1}{k}+\frac{h}{Mg}\right)=\frac{4\pi^2}{g}h+\frac{4\pi^2M}{k}$$

which is a straight line in h. The load has dropped out of the first term and stayed in the second, and that is the whole reason the graph can be made to give two answers at once.

(iv) Taking A (0.07m, $0.63s^2$) and B (0.43m, $2.08s^2$) on the line:

$$\text{slope}=\frac{2.08s^2\text{-}0.63s^2}{0.43\text{m-}0.07\text{m}}=\frac{1.45s^2}{0.36\text{m}}=4.03s^2m^{-1}$$

and the line meets the T^2 axis at $0.348s^2$.

(v) The slope is $4\pi^2/g$, so

$$g=\frac{4\times3.14^2}{4.03s^2m^{-1}}=9.79ms^{-2}$$

against the accepted $9.81ms^{-2}$, a difference of two parts in a thousand. Do the arithmetic before you name a cause for it. The points scatter about the line by half a small square at worst, which is by itself worth more than the whole difference. The swing angle is the systematic error here most likely to be worth computing, and it is under five degrees; it lengthens the short-thread periods more than the long ones, so it lowers the slope and raises the value of g by about a tenth of that difference. It works the wrong way to explain a shortfall, and it is too small to explain anything. Say that the difference is inside the scatter and leave it there.

(vi) The intercept is $4\pi^2M/k$, so

$$k=\frac{4\times3.14^2\times0.400kg}{0.348s^2}=45.3Nm^{-1}$$

which is the horizontal force, in newtons, that would move the free end of the rule through one metre if it could be moved that far. Then, from $k=Ebd^3/4L^3$,

$$E=\frac{4kL^3}{bd^3}=\frac{4\times45.3Nm^{-1}\times0.010648m^3}{6.414\times10^{-10}m^4}=3.01\times10^9Nm^{-2}$$

using $bd^3=0.0286m\times(2.82\times10^{-3}m)^3$. Three thousand million newtons per square metre is the right order for a moulded plastic, and about a third of the figure Experiment 11.1 found for wood. Both this and the stiffness above came off the intercept, so write the third figure down but do not defend it.

(vii) Because a rule on edge is stiff in the direction the load hangs and soft in the direction it swings, and the experiment needs exactly that. The stiffness holds bd^3, so turning the rule from flat to edgewise multiplies the stiffness in the vertical direction by the square of $28.6mm/2.82mm$, which is about a hundred. Clamped on edge the rule carries the $400g$ with a droop of less than a millimetre and stays where it is put, while the sideways bending it does allow is what the intercept measures.

Clamped flat, the same rule would sag several centimetres under the load before you began, and worse, it would be soft in the vertical direction and stiff in the horizontal one, so the load would bounce up and down as readily as it swung. The two motions have different periods; you would be timing whichever of the two had the larger amplitude at that moment, and no straight line would come of it.

Examiner's eye

- Two answers come off one line, and they come off different parts of it. The slope knows nothing about the rule and the intercept knows nothing about g. Say which is which in your comment, because a candidate who divides the intercept by the slope has found a length and thinks it is a modulus.
- The stiffness k belongs to that rule at that projecting length. Loosen the clamp between readings and you have two experiments on one sheet of paper, and the intercept of the line through them means nothing at all.
- Leave h as a length. It is a length in metres on the horizontal axis and it stays one; it is the ordinate that has been squared. Square both and the line bends into an obvious curve, which is at least the kind of mistake that shows.
- Measure h to the centre of the load and not to the top of it. A load two centimetres deep, measured to its top, makes every abscissa short by a centimetre, which slides the whole line sideways: the slope survives it, and the intercept does not.

Practice 11.2

A notice appeared on the laboratory door on the Tuesday of the second week, in **Mr. Akilikubwa**'s handwriting.

PLASTIC RULES ARE FOR MEASURING AND FOR BENDING. THIS TERM, IN THIS ROOM, THEY HAVE ALSO BEEN USED AS: (a) A CATAPULT. (b) A SPOON. (c) A DEVICE FOR PUTTING A THREAD INTO THE CEILING FAN. (d) A RULE. THE LAST OF THESE IS PERMITTED.

Kipanga read it twice and asked which of them was his. *"The ceiling fan,"* said **Mr. Akilikubwa**, *"and since you and **Kipute** are so fond of hanging things from the ends of rules, you may both spend this afternoon doing it properly."*

Properly, this time, means leaving the thread alone and changing the load instead.

Procedure

Proceed as follows:

(a) Clamp a plastic rule 50cm long on edge to the bench with a G-clamp, at the edge of the bench, so that a length L of 24.0cm projects beyond the edge of the clamp, and tighten the clamp firmly.

(b) Tie a thread to the free end of the rule and hang from it a total mass M of 100g, the hanger included, so that h, the distance from the free end of the rule to the centre of the load, is 30.0cm.

(c) Draw the load about two centimetres to one side, in the direction at right angles to the rule, and release it. Allow two oscillations to settle, then record the time t for 25 complete oscillations.

(d) Repeat the procedure in P11.2 (b) and (c) with M equal to 200g, 300g, 400g, 500g and 600g, the thread length and the projecting length being left untouched throughout.

Questions

(i) Tabulate your results, including T and T^2.

(ii) Plot a graph of T^2 (s^2) against M (kg).

(iii) Determine the slope of your graph and the intercept it makes on the T^2 axis.

(iv) For this arrangement $T^2=(4\pi^2/g)h+4\pi^2M/k$, where k is the horizontal force needed to move the free end of the rule through one metre. Use your slope to determine k, and hence Young's modulus for the material of the rule, given that $k=Ebd^3/4L^3$, that b is 30.0mm and that d is 2.50mm.

(v) Use your intercept to determine the acceleration due to gravity.

(vi) The rule itself has to be set moving as well as the load. State which of your two answers that affects, and in which direction.

(vii) The experiment is repeated with the same rule clamped so that only 12.0cm projects, the thread and the loads being unchanged. State how the slope and the intercept of the new graph would compare with yours.

Experiment 11.3

The carpenter whose shop backs on to the school field builds bookshelves to order, and he has a policy about them he will not explain. Ask for a shelf longer than about three feet and he will build it in two, with an upright down the middle, and he will not be argued out of it even by a customer who wants one long shelf and is holding the money.

Kipanga had a theory. The school library has one long shelf, put up by somebody else, and it dips in the middle like a hammock; and **Kipanga**'s theory was that it had been fitted upside down, and that taking it out, turning it over and putting it back would set the sag going the other way and cancel it.

Mr. Akilikubwa agreed that this was worth trying, and it was tried, and the shelf now dips in the middle exactly as much as it did before, and the books are back on it. What the carpenter knows and neither of them did is how fast the sag grows with the length, and it is not a thing you would guess.

Aim

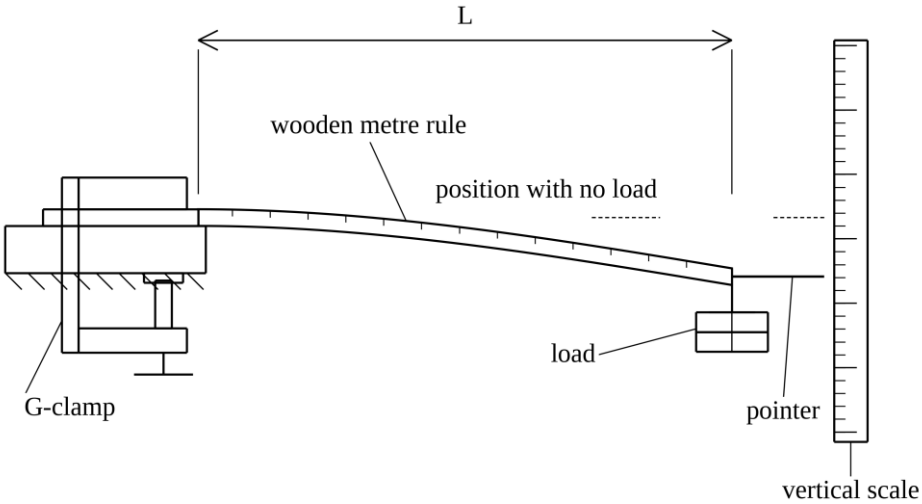
To determine Young's modulus for the wood of a metre rule from the static depression of a loaded cantilever.

Apparatus

The wooden metre rule of Experiment 11.1, a G-clamp, a bench with a straight edge, a mass hanger with slotted masses making up a total of 250g, a short loop of thread, a pin fixed to the free end of the rule with a little plasticine to serve as a pointer, a half-metre rule clamped vertically beside it, a micrometer screw gauge and a vernier calliper.

Setup

The rule is clamped flat to the bench at its edge, exactly as in Experiment 11.1, with a length L projecting. The pin at the free end moves against the vertical half-metre rule, which is clamped so that it does not move when the bench is knocked. For each projecting length the position of the pointer is read twice, once with nothing on the rule and once with the load hanging from the free end, and the depression s is the second reading less the first. The clamp stays where it is and the rule is slid through it, so that the free end stays opposite the vertical scale.



Procedure

Proceed as follows:

(a) Measure the thickness of the rule with the micrometer screw gauge and its breadth with the vernier calliper.

(b) Clamp the rule flat to the bench with the G-clamp, at the edge of the bench, so that a length L of 40.0cm projects beyond the edge of the clamp, and tighten the clamp firmly.

(c) With nothing hanging on the rule, read the position of the pointer against the vertical half-metre rule, to the nearest half millimetre, with your eye level with the pin.

(d) Hang the loop of thread over the free end and hang from it a total mass M of 250g, the hanger included. Allow the rule to come to rest and read the position of the pointer again.

(e) Remove the load, slide the rule through the clamp and repeat the procedure in 11.3 (c) and (d) with L equal to 45.0cm, 50.0cm, 55.0cm, 60.0cm and 65.0cm.

Questions

(i) Tabulate your results, including L^3 and the depression s.

(ii) Plot a graph of s (mm) against L^3 (m^3).

(iii) State the relationship between s and L shown by your graph.

(iv) Determine the slope of your graph and state its units.

(v) The breadth of the rule is 24.8mm and its thickness 6.05mm. Use your slope to determine Young's modulus for the wood, given that $s=4MgL^3/Ebd^3$.

(vi) A reading with no load was taken at every length and not once at the start. State what would have gone wrong had only the first been taken, and in which direction it would have moved your answer.

(vii) Calculate the depression the same rule and the same load would give at a projecting length of 0.700m, and say why the answer should be treated with caution.

Table of results

The mass hung at the free end was 250g throughout, and the readings are the position of the pointer down the vertical scale.

L (cm)	L^3 (m^3)	Reading, no load (mm)	Reading, loaded (mm)	s (mm)
40.0	0.0640	121.0	132.0	11.0
45.0	0.0911	121.5	137.0	15.5
50.0	0.1250	122.0	145.0	23.0
55.0	0.1664	123.0	153.5	30.5
60.0	0.2160	124.0	162.0	38.0
65.0	0.2746	125.5	173.5	48.0

Making the measurement count

Practical tip 1: *The newtons in this experiment were never measured.*

The formula holds a force, and you did not measure one. What you did was read a number stamped on a piece of brass, turn it into kilograms and multiply by 9.81ms^{-2}, because a weight in newtons is a mass in kilograms times the acceleration due to gravity. The hanger carries its own mass stamped on it and it is part of the load; the loop of thread is not, because it weighs a fraction of a gram against two hundred and fifty of them.

This is the first of the two measurements in this chapter that cannot be made at all without a value of g being carried into them; the other is the beam on two knife edges. Notice what that costs. A static depression alone can never test a value of g, because one has had to be carried into it before it could start, in order to turn the masses into forces. A period needs none, which is why Experiment 11.2 can hand one back.

Practical tip 2: *A depression is a difference, so it costs two readings and carries the errors of both.*

The quantity you plot is not measured at all. It is the distance the pointer moved, and the only way to get it is to read the scale twice and subtract, so half a millimetre out in either reading is half a millimetre out in the depression. Keep the two readings in two columns of their own and do the subtraction in a third. Do it over the top of the readings and the column ends up holding two kinds of number with nothing to tell them apart.

Get your eye level with the pin, every time, and read the scale where the pin crosses it rather than where you think it ought to. A pin looked down on from above gives a reading further down the scale than the truth, and one looked up at gives a reading further up, and the two cancel in a difference only if you made them from the same place. That is also why the vertical scale is clamped and not held.

Practical tip 3: *Two limits, and how to tell you have passed them.*

The formula assumes the depression stays small compared with the projecting length, because it takes the free end to move straight down. Bend the rule far enough and the free end swings in towards the clamp as well, the length that is bending is no longer the length you laid off along the bench, and the formula begins to overestimate the sag. Part (vii) asks you to decide whether that has begun to happen.

Beyond that is the elastic limit, where the wood takes a permanent set and does not come back. The test costs nothing: take the load off and read the pointer again, and if it has not returned to the no-load reading, the rule has been bent and every reading after the one that bent it was measured from a position the pointer had already left.

Practical tip 4: *The sag and the period are the same measurement.*

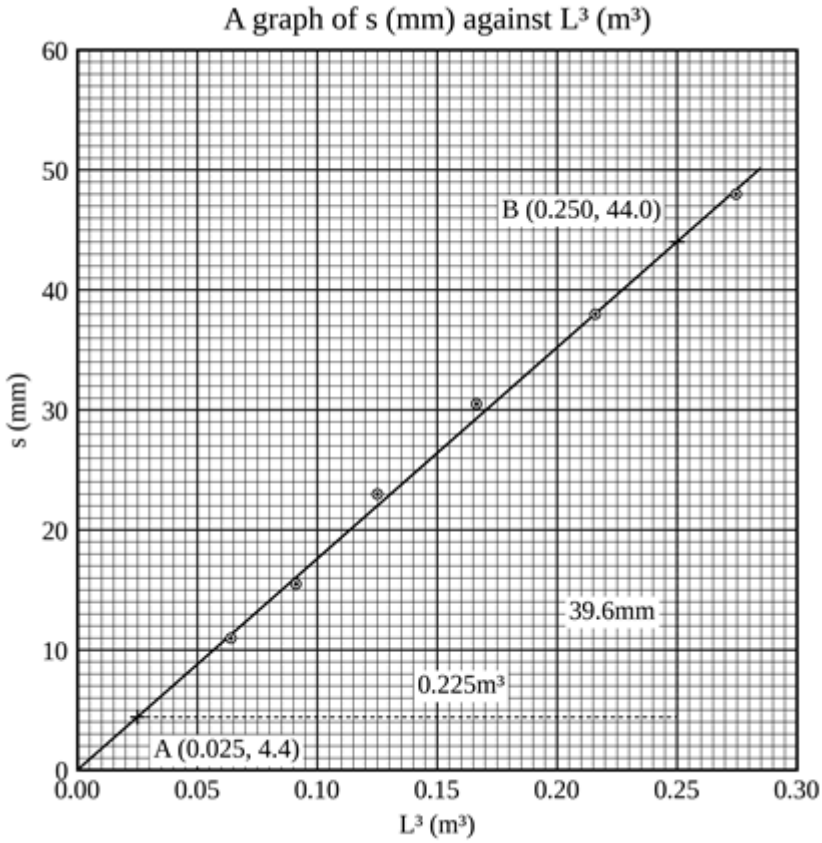
Put the two cantilever equations side by side. The depression is $s=4MgL^3/Ebd^3$ and the period satisfies $T^2=16\pi^2ML^3/Ebd^3$. Everything except g and the numbers cancels between them, and what is left is

$$s=\frac{g}{4\pi^2}T^2$$

which holds for any cantilever whatever, of any material and any shape, provided the rule's own mass is small beside the load. It says that a cantilever with a period you can count must droop several centimetres, and that a cantilever stiff enough to stay nearly straight vibrates too fast to time. That is not a coincidence to be admired and passed over; it is why the two experiments in this chapter that use the same rule use it at the lengths they do.

Answers to the questions

(i) and (ii). The table is printed above. s is the loaded reading less the no-load reading of the same row. Give the graph the title A graph of s (mm) against L^3 (m^3). Take L^3 at 2cm to $0.05m^3$, running from $0m^3$ to $0.30m^3$, and s at 2cm to 10mm, running from 0mm to 60mm. That is 12cm by 12cm, which fits an A4 sheet.



(iii) The points lie on a straight line through the origin, so $s \propto L^3$: the depression is proportional to the cube of the projecting length, and a rule pulled out to one and a half times its length sags not half again as far but three and a third times as far. That is what the carpenter has learned, and it is why he builds the shelf in two.

(iv) Taking A ($0.025m^3$, 4.4mm) and B ($0.250m^3$, 44.0mm) on the line:

$$\text{slope}=\frac{44.0\text{mm}-4.4\text{mm}}{0.250\text{m}^3-0.025\text{m}^3}=\frac{39.6\text{mm}}{0.225\text{m}^3}=176(\text{mm})\text{m}^{-3}$$

The unit is millimetres per cubic metre, which is an awkward thing to carry into a substitution, so turn it round first: 176mmm^{-3} is 0.176m^{-2}, a depression in metres for each cubic metre of length cubed.

(v) From $s=4MgL^3/Ebd^3$, the slope of a graph of s against L^3 is $4Mg/Ebd^3$, so

$$E=\frac{4Mg}{\text{slope}\times bd^3}=\frac{4\times0.250\text{kg}\times9.81\text{ms}^{-2}}{0.176\text{m}^{-2}\times5.492\times10^{-9}\text{m}^4}=1.01\times10^{10}\text{Nm}^{-2}$$

using $bd^3=0.0248\text{m}\times\left(6.05\times10^{-3}\text{m}\right)^3$ as before. Experiment 11.1 used this same rule and found $9.84\times10^9\text{Nm}^{-2}$, three percent lower. The two are not measuring quite the same thing: the vibrating rule has to carry a share of its own mass as well as the load, which lengthens every period and pushes the modulus down, while a static depression, taken as a difference of two readings at one length, subtracts the rule's own sag away. Three percent is about the size of that share, and Practice 11.1 measures it directly.

(vi) Every depression after the first would have been too large, and by an amount that grew as the rule was pulled further out, so the slope would have come out too large and E too small. The rule sags under its own weight before anything is hung on it at all, and that sag grows with the fourth power of the projecting length: about 0.8mm at 0.40m and about 5.6mm at 0.65m. Take the no-load reading at every length and that sag sits in both readings of the row, where it cancels; take it once at the shortest length and the difference of the two sags is left inside every later depression. The no-load column above shows what that costs: the last row would have come out four and a half millimetres too deep.

(vii) The depression is the slope multiplied by the cube of the length:

$$s=176\text{mmm}^{-3}\times(0.700\text{m})^3=176\text{mmm}^{-3}\times0.343\text{m}^3=60.4\text{mm}$$

Treat it with caution because 60mm is nearly a tenth of 700mm, and the whole derivation assumes the depression is small beside the projecting length. At that angle the free end has moved measurably in towards the clamp as well as downwards, so the length that is bending is no longer the length you measured along the bench, and the formula overestimates the sag. It is also an extrapolation: the furthest length actually measured was 0.65m, and a line is only evidence over the range that produced it.

Examiner's eye

- The quantity to be plotted is the depression, not the reading. The pointer readings are large numbers with a great deal in common and very little difference between them; the difference is the whole experiment.
- A slope in millimetres per cubic metre put straight into a formula written in metres gives a modulus a thousand times too small, and $1\times10^7\text{Nm}^{-2}$ for a wooden rule is the modulus of something you could push your thumb through.
- The no-load reading belongs to its own row. It is not a constant of the apparatus, because the rule sags further under its own weight at every new length, and part (vi) is about exactly that.
- Name the quantity that had to be brought in from outside. This experiment cannot be done without a value for the acceleration due to gravity, and an answer that never mentions it has left the newtons unaccounted for.

Practice 11.3

Mr. Akilikubwa carries a sack of maize home on the back of his bicycle once a month, and once a month the carrier over the back wheel bows in the middle under it and springs back when the sack comes off. This month it did not spring all the way back, so the bicycle went to the repairer at the corner, who looked at it for some time and then offered two remedies. The first was a thicker carrier. The second was less maize.

Kipute wanted to know which of the two the repairer would charge for. **Kipanga** wanted to know something more useful: the carrier rests on two struts and the sack sits between them, so if the sag went by what was on it, the carrier would be a weighing machine and would have been one all along. Nobody at the corner was in a position to say.

Settling it needs a scale to read the sag against and something whose mass is known to put on top, and the laboratory has both, with a metre rule for the carrier and slotted masses for the maize.

Procedure

Proceed as follows:

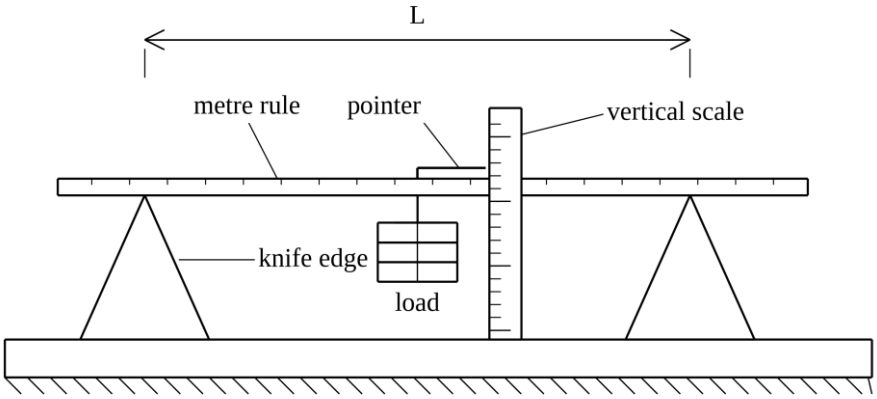
(a) Rest a wooden metre rule flat on two knife edges placed on the bench 80.0cm apart, with the mid-point of the rule half way between them.

(b) Fix a pin to the rule at its mid-point with a little plasticine to serve as a pointer, and clamp a vertical scale beside it.

(c) With nothing hanging on the rule, read the position of the pointer against the vertical scale, to the nearest half millimetre, with your eye level with the pin.

(d) Hang a loop of thread over the rule at its mid-point and hang from it a total mass M of 400g, the hanger included. Allow the rule to come to rest and read the position of the pointer again.

(e) Repeat the procedure in P11.3 (d) with M equal to 600g, 800g, 1000g, 1200g and 1400g, the knife edges being left untouched throughout.



Questions

(i) Tabulate your results, including the depression s of the mid-point.

(ii) Plot a graph of s (mm) against M (kg).

(iii) State the relationship between s and M shown by your graph, and determine the slope, with its units.

(iv) For a beam of breadth b and thickness d resting on two supports a distance L apart and loaded at its mid-point with a mass M, the depression of the mid-point is $s = MgL^3/4Ebd^3$. The breadth of the rule is 25.2mm and its thickness 5.40mm. Use your slope to determine Young's modulus for the wood.

(v) The knife edges were 80.0cm apart on a rule one metre long, so that 10.0cm projected beyond each of them. State, with a reason, whether those projecting ends affect your result.

(vi) Only one reading with no load was taken, at the beginning. State why one was enough here.

(vii) Calculate the mass that would depress the mid-point by 25.0mm, and state whether your experiment supports the answer.

Chapter 12

MOMENTS, EQUILIBRIUM AND APPLICATIONS

The whole of this chapter is done with little more than a metre rule, a knife edge and a few known masses. There is no stopwatch in it and no spring, and every reading you take is a distance along a rule. What that buys you is a surprising list: the mass of a body heavier than anything in the box of masses, the mass of the rule itself, the place where its centre of gravity really is, and the relative density of a solid and of a liquid.

All of it rests on one statement. *A body that is balanced does not turn, and it does not turn because the turning effects on the two sides of the pivot are equal.* **The turning effect of a force about a point** *is the force multiplied by the perpendicular distance from the point to the line of the force,* and it is called the **moment of the force.** So for a rule resting on a knife edge with a mass X hung a distance a from the knife edge on one side and a mass m hung a distance b from it on the other, the condition for balance is

$$Xa = mb$$

Two habits of thought will carry you through the four experiments below. The first is to ask, before writing anything down, what is pressing on the knife edge and where each of those things acts. A rule has a mass of its own and that mass acts at one point, the centre of gravity; whether it appears in your equation depends entirely on where the knife edge is, and the experiments below are built on both answers, three of them putting the knife edge at the centre of gravity and one putting it somewhere else on purpose. The second habit is to distrust a single pair of readings. One balance point gives you an answer and no way of knowing whether it is a good one; six of them, plotted, tell you both.

One warning belongs at the front, as it has in each of the last two chapters. In Chapter 10 it was a zero and in Chapter 11 it was a cube. Here it is subtraction. The last two of the four experiments end by subtracting one measured quantity from another very like it, and a difference is always worth less than the numbers that made it. Two slopes, each read to better than one part in a hundred, can differ by a third of their own size, and that difference is then worth only two or three parts in a hundred. Watch for the subtractions as they arrive.

The four varieties at a glance

	What is different about it	What the graph gives you
12.1	The rule is balanced at its own centre of gravity first, and both distances are then changed together	the mass of a body, from a straight line
12.2	The load stays at the end of the rule and the knife edge is moved to meet it	the mass of the rule and the position of its centre of gravity, from one line
12.3	The body hangs on a thread and is lowered into water	the mass of the body and its relative density, from two lines
12.4	A second liquid takes the place of the water	the relative density of that liquid, from three lines

All four are worked below in full. The second variety takes the knife edge away from the centre of gravity and makes the rule one of the loads; the third and fourth hang the body on a long thread so that it can be balanced while it is under a liquid.

Experiment 12.1

The school's box of masses came back from the chemistry laboratory at the end of term with the 500g cylinder missing out of the middle of it. **Mr. Akilikubwa** found it a fortnight later, holding the store-room door open against the wind, and by then he had stopped looking.

What he wanted it for was a lump of brass off the workshop floor, which somebody was going to have to pay for. **Kipute** said the thing could not be done. *"The heaviest one left in the box is a hundred grams, sir. You cannot weigh anything with masses smaller than it is."*

Mr. Akilikubwa laid a metre rule across a knife edge, slid it until it sat level, and hung the hundred on one side of the knife edge and the brass on the other. Then he moved the two of them about until the rule came level again.

"Now tell me what I have just done," he said, *"and then tell me why the door is standing open."*

Aim

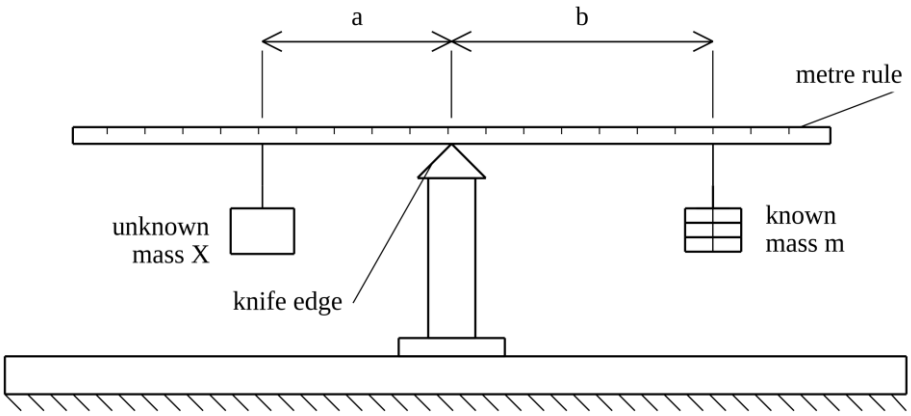
To determine the mass of a body by the principle of moments.

Apparatus

A metre rule, a knife edge mounted on a stand, the body whose mass is wanted, a single known mass of 100g and two short loops of fine thread.

Setup

The rule rests across the knife edge and is slid along until it balances with nothing hanging on it. That point is the centre of gravity of the rule, and the knife edge stays there for the whole of this experiment. The unknown mass X hangs from a loop of thread a distance a from the knife edge, and the known mass m of 100g hangs from a second loop on the other side, a distance b from it. Both distances are measured from the knife edge to the centre of the loop that carries the mass.



Procedure

Proceed as follows:

(a) Rest the metre rule across the knife edge and slide it until it balances horizontally with nothing hanging on it. Note the mark at which it balances and leave the knife edge at that point.

(b) Hang the unknown mass X from a loop of thread at a distance a of 10.0cm from the knife edge.

(c) Hang the known mass m of 100g from a second loop on the other side of the knife edge, and slide that loop along the rule until the rule balances horizontally again. Record the distance b from the knife edge to the loop.

(d) Repeat the procedure in 12.1 (b) and (c) with a equal to 14.0cm, 18.0cm, 22.0cm, 26.0cm and 30.0cm.

Questions

(i) Tabulate your results.

(ii) Plot a graph of b (cm) against a (cm).

(iii) State the relationship between b and a shown by your graph.

(iv) Determine the slope of your graph and state its unit.

(v) Use your slope to determine the mass X of the unknown body.

(vi) A moment is a force multiplied by a distance, yet your working in part (v) used masses throughout and no value of the acceleration due to gravity. Explain why the mass you obtain is unaffected.

(vii) Suppose the knife edge had been left a few millimetres to one side of the balance point found in 12.1 (a). State what would happen to your line, and whether the mass obtained from it would then be too large, too small or unchanged.

Table of results

The known mass was 100g throughout, and the rule balanced at the 50.3cm mark with nothing hanging on it.

a (cm)	b (cm)
10.0	14.1
14.0	19.0
18.0	24.5
22.0	30.6
26.0	36.0
30.0	41.4

Making the measurement count

Practical tip 1: *One equation, and what has been left out of it.*

Take moments about the knife edge. The unknown mass turns the rule one way with a moment Xga and the known mass turns it the other with a moment mgb, and when the rule is level the two are equal. There is a third mass on the knife edge, the rule's own, and it is the reason for step (a): a body's weight acts through its centre of gravity, so if the knife edge is at that point the rule's weight has no lever arm and no moment about it. Balance the rule empty, and the equation has two terms in it instead of three.

Written for the graph the equation is

$$b=\frac{X}{m}a$$

so the slope carries the two masses and nothing else. Part (v) asks you to turn it into a mass, part (vi) asks what happened to the g in the two moments, and part (vii) asks what step (a) was worth.

Practical tip 2: *Finding the balance point, and keeping it.*

Rest the rule on the knife edge and slide it a few millimetres at a time. Near the balance point the rule tips slowly instead of falling, and there is a short stretch of rule over which it will sit level at all; take the middle of that stretch. Read the mark, write it down, and do not move the knife edge again. If the rule is knocked off, balance it empty a second time before going on, and check that you get the same mark.

Nothing here requires the rule to balance at the 50.0cm mark, and nothing goes wrong if it does not: varnish, a chipped end and a metal cap are each enough to move the point half a centimetre. What matters is only that the knife edge is at the point, wherever the point turns out to be.

Practical tip 3: *Where a distance is measured from.*

Both distances run from the knife edge to the thread that carries the mass, not to the near edge of the loop and not to the mass hanging below it. A loop a centimetre wide leaves you unsure of the distance by half a centimetre, which is one part in twenty at 10.0cm and one part in sixty at 30.0cm. That is the reason the smallest distance in the ladder is ten centimetres and not three. Tie the loops small, tie them in fine thread, and pull them tight before you read anything.

Practical tip 4: *Reading a balance instead of guessing at one.*

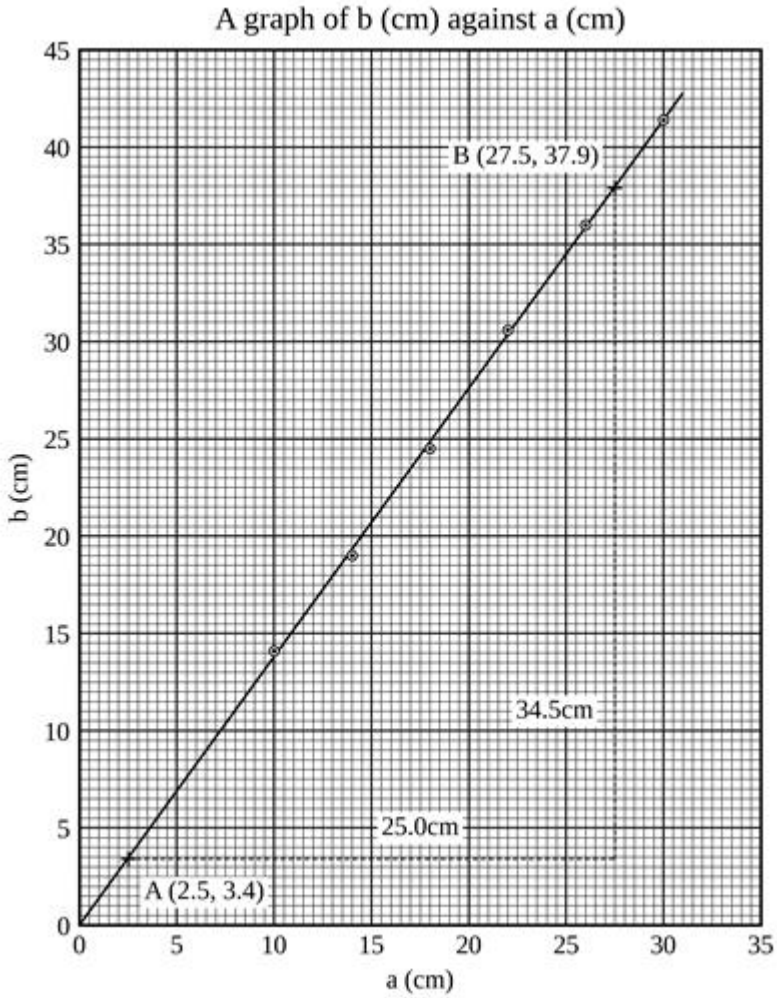
The rule is balanced when it will rest level, not when it happens to be level. Slide the loop until the rule tips one way, note the mark; slide it back until it tips the other way, note that mark; the balance point is halfway between

them. On a good knife edge those two marks are half a centimetre apart at most, and the halfway rule turns a reading you were going to estimate into one you have measured.

Do not press the rule down to level it and do not hold it there. Let go of it, tap the bench once, and see where it settles.

Answers to the questions

(i) and (ii). The table is printed above. Take a at 2cm to 5cm, running from 0cm to 35cm, and b at 2cm to 5cm, running from 0cm to 45cm. That is 14cm by 18cm, which fits an A4 sheet upright. Give the graph the title A graph of b (cm) against a (cm). Both axes must start at zero, because part (vii) turns on where the line meets them.



(iii) The six points lie on a straight line through the origin, so b∝a: double the distance of the unknown mass from the knife edge and the known mass has to go twice as far out to balance it. The line through the origin is worth a sentence of its own. It says that when one distance is nothing the other is nothing too, which is another way of saying that with the rule balanced empty there is no third moment left over to be propped up by either mass.

(iv) Taking the two marked corners A (2.5cm, 3.4cm) and B (27.5cm, 37.9cm) from the line,

$$\text{slope}=\frac{37.9\text{cm}-3.4\text{cm}}{27.5\text{cm}-2.5\text{cm}}=\frac{34.5\text{cm}}{25.0\text{cm}}=1.38$$

The slope is a length divided by a length, so it has no unit. Quote it as a bare number and resist the temptation to write centimetres after it.

(v) From Xa=mb, X=mb/a, and b/a is the slope, so

$$X=100\text{g}\times1.38=138\text{g}$$

The unknown mass is 138g, which is more than any mass left in the box, and it was found with one smaller than itself.

(vi) Every term in the balance is a weight multiplied by a distance. Written in full the condition is $Xga=mgb$, and the same g stands in both moments because both masses are in the same place on the same afternoon. Divide it out and $Xa=mb$ is left, which contains no g at all. The experiment therefore compares one mass with another and never needs to know what either of them weighs; that is why the answer is a mass in grams and not a force in newtons, and why nothing in this chapter has to be told the value of the acceleration due to gravity.

(vii) Suppose the knife edge sits a distance δ from the centre of gravity, on the side the unknown mass hangs, so that the centre of gravity is left on the known mass's side of the pivot. The rule's own mass M_r now has a lever arm and turns the rule the same way as the known mass, so the balance reads $Xa=mb+M_r\delta$, that is $b=Xa/m-M_r\delta/m$. The slope is untouched and the whole line slides down by the same amount at every point, cutting the b axis below the origin; with the knife edge displaced the other way the line rises above it. So the mass obtained is unchanged, because it comes from the slope alone.

Two things follow, and the second is the more important. First, the fault is small: a rule of about 100g displaced by 5mm moves the line by half a centimetre, one small square, which is at the edge of what these six points would show. Second, a single pair of readings has no defence against it at all. Work out mb/a from the first row of the table and the displaced knife edge goes straight into the answer; take the slope of six rows and it does not.

Examiner's eye

- Balance the rule before you hang anything on it, and say in your account that you did. An answer that begins with the loads on the rule has three moments in it and only writes down two.
- The slope here has no unit. Marks go for saying so, and go missing for a slope quoted in centimetres or in grams per centimetre.
- Distances are measured from the knife edge, not from the end of the rule and not from the 50cm mark. The knife edge is the pivot, and a moment is reckoned about the pivot.
- Do not write that the acceleration due to gravity has been assumed or taken as any particular value. It has been divided out, which is a different thing altogether.

Practice 12.1

A board at the entrance to the market, hand-painted and repainted, says that the Weights and Measures Officer calls on the first Tuesday of every month and that all scales are to be set out for him. Underneath, somebody has added in a different hand that he also calls on other days.

Mr. Akilikubwa stopped in front of it long enough for **Kipute** to read it twice. What the officer carries, it turns out, is a wooden rule, a knife edge and a nest of small brass masses that screw inside one another in his coat pocket, and with those three things he can sit down at any stall in the market and settle what any of its weights really are. The heavy standards stay at the office. He has never needed them, he says, and neither has anybody who could carry them.

The method below is his, with one change: this time it is the known mass that is changed from reading to reading, and the body whose mass is wanted that is slid along the rule.

Procedure

Proceed as follows:

(a) Rest a metre rule across a knife edge and slide it until it balances horizontally with nothing hanging on it. Note the mark at which it balances and leave the knife edge at that point.

(b) Hang a known mass m of 40g from a loop of thread at a fixed distance b of 15.0cm from the knife edge.

(c) Hang the unknown mass X from a second loop on the other side of the knife edge, and slide that loop along the rule until the rule balances horizontally. Record the distance a from the knife edge to the loop.

(d) Repeat the procedure in P12.1 (b) and (c) with m equal to 60g, 80g, 100g, 120g and 140g, the distance b being left untouched throughout.

Questions

(i) Tabulate your results.

(ii) Plot a graph of a (cm) against m (g).

(iii) State the relationship between a and m shown by your graph.

(iv) Determine the slope of your graph and state its unit.

(v) Use your slope to determine the mass X of the unknown body.

(vi) The largest known mass used was 140g. Estimate the largest known mass this arrangement could have taken, and state what sets the limit.

Experiment 12.2

The physics store has a drawer marked FAULTY, and in the drawer are three metre rules. Nobody now at the school knows who put them there or what is supposed to be wrong with them. During the stock-take **Kipanga** balanced one of them across a finger, found that it did not sit level at the 50cm mark, and announced that he had found the fault.

Kipute took two of the good ones off the shelf and balanced those. Neither of them sat level at the 50cm mark either, and the one out of the drawer and the two off the shelf were now the only rules in the school that anybody had ever measured. **Mr. Akilikubwa** let the argument run for a while and then asked the two of them what a metre rule weighs, which stopped it, because there is no balance in the physics laboratory with a pan long enough to take one.

So a finger settles the easier of the two questions in a moment and cannot touch the harder one at all. Hang a load at one end of the rule and move the knife edge along it, six times over, and one line gives you both at once, the mass included.

Aim

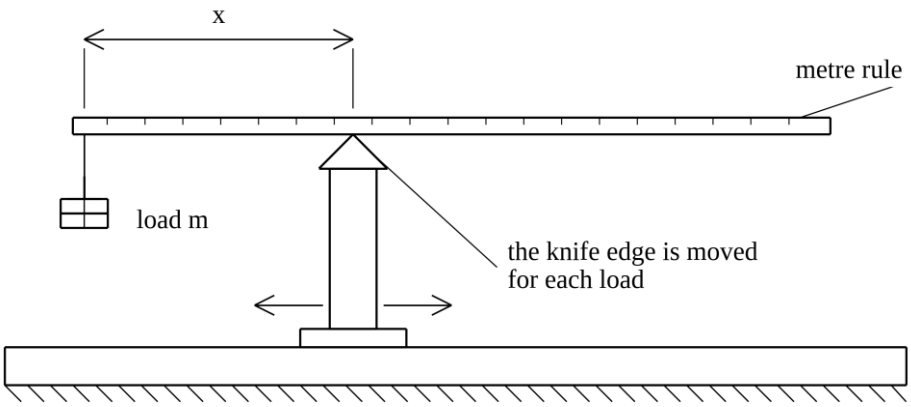
To determine the mass of a metre rule and the position of its centre of gravity by the principle of moments.

Apparatus

A metre rule, a knife edge mounted on a stand, a mass hanger with slotted masses making up 300g in the steps given below, and a loop of fine thread.

Setup

The load hangs from a loop of thread at one end of the rule, over the mark at which the scale begins, and it stays there. The knife edge is moved: for each load it is slid along until the rule balances horizontally, and the distance x from the loop to the knife edge is recorded. The rule now has its centre of gravity on the far side of the knife edge, so the rule itself is one of the two things being balanced, and x_G is written for the distance from the loop to that centre of gravity.



Procedure

Proceed as follows:

(a) Hang the mass hanger from a loop of fine thread over the end of a metre rule, at the mark at which the scale begins, and pull the loop tight so that it cannot slide.

(b) Load the hanger to a total mass m of 60g, the hanger included.

(c) Rest the rule across the knife edge and move the knife edge along the rule until the rule balances horizontally. Record the distance x from the loop to the knife edge.

(d) Repeat the procedure in 12.2 (b) and (c) with m equal to 90g, 130g, 180g, 240g and 300g.

Questions

(i) Tabulate your results, including the product mx in gcm.

(ii) Plot a graph of mx (gcm) against x (cm).

(iii) State the relationship between mx and x shown by your graph.

(iv) Determine the slope of your graph and state its unit.

(v) Determine the intercept your line makes on the mx axis, and use it together with your slope to find the mass of the rule and the distance of its centre of gravity from the loop.

(vi) Explain why the slope of this graph is negative.

(vii) The rule is 101.0cm long overall, and its scale begins 0.5cm from the end at which the load was hung. State, with a reason, whether your result shows the rule to be non-uniform.

Table of results

The load hung at the mark at which the scale begins throughout, and x was measured from that loop to the knife edge.

m (g)	x (cm)	mx (gcm)
60	32.6	1956
90	28.3	2547
130	23.5	3055
180	19.7	3546
240	16.1	3864
300	13.6	4080

Making the measurement count

Practical tip 1: *The rule is now one of the loads.*

Take moments about the knife edge. On one side the load turns the rule with a moment mgx, and on the other the rule's own weight acts at its centre of gravity, a distance x_G-x beyond the knife edge, so that its moment is that distance multiplied by M_rg. Cancel the g and

$$mx=M_r(x_G-x)$$

Multiply out and the right-hand side is a straight line in x:

$$mx=-M_rx+M_rx_G$$

which is why part (ii) asks for the product mx and not for the load by itself. Both of the constants this experiment is after are somewhere in that line, and parts (iv) to (vi) are the business of getting them out of it.

Practical tip 2: *A load that stays where it is put.*

Everything here is measured from the loop, so the loop is the zero of the experiment and it must not move. Tie it, do not hook it; pull it tight over the end of the rule; and check before each reading that it is still where it was. If the loop slides a millimetre towards the knife edge, every distance after it is a millimetre wrong and the error is not random, so the line will still look straight.

Measure x with a second rule laid alongside, from the thread to the edge of the knife. Read it off the graduations of the rule you are balancing instead and you are quicker by a few seconds and right only if the loop sits exactly on the mark at which the scale begins.

Practical tip 3: *Why the loads climb the way they do.*

The balance point runs in towards the load as the load grows, and it does not run in evenly: going from 60g to 90g moves it four centimetres and going from 240g to 300g moves it two and a half. That is the shape of the relationship, and it is the reason the loads are not in equal steps: equal steps of load would crowd the last four points into the last few centimetres of the graph and leave the first two on their own.

It also sets the two ends of the ladder. Too small a load and the knife edge runs out past the centre of gravity, where the rule will not balance at all; too large and it comes in so close to the loop that the width of the loop is a noticeable part of the distance.

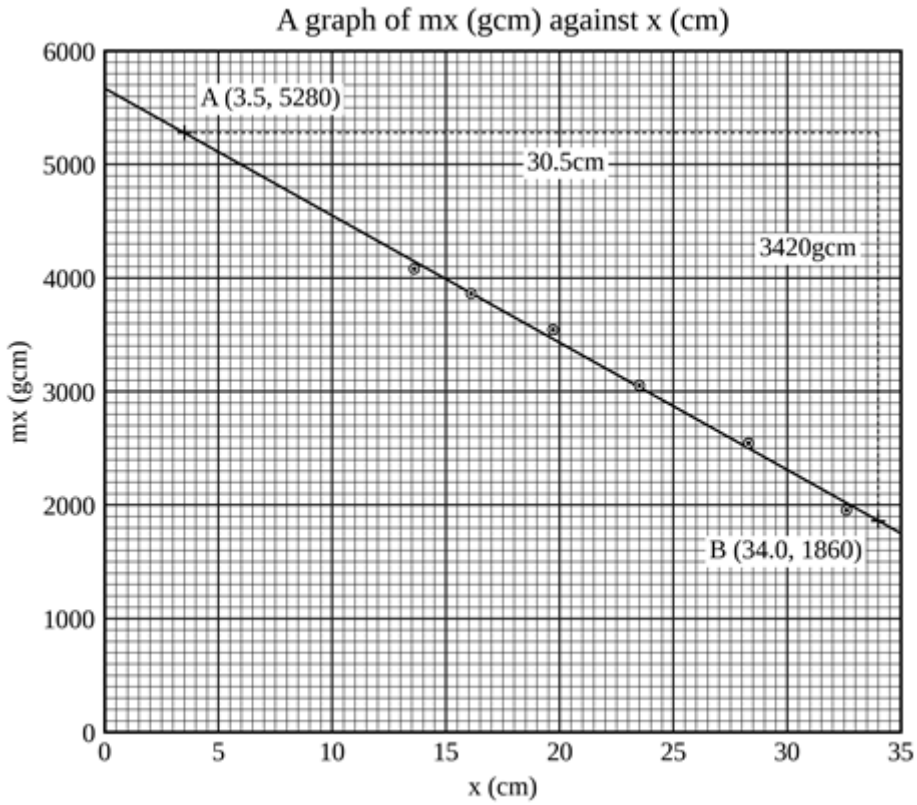
Practical tip 4: *Multiplying before you plot.*

The first column of the graph is a product you work out yourself, so work it out from the numbers you wrote down and not from the ones you meant to write down. A load of 130g at 23.5cm gives 3055gcm, and the unit of the product is the gram centimetre, which looks odd on a page and is perfectly respectable: it is a moment with the g divided out of it.

Because the product carries x in it, an error in a distance appears twice on the graph, once along each axis. Take the six distances carefully and the multiplication afterwards is arithmetic, which nobody ever got wrong twice in the same direction.

Answers to the questions

(i) and (ii). The table is printed above. Take x at 2cm to 5cm, running from 0cm to 35cm, and mx at 2cm to 1000gcm, running from 0gcm to 6000gcm. That is 14cm by 12cm, which fits an A4 sheet upright. Give the graph the title A graph of mx (gcm) against x (cm). The vertical axis must start at zero, because part (v) asks for the intercept on it.



(iii) The six points lie on a straight line of negative slope: mx falls by the same amount for every centimetre the knife edge is moved away from the load, and the line does not pass through the origin.

(iv) Taking the two marked corners A (3.5cm, 5280gcm) and B (34.0cm, 1860gcm) from the line,

$$\text{slope}=\frac{1860\text{gcm}-5280\text{gcm}}{34.0\text{cm}-3.5\text{cm}}=\frac{-3420\text{gcm}}{30.5\text{cm}}=-112\text{g}$$

A gram centimetre divided by a centimetre is a gram, so the slope is a mass, and it is negative.

(v) The line cuts the vertical axis at 5670gcm. Comparing with $mx=-M_r x+M_r x_G$, the slope is $-M_r$ and the intercept is $M_r x_G$. Hence

$$M_r=112\text{g} \quad \text{and} \quad x_G=\frac{5670\text{gcm}}{112\text{g}}=50.6\text{cm}$$

The rule has a mass of 112g and its centre of gravity lies 50.6cm from the loop. The mass is the prize here, because no balance in the laboratory will take a metre rule. The centre of gravity comes out of the same line as a by-product, and part (vii) asks what that by-product is worth.

(vi) The load's moment about the knife edge has to match the rule's, and the rule's moment is its weight multiplied by the length of arm it has left, x_G-x. Slide the knife edge away from the load and towards the centre of gravity and that arm shortens, so there is less for the load to balance and mx must come down. The steepness of the fall is the rule's own mass, because that is the one thing on the far side that never changes.

(vii) It does not. A uniform rule has its centre of gravity at the middle of the wood, which is 50.5cm from either end of 101.0cm of it. The loop hangs 0.5cm in from that end, so a perfectly uniform rule would give x_G equal to 50.0cm, and the measurement gives 50.6cm. The difference is six millimetres, and six millimetres is well inside what this graph can support.

Say how far inside. The intercept can be read to about half a small square, which is 50gcm and moves x_G by half a centimetre on its own; the slope that divides it is good to about four grams in 112g, which is another one and four fifths of a centimetre. Quote the centre of gravity as 50.6cm with an uncertainty of one to two centimetres, and say that it is consistent with a uniform rule. Notice what that means for the method: this line would have to put the centre of gravity two or three centimetres out of place before it could show anything at all, while a rule rested empty on the knife edge gives the same point to a millimetre or two. The graph is here for the mass.

Examiner's eye

- The intercept is read off the graph, not calculated from a row of the table. Every row satisfies the equation on its own, and a row cannot tell you that it does.
- Two answers come out of one line here, so label which is which before you start substituting. The slope is a mass and the intercept is a mass multiplied by a distance, and their units say so.
- A negative slope is not a mistake to be tidied away by dropping the sign. Quote it as -112g and then say that the mass of the rule is 112g.
- Work out where the centre of gravity ought to be before saying that yours is in the wrong place. It is the middle of the wood you want, and your x_G is measured neither from the middle nor from the 50cm mark but from the loop.

Practice 12.2

Between the school and the main road there is a ditch, and across the ditch there is a plank. It is not fixed at either end; it lies across the two banks with a good deal of it projecting past the near one, out over the path, because the plank is longer than the ditch is wide. The children who use it twice a day know, without ever having written anything down, exactly how far along the overhanging end each of them may walk before the far end lifts, and they know that the answer is different for each of them, and they have arranged themselves in order of it.

Mr. Akilikubwa watched the smallest of them go three paces further than the rest and understood the ranking. **Kipanga** wanted to know something the ranking does not contain: not the order the children stand in, which the plank gives away every morning, but how heavy the plank itself is. Nobody has ever picked it up. The two who once tried were carrying it back the same afternoon, having found out that a plank lifted off the banks is a plank somebody has to lay across a ditch again.

The measurement below is the same as Experiment 12.2's and the readings are taken the same way. What is different is the plot: the plotting is done in kilograms and metres, and the graph is drawn against the reciprocal of the distance instead of against the distance itself.

Procedure

Proceed as follows:

(a) Hang a mass hanger from a loop of fine thread over the end of a metre rule, at the mark at which the scale begins, and pull the loop tight so that it cannot slide.

(b) Load the hanger to a total mass m of 0.020kg, the hanger included.

(c) Rest the rule across a knife edge and move the knife edge along the rule until the rule balances horizontally. Record the distance x from the loop to the knife edge.

(d) Repeat the procedure in P12.2 (b) and (c) with m equal to 0.040kg, 0.060kg, 0.080kg, 0.100kg and 0.120kg.

Questions

(i) Tabulate your results, including x in metres and $1/x$ in m^{-1}.

(ii) Plot a graph of $1/x$ (m^{-1}) against m (kg).

(iii) State the relationship between $1/x$ and m shown by your graph.

(iv) Determine the slope of your graph and the intercept it makes on the $1/x$ axis, and state the unit of each.

(v) Use them to determine the mass of the rule and the distance of its centre of gravity from the loop.

(vi) Use your line to find the load that would balance the rule with the knife edge 0.250m from the loop.

Experiment 12.3

The chain came back from a fair on the coast, and the boy who won it had been told at the stall that it was gold. By the second week it had left a grey line round his neck, which settled the matter for everybody except its owner, who pointed out that everything in the school leaves a grey line on you, chalk included.

Kipanga said that you can tell gold by biting it, and was invited to bite it, and declined. **Kipute** asked the question that got the chain into the laboratory: whether the school could settle it without cutting the chain open, since the owner was not going to allow that either.

"We can weigh it," said **Mr. Akilikubwa**, *"and then we can weigh it again with it under water, and the difference between the two will tell us what it is made of. The chain will not mind. Take it off him before he changes his mind."*

Aim

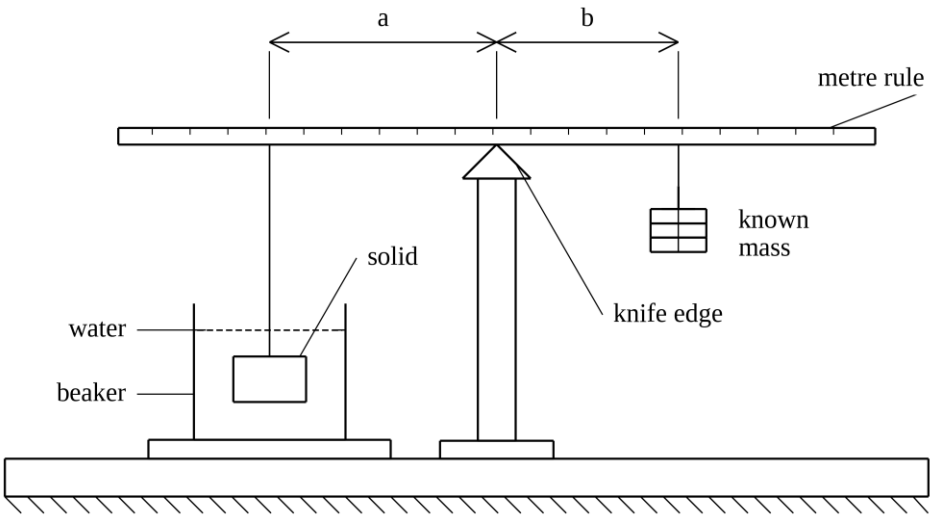
To determine the mass and the relative density of a solid by the principle of moments.

Apparatus

A metre rule, a knife edge mounted on a stand, the solid whose relative density is wanted, a known mass of 100g, a beaker of water standing on a wooden block, a length of fine thread and a short loop of the same thread.

Setup

The rule is balanced empty on the knife edge as before, and the knife edge stays at that point throughout. The solid hangs from a long thread a distance a from the knife edge, well below the rule, and the known mass m_k hangs from a loop a distance b from it on the other side. Every reading is taken twice over: once with the solid hanging in air, and once with the beaker standing under it on the block so that the solid is completely under the water and touching neither the bottom nor the sides. The beaker is moved along the bench each time a is changed, because the solid moves with the thread.



Procedure

Proceed as follows:

(a) Rest the metre rule across the knife edge and slide it until it balances horizontally with nothing hanging on it. Note the mark at which it balances and leave the knife edge at that point.

(b) Hang the solid from the long thread at a distance a of 12.0cm from the knife edge, so that it swings clear of the bench.

(c) Hang the known mass m_k of 100g from a loop on the other side of the knife edge, and slide that loop along the rule until the rule balances horizontally. Record the distance b from the knife edge to the loop.

(d) Repeat the procedure in 12.3 (b) and (c) with a equal to 16.0cm, 20.0cm, 24.0cm, 28.0cm and 32.0cm.

(e) Stand the beaker of water on the block below the solid so that the solid is completely immersed and touches neither the bottom nor the sides of the beaker, and repeat 12.3 (b) to (d) for the same six distances, moving the beaker along the bench each time the solid is moved.

Questions

(i) Tabulate your results.

(ii) Plot graphs of b (cm) against a (cm) for the solid in air and for the solid in water, using the same axes.

(iii) State what the two graphs have in common and how they differ.

(iv) Determine the slope of each of your two lines and state the unit of each.

(v) Use your slopes to determine the mass of the solid and its apparent mass in water.

(vi) Hence determine the volume of the solid and its relative density, taking the density of water as $1.00gcm^{-3}$.

(vii) State, with a reason, whether the solid has to hang at the same depth below the surface of the water for every reading.

Table of results

The known mass was 100g throughout, and the rule balanced at the 50.1cm mark with nothing hanging on it.

a (cm)	b in air (cm)	b in water (cm)
12.0	9.4	5.8
16.0	13.2	8.2
20.0	16.3	10.5
24.0	19.4	12.4
28.0	23.0	14.4
32.0	25.6	16.0

Making the measurement count

Practical tip 1: *The same balance twice, with something taken away in between.*

In air the thread carries the whole of the solid. Take moments about the knife edge and$m_s a = m_k b$. Lower the solid into the water and the water pushes up on it with an upthrust, so the thread has less to carry and the known mass comes in closer. Write the upthrust as $\rho_w Vg$, with V the volume of the solid, and the second balance is

$$(m_s - \rho_w V)a = m_k b$$

Each slope is therefore a mass divided by m_k. What the second set of readings measures is therefore not the upthrust itself but the mass the thread appears to be carrying while the solid is under water, and part (vi) asks what the difference between the two is worth.

Practical tip 2: *Immersed means immersed.*

The solid has to be completely under the surface, clear of the bottom and clear of the sides. Let it rest on the bottom and the beaker carries part of the solid, the thread carries the rest, and the balance reports whatever fraction the beaker has been kind enough to leave; let it lean on a side and you have that fault with friction added to it.

Look at the solid before each reading. A bubble clinging to it adds its own volume to the volume displaced, so slide the solid up and down once in the water to shake the bubbles off and let it come to rest before reading. Wipe the wet thread where it passes over the rule; a wet thread carries water that is not part of the solid.

Practical tip 3: *One knife edge, one balance point, two sets of readings.*

Balance the rule empty once, at the start, and do not touch the knife edge again, not even between the air readings and the water readings. The two lines are being compared with one another, so anything that moves between them turns up in the comparison. For the same reason, use the same loop and the same known mass for both sets, and hang the solid at the same six distances rather than at six convenient new ones.

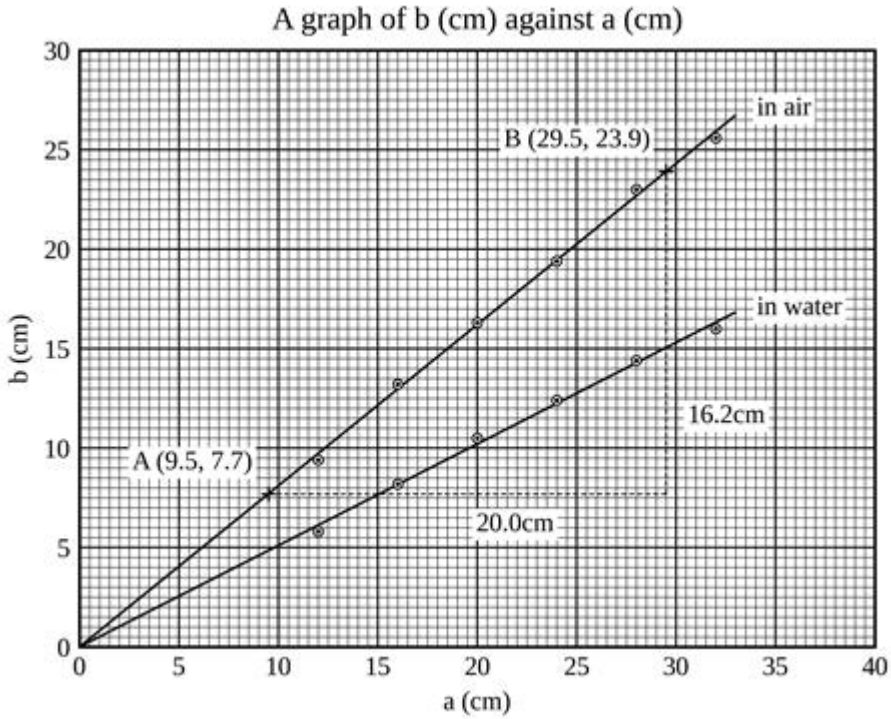
Practical tip 4: *The thread, and the two ways it can lie to you.*

What enters the balance is the perpendicular distance from the knife edge to the line of the thread, and that is the distance you measured only while it hangs upright. Leave the beaker where it was when the solid was moved and the solid comes to rest against the inside of the wall, which holds the thread over at an angle and shortens the arm without changing anything you have written down. So move the beaker to meet the solid, never the solid to meet the beaker, and look along the thread against the edge of the bench before each reading.

The length of thread under the water displaces water of its own, and it is worth knowing how little. Fine thread half a millimetre across, with five centimetres of it wet, displaces about 0.01cm^3 of water, which is 0.01g against an upthrust of tens of grams. Work a correction out before you decide to worry about it: this one has earned the right to be ignored, and the thread out of the upright has not.

Answers to the questions

(i) and (ii). The table is printed above. Take a at 2cm to 5cm, running from 0cm to 40cm, and b at 2cm to 5cm, running from 0cm to 30cm. That is 16cm by 12cm, which fits an A4 sheet upright. Give the graph the title A graph of b (cm) against a (cm), and label the two lines in air and in water as you draw them.



(iii) Both sets of points lie on straight lines through the origin, so $b \propto a$ whether the solid is in air or in water: the balance is a comparison of two moments and neither line has anything left over at $a=0$. They differ in slope alone. The water line is the less steep of the two, so the known mass does not have to go so far out to balance the solid once the solid is under water.

(iv) For the line in air, taking the marked corners A (9.5cm, 7.7cm) and B (29.5cm, 23.9cm),

$$\text{slope}=\frac{23.9\text{cm}-7.7\text{cm}}{29.5\text{cm}-9.5\text{cm}}=\frac{16.2\text{cm}}{20.0\text{cm}}=0.810$$

and constructing a triangle in the same way on the line in water, between P (10.5cm, 5.4cm) and Q (30.5cm, 15.6cm),

$$\text{slope}=\frac{15.6\text{cm}-5.4\text{cm}}{30.5\text{cm}-10.5\text{cm}}=\frac{10.2\text{cm}}{20.0\text{cm}}=0.510$$

Both slopes are lengths divided by lengths and have no unit.

(v) From $m_s a=m_k b$, the slope b/a in air is m_s/m_k, so

$$m_s=100\text{g}\times0.810=81.0\text{g}$$

The same argument applied to the second line gives the apparent mass in water,

$$m_w=100\text{g}\times0.510=51.0\text{g}$$

(vi) The solid appears to lose 81.0g-51.0g=30.0g when it is put under the water, and what it has lost is the upthrust. The upthrust is the weight of the water pushed out of the way, so the water displaced has a mass of 30.0g and, at 1.00gcm^{-3}, a volume of 30.0cm^3. The solid displaces its own volume, so that is the volume of the solid, and

$$\text{density}=\frac{81.0\text{g}}{30.0\text{cm}^3}=2.70\text{gcm}^{-3}$$

The relative density is the density of the solid divided by the density of water, which is the same arithmetic done without the units:

$$\text{relative density}=\frac{m_s}{m_s-m_w}$$

giving 81.0g/30.0g=2.70. A relative density of 2.70 is aluminium, or a stone; gold is seven times as dense as that, so the chain at the head of this experiment was never in any danger of being gold.

(vii) No. The upthrust is the weight of the water displaced, and once the solid is completely under the surface it displaces its own volume however deep it is: pushing it further down displaces no more water, because the water it would displace is already out of the way. What matters is that it is completely covered, that it hangs clear of the bottom and the sides, and that it carries no air bubbles. Practical tip 4 has already put a number on the one thing that does change with the depth, the water displaced by the wet thread, and it came to a hundredth of a gram.

Examiner's eye

- Both lines go on one grid, to one scale. Two sheets mean two scales, and the answer is a comparison of the two slopes.
- The quantity the second line gives is an apparent mass, not a mass. The solid has not lost anything; it is being held up by the water as well as by the thread, and saying so is worth a mark.
- The upthrust in this experiment is a mass loss of 30.0g and a volume of 30.0cm^3 only because water has a density of 1.00gcm^{-3}. Write the division down with its units rather than moving the number across unchanged, or the same step in a liquid that is not water will catch you out.
- A relative density has no unit. A density has one, and it is not the same number in gcm^{-3} as in kgm^{-3}.

Practice 12.3

Kipute built a density machine out of a metre rule, a knife edge, a bucket and a stone, and announced at the end of the first afternoon that he had found a stone that weighed nothing at all. The known mass balanced it at no distance from the knife edge whatever; the rule sat level with the mass hanging against the knife edge itself.

Mr. Akilikubwa came and looked into the bucket. The stone was sitting on the bottom of it, and had been all afternoon, so that the bucket was holding the stone up and the thread was holding nothing but itself. **Kipanga** said that this was the best result anyone in the class had obtained all term, since it was the only one that was exactly right about something, and would not say what.

Lower the bucket, shorten the thread, and the machine works. The measurement below takes the same two sets of readings as Experiment 12.3 and then plots one against the other instead of both against the distance, which turns out to need one number fewer.

Procedure

Proceed as follows:

(a) Rest a metre rule across a knife edge and slide it until it balances horizontally with nothing hanging on it. Note the mark at which it balances and leave the knife edge at that point.

(b) Hang the solid from a long thread at a distance a of 10.0cm from the knife edge, so that it swings clear of the bench.

(c) Hang a known mass of 100g from a loop on the other side of the knife edge, and slide that loop along the rule until the rule balances horizontally. Record the distance b_a from the knife edge to the loop.

(d) Repeat the procedure in P12.3 (b) and (c) with a equal to 14.0cm, 18.0cm, 22.0cm, 26.0cm and 30.0cm.

(e) Stand a beaker of water on a block below the solid so that the solid is completely immersed and touches neither the bottom nor the sides of the beaker, and repeat P12.3 (b) to (d) for the same six distances, moving the beaker along the bench each time the solid is moved and recording the balancing distance b_w in each case.

Questions

(i) Tabulate your results.

(ii) Plot a graph of b_w (cm) against b_a (cm).

(iii) State the relationship between b_w and b_a shown by your graph.

(iv) Determine the slope of your graph and state its unit.

(v) Use your slope to determine the relative density of the solid.

(vi) The volume of the solid is $30.0cm^3$. Determine its mass, taking the density of water as $1.00gcm^{-3}$.

Experiment 12.4

The school swims twice a year, once in the river above the bridge and once at the coast, and the boys who go both times come back with the same complaint: the sea holds you up and the river will not. It is not a complaint about anybody's swimming. Float on your back in the river and you sink to your ears; float on your back in the sea and you can lie there with your hands behind your head and watch the sky.

Kipanga wanted a number for it, on the reasonable ground that everybody agrees the sea is different and nobody can say by how much. The guesses that came back from the rest of the class ran from a tenth to twice, which is the sort of agreement that is worse than none at all. **Mr. Akilikubwa** said that the sea was not available in the physics laboratory on a Tuesday, but that a beaker of something else was, and that the measurement would be the same measurement.

It takes three sets of balancing distances instead of two, and the third set is taken in the liquid whose relative density is wanted.

Aim

To determine the relative density of a liquid by the principle of moments.

Apparatus

A metre rule, a knife edge mounted on a stand, a solid that sinks in both liquids, a known mass of 100g, a beaker of water and a beaker of the liquid L, a wooden block, a length of fine thread and a short loop of the same thread.

Setup

The arrangement is the one drawn in Experiment 12.3, and the readings are taken in the same way: the rule is balanced empty, the solid hangs from a long thread a distance a from the knife edge and the known mass m_k balances it a distance b away on the other side. The difference is that the six distances are worked through three times over, with the solid hanging in air, then under water, then under the liquid L, the solid being dried between one liquid and the next.

Procedure

Proceed as follows:

(a) Rest the metre rule across the knife edge and slide it until it balances horizontally with nothing hanging on it. Note the mark at which it balances and leave the knife edge at that point.

(b) Hang the solid from the long thread at a distance a of 10.0cm from the knife edge, so that it swings clear of the bench.

(c) Hang the known mass of 100g from a loop on the other side of the knife edge, and slide that loop along the rule until the rule balances horizontally. Record the distance b from the knife edge to the loop.

(d) Repeat the procedure in 12.4 (b) and (c) with a equal to 14.0cm, 18.0cm, 22.0cm, 26.0cm and 30.0cm.

(e) Stand the beaker of water on the block below the solid so that the solid is completely immersed and touches neither the bottom nor the sides of the beaker, and repeat 12.4 (b) to (d) for the same six distances, moving the beaker along the bench each time the solid is moved.

(f) Take the solid out and dry it, stand the beaker of the liquid L on the block in place of the beaker of water, and repeat 12.4 (b) to (d) once more.

Questions

(i) Tabulate your results.

(ii) Plot graphs of b (cm) against a (cm) for the solid in air, in water and in the liquid L, using the same axes.

(iii) State what your three lines have in common and how they differ.

(iv) Determine the slope of each line and state the unit of each.

(v) Use your slopes to determine the mass of the solid and its volume, taking the density of water as $1.00gcm^{-3}$.

(vi) Hence determine the relative density of the liquid L.

(vii) The solid is taken out of the water and lowered into the liquid L without being dried. State, with a reason, whether the relative density obtained would be too large or too small.

Table of results

The known mass was 100g throughout, and the rule balanced at the 49.6cm mark with nothing hanging on it.

a (cm)	b in air (cm)	b in water (cm)	b in liquid L (cm)
10.0	9.9	5.9	6.3
14.0	13.2	7.5	9.3
18.0	17.2	10.2	11.5
22.0	20.8	12.0	13.9
26.0	25.3	14.6	16.3
30.0	28.8	17.0	19.5

Making the measurement count

Practical tip 1: *Three lines, and the two differences between them.*

Each of the three sets obeys the same balance, $ma=m_k b$, with m the mass the thread is carrying: the whole mass of the solid in air, and the mass less the upthrust in each of the liquids. So the three slopes are three masses divided by the same known mass, and what separates one line from another is an upthrust. Part (vi) asks what two of those separations, taken together, are worth, so write down which pair of lines each of your subtractions belongs to before you start.

Practical tip 2: *Drying between liquids, and the order of the runs.*

Take the readings in air first, and dry the solid with a cloth between one liquid and the next. The drying that matters is the one after the water: water clings to a solid more stubbornly than most laboratory liquids do, and a drop of it carried into the liquid L is both a mass that does not belong to the solid and a contamination of the liquid. Part (vii) asks which way that error runs.

Use a separate beaker for each liquid rather than emptying and refilling one, and keep the two beakers apart on the bench. Then nothing crosses from one to the other but the solid and the thread it hangs on, and the thread you have already been told to wipe.

Practical tip 3: *Choosing a solid and enough liquid to cover it.*

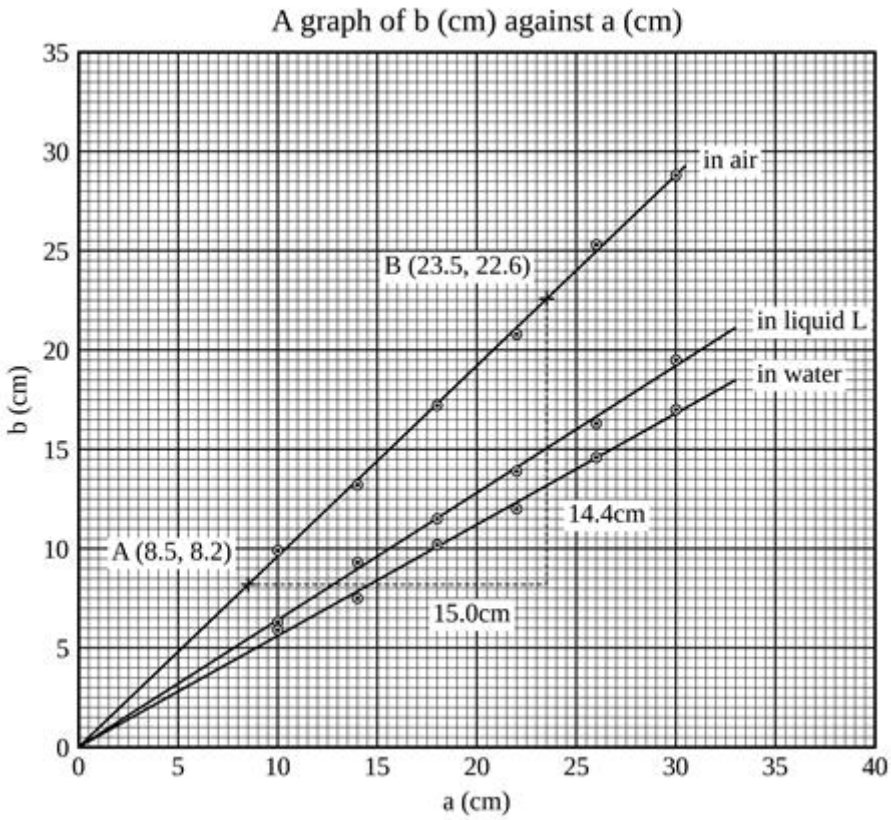
The solid has to sink in both liquids, so it must be denser than either of them, and it has to be small enough to hang clear of the bottom and the sides of the beaker with the surface well above it. A dense solid of small volume gives a small upthrust and two slopes that are hard to tell apart; a large solid of low density gives a fine separation between the lines and a beaker of liquid you may not have. Something the size of a small stone and heavy for its size, which is to say a stone, a lump of glass or a short metal cylinder, is what to look for.

Practical tip 4: *What three figures on a slope are worth after two subtractions.*

The answer is a ratio of two differences of slopes, and each subtraction throws away most of the precision that went into it. Take the three slopes as good to 0.005 each. The two differences are then good to about 0.007 each, which on differences of the size found here is about one part in fifty. The two differences share the line taken in air, so their errors do not simply add; carried through the ratio they leave the relative density of the liquid worth about two parts in a hundred. Quote it to three figures by all means, but do not defend the third one.

Answers to the questions

(i) and (ii). The table is printed above. Take a at 2cm to 5cm, running from 0cm to 40cm, and b at 2cm to 5cm, running from 0cm to 35cm. That is 16cm by 14cm, which fits an A4 sheet upright. Give the graph the title A graph of b (cm) against a (cm), and label each line as you draw it.



(iii) All three sets of points lie on straight lines through the origin. They differ only in slope, and they lie in the order air, liquid L, water, from the steepest downwards: the greater the upthrust, the less the thread carries and the less steep the line.

(iv) For the line in air, taking the marked corners A (8.5cm, 8.2cm) and B (23.5cm, 22.6cm),

$$\text{slope}=\frac{22.6\text{cm}-8.2\text{cm}}{23.5\text{cm}-8.5\text{cm}}=\frac{14.4\text{cm}}{15.0\text{cm}}=0.960$$

Construct a triangle in the same way on each of the other two lines and the slopes come out at 0.560 for the line in water and 0.640 for the line in the liquid L. None of the three has a unit.

(v) Each slope is the mass the thread carries divided by the known mass, so multiplying by 100g turns the three slopes into three masses: 96.0g in air, 56.0g in water, which is m_w, and 64.0g in the liquid L, which is m_L. The mass of the solid is the first of these, 96.0g. The upthrust in water is

$$96.0g-56.0g=40.0g$$

so the water displaced has a mass of 40.0g and, at $1.00gcm^{-3}$, a volume of $40.0cm^3$. The solid displaces its own volume, so the volume of the solid is $40.0cm^3$.

(vi) In the liquid L the upthrust is 96.0g-64.0g=32.0g, so $40.0cm^3$ of the liquid L has a mass of 32.0g where the same volume of water has a mass of 40.0g. The relative density of the liquid is the ratio of the two upthrusts:

$$\frac{\rho_L}{\rho_w}=\frac{m_s-m_L}{m_s-m_w}$$

$$\frac{\rho_L}{\rho_w}=\frac{32.0g}{40.0g}=0.800$$

The volume never had to be worked out to reach that number, because it is the same volume in both liquids and cancels; part (v) needed it, part (vi) does not. Accept 0.77 to 0.83. That is a marking tolerance, not the precision of one carefully drawn graph. A liquid of relative density 0.800 is an oil or a spirit, and the solid at $96.0g/40.0cm^3=2.40gcm^{-3}$ sinks in both, as it must for the experiment to work at all. As for the sea, the same measurement made on sea water gives about 1.03 against the river's 1.00, and this method would tell you which beaker was which and not a great deal more. Three parts in a hundred is the whole of the difference, and it is enough to put a swimmer's ears above the water instead of below.

(vii) Too small. A film of water carried over on the solid is a mass that rides with it into the liquid L, and it is a mass the water's own upthrust barely touches, so the thread appears to be carrying more than it should while the solid is in the liquid L. That makes the apparent mass in the liquid L too large, the upthrust m_s-m_L too small, and the ratio of that upthrust to the one in water too small with it. Size it before you dismiss it. A gram of water riding on the solid brings its own volume with it, and the liquid L pushes up on that volume with 0.8g of upthrust, so the balance feels about a fifth of a gram: one part in a hundred and sixty of the upthrust, against the two parts in a hundred the slopes themselves are worth. What makes it worth a sentence is that it runs the same way every time, and that a solid lifted out dripping rather than merely damp carries a great deal more than a gram.

Examiner's eye

- Three lines on one grid need three labels, written on the lines as they are drawn. A fan of unlabelled lines will tell you which upthrust was the largest and nothing at all about which liquid produced it, and by then the beakers have been emptied.
- The relative density of the liquid is a ratio of two upthrusts, and an upthrust is a difference between two of your own slopes. Divide by the mass of the solid instead of by the upthrust in water and what you have worked out is how much of the solid's own mass the liquid supports, which is a different quantity with a different name.
- Dry the solid between the liquids, and put that in your account. It is the difference between three figures and two.
- Check that the solid sinks in both liquids before starting. A solid that floats in the liquid L has the thread slack and the balance reports the known mass alone, and no amount of care afterwards recovers the run.

Practice 12.4

A letter arrived at the school from a former pupil who now works at a dairy, where the milk comes in by bicycle from thirty smallholders and goes out by tanker. The dairy pays for what comes in and is paid for what goes out, and between the two there is a persistent difference that nobody at the dairy can account for and everybody at the dairy has an opinion about.

The letter asks how a dairy is supposed to tell whether milk has had water added to it. **Kipute** wanted to know what the dairy does about it now, which is taste it, and how thirty smallholders take being told their milk tastes thin. **Kipanga** pointed out that the letter is not really asking how to tell; it is asking how to prove it to a man standing in front of you with a churn and an opinion. **Mr. Akilikubwa** read the postscript out, which asks for an answer before Friday, as the milk will not keep.

What settles an argument like that is a number, measured the same way for every churn that comes in, and the number wanted is a relative density. The method below reaches one by a shorter road than Experiment 12.4 took, and the shorter road is worth seeing for what it leaves out.

Procedure

Proceed as follows:

(a) Rest a metre rule across a knife edge and slide it until it balances horizontally with nothing hanging on it. Note the mark at which it balances and leave the knife edge at that point.

(b) Hang the solid from a long thread at a distance a of 10.0cm from the knife edge, so that it swings clear of the bench.

(c) Hang a known mass of 100g from a loop on the other side of the knife edge, and slide that loop along the rule until the rule balances horizontally. Record the distance b_a from the knife edge to the loop.

(d) Repeat the procedure in P12.4 (b) and (c) with a equal to 14.0cm, 18.0cm, 22.0cm, 26.0cm and 30.0cm.

(e) Stand a beaker of water on a block below the solid so that the solid is completely immersed and touches neither the bottom nor the sides of the beaker, and repeat P12.4 (b) to (d) for the same six distances, moving the beaker along the bench each time the solid is moved and recording the balancing distance b_w in each case.

(f) Take the solid out and dry it, stand a beaker of the liquid Q on the block in place of the beaker of water, and repeat P12.4 (b) to (d) once more, recording the balancing distance b_Q in each case.

Questions

(i) Tabulate your results, including b_a-b_w and b_a-b_Q.

(ii) Plot a graph of b_a-b_Q (cm) against b_a-b_w (cm).

(iii) State the relationship between b_a-b_Q and b_a-b_w shown by your graph.

(iv) Determine the slope of your graph and state its unit.

(v) State what the slope measures, and give the relative density of the liquid Q.

(vi) The known mass appears nowhere in your answer, and neither does the volume of the solid. Explain how the plot has disposed of them.

PART THREE

HEAT

Chapter 13

COOLING CURVES AND NEWTON'S LAW OF COOLING

Everything in this chapter is a vessel of hot water standing on a bench, a thermometer in it and a stopwatch running. There is nothing to hang, nothing to wind and nothing to time to a hundredth of a second. You take one column of temperatures against one column of times, and one sheet of readings of that kind will give you three separate things: the rate at which heat is leaving the vessel at one named instant, a constant that governs the whole of its cooling, and a way of getting at the temperature of the room without reading it.

A hot body left in a cool room loses heat to the room and its temperature falls. How fast it falls, and what that rate depends on, is the business of the pages ahead. The answer is not visible in the readings themselves: a column of temperatures dropping by 3K, then by 2.5K, then by 2K is telling you something, but not in a form you can quote, check or use.

Two difficulties arrive with the first graph, and neither of them appeared anywhere in Part Two. The first is that the graph is a curve. Every graph you have drawn so far was a straight line, and a straight line has one slope which is the same all the way along it. A curve has a fresh slope at every point, so before you can quote a rate at all you have to say at which instant you mean it, and then draw a tangent there. You have to draw that tangent yourself, it is a skill in itself, it carries marks of its own, and it is the least reliable step in the whole experiment.

The second is that a rate is not a total. The heat that leaves the calorimeter while it stands, and the speed at which it is losing it at the end of that time, are two quantities with unlike units, and they are not found in the same way. Reach for the wrong one and the last marks of a Heat question go with it.

There is a way round the tangent altogether, and Experiment 13.2 is it. Take logarithms of the right quantity and the curve straightens, and the awkward job of laying a ruler against a bend by eye becomes the familiar job of drawing a best line through twelve points. What that costs you is the instant: a logarithmic plot describes the whole run and says nothing directly about any one moment of it. Experiment 13.3 takes the tangents seriously instead, six of them on a single curve, and finds out what the rate of cooling actually depends on.

The three varieties at a glance

	What is different about it	What the graph gives you
13.1	One cooling run, and one tangent drawn on the curve at a stated temperature	the rate of cooling there, and the rate at which heat is leaving the calorimeter
13.2	The same apparatus, with the logarithm of the excess temperature plotted against time	the cooling constant of the calorimeter and its contents, from a straight line
13.3	Six tangents on one curve instead of one	the cooling constant again, and a test of the law the cooling obeys

All three are worked below in full, and all three use the same copper calorimeter holding the same mass of water, so that the constant which comes out of the second can be set beside the constant which comes out of the third. The practice questions change the calorimeter, the quantity of water and the room.

Experiment 13.1

Above the urn in the school dining hall there is a card, lettered by hand and lettered again where the first hand had faded:

PORRIDGE IS POURED AT SIX FORTY

IT IS TOO HOT AT SIX FORTY

DO NOT ASK THE COOK

Underneath, in a third hand and in pencil, somebody has added *"and by seven it is cold."*

Mr. Akilikubwa read all four lines twice and said that the cook had been answering that question every morning for eleven years without once writing anything down. She poured, she scoured the pot, she called the school in, and she was never wrong by more than a minute. **Kipanga** said that was guessing. **Kipute** said that guessing which has not been wrong in eleven years has another name, and that nobody has ever asked the cook the harder question anyway.

The harder question is not how long the porridge takes to become drinkable. It is what is happening in any one of those minutes while it does.

Aim

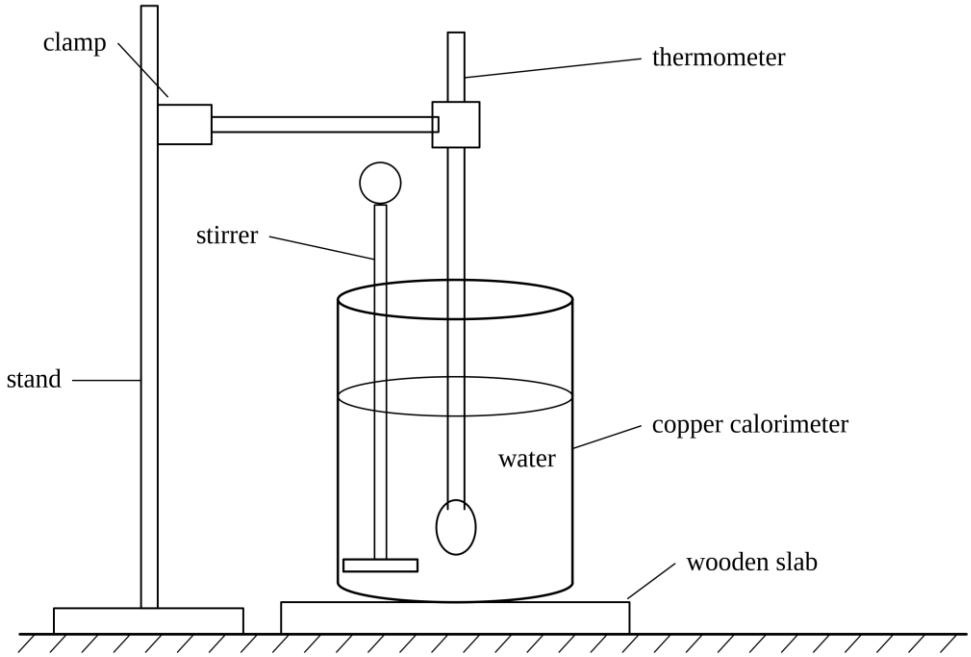
To determine the rate at which a calorimeter of hot water is losing heat when its temperature has fallen to a stated value.

Apparatus

A copper calorimeter with its stirrer, of total mass 88.0g, a thermometer reading from 0 to 100°C in half degrees, a stopwatch, a wooden slab, a retort stand with a clamp, a beaker of hot water, a balance and a transparent ruler.

Setup

The calorimeter stands on a wooden slab on the bench, away from any window or doorway, and nothing is wrapped round it. The thermometer is held in the clamp with its bulb well below the surface of the water and clear of the bottom and the sides of the calorimeter, so that it reads the water and not the copper. The stirrer stays in the water throughout. The temperature of the room is taken with the same thermometer before the hot water is poured.



Procedure

Proceed as follows:

(a) Find the mass of the calorimeter together with its stirrer, and record the temperature of the room.

(b) Pour hot water at about 90°C into the calorimeter until it is about two thirds full, and find the mass of water it holds.

(c) Stand the calorimeter on the wooden slab and lower the thermometer into the water so that its bulb is clear of the bottom and the sides.

(d) Stirring gently and continuously, start the stopwatch at the instant the thermometer reads 80.0°C, and record the temperature θ every 2 minutes for 24 minutes.

Questions

(i) Tabulate your results.

(ii) Plot a graph of θ (°C) against t (min) and draw a smooth curve through your points.

(iii) Draw the tangent to your curve at the point where the temperature is 70.0°C, determine its slope and state the unit.

(iv) The specific heat capacity of water is 4.2J/gK and that of copper is 0.38J/gK. Use these, with your answer to (iii), to determine the rate at which the calorimeter and its contents are losing heat at 70.0°C.

(v) Find from your table the average rate of fall of temperature over the first 12 minutes of the run, compare it with your answer to (iii), and account for what you find.

(vi) State, with a reason, whether a tangent drawn to your curve at 50.0°C would be steeper or less steep than the one you drew.

(vii) Mention one source of error in this experiment and one precaution to be taken against it.

Table of results

The temperature of the room was 26.0°C throughout, and the calorimeter held 165g of water.

t (min)	θ (°C)	t (min)	θ (°C)
0	80.0	14	60.0
2	77.0	16	57.0
4	73.5	18	55.0
6	70.5	20	53.5
8	67.0	22	51.5
10	64.0	24	49.5
12	62.0		

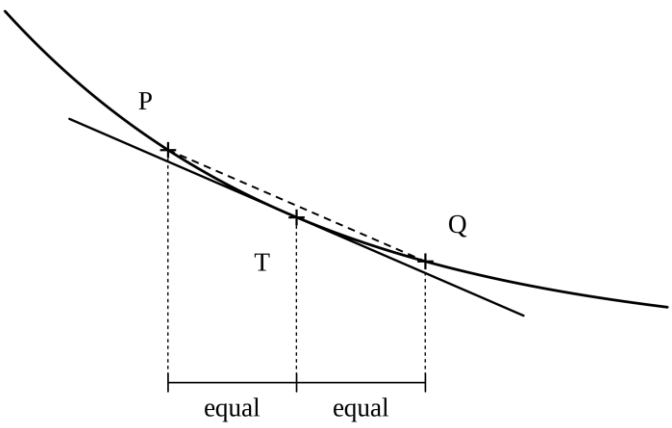
Making the measurement count

Practical tip 1: *Reading a thermometer while the reading is moving.*

Water in a calorimeter does not cool evenly. The layer against the copper is colder than the middle, and a bulb in the middle of an unstirred calorimeter reads the warmest part of it. Stir gently and without stopping and you take that difference away. The resolution of this thermometer is half a degree, so read to the nearest half degree and no finer, with your eye level with the top of the mercury; a reading taken from above or from below is out by more than the graduation is worth.

Practical tip 2: *Drawing a tangent, rather than guessing at one.*

Lay a ruler against the curve, turn it until it looks about right, and you have used no method at all; that is where most of the marks in a tangent question go. Use the construction below instead. Mark the point T on the curve where the temperature is the one you want. Mark two points P and Q on the curve at equal distances on either side of T along the time axis, about 5 minutes each way, or less where the contact is near an end of the run, and join them with a faint chord. Draw the tangent through T parallel to that chord. The chord fixes the direction for you, and a direction copied from a chord is worth several times a direction judged by eye.



Two things to watch. Extend the tangent well beyond P and Q, and read it between two points as far apart as the drawn line allows; two ordinates 2cm apart are too close together to divide. And it must touch at T and nowhere else. A line that crosses the curve twice is a chord, and on a curve that bends the way a cooling curve

bends, a chord is a shade steeper than the tangent at the middle of it, so a tangent drawn short is a tangent read too steep.

Practical tip 3: *What the two specific heat capacities are for.*

The thermometer follows the water, but the copper is cooling with it, and the heat leaving the calorimeter comes out of both. What matters is therefore the heat capacity of the whole system, the mass of each part multiplied by its own specific heat capacity and the two added:

$$m_c c_c + m_w c_w$$

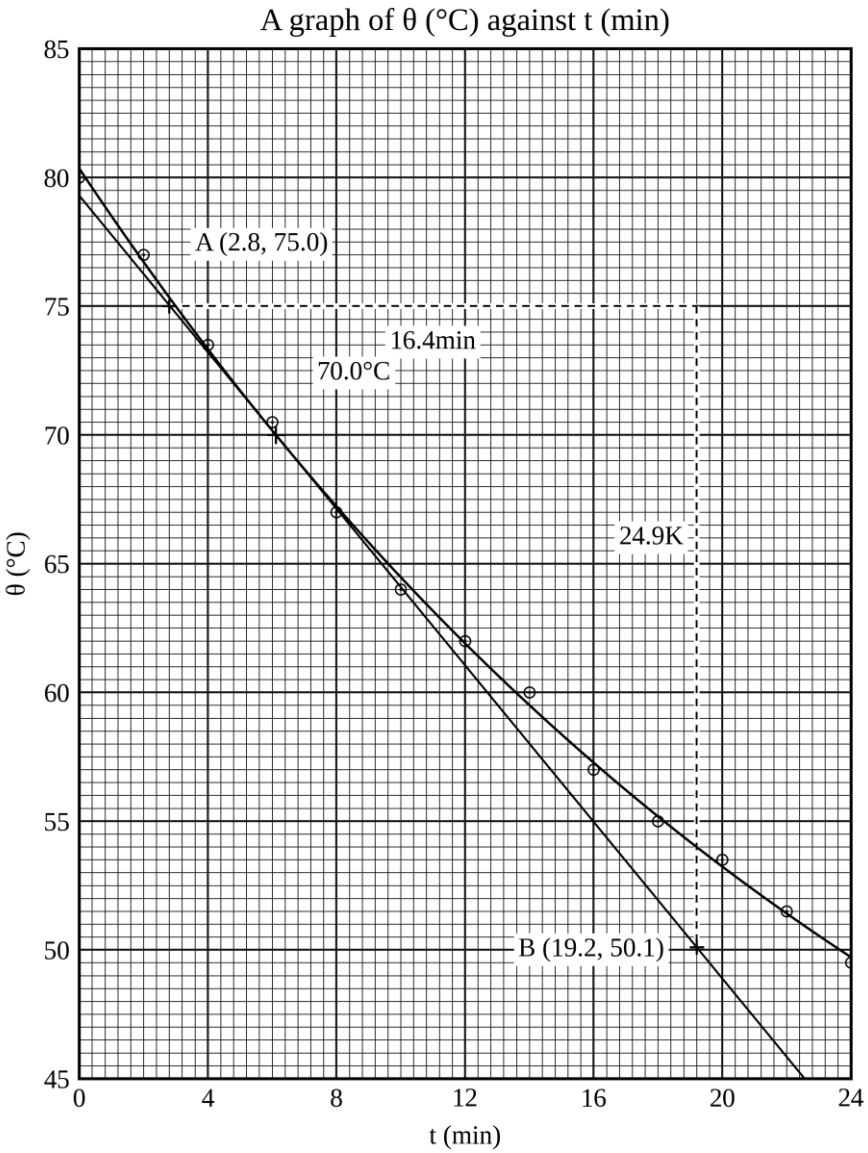
That sum is a number of joules per kelvin, and part (iv) asks you to multiply it by something. Notice that the water carries the greater part of it by a long way, which is why the mass of water is the reading in this experiment worth taking most carefully. Notice also that the thermometer itself is in the water and is not in the sum; the correction it would make is smaller than the scatter of the readings and the question does not ask for it.

Practical tip 4: *What a hand-drawn tangent is worth.*

Suppose the tangent is laid half a degree out at each end of a run of 16 minutes, in opposite directions. The rise you read is then wrong by 1.0K in about 25K, and the slope with it: a percentage uncertainty of 4%. Drawn with a sharp pencil through a well-made construction, and read over the longest base the line allows, a tangent is worth about half of that. Either way, quote your slope to three significant figures because the graph paper will carry three, and do not believe the third one: every quantity computed from the slope propagates that uncertainty into itself, and no amount of care with the arithmetic afterwards will take it out again. That is what the instant costs, and Experiment 13.2 is about not paying it.

Answers to the questions

(i) and (ii). Take t at 2cm to 4 minutes, running from 0 to 24 minutes, and θ at 2cm to 5°C, running from 45°C to 85°C. That is 12cm by 16cm, which fits an A4 sheet upright. Give the graph the title A graph of θ (°C) against t (min).



(iii) The curve reaches 70.0°C a little after 6 minutes. Taking the two marked corners A (2.8, 75.0) and B (19.2, 50.1) on the tangent drawn there,

$$\text{slope}=\frac{50.1°\text{C}-75.0°\text{C}}{19.2\text{min}-2.8\text{min}}=\frac{-24.9\text{K}}{16.4\text{min}}=-1.52\text{Kmin}^{-1}$$

The minus sign says the temperature is falling, which it plainly is. The quantity asked for is the rate of cooling, and it is 1.52Kmin^{-1}.

(iv) The heat capacity of the calorimeter and its contents is

$$88.0\text{g}\times0.38\text{J/gK}+165\text{g}\times4.2\text{J/gK}=33.44\text{JK}^{-1}+693\text{JK}^{-1}=726.44\text{JK}^{-1}$$

Every kelvin the system falls therefore sends 726.44J to the room, and at 70.0°C it is falling at 1.52Kmin^{-1}. So

$$\text{rate of loss of heat}=726.44\text{JK}^{-1}\times1.52\text{Kmin}^{-1}=1104\text{Jmin}^{-1}$$

A minute is 60s, so that figure has to be turned into joules per second before it can be called a power:

$$\frac{1104\text{J}}{60\text{s}}=18.4\text{Js}^{-1}$$

The calorimeter and its contents are losing heat at 18.4W at the moment the water passes 70.0°C, and at no other moment.

(v) The water was at 80.0°C when the watch was started and at 62.0°C after 12 minutes, so the average rate of fall over that stretch is 18.0K/12min=1.50Kmin^{-1}. The tangent gave 1.52Kmin^{-1}, which is larger by a little over 1%.

They agree because of where 70.0°C falls. An average rate over an interval is the slope of the chord joining its two ends, and the chord of a smooth curve has the same slope as the tangent at some point inside the interval, close to the middle of it. The curve passes 70.0°C at very nearly 6 minutes, which is the middle of the first 12, so the two numbers are two measurements of nearly the same thing, and the second of the two is free: an average rate costs readings and no drawing at all, and it is a free check on a tangent you have spent 5 minutes over. It is only a check, though, and only because the interval was chosen to straddle the point: take the average over the whole 24 minutes instead and it comes out at 1.27Kmin^{-1}, which belongs to the middle of the run and not to 70.0°C at all. Notice which of the two came out the larger, too. A chord ought in principle to be a shade steeper than the tangent at its middle, by about 1% over an interval this long; here it is a shade less steep, because the chord hangs on two readings, and half a degree of resolution at each end of a fall of 18.0K is worth about 3%, which swallows the 1% whole.

(vi) Less steep. At 50.0°C the water is nearer the temperature of the room than it was at 70.0°C, and the curve is visibly flattening as it goes; over the last three intervals of the table the temperature falls by about 2.0K in 2 minutes, where in the first it fell by 3.0K. Whatever drives the cooling is weakening as the difference in temperature closes, and the tangent lies down accordingly.

(vii) A source of error: the calorimeter is open at the top, so heat leaves the water by evaporation as well as through the copper, and evaporation is not governed by the excess temperature in the way the rest of the loss is. A precaution: cover the calorimeter with a light lid pierced for the thermometer and the stirrer, which stops most of the evaporation without wrapping the calorimeter in anything that would change what it loses through its walls.

Examiner's eye

- The question names a temperature, not a time. Find where the curve crosses 70.0°C and draw the tangent there. A tangent drawn at the plotted point at 6 minutes instead answers a slightly different question and loses the mark for the one that was asked.
- Show the tangent on the graph, mark the two points you read it between, and write their coordinates down. An unsupported slope is a number the examiner cannot check, and it is marked as such.
- Start the temperature axis at a round value below your lowest reading, not at zero. Nothing in this question is read at the origin, and thirteen readings crowded into the top third of a sheet cannot be joined by a curve anybody can lay a tangent against.
- Watch the minutes, and watch the unit. The slope comes off the graph in Kmin^{-1} and a rate of loss of heat is wanted in watts, so a factor of 60 has to appear somewhere in your working; write the conversion down as a line of its own. Leave the calorimeter out of the heat capacity altogether and you are nearly 5% short, which is a gross error and not an uncertainty.

Practice 13.1

The preparation room has a kettle, a stopwatch and no thermostat bath, and it has had all three for as long as anybody can remember. When a demonstration wants water at a stated temperature at a stated minute, the kettle goes on early and somebody is posted beside the calorimeter to shout.

Kipute offered to put a stop to the shouting. His plan was to boil the water later, so that there would be less waiting to do, which is a plan with the arithmetic taken out of it: water poured at 90°C and wanted at 60°C has to shed 30K whenever it is poured, and boiling later moves the whole afternoon along with it. **Kipanga** said the shouting could be done away with altogether if somebody drew the curve once and kept it, and **Mr. Akilikubwa** said that was the first labour-saving suggestion the class had produced all term.

One curve and one tangent settle it, for that calorimeter and that quantity of water, once and for all.

Procedure

Proceed as follows:

(a) Find the mass of the calorimeter together with its stirrer, and record the temperature of the room.

(b) Pour hot water at about 95°C into the calorimeter until it is about two thirds full, and find the mass of water it holds.

(c) Stand the calorimeter on the wooden slab and lower the thermometer into the water so that its bulb is clear of the bottom and the sides.

(d) Stirring gently and continuously, start the stopwatch at the instant the thermometer reads 85.0°C, and record the temperature θ every 2 minutes for 24 minutes.

Questions

(i) Tabulate your results.

(ii) Plot a graph of θ (°C) against t (min) and draw a smooth curve through your points.

(iii) Draw the tangent to your curve at the point where the temperature is 60.0°C, determine its slope and state the unit.

(iv) The specific heat capacity of water is 4.2J/gK and that of copper is 0.38J/gK. Determine the rate at which the calorimeter and its contents are losing heat at 60.0°C.

(v) Determine the total heat lost by the calorimeter and its contents during the first 8 minutes of the run.

(vi) Explain why your answer to (v) is not equal to your answer to (iv) multiplied by 8 minutes.

(vii) State one advantage of recording the temperature every 2 minutes rather than every 5 minutes, and one disadvantage of recording it every 30 seconds.

Experiment 13.2

A letter came to the school from a former student who is now teaching in a school of his own, two districts away. **Kipute** was given it to read out.

> Sir,
>
> My class recorded a cooling run last Thursday and then spent the rest of the lesson arguing about what to do with the numbers. One group averaged the drops. A second took the ratio of each drop to the one before it, which I could not fault and could not use either. A third drew the curve, and then could not agree where to lay the ruler, and one of them drew a line that cut the curve in two places and called it a tangent. Is there a way of getting a straight line out of a cooling curve without drawing a single tangent?
>
> Your former student,
>
> P.S. Please do not tell them how many ways I tried it that evening before I gave up.

Mr. Akilikubwa said there is, that it has been known for three hundred years, and that the whole of it lives in one column of the table which nobody in that lesson had thought to write down.

Aim

To determine the cooling constant of the calorimeter and its contents from a logarithmic plot.

Apparatus

A copper calorimeter with its stirrer, of total mass 88.0g, a thermometer reading from 0 to 100°C in half degrees, a stopwatch, a wooden slab, a retort stand with a clamp, a beaker of hot water, a balance, a transparent ruler and a table of logarithms.

Setup

The arrangement is the one drawn in Experiment 13.1, with the same calorimeter and the same mass of water in it. Take the temperature of the room before the hot water is poured and write it at the head of your table, where you and the examiner can both see it: the third column of this experiment cannot be worked out without it, and an account that does not record it is an account nobody can check.

Procedure

Proceed as follows:

(a) Find the mass of the calorimeter together with its stirrer, and record the temperature of the room.

(b) Pour hot water at about 90°C into the calorimeter until it is about two thirds full, and find the mass of water it holds.

(c) Stand the calorimeter on the wooden slab and lower the thermometer into the water so that its bulb is clear of the bottom and the sides.

(d) Stirring gently and continuously, start the stopwatch at the instant the thermometer reads 76.0°C, and record the temperature θ every 2 minutes for 22 minutes.

Questions

(i) Tabulate your results, including for each reading the excess temperature θ-θ_r and its logarithm to base ten.

(ii) Plot a graph of $\log_{10}(\theta\text{-}\theta_r)$ against t (min).

(iii) Given that $\log_{10}(\theta\text{-}\theta_r)$=A-Kt, where A and K are constants, use your graph to determine A and K.

(iv) The cooling constant k of the calorimeter and its contents is given by k=2.303K. Determine k and state its unit.

(v) Use your graph to predict the temperature of the water 26 minutes after timing began.

(vi) State what the fact that your graph is a straight line shows about the way the excess temperature of the water falls with time, and support your statement with figures taken from your table.

(vii) The experiment is repeated in a cooler room, the water again being at 76.0°C when timing begins. State, with reasons, how the slope and the intercept of the new graph would compare with yours.

(viii) Mention one source of error in this experiment and one precaution to be taken against it.

Table of results

The temperature of the room was 25.5°C throughout, and the calorimeter held 165g of water.

t (min)	θ (°C)	$\theta - \theta_r$ (K)	$\log_{10}(\theta - \theta_r)$
0	76.0	50.5	1.703
2	73.0	47.5	1.677
4	70.0	44.5	1.648
6	66.5	41.0	1.613
8	64.0	38.5	1.585
10	61.5	36.0	1.556
12	58.5	33.0	1.519
14	56.5	31.0	1.491
16	55.0	29.5	1.470
18	53.0	27.5	1.439
20	51.0	25.5	1.407
22	49.0	23.5	1.371

Making the measurement count

Practical tip 1: *Where the straight line comes from.*

Suppose, for the moment, that the excess temperature falls away in the manner a great many things in nature fall away, so that

$$\theta\text{-}\theta_r=(\theta_0\text{-}\theta_r)e^{-kt}$$

with θ_0 the temperature at which the watch was started and k a constant belonging to this calorimeter, this quantity of water and the stillness of the air around it. Take logarithms to base ten of both sides. The exponential becomes a multiple of t, and

$$\log_{10}(\theta\text{-}\theta_r)=\log_{10}(\theta_0\text{-}\theta_r)\text{-}\frac{k}{2.303}t$$

which is a straight line of slope $-k/2.303$ against time, with an intercept that does not contain t at all. Compare it with the relation printed in part (iii): $K=k/2.303$, which is why part (iv) multiplies by 2.303 and not by anything else. Nothing here says why the fall should be of that kind; part (vi) asks what your own graph has to say about it.

Practical tip 2: *Three decimal places in the logarithm column, and no fewer.*

Over the whole of this run the logarithm falls by about 0.33, which on the scale below is about three quarters of the height of the grid. Round the column to 2 decimal places and every value is free to move by up to 0.005, a whole small square. On these readings the line would still give the same slope to 3 figures, but the points would lie half as far again from it, and a slope you could defend to 1% becomes one you could defend to 1.5%. Work to 3 decimal places, keep the third figure even when it is a zero, and write the excess temperature down before you take its logarithm so that a mistake in one column can be found without redoing the other.

Practical tip 3: *Reading beyond the last point.*

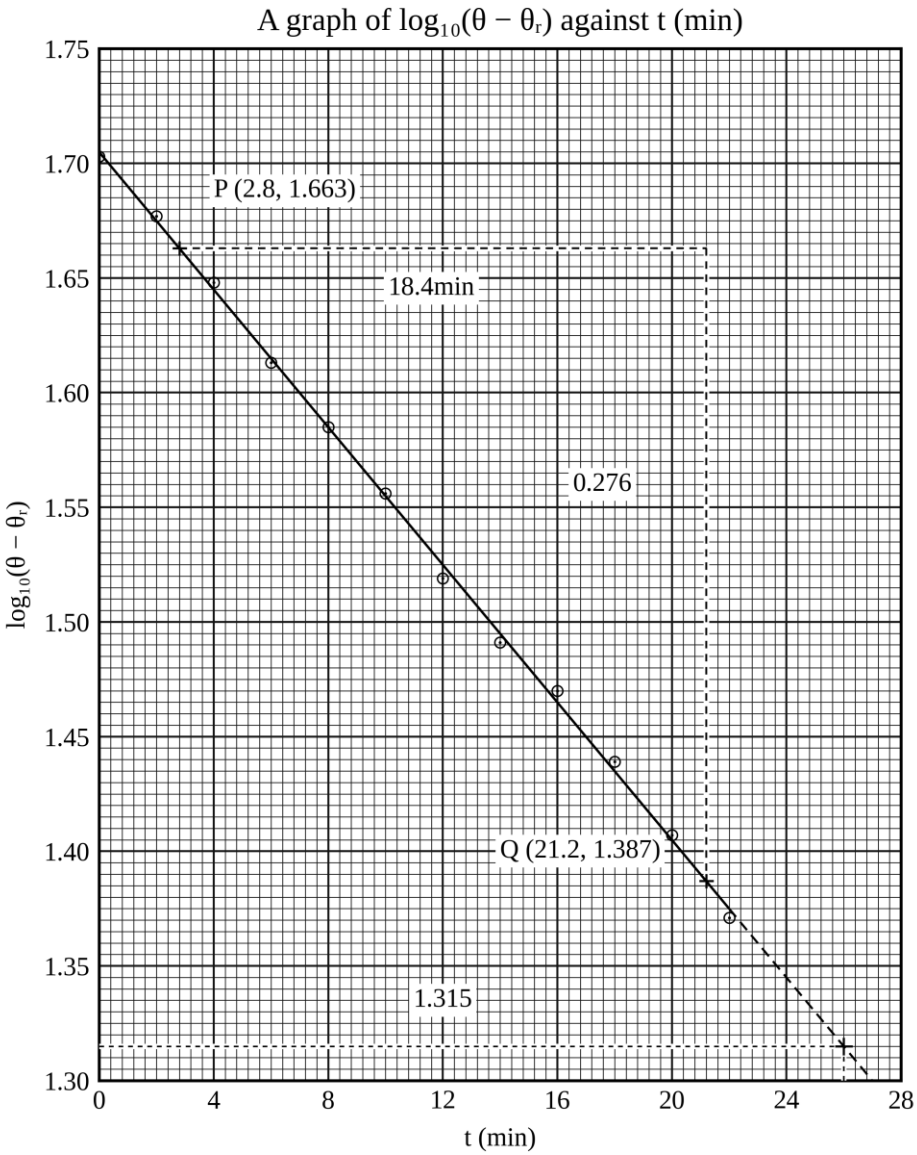
Part (v) asks for a temperature at a time later than any you measured, which means extending the line. Draw the extension as a broken line so that it cannot be mistaken for data, take the reading off the extended line rather than computing it from the last plotted point, and make sure the axes were drawn long enough in the first place to reach it. What you read off is a logarithm, and part (v) asks for a temperature, so there is work to do afterwards; the temperature of the room comes back into it before you are finished.

Practical tip 4: *What the logarithm buys, and what it costs.*

Twelve points and a transparent ruler will place this line to within about 1.5%, which is better than a single hand-drawn tangent by a factor of two or three, and the whole of the improvement comes from having twelve readings share the work instead of two. That is the bargain. What you give up is the instant: this graph describes the entire run and points at no particular minute of it, so if what you want is the rate of loss of heat at one named temperature you are back to a tangent, or to Experiment 13.3.

Answers to the questions

(i) and (ii). Take t at 2cm to 4 minutes, running from 0 to 28 minutes, which reaches past the last reading, and $\log_{10}(\theta-\theta_r)$ at 2cm to 0.05, running from 1.30 to 1.75. That is 14cm by 18cm, which fits an A4 sheet upright. Give the graph the title A graph of $\log_{10}(\theta-\theta_r)$ against t (min).



(iii) The twelve points lie on a straight line, and the corners of the slope triangle are lettered P and Q here because the letter A already belongs to the intercept. Taking P (2.8, 1.663) and Q (21.2, 1.387) from the line,

$$\text{slope}=\frac{1.387\text{-}1.663}{21.2\text{min-}2.8\text{min}}=\frac{\text{-}0.276}{18.4\text{min}}=\text{-}0.0150\text{min}^{-1}$$

The slope is -K, so K is 0.0150min^{-1}. A logarithm has no unit, so the slope carries the unit of one over time and no other. The intercept is read where the line meets the axis at t=0, and it is 1.705. Check it against the triangle:

$$1.663+0.0150\text{min}^{-1}\times 2.8\text{min}=1.705$$

(iv) k=2.303K, so

$$k=2.303\times 0.0150\text{min}^{-1}=0.0345\text{min}^{-1}$$

The cooling constant of the calorimeter and its contents is 0.0345min^{-1}. Accept 0.0335min^{-1} to 0.0355min^{-1}.

(v) Extend the line as a broken line to 26min and read across from it to the vertical axis. The reading is 1.315, which the equation of the line confirms:

$$\log_{10}(\theta\text{-}\theta_r)=1.705\text{-}0.0150\text{min}^{-1}\times 26\text{min}=1.315$$

The antilogarithm of 1.315 is 20.7, so the excess temperature is 20.7K and the temperature of the water is 25.5°C+20.7K=46.2°C.

(vi) A straight line on this plot means that the logarithm of the excess temperature falls by equal amounts in equal times. Equal steps in a logarithm are equal ratios in the quantity itself, so the excess temperature falls by

the same fraction of itself in every interval of the same length: it is multiplied by a fixed number every two minutes rather than reduced by a fixed number of degrees. Over the first ten minutes of this run the excess fell from 50.5K to 36.0K, and over the last ten from 33.0K to 23.5K. The first is a loss of 14.5K and the second of only 9.5K, but each is a fall to 71% of what it was. An exponential fall is a fall by a fixed fraction, and a straight line on this plot means, and a straight line on this plot is what it looks like.

(vii) The slope would be the same, or as near the same as makes no difference. The slope is -K, and k is fixed by the surface of the calorimeter, the quantity of water in it and the stillness of the air around it, none of which the temperature of the room alters. The intercept would be higher. It is the logarithm of the excess temperature at the moment the watch was started, and starting from the same 76.0°C in a room at 20.0°C instead of 25.5°C makes that excess 56.0K instead of 50.5K. The intercept is the logarithm of that excess, and $\log_{10}56.0$-$\log_{10}50.5=0.045$, so the whole line is lifted 0.045 up the sheet, from the 1.705 you read to about 1.750, without being tilted at all.

(viii) A source of error: the temperature of the room may drift during a run of more than 20 minutes, and every value in the third column is computed from the single figure recorded at the start. A precaution: take the room temperature at the beginning and again at the end with the same thermometer, and if the two differ use their mean and say so in your account.

Examiner's eye

- Use one thermometer for the room and for the water. A second instrument reading half a degree differently shifts every excess in the third column the same way, which tilts the line and changes K without leaving a mark anybody can see.
- Subtract before you take the logarithm. $\log_{10}(\theta\text{-}\theta_r)$ is not $\log_{10}\theta\text{-}\log_{10}\theta_r$. A table built the second way is the dangerous kind of wrong: the line looks perfectly straight and the cooling constant comes out at little more than half its true size.
- Start the time axis at zero whatever you do with the other one. This is a graph on which one axis is suppressed and the other cannot be, because A lives at $t=0$.
- Give the slope its sign, then take the sign off K. The line falls, so the slope is negative and K is the positive number left when the minus is dropped. Carry the minus into K and the equation you have written describes a calorimeter that is getting hotter.

Practice 13.2

Two men were arguing outside the tailor's stall at the corner of the market, and neither of them was the tailor. The charcoal iron was the subject. One held that a filling is good for the length of a football match and always has been. The other held that it depends entirely on how hot you get the thing to start with, and that anybody who has ever ironed a shirt knows as much.

Kipanga took the first man's side, on the ground that the iron loses its heat at a steady rate and a steady rate cannot care where it started from. **Kipute** asked whether the iron was still losing heat after it had gone cold, and if not, at what point in the afternoon a steady rate had stopped being steady.

Mr. Akilikubwa said that putting a number on how fast a hot thing loses its heat is not hard, and that the number does not even need a tangent, provided you know which column to take the logarithm of.

Procedure

Proceed as follows:

(a) Find the mass of the calorimeter together with its stirrer, and record the temperature of the room.

(b) Pour hot water at about 90°C into the calorimeter until it is about two thirds full, and find the mass of water it holds.

(c) Stand the calorimeter on the wooden slab and lower the thermometer into the water so that its bulb is clear of the bottom and the sides.

(d) Stirring gently and continuously, start the stopwatch at the instant the thermometer reads 78.0°C, and record the temperature θ every 2 minutes for 22 minutes.

Questions

(i) Tabulate your results, including for each reading the excess temperature θ-θ_r and its logarithm to base ten.

(ii) Plot a graph of $\log_{10}(\theta\text{-}\theta_r)$ against t (min).

(iii) Given that $\log_{10}(\theta\text{-}\theta_r)$=A-Kt, where A and K are constants, use your graph to determine A and K.

(iv) The cooling constant k is given by k=2.303K. Determine k in min^{-1} and in s^{-1}.

(v) Use your graph to find the time taken for the excess temperature to fall to half its value at t=0.

(vi) Show that this time does not depend on the temperature at which the stopwatch was started.

(vii) State which end of your graph carries the least reliable points, and give a reason.

Experiment 13.3

Kipute had drawn a cooling curve worth looking at and he knew it. He carried it to the front, laid a tangent on it, read off the slope and waited to be told it was right.

"*Good,*" said **Mr. Akilikubwa**. "*Now another one, lower down.*"

Kipute drew another one lower down.

"*And another.*"

Kipanga asked from the back whether this was going to go on all afternoon. He was told that it was, that there would be six of them before it was over, and that not one of the six would tell them anything on its own. **Kipute** said that in that case he could not see why anybody would draw them. **Kipanga** said he thought he could see why, but could not have said what it was, which was where most of the class had got to.

Aim

To find what the rate of cooling of a hot liquid depends on, and to determine the cooling constant of the calorimeter and its contents from six tangents drawn to one curve.

Apparatus

A copper calorimeter with its stirrer, of total mass 88.0g, a thermometer reading from 0 to 100°C in half degrees, a stopwatch, a wooden slab, a retort stand with a clamp, a beaker of hot water, a balance and a transparent ruler.

Setup

The arrangement is the one drawn in Experiment 13.1, with the same calorimeter and the same mass of water in it. The run is started higher and carried on longer than in the two experiments before it, because six tangents need six well-separated temperatures to be drawn at, and a short run does not offer them. Leave 5 minutes of curve clear beyond the first tangent you mean to draw and 5 beyond the last: a tangent at the very edge of a curve has too little on one side of it to be guided by.

Procedure

Proceed as follows:

(a) Find the mass of the calorimeter together with its stirrer, and record the temperature of the room.

(b) Pour hot water at about 95°C into the calorimeter until it is about two thirds full, and find the mass of water it holds.

(c) Stand the calorimeter on the wooden slab and lower the thermometer into the water so that its bulb is clear of the bottom and the sides.

(d) Stirring gently and continuously, start the stopwatch at the instant the thermometer reads 90.0°C, and record the temperature θ every 4 minutes for 40 minutes.

Questions

(i) Tabulate your results.

(ii) Plot a graph of θ (°C) against t (min) and draw a smooth curve through your points.

(iii) Draw tangents to your curve at the six temperatures 82.0, 74.0, 66.0, 58.0, 52.0 and 48.0°C. Determine the magnitude S of the slope of each, and tabulate S against the excess temperature θ-θ_r.

(iv) Plot a graph of S (K/min) against θ-θ_r (K).

(v) State the relationship between S and θ-θ_r shown by your second graph, and say what it establishes about the rate at which the calorimeter loses heat.

(vi) Determine the slope of your second graph and state its unit.

(vii) The same calorimeter holding the same mass of water was used in Experiment 13.2, where the cooling constant was found to be $0.0345 min^{-1}$. Compare that value with yours and comment on the agreement.

Table of results

The temperature of the room was 26.5°C throughout, and the calorimeter held 165g of water.

t (min)	θ (°C)	t (min)	θ (°C)
0	90.0	24	54.5
4	82.5	28	50.5
8	74.5	32	47.0
12	69.0	36	45.0
16	63.0	40	42.5
20	58.0		

Making the measurement count

Practical tip 1: *Six tangents on one curve, without ruining the curve.*

Draw the curve first, in a fine line, and do not touch it again. Mark the six temperatures on the vertical axis, run a faint horizontal across from each one to meet the curve, and mark the six points of contact before you draw anything else; that way the six tangents are all drawn to the same curve rather than to six slightly different curves you have been improving as you went. Use the equal-chord construction of Experiment 13.1 for every one of them, with the two chord points about 5 minutes on either side of the contact, and less than that where the contact falls near the beginning or the end of the run; what matters is that the two are equally spaced about it. Then extend each tangent well past its chord points, so that there is a long base to read it over.

Practical tip 2: *Why six and not one.*

A single tangent is good to about 2%, as Experiment 13.1 said, and drawing it more slowly will not improve on that. Six of them plotted on a second graph do improve on it, because the line you then draw through six points splits the difference between six separate errors of placement and lands somewhere near the middle of them. The gain goes roughly as the square root of the number of tangents, so six of them bring about 2% down to about 1%. It stops there, and for two reasons: draw twelve and you gain a factor of only 1.4 for an afternoon's work, and the six tangents are not fully independent anyway, because they are all drawn to the one curve and share whatever is wrong with it.

Practical tip 3: *A chord is steeper than the tangent at the middle of it.*

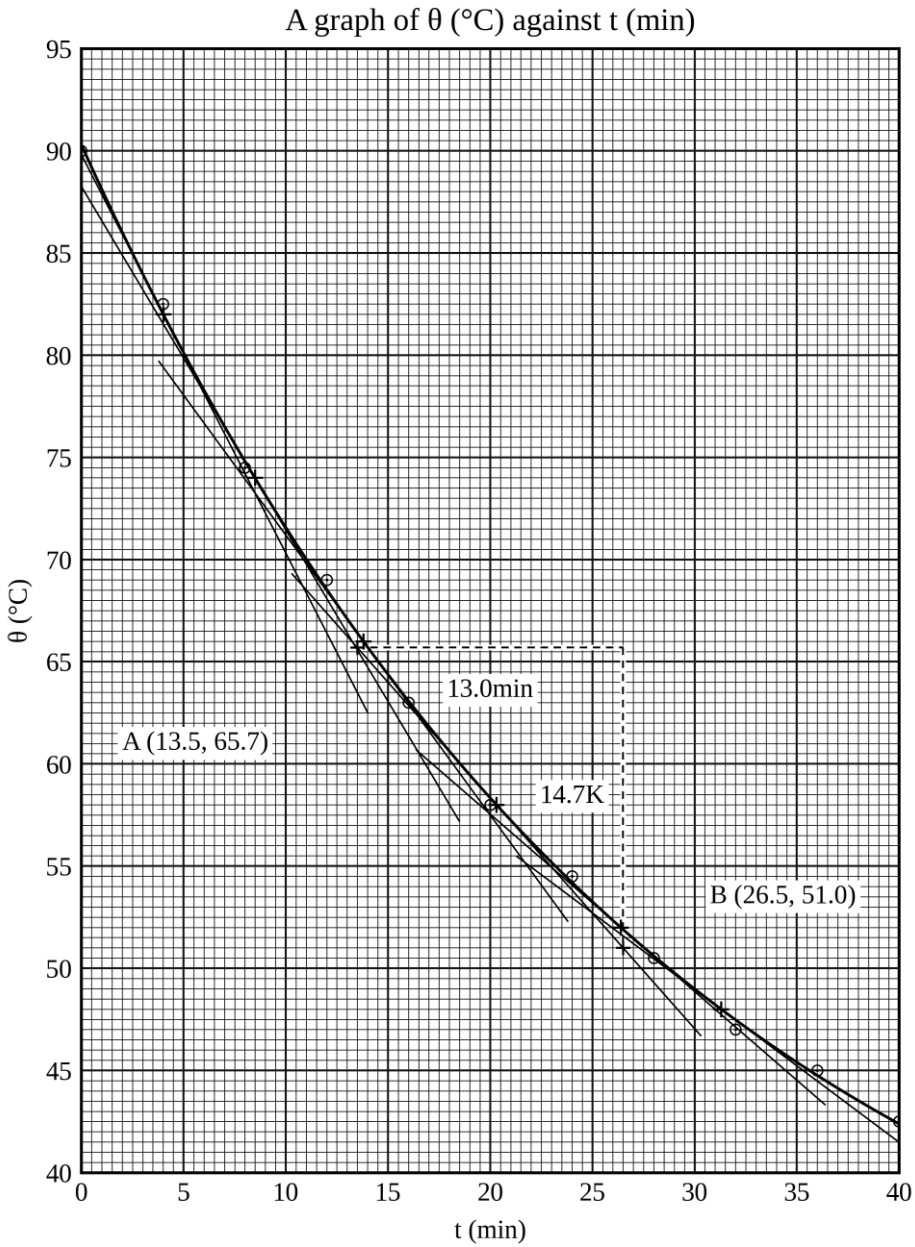
The construction of Experiment 13.1 draws the tangent parallel to a chord, and on a curve that bends the way this one does the chord is the steeper of the two. That makes it a bias and not a scatter: it goes the same way every time, so it does not average out over six tangents the way errors of placement do. Its size depends entirely on how far apart you put the chord points. At 5 minutes either side of the contact it is about 0.5% on this curve, which is small beside everything else; at 10 minutes either side it is about 2%, which is not. That is the whole reason Practical tip 1 asks for the closer pair, and the reason for taking them no closer than you can judge a direction over.

Practical tip 4: *Reading a slope you have already agreed to call positive.*

Every one of these tangents falls, so every slope is negative, and the quantity S that the question asks you to plot is the magnitude of it. Say so once, at the head of your table, and then work in positive numbers throughout. What you must not do is keep the minus signs in the third column and lose them in the graph, or the slope of the second graph will come out negative, and no calorimeter on this bench has a negative cooling constant.

Answers to the questions

(i) and (ii). Take t at 2cm to 5 minutes, running from 0 to 40 minutes, and θ at 2cm to 5°C, running from 40°C to 95°C. That is 16cm by 22cm, which fits an A4 sheet upright. Give the graph the title A graph of θ (°C) against t (min).



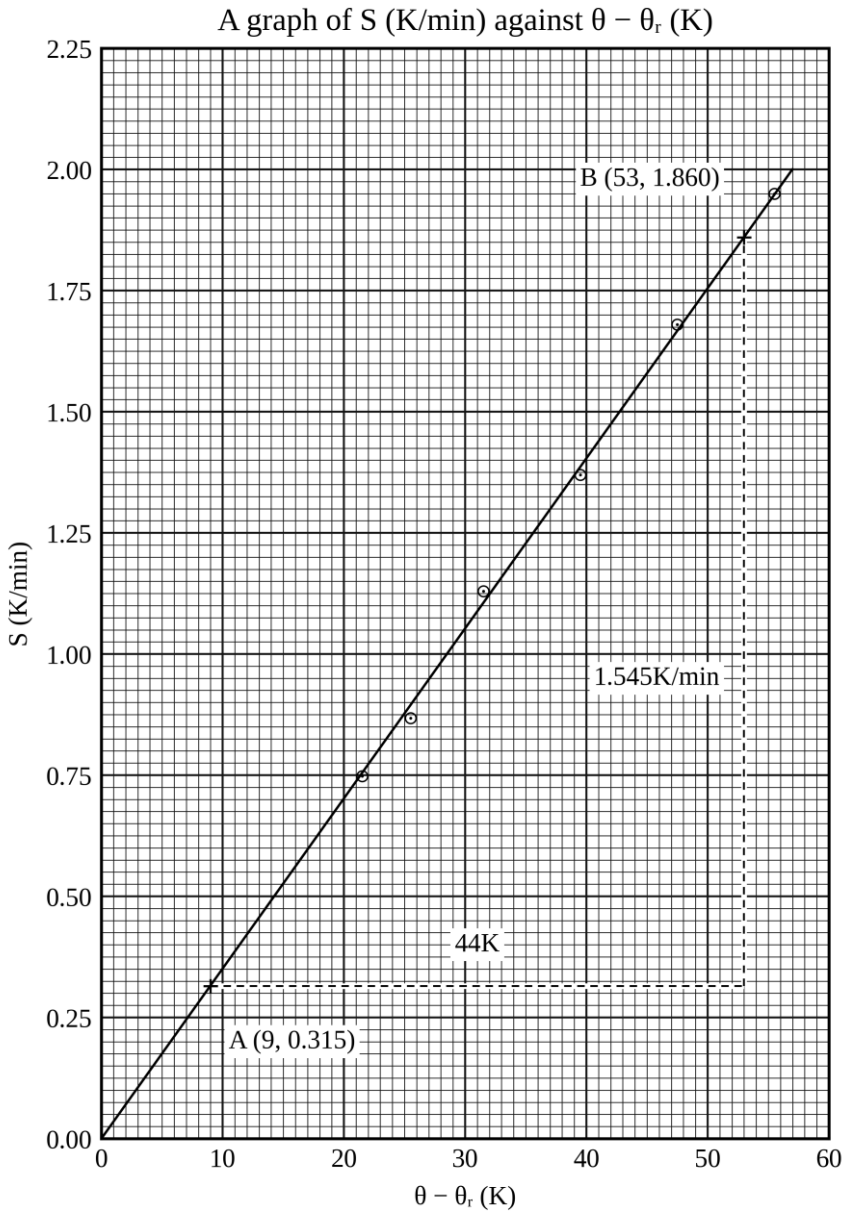
(iii) The six points of contact are marked on the curve. Only one tangent is lettered here, the one at 58.0°C, so that the drawing stays readable; every other one was read the same way, between two points as far apart as the drawn tangent allowed. For that one, with A (13.5, 65.7) and B (26.5, 51.0) on it,

$$S=\frac{65.7°C-51.0°C}{26.5min-13.5min}=\frac{14.7K}{13.0min}=1.13Kmin^{-1}$$

The six together are:

θ (°C)	$\theta - \theta_r$ (K)	S (K/min)
82.0	55.5	1.95
74.0	47.5	1.68
66.0	39.5	1.37
58.0	31.5	1.13
52.0	25.5	0.868
48.0	21.5	0.748

(iv) Take θ-θ_r at 2cm to 10K, running from 0 to 60K, and S at 2cm to 0.25K/min, running from 0 to 2.25K/min. That is 12cm by 18cm, which fits an A4 sheet upright. Give the graph the title A graph of S (K/min) against θ-θ_r (K). Both axes start at zero.



(v) The six points lie on a straight line through the origin, so $S \propto \theta$-θ_r: the rate at which the temperature falls is directly proportional to the amount by which the water is hotter than the room. The line through the origin says the rest of it. When the excess is nothing the rate is nothing, which is only to say that a body at the temperature of its surroundings does not cool. The result carries a condition with it, and the condition belongs in the account: the excess temperature in this run never rose above 64K. Over a much wider range the six points would begin to bend away from the line, because a loss that is partly convection and partly radiation grows a little faster than the excess itself, so the proportionality is an approximation and not a law of nature.

The rate of loss of heat follows at once. The heat leaving the calorimeter each minute is the heat capacity of the whole system multiplied by S, and the heat capacity of the calorimeter and its contents hardly changes during the run, so the rate of loss of heat is proportional to S and therefore to the excess temperature as well. That is Newton's law of cooling, and this graph tests it rather than assuming it: nothing in the plotting supposed the answer, and the points were free to lie on a curve.

(vi) Read the line between the two marked corners A (9, 0.315) and B (53, 1.860):

$$\text{slope}=\frac{1.860\text{K/min}-0.315\text{K/min}}{53\text{K}-9\text{K}}=\frac{1.545\text{K/min}}{44\text{K}}=0.0351\text{min}^{-1}$$

A rate in kelvin per minute divided by a temperature in kelvin leaves one over a minute, so the slope is 0.0351min^{-1}, the cooling constant of the calorimeter and its contents. Accept 0.0335min^{-1} to 0.0365min^{-1}. That is a marking tolerance, not the precision of one carefully drawn graph.

It is the same constant Experiment 13.2 found, and one line shows why. S is the rate at which the excess temperature is falling, so the relationship your graph has just established is

$$-\frac{d(\theta-\theta_r)}{dt}=k(\theta-\theta_r)$$

which says that the excess loses a fixed fraction of itself in every second, and a quantity that does that falls away exponentially. That exponential is what Experiment 13.2 assumed and took the logarithm of, with this k in it. The two experiments therefore measure one number by two routes: this one takes the rate directly and infers the law, and that one assumes the law and never measures a rate at all.

(vii) The logarithmic method gave 0.0345min^{-1} and six tangents give 0.0351min^{-1}, a difference of 1.7%. The two agree, but only just, and it is worth saying so rather than more. The six points sit within about 2% of the line drawn through them, which places that line to about 1%; the logarithmic line is worth about 1.5%; put those together and 1.7% is about what two honest determinations by these two methods should be expected to differ by. No cause needs to be invented for it. What would be worth investigating is a difference of the same size and the same sign on run after run, because the chord bias of Practical tip 3 is the one effect here that does not average away.

Examiner's eye

- Read the question before choosing the horizontal axis. This one asks for S against the excess temperature, which is not the same quantity as the temperature and does not give the same line.
- Six tangents mean six separate constructions, not one tangent slid along the curve. If two of the six slopes come out identical to three figures, one of them has not been drawn independently, and the second graph is worth less than it looks.
- Two graphs mean two titles, two pairs of labelled axes and two stated scales. The second sheet is where the difficult work gets thrown away for want of a heading.
- Draw the line through the six points and let it fall where it falls. Passing within a small square of the origin is passing through it; missing it by five is a result, and one to be reported rather than ruled away.

Practice 13.3

Pinned to the laboratory noticeboard, high up where nobody has been able to reach it since the ladder was borrowed, there is a graph left behind by a class that has long since left the school. It has a rate of cooling up the side and a temperature along the bottom, six points on it, and a line drawn through them with a ruler.

The line has been dragged down to the origin. You can see where it was first drawn, because the pencil went twice, and the second time it went through the corner of the sheet, missing four of the six points to get there. Underneath, in pencil, three answers have been written and two of them crossed out.

Kipanga said that somebody had been taught that a straight line through experimental points has to go through the origin, and had believed it. **Kipute** said that what he would like to know is which of the three answers the class settled on in the end, and what the second line was supposed to be telling anybody.

Procedure

Proceed as follows:

(a) Find the mass of the calorimeter together with its stirrer.

(b) Pour hot water at about 95°C into the calorimeter until it is about two thirds full, and find the mass of water it holds.

(c) Stand the calorimeter on the wooden slab and lower the thermometer into the water so that its bulb is clear of the bottom and the sides.

(d) Stirring gently and continuously, start the stopwatch at the instant the thermometer reads 88.0°C, and record the temperature θ every 3 minutes for 33 minutes.

Questions

(i) Tabulate your results.

(ii) Plot a graph of θ (°C) against t (min) and draw a smooth curve through your points.

(iii) Draw tangents to your curve at the six temperatures 80.0, 72.0, 64.0, 56.0, 50.0 and 47.0°C. Determine the magnitude S of the slope of each, and tabulate S against θ.

(iv) Plot a graph of S (K/min) against θ (°C) and draw the best straight line through your points.

(v) Determine the slope of that line and state its unit.

(vi) The temperature of the surroundings was not recorded. Use your graph to find it, and state which feature of the graph gives it.

(vii) Explain why this graph should not be expected to pass through the origin.

Chapter 14

COMPARATIVE COOLING

Chapter 13 put one calorimeter of hot water on a slab and asked how fast it cooled. This chapter puts two of them side by side and asks a different question: what does that rate depend on? Nothing here is harder to measure than a temperature and a time, and every graph is one you have already drawn. What is new is that the answer you are after is never one run. It is what happens to the cooling when one thing about the calorimeter is changed and nothing else is.

Two quantities fix how quickly a hot body cools. One is how much heat it has to give up for every kelvin it falls, which is a property of what is inside it. The other is how readily its surface hands that heat to the room, which is a property of the outside. Every experiment in this chapter sets out to alter one of the two and leave the other alone: the mass of water in the vessel, the finish on its outside, a jacket of lagging round it, a wet cloth on it, and the liquid it holds.

The design that makes such a comparison worth anything is the same every time, and it is worth stating once before you meet it five times. Two runs are made that are alike in every particular you can control: the same room, the same slab, the same starting temperature, the same stirring, the same thermometer resolution. One thing is different. Whatever comes out of the comparison then belongs to that one thing, and to nothing else. Change two at a time and you cannot say which of them the answer belongs to, however careful the readings are.

Notice what that buys you. Divide one run by the other and a great deal of what you never measured divides out with it, exactly rather than approximately, and what is left is a pure number that can be quoted and argued with. Most of the answers in this chapter pass through a ratio for that reason. Which quantities go and which stay is not the same in every experiment, and several of the questions ahead turn on exactly that; where a ratio will not serve, the chapter says so and pays for it in the error budget.

The five varieties at a glance

	What is changed between the two runs	What the comparison gives you
14.1	The mass of water, in two identical calorimeters	the mass of water in the second of them
14.2	The outside of the calorimeter, blackened against polished	a cooling constant for each surface, and their ratio
14.3	A jacket of cotton wool round the calorimeter	how much of the heat loss the lagging removes
14.4	A wet cloth wrapped round the calorimeter	the extra rate of loss, and the water that leaves as vapour
14.5	The liquid in the calorimeter, water against liquid A	the specific heat capacity of liquid A

All five are worked below in full. The practice questions change the calorimeter, the quantities and the room, and the last of them returns to the first factor by a different route.

Experiment 14.1

The tea stall outside the school gate keeps two vacuum flasks on the counter, a tall one and a short one, and by the middle of the afternoon the tea in one of them has gone tepid while the other is still worth buying. Which one is which has been argued about at that counter for years.

Kipanga put his money on the tall flask going cold first, on the grounds that there is more tea in it and therefore more heat to get rid of. **Kipute** put his money the other way, on the grounds that there is more tea in it and therefore more heat to get rid of, which is the same sentence twice and the opposite prediction. The stall owner, who had heard the argument before, sold them a cup from each and told them to settle it themselves.

Mr. Akilikubwa pointed out on the way back that the two flasks were not the same size, not the same age and not filled at the same hour, so whichever boy turned out to be right would have been right by accident. The question underneath the argument is what more tea in a vessel does to the rate at which it cools.

Aim

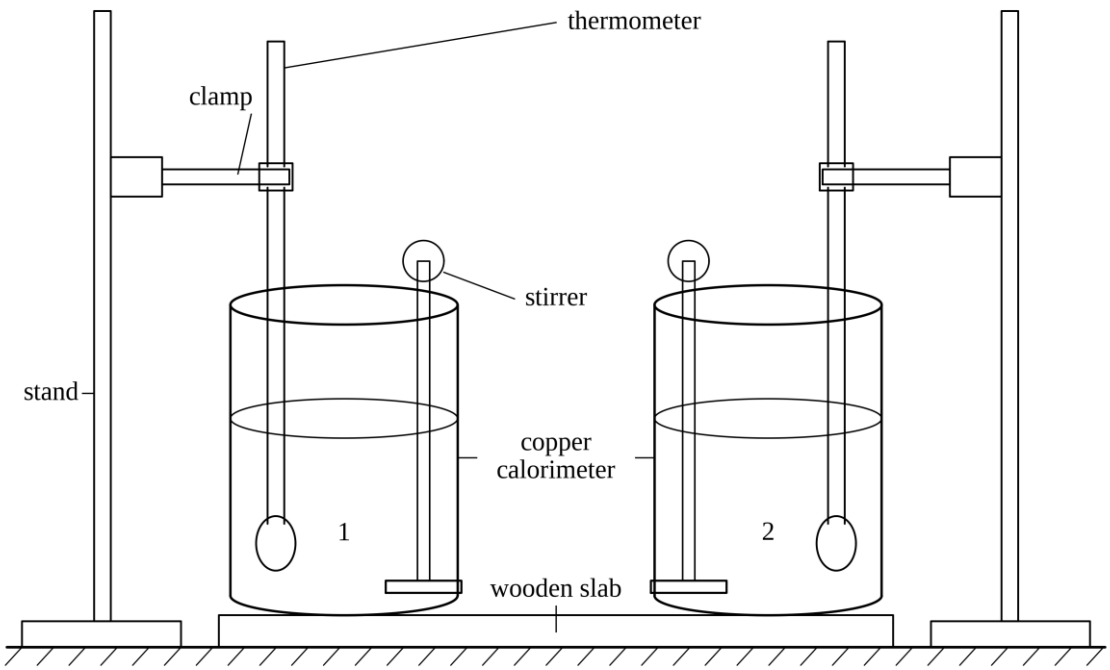
To determine the mass of water in one calorimeter by comparing the times it and an identical calorimeter take to cool through the same temperatures.

Apparatus

Two identical copper calorimeters with their stirrers, each of total mass 96.0g, two thermometers reading from 0 to 100°C in steps of 0.5°C, two stopwatches, a wooden slab, a retort stand with two clamps, a beaker of hot water, a balance and a transparent ruler.

Setup

The two calorimeters stand side by side on one wooden slab, away from any window or doorway, and nothing is wrapped round either of them. Each thermometer is held in a clamp with its bulb well below the surface of the water and clear of the bottom and the sides. Both stirrers stay in the water throughout. Calorimeter 1 is filled with a measured 200g of hot water. Calorimeter 2 is filled from the same beaker, and how much it holds is not recorded. The temperature of the room is taken with one of the two thermometers before the water is poured. The two vessels are drawn alike because they are alike; the drawing says nothing about how much water is in either of them.



Procedure

Proceed as follows:

(a) Find the mass of each calorimeter together with its stirrer, and record the temperature of the room.

(b) Pour 200g of water at about 90°C into calorimeter 1, and hot water from the same beaker into calorimeter 2 without measuring it.

(c) Stand both calorimeters on the wooden slab and lower a thermometer into each so that its bulb is clear of the bottom and the sides.

(d) Stirring both gently and continuously, start one stopwatch at the instant its own thermometer reads 80.0°C, and start the other the same way.

(e) Record the time t_1 on the first watch and t_2 on the second at which each thermometer reaches each of the temperatures in the table.

Questions

(i) Tabulate your results.

(ii) Plot a graph of t_1 (min) against t_2 (min) and draw the best straight line through your points.

(iii) Determine the slope of your line and state its unit.

(iv) The two calorimeters are alike in shape, size and finish, and they stand in the same room, so at any stated temperature each is losing heat at the same rate as the other. Show that the slope of your graph is therefore the ratio of the heat capacities of the two systems.

(v) The specific heat capacity of water is 4.2J/gK and that of copper is 0.38J/gK. Use these, with your slope, to determine the mass of water in calorimeter 2.

(vi) State, with a reason, what your line does at the origin.

(vii) The temperature of the room appears nowhere in your working. Explain why it does not need to.

Table of results

The temperature of the room was 26.0°C throughout, and calorimeter 1 held 200g of water.

θ (°C)	t_1 (min)	t_2 (min)
75.0	2.8	1.9
70.0	6.2	3.7
65.0	9.4	5.7
60.0	13.5	8.4
55.0	18.3	11.0
50.0	23.6	14.8

Making the measurement count

Practical tip 1: *Two runs at once, and two watches.*

The two calorimeters do not reach 80.0°C at the same instant, and there is no reason why they should: the one with less water in it gets there first. Give each its own watch and start each at its own moment. What the two runs must share is the temperature they start from and the temperatures they are timed to, not the reading on any clock. Start both from the same temperature and you have taken the starting temperature out of the comparison altogether.

Practical tip 2: *One difference, and no others.*

Stand both calorimeters on the same slab, the same distance from the window, and stir both at the same rate. Do not put a lid on one and leave the other open, and do not stand one on the bench and the other on the slab. Each of those is a second difference, and a second difference makes the comparison worthless without making it look wrong: the graph comes out just as straight as before.

Practical tip 3: *The heat capacity of the whole system.*

The thermometer follows the water, but the copper is cooling with it, and the heat that leaves comes out of both. What governs the cooling is therefore the heat capacity of the whole system, each mass multiplied by its own specific heat capacity and the two added, as in Chapter 13:

$$C = m_c c_c + m_w c_w$$

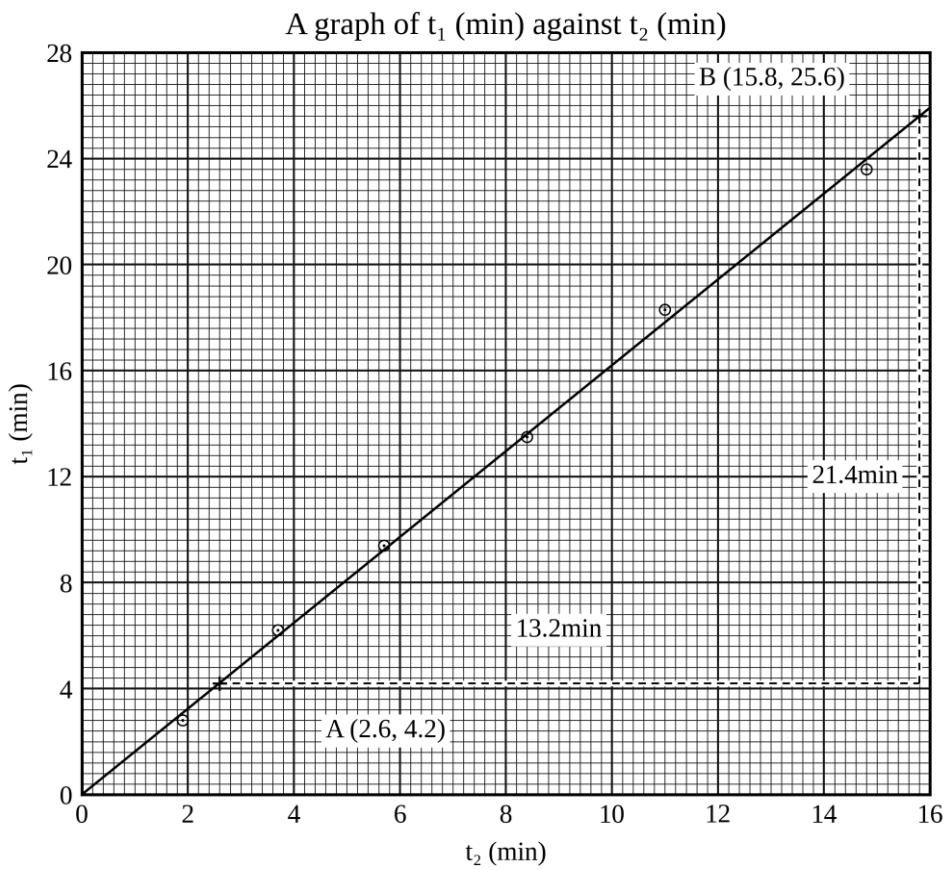
The mass of water is therefore the one reading in this experiment that has to be taken with real care, because it carries the greater part of that sum by a long way. The copper is the smaller part, and part (v) asks you what to do about it.

Practical tip 4: *Six pairs of readings rather than one.*

One fall from 80.0°C to 50.0°C timed in each calorimeter would give you the ratio in two readings. Six pairs give the same ratio from a line, and a line drawn through six points is worth about half as much again as the best single pair, because the scatter of the individual times averages down. Six pairs also tell you something a single pair cannot: whether the two watches really were started at the same temperature. Part (vi) is about how they tell you.

Answers to the questions

(i) and (ii). Take t_2 at 2cm to 2 minutes, running from 0 to 16 minutes, and t_1 at 2cm to 4 minutes, running from 0 to 28 minutes. That is 16cm by 14cm, which fits an A4 sheet upright. Give the graph the title A graph of t_1 (min) against t_2 (min).



(iii) Taking the two marked corners A (2.6, 4.2) and B (15.8, 25.6) on the drawn line,

$$\text{slope}=\frac{25.6\text{min}-4.2\text{min}}{15.8\text{min}-2.6\text{min}}=\frac{21.4\text{min}}{13.2\text{min}}=1.62$$

The slope is 1.62. It is a time divided by a time, so it carries no unit at all.

(iv) Write C_1 and C_2 for the heat capacities of the two systems, and P for the rate at which either of them loses heat when it stands at temperature θ. Take one small step of the descent, a fall δθ at that temperature. Calorimeter 1 must give up $C_1\delta\theta$ to make it and is losing heat at P, so the step takes it $C_1\delta\theta/P$. Calorimeter 2 makes the same step in $C_2\delta\theta/P$, with the same P, because it is at the same temperature in the same room with the same surface. Every step of the descent is therefore in the ratio C_1:C_2, and so is the sum of them all:

$$\frac{t_1}{t_2}=\frac{C_1}{C_2}$$

That holds at every temperature in the table, which is why the points lie on a straight line and not merely near one. Notice what the argument did not need: no law of cooling was used in it anywhere. Besides the common

starting temperature, all it assumes is that the rate of loss depends on the temperature and on nothing else. There is one place where that assumption is strained. The two vessels hold different amounts, so the water stands at different depths and the copper above the surface is a little cooler than the copper below it. That is the largest approximation in the method, and an account that names it has understood the method.

(v) The heat capacity of calorimeter 1 and its contents is

$$C_1 = 96.0\text{g} \times 0.38\text{J/gK} + 200\text{g} \times 4.2\text{J/gK} = 36.48\text{JK}^{-1} + 840\text{JK}^{-1} = 876.48\text{JK}^{-1}$$

From the slope, $C_1/C_2 = 1.62$, so

$$C_2 = \frac{876.48\text{JK}^{-1}}{1.62} = 541.04\text{JK}^{-1}$$

The copper accounts for 36.48JK^{-1} of that, exactly as it does in calorimeter 1, so the water in calorimeter 2 accounts for the rest:

$$m_w = \frac{541.04\text{JK}^{-1} - 36.48\text{JK}^{-1}}{4.2\text{J/gK}} = \frac{504.56\text{JK}^{-1}}{4.2\text{J/gK}} = 120.1\text{g}$$

Calorimeter 2 held 120.1g of water.

(vi) It passes through the origin. Both watches were started at 80.0°C, so t_1 and t_2 are both zero at that temperature, and the point (0, 0) is a reading of the experiment like any other. Draw the line through it.

(vii) A ratio of two times over the same fall does not contain it. Each calorimeter is cooling in the same room, so a change in the room temperature lengthens both times together and leaves the quotient where it was. It would be needed for a single cooling constant, which is read from an excess temperature, and it is not needed for a ratio.

Examiner's eye

- Put t_2 on the horizontal axis, which is what the question asks for, and check it twice. Both columns are times in minutes and both start at zero, so a graph plotted the wrong way round looks perfectly respectable and gives 0.617 where 1.62 was wanted.
- Draw the line through the origin. It is a seventh reading, and it is fixed by the way the two watches were started rather than by your eye on a moving thermometer.
- Show the working even though the slope has no unit. Write the two corners down, write the subtraction down, and let the minutes cancel on the page. An answer of 1.62 on its own is a number the examiner cannot check.
- Keep the copper out of the water and in the sum. Its 36.48JK^{-1} is nearly 7% of the calorimeter you subtract it from, and subtract it you must, because that is the step that turns a heat capacity into a mass of water. Forget it and the mass comes out 8.7g too large.

Practice 14.1

Taped inside the preparation room door, on card that has gone the colour of weak tea, is the laboratory assistant's timetable for the afternoon:

ALLOW 20 MINUTES FOR THE SMALL BEAKER

AND 35 FOR THE LARGE

The 35 has been struck out and 34 written above it in a second hand, and under the whole thing a third hand has added, in pencil, "depends what is in it."

Mr. Akilikubwa said the pencil was the only one of the three that had understood the problem, and that a card cannot carry the answer because the answer is not a number of minutes but a ratio. **Kipute** asked a ratio of what to what, which is the one thing none of the three hands had written down.

Procedure

Proceed as follows:

(a) Find the mass of each calorimeter together with its stirrer, and record the temperature of the room.

(b) Pour 240g of water at about 92°C into calorimeter 1, and hot water from the same beaker into calorimeter 2 without measuring it.

(c) Stand both calorimeters on the wooden slab and lower a thermometer into each so that its bulb is clear of the bottom and the sides.

(d) Stirring both gently and continuously, start one stopwatch at the instant its own thermometer reads 81.0°C, and start the other the same way.

(e) Record the time t_1 on the first watch and t_2 on the second at which each thermometer reaches each of the temperatures in the table.

Questions

(i) Tabulate your results.

(ii) Plot a graph of t_1 (min) against t_2 (min) and draw the best straight line through your points.

(iii) Determine the slope of your line.

(iv) The specific heat capacity of water is 4.2J/gK and that of copper is 0.38J/gK. Determine the heat capacity of calorimeter 1 and its contents.

(v) Hence determine the mass of water in calorimeter 2.

(vi) One of the two watches was in fact started at 80.0°C by mistake, and the error was noticed afterwards. State what this does to the plotted points, and whether the slope you found is still usable.

(vii) State one advantage of this method over pouring the water out of calorimeter 2 and finding its mass on the balance.

Experiment 14.2

The staff room has two kettles of the same pattern, bought together. One of them spent three terms on a paraffin stove in the old kitchen and came back the colour of a chimney. The other has been wiped every Friday since it arrived and still shows a face in it. Tea made in one of the two is cold before the second cup is poured, and the standing explanation is that its lid does not sit properly.

Kipute repeated the explanation about the lid and offered to bend it straight. **Kipanga** asked why anybody thought the lid was the difference when the outside of the two kettles could hardly look less alike, and pointed out that the lid had been blamed for three terms without once being measured.

Mr. Akilikubwa said that both of them had now made a claim about a kettle and neither had made a reading, and that a claim nobody has measured is not worth defending for three terms.

Aim

To determine the cooling constant of a calorimeter with a blackened outside and of an identical calorimeter with a polished outside.

Apparatus

Two identical copper calorimeters with their stirrers, each of total mass 96.0g, two thermometers reading from 0 to 100°C in steps of 0.5°C, two stopwatches, a wooden slab, a retort stand with two clamps, a candle, metal polish and a cloth, a beaker of hot water, a balance and a transparent ruler.

Setup

The arrangement is the one drawn in Experiment 14.1. The outside of calorimeter 1 has been blackened in the smoky part of a candle flame and allowed to cool and dry; the outside of calorimeter 2 has been polished until it is bright. Each holds 180g of water. Both stand on the same slab at the same distance from the window, both are open, and both are stirred throughout. The temperature of the room is taken before the water is poured and again at the end of the run.

Procedure

Proceed as follows:

(a) Blacken the outside of calorimeter 1 in a candle flame and polish the outside of calorimeter 2, and let both stand until they are at room temperature.

(b) Find the mass of each calorimeter together with its stirrer, and record the temperature of the room.

(c) Pour 180g of water at about 88°C into each calorimeter, stand both on the wooden slab and lower a thermometer into each so that its bulb is clear of the bottom and the sides.

(d) Stirring both gently and continuously, start one stopwatch at the instant its own thermometer reads 78.0°C, and start the other the same way.

(e) Record the temperature in each calorimeter every 2 minutes for 22 minutes, each on its own watch, and record the temperature of the room again at the end.

Questions

(i) Tabulate your results, and for each calorimeter add a column of the excess temperature over the room and a column of its logarithm to base 10.

(ii) Plot $\log_{10}(\theta\text{-}\theta_r)$ against t (min) for both calorimeters on the same pair of axes, and draw the best straight line through each set of points.

(iii) Determine the slope of each line.

(iv) The cooling of each calorimeter obeys $\log_{10}(\theta\text{-}\theta_r)=A\text{-}Kt$, where $k=2.303K$ and k is the cooling constant. Determine k for each calorimeter and state its unit.

(v) Determine the ratio of the cooling constant of the blackened calorimeter to that of the polished one, and state what it tells you about the two surfaces.

(vi) Your two lines meet the vertical axis at very nearly the same point. State what that intercept is and why the two runs share it.

(vii) The blackened calorimeter was polished bright and the experiment repeated. State what you would expect the two lines to do, and what that would tell you about the blackening.

Table of results

The temperature of the room was 25.0°C throughout, calorimeter 1 was the blackened one and calorimeter 2 the polished one, and each held 180g of water.

t (min)	θ_1 (°C)	$\theta_1 - \theta_r$ (K)	$\log_{10}(\theta_1 - \theta_r)$	θ_2 (°C)	$\theta_2 - \theta_r$ (K)	$\log_{10}(\theta_2 - \theta_r)$
0	78.0	53.0	1.724	78.0	53.0	1.724
2	74.0	49.0	1.690	74.0	49.0	1.690
4	69.0	44.0	1.643	71.5	46.5	1.667
6	66.0	41.0	1.613	68.5	43.5	1.638
8	61.5	36.5	1.562	65.0	40.0	1.602
10	58.5	33.5	1.525	63.5	38.5	1.585
12	56.0	31.0	1.491	60.5	35.5	1.550
14	53.0	28.0	1.447	58.0	33.0	1.519
16	51.0	26.0	1.415	56.5	31.5	1.498
18	48.5	23.5	1.371	54.5	29.5	1.470
20	46.5	21.5	1.332	52.0	27.0	1.431
22	44.5	19.5	1.290	50.5	25.5	1.407

Making the measurement count

Practical tip 1: *Blackening a calorimeter, and polishing one.*

Hold the calorimeter in the smoky yellow part of a candle flame and turn it slowly until the outside carries an even film of soot. Keep the flame off the rim and off the inside; soot inside the vessel comes away into the water and changes the mass you weighed. The other calorimeter is polished with metal polish and a dry cloth until it takes a reflection, and it must be wiped completely dry, because a film of polish left on it is a third surface and not one of the two you meant to compare.

Practical tip 2: *One room temperature, taken twice.*

Every number in the two logarithm columns is built on the room temperature, so an error in that one reading tilts both lines. Take it with one thermometer before the run and again at the end, and say so with your results if the room has moved by more than 0.5°C. A drifting room bends a logarithmic plot at the far end, where the excess is small and the drift is the largest part of it, which is why the last two or three points are the least entitled to their places on the line.

Practical tip 3: *Two lines on one sheet.*

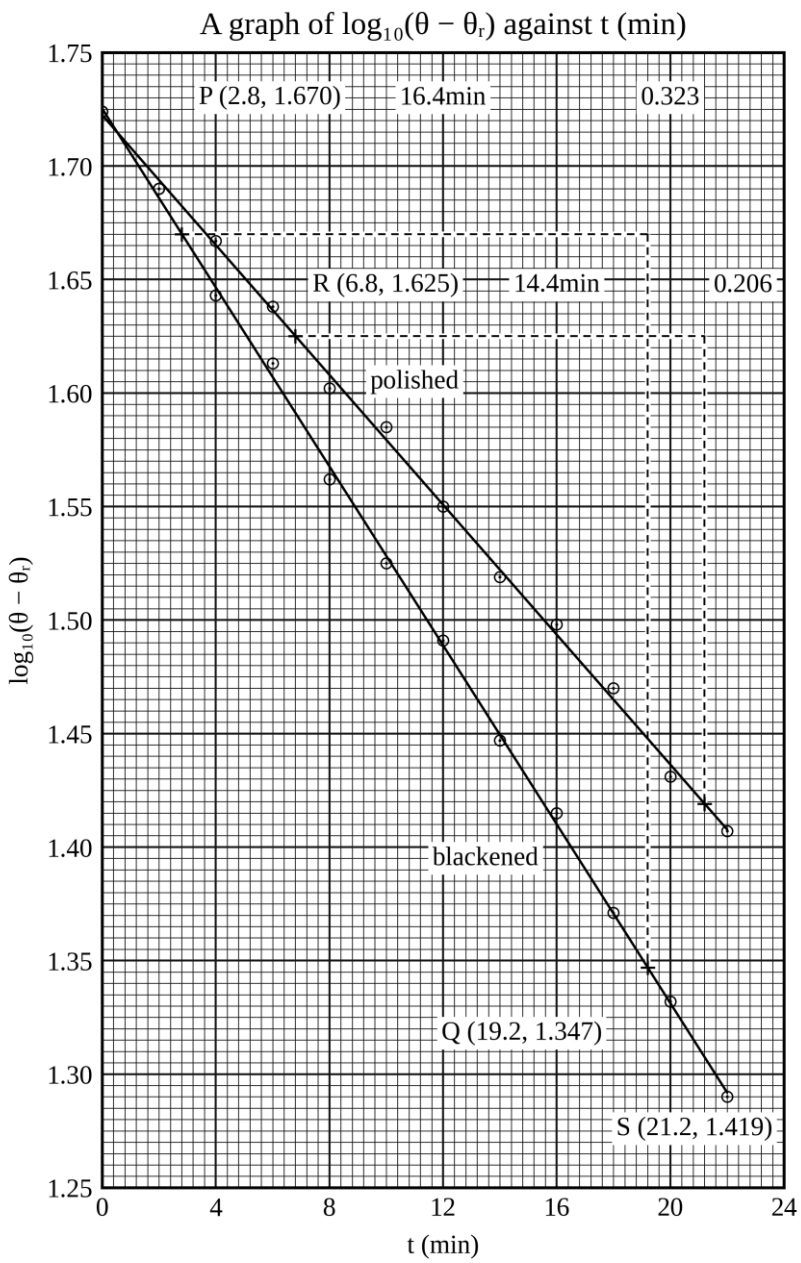
Both sets of points go on one pair of axes, to one scale laid out for whichever set runs furthest. Plot them all before you draw anything, then draw each line and write what it is against the end of it; two unlabelled lines on a sheet are two lines nobody can mark. Keep the slope triangles apart as well, one high on its line and the other low on its own, so that the legs of the first miss the corners of the second.

Practical tip 4: *What a ratio of two slopes is worth.*

Each line here is fixed by twelve points, and a ruler laid through twelve points on a logarithmic plot is worth about 1.5%. The ratio of two such slopes carries both uncertainties, and percentages add when quantities are divided, so the answer to part (v) is good to about 3%, which on a ratio of this size is a few units in the second decimal place. Part (v) asks what the ratio says, and 3% is what you are entitled to say it with.

Answers to the questions

(i) and (ii). Take t at 2cm to 4 minutes, running from 0 to 24 minutes, and $\log_{10}(\theta\text{-}\theta_r)$ at 2cm to 0.05, running from 1.25 to 1.75. That is 12cm by 20cm, which fits an A4 sheet upright. Give the graph the title A graph of $\log_{10}(\theta\text{-}\theta_r)$ against t (min).



(iii) On the blackened calorimeter's line, taking the corners P (2.8, 1.670) and Q (19.2, 1.347),

$$\text{slope}=\frac{1.347-1.670}{19.2\text{min}-2.8\text{min}}=\frac{-0.323}{16.4\text{min}}=-0.0197\text{min}^{-1}$$

and on the polished calorimeter's line, taking R (6.8, 1.625) and S (21.2, 1.419),

$$\text{slope}=\frac{1.419-1.625}{21.2\text{min}-6.8\text{min}}=\frac{-0.206}{14.4\text{min}}=-0.0143\text{min}^{-1}$$

A logarithm carries no unit, so each slope is a reciprocal minute. Compare the two lines with $\log_{10}(\theta-\theta_r)=A-Kt$, and $K=0.0197\text{min}^{-1}$ for the blackened calorimeter and 0.0143min^{-1} for the polished one.

(iv) Since $k=2.303K$, the cooling constant of the blackened calorimeter is

$$k=2.303\times0.0197\text{min}^{-1}=0.0454\text{min}^{-1}$$

and that of the polished one is

$$k=2.303\times0.0143\text{min}^{-1}=0.0329\text{min}^{-1}$$

(v) The ratio is

$$\frac{0.0454\text{min}^{-1}}{0.0329\text{min}^{-1}}=1.38$$

The two calorimeters hold the same mass of water in the same copper, so they have the same heat capacity, and they are the same size and shape standing in the same room. Everything that could make the two constants differ has been held fixed except the outside surface, so the whole of the difference belongs to that surface: the vessel with the blackened outside loses heat 1.38 times as fast at the same temperature. That figure is a floor and not a ceiling, because both vessels are open and the heat leaving through the mouth is common to the two of them. As for the staff room, a candle and an afternoon have shown that a sooty outside alone will do this much; the lid has still never been measured.

(vi) Put $t=0$ in $\log_{10}(\theta\text{-}\theta_r)=A\text{-}Kt$ and the intercept is $A=\log_{10}(\theta_0\text{-}\theta_r)$, the logarithm of the excess temperature at the instant each watch was started. Both watches were started at 78.0°C in a room at 25.0°C, an excess of 53.0K in both cases, and

$$\log_{10}53.0=1.724$$

The drawn lines give 1.725 and 1.722, one on each side of it. The intercept says nothing about either surface; it is a record of where the two runs began, and the two runs began together.

(vii) The two lines would come together. Both calorimeters would then have the same surface as well as the same contents, so both would have the same cooling constant and the ratio would fall to 1.00. That is the control the experiment needs: it shows that the difference measured in part (v) was carried by the soot and not by some other difference between two vessels that were never quite identical.

Examiner's eye

- The room comes off before the logarithm goes on. $\log_{10}(\theta\text{-}\theta_r)$ is the logarithm of the excess, and the excess is what is left after the room has been taken away. A column of $\log_{10}\theta$ gives a line so nearly straight that nothing on the page warns you, and a cooling constant a little over half the true one.
- Three decimal places in the logarithm column, no more and no fewer. Two is too coarse to plot on this scale and four claims a temperature read to 0.01°C, which the thermometer cannot deliver.
- Each slope keeps its minus sign in the working, and K does not. K is defined by $\log_{10}(\theta\text{-}\theta_r)=A\text{-}Kt$ with the minus already in it, and a cooling constant reported as negative is a cooling constant reported for a body that is warming up.
- Divide the two constants the way round the question asks and say which is on top. A ratio quoted as 0.725 is the same measurement as 1.38 and answers a different question; the sentence that follows it has to match.

Practice 14.2

At the bus stand a woman sells buns from a basket, and every bun goes out wrapped in a square of kitchen foil. Half the stand wraps them shiny side outwards and half shiny side in, and the two halves have been telling each other they are wrong since before the school was built.

Kipanga said the shiny side belongs outside, so that the heat is turned back. **Kipute** said the shiny side belongs inside, so that the heat is turned back, which is the same reason twice and cannot be right twice. Neither of them had bought a bun, and neither of them had a thermometer.

Mr. Akilikubwa heard the whole of it at the queue and asked the question underneath: whether a sheet of foil round a hot thing makes any difference at all, and if so how much. Which way the shiny side faces is the second question, and nobody at that stand has answered the first.

Procedure

Proceed as follows:

(a) Find the mass of each calorimeter together with its stirrer, and record the temperature of the room.

(b) Wrap the outside of calorimeter 2 closely in aluminium foil, leave calorimeter 1 bare, and let both stand until they are at room temperature.

(c) Pour 160g of water at about 86°C into each calorimeter, stand both on the wooden slab and lower a thermometer into each so that its bulb is clear of the bottom and the sides.

(d) Stirring both gently and continuously, start one stopwatch at the instant its own thermometer reads 76.0°C, and start the other the same way.

(e) Record the temperature in each calorimeter every 2 minutes for 22 minutes, each on its own watch, and record the temperature of the room again at the end.

Questions

(i) Tabulate your results, and for each calorimeter add a column of the excess temperature over the room and a column of its logarithm to base 10.

(ii) Plot $\log_{10}(\theta\text{-}\theta_r)$ against t (min) for both calorimeters on the same pair of axes.

(iii) Determine the slope of each line.

(iv) Taking $\log_{10}(\theta\text{-}\theta_r)$=A-Kt with k=2.303K, determine the cooling constant of each calorimeter.

(v) Determine the ratio of the cooling constant of the bare calorimeter to that of the wrapped one.

(vi) Determine the percentage by which the foil reduces the rate at which the calorimeter loses heat, and state how many figures of that percentage you are entitled to quote.

(vii) Give one reason why wrapping a calorimeter in foil is not the same experiment as polishing its outside.

Experiment 14.3

The vacuum flask that used to live in the physics store was broken years ago, by a boy who wanted a hammer and could not find one, and its replacement has never arrived. The store now keeps hot water for demonstrations in a beaker with a saucer on top.

Kipute proposed a repair, which was to build the flask again out of what the store does have: an old paint tin, and the bale of cotton wool that arrives every year for the first-aid box and is never opened. Pack the wool round the beaker, put the tin over the lot, and see. **Kipanga** said it would not work, because a flask is a flask and cotton wool is bedding.

Mr. Akilikubwa said the interesting word in that sentence was "work," since neither boy had said how much good the wool would have to do before the repair counted. Until somebody puts a figure on it there is nothing to be right or wrong about.

Aim

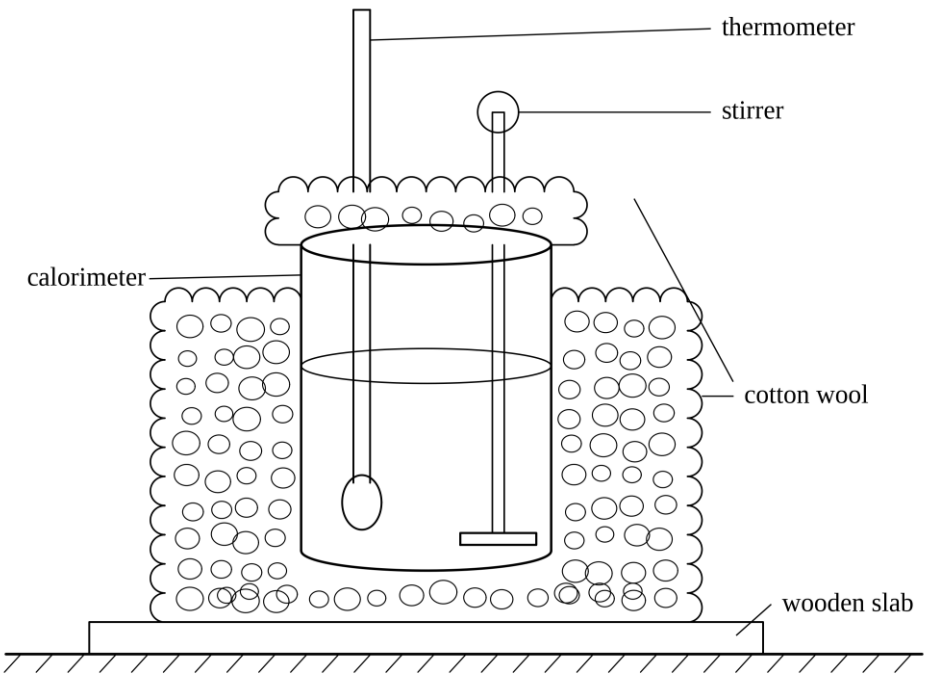
To determine the factor by which a jacket of cotton wool changes the rate of cooling of a calorimeter of hot water.

Apparatus

A copper calorimeter with its stirrer, of total mass 96.0g, a thermometer reading from 0 to 100°C in steps of 0.5°C, a stopwatch, a wooden slab, a retort stand with a clamp, a quantity of cotton wool, a beaker of hot water, a balance and a transparent ruler.

Setup

The first run is made with the calorimeter bare, standing on the wooden slab exactly as in Experiment 14.1. For the second run the same calorimeter is emptied, dried and allowed to come back to room temperature, and cotton wool is then packed round its sides, under its base and over its mouth, with the thermometer and the stirrer passing through the pad on top. Both runs use the same calorimeter, the same mass of water and the same room, and both are started from the same temperature.



Procedure

Proceed as follows:

(a) Find the mass of the calorimeter together with its stirrer, and record the temperature of the room.

(b) Pour 180g of water at about 88°C into the calorimeter, stand it on the wooden slab and lower the thermometer into it so that its bulb is clear of the bottom and the sides.

(c) Stirring gently and continuously, start the stopwatch at the instant the thermometer reads 80.0°C, and record the temperature every 5 minutes for 40 minutes.

(d) Empty and dry the calorimeter, let it return to room temperature, and pack cotton wool round its sides, under its base and over its mouth.

(e) Repeat steps (b) and (c) with the lagged calorimeter, recording the temperature of the room again before you begin.

Questions

(i) Tabulate your results.

(ii) Plot both cooling curves on the same pair of axes, with θ (°C) against t (min), and draw a smooth curve through each set of points.

(iii) Read from your graph the time each run took to fall from 75.0°C to 60.0°C, and determine the ratio of the lagged time to the bare one.

(iv) The excess temperature of each run falls according to $\theta-\theta_r=(\theta_0-\theta_r)e^{-kt}$, where k is its cooling constant. Show that the ratio in part (iii) is the ratio of the two cooling constants the other way up.

(v) Determine the cooling constant of each run and state its unit.

(vi) The specific heat capacity of water is 4.2J/gK and that of copper is 0.38J/gK. Determine the rate at which heat is leaving the calorimeter at 70.0°C in each run, and hence the heat the lagging saves in one minute at that temperature.

(vii) State one source of error that the ratio in part (iii) removes, and one that it does not.

Table of results

The temperature of the room was 25.0°C on both afternoons, and the calorimeter held 180g of water in both runs.

t (min)	θ bare (°C)	θ lagged (°C)
0	80.0	80.0
5	70.0	76.0
10	62.0	72.5
15	55.5	69.5
20	50.5	66.5
25	46.0	63.0
30	42.0	60.5
35	39.0	58.0
40	36.5	56.0

Making the measurement count

Practical tip 1: *Packing lagging so that it lags.*

Cotton wool works because of the air held still inside it, so pack it loosely and evenly and do not press it down. Wool crushed hard against the copper on one side and left thin on the other gives a jacket of two thicknesses and a constant that belongs to neither. Cover the base, which is where a calorimeter loses heat to whatever it stands on, and cover the mouth, which is where it loses heat to the air above. The wool must be dry: damp wool carries heat several times as fast and evaporates as well, which is a different experiment altogether.

Practical tip 2: *What the cooling constant is made of.*

Experiments 14.1 and 14.2 between them have taken the constant apart. The rate at which heat leaves a vessel at a stated excess temperature depends on the area A of its surface and on a quantity h that describes how readily that surface passes heat to the room. The rate at which its temperature then falls depends on that, divided by the heat capacity C of everything inside:

$$k=\frac{hA}{C}$$

Experiment 14.1 changed C and left hA alone. Experiment 14.2 changed h and left the rest alone. This one changes h again, and by a much larger factor.

Practical tip 3: *Reading a time off a curve at a stated temperature.*

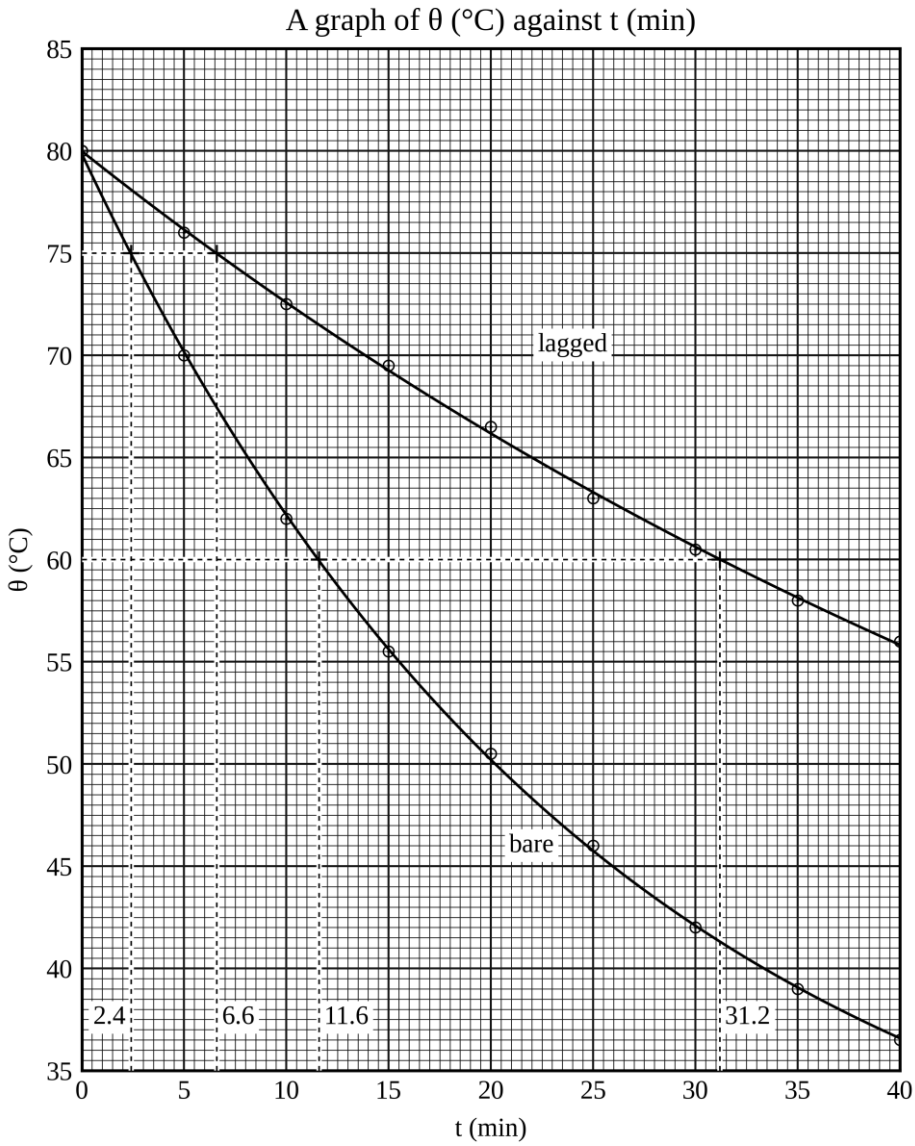
Rule a faint horizontal line across the sheet at the temperature you want, mark where it cuts each curve, and drop a vertical from each mark to the time axis. Read the two times where those verticals land. On this sheet one small square is 0.5min, so a time can be read to about 0.1min, and both times are wanted to that. Draw the construction lines and leave them on the graph: an examiner marks the construction as well as the number.

Practical tip 4: *Two runs on two afternoons.*

This is the first experiment in the chapter whose two runs cannot be made side by side, because the same calorimeter has to be used twice. Everything the two runs are supposed to share therefore has to be checked twice rather than arranged once: the mass of water, the starting temperature, the position on the bench and the temperature of the room. Record the room before each run and put both figures with your results. Part (vii) asks which of the errors left over survive the division.

Answers to the questions

(i) and (ii). Take t at 2cm to 5 minutes, running from 0 to 40 minutes, and θ at 2cm to 5°C, running from 35°C to 85°C. That is 16cm by 20cm, which fits an A4 sheet upright. Give the graph the title A graph of θ (°C) against t (min).



(iii) The bare run passes 75.0°C at 2.4min and 60.0°C at 11.6min, a fall lasting 9.2min. The lagged run passes the same two temperatures at 6.6min and 31.2min, a fall lasting 24.6min. The ratio is

$$\frac{24.6\text{min}}{9.2\text{min}}=2.67$$

The lagging makes the same fall take 2.67 times as long.

(iv) Rearrange $\theta-\theta_r=(\theta_0-\theta_r)e^{-kt}$ for the time at which the excess has fallen to a stated value:

$$kt=\log_e\frac{\theta_0-\theta_r}{\theta-\theta_r}$$

so the time taken to go from θ_1 to θ_2 is

$$t=\frac{1}{k}\log_e\frac{\theta_1-\theta_r}{\theta_2-\theta_r}$$

The logarithm on the right holds nothing but the two temperatures and the room, and both runs were made between the same two temperatures in the same room, so it has the same value in both. What is left is

$$\frac{t_{lagged}}{t_{bare}}=\frac{k_{bare}}{k_{lagged}}$$

(v) The excess falls from 50.0K to 35.0K in both runs, so for either of them

$$kt=2.303\times\log_{10}\frac{50.0K}{35.0K}=2.303\times0.1549=0.3567$$

Divide by each of the two times:

$$k_{bare}=\frac{0.3567}{9.2min}=0.0388min^{-1}$$

$$k_{lagged}=\frac{0.3567}{24.6min}=0.0145min^{-1}$$

(vi) The heat capacity of the calorimeter and its contents is

$$C=96.0g\times0.38J/gK+180g\times4.2J/gK=36.48JK^{-1}+756JK^{-1}=792.48JK^{-1}$$

At 70.0°C the excess over the room is 45.0K, and the rate at which heat leaves is rate of loss of heat$=Ck(\theta-\theta_r)$. For the bare run,

$$792.48JK^{-1}\times0.0388min^{-1}\times45.0K=1384Jmin^{-1}$$

and for the lagged run,

$$792.48JK^{-1}\times0.0145min^{-1}\times45.0K=517Jmin^{-1}$$

so the lagging saves

$$1384Jmin^{-1}-517Jmin^{-1}=867Jmin^{-1}$$

at that temperature, which is 63% of what the bare calorimeter was losing. Whether that is enough for the store to call the result a flask is now a question with a number in it. One reservation goes with the figure: the wool is cooling along with the calorimeter and is not in the $792.48JK^{-1}$, so the saving is a little over-stated.

(vii) The division removes an error in the temperature of the room, which enters both times through the same logarithm and cancels out of the quotient. It does not remove a room that stood at two different temperatures on the two afternoons, because then there are two logarithms and nothing cancels. That is what running the two halves one after the other costs.

Examiner's eye

- Lay the sheet out for the run that lasts longest and reaches lowest, then plot the other on the same axes. Split the two runs between two sheets and there is no way left to read one against the other, which is what the question asks you to do.
- Units belong on the constants and not on the ratio. Write min^{-1} after each constant in part (v) and write nothing at all after the two times divided in part (iii).
- Write the ratio to three figures and defend two of them. Each of the four times is fixed by a curve drawn through nine readings rather than by one of them, but the shorter interval is the weaker of the two, and between them the quotient is good to about 3%.
- Multiply by the excess temperature in part (vi), not by the temperature. A rate of loss is fixed by how far above the room the calorimeter is standing, and 70.0°C in a room at 25.0°C is an excess of 45.0K. Use the 70.0 and every answer in the part is out by more than half again.

Practice 14.3

The wool from Experiment 14.3 went back into the first-aid box, and the argument about it did not.

Kipanga wanted the answer given as minutes saved, because minutes are what a cook cares about. **Mr. Akilikubwa** said minutes saved is a fair answer to a cook and no answer at all to a physicist, because it depends on how hot the water started and how cold you will settle for, and asked what quantity does not.

Procedure

Proceed as follows:

(a) Find the mass of the calorimeter together with its stirrer, and record the temperature of the room.

(b) Pour 200g of water at about 92°C into the calorimeter, stand it on the wooden slab and lower the thermometer into it so that its bulb is clear of the bottom and the sides.

(c) Stirring gently and continuously, start the stopwatch at the instant the thermometer reads 84.0°C, and record the temperature every 5 minutes for 40 minutes.

(d) Empty and dry the calorimeter, let it return to room temperature, and pack cotton wool round its sides, under its base and over its mouth.

(e) Repeat steps (b) and (c) with the lagged calorimeter, recording the temperature of the room again before you begin.

Questions

(i) Tabulate your results.

(ii) Plot both cooling curves on the same pair of axes, with θ (°C) against t (min).

(iii) Read from your graph the time each run took to fall from 78.0°C to 58.0°C, and determine the ratio of the lagged time to the bare one.

(iv) Determine the cooling constant of each run, taking $\theta-\theta_r=(\theta_0-\theta_r)e^{-kt}$.

(v) The specific heat capacity of water is 4.2J/gK and that of copper is 0.38J/gK. Determine the rate of loss of heat at 68.0°C in each run, and the percentage of the loss that the lagging removes.

(vi) State, with a reason, whether your ratio in part (iii) would change if the calorimeter had held 300g of water instead, and whether it would change if a calorimeter twice the size had been used.

(vii) State one precaution that matters in this experiment and does not matter when the two runs are made side by side in two calorimeters.

Experiment 14.4

On the hottest afternoon of the term the soda seller at the market had his crates under a sack that he soaked from a bucket every half hour, and his bottles came out cold enough to sting. Two stalls along, a rival had bought a bigger sack, a thicker one, and had draped it over his crates dry, and his soda was warm by noon.

Kipanga said the second man had the better sack and the worse idea, and that the bucket was doing the work. **Kipute** asked how much work, since a wet sack is also a heavier sack and a darker one, and any of the three could be the reason.

Mr. Akilikubwa said that was the right question in the right order, and that the laboratory could put a number on it before the ice in the staff room fridge had finished forming.

Aim

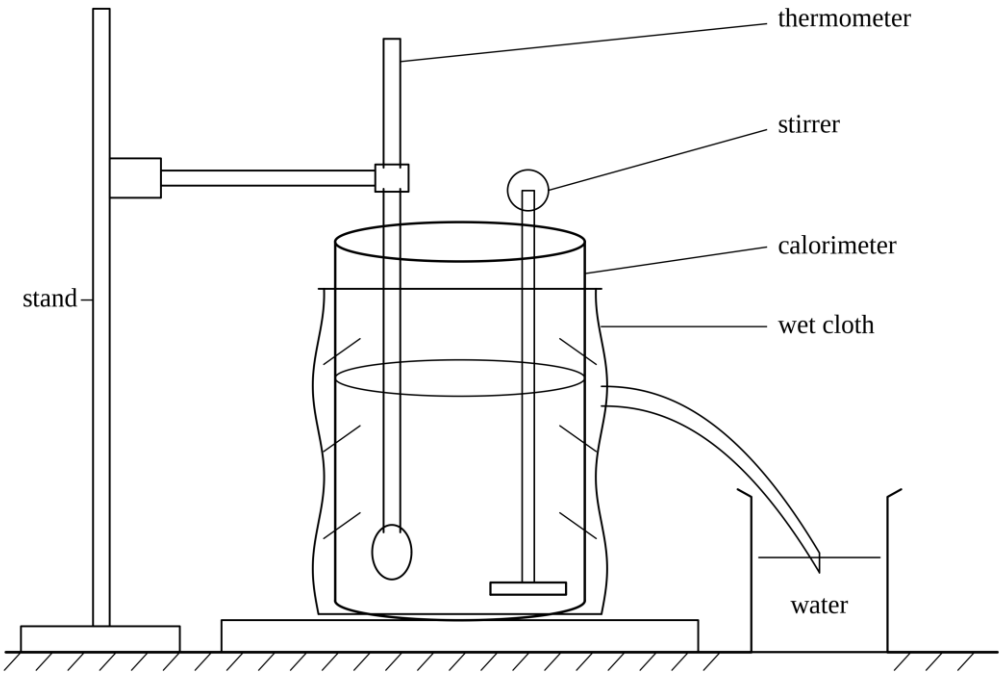
To compare the cooling of a calorimeter wrapped in a wet cloth with its cooling when dry, and to estimate the water the cloth loses as vapour.

Apparatus

A copper calorimeter with its stirrer, of total mass 96.0g, a thermometer reading from 0 to 100°C in steps of 0.5°C, a stopwatch, a wooden slab, a retort stand with a clamp, a square of thin cotton cloth, a small beaker of water, a beaker of hot water, a balance and a transparent ruler.

Setup

The first run is made with the calorimeter dry and bare on the wooden slab, exactly as in Experiment 14.1. For the second run the same calorimeter, emptied, dried and back at room temperature, is wrapped closely in a cotton cloth that has been wetted and wrung out, and a tail of that cloth is left hanging into a small beaker of water standing beside the slab. Water travels up the tail and keeps the cloth wet without anybody touching it during the run. The calorimeter itself stands on the same slab in both runs.



Procedure

Proceed as follows:

(a) Find the mass of the calorimeter together with its stirrer, and record the temperature of the room.

(b) Pour 180g of water at about 88°C into the calorimeter, stand it on the wooden slab and lower the thermometer into it so that its bulb is clear of the bottom and the sides.

(c) Stirring gently and continuously, start the stopwatch at the instant the thermometer reads 80.0°C, and record the temperature every 2 minutes for 20 minutes.

(d) Empty and dry the calorimeter, let it return to room temperature, wrap it in the wetted cloth and set the tail of the cloth in the small beaker of water.

(e) Repeat steps (b) and (c) with the wrapped calorimeter, recording the temperature of the room again before you begin.

Questions

(i) Tabulate your results.

(ii) Plot both cooling curves on the same pair of axes, with θ (°C) against t (min), and draw a smooth curve through each set of points.

(iii) Read from your graph the time each run took to fall from 75.0°C to 55.0°C, and determine the ratio of the dry time to the wet one.

(iv) Determine the cooling constant of each run, taking $\theta-\theta_r=(\theta_0-\theta_r)e^{-kt}$.

(v) The specific heat capacity of water is 4.2J/gK and that of copper is 0.38J/gK. Determine the extra rate at which the wrapped calorimeter is losing heat when its temperature is 60.0°C.

(vi) The specific latent heat of vapourisation of water is 2260J/g. Determine the mass of water leaving the cloth as vapour each minute at that temperature.

(vii) Your answer to (vi) treats every joule of the extra loss as a joule spent on evaporation. Give one reason why that makes the answer too large.

Table of results

The temperature of the room was 25.0°C on both afternoons, and the calorimeter held 180g of water in both runs.

t (min)	θ dry (°C)	θ wet cloth (°C)
0	80.0	80.0
2	76.0	73.0
4	72.5	67.0
6	69.5	62.0
8	66.0	57.0
10	63.5	53.0
12	60.0	49.0
14	57.5	46.0
16	55.5	44.0
18	53.5	41.0
20	51.5	39.5

Making the measurement count

Practical tip 1: *Wetting the cloth, and keeping it wet.*

Wet the cloth and wring it out before you wrap it, so that it is damp all over and dripping nowhere. Water running down the outside of the calorimeter and off the slab carries heat away in a way nothing in your working allows for. Leave the tail in the beaker and do not touch the cloth again during the run: a hand on it at minute 9 puts a step in the curve, and a cloth that has dried out by minute 15 puts a bend in it.

Practical tip 2: *What the cloth adds that the arithmetic does not own.*

The heat capacity you will work out in part (v) is the copper and the water inside the calorimeter, and nothing else. The cloth and the water it holds are cooling too, and they are not in that sum. Keep the cloth thin and keep it damp rather than sodden, and what they contribute stays small beside the heat capacity you are about to work out.

Practical tip 3: *A difference of two constants.*

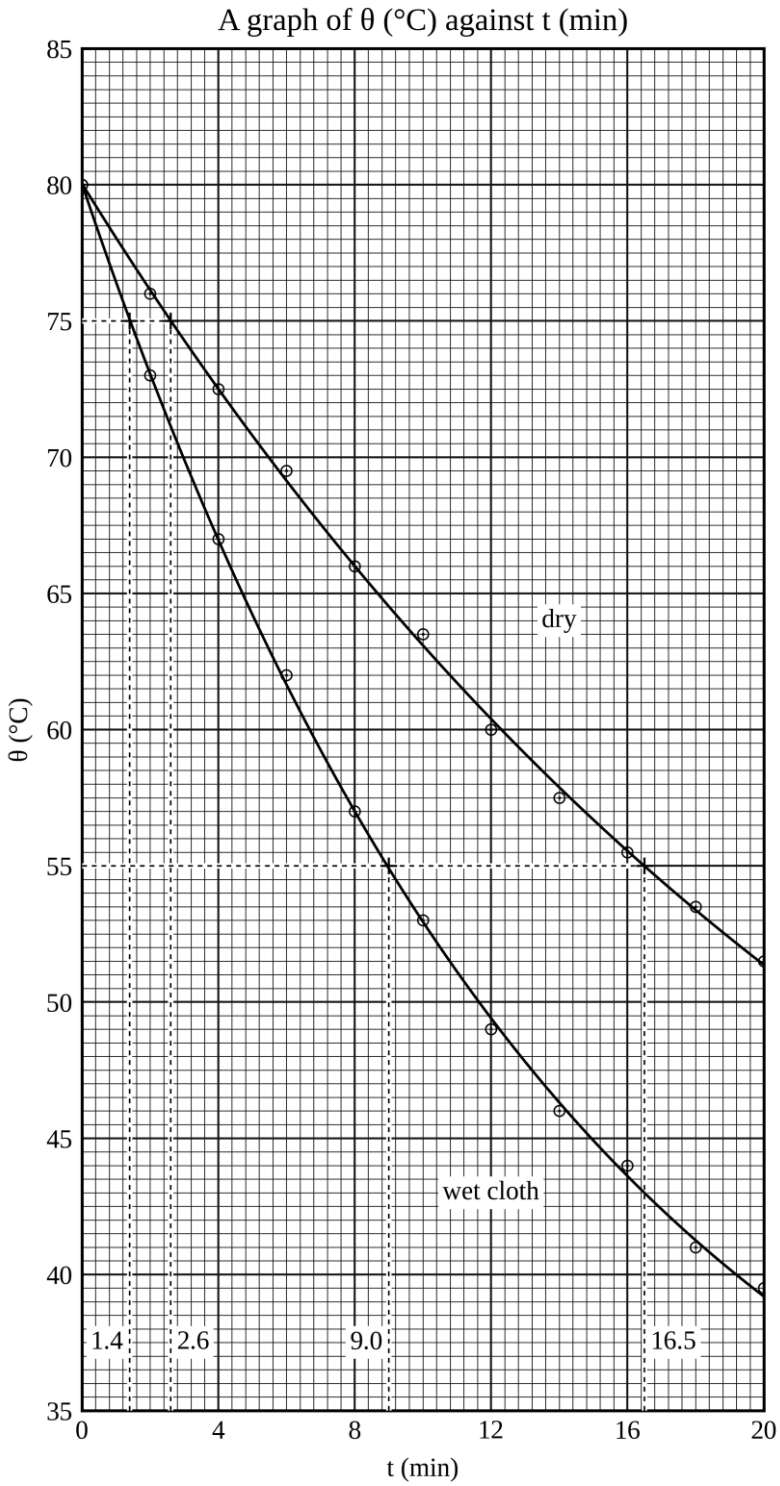
Part (v) wants the extra loss, which is what the wet run loses beyond what the dry run loses at the same temperature. That is a difference, and a difference always carries a larger percentage than either of the numbers that made it. How much larger depends on how far apart they are: two constants that sit close together leave a difference with almost nothing in it, and two that are far apart leave one that keeps most of its precision. Work out which case you are in before you decide how many figures to write down.

Practical tip 4: *From joules a minute to grams a minute.*

The specific latent heat of vapourisation is a number of joules for every gram turned to vapour, so it is the quantity that turns a rate of loss of heat into a rate of loss of mass. Write the unit down at each step and let the unit tell you which way round to divide. That matters more here than it usually does, because the wrong way round is only a few times the right one and nothing about the size of it will warn you.

Answers to the questions

(i) and (ii). Take t at 2cm to 4 minutes, running from 0 to 20 minutes, and θ at 2cm to 5°C, running from 35°C to 85°C. That is 10cm by 20cm, which fits an A4 sheet upright. Give the graph the title A graph of θ (°C) against t (min).



(iii) The dry run passes 75.0°C at 2.6min and 55.0°C at 16.5min, a fall lasting 13.9min. The wrapped run passes them at 1.4min and 9.0min, a fall lasting 7.6min. The ratio is

$$\frac{13.9\text{min}}{7.6\text{min}} = 1.83$$

The dry run takes 1.83 times as long over the same fall as the wrapped one.

(iv) The excess falls from 50.0K to 30.0K in both runs, so

$$kt=2.303\times\log_{10}\frac{50.0K}{30.0K}=2.303\times0.2218=0.5108$$

$$k_{dry}=\frac{0.5108}{13.9min}=0.0367min^{-1}$$

$$k_{wet}=\frac{0.5108}{7.6min}=0.0672min^{-1}$$

(v) The heat capacity of the calorimeter and its contents is

$$C=96.0g\times0.38J/gK+180g\times4.2J/gK=36.48JK^{-1}+756JK^{-1}=792.48JK^{-1}$$

The two constants differ by

$$0.0672min^{-1}-0.0367min^{-1}=0.0305min^{-1}$$

and at 60.0°C the excess over the room is 35.0K, so the extra rate of loss is

$$792.48JK^{-1}\times0.0305min^{-1}\times35.0K=846Jmin^{-1}$$

(vi) Every gram of water that leaves the cloth as vapour carries 2260J away with it, so

$$\frac{846Jmin^{-1}}{2260J/g}=0.37g/min$$

About 0.37g of water is leaving the cloth every minute while the calorimeter stands at 60.0°C.

(vii) Not all of the extra loss is evaporation. A wet cloth conducts heat outward far better than the dry air it replaced, so part of the difference between the two runs is ordinary conduction through a wetted layer and would be there even if nothing evaporated. Charge that part to the latent heat as well and the mass comes out too large.

Examiner's eye

- One number is asked for in part (v), and $C(k_2-k_1)(\theta-\theta_r)$ reaches it in one line. Two full rates worked out separately and then subtracted reach the same place with twice as many chances of an arithmetical slip.
- Watch which run is which in the ratio. The wet cloth makes the calorimeter cool faster, so its time is the shorter and its constant the larger. A ratio quoted upside down turns an answer of 1.83 into 0.55 and the sentence after it into nonsense.
- Where the latent heat came from is worth a sentence: the question supplied it precisely because the experiment cannot measure it, so do not present it as something your graph produced.
- Sources of error in general are not what part (vii) is after. It names one assumption, gives you the direction the answer moves, and asks for the reason; so give one reason and give it in full. A list of careless mistakes answers a question from a different paper.

Practice 14.4

A letter came for **Mr. Akilikubwa** from a student who left three years ago and now works at a dairy on the Iringa road. He read it to the class, all but the first paragraph, which was about somebody's wedding.

> Dear Sir,
>
> The milk comes in at nine and the cooling plant is down again. We stand the cans in the yard and put wet sacking over them, and the man before me swore by it, and I can see for myself that it works. What I cannot tell anybody is how much it is worth. The manager wants to know whether to buy a second cooling plant or a lorry-load of sacks.
>
> Yours faithfully,
>
> Your old student of Form Six
>
> P.S. It is the same sacking we used for the maize, and nobody at the dairy will tell me why it works better wet.

Kipute said the postscript was where the physics had got to. **Mr. Akilikubwa** said the manager had asked for a figure and been sent a conviction, and that a figure is what the class had better send back.

Procedure

Proceed as follows:

(a) Find the mass of the calorimeter together with its stirrer, and record the temperature of the room.

(b) Pour 170g of water at about 90°C into the calorimeter, stand it on the wooden slab and lower the thermometer into it so that its bulb is clear of the bottom and the sides.

(c) Stirring gently and continuously, start the stopwatch at the instant the thermometer reads 82.0°C, and record the temperature every 2 minutes for 20 minutes.

(d) Empty and dry the calorimeter, let it return to room temperature, wrap it in a wetted cloth and set the tail of the cloth in a small beaker of water.

(e) Repeat steps (b) and (c) with the wrapped calorimeter, recording the temperature of the room again before you begin.

Questions

(i) Tabulate your results.

(ii) Plot both cooling curves on the same pair of axes, with θ (°C) against t (min).

(iii) Read from your graph the time each run took to fall from 77.0°C to 57.0°C, and determine the ratio of the dry time to the wet one.

(iv) Determine the cooling constant of each run.

(v) The specific heat capacity of water is 4.2J/gK and that of copper is 0.38J/gK. Determine the extra rate at which the wrapped calorimeter loses heat at 62.0°C.

(vi) Taking the specific latent heat of vapourisation of water as 2260J/g, determine the mass of water evaporating each minute at that temperature.

(vii) The dairy in the letter cools cans holding 40kg of milk. State one reason why the answer to (vi) cannot simply be scaled up to tell the manager how much water the sacking will use.

Experiment 14.5

The mechanic at the garage by the bus stand drains the engine oil hot into an open tin and will not go near that tin for an hour. He drains the radiator into a bucket at the same moment and dips a finger in it long before that, and he says the oil holds the heat.

Kipanga agreed with him and offered a reason: oil is thicker, so the heat cannot get out of it. **Kipute** asked what "holds the heat" was supposed to mean, since the two vessels were a tin and a bucket, of different sizes, standing in different corners of the yard.

Mr. Akilikubwa said the yard had four differences in it and one question, which is three differences too many, and asked what would be left if the tin and the bucket were the same vessel to the gram.

Aim

To determine the specific heat capacity of a liquid by comparing the time it and an equal mass of water take to cool side by side through the same temperatures.

Apparatus

Two identical copper calorimeters with their stirrers, each of total mass 96.0g, two thermometers reading from 0 to 100°C in steps of 0.5°C, two stopwatches, a wooden slab, a retort stand with two clamps, a supply of liquid A, a water bath, a balance and a transparent ruler.

Setup

The arrangement is the one drawn in Experiment 14.1. Calorimeter 1 holds 150g of liquid A and calorimeter 2 holds 150g of water, and the two are filled to nearly the same depth. Both are heated in a water bath rather than over a flame, both stand on the same slab, and both are stirred throughout. Neither is covered. The temperature of the room is taken before either liquid is poured.

Procedure

Proceed as follows:

(a) Find the mass of each calorimeter together with its stirrer, and record the temperature of the room.

(b) Warm liquid A and water separately in the water bath to about 88°C, and pour 150g of liquid A into calorimeter 1 and 150g of water into calorimeter 2.

(c) Stand both calorimeters on the wooden slab and lower a thermometer into each so that its bulb is clear of the bottom and the sides.

(d) Stirring both gently and continuously, start one stopwatch at the instant its own thermometer reads 80.0°C, and start the other the same way.

(e) Record the time t_1 on the first watch and t_2 on the second at which each thermometer reaches each of the temperatures in the table.

Questions

(i) Tabulate your results.

(ii) Plot a graph of t_1 (min) against t_2 (min) and draw the best straight line through your points.

(iii) Determine the slope of your line.

(iv) The specific heat capacity of water is 4.2J/gK and that of copper is 0.38J/gK. Determine the heat capacity of calorimeter 2 and its contents.

(v) Hence determine the specific heat capacity of liquid A.

(vi) State the assumption about the two calorimeters on which this method rests, and one precaution you would take to make it true.

(vii) Give one reason why the method would fail if liquid A were volatile.

Table of results

The temperature of the room was 25.0°C throughout, calorimeter 1 held 150g of liquid A and calorimeter 2 held 150g of water.

θ (°C)	t_1 (min)	t_2 (min)
75.0	1.6	2.5
70.0	3.0	5.5
65.0	5.3	8.3
60.0	7.3	11.8
55.0	9.5	16.2
50.0	12.3	20.9

Making the measurement count

Practical tip 1: *Equal masses, and nearly equal depths.*

The balance decides how much of each liquid goes in, because the mass is what the arithmetic uses. But a liquid whose density differs from water's will stand at a different depth for the same mass, and a different depth wets a different area of copper. Choose a liquid close enough in density that the two levels are within a few millimetres of each other, and if they are not, match the volumes instead and put both calorimeters on the balance to find out what you poured.

Practical tip 2: *Warming a liquid that is not water.*

Stand the vessel of liquid A in a bath of hot water and let it come up slowly, and keep every flame out of the room. A liquid heated directly over a burner reaches temperatures at the bottom of the vessel far above the reading on the thermometer, and many liquids that are perfectly safe at 80°C are not safe at 200°C. The water bath also gives you both liquids at the same starting temperature without any watching.

Practical tip 3: *What the answer is worth.*

Two things fix what the answer is good to. The slope is read off a triangle whose two legs can each be measured to a fifth of a small square, and on these scales that comes to about 2.5% once the two percentages are added. The heat capacity of the water side is better than that, because masses on a balance are the surest readings in the experiment. Then part (v) takes the copper out by subtraction, which costs a little more, and the specific heat

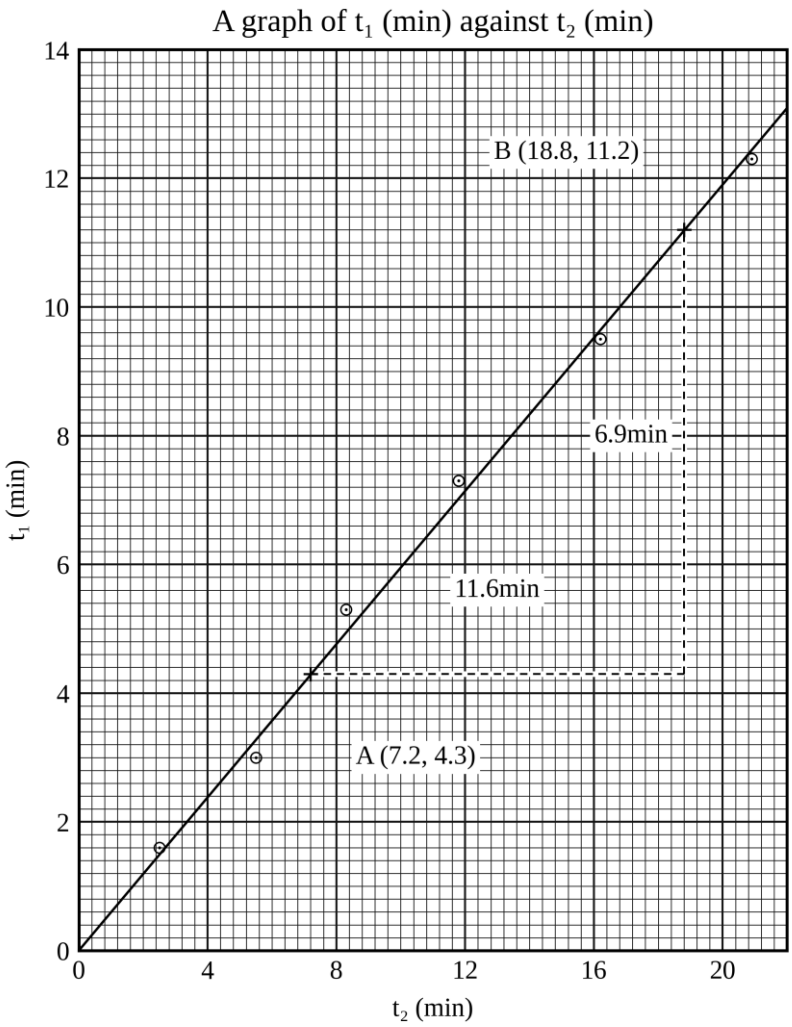
capacity ends up good to about 3%, which on an answer of this size is a few hundredths of a J/gK: write three figures and do not defend the third.

Practical tip 4: *Test the pair before you trust them.*

Put water in both calorimeters, run them side by side and plot the same graph, before any liquid A goes near either of them. That line should come out at a slope of 1.00 through the origin, and if it does not, everything you do afterwards carries the difference. Do the same for the two thermometers by standing them in one beaker and reading them together; two thermometers that disagree by 0.5°C are timing the two runs to two different temperatures.

Answers to the questions

(i) and (ii). Take t_2 at 2cm to 4 minutes, running from 0 to 22 minutes, and t_1 at 2cm to 2 minutes, running from 0 to 14 minutes. That is 11cm by 14cm, which fits an A4 sheet upright. Give the graph the title A graph of t_1 (min) against t_2 (min).



(iii) Taking the marked corners A (7.2, 4.3) and B (18.8, 11.2) on the line,

$$\text{slope}=\frac{11.2\text{min}-4.3\text{min}}{18.8\text{min}-7.2\text{min}}=\frac{6.9\text{min}}{11.6\text{min}}=0.595$$

The slope is 0.595, and it has no unit.

(iv) Calorimeter 2 holds water:

$$C_2=96.0\text{g}\times0.38\text{J/gK}+150\text{g}\times4.2\text{J/gK}=36.48\text{JK}^{-1}+630\text{JK}^{-1}=666.48\text{JK}^{-1}$$

(v) The two calorimeters are identical and stand in the same room, so the slope is the ratio of the heat capacities of the two systems, C_1/C_2, exactly as it was in Experiment 14.1. Then

$$C_1 = 0.595 \times 666.48\text{JK}^{-1} = 396.56\text{JK}^{-1}$$

The copper in calorimeter 1 is the same 36.48JK^{-1} as in calorimeter 2, so what is left belongs to the 150g of liquid A:

$$c_A = \frac{396.56\text{JK}^{-1} - 36.48\text{JK}^{-1}}{150\text{g}} = \frac{360.08\text{JK}^{-1}}{150\text{g}} = 2.40\text{J/gK}$$

The specific heat capacity of liquid A is 2.40J/gK, a little over half that of water. So much for holding the heat: for a given mass and a given fall, this liquid has less heat to give up than water has, not more. Why the mechanic's tin stayed hot is a separate question, and nothing in this experiment answers it, because the tin and the bucket differed in four things at once.

(vi) The method assumes that the two calorimeters lose heat at the same rate when they stand at the same temperature, which needs them to have the same area, the same outside finish and the same surroundings. Take them from the same set so that they are of one pattern, and clean and dry both outsides before the run; a smear of grease on one of them is a difference the experiment cannot tell from the liquid.

(vii) A volatile liquid takes heat away as vapour as well as through the walls, so calorimeter 1 would be losing heat by a route calorimeter 2 has very much less of, and the two would no longer be losing at the same rate at the same temperature. The slope would then be a mixture of the heat capacity and the evaporation, and the mass in the calorimeter would be falling as the run went on, so there would be no single figure to divide by at the end. Neither vessel is covered, so water evaporates a little from its own surface too and the match is never perfect; what the method cannot survive is one liquid doing it much faster than the other.

Examiner's eye

- Which calorimeter is which is settled by the axes and not by the apparatus. The liquid with the smaller heat capacity reaches every temperature first, so its times are the shorter ones, and a slope below 1.00 means the liquid on the vertical axis is the one that cools faster.
- Subtract the copper at the end, once, and then divide by the mass of liquid. Leave the copper in and the specific heat capacity comes out at 2.64J/gK, which is 10% high and looks perfectly reasonable on the page.
- Answer in the units the question gave you, J/gK. A specific heat capacity converted to J/kgK is not wrong, but it invites a factor of 1000 that nothing in the question needed.
- Part (vii) is about a route, not about a risk. The danger of a volatile liquid in a warm laboratory is real and it is not what is being asked; the question wants to know what evaporation does to the comparison the method depends on.

Practice 14.5

In the cookery room two identical pans went on two identical burners at the same minute, one holding water and the other the same mass of cooking oil, with a thermometer standing in each. The oil passed $90°\text{C}$ while the water was barely past $60°\text{C}$.

Kipute said the burner under the oil must be turned up higher, and went to check, and it was not. **Kipanga** said the pans were the same, the burners were the same and the masses were the same, so whatever was left had to be something about the oil itself, and he could not name it.

Name it and you have answered this question.

Procedure

Proceed as follows:

(a) Find the mass of each calorimeter together with its stirrer, and record the temperature of the room.

(b) Warm liquid B and water separately in a water bath to about $90°\text{C}$, and pour 160g of liquid B into calorimeter 1 and 160g of water into calorimeter 2.

(c) Stand both calorimeters on the wooden slab and lower a thermometer into each so that its bulb is clear of the bottom and the sides.

(d) Stirring both gently and continuously, start one stopwatch at the instant its own thermometer reads 82.0°C, and start the other the same way.

(e) Record the time t_1 on the first watch and t_2 on the second at which each thermometer reaches each of the temperatures in the table.

Questions

(i) Tabulate your results.

(ii) Plot a graph of t_1 (min) against t_2 (min) and draw the best straight line through your points.

(iii) Determine the slope of your line.

(iv) The specific heat capacity of water is 4.2J/gK and that of copper is 0.38J/gK. Determine the heat capacity of calorimeter 2 and its contents.

(v) Hence determine the specific heat capacity of liquid B.

(vi) Calorimeter 1 had been used the term before for a demonstration and its outside was left dull, while calorimeter 2 was bright. State which way this moves your answer to (v).

(vii) Liquid B is expensive. Suggest one change that would let you use less of it, and state what that change would cost you.

Practice 14.6

The health post keeps its vaccines in a cold box with a set of frozen packs, and the nurse fills whatever room is left with crumpled newspaper before she shuts the lid. On the days when there is little to carry the box comes back warm by afternoon, and on the days when it is packed to the lid it is still cold at five.

Kipute watched this happen twice and concluded that the newspaper was doing the cooling. **Mr. Akilikubwa** said the frozen packs would like a word, and that the question to ask is not what keeps the box cold but why a full box holds out longer than an empty one. A box warming up and a calorimeter cooling down are the same question turned over.

Procedure

Proceed as follows:

(a) Find the mass of each calorimeter together with its stirrer, and record the temperature of the room.

(b) Pour hot water at about 88°C into calorimeter 1 without measuring it, and 150g of water from the same beaker into calorimeter 2.

(c) Stand both calorimeters on the wooden slab and lower a thermometer into each so that its bulb is clear of the bottom and the sides.

(d) Stirring both gently and continuously, start one stopwatch at the instant its own thermometer reads 80.0°C, and start the other the same way.

(e) Record the temperature in each calorimeter every 3 minutes for 24 minutes, each on its own watch.

Questions

(i) Tabulate your results.

(ii) Plot both cooling curves on the same pair of axes, with θ (°C) against t (min).

(iii) Read from your graph the time each calorimeter took to fall from 75.0°C to 60.0°C, and determine the ratio of the longer time to the shorter.

(iv) Show that this ratio is the ratio of the heat capacities of the two systems.

(v) The specific heat capacity of water is 4.2J/gK and that of copper is 0.38J/gK. Determine the mass of water in calorimeter 1.

(vi) Experiment 14.1 found the same quantity from the slope of a straight line. State which of the two methods gives the better answer, with a reason.

(vii) State one advantage of reading the two times off drawn curves rather than timing the two falls with a watch as they happen.

Chapter 15

SPECIFIC HEAT CAPACITY

The two chapters before this one asked how fast a hot body loses its heat. This one asks a different question about the same calorimeter on the same slab: how much heat is in it. Every substance has its own answer, and the number that carries it is the specific heat capacity, the heat one gram of the substance takes in for every kelvin it rises.

Four methods are worked here. In 15.1 and 15.4 heat leaves one body and enters another in front of you, with no third party allowed, and one unknown is left in the equation when everything else has been weighed. In 15.2 the heat lost to the room, which the other two spend their time fighting, becomes the measurement itself; in 15.3 the same vessel loses heat by the same law in both runs, so whatever that law's constant is it divides out and never has to be measured at all. One of the four needs no graph, which makes it the quickest to work and the one with the fewest places to hide a mistake.

There is one difficulty they share, and it should be named before you meet it. A calorimeter warmer than the room is losing heat to the room while you are working, and one cooler than the room is gaining it. No apparatus stops that. Three things are done about it here: measure the rate of loss beforehand and let it do the measuring for you, follow the loss while it happens and put it back, or arrange two runs so alike that what governs the loss is common to both and drops out between them. Experiment 15.1 does none of the three and pays for it in the last part of its question; the other three take one each, in the order 15.2, 15.4, 15.3.

The four varieties at a glance

	What you do	What comes out of it
15.1	A hot solid of known mass is dropped into cold water in a calorimeter, and the highest temperature is read	the specific heat capacity of the solid, from a heat balance and no graph at all
15.2	One liquid is left to cool and a tangent is drawn to its cooling curve at a stated temperature, with the rate of loss of heat at that temperature supplied	the specific heat capacity of the liquid
15.3	The same calorimeter is cooled twice, filled to the same mark with water alone and with water and a solid in it	the specific heat capacity of the solid, from the greatest separation of the two curves
15.4	A hot solid is dropped in as in 15.1, but the temperature is followed before and after the transfer	the specific heat capacity of the solid, with the cooling of the mixture allowed for

All four are worked below in full. The practice questions change the vessel and the substance, and two of them change the unknown as well.

Experiment 15.1

There is a welding shop just outside the school gate, and the man who runs it keeps a quenching trough of water beside his bench. Every bar he heats goes into that trough when he has finished with it. By the middle of the afternoon the water is warm enough to wash in, and in 11 years he has never once thought of the trough as an instrument.

Kipute noticed the steam and asked how many bars it took. **Kipanga** had the answer ready: the water must finish as hot as the bar was, because all the heat in the bar had gone into it and heat does not vanish. The welder, without looking up from his work, invited him to put one hand in the trough and the other on the bar he had just taken out of it.

That settled the afternoon but not the question, and the question is one the workshop cannot answer by hand. The same bar dropped into a bucket instead of a trough would leave the water hotter, so the rise belongs to the water as much as to the bar. What belongs to the bar alone is a number you can get on a bench with a balance, a thermometer and one careful transfer.

Aim

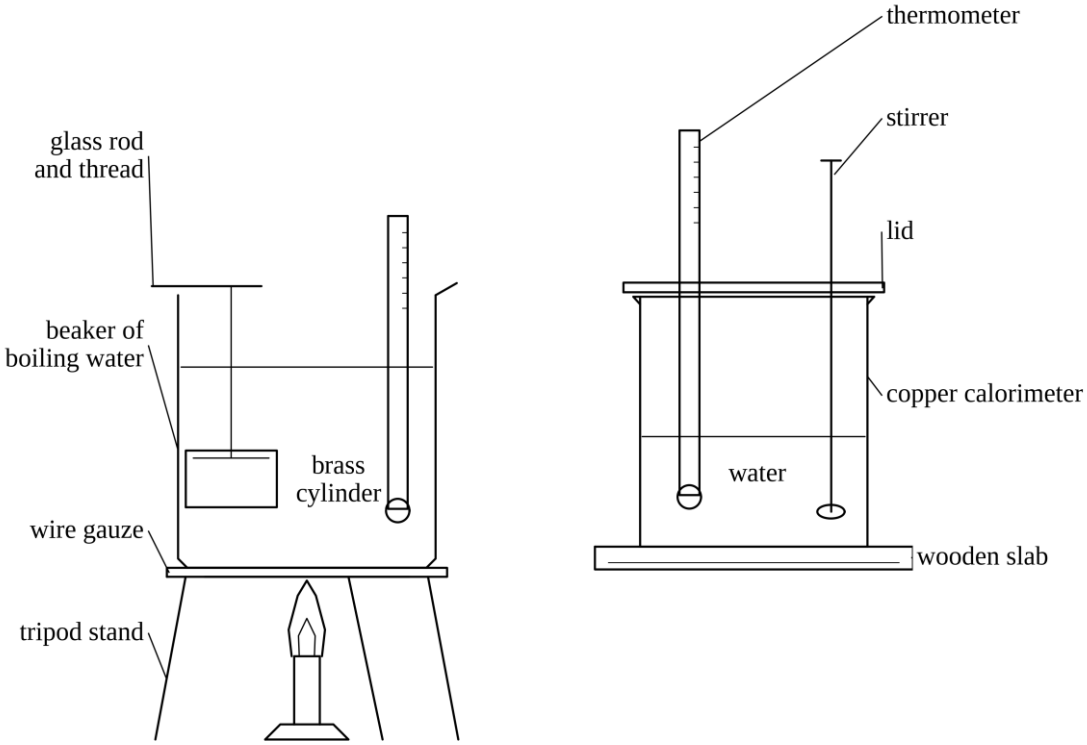
To determine the specific heat capacity of brass by the method of mixtures.

Apparatus

A copper calorimeter with its stirrer and a lid pierced for the thermometer and the stirrer, of total mass 94.0g with the stirrer; a brass cylinder of mass 248.0g with a thread tied round it; two thermometers reading from 0 to 100°C in steps of 0.5°C, one for the bath and one for the calorimeter; a beaker of water on a wire gauze and a tripod over a burner; a glass rod; a balance reading to 0.1g; a clock; a wooden slab; cold water.

Setup

The calorimeter stands on the wooden slab, away from any window or doorway, with its lid on and the thermometer and the stirrer through the two holes in the lid. The brass cylinder hangs from a thread tied to a glass rod laid across the mouth of the beaker, so that it is well under the surface and clear of the bottom. The thermometer in the beaker is what tells you the temperature of the cylinder, and the calorimeter is drawn about half full because that is how it is used.



Procedure

Proceed as follows:

(a) Find the mass of the calorimeter together with its stirrer, and record the temperature of the room.

(b) Half fill the calorimeter with cold water, find the mass of the calorimeter, the stirrer and the water together, and record it.

(c) Tie the thread to the brass cylinder, hang it in the beaker from the glass rod so that it is well under the surface and clear of the bottom, and bring the water to the boil.

(d) Keep the water boiling for 5 minutes, then read and record the temperature of the boiling water.

(e) Stand the calorimeter on the wooden slab, put the lid on with the thermometer and the stirrer through it, stir gently and record the temperature of the water in it.

(f) Lift the cylinder out by its thread, shake the drops from it, lower it into the calorimeter without splashing and replace the lid at once.

(g) Stirring gently and continuously, watch the thermometer and record the highest temperature it reaches.

Questions

(i) Record your readings in a table, including the mass m_w of the water in the calorimeter.

(ii) Write down an expression for the heat taken in by the calorimeter and the water as their temperature rises from θ_1 to θ_f, in terms of the mass m_c of the calorimeter and its stirrer, the specific heat capacity c_c of copper, the mass m_w of the water, the specific heat capacity c_w of water, and the two temperatures.

(iii) Write down an expression for the heat given up by the brass cylinder as it falls from θ_2 to θ_f, in terms of its mass m_s, its specific heat capacity c and the two temperatures.

(iv) The specific heat capacity of water is 4.2J/gK and that of copper is 0.38J/gK. Use these, with your readings, to determine the specific heat capacity c of brass.

(v) State the assumption you make in putting your answers to (ii) and (iii) equal to one another.

(vi) The accepted value for brass is 0.380J/gK. By what percentage does your value differ from it, and in which direction?

(vii) The difference in (vi) is larger than the reading errors of this experiment can account for. Suggest two places where heat has gone astray, and say for each of them which way it moves your answer.

(viii) Name one change to the procedure that would make the difference smaller, and explain how it acts.

Table of results

The temperature of the room was 26.5°C throughout.

Reading	Value
Mass of the calorimeter and its stirrer, m_c	94.0g
Mass of the calorimeter, the stirrer and the water	176.4g
Mass of the water, m_w	82.4g
Mass of the brass cylinder, m_s	248.0g
Temperature of the water in the calorimeter, θ_1	26.0°C
Temperature of the boiling water, θ_2	98.0°C
Highest temperature of the mixture, θ_f	39.7°C

Making the measurement count

Practical tip 1: *One gram and one kelvin.*

A specific heat capacity belongs to one gram of a substance. A heat capacity belongs to the whole of the thing in front of you, and carries the unit JK^{-1} rather than J/gK. A calorimeter with water in it is two substances and therefore has two contributions to its heat capacity, and part (ii) asks you to build the second quantity out of the first. Run the two units together and the answer is wrong by a factor of a mass.

Practical tip 2: *A flame does not set a temperature; boiling water does.*

Boiling water is at one temperature all through it, whatever the flame is doing underneath, and that is the whole reason for heating the cylinder in water rather than over the burner. Give it 5 minutes in the boiling water and the brass is at the temperature the thermometer in the beaker reads, to better than the thermometer can show. Keep the cylinder clear of the bottom of the beaker as well: the glass there is hotter than the water, and a cylinder resting on it is not at the temperature you recorded.

Practical tip 3: *Shake the drops off, and go quickly.*

The cylinder comes out of the beaker wet, and the film of water on it is at 98°C with a great deal of heat to give up. Some of that film goes in with the cylinder and some evaporates on the way, and a film too thin to see carries off more heat as it goes than its size suggests, because turning water to vapour is expensive. Shake the drops off, and let the transfer take seconds rather than half a minute.

Practical tip 4: *Read the highest temperature, and read it to a tenth.*

The temperature climbs for about a minute after the cylinder goes in, stands still for a few seconds and then begins to fall. The reading you want is the one at the top of that climb, so watch the thermometer from the moment the lid goes back on rather than looking at it once when the stirring is finished. Read it to the nearest 0.1°C by estimating fifths of the 0.5°C divisions, as you would read fifths of a small square on graph paper.

Chapters 13 and 14 read this thermometer to the half degree, which is enough when the answer is a time. Here the whole answer sits in a rise of about 14K, and half a degree lost off the top of it costs the answer 4.5%.

Answers to the questions

(i) The readings are those in the table above. The mass of water is the difference of the two weighings,

$$m_w = 176.4g - 94.0g = 82.4g$$

(ii) The calorimeter and the water rise through the same temperature interval, so

$$Q_{gained} = (m_c c_c + m_w c_w)(\theta_f - \theta_1)$$

(iii) The brass falls from the temperature of the bath to the temperature the mixture settles at, so

$$Q_{lost} = m_s c(\theta_2 - \theta_f)$$

(iv) Putting the two equal,

$$m_s c(\theta_2 - \theta_f) = (m_c c_c + m_w c_w)(\theta_f - \theta_1)$$

The heat capacity of the calorimeter and its water is

$$C = 94.0g \times 0.38J/gK + 82.4g \times 4.2J/gK = 35.72JK^{-1} + 346.08JK^{-1} = 381.80JK^{-1}$$

The rise is 39.7°C − 26.0°C = 13.7K and the fall of the brass is 98.0°C − 39.7°C = 58.3K, so

$$c = \frac{381.80JK^{-1} \times 13.7K}{248.0g \times 58.3K} = \frac{5230.7J}{14458.4gK} = 0.362J/gK$$

(v) That the heat given up by the brass is the heat taken in by the calorimeter and the water, and nothing else takes any: the room, the lid, the thermometer bulb and the air above the water all end the experiment as they began it, and the highest reading is the temperature the mixture settles at.

(vi) Your value is below the accepted one by

$$\frac{0.380 - 0.362}{0.380} \times 100 = 4.7\%$$

(vii) Heat has gone astray in two places, and both of them make the answer low. The first is the transfer: the cylinder is out of the bath for a second or two, wet, and what it loses on the way did not go into the water, so it entered a little below 98.0°C and the fall you divide by is too big. The second is the minute in which the mixture climbs to its highest reading, at up to 13K above the room and losing heat to it for all but the first seconds, so that reading falls short of the temperature the mixture would have reached and the rise you multiply by is too small. Both faults pull the same way, and 4.7% between them is what a bare calorimeter and a wet cylinder are worth.

(viii) Lag the calorimeter, by standing it in a larger vessel on three corks with cotton wool packed between the two. That does nothing about the transfer, but it cuts the second loss by most of itself, because what a body loses to the room depends on how easily its surface can hand the heat over, and cotton wool is very bad at that.

Examiner's eye

- Weigh the calorimeter before you put anything in it. The mass of water is a difference of two weighings and both have to be on the page; an answer that begins with 82.4g and no arithmetic to reach it has thrown away the easiest mark in the question.
- C dies when you pour the water away and c does not. A cylinder of brass twice this one has the same c and twice the heat to give, which is why the mass sits in the denominator and not in the answer.
- Every temperature in (iv) enters as part of a difference, so a thermometer half a degree high everywhere does no harm at all; one that is half a degree out at 98°C but true at 40°C is worth 0.9%, because it reaches the answer through the 58.3K fall and nowhere else.
- An error you cannot sign is not worth writing down. Two real sources named without a direction count for less than one named with it, and part (vii) is marked on the directions.

Practice 15.1

There is a card taped to the shelf in the preparation room, above the row of stoppered bottles, printed in the technician's square capitals:

COOKING OIL

DO NOT USE FOR WATER BATHS

IT HOLDS LESS HEAT THAN WATER

Kipute read it twice and asked which of the two lower lines was the reason for the other. **Kipanga** put it to the technician, who said the card had been there when he came and that he had assumed a physics teacher wrote it. **Mr. Akilikubwa** said that anybody with a pen can give a card an opinion, and that a bottle can only be weighed. Then he took the bottle down and stood it beside the balance.

Procedure

Proceed as follows:

(a) Find the mass of the calorimeter together with its stirrer, and record the temperature of the room.

(b) Pour cooking oil into the calorimeter until it is about half full, find the mass of the calorimeter, the stirrer and the oil together, and record it.

(c) Tie the thread to the brass cylinder, hang it in the beaker from the glass rod so that it is well under the surface and clear of the bottom, and bring the water to the boil.

(d) Keep the water boiling for 5 minutes, then read and record the temperature of the boiling water.

(e) Stand the calorimeter on the wooden slab, put the lid on with the thermometer and the stirrer through it, stir gently and record the temperature of the oil in it.

(f) Lift the cylinder out by its thread, shake the drops from it, lower it into the calorimeter without splashing and replace the lid at once.

(g) Stirring gently and continuously, watch the thermometer and record the highest temperature it reaches.

Questions

(i) Record your readings in a table, including the mass of the oil in the calorimeter.

(ii) The specific heat capacity of brass is $0.380\,\mathrm{J/gK}$ and that of copper is $0.38\,\mathrm{J/gK}$. Determine the heat given up by the brass cylinder.

(iii) Determine the heat taken in by the calorimeter alone.

(iv) Hence determine the specific heat capacity of the cooking oil.

(v) Published values for vegetable oils are close to $2.0\,\mathrm{J/gK}$. By what percentage does your value differ from that, and in which direction?

(vi) In Experiment 15.1 the same neglected losses made the answer come out low. State which way they move the answer here, and explain why.

(vii) The card on the shelf says that oil holds less heat than water. State what your result says about that claim, and state what it does not say.

(viii) Cooking oil is far more viscous than water. Say what the stirring is for, and what your value would do if the stirring were poor.

Experiment 15.2

A letter reached the school in the second week of term, in an envelope with a sunflower printed on the flap.

> Dear **Mr. Akilikubwa**,
>
> I am the one who sat at the back and broke the burette. I work now at the oil press, where the crude oil is warmed before it goes to the filters. The manager wants to know what the warming ought to cost in firewood, and I told him it turns on a number I can no longer put a name to. We have here a thermometer, a clock and a balance, and nothing else that anybody would call an instrument. Can the number be got with those three?
>
> Yours faithfully,
>
> a former student of yours
>
> P.S. Please do not put this to Kipute. He told me in Form Six that a thermometer knows nothing about heat and only about temperature, and I have carried it with me ever since.

The postscript is half right, and half right is the dangerous kind. A thermometer on its own tells you nothing about heat. A thermometer and a clock together tell you how fast a temperature is falling, which is a different matter altogether, and the balance turns that fall into an answer. Three instruments, one figure measured beforehand, and the letter can be answered.

Aim

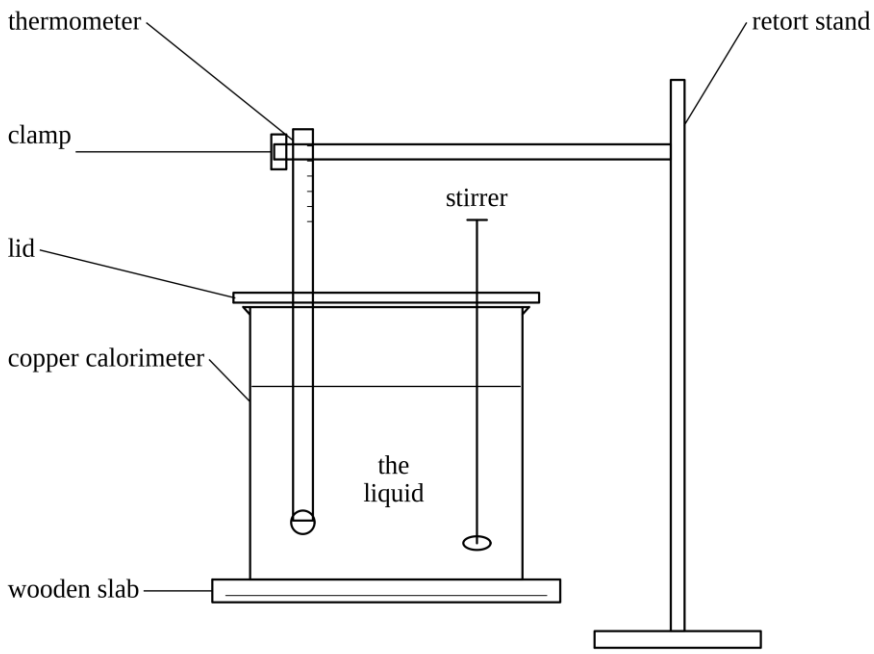
To determine the specific heat capacity of liquid D.

Apparatus

A copper calorimeter with its stirrer and a lid pierced for the thermometer and the stirrer, of total mass 96.0g with the stirrer; about 200g of liquid D; a thermometer reading from 0 to 100°C in steps of 0.5°C; a beaker of water on a wire gauze and a tripod over a burner; a stopwatch; a retort stand with a clamp; a wooden slab; a balance reading to 0.1g.

Setup

The calorimeter stands on the wooden slab, away from any window or doorway, with nothing wrapped round it and its lid on. The thermometer is held in the clamp and passes through one hole in the lid, with its bulb well below the surface of the liquid and clear of the bottom and the sides; the stirrer passes through the other and stays in the liquid throughout. The lid is on from the first reading to the last.



Procedure

Proceed as follows:

(a) Find the mass of the calorimeter together with its stirrer, and record the temperature of the room.

(b) Pour liquid D into the calorimeter until it is about three quarters full, find the mass of the calorimeter, the stirrer and the liquid together, and record it.

(c) Warm the liquid to about 95°C, stand the calorimeter on the wooden slab and put the lid on with the thermometer and the stirrer through it.

(d) Stirring gently and continuously, start the stopwatch at the instant the thermometer reads 90.0°C.

(e) Read and record the temperature every 2 minutes for 24 minutes.

Questions

(i) Tabulate your results.

(ii) Plot a graph of θ (°C) against t (min).

(iii) Draw a tangent to your curve at 70.0°C and determine its slope S.

(iv) This calorimeter, measured beforehand in this room, loses heat at the rate $P = 13.4Js^{-1}$ when its outside is at 70.0°C and the room is at 25.0°C. Use this, with your slope, to determine the heat capacity C of the calorimeter and its contents.

(v) The specific heat capacity of copper is 0.38J/gK. Hence determine the specific heat capacity of liquid D.

(vi) State the assumption you make in applying the given rate of loss of heat to a calorimeter filled with liquid D.

(vii) Predict the slope of the tangent at 50.0°C, and name the law you have used.

(viii) Give one advantage this method has over the method of mixtures when the substance is a liquid.

Table of results

The temperature of the room was 25.0°C throughout. The calorimeter and its stirrer had a mass of 96.0g and the calorimeter with the stirrer and the liquid had a mass of 276.0g, so the calorimeter held 180.0g of liquid D.

t (min)	θ (°C)	t (min)	θ (°C)
0.0	90.0	14.0	62.3
2.0	85.1	16.0	59.5
4.0	80.5	18.0	56.4
6.0	76.0	20.0	54.4
8.0	72.1	22.0	51.9
10.0	68.8	24.0	49.7
12.0	65.0		

Making the measurement count

Practical tip 1: *Lay the tangent along a chord, and do not trust your eye.*

Find where the curve crosses 70.0°C and mark it. Mark two more points on the curve 6 minutes on either side of it, join those two with a ruler, and then draw the tangent through the marked point parallel to that chord, as Chapter 13 sets out. Produce the tangent well beyond both ends before you measure it, because a slope taken over a short base is a poorer measurement than the same slope taken over a long one. On this curve a chord over 6 minutes each way runs about 1% steeper than the tangent at its middle, so lay the tangent a shade flatter than the ruler suggests.

Practical tip 2: *A rate measured on one filling is being used on another.*

The figure you are given for the rate of loss of heat was found for this calorimeter standing in this room, and it was not found with liquid D in it. That is why it can be handed to you at all, and it is also why it comes with a condition attached. Part (vi) asks what has to be true before a number measured under one set of circumstances may be used under another.

Practical tip 3: *Joules per second on one side, kelvin per minute on the other.*

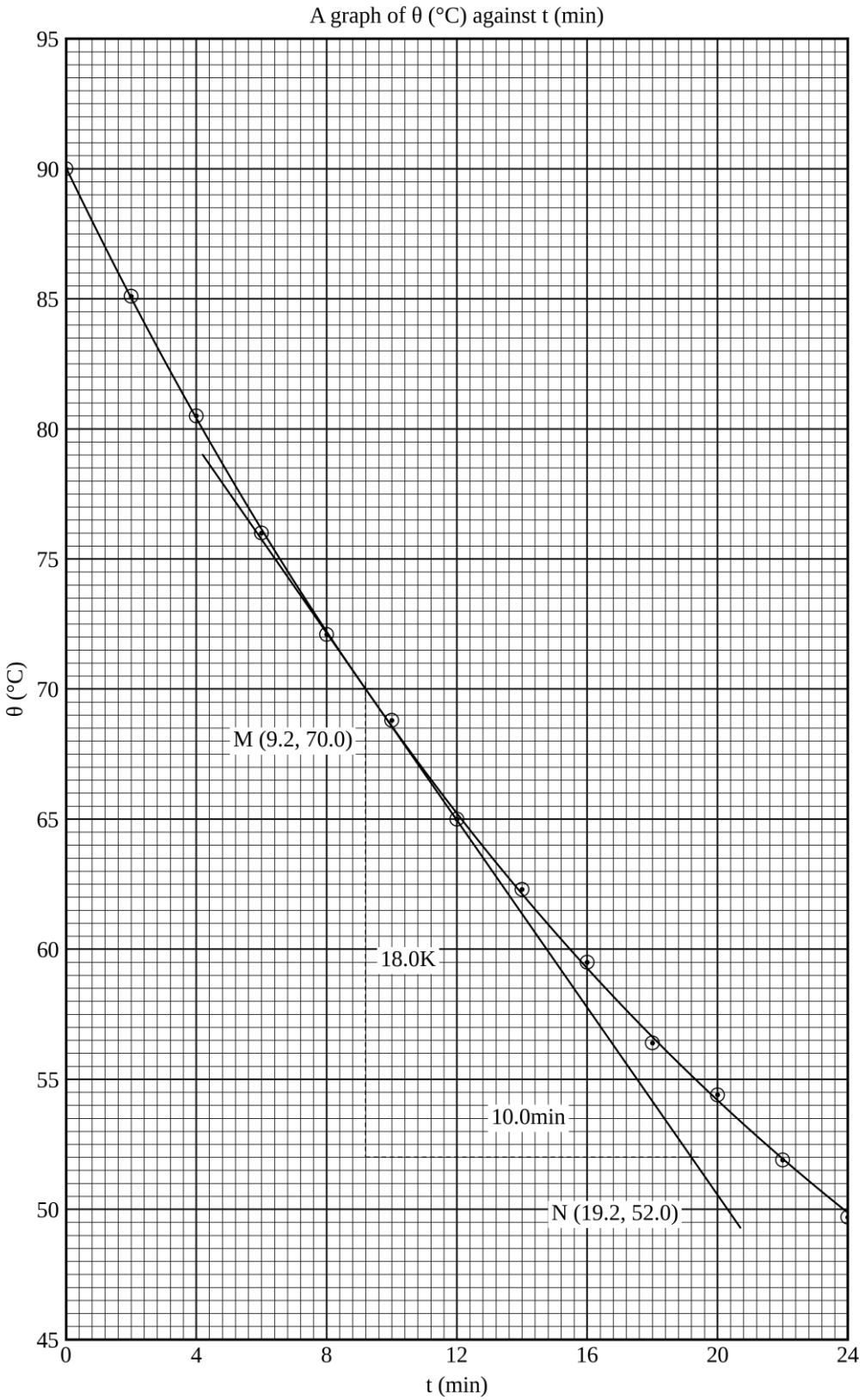
The rate of loss of heat is given in joules per second and the slope of your tangent comes off the graph in kelvin per minute. Put the two together as they stand and the answer comes out 60 times too small. Write the conversion on its own line, with both units on the page, before you divide anything.

Practical tip 4: *The answer is only as good as the number you were handed.*

Everything in this method passes through the supplied rate of loss. A tangent drawn with real care off a graph of this size is good to about 2%; the rate of loss is no better than the measurement that produced it, and no care with the tangent can improve on that. Carry the working to one figure more than you mean to keep, and quote the specific heat capacity to three significant figures and no more.

Answers to the questions

(i) and (ii). Take t at 2cm to 4 minutes, running from 0 to 24 minutes, and θ at 2cm to 5K, running from 45°C to 95°C. That is 12cm by 20cm, which fits an A4 sheet upright. Give the graph the title A graph of θ (°C) against t (min).



(iii) The tangent at 70.0°C touches the curve near t = 9.2 minutes. Taking the two marked corners M (9.2, 70.0) and N (19.2, 52.0) on the drawn tangent,

$$S=\frac{70.0°C-52.0°C}{19.2min-9.2min}=\frac{18.0K}{10.0min}=1.80K/min$$

The temperature is falling, so the slope of the curve itself is negative; S is its size.

(iv) The heat leaving the calorimeter every second is the heat capacity of the whole system multiplied by the number of kelvin it falls in a second, so

$$P=CS$$

The slope has first to be put into kelvin per second:

$$S=\frac{1.80K}{60s}=0.0300K/s$$

and then

$$C=\frac{13.4Js^{-1}}{0.0300K/s}=447JK^{-1}$$

(v) The copper is part of that heat capacity:

$$m_c c_c=96.0g\times0.38J/gK=36.48JK^{-1}$$

so the liquid accounts for the rest,

$$447JK^{-1}-36.48JK^{-1}=410.52JK^{-1}$$

and dividing by the mass of liquid,

$$c=\frac{410.52JK^{-1}}{180.0g}=2.28J/gK$$

(vi) That the rate at which the vessel loses heat depends on the temperature of its outside and on nothing else. The copper is thin and the liquid is stirred, so the outside sits at the temperature the thermometer shows whichever liquid is inside, and the measurement carries across. It would not carry across to an open vessel of water, where evaporation adds a second route out that the figure misses.

(vii) For the modest excess temperatures this experiment works in, the rate at which a body loses heat is proportional to how far it stands above its surroundings, which is Newton's law of cooling, so the slope falls in the same proportion. The excess has fallen from 45.0K to 25.0K, so

$$S=1.80K/min\times\frac{25.0K}{45.0K}=1.00K/min$$

(viii) Nothing is transferred. The liquid is weighed in the vessel it is measured in and never leaves it: none is spilt, none is left clinging to a second beaker, and no hot body crosses the air with a wet film on it. The method of mixtures asks you to move a hot solid in a second or two and lose nothing on the way.

Examiner's eye

- The subtraction in part (v) takes out the copper and nothing else. The lid and the thermometer bulb cool with the vessel too, and their share stays inside the figure you quote for the liquid.
- Find 70.0°C on the curve, not on the line you have drawn. A tangent laid down carelessly touches the curve above or below the temperature you were asked for, and the slope then belongs to a different question.
- Heat capacity in part (iv), specific heat capacity in part (v). The step between them is the subtraction of the copper and then a division by the mass, and a jump straight from $447JK^{-1}$ to a value for the liquid has done neither.
- Part (vii) wants a calculation. A second tangent drawn on the graph is 5 minutes of work spent on a poorer answer.

Practice 15.2

A drum stands in the corner of the hardware shop by the market, and its label has been in the sun since before the present owner bought the place. One letter of it survives, and the shop sells the contents as liquid F on the strength of that letter. It is sold for oiling hinges, for quieting bearings and, once, for stopping a roof from squeaking, and nobody has ever asked it for anything else.

Mr. Akilikubwa had sent the two of them for a metre of rubber tubing. **Kipute** wanted to know what was in the drum. **Kipanga** said the shop would not know either, and that the drum had been answering a different question for years without anybody noticing which one. Kipute bought some of it with his own money, and they came back without the tubing.

Procedure

Proceed as follows:

(a) Find the mass of the calorimeter together with its stirrer, and record the temperature of the room.

(b) Pour liquid F into the calorimeter until it is about three quarters full, find the mass of the calorimeter, the stirrer and the liquid together, and record it.

(c) Warm the liquid to about 90°C, stand the calorimeter on the wooden slab and put the lid on with the thermometer and the stirrer through it.

(d) Stirring gently and continuously, start the stopwatch at the instant the thermometer reads 85.0°C.

(e) Read and record the temperature every 2 minutes for 24 minutes.

Questions

(i) Tabulate your results.

(ii) Plot a graph of θ (°C) against t (min).

(iii) Draw a tangent to your curve at 60.0°C and determine its slope.

(iv) The calorimeter and its contents lose heat at the rate of $15.4Js^{-1}$ when the liquid in it is at 80.0°C. The room was at 24.0°C. Determine the rate at which they lose heat when the liquid is at 60.0°C.

(v) Hence determine the heat capacity of the calorimeter and its contents.

(vi) The specific heat capacity of copper is 0.38J/gK. Determine the specific heat capacity of liquid F.

(vii) A second bottle of liquid F is found to have water in it. State, with a reason, whether the value you have found would come out larger or smaller if the experiment were repeated with that bottle.

(viii) The shop wants to know what the liquid is. Say how far your measurement takes that question and where it stops.

Experiment 15.3

Behind the house where Kipute's aunt lives there is a drum of water that stands in the sun all day and is used for washing when the light has gone. Somebody, long before anyone now living in the house, put a layer of round stones in the bottom of it, and nobody has ever taken them out.

Kipute held that the stones were doing good service: they sit in the sun all afternoon like everything else and give their heat back after dark. **Kipanga** held that they were doing harm, because every stone in the bottom is a stone's worth of water that is not there, and water is the better keeper of the two. Kipute asked how he knew. Kipanga said he did not, and that this was exactly his objection.

Mr. Akilikubwa pointed out that neither of them had said how much stone, and that the whole argument turns on it: a handful and a barrowload are not the same case put twice. What is needed is a figure for the exchange, so many joules of stone against so many joules of the water it pushed aside. Nobody in the yard has a second drum without stones in it, and that is the whole difficulty; a calorimeter has no such trouble, because it can be emptied.

Aim

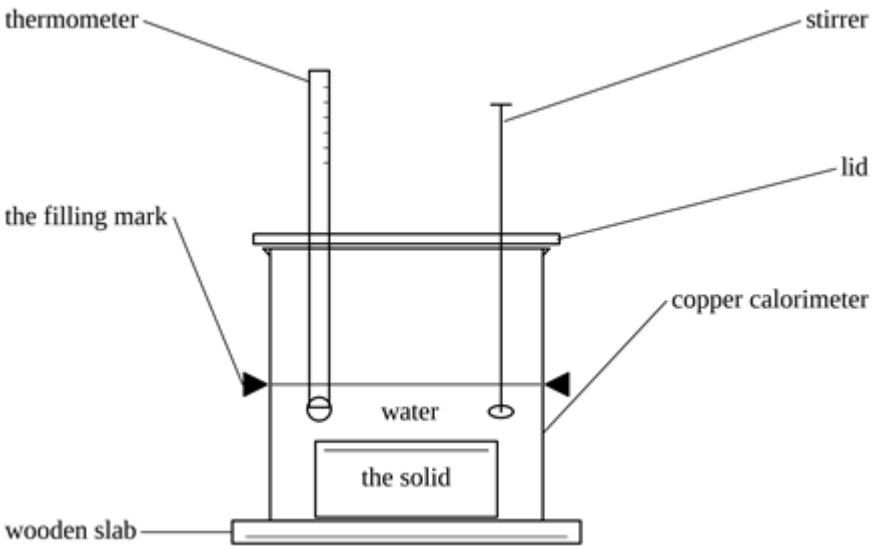
To determine the specific heat capacity of solid X.

Apparatus

A copper calorimeter with its stirrer and a lid pierced for the thermometer and the stirrer, of total mass 92.0g with the stirrer, and carrying a mark that shows the level to fill it to; solid X, a metal block small enough to lie under that level; a thermometer reading from 0 to 100°C in steps of 0.5°C; a stopwatch; a wooden slab; a balance reading to 0.1g; a beaker of water on a wire gauze and a tripod over a burner; a pair of tongs.

Setup

The calorimeter stands bare on the wooden slab, out of any draught from a window or a doorway, with its lid on. The thermometer passes through one hole in the lid with its bulb well below the surface and clear of the bottom and the sides, and the stirrer through the other. The experiment is two runs of the same vessel: the first with water alone, the second with the block lying in the bottom and water added to the same mark. The block is warmed in the hot water before it goes in, so that it does not quench the run before the watch is started. It takes up room, so the second run holds less water than the first, and both amounts are found on the balance rather than judged by eye.



Procedure

Proceed as follows:

(a) Find the mass of the calorimeter together with its stirrer, and record the temperature of the room.

(b) Fill the calorimeter to the mark with hot water, stand it on the wooden slab and put the lid on with the thermometer and the stirrer through it.

(c) Stirring gently and continuously, start the stopwatch at the instant the thermometer reads 80.0°C, and record the temperature every 2 minutes for 24 minutes.

(d) Find the mass of the calorimeter, the stirrer and the water together, record it, and pour the water away.

(e) Find the mass of solid X and stand it in the beaker of hot water for 5 minutes, so that it is not cold when it goes in.

(f) Lift the solid out with the tongs, lower it into the calorimeter, add hot water to the same mark, stand the calorimeter on the wooden slab and put the lid on with the thermometer and the stirrer through it.

(g) Stir until the temperature is falling slowly and steadily, then repeat step (c).

(h) Find the mass of the calorimeter, the stirrer, the solid and the water together, and record it.

Questions

(i) Tabulate your results for the two runs, including the difference between the two temperatures at each time.

(ii) Plot the two cooling curves on the same axes, marking the water-only run A and the run with the solid in it B, and satisfy yourself that A is the upper of the two.

(iii) Determine the mass of water in each run.

(iv) Determine the time at which the difference between the two temperatures is greatest, and read the temperature of each curve at that time.

(v) Explain why the two curves are falling at the same rate at that time.

(vi) The specific heat capacity c of the solid is given by

$$m_s c = (m_A - m_B)c_w - \frac{C_1 \Delta\theta}{\theta_A - \theta_r}$$

where m_s is the mass of the solid, m_A and m_B are the masses of water in the two runs, C_1 is the heat capacity of the calorimeter and the water in the first run, θ_A is the temperature of the first curve at the time found in (iv),

$\Delta\theta$ is the difference between the two curves at that time and θ_r is the temperature of the room. Taking c_w = 4.2J/gK and c_c = 0.38J/gK, determine c.

(vii) Solid X is a block of aluminium, for which the accepted value is 0.900J/gK. Comment on your result.

(viii) State which single reading your answer is most sensitive to, and why.

Table of results

The temperature of the room was 25.0°C throughout. The calorimeter and its stirrer had a mass of 92.0g, the calorimeter with the stirrer and the water of the first run had a mass of 196.0g, solid X had a mass of 148.5g, and the calorimeter with the stirrer, the solid and the water of the second run had a mass of 291.2g.

t (min)	θ_A (°C)	θ_B (°C)	$\theta_A - \theta_B$ (K)
0	80.0	80.0	0.0
2	73.3	71.9	1.4
4	67.7	65.4	2.3
6	62.8	59.4	3.4
8	58.2	54.5	3.7
10	54.3	50.2	4.1
12	50.7	46.5	4.2
14	47.7	43.4	4.3
16	45.0	40.8	4.2
18	42.7	38.6	4.1
20	40.6	36.5	4.1
22	38.7	34.9	3.8
24	36.9	33.2	3.7

Making the measurement count

Practical tip 1: *Fill to the mark, not to the same mass.*

The mark is not a convenience, and the balance is no substitute for it. Fill to the mark in both runs and find the mass of water afterwards, rather than setting out to pour the same mass twice. The two masses will then differ, and the first term of the equation in part (vi) is there to take the difference.

Practical tip 2: *Let the block catch up before you start the watch.*

Even a warmed block comes out of the beaker cooler than the water you pour on it, and for the first half minute the thermometer falls quickly while the two settle together. That fall is not cooling and does not belong in the table. Stir until the temperature is coming down slowly and evenly, and only then wait for 80.0°C and start the watch. From that instant the block and the water are at one temperature and stay so. A block put in cold would take the run below 80.0°C before the watch could be started at all.

Practical tip 3: *Find the moment in the table, then check it on the graph.*

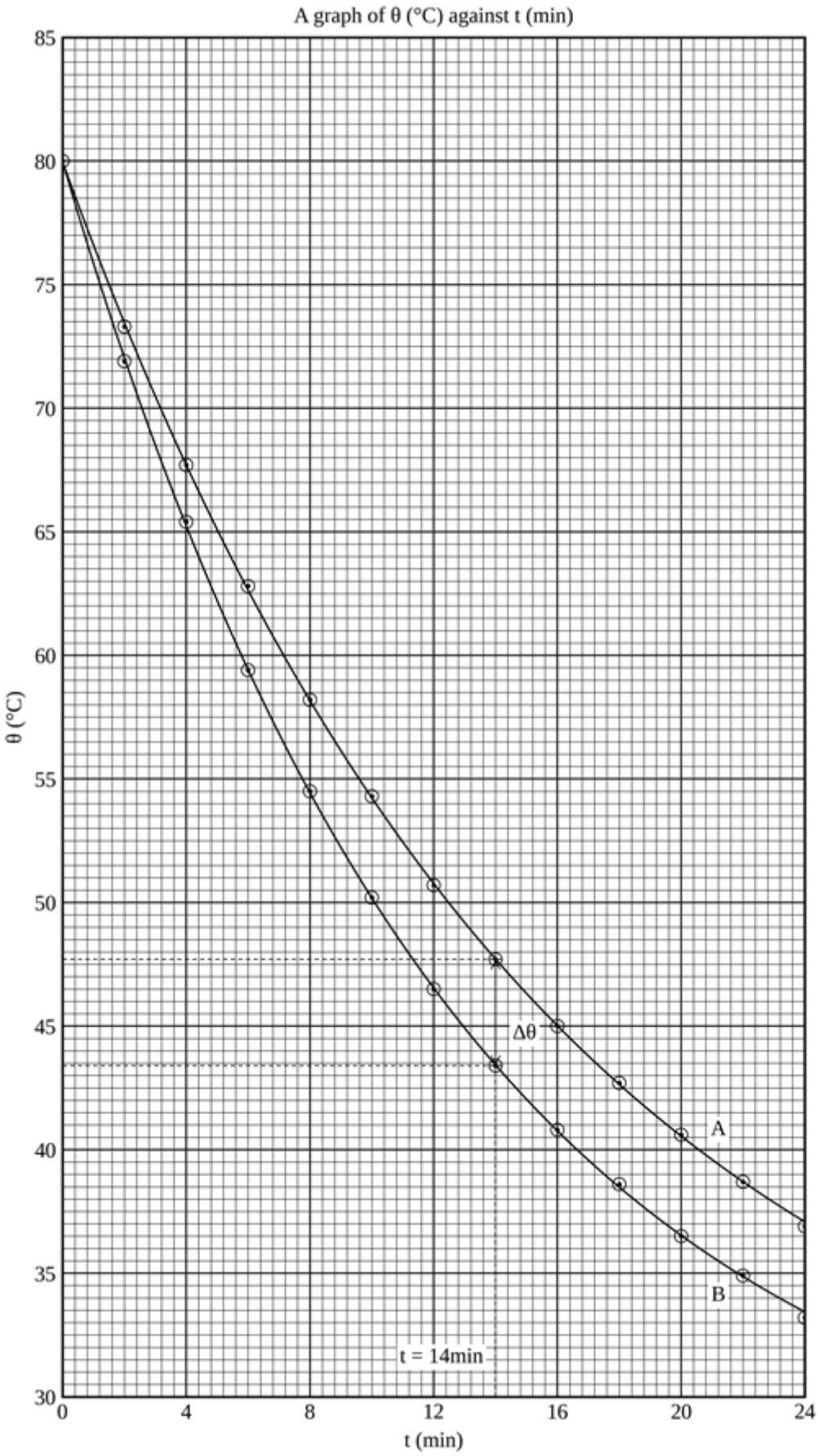
The difference column falls out of the table with no drawing at all, and it beats the eye every time: two curves a few kelvin apart open and close so gently across the sheet that nobody can pick the widest place by looking. Work down the column, find where it turns, and then look at the graph to confirm that the two curves really are running parallel there. Part (iv) asks for both, and the two agreeing is the check.

Practical tip 4: *One vessel, two runs, one difference between them.*

Stand the calorimeter in the same place on the same slab for both runs, stir at the same rate, start both at the same temperature and use the same thermometer. Do not fan one and leave the other, and do not do the first run with the window open and the second with it shut. Every one of those is a second difference, and a second difference does not make the graph look wrong; it makes the answer wrong while the graph goes on looking perfectly respectable. What the discipline buys is worth the trouble: Experiment 15.2 can be no better than the figure it was handed for the rate of loss of heat, and this one is handed nothing.

Answers to the questions

(i) and (ii). Take t at 2cm to 4 minutes, running from 0 to 24 minutes, and θ at 2cm to 5K, running from 30°C to 85°C. That is 12cm by 22cm, which fits an A4 sheet upright. Give the graph the title A graph of θ (°C) against t (min).



(iii) By subtraction. In the first run,

$$m_A = 196.0g - 92.0g = 104.0g$$

and in the second the calorimeter and the solid together weigh 92.0g + 148.5g = 240.5g, so

$$m_B = 291.2g - 240.5g = 50.7g$$

(iv) The difference column rises to 4.3K at t = 14 minutes, with 4.2K on either side of it. At that time curve A reads 47.7°C and curve B reads 43.4°C. Accept 12 to 16 minutes for the time.

(v) The difference between two quantities stops changing at the moment they are changing at the same rate, and that is all a greatest separation means. B starts level with A, so at the first instant the two are losing heat at the same rate; B has the smaller heat capacity, so that same loss takes it down further and the gap opens. From then on B is the cooler of the two and loses heat the more slowly. By the middle of the run it has fallen close enough to the room for the smaller loss exactly to offset the smaller heat capacity. After that the smaller loss wins, B falls the more slowly, and the gap closes. In between there is one instant at which the two curves run parallel.

(vi) Write C_1 for the heat capacity of the calorimeter and the water of run A and C_2 for that of run B. It is one vessel with one outside standing in one room. Newton's law of cooling has to hold only at the one instant used, and only between the 18.4K and the 22.7K of excess the two curves stand at there, so the rates of loss of heat are as the excess temperatures while the rates of fall are equal. A rate of fall is a rate of loss divided by a heat capacity, so

$$\frac{\theta_A - \theta_r}{C_1} = \frac{\theta_B - \theta_r}{C_2}$$

Notice what has left that line: whatever the constant of proportionality is, it stood on both sides and has divided out. The heat capacity of the first run is

$$C_1 = 92.0g \times 0.38J/gK + 104.0g \times 4.2J/gK = 34.96JK^{-1} + 436.8JK^{-1} = 471.8JK^{-1}$$

and rearranging the relation in the question with the readings of part (iv),

$$(m_A - m_B)c_w = 53.3g \times 4.2J/gK = 223.9JK^{-1}$$

$$\frac{C_1 \Delta\theta}{\theta_A - \theta_r} = \frac{471.8JK^{-1} \times 4.3K}{47.7°C - 25.0°C} = \frac{471.8JK^{-1} \times 4.3K}{22.7K} = 89.4JK^{-1}$$

$$m_s c = 223.9JK^{-1} - 89.4JK^{-1} = 134.5JK^{-1}$$

$$c = \frac{134.5JK^{-1}}{148.5g} = 0.906J/gK$$

(vii) The accepted value is 0.900J/gK and this is 0.906J/gK, which is 0.7% above it. A difference of that size is well inside what the reading of two curves a few kelvin apart can settle, so the result agrees with the accepted value and there is nothing in the difference to explain. It also settles the drum in the yard, or would, once the same pair of runs had been done on a stone. This block holds $134.5JK^{-1}$ against the $223.9JK^{-1}$ held by the water it pushed out of the way, so it keeps about three fifths of what the water it displaced would have kept, and water is the better keeper of the two. Aluminium is not stone, but the sum runs the same way, and Kipanga had the sum right.

(viii) The time in part (iv). The greatest separation itself is flat: 2 minutes either side of it the difference has moved by only a tenth of a kelvin, so $\Delta\theta$ is well determined. The temperature of curve A at that instant is not flat at all; it is falling at about 1.4K/min there. Take the time 2 minutes late and θ_A comes out 2.7K low, which raises the quotient and drops the answer by about 7%; take it 2 minutes early and the answer rises by about 9%. Everything else is a weighing or a temperature, and the temperatures are forgiving, because the answer depends only on a difference and on an excess: a fault shared by all three temperature readings cancels outright, and half a kelvin between the room and the curves is worth 1.5%.

Examiner's eye

- Read both curves at the same instant. The two temperatures in part (iv) belong to one vertical line on the sheet; take θ_A from the top of one hump and θ_B from a convenient plotted point 2 minutes away and you have lost the question in the commonest way there is.
- There are four weighings in this experiment. The second run needs its own weighing after it, because the mass of water in it is not the mass in the first run and is not something you can predict without the volume of the block.

- Both curves share one pair of axes, because both are temperature against time. The letters A and B belong beside the curves themselves, and a graph with two vertical scales on it has misread the question.
- Neither curve may be drawn through every point. Put a smooth curve through the middle of the scatter and let two or three points sit off it on either side, exactly as in Chapter 13; a curve bent to reach every plotted cross will wander at the very place where the two are compared.

Practice 15.3

Two blocks live in the cupboard at the back of the laboratory, the same size to within a millimetre and labelled nowhere. One of the two is noticeably the heavier, and somebody, at some point, sawed a corner off the lighter one to see what was inside, which settled nothing and spoiled a good block.

Mr. Akilikubwa set them side by side and asked which would take more heat to warm it through 10 kelvin. **Kipanga** said the heavier one, because there is more of it there. **Kipute** agreed, and then did not: more of it, certainly, but more of what? The balance says how many grams there are. It has never yet said what one gram is good for.

Procedure

Proceed as follows:

(a) Find the mass of the calorimeter together with its stirrer, and record the temperature of the room.

(b) Fill the calorimeter to the mark with hot water, stand it on the wooden slab and put the lid on with the thermometer and the stirrer through it.

(c) Stirring gently and continuously, start the stopwatch at the instant the thermometer reads 78.0°C, and record the temperature every 2 minutes for 24 minutes.

(d) Find the mass of the calorimeter, the stirrer and the water together, record it, and pour the water away.

(e) Find the mass of solid Y and stand it in the beaker of hot water for 5 minutes, so that it is not cold when it goes in.

(f) Lift the solid out with the tongs, lower it into the calorimeter, add hot water to the same mark, stand the calorimeter on the wooden slab and put the lid on with the thermometer and the stirrer through it.

(g) Stir until the temperature is falling slowly and steadily, then repeat step (c).

(h) Find the mass of the calorimeter, the stirrer, the solid and the water together, and record it.

Questions

(i) Tabulate your results for the two runs, including the difference between the two temperatures at each time.

(ii) Plot the two cooling curves on the same axes, marking the water-only run A and the run with the solid in it B.

(iii) Determine the mass of water in each run.

(iv) Determine the time at which the difference between the two temperatures is greatest, and read the temperature of each curve at that time.

(v) Using the relation given in Experiment 15.3, with $c_w = 4.2\text{J/gK}$ and $c_c = 0.38\text{J/gK}$, determine the specific heat capacity of solid Y.

(vi) The difference between the two masses of water is the mass of water the block pushed out of the way. Water at the temperature the vessel was filled at has a density of 0.97gcm^{-3}. Determine the volume of solid Y, and hence its density.

(vii) Use both of your results to suggest what solid Y is made of, and say which of the two results does the deciding.

(viii) State one reason why the two runs must be filled to the same mark and not with the same mass of water.

Experiment 15.4

The tap at the corner of the compound runs slowly, and the tank it fills carries no gauge, so the morning's water is measured in buckets. The counting was given to **Kipute**, who is reliable.

He counted 34 buckets in and worked out what the tank ought to be holding. The marked stick they use for a gauge said a good deal less. He counted again the next morning, slowly, and got the same disagreement to within a bucket. **Kipanga** offered to check the stick, on the grounds that the stick was the only part of the arrangement nobody had suspected of anything. Then he looked at the ground instead and pointed to the wet ring round the tank's foot, which had been keeping its own record every morning of the term.

The arithmetic was not what was wrong with it, **Mr. Akilikubwa** said. What goes in is not what stays in, and the shortfall is not a blunder to be found and removed but a rate to be measured and allowed for. Experiment 15.1 left the same kind of shortfall out of its books altogether. This one keeps the watch running and puts it back.

Aim

To determine the specific heat capacity of brass by the method of mixtures, with the cooling of the mixture allowed for.

Apparatus

The apparatus of Experiment 15.1, the same calorimeter and the same brass cylinder, with a stopwatch added.

Setup

The arrangement is the one drawn in Experiment 15.1 and is set up in the same way. Two things are different in the handling. The water put into the calorimeter is drawn straight from the tap, so that it starts a few kelvin below the temperature of the room, and the stopwatch is started before the cylinder is moved and runs without stopping until the last reading is taken. The cylinder goes in at a stated time on that watch, and the temperature is recorded on both sides of that moment.

Procedure

Proceed as follows:

(a) Find the mass of the calorimeter together with its stirrer, and record the temperature of the room.

(b) Half fill the calorimeter with cold water from the tap, find the mass of the calorimeter, the stirrer and the water together, and record it.

(c) Tie the thread to the brass cylinder, hang it in the beaker from the glass rod so that it is well under the surface and clear of the bottom, and bring the water to the boil.

(d) Keep the water boiling for 5 minutes, then read and record the temperature of the boiling water.

(e) Stand the calorimeter on the wooden slab, put the lid on with the thermometer and the stirrer through it, start the stopwatch and record the temperature every half minute.

(f) At exactly 2.0 minutes lift the cylinder out by its thread, shake the drops from it, lower it into the calorimeter without splashing and replace the lid at once. Do not stop the watch.

(g) Go on stirring gently and recording the temperature every half minute until the watch reads 7.0 minutes.

Questions

(i) Tabulate your results.

(ii) Plot a graph of θ (°C) against t (min).

(iii) Draw the best straight line through the readings taken before the transfer, produce it to t = 2.0 minutes and read θ_1, the temperature of the water at the instant the cylinder entered it.

(iv) Draw the best smooth curve through the readings taken from t = 3.5 minutes onwards, produce it backwards to t = 2.0 minutes and read θ_f at that time.

(v) Say what θ_f is the temperature of, and explain why it lies above the highest temperature the thermometer actually showed.

(vi) The specific heat capacity of water is 4.2J/gK and that of copper is 0.38J/gK. Determine the specific heat capacity of brass.

(vii) Determine what your answer would have been had you used the highest recorded temperature θ_m in place of θ_f, and compare both figures with the accepted value 0.380J/gK.

(viii) Name one loss that this correction cannot put back, and say why not.

Table of results

The temperature of the room was 25.5°C throughout. The calorimeter and its stirrer had a mass of 94.0g, the calorimeter with the stirrer and the water had a mass of 164.0g, the brass cylinder had a mass of 248.0g and the boiling water stood at 98.0°C. The cylinder was transferred at t = 2.0 minutes.

t (min)	θ (°C)	t (min)	θ (°C)
0.0	21.9	4.0	38.3
0.5	22.2	4.5	38.0
1.0	22.2	5.0	38.0
1.5	22.1	5.5	37.8
2.0	22.2	6.0	37.6
2.5	37.6	6.5	37.3
3.0	38.5	7.0	37.2
3.5	38.3		

Making the measurement count

Practical tip 1: *Three pieces of curve, and two of them are drawn.*

What comes out of this experiment is not one curve but three pieces of one. Before the transfer the water is drifting slowly towards the temperature of the room. During the transfer it climbs steeply for about a minute while the brass gives its heat up. After that it is an ordinary cooling curve of the kind Chapter 13 is full of. Only the first piece and the third are drawn as lines through their own points; the climb between them is joined freehand, and nothing is produced forwards or backwards from that join. One number does come out of the middle of the record later on, but it is read off the table where it stands and not off a line.

Practical tip 2: *Everything on one stopwatch.*

Start the stopwatch when the calorimeter goes on the slab, not when the cylinder goes in, and let it run through the transfer and on to the end. Every reading in the table then belongs to the same clock, and the instant of transfer is a time on that clock like any other. Stop it and restart it at the transfer and the two halves of the graph have nothing to join them.

Practical tip 3: *Produce the curve, not a chord across it.*

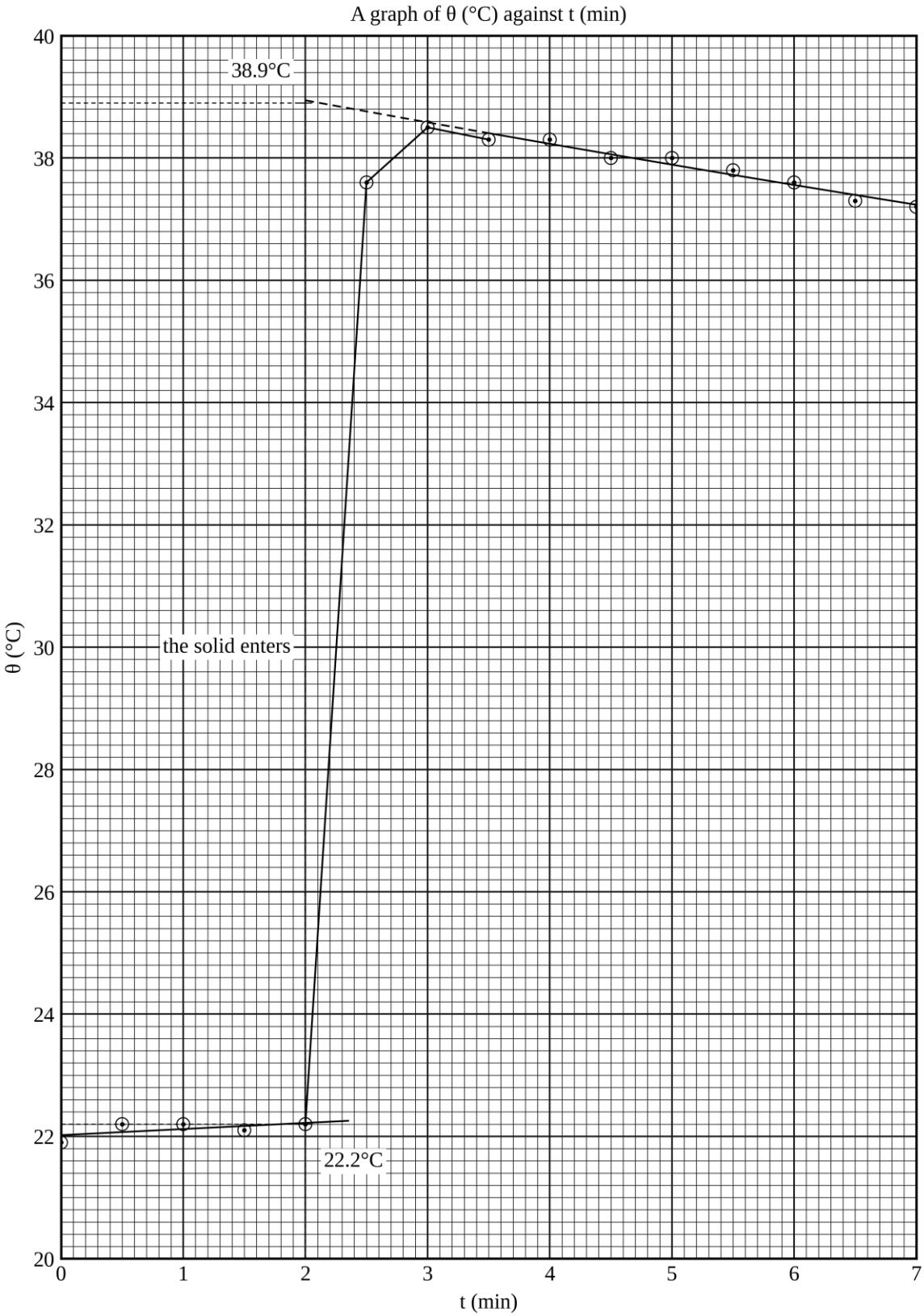
The cooling branch is bending, gently but unmistakably, so produce it backwards by continuing its shape and not by laying a ruler along its last two points. A straight chord across a curve that falls and flattens always lands below the curve, and lands further below the further it is carried; here it is carried a minute and a half, which is far. Take the curvature from the part of the branch you have already drawn and keep it all the way to the transfer. A vessel that loses heat quickly bends its branch harder and leaves more to be put back, so a quick cooler wants the more careful hand on this construction, not the less.

Practical tip 4: *The readings before the transfer are not decoration.*

Five readings of a temperature that is barely moving look like time wasted, and they are what give you the temperature the water had at the instant the cylinder went in. The last reading before the transfer is one reading, good to about 0.1K; a line drawn through five of them and produced to the transfer does better than that, and it also shows you whether the water was drifting fast enough to matter.

Answers to the questions

(i) and (ii). Take t at 2cm to 1 minute, running from 0 to 7 minutes, and θ at 2cm to 2K, running from 20°C to 40°C. That is 14cm by 20cm, which fits an A4 sheet upright. Give the graph the title A graph of θ (°C) against t (min).



(iii) The five readings before the transfer climb from 21.9°C to 22.2°C and a straight line lies well through them. Produced to t = 2.0 minutes it gives θ_1 = 22.2°C.

(iv) The readings from t = 3.5 minutes onwards fall from 38.3°C to 37.2°C along a shallow curve. Produced backwards to t = 2.0 minutes it gives θ_f = 38.9°C.

(v) It is the temperature the mixture would have reached if all the heat in the brass had arrived at the instant the cylinder went in and none of it had gone anywhere else. The real transfer takes about a minute. The water crosses room temperature in the first few seconds of it, and spends the rest of the minute above the room and losing

heat, so what it loses over that minute outweighs the little it gained before. The thermometer therefore turns over at 38.5°C. Had all the heat arrived at once, it would have turned over higher. Produce the cooling curve backwards and you are asking what that curve would have started from.

(vi) The heat capacity of the calorimeter and its water is

$$C=94.0g\times0.38J/gK+70.0g\times4.2J/gK=35.72JK^{-1}+294.0JK^{-1}=329.72JK^{-1}$$

The corrected rise is 38.9°C − 22.2°C = 16.7K and the fall of the brass is 98.0°C − 38.9°C = 59.1K, so

$$c=\frac{329.72JK^{-1}\times16.7K}{248.0g\times59.1K}=\frac{5506.3J}{14656.8gK}=0.376J/gK$$

(vii) The highest reading on the thermometer was 38.5°C, giving a rise of 16.3K and a fall of 59.5K:

$$c=\frac{329.72JK^{-1}\times16.3K}{248.0g\times59.5K}=0.364J/gK$$

Against the accepted 0.380J/gK the corrected value is 1.1% low and the uncorrected one is 4.2% low, so the correction has removed three quarters of the shortfall for the price of a graph.

(viii) Whatever the cylinder lost between the bath and the water. That loss happened before t = 2.0 minutes and outside the calorimeter altogether, so no amount of producing the calorimeter's own curve backwards can see it: the curve knows only about heat that arrived. It is the reason the corrected answer is still low rather than exactly right.

Examiner's eye

- Produce two lines, and carry both of them to the same instant. The rise you want is measured up one vertical line on the sheet, from the drift line to the produced curve, and a rise taken from the first plotted point to the produced curve mixes a corrected temperature with an uncorrected one.
- Number the readings on the graph as well as in the table. The two at 2.5 and 3.0 minutes belong to neither of the lines you draw, and a cross that has strayed into the wrong branch is the one mistake nobody can see once the ink is dry.
- Half a minute is not half a degree. The readings are taken every half minute and estimated to 0.1°C, and both of those are needed: at half-degree readings the whole correction found here would be one step of the thermometer.
- Set the uncorrected figure beside Experiment 15.1, which was the method of mixtures on the same brass with no graph and no correction and came out 4.7% low. Two afternoons, different water, different starting temperatures, and the two uncorrected values short by 4.7% and 4.2%: scatter does not do that. One fault has worked the same way twice, and a fault that repeats can be hunted down.
- Say that the corrected value and the accepted one agree to about 1%, name the one loss the method still does not catch, and stop. 0.376J/gK against an accepted 0.380J/gK wants no paragraph of excuses, and a list of six sources of error for a 1% difference is a list drawn up without a glance at the size of it.

Practice 15.4

Kipanga bet his afternoon bread that the same experiment done in a warmer room would give a different answer, since a calorimeter with a shorter way to fall loses heat more slowly and everybody knows it.

Kipute would not take the bet. He said a specific heat capacity belongs to the metal, and that a number belonging to a piece of steel cannot depend on what the weather is doing in the room where somebody happens to be weighing it. Kipanga said he was not talking about the metal, he was talking about the answer, and that the two had better not be confused.

Mr. Akilikubwa remarked that they were not disagreeing about anything yet, because neither of them had said which of the two he meant, and that he would hold the bread himself until one of them could tell the metal from the answer.

Procedure

Proceed as follows:

(a) Find the mass of the calorimeter together with its stirrer, and record the temperature of the room.

(b) Half fill the calorimeter with cold water from the tap, find the mass of the calorimeter, the stirrer and the water together, and record it.

(c) Tie the thread to the steel block, hang it in the beaker from the glass rod so that it is well under the surface and clear of the bottom, and bring the water to the boil.

(d) Keep the water boiling for 5 minutes, then read and record the temperature of the boiling water.

(e) Stand the calorimeter on the wooden slab, put the lid on with the thermometer and the stirrer through it, start the stopwatch and record the temperature every half minute.

(f) At exactly 2.0 minutes lift the block out by its thread, shake the drops from it, lower it into the calorimeter without splashing and replace the lid at once. Do not stop the watch.

(g) Go on stirring gently and recording the temperature every half minute until the watch reads 7.0 minutes.

Questions

(i) Tabulate your results.

(ii) Plot a graph of θ (°C) against t (min).

(iii) Produce the line through the readings taken before the transfer to t = 2.0 minutes and read θ_1.

(iv) Produce the cooling curve backwards to t = 2.0 minutes and read θ_f.

(v) The specific heat capacity of water is $4.2 J/gK$ and that of copper is $0.38 J/gK$. Determine the specific heat capacity of the steel.

(vi) Determine what the answer would have been from the highest recorded temperature, and state by what percentage the correction changed it.

(vii) The accepted value for steel is $0.460 J/gK$. Comment on your result.

(viii) The room here was 0.5K warmer than the room in Experiment 15.4. State what that changes in the working and what it does not, and hence settle the argument in the opening.

Chapter 16

HEATING CURVES AND CHANGE OF STATE

Everything in the two chapters before this one cooled. A calorimeter of hot water was set down on a slab, the thermometer came back towards the room, and the only question was how quickly. Two things in this chapter break that pattern. The first is that the temperature is driven the other way, by standing the vessel in a bath hotter than itself. The second is that the substance is taken through a change of state, and there the reasoning you have been using needs one term it has not needed before.

One sentence carries every question set that follows, and it is worth having in advance rather than meeting it six times. The rate at which the thermometer moves is the rate at which heat crosses the wall of the vessel, divided by the thermal capacity of everything that heat has to warm. Write it as $S=P/C$, with S the rate of change of temperature in kelvin per minute, P the rate at which heat is crossing in joules per minute, and C the thermal capacity in joules per kelvin. Read forwards, it tells you how fast a temperature will change. Read backwards, a rate of change you have measured off a graph becomes a quantity of heat you have measured. Half the determinations in this chapter are that sentence read backwards; the rest are heat balances, the accounting of Chapter 15 with one term added for the change of state, done over a whole run instead of at one instant.

A change of state is where that reasoning has to be handled with care, because while a substance is melting or freezing it takes in or gives out heat and the thermometer does not move at all. Three of the six question sets are built on measuring exactly how much heat that is. One of them is the only set in the chapter that asks for no graph whatever: it is a weighing and a heat balance, and it is set that way because the paper sets it that way.

The apparatus is the apparatus of Chapter 13 with a source of heat added. A copper calorimeter, a stirrer, a thermometer reading to 0.5K, a stopwatch and a balance do most of it; a beaker of water on a tripod is the bath; a test tube inside a wider tube is the vessel for the two runs that use it. The instructions page supplies the specific heat capacity of water as $4.2\,J/gK$ and of copper as $0.38\,J/gK$, and, where a question needs it, of ice as $2.1\,J/gK$. Masses stay in grams from the first line to the last, so that those numbers can be used exactly as they are given.

The three varieties at a glance

	What you do	What comes out of it
16.1	Water and then the same mass of liquid Y are warmed in one bath, and a tangent is taken to each curve at one temperature	the specific heat capacity of liquid Y, from the two rates of warming at that temperature
16.2	Naphthalene is lifted out of the bath above its freezing point and left to cool in still air	the melting point from the flat of the curve, and the specific heat capacity of the solid from the tangents on either side of it
16.3	Dried ice is melted in warm water in a lagged calorimeter, with no graph anywhere in the question	the specific latent heat of fusion of ice, from a heat balance and a set of weighings

All three are worked below in full. The practice questions stand a cold calorimeter in a hot can and read the two together, warm an unnamed solid through its melting point instead of cooling a known one, and repeat the ice with the lagging taken away, so that the heat that leaks to the room has to be taken out on the graph.

Experiment 16.1

Mr. Akilikubwa had been invited to a wedding, and had brought **Kipute** and **Kipanga** with him on the strength of a rumour about pilau. What there was in the cooking yard, when they reached it, was an argument. The cook had stood her great pot of stew inside a still greater pot of water, the water was over the fire, and the stew was not ready.

"The inside pot must be the hotter of the two," said **Kipanga**, who was hungry. *"It has the fire below it and the water all round it."*

"Then why is my water jumping," said the cook, without turning round, *"and my stew is not?"*

Kipute laughed so much that he had to sit down on a crate of bottles, and the man whose crate it was moved him off it. **Mr. Akilikubwa** stood looking at the two pots for a long time, and on the way home he told the boys to bring a stopwatch to Thursday's lesson.

Aim

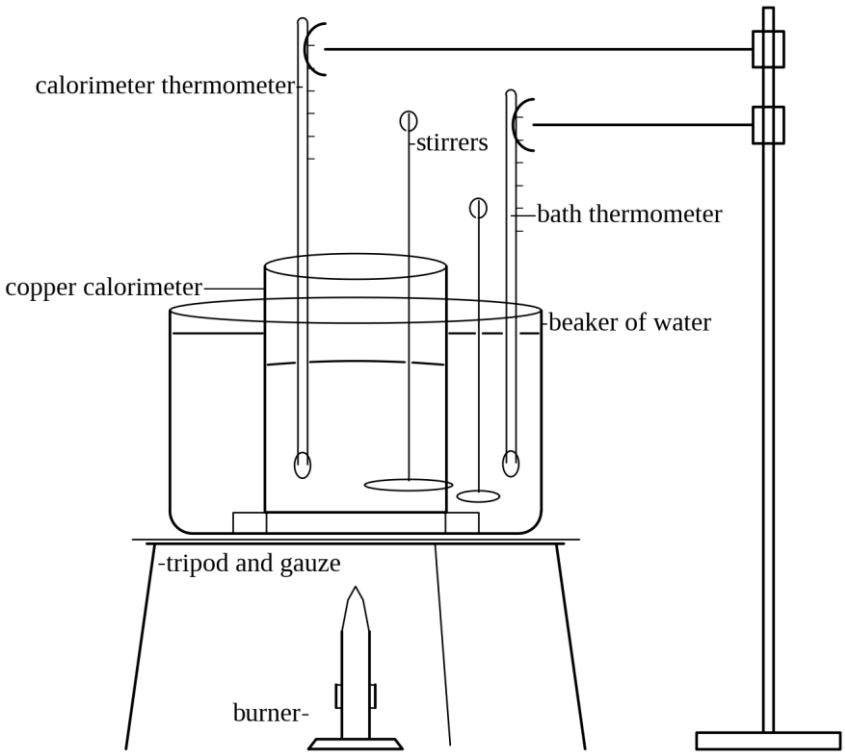
To determine the specific heat capacity of a liquid Y from the rate at which it is warmed by a bath.

Apparatus

A copper calorimeter with its stirrer, of mass 78.0g; a beaker of about $600cm^3$ as the bath; a tripod, a gauze and a burner; two thermometers reading to 0.5K; a stopwatch; a retort stand with two clamps; a small cork ring; a stirring rod for the bath; a balance; 120g of water and 120g of a liquid labelled Y.

Setup

The arrangement is the one drawn below. The calorimeter stands on a small cork ring inside the beaker so that its base does not touch the glass, and one thermometer is clamped with its bulb in the middle of the liquid in the calorimeter. The second thermometer is clamped in the water of the bath, well clear of the calorimeter. The burner is turned low, and the bath is stirred and its temperature held at 70.0°C throughout by adjusting the flame. The room is at 26.0°C.



Procedure

Proceed as follows:

(a) Pour 120g of water into the dry calorimeter and record the temperature of the water.

(b) Bring the bath to 70.0°C and hold it there.

(c) Lower the calorimeter into the bath on its cork ring, start the stopwatch at the same instant, and clamp the thermometer with its bulb in the middle of the liquid.

(d) Stir the liquid gently and continuously, and read its temperature every 1min for 12min. Read the bath at the same moments and correct the flame if it has moved off 70.0°C.

(e) Empty and dry the calorimeter, leave it standing until it is cool, and repeat steps (a), (c) and (d) with 120g of liquid Y in place of the water. Record the starting temperature of the liquid whatever it turns out to be.

(f) Record all your readings in a suitable table.

Questions

(i) Tabulate your readings for the two runs.

(ii) On the same axes plot a graph of temperature θ against time t for both runs.

(iii) Draw a tangent to each curve at 55.0°C and determine from it the rate of rise of temperature of each liquid at that temperature.

(iv) State, with a reason, why the rate at which heat enters the calorimeter at 55.0°C is the same in the two runs.

(v) Calculate the thermal capacity of the calorimeter together with the water, and hence the rate at which heat enters it at 55.0°C.

(vi) Determine the specific heat capacity of liquid Y.

(vii) Explain why the rate of rise of temperature falls as each run goes on.

(viii) Give one reason why your value in (vi) may be in error, and suggest an improvement to the arrangement.

Table of results

The two runs go in one table, since both were read at the same clock times, and both liquids were warmed in the one bath. The bath was read at each of these instants too and never moved from 70.0°C by as much as half a division, so it carries no column of its own.

t (min)	water (°C)	liquid Y (°C)	t (min)	water (°C)	liquid Y (°C)
0	28.0	28.0	7	56.0	64.0
1	34.5	37.5	8	58.0	65.5
2	39.0	45.5	9	60.0	66.0
3	43.5	51.0	10	61.0	67.0
4	48.0	56.0	11	62.5	67.5
5	50.5	59.0	12	64.0	68.0
6	54.0	62.0			

Making the measurement count

Practical tip 1: *A tangent is only as good as the curve you drew it on.* Plot every point, draw one smooth line through them with the scatter falling on both sides of it, and then lay the ruler along that line when the moment comes to draw a tangent. Take the two corners of the slope triangle on ruled lines, well apart, and write their coordinates on the sheet where the examiner can find them.

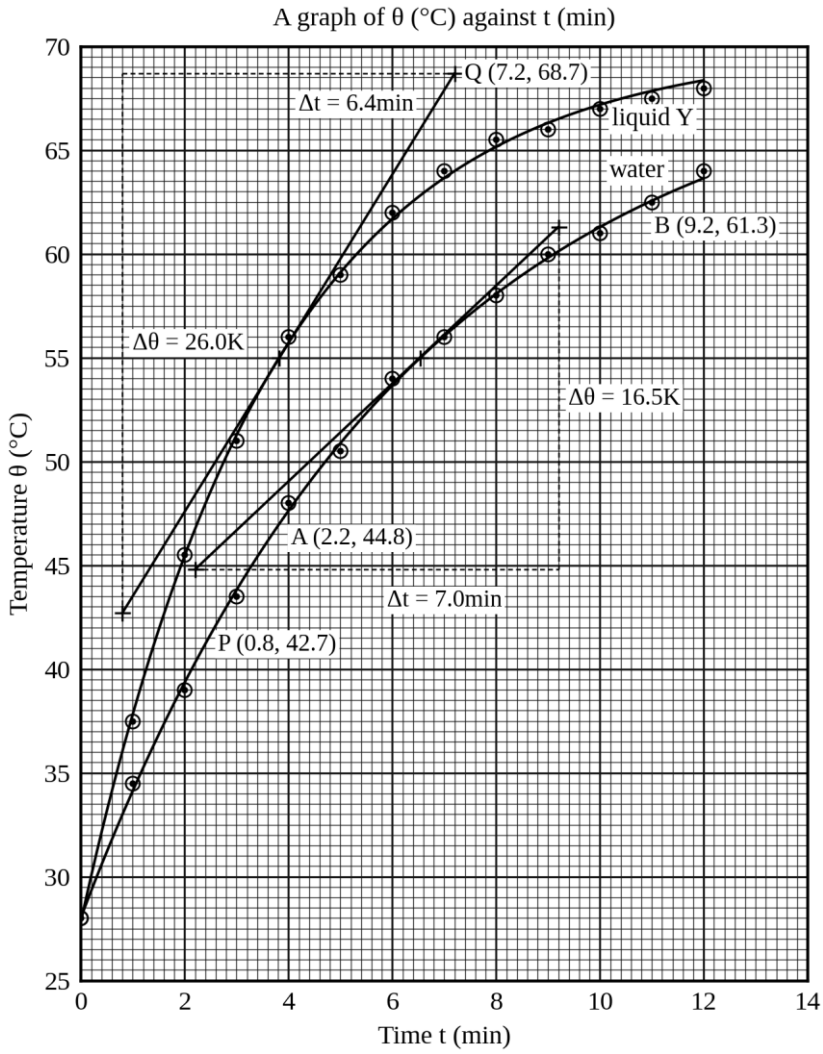
Practical tip 2: *Both runs go on one sheet, and the sheet has to hold the taller of the two curves.* Work the scale out from the largest reading in either table before you draw a single axis. The table you happen to pick up first is not always the one that reaches highest.

Practical tip 3: *Copper is warmed along with the liquid it holds, and it carries a real share of the heat.* With a vessel of 78.0g it accounts for about a seventeenth of the thermal capacity of the water it holds, and part (vi) has to take it out again before the specific heat capacity of liquid Y can be reached.

Practical tip 4: *Stir at every reading, and read at eye level.* Heat arrives through the wall, so the layer of liquid against the copper is always ahead of the middle, and how far ahead depends on the liquid. Stir, and the thermometer speaks for the whole of the contents; and it has to be done the same way in both runs if the comparison is to mean anything.

Answers to the questions

(i) and (ii). Take t along the horizontal axis at 2cm to 2min, running from 0 to 14min, and θ up the vertical axis at 2cm to 5K, running from 25°C to 70°C. That is 14cm by 18cm, which fits an A4 sheet upright. Give the graph the title A graph of θ (°C) against t (min) and label each curve with the liquid it belongs to.



(iii) Both tangents are drawn at 55.0°C, where each curve crosses that ruling. For water the tangent passes through A (2.2, 44.8) and B (9.2, 61.3), so its slope is

$$S_w = \frac{61.3°C - 44.8°C}{9.2min - 2.2min} = \frac{16.5K}{7.0min} = 2.36Kmin^{-1}$$

For liquid Y the tangent passes through P (0.8, 42.7) and Q (7.2, 68.7), so

$$S_Y = \frac{68.7°C - 42.7°C}{7.2min - 0.8min} = \frac{26.0K}{6.4min} = 4.06Kmin^{-1}$$

(iv) Heat reaches the contents by crossing the wall of the calorimeter, and the rate at which it crosses depends on three things: how much hotter the bath is than the contents, how much of the wall is wetted on both sides, and how readily the copper and the layers of liquid against it pass heat. At 55.0°C the first is the same in the two runs, because the bath is held at 70.0°C for both. The second and third are the same to the extent that the vessel is the same and both liquids are stirred alike, which is why the Procedure insists on stirring; part (viii) takes up what is left of the difference. Grant that much and the rate of entry is the same, and it is that shared rate which makes the two slopes comparable at all.

(v) For the water run the thermal capacity of the calorimeter together with its contents is

$$C_w = 78.0g \times 0.38J/gK + 120g \times 4.2J/gK = 29.6JK^{-1} + 504JK^{-1} = 533.6JK^{-1}$$

and the rate at which heat is entering at 55.0°C follows from $S = P/C$ rearranged:

$$P = C_w S_w = 533.6JK^{-1} \times 2.36Kmin^{-1} = 1260Jmin^{-1}$$

(vi) The same $1260Jmin^{-1}$ is entering during the second run at 55.0°C, so the thermal capacity of the calorimeter with liquid Y in it is

$$C_Y = P/S_Y = 1260Jmin^{-1}/4.06Kmin^{-1} = 310JK^{-1}$$

The copper accounts for 29.6JK^{-1} of that, so the 120g of liquid Y accounts for 280.4JK^{-1}, and

$$c_Y = \frac{310\text{JK}^{-1} - 29.6\text{JK}^{-1}}{120\text{g}} = \frac{280.4\text{JK}^{-1}}{120\text{g}} = 2.34\text{J/gK}$$

Liquid Y therefore has a specific heat capacity of about 2.3J/gK, a little more than half that of water.

Accept 2.20J/gK to 2.50J/gK.

(vii) Heat crosses the wall at a rate set by how much hotter the bath is than the contents. As the contents warm, that difference shrinks: it is 42K at the start and only 15K by the time the thermometer reads $55.0°\text{C}$. The thermal capacity being warmed has not changed, so by $S = P/C$ the rate of rise falls in the same proportion, and the curve flattens as it approaches the bath temperature.

(viii) Equal masses of two liquids are not equal depths. If liquid Y is the less dense of the two, its 120g stands higher up the calorimeter and wets more copper, so more heat crosses in the second run than in the first. The method has no way of knowing that: it pins the rate of entry at the 1260Jmin^{-1} measured with water, and blames the faster warming on a smaller thermal capacity. The answer to (vi) therefore comes out too small. Take equal volumes instead of equal masses, fill to a scratch mark on the calorimeter, and find each mass by weighing after the run rather than before it. Then the wetted area is the same in both runs and only the contents have changed.

Examiner's eye

- Both slopes enter the answer only through their ratio, which is the thing to keep in mind as you draw them. A habit that flattens both tangents by the same fraction cancels out; one that flattens both by the same amount does not, because the two slopes are far apart; and a slip on one tangent alone costs the whole answer.
- Nothing requires the two runs to start at the same temperature. The comparison is made at one temperature and not at one time, so a run that began a kelvin warmer simply reaches $55.0°\text{C}$ a little sooner. A second run that set out from $28.5°\text{C}$ is a perfectly good run, and a answer that throws one away has lost the question for no reason at all.
- A calorimeter with an open mouth loses heat as well as gaining it, and what P really measures is the difference. Evaporation from 120g of water at $60°\text{C}$ can carry off a tenth of what the wall lets in, and it will not carry off the same fraction from liquid Y. A lid of card with two holes in it is the work of a minute and removes most of the objection.
- The immersed stem of the thermometer is warmed along with the liquid and appears in nobody's arithmetic. For a stem in 120g of water it is worth roughly 5JK^{-1} against 533.6JK^{-1}, which is about 1%: not fatal, but say so in your account and the mark is yours.

Practice 16.1

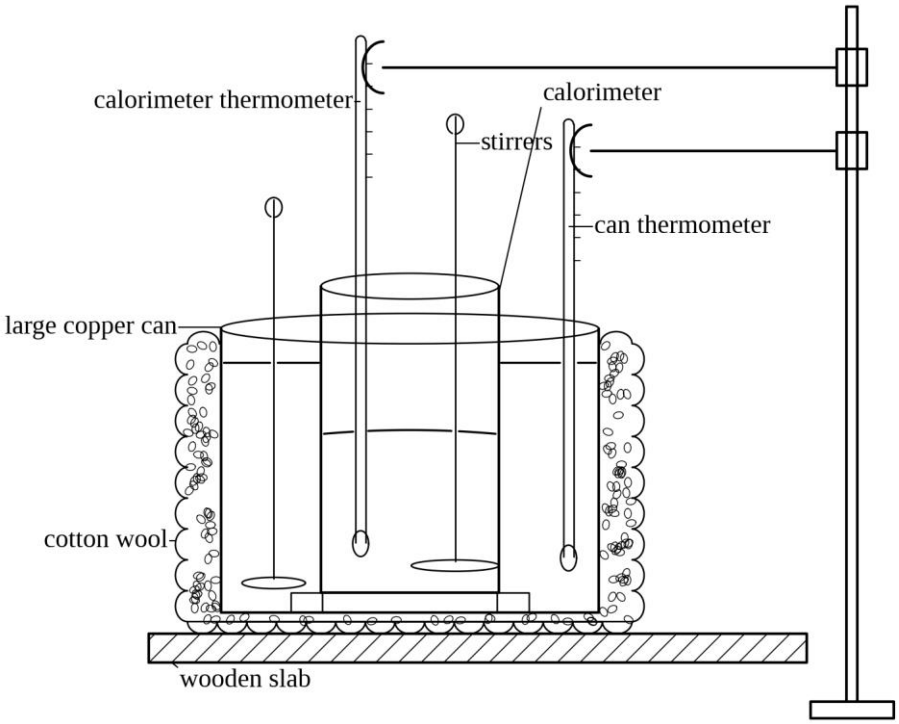
At the bus stand there is an urn of tea the size of an oil drum, and a notice tied to its handle with sisal string.

TEA HOT ALL DAY

DO NOT LIFT THE LID

THE STEAM IS THE TEA

Kipanga wanted to know how much was left in it, on the grounds that he was third in the queue. **Kipute** said there was one way to find that out and it began with lifting the lid. The man with the urn folded his arms and looked at the notice, and then at **Kipute**, and then at the notice again. **Mr. Akilikubwa** bought three cups and did not argue. He held his own cup against the flank of the urn for a moment, and told **Kipute** that a man who may not lift a lid can still weigh what is under it.



Procedure

Proceed as follows:

(a) Pour 150g of cold water into the dry copper calorimeter, whose own mass with its stirrer is 78.0g, and record its temperature.

(b) Record the temperature of the water in the large copper can, whose own mass with its stirrer is 140g, and the temperature of the room.

(c) Lower the calorimeter into the can on its cork ring and start the stopwatch at the same instant.

(d) Stir both vessels gently and continuously, and read both thermometers at the same instant every 1min for 24min.

(e) Record all your readings in a suitable table.

Questions

(i) Tabulate your readings.

(ii) On the same axes plot a graph of temperature θ against time t for both vessels.

(iii) Read from your graph the temperature at which the two curves run together, and the time at which they first do so.

(iv) Calculate the heat gained by the calorimeter and its water up to that instant.

(v) Hence determine the thermal capacity of the large can together with its contents, and the mass of hot water it held. The can is lagged all round and stands on a wooden slab.

(vi) Explain why both curves continue to fall, together, after they have met.

(vii) State, with a reason, whether your value in (v) is above or below the true mass.

(viii) Suggest one change to the apparatus that would improve the determination.

Experiment 16.2

Kipute's mother keeps her good cloth in a wooden chest with a handful of white balls scattered among the folds, and she buys the balls in the market by the twist of newspaper. Twice a year she complains that they have got smaller. Nothing has spilled and none has leaked; the chest has stood in the same corner since before **Kipute** was born; and the balls are half the size they were.

Kipute told her it was his sister, who liked the smell.

Mr. Akilikubwa heard the story on Monday. *"Then she has been at them since before you were born,"* he said, *"through the lid of a shut chest, and she has taken the same share off every ball in it."* He added that he would be glad of one of the balls if there were any left. **Kipanga** offered to fetch a whole twist of them from the market for the price of the bus fare there, and was taken up on it. On Thursday there was a jar of white balls on the front bench, and beside the jar a test tube, a wire stirrer and a thermometer.

Aim

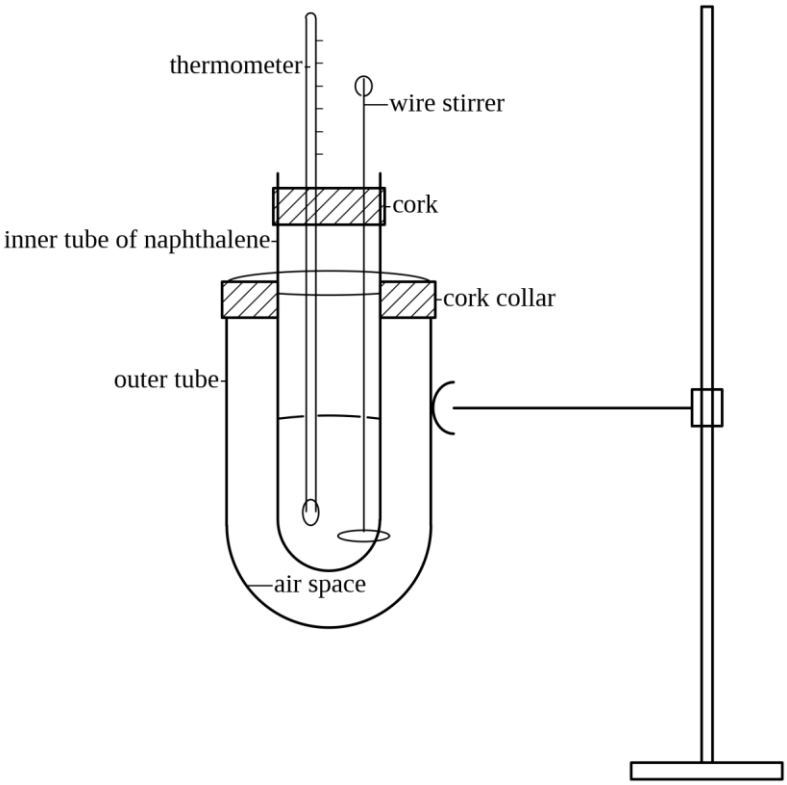
To determine the melting point of naphthalene and the specific heat capacity of solid naphthalene from a cooling curve.

Apparatus

12.0g of naphthalene in a test tube; a wider boiling tube to serve as an air jacket, with a cork collar to hold the test tube upright and central in its mouth; a cork carrying a thermometer reading to 0.5K and a wire stirrer; a retort stand with a clamp; a stopwatch; a beaker of water on a tripod with a burner, brought to the boil and used only to melt the sample. The thermal capacity of the tube, the stirrer and the immersed part of the thermometer is 26.0JK^{-1}, and the specific heat capacity of liquid naphthalene is 1.70J/gK.

Setup

The arrangement is the one drawn below. The tube of naphthalene hangs in the mouth of the wider tube on a cork collar, clear of its closed bottom, so that a layer of air separates the sample from the room; the outer tube is clamped upright and stands in still air, away from any window or doorway. The thermometer bulb is in the middle of the naphthalene and the stirrer reaches to the bottom of it. Before the run the assembly is warmed in a beaker of hot water until the naphthalene is completely liquid and reads 98.0°C, and is then lifted out and clamped as drawn. The room is at 26.0°C.



Procedure

Proceed as follows:

(a) Weigh the naphthalene into the inner tube and fit the cork carrying the thermometer and the stirrer.

(b) Stand the assembly in the boiling water bath until all the naphthalene has melted and the thermometer reads 98.0°C.

(c) Lift the assembly out, clamp it upright in still air, and start the stopwatch at the same instant.

(d) Stir the naphthalene gently and continuously, and read its temperature every 2min for 42min. Go on stirring after the sample begins to set, for as long as the stirrer will move.

(e) Record the temperature of the room, and weigh the tube and its contents again at the end of the run.

(f) Record all your readings in a suitable table.

Questions

(i) Tabulate your readings.

(ii) Plot a graph of temperature θ against time t.

(iii) Read from your graph the melting point of naphthalene.

(iv) Explain why the temperature stays constant while the naphthalene is solidifying.

(v) Draw a tangent to the curve where it meets the flat part from above, and another where it leaves the flat part below, and determine the rate of fall of temperature in each case.

(vi) Hence determine the specific heat capacity of solid naphthalene.

(vii) Account for the small dip in the curve just before the flat part begins.

(viii) State one precaution you took while the sample was setting, and give the reason for it.

Table of results

The tube was lifted out of the bath at 98.0°C and the thermometer read every 2min while the naphthalene was stirred, for as long as the stirrer would move. The naphthalene weighed 12.0g into the tube and 11.9g at the end of the run, so less than 1% of it had gone off as vapour and the starting mass is the one used below. The room stood at 26.0°C throughout.

t (min)	θ (°C)	t (min)	θ (°C)
0	98.0	22	80.0
2	93.5	24	80.0
4	89.5	26	80.0
6	85.5	28	80.0
8	82.0	30	80.0
10	79.0	32	79.5
12	80.0	34	76.5
14	80.0	36	73.0
16	80.0	38	70.0
18	80.0	40	67.0
20	80.0	42	64.0

Making the measurement count

Practical tip 1: *A run of identical readings has to be plotted, every one of them.* There are 22 points in this table and 10 of them carry the same number. The examiner is looking for the flat, and a flat drawn through two points is only a line drawn through two points.

Practical tip 2: *Naphthalene evaporates slowly even when solid, and freely when hot.* That is why the balls in a clothes chest grow smaller with nothing spilled, and it is why the tube is weighed at both ends of the run. Melt

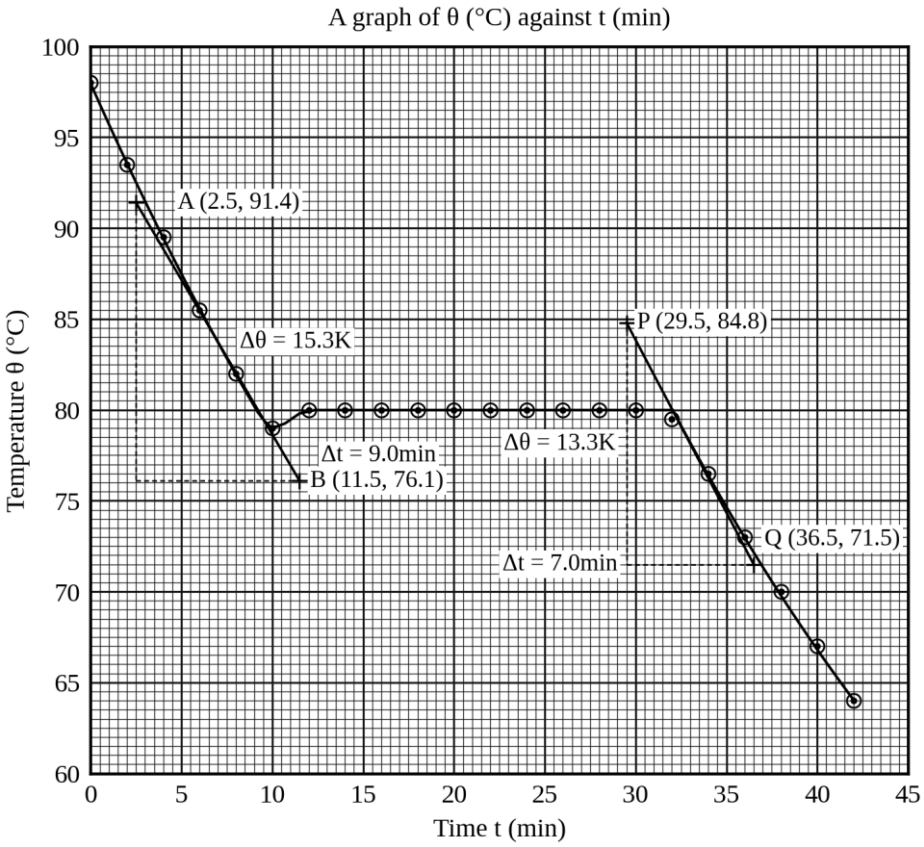
the sample near an open window and keep the tube stoppered between runs; the cooling itself has to be done in still air, so do that part of it away from the window and do not lean over the mouth of the tube.

Practical tip 3: *Where you take a tangent matters as much as how you draw it.* Part (v) names the two places and part (vi) turns on your having used them and no others. Take each tangent where the curve actually touches the flat, not a division away from it, and lay the ruler along the smooth line you drew rather than along the last two points.

Practical tip 4: *Errors in the two slopes do not stay the size you made them.* The thermal capacity of the tube and its fittings is supplied because you cannot measure it, and it is larger than the sample's own. Part (vi) ends by subtracting one from the other, so a ratio read to 1% arrives in the answer as about 3%. Draw both tangents with the same care, and do not quote the answer to more than three figures.

Answers to the questions

(i) and (ii) Take t along the horizontal axis at 2cm to 5min, running from 0 to 45min, and θ up the vertical axis at 2cm to 5K, running from 60°C to 100°C. That is 18cm by 16cm, which fits an A4 sheet upright. Give the graph the title A graph of θ (°C) against t (min).



(iii) The curve reaches 80.0°C at about the ninth minute and holds there, dip apart, until about the thirty-second, so the melting point of naphthalene is 80.0°C. Accept 79.5°C to 80.5°C. The accepted value is 80.2°C, which the reading matches to within one division of the thermometer.

(iv) The tube is losing heat to the room throughout, before, during and after the flat, because it is hotter than the room throughout. While the naphthalene is turning from liquid to solid, the heat it gives up is the latent heat released by that change, and it is released at just the rate at which the tube is losing heat. Nothing is being drawn from the store of energy that shows itself as temperature, so the thermometer does not move until the last of the liquid has set.

(v) Above the flat the tangent passes through A (2.5, 91.4) and B (11.5, 76.1), giving a rate of fall of

$$S_l = \frac{15.3\text{K}}{9.0\text{min}} = 1.70\text{Kmin}^{-1}$$

Below the flat the tangent passes through P (29.5, 84.8) and Q (36.5, 71.5), giving

$$S_s = \frac{13.3\text{K}}{7.0\text{min}} = 1.90\text{Kmin}^{-1}$$

(vi) The solid cools faster than the liquid did, at the same temperature and in the same room. Just above and just below the flat the tube is at 80.0°C and the room is at 26.0°C, and the outside of the assembly is in much the same state in the two cases, so heat is leaving at the same rate P. Apply S=P/C on each side of the flat and equate the two rates of loss:

$$C_s S_s = C_l S_l \quad \text{so} \quad C_s = C_l \frac{S_l}{S_s}$$

The thermal capacity of the tube with the liquid in it is

$$C_l = 12.0\text{g} \times 1.70\text{J/gK} + 26.0\text{JK}^{-1} = 20.4\text{JK}^{-1} + 26.0\text{JK}^{-1} = 46.4\text{JK}^{-1}$$

and the ratio of the two slopes is 1.70Kmin^{-1} divided by 1.90Kmin^{-1}, which is 0.895, so

$$C_s = 46.4\text{JK}^{-1} \times 0.895 = 41.5\text{JK}^{-1}$$

Take out the tube once more and the solid naphthalene is left:

$$c_s = \frac{41.5\text{JK}^{-1} - 26.0\text{JK}^{-1}}{12.0\text{g}} = \frac{15.5\text{JK}^{-1}}{12.0\text{g}} = 1.29\text{J/gK}$$

Solid naphthalene therefore has a specific heat capacity of 1.29J/gK, appreciably less than the 1.70J/gK of the liquid. The specific heat capacity of a solid climbs with temperature, so a determination made at the melting point should sit a little above the figure tabulated for a cool room, and the band allows for it. Accept 1.15J/gK to 1.55J/gK. That is a marking tolerance, not the precision of one carefully drawn graph.

(vii) The liquid was carried about 1K below its freezing point before the first crystals appeared. Until a crystal exists there is nothing for the rest of the liquid to freeze on to, so the sample goes on cooling past the temperature at which it ought to have set. The moment the first crystals do form, the latent heat they release warms the sample back to 80.0°C, and the flat begins from there. Stir, and the dip is smaller, because you give the crystals somewhere to start; but no amount of stirring abolishes it.

(viii) Stir continuously for as long as the stirrer will move, and go on stirring well into the flat. A crust of solid forms first against the glass, where the naphthalene is coolest, and it insulates the liquid in the middle from the wall. An unstirred sample gives a flat that sags all the way along instead of one that is level, and the melting point read from it is too low. Half a division at the very end, where the last of the liquid sets round the bulb, is normal even in a stirred sample.

Examiner's eye

- Naphthalene shrinks as it sets, by about a tenth of its volume, and the solid pulls away from the glass as it does so. That puts a film of air where there was none above the flat, and it is the one respect in which the tube is genuinely not the same vessel on the two sides of the plateau. Say so, and say which way it pushes the answer: less heat escapes, the solid cools more slowly than it should, and the ratio makes its thermal capacity look large.
- Very little of the answer hangs on the second weighing. The mass enters part (vi) twice, once inside C_l and once in the final division, and the two entries pull against each other, so 3% of the naphthalene can go off as vapour and move the specific heat capacity by only about 0.5%. What the weighing at the end of the run buys you is the size of the loss: 3% is one thing and 30% quite another, and only the balance will tell you which of them you had.
- Which way up is the ratio? The solid cools faster, so the solid must be the one with the smaller thermal capacity, and C_s must come out below C_l. A value of 51.9JK^{-1} for the solid means the fraction went in upside down; the arithmetic will not tell you so, but the physics will.
- A flat that slopes is a real observation and deserves a sentence. An impure sample does not have one melting point but a range, and its curve falls slowly all the way along instead of holding level. Say which of the two you saw.

Practice 16.2

The school bought a box of coloured candles for the end-of-year concert and somebody stood the box on the windowsill of the physics room, where the sun comes in after eleven. By the evening the candles had settled into one another and set again as one object, and no amount of prising would tell you where one of them ended and the next began.

Kipanga was certain the sun could not have done it. A candle, he said, burns at the very top and stays hard everywhere else, so wax plainly needs a flame under it and a windowsill is not a flame; somebody had stood the box against the wall of the boiler house and would not own up to it. **Kipute** bet him a soda that the windowsill was enough, and went off to fetch a second box and settle it. He came back empty handed. The school had bought one box, and the one box was on the bench in front of them.

The fused block stayed there for a week. **Mr. Akilikubwa** weighed it on the Friday, wrote the mass on the board, and left it standing with nothing beside it. The block will tell anybody who looks at it that the wax melted. It will not say how warm the sill had to get, and it will not say how much heat went in before the first drop ran.

Procedure

Proceed as follows:

(a) Weigh 15.0g of solid X into the inner tube of the assembly of Experiment 16.2 and fit the cork carrying the thermometer and the stirrer. The thermal capacity of the tube, the stirrer and the immersed part of the thermometer is 26.0JK^{-1}. Solid X is a powder; pack it down round the thermometer bulb so that no air pocket is left against it.

(b) Bring the bath to 95.0°C and hold it there.

(c) Lower the assembly into the bath and start the stopwatch at the same instant.

(d) Read the temperature of the solid every 2min for 38min, and stir as soon as there is enough liquid for the stirrer to move in.

(e) Record all your readings in a suitable table.

Questions

(i) Tabulate your readings.

(ii) Plot a graph of temperature θ against time t.

(iii) Read from your graph the melting point of solid X, and the length of time for which the temperature held steady.

(iv) The specific heat capacity of solid X is 1.60J/gK. Draw a tangent to the curve where it meets the flat part from below, determine the rate of rise of temperature there, and hence the rate at which heat is entering the tube.

(v) Determine the specific latent heat of fusion of solid X.

(vi) Explain why the rate of rise just above the flat part is smaller than the rate just below it.

(vii) Estimate the uncertainty in your value in (v), and state which of your measurements contributes most to it.

(viii) State one way in which this curve would differ from a cooling curve of the same substance, and give the reason.

Experiment 16.3

A letter came to the school from the *soda* depot at the crossroads, written on the back of an invoice.

> Dear Sir,
>
> We are told you teach the boys about heat. We buy ice from the town and we put it in the crates with the *sodas*. Some days we buy too much and it melts in the store, and some days we buy too little and the drivers complain. Nobody here can tell us how much we ought to buy, and the man who sells the ice is not a person we would ask.
>
> Yours faithfully,
>
> J. M. Mwakabana, Depot Manager
>
> P.S. Please do not send the answer in degrees. We buy ice by the kilogram.

Mr. Akilikubwa read the whole of it aloud. **Kipute** wanted to know how many crates there were to a lorry, and **Kipanga** said the depot should send the lorry to the school and let the physics room deal with it. Nobody in the room could say how much heat a kilogram of ice is worth. **Mr. Akilikubwa** turned the invoice over and read the postscript out a second time.

Aim

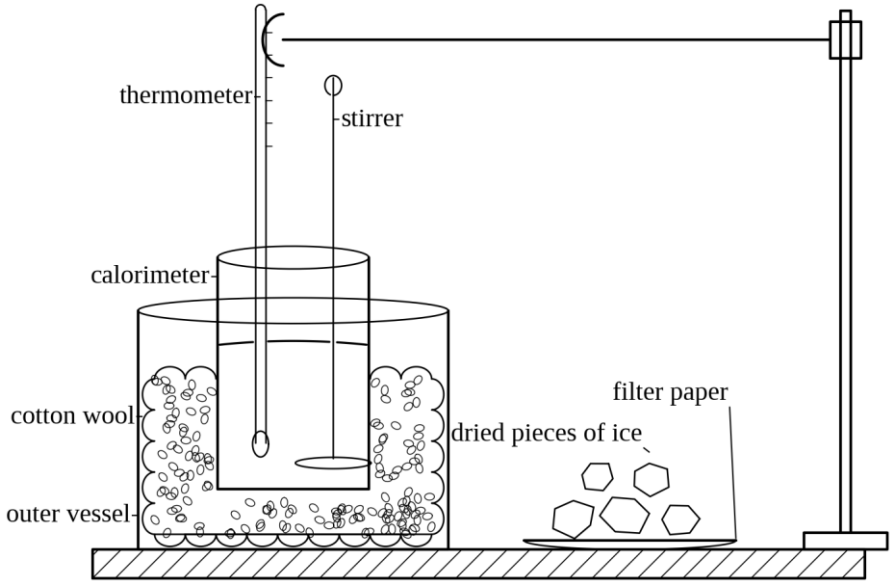
To determine the specific latent heat of fusion of ice by the method of mixtures.

Apparatus

A copper calorimeter with its stirrer; an outer vessel with cotton wool lagging; a thermometer reading to 0.5K; a balance reading to 0.1g; a retort stand with a clamp; a quantity of ice from the icebox of a refrigerator, broken small; filter paper; warm water. The specific heat capacity of ice is 2.1J/gK.

Setup

The arrangement is the one drawn below. The calorimeter stands inside the outer vessel with cotton wool packed all round it and under it, and the thermometer is clamped with its bulb in the middle of the water. The ice is broken into pieces the size of a thumbnail and each piece is pressed between two sheets of filter paper immediately before it goes in, so that no water is carried in with it. The room is at $26.0°\text{C}$ and the water is warmed to $42.0°\text{C}$ before the run begins; do not warm it further than that.



Procedure

Proceed as follows:

(a) Find the mass of the dry calorimeter together with its stirrer.

(b) Pour in about 120g of warm water at $42°\text{C}$, and find the mass again.

(c) Place the calorimeter in the lagging, clamp the thermometer in it, stir, and record the temperature of the water and the temperature of the room.

(d) Dry each piece of ice between filter papers and drop it in, stirring all the time, until a piece takes a long while to disappear and the thermometer is barely moving. Add no more ice after that.

(e) Go on stirring until the temperature stops falling, and record the lowest steady reading.

(f) Find the mass of the calorimeter, the stirrer and all the water in it.

(g) Record all your readings in a suitable table.

Questions

(i) Tabulate the masses and the temperatures you recorded.

(ii) State the mass of warm water and the mass of ice used.

(iii) Calculate the heat given out by the calorimeter and its water.

(iv) Calculate the heat taken in by the melted ice in warming from 0°C to the final temperature.

(v) Hence determine the specific latent heat of fusion of ice.

(vi) The accepted value is 334J/g. Using the specific heat capacity of ice, estimate how far below 0°C the ice must have been when it was added.

(vii) Explain why the water is warmed to about as far above room temperature as the mixture is expected to finish below it.

(viii) Suggest one improvement to the procedure.

Table of results

The masses were found on one balance and the temperatures on one thermometer, read to half a division.

Quantity	Reading
mass of calorimeter and stirrer	78.0g
mass of calorimeter, stirrer and warm water	198.0g
mass of calorimeter, stirrer and all water at the end	238.2g
initial temperature of the water θ_1	42.0°C
final temperature of the mixture θ_2	12.0°C
temperature of the room	26.0°C

Making the measurement count

Practical tip 1: *Ice will not stay still on a balance pan, so it is weighed as part of something else, twice over.* What the balance gives you is the calorimeter and its water before, and the calorimeter and all its water after; the difference is the ice, and it is the only way of getting at it that does not involve holding melting ice on a balance pan.

Practical tip 2: *Wet ice is not ice.* A film of water at 0°C clinging to a lump weighs the same as ice and needs no latent heat at all, so every gram of it you carry in makes the answer smaller. Two sheets of filter paper and a firm press, one lump at a time.

Practical tip 3: *Small pieces here, and one at a time.* A lump the size of an egg is still melting when you take the final reading, and a mixture that is still changing has no final reading. Break the ice small, add it a piece at a time, and let each piece disappear before the next goes in. Practice 16.3 asks for the opposite and explains why it can afford to.

Practical tip 4: *The heat balance has three terms and only two of them are obvious.* The water and the copper give out heat as they cool; the ice takes in heat to melt; and the water that the ice becomes then takes in more heat to warm from 0°C up to the final temperature. Part (iv) is that third term. Leave it out and you lose the question, which is the commonest way of losing it.

Answers to the questions

(i) and (ii). The mass of warm water is 198.0g less 78.0g, which is 120.0g, and the mass of ice is 238.2g less 198.0g, which is 40.2g.

(iii) The thermal capacity of the calorimeter with its water is

$$C=78.0g\times0.38J/gK+120.0g\times4.2J/gK=29.6JK^{-1}+504JK^{-1}=533.6JK^{-1}$$

and it cooled from 42.0°C to 12.0°C, a fall of 30.0K, so the heat it gave out is

$$533.6JK^{-1}\times30.0K=16008J$$

(iv) The 40.2g of melted ice is water at 0°C which then has to be warmed to 12.0°C:

$$40.2g\times4.2J/gK\times12.0K=2026J$$

(v) Everything the calorimeter gave out went either into melting the ice or into warming the water it became, so the heat used in melting alone is

$$16008J-2026J=13982J$$

and that melted 40.2g of ice, so

$$L=\frac{13982J}{40.2g}=348J/g$$

Accept 330J/g to 365J/g. The band has to reach down to the accepted value, because ice that had been standing in water would bring the answer down to it.

(vi) The value found is 348J/g against an accepted 334J/g, an excess of 14J/g. Ice that starts below zero has to be warmed to 0°C before any of it melts, and that heat comes out of the same calorimeter; the arithmetic in (v) has no way of telling it apart from latent heat and credits all of it to L. Ice takes 2.1J/gK to warm, so

$$\Delta\theta=\frac{14J/g}{2.1J/gK}=6.7K$$

The ice was about 7K below zero when it went in, and that is the excess accounted for.

(vii) The calorimeter is exchanging heat with the room throughout the run, and the lagging slows that exchange without stopping it. While the mixture is above 26.0°C it loses heat to the room; while it is below, it gains heat from the room; and the two effects have opposite signs. Begin 16.0K above the room and finish 14.0K below it, and the second nearly cancels the first; what is left is far smaller than the 14J/g that part (vi) has already accounted for.

(viii) Stand the ice in a dish with a little water in it for a few minutes before the run, so that its surface is visibly wet and it is certainly at 0°C, and dry each piece between filter papers only at the instant it goes in. That removes the whole of the 14J/g the answer is carrying, at the cost of a little more care with the filter paper.

Examiner's eye

- There is one instant in the whole run that carries the answer, and it is the lowest steady reading. While any ice is left the thermometer is still coming down; the moment the last of it goes, the room begins to warm the mixture back up again. Watch the column continuously through that turning point instead of glancing at it once a minute. Practice 16.3 exists because that instant is easy to miss.
- Ice at 0°C is an assumption, and this question is built to expose it. Part (vi) asks for more than the word "losses": the arithmetic points at one cause and gives its size, and an answer that names a cause without a number has not used the figures it was given.
- This question needs no graph and no stopwatch. What it needs is a balance read to 0.1g and the discipline to record every weighing at the moment it is made, because the mass of ice is a difference of two masses and neither of them can be recovered afterwards.
- Room temperature belongs in the table even though no part of the determination of L uses it. It is the figure that justifies the choice of 42.0°C to start with, and part (vii) cannot be answered without it.

Practice 16.3

There is an exercise book in **Mr. Akilikubwa**'s cupboard with his own name on it in a schoolboy hand, and once a year he takes it out and opens it at the same page. The page carries a heading, a table of masses, four lines of faultless arithmetic and an answer, and beside the answer a red line and six words: *See me. Where was the room?*

Kipute read the four lines twice and could find no fault in them. **Kipanga** checked the multiplication on his fingers and got the same figure. **Mr. Akilikubwa** said that his teacher had been right, and that it had taken him a fortnight to see why. **Kipute** asked him to save them the fortnight. He said no, and handed over the stopwatch.

Procedure

Proceed as follows:

(a) Find the mass of the dry calorimeter together with its stirrer, pour in warm water at about 52°C, and find the mass again.

(b) Stand the calorimeter bare on the bench, with none of the lagging of Experiment 16.3 round it, record the temperature of the room, clamp the thermometer in the water, start the stopwatch, and read the temperature every 1min, stirring gently throughout.

(c) At the fifth minute exactly, add all the dried ice at once and go on reading every minute and stirring.

(d) Continue reading for 8min after the temperature has passed its lowest value.

(e) Find the mass of the calorimeter, the stirrer and all the water in it.

(f) Record all your readings in a suitable table.

Questions

(i) Tabulate your readings.

(ii) Plot a graph of temperature θ against time t.

(iii) Draw the best straight line through the readings taken before the ice was added and another through the lowest reading and all those after it. Erect a vertical line at the middle of the time during which the temperature fell rapidly, produce each straight line to meet it, and read the two temperatures θ_1 and θ_2 there.

(iv) Calculate the heat given out by the calorimeter and its water between those two temperatures.

(v) Hence determine the specific latent heat of fusion of ice.

(vi) Work out the value you would have obtained from the last reading before the ice was added and the lowest reading after it, and state by how much the two answers differ.

(vii) Explain why the corrected value is the better of the two.

(viii) The accepted value is $334 J/g$ and the specific heat capacity of ice is $2.1 J/gK$. Account for the difference that remains.

PART FOUR

CURRENT ELECTRICITY

Chapter 17

OHM'S LAW AND DIRECT RESISTANCE

Everything in the question sets that follow is one measurement made over and over: a current and a potential difference, read at the same instant. What changes is where the two meters are connected and what is done with the quotient afterwards. Learn to see that one measurement inside each arrangement and the chapter stops being a list of experiments and becomes one, tilted eight ways.

The quotient itself is the resistance, $R=V/I$, and the graph the paper asks for is almost always V against I. What can be read off such a graph, and what cannot, is the business of the question sets that follow.

One idea runs under the whole of Part Four and this chapter is where it is met first. **The meters are part of the circuit.** An ammeter has resistance of its own and a voltmeter draws a current of its own, so a meter changes the thing it is put there to measure. Half the sets below turn on it: two measure the resistance of a voltmeter, one has to correct for the resistance of an ammeter, and one gets a meter's own current to cancel itself. In the other four it is small enough to leave alone, and the first of the four says so and says which way.

The same few instruments serve throughout: dry cells in a holder, a key, a resistance box, a rheostat, an ammeter reading to 0.01A in six of the eight sets, and a voltmeter reading to 0.05V in all eight. What changes is what they are connected to. Resistances stay in ohms and currents in amperes from the first line to the last. Where a question needs a constant the question supplies it, exactly as the instructions page does.

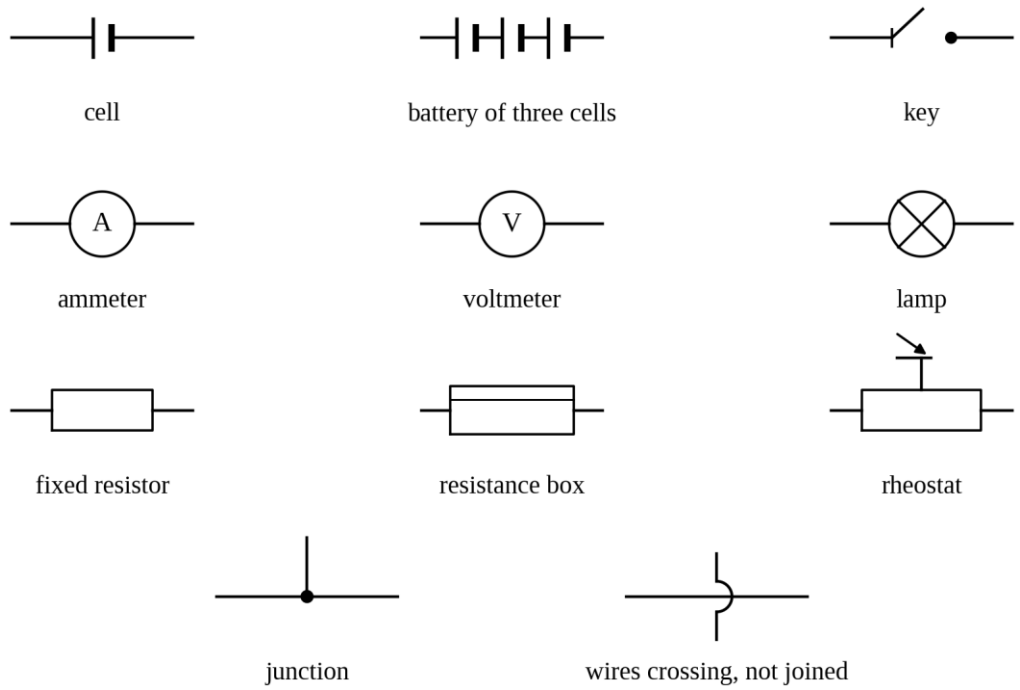
The four varieties at a glance

	What you do	What comes out of it
17.1	A resistance box steps the current through a coil, with the voltmeter across the coil alone	the resistance of the coil, from the slope of V against I
17.2	A rheostat sweeps the current through two coils cut from one reel of wire, one twice as long as the other	Ohm's law verified for a metal, and what the slope of a line belongs to
17.3	A resistance box in series with the voltmeter itself, and no ammeter anywhere in the loop	the emf of the battery and the resistance of the voltmeter, from one graph of 1/V against R
17.4	Two resistors between one pair of terminals, singly and then joined the two ways	the series and the parallel rule, each tested against a measurement of the pair

All four are worked below in full. The practice questions move one lead of the voltmeter, put a torch bulb where a coil was, change both the meter and the battery, and the last of them measures a resistance the ammeter reads too coarsely by putting a known one beside it.

Drawing the circuit

Every question in Part Four rests on a circuit diagram, whether or not it asks you to draw one, and more marks are lost on that one drawing than on any other single item in the paper. The remedy is on this page. The symbols are these.



Six rules turn those symbols into a diagram that earns its marks.

1. Draw with a ruler and a sharp pencil. Conductors are straight lines meeting at square corners, and a diagram drawn freehand is read as a diagram drawn carelessly.
2. Draw the outline of the circuit first, as one closed loop, and put the components into it afterwards. Then no component can end up in a branch it does not belong to.
3. The ammeter goes in the line, so that the current you are measuring has to pass through it. The voltmeter goes across the thing whose potential difference you want, and never in the line when a potential difference is what you are after. Twice in this chapter one is put in the line on purpose, and the question that does it says why.
4. Put a dot wherever three or more conductors meet, and no dot where two conductors merely cross. A missing dot turns a parallel combination into a series one on the page.
5. Label every component, and give the cell its long plate on the positive side. A rectangle with nothing beside it could be a resistor, a box or a rheostat.
6. Check the drawing by tracing the loop with a finger from one terminal of the cell back to the other. Anything your finger does not reach is not connected.

Experiment 17.1

The man who mends radios at the end of the market keeps his spare coils in a biscuit tin, and the paint has come off most of them. When **Kipanga** asked for one of about 10Ω the man tipped the tin out on the counter, picked three up in turn, and held each one against the back of his hand for a moment while a cell and a lamp were joined across it.

"This one is small," he said, putting it down. *"This one is too big, you will get nothing through it. Take this one."*

Kipute asked how near 10Ω it was. The man said it was near enough for a radio, and that a radio has never yet complained. **Mr. Akilikubwa** bought the coil, paid for it without arguing, and carried it to Thursday's lesson still in its twist of newspaper.

Aim

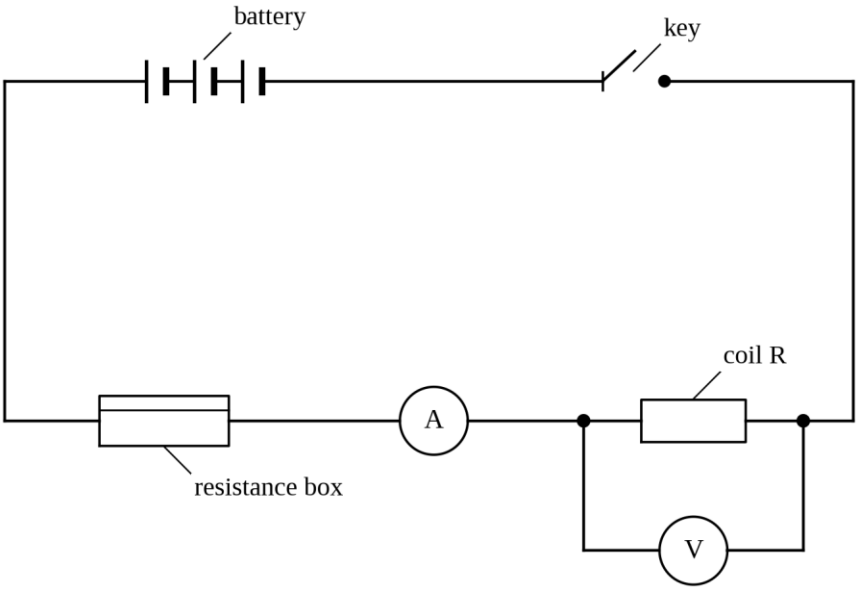
To determine the resistance of a coil from a graph of potential difference against current.

Apparatus

A battery of three dry cells in a holder; a key; a resistance box; an ammeter reading to 0.01A; a voltmeter reading to 0.05V; the coil of unknown resistance, labelled R; connecting wires with crocodile clips.

Setup

The circuit is the one drawn below. The resistance box is in series with the battery, the key, the ammeter and the coil, so that its setting fixes the current everywhere in the loop. The voltmeter is connected across the coil alone, and across no other part of the loop. Keep the key open between readings, so that no current flows except while a reading is being taken.



Procedure

Proceed as follows:

(a) Connect the circuit as drawn, with the resistance box set to 17Ω, and check it by tracing the loop before you close the key.

(b) Close the key, read the ammeter and the voltmeter together, and open the key again.

(c) Set the box to 14Ω and repeat step (b).

(d) Repeat step (b) with the box at 11Ω, 8Ω, 5Ω and 2Ω.

(e) Record all your readings in a suitable table.

Questions

(i) Tabulate your readings.

(ii) Plot a graph of the potential difference V against the current I.

(iii) Draw the best straight line through your points and determine its slope.

(iv) State what the slope represents, and give the resistance of the coil.

(v) Explain why the line should pass through the origin.

(vi) The battery ran down slightly while the readings were being taken. Explain why this does not disturb your answer to (iv).

(vii) The voltmeter takes a small current of its own. State, with a reason, whether this makes your answer to (iv) larger or smaller than the true resistance of the coil.

(viii) State one change you would make to this circuit before using it on a coil of about 500Ω, and give the reason.

Table of results

The box was stepped from 17Ω down to 2Ω, and the two meters were read at the same instant, the ammeter to 0.01A and the voltmeter to 0.05V.

box (Ω)	I (A)	V (V)
17	0.20	0.95
14	0.23	1.05
11	0.27	1.25
8	0.32	1.50
5	0.41	1.95
2	0.56	2.65

Making the measurement count

Practical tip 1: *Both meters are read at the same instant, and the key is open in between.* Current warms a coil, a warm coil has a different resistance, and a run taken with the key held down for 2 minutes measures a coil whose resistance drifted while you worked. Close, read the pair, open.

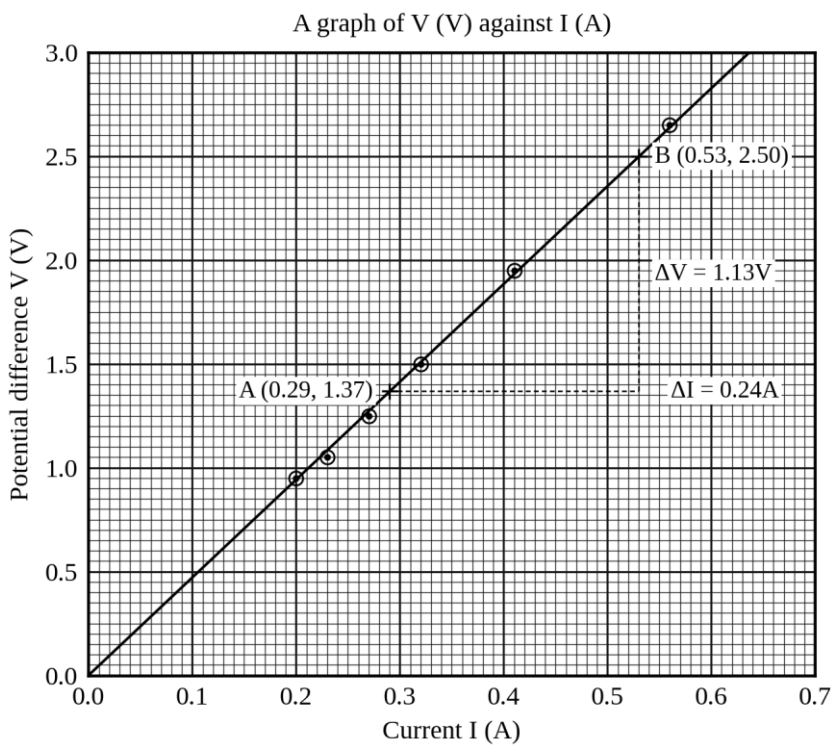
Practical tip 2: *The resistance box is not the thing being measured.* It is there to move the current, and its settings never appear in the analysis at all. Do record them, because they are what makes the run repeatable, but do not be tempted to plot against them.

Practical tip 3: *A line forced through the origin is not a line fitted to the points.* Part (v) asks what entitles you to the origin, and you should have that answer in your head before the ruler touches the paper. Draw the line through the origin and through the middle of the scatter, and if the points plainly want a line that misses it, say so in your account rather than forcing it.

Practical tip 4: *Take the slope from the line and never from one pair of readings.* A quotient formed from the largest current uses one row out of six and throws the other five away, and part (iii) is where the marks for the graph are actually paid.

Answers to the questions

(i) and (ii). Take I along the horizontal axis at 2cm to 0.1A, running from 0 to 0.7A, and V up the vertical axis at 2cm to 0.5V, running from 0 to 3.0V. That is 14cm by 12cm, which fits an A4 sheet upright. Title the graph "A graph of V (V) against I (A)".



(iii) The points lie on a straight line through the origin. Taking A (0.29, 1.37) and B (0.53, 2.50) on that line,

$$\text{slope}=\frac{2.50\text{V}-1.37\text{V}}{0.53\text{A}-0.29\text{A}}=\frac{1.13\text{V}}{0.24\text{A}}=4.71\Omega$$

(iv) For a conductor obeying $V=IR$ the quotient $R=V/I$ is the resistance, so the slope of a graph of V against I is the resistance itself. The coil has a resistance of 4.71Ω. Accept 4.55Ω to 4.90Ω.

(v) With the key open there is no current in the coil and no potential difference across it, so the point (0, 0) belongs to the graph as surely as any reading in the table. Any conductor whose resistance does not depend on the current gives a line through that point.

(vi) A cell that has run down drives a smaller current, and a smaller current makes a smaller potential difference across the same coil. Both readings of a pair fall together and their quotient does not move, so the point simply slides down the same line towards the origin instead of leaving it. What the run would lose is range, not slope, and the range here is wide enough to spare.

(vii) Smaller. The ammeter is in the main line, so what it reads is the current in the coil together with the small current the voltmeter takes for itself. The plotted I is therefore larger than the current actually in the coil while V is right, and a quotient with too large a denominator is too small. Practice 17.1 puts a number on it for a coil of 500Ω.

(viii) Use a more sensitive ammeter, one reading to 0.001A rather than 0.01A. With 500Ω in the loop the largest current three cells can drive through the coil is under 0.01A, which is one division of the present meter, and a table of readings all equal to one or two divisions cannot support a line at all. A larger battery is the wrong answer: it would raise the current, and it would also raise the potential difference beyond the range of the voltmeter.

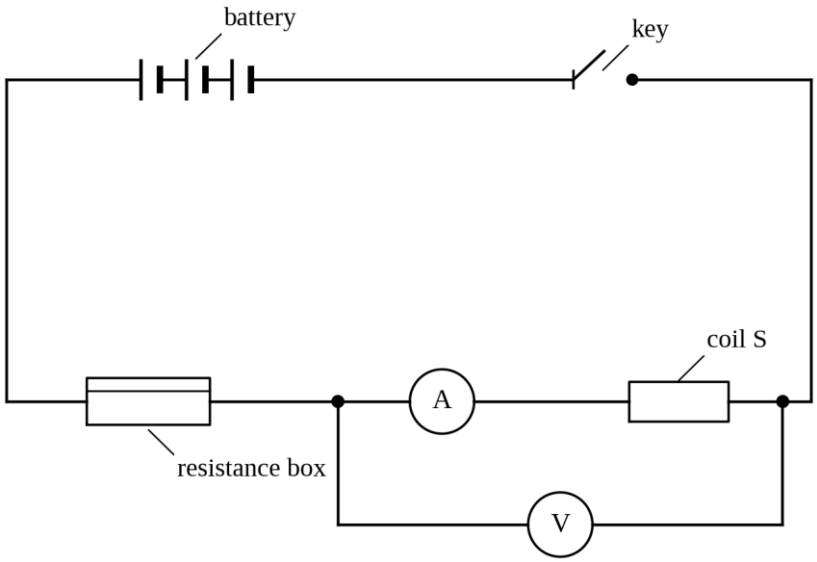
Examiner's eye

- Number the rows of your table. When one point plots badly the only way back to it is the row it came from, and a table with no row numbers costs you a re-reading of all six rows to find the one you want.
- A graph of V against I and a graph of I against V are both correct and their slopes are reciprocals. Read the question and plot what it names first up the vertical axis, then say in your answer which way round you have it. Half a mark hangs on the sentence and the whole answer hangs on the slope.
- Only one of the two meters is in the main line, and only one of them is reading exactly the quantity you want. Anything the chapter says about a meter disturbing a measurement comes back to that: a meter you can ignore is a meter whose effect you have estimated and found small, and an estimate you have not made is not an estimate.
- Six points and a ruler are worth more than twelve points and no graph. If the apparatus fails half way, plot what you have and say how many points the line rests on; a line through four points honestly labelled scores better than a slope quoted with no graph behind it.

Practice 17.1

Two notebooks came in on the Friday with one coil measured in both of them and answers that differed by almost a tenth. **Kipute** had checked his arithmetic twice, had found nothing wrong with it either time, and said that one of the two of them must have misread the meter.

Mr. Akilikubwa did not look at the arithmetic at all, which **Kipute** said had taken him the best part of an hour. He asked each of them instead to draw the circuit exactly as it had stood on the bench, and put the two drawings side by side on the board. They differed in one place: the wire from the left-hand terminal of the voltmeter. In **Kipanga**'s drawing it went to the coil, and in **Kipute**'s it went to the far side of the ammeter.



Procedure

Proceed as follows:

(a) Connect the circuit as drawn, with the resistance box set to 14Ω, and check it by tracing the loop before you close the key.

(b) Close the key, read the ammeter and the voltmeter together, and open the key again.

(c) Set the box to 11Ω and repeat step (b).

(d) Repeat step (b) with the box at 8Ω, 6Ω, 4Ω and 2Ω.

(e) Record all your readings in a suitable table.

Questions

(i) Tabulate your readings.

(ii) Plot a graph of V against I.

(iii) Determine the slope of your graph.

(iv) The ammeter reads to 0.01A on a range of 1A and has a resistance of 0.40Ω. Explain what the slope represents in this arrangement, and hence determine the resistance of the coil S.

(v) Express the correction you made in (iv) as a percentage of the slope.

(vi) State which of the two connections, this one or that of Experiment 17.1, you would use to measure a coil of about 500Ω, and give the reason.

(vii) Explain why the line still passes through the origin even though the slope is not the resistance of the coil.

(viii) Suggest one measurement you could make to check the figure you were given for the resistance of the ammeter.

Experiment 17.2

Two men rewind motors in a shed by the bus stand, and on Saturday they were disagreeing at some volume about a coil of wire. One of them wanted the thick wire for the job and the other said the thin would do and cost less.

"Thick wire carries more current," said the first. *"Everybody knows it."*

"More current than what?" said the second. *"You have not told me what you are going to put across it."*

Kipanga, who had stopped to watch with **Kipute**, said the first man was right and that thick wire simply carries more. **Mr. Akilikubwa** asked him how much more, and how he proposed to find out, and the five of them stood in the road until the two winders went back inside. On Thursday there were two coils on the bench, wound from one reel, one of them twice as long as the other.

Aim

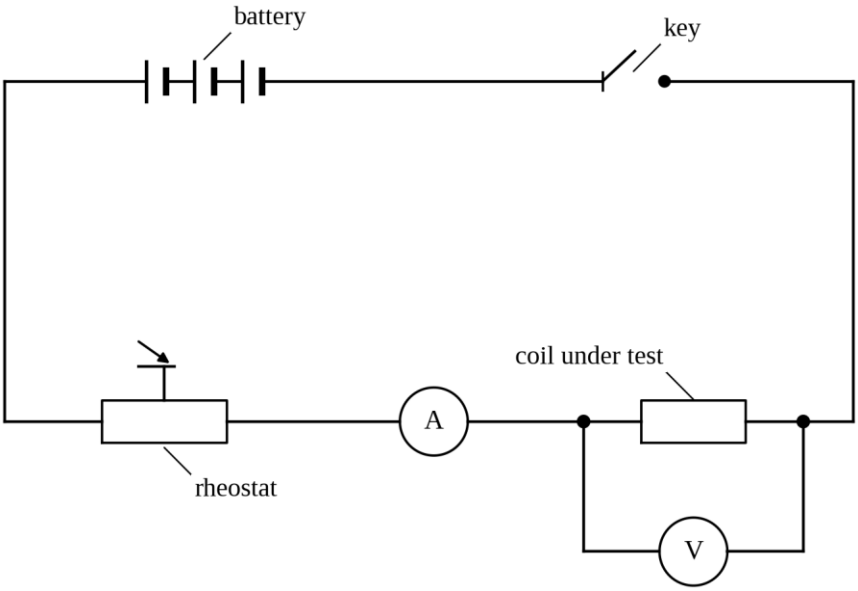
To test whether the resistance of a metallic conductor is the same at every current, and to compare the resistances of two coils cut from one length of wire.

Apparatus

A battery of three dry cells in a holder; a key; a rheostat of about 50Ω; an ammeter reading to 0.01A; a voltmeter reading to 0.05V; two coils wound from the same reel of wire, labelled P and Q, of which Q is twice as long as P; connecting wires.

Setup

The circuit is the one drawn below. The rheostat is in series with the battery, the key, the ammeter and the coil under test, and its slider is moved until the ammeter reads whatever value is wanted. The voltmeter is across the coil alone. Coil P is connected first and coil Q afterwards, and no other connection is disturbed. The coils are left where the air can reach them and the key is open between readings.



Procedure

Proceed as follows:

(a) Connect coil P into the circuit as drawn, and check it by tracing the loop before you close the key.

(b) Close the key, move the slider of the rheostat until the ammeter reads exactly 0.10A, read the voltmeter, and open the key again.

(c) Repeat step (b) with the ammeter set to 0.20A, 0.30A, 0.40A, 0.50A and 0.60A.

(d) Take coil P out, put coil Q in its place, and repeat steps (b) and (c).

(e) Record all your readings in a suitable table.

Questions

(i) Tabulate your readings for the two coils.

(ii) On the same axes plot a graph of V against I for both coils.

(iii) State the two features of your graphs which show that each coil obeys Ohm's law.

(iv) Determine the slope of each line.

(v) State what the slope of each line measures.

(vi) Coil Q is twice as long as coil P and cut from the same wire. Compare your two slopes and say what the comparison shows.

(vii) Explain why the current is set to chosen values and the potential difference read, rather than the other way about.

(viii) State one precaution you took to keep the coils at a steady temperature, and say which way your answer would move without it.

Table of results

Each current was set on the ammeter with the rheostat and the potential difference read, coil P first and coil Q afterwards.

I (A)	V for coil P (V)	V for coil Q (V)
0.10	0.25	0.45
0.20	0.45	0.85
0.30	0.65	1.30
0.40	0.85	1.75
0.50	1.10	2.20
0.60	1.30	2.65

Making the measurement count

Practical tip 1: *One line tests the law for one coil and says nothing about where the resistance lives.* What a second line of quite a different slope settles, drawn with the same battery and the same rheostat, is the question in part (v), and its answer earns more than the two slopes do.

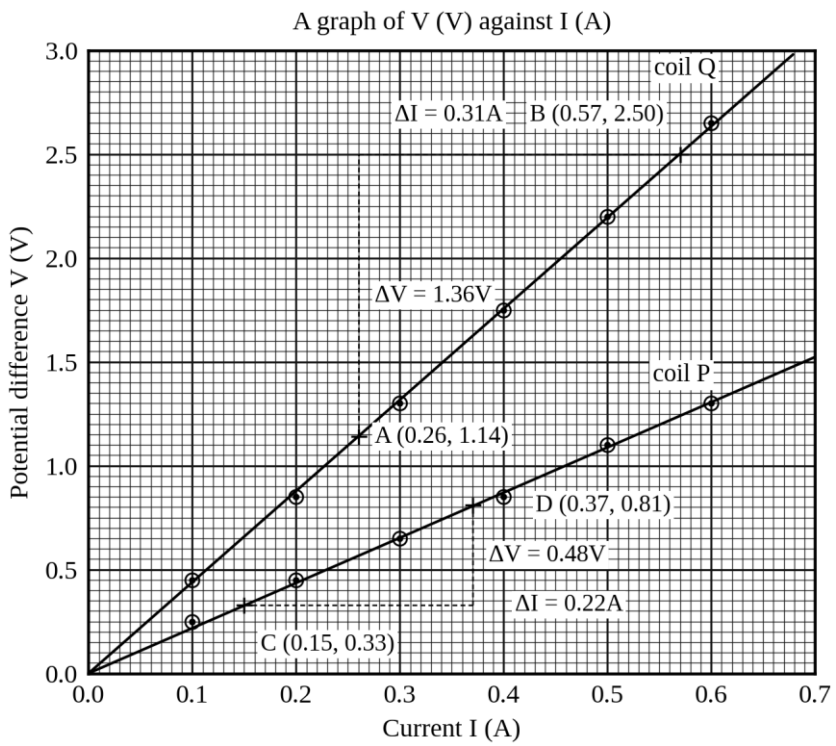
Practical tip 2: *One column of a good table is chosen and one is measured.* Here the currents are the round numbers and the potential differences fall where they fall. Part (vii) asks which of the two ought to be the chosen column, and it is the apparatus rather than the physics that decides it.

Practical tip 3: *Two lines on one sheet share a scale, and the scale has to suit the steeper of them.* Work out the largest potential difference in either column before you draw a single axis. A sheet chosen to fit coil P alone will send the last three readings of coil Q off the top of the paper.

Practical tip 4: *Find the two things in this run that hold the temperature steady, one in the Setup and one in the order of the readings.* Part (viii) asks for one of them, and for what the slope does when neither is done.

Answers to the questions

(i) and (ii). Take I along the horizontal axis at 2cm to 0.1A, running from 0 to 0.7A, and V up the vertical axis at 2cm to 0.5V, running from 0 to 3.0V. That is 14cm by 12cm, which fits an A4 sheet upright. Title the graph "A graph of V (V) against I (A)" and label each line with the coil it belongs to.



(iii) Each set of points lies on a straight line, so V and I are joined by a straight line law; and each line passes through the origin, so there is no fixed part added on and V is proportional to I. Straightness on its own would allow a fixed part; the origin is what rules it out. Those two together are Ohm's law for a metallic conductor at constant temperature.

(iv) For coil Q the line passes through A (0.26, 1.14) and B (0.57, 2.50), so

$$\text{slope}_Q = \frac{1.36\text{V}}{0.31\text{A}} = 4.39\Omega$$

For coil P the line passes through C (0.15, 0.33) and D (0.37, 0.81), so

$$\text{slope}_P = \frac{0.48\text{V}}{0.22\text{A}} = 2.18\Omega$$

Accept 2.05Ω to 2.30Ω for coil P and 4.20Ω to 4.60Ω for coil Q.

(v) The slope of each line is the resistance of the coil that line belongs to. It is a property of the coil and not of the battery or the rheostat, which are common to the two runs while the two slopes are quite different.

(vi) The two slopes are in the ratio

$$\frac{4.39\Omega}{2.18\Omega} = 2.01$$

and coil Q is twice as long as coil P. Within the reading of the graph the resistance has doubled with the length, so for one wire of one thickness the resistance is proportional to the length. Accept 1.80 to 2.25 for the ratio, which is what the two slope bands above allow between them. That is as far as these two coils can take the argument, and it is not as far as the two winders wanted to go.

(vii) The rheostat controls the current directly, so the slider can be moved until the ammeter needle sits exactly on a scale mark; there is no comparable way of setting the voltmeter, which reads whatever the circuit gives it. Currents chosen in advance also space the points evenly along the line, which is what a line drawn by eye needs, and they make the two runs comparable row by row.

(viii) Keep the key open between readings, so that current is in the coil only while the pair is being read. A coil left carrying 0.60A warms, and its resistance rises with its temperature, so the last readings would sit above the line the first ones set and the graph would bend upwards. A slope taken across the whole of such a graph comes out too large.

Examiner's eye

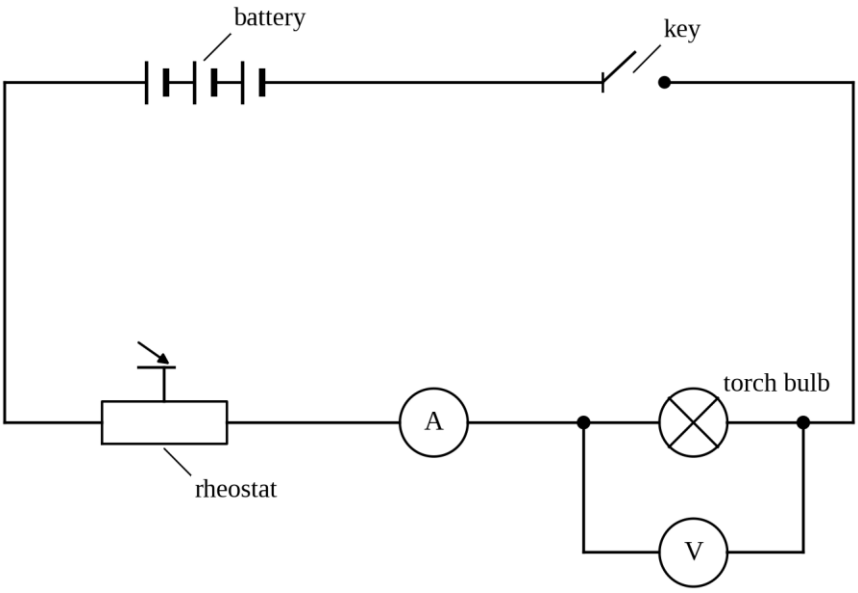
- Two lines on one sheet need two labels, and the labels are worth a mark. Write "coil P" and "coil Q" against the lines themselves, not in a key in the corner, because a key can be matched to the wrong line and a label on the line cannot.
- Watch what part (iii) is asking for. It wants the two features of the graph named, straight and through the origin, and each one turned into a sentence about the physics. A script that draws a beautiful pair of lines and then writes only "Ohm's law is verified" has done the work and left it unreported.
- Resistance rises with temperature for a metal and the effect is perfectly measurable: a coil that warms by 20K has gained about 8% for copper. It is small here only because the key is open between readings, and a conductor that is not given that treatment will show it.
- Coils of one thickness and two lengths settle the length, and no more than the length. A claim about thickness needs two coils of one length and two thicknesses, and a claim about the metal needs two coils of one shape and two wires; each of the three is a separate afternoon at the bench.

Practice 17.2

The night watchman's torch is famous. It comes on the colour of a cooking fire, stays that way while he counts to five, and then puts up a white light you can read by. **Kipute** said the filament was warming up, and that a hot filament is harder to push a current through than a cold one, so the torch was fighting itself for the first five seconds.

Kipanga said that a resistance is a resistance. It is a number written on the thing, he said, in the same way that a mass is written on a bag of cement, and a number does not change its mind halfway through the evening.

The watchman, who had not asked, said the torch had been like that since he bought it and that he had stopped minding. **Mr. Akilikubwa** heard the argument repeated on Monday, agreed with neither boy out loud, and put a torch bulb in a holder on the bench on Thursday with a rheostat beside it.



Procedure

Proceed as follows:

(a) Connect the circuit as drawn, with a torch bulb marked 2.5V and 0.30A in the holder and the rheostat at its largest resistance, and check it by tracing the loop before you close the key.

(b) Close the key, move the slider of the rheostat until the ammeter reads exactly 0.06A, wait a few seconds, and read the voltmeter.

(c) Repeat step (b) at 0.10A, 0.14A, 0.18A, 0.22A, 0.26A and 0.30A, taking the currents in that order.

(d) Open the key as soon as the last reading is taken.

(e) Record all your readings in a suitable table.

Questions

(i) Tabulate your readings.

(ii) Plot a graph of V against I.

(iii) Read from your graph the potential difference across the bulb when the current is 0.24A.

(iv) Calculate R=V/I for each of your readings and add the values to your table.

(v) State whether the filament obeys Ohm's law, and give the reason.

(vi) Give the resistance of the filament at 0.10A and at 0.30A, and the ratio of the two.

(vii) Ohm's law is often written V=IR. Explain in what sense that equation still applies to this bulb.

(viii) State one reason why the readings were taken from the smallest current upwards rather than downwards.

Experiment 17.3

A card has been taped to the door of the apparatus cupboard for as long as anybody can remember. It is in the handwriting of a teacher who left before **Kipute** was born, and it reads:

THE MOVING COIL VOLTMETERS ARE NOT TOYS

NEVER CONNECT ONE IN SERIES

A VOLTMETER IS NOT A PIECE OF WIRE

Underneath the last line somebody has written, in pencil and in a schoolboy hand, "why not." Underneath that, in a hand the whole school knows, "come on Thursday and find out."

Kipanga wanted to know which of the two hands on the door was going to be proved right. **Kipute** said the card had been on that door longer than the meters had been in the cupboard, and that somebody should have asked years ago. **Mr. Akilikubwa** set a voltmeter, a resistance box and three cells on the bench and joined them

in one series loop, which is the thing the card forbids, and asked the class what the meter could possibly be telling them.

Aim

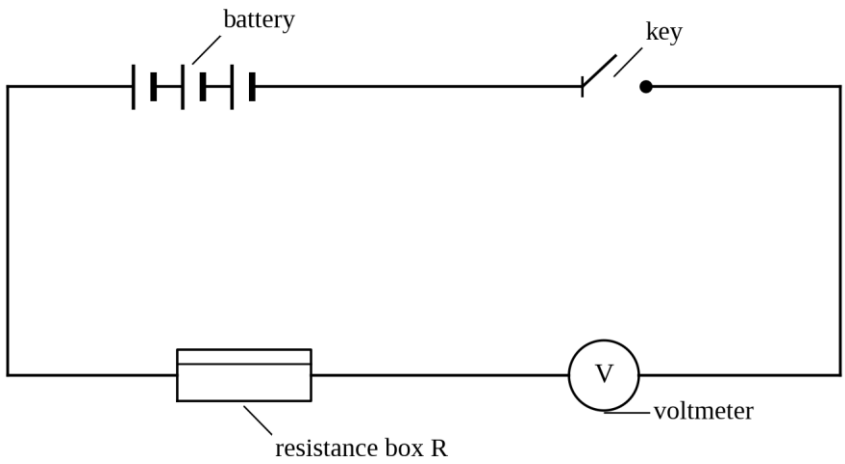
To determine the resistance of a voltmeter and the emf of a battery from one graph.

Apparatus

A battery of three dry cells in a holder; a key; a resistance box reading up to 10000Ω; a voltmeter reading to 0.05V on a range of 5V; connecting wires. There is no ammeter in this experiment.

Setup

The circuit is the one drawn below, and it is a single series loop: the battery, the key, the resistance box and the voltmeter, one after another, and nothing else anywhere in it. Use a battery that has not been used earlier in the day, and take the six readings briskly.



Procedure

Proceed as follows:

(a) Connect the circuit as drawn, with the resistance box set to 1000Ω, and check it by tracing the loop before you close the key.

(b) Close the key, read the voltmeter, and open the key again.

(c) Repeat step (b) with the box set to 2000Ω, 3000Ω, 4000Ω, 5000Ω and 6000Ω.

(d) Record all your readings in a suitable table, and add a column of $1/\text{V}$.

Questions

(i) Tabulate R, V and $1/\text{V}$.

(ii) Plot a graph of $1/\text{V}$ against R.

(iii) Determine the slope of your graph and the intercept it makes on the vertical axis.

(iv) Show that the readings should satisfy

$$\frac{1}{\text{V}} = \frac{1}{\text{E}} + \frac{\text{R}}{\text{ER}_\text{V}}$$

where E is the emf of the battery and R_V the resistance of the voltmeter.

(v) Hence determine the emf of the battery.

(vi) Hence determine the resistance of the voltmeter.

(vii) Explain why no ammeter is needed, and state what the current in the loop was at the largest setting of the box.

(viii) The cells have an internal resistance of about 1Ω altogether. Explain why it has been left out of the analysis.

Table of results

The box was stepped from 1000Ω to 6000Ω, the voltmeter was read to 0.05V, and the third column is 1 divided by each reading.

R (Ω)	V (V)	1/V (per volt)
1000	3.40	0.294
2000	2.70	0.370
3000	2.25	0.444
4000	1.95	0.513
5000	1.70	0.588
6000	1.50	0.667

Making the measurement count

Practical tip 1: *A reciprocal column magnifies the uncertainty at the small end.* Half a division on a reading of 3.40V moves 1/V by 0.002, and half a division on 1.50V moves it by 0.011, five times as much. That is why the box stops at 6000Ω.

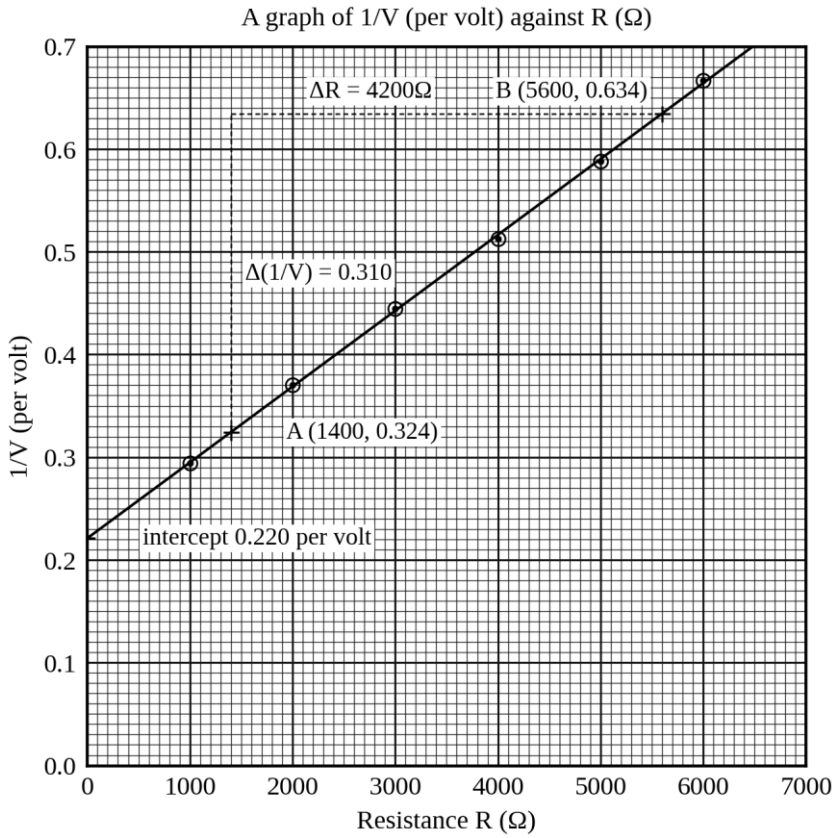
Practical tip 2: *Start the vertical axis at zero, because the intercept is one of the answers.* There is no choice about it here and no marks for a clever scale. Run the vertical axis from 0 and let the points sit where they fall.

Practical tip 3: *A voltmeter in series is not being misused; it is being measured.* The card on the cupboard is good advice to somebody who wants a reading of a potential difference, because a voltmeter in series reads a fraction of what they think it does. It is exactly the wrong advice to somebody who wants the resistance of the meter itself.

Practical tip 4: *Two quantities come out of one line here, and they come from different parts of it.* One is read where the line crosses the axis and the other from how steeply it climbs, so a line pivoted a little about the middle of the points moves both of them, and moves them opposite ways. Draw it once, carefully, and use the same line for both.

Answers to the questions

(i) and (ii). Take R along the horizontal axis at 2cm to 1000Ω, running from 0 to 7000Ω, and 1/V up the vertical axis at 2cm to 0.1 per volt, running from 0 to 0.7 per volt. That is 14cm by 14cm, which fits an A4 sheet upright. Title the graph "A graph of 1/V (per volt) against R (Ω)".



(iii) Taking A (1400, 0.324) and B (5600, 0.634) on the line,

$$\text{slope}=\frac{0.634\text{ per volt}-0.324\text{ per volt}}{5600\Omega-1400\Omega}=\frac{0.310\text{ per volt}}{4200\Omega}=7.38\times10^{-5}\text{ per volt per ohm}$$

and the line cuts the vertical axis at 0.220 per volt.

(iv) The box and the voltmeter are in series across the battery, so the same current is in both and the potential difference across the voltmeter is its share of the emf:

$$V=\frac{ER_V}{R+R_V}$$

Turn both sides upside down and separate the two terms on the right:

$$\frac{1}{V}=\frac{R+R_V}{ER_V}=\frac{1}{E}+\frac{R}{ER_V}$$

which is a straight line of $1/V$ against R. The intercept is $1/E$ and the slope is 1 divided by the product ER_V.

(v) The intercept is $1/E$, so

$$E=\frac{1}{0.220\text{ per volt}}=4.55V$$

The battery has an emf of 4.55V, which is about 1.52V a cell. Accept 4.35V to 4.75V.

(vi) Divide the intercept by the slope and the emf cancels:

$$\frac{1/E}{1/ER_V}=R_V$$

so

$$R_V=\frac{0.220\text{ per volt}}{7.38\times10^{-5}\text{ per volt per ohm}}=2980\Omega$$

The voltmeter has a resistance of about 2980Ω. Accept 2700Ω to 3300Ω.

(vii) Nothing in the loop carries a resistance worth counting except the box and the voltmeter, so the current is fixed by the two of them together and can be worked out from the reading instead of measured. At the largest setting the reading was 1.50V across a meter of 2980Ω, so

$$I=\frac{1.50V}{2980\Omega}=0.000503A$$

The smallest division of an ammeter reading to 0.01A is 20 times that current, so the needle would not have moved off its stop.

(viii) The internal resistance is in series with the box and the voltmeter, so strictly the R in the analysis should be the box setting plus about 1Ω. The smallest setting used is 1000Ω, so the largest error this makes is 1 part in 1000, which is a hundredth of a small square on the graph and far inside the width of the line. It has been left out because it cannot be seen, and the same argument would fail at once if the box were set to a few ohms.

Examiner's eye

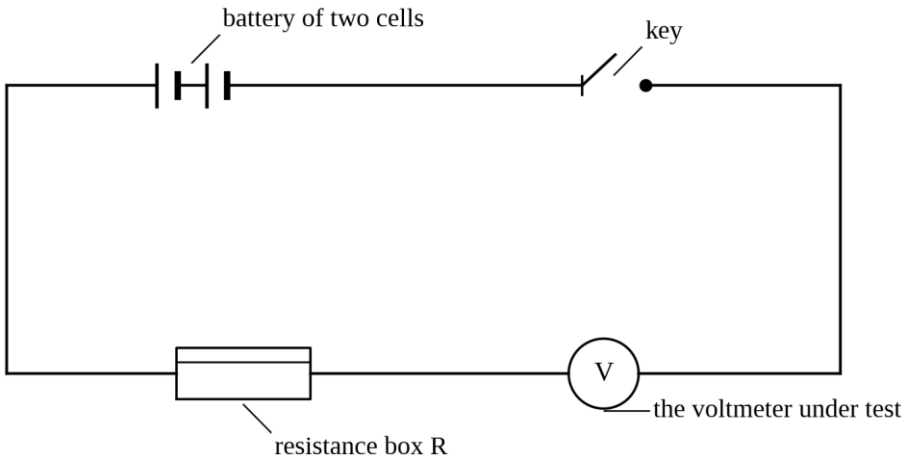
- Quote the unit of the slope. This one is per volt per ohm, which is nobody's favourite unit and is exactly what the axes make it. A slope written as a bare number here is a slope the examiner cannot tell from the intercept.
- Learn the intercept divided by the slope as a move in its own right. Whenever a line gives you two quantities and the unknown you want appears in both, the quotient of the two throws away everything they have in common. Here it throws away the emf, which is why part (vi) does not need part (v).
- Say where the rest of the potential difference has gone. The meter reads 3.40V with 1000Ω in series with it, which is three quarters of the emf, and the missing quarter is across the box. One sentence to that effect shows the reader that you have followed the circuit as well as the graph.
- A moving coil voltmeter carries this figure on its dial, as so many ohms for each volt of the range. Divide your 2980Ω by the 5V range and compare the two after you have finished. The dial figure is a nominal one for the whole instrument, so do not expect the two to agree closely; expect them to agree in order of size, and say in your account which you would trust.

Practice 17.3

Two equal resistors were joined end to end across a battery and the class was asked what the potential difference across one of them would be. Everybody said half, and everybody was right. **Kipanga** was sent up to the bench to show it, and came back to his place unhappy.

"It says nothing like half," he said. *"It says a good deal less than half."*

Kipute offered to go and read it properly. He got the same figure, and then, because the meter was in his hand and the battery was there, he put it across the other resistor and got the same reading again. The two together did not come to the battery. **Mr. Akilikubwa** said the meter was telling the exact truth about a circuit that was not the one on the board, and handed them a resistance box.



Procedure

Proceed as follows:

(a) Connect the circuit as drawn, with the resistance box set to 500Ω, and check it by tracing the loop before you close the key.

(b) Close the key, read the voltmeter, and open the key again.

(c) Repeat step (b) with the box set to 900Ω, 1300Ω, 1700Ω, 2100Ω and 2500Ω.

(d) Record all your readings in a suitable table, and add a column of 1/V.

Questions

(i) Tabulate R, V and 1/V.

(ii) Plot a graph of 1/V against R.

(iii) Determine the slope and the intercept on the vertical axis.

(iv) Determine the emf of the battery.

(v) Determine the resistance of the voltmeter.

(vi) Two equal resistors of 2000Ω are joined end to end across the same battery and this voltmeter is connected across one of them. Calculate what it will read.

(vii) Express the difference between that reading and the true potential difference as a percentage of the true value.

(viii) The meter carries a 10V range as well as the 5V range used here. State one way of making that error small without buying a different meter, and say what the other range would and would not do about it.

Experiment 17.4

Mr. Akilikubwa put two resistors on the bench, said not one word about what was printed on them, and offered a mark to anybody who could say, without touching a meter, which of the two ways of joining them gives the larger resistance and by how much.

"End to end," said **Kipanga**, before the sentence was quite finished.

"He has not looked at them," said **Kipute**. *"He answered like that about the bell last week, and the bell has been silent ever since."*

Mr. Akilikubwa put a mark beside neither name. He said that the first half of the question can be guessed and the second half cannot, that the second half is the whole of the physics, and that the afternoon would be spent on it with two resistors and a graph.

Aim

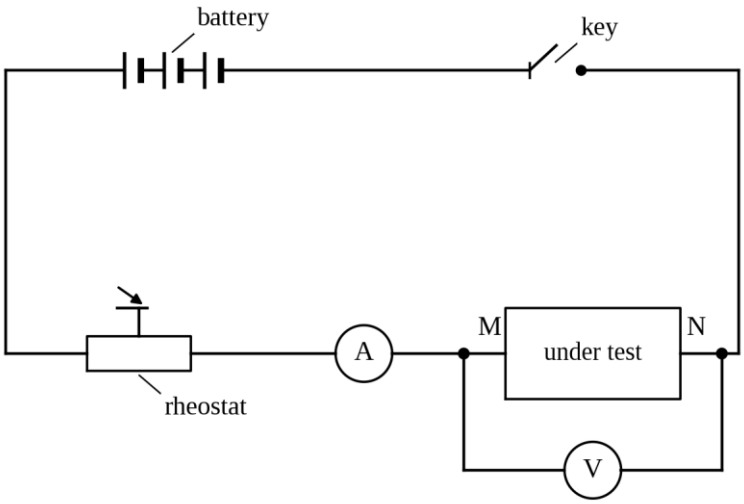
To test the rules for the resistance of two resistors joined in series and joined in parallel.

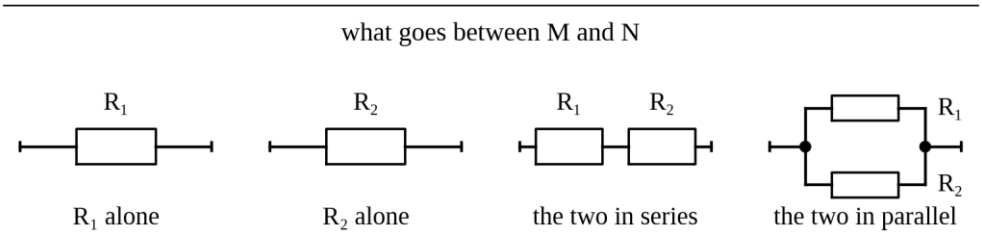
Apparatus

A battery of three dry cells in a holder; a key; a rheostat of about 100Ω; an ammeter reading to 0.01A on a range of 1A; a voltmeter reading to 0.05V; two resistors labelled R_1 and R_2; connecting wires.

Setup

The circuit is the one drawn below. Between the terminals M and N goes whatever is being measured, and the four runs put four different things there in turn: R_1 by itself, R_2 by itself, the two in series and the two in parallel. Nothing else in the circuit is touched between the runs, and the voltmeter stays across M and N throughout.





Procedure

Proceed as follows:

(a) Connect the circuit as drawn, with R_1 alone between M and N, and check it by tracing the loop before you close the key.

(b) Close the key, move the slider of the rheostat until the ammeter reads exactly 0.10A, read the voltmeter, and open the key again.

(c) Repeat step (b) at 0.20A, 0.30A, 0.40A, 0.50A and 0.60A.

(d) Take R_1 out, put R_2 between M and N, and repeat steps (b) and (c) at 0.10A, 0.18A, 0.26A, 0.34A, 0.42A and 0.50A.

(e) Repeat with the two in series between M and N, at 0.06A, 0.10A, 0.14A, 0.18A, 0.22A and 0.26A, and then with the two in parallel, at 0.20A, 0.34A, 0.48A, 0.62A, 0.76A and 0.90A.

(f) Record all your readings in a suitable table.

Questions

(i) Tabulate your readings for the four arrangements.

(ii) On the same axes plot a graph of V against I for all four.

(iii) Determine the resistance of each arrangement from its line.

(iv) Test the series rule against your measurements.

(v) Test the parallel rule against your measurements.

(vi) Explain why all four lines pass through the origin.

(vii) Explain why the four runs do not cover the same range of current.

(viii) State one precaution that matters more in the parallel run than in the other three, and give the reason.

Table of results

The four arrangements were put between M and N in turn, and each was swept over its own set of currents with the rheostat.

I (A)	V for R_1 (V)	I (A)	V for R_2 (V)
0.10	0.45	0.10	0.55
0.20	0.95	0.18	1.00
0.30	1.40	0.26	1.45
0.40	1.90	0.34	1.85
0.50	2.35	0.42	2.35
0.60	2.80	0.50	2.80

I (A)	V in series (V)	I (A)	V in parallel (V)
0.06	0.60	0.20	0.50
0.10	1.00	0.34	0.85
0.14	1.45	0.48	1.25
0.18	1.85	0.62	1.60
0.22	2.25	0.76	1.95
0.26	2.65	0.90	2.30

Making the measurement count

Practical tip 1: *Four lines on one sheet are four chances to plot a point in the wrong set.* Plot one run completely, draw its line and label it, and only then start the next. A sheet with 24 points on it and no lines yet is a sheet nobody can sort out afterwards, including you.

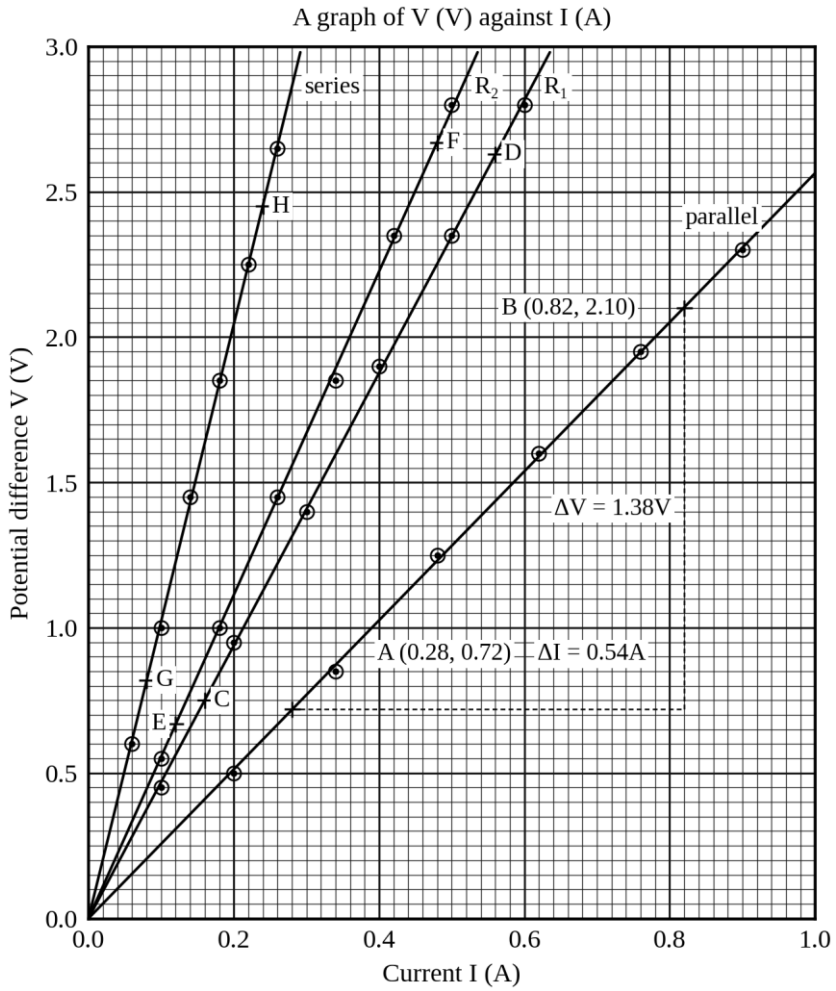
Practical tip 2: *The parallel combination is the shallowest line and the series combination is the steepest, and between them they set the sheet.* Whatever two resistors you are given, the series pair is about 4 times the parallel pair or more, and a pair of nearly equal resistors sits at the bottom of that. Work the scale out from those two before you plot anything.

Practical tip 3: *Four triangles on four lines want four separate pieces of clear paper.* Take each triangle in the stretch of its own line where the neighbouring lines are furthest away, which for the steep ones is low down and for the shallow one is far out to the right. Letter the eight corners and write the coordinates beside them, or the examiner cannot tell which triangle belongs to which line.

Practical tip 4: *The rule for parallel resistors is a rule about reciprocals, and reciprocals of three-figure numbers want four figures.* A reciprocal such as 1/6.9 rounded to 0.14 is already 3% wrong before the addition starts. Carry the extra figure through the addition and round only at the end.

Answers to the questions

(i) and (ii). Take I along the horizontal axis at 2cm to 0.2A, running from 0 to 1.0A, and V up the vertical axis at 2cm to 0.5V, running from 0 to 3.0V. That is 10cm by 12cm, which fits an A4 sheet upright. Title the graph "A graph of V (V) against I (A)" and label each line with the arrangement it belongs to.



(iii) For the parallel combination the line passes through A (0.28, 0.72) and B (0.82, 2.10), so

$$R_p = \frac{1.38\text{V}}{0.54\text{A}} = 2.56\Omega$$

The other three lines are read the same way, from the corners marked C to H on the graph. C (0.16, 0.75) and D (0.56, 2.63) give R_1 = 4.70Ω; E (0.12, 0.67) and F (0.48, 2.67) give R_2 = 5.56Ω; G (0.08, 0.82) and H (0.24, 2.45) give 10.2Ω for the two in series. The dashed legs are drawn on the parallel triangle only, to keep the sheet readable; draw all four sets on your own. Accept 4.45Ω to 4.90Ω for R_1, 5.35Ω to 5.85Ω for R_2, 9.7Ω to 10.7Ω for the series pair and 2.44Ω to 2.68Ω for the parallel pair.

(iv) The series rule says $R_s = R_1 + R_2$. From the two single lines,

$$4.70\Omega + 5.56\Omega = 10.26\Omega$$

against 10.2Ω measured directly on the pair, a difference of 0.06Ω in 10.2Ω, which is about 0.6%. The rule is confirmed, and comfortably inside what the graph can be read to.

(v) The parallel rule says $1/R_p = 1/R_1 + 1/R_2$. From the two single lines,

$$\frac{1}{4.70\Omega} + \frac{1}{5.56\Omega} = 0.2128 \text{ per ohm} + 0.1799 \text{ per ohm} = 0.3927 \text{ per ohm}$$

so the rule predicts

$$R_p = \frac{1}{0.3927 \text{ per ohm}} = 2.55\Omega$$

against 2.56Ω measured directly, a difference of about 0.4%. Both rules stand. Accept a difference of up to 3% between your own predicted and measured figures.

(vi) Each arrangement is a set of metallic resistors at one temperature, so each obeys Ohm's law and has one resistance at every current. With the key open no current passes and no potential difference appears across M and N, whatever is between them, so (0, 0) belongs to all four graphs.

(vii) The voltmeter has one useful range and the four arrangements differ in resistance by a factor of about 4, so a current that gives a comfortable reading on the parallel pair would drive the series pair off the top of the scale. Each run was therefore swept over the currents that kept the reading between about 0.5V and 2.8V. Nothing is lost by it: a slope is a slope wherever along the line you measure it.

(viii) Make the two joints of the parallel pair tight and clean before you start. The parallel run carries the largest currents in the experiment, up to 0.90A against 0.60A in the next largest, so a poor joint there dissipates more than twice the power, warms while the readings are being taken, and climbs in resistance as it does. The points then bend away from the line instead of scattering about it, and a bend is the one fault a straight line drawn by eye will hide.

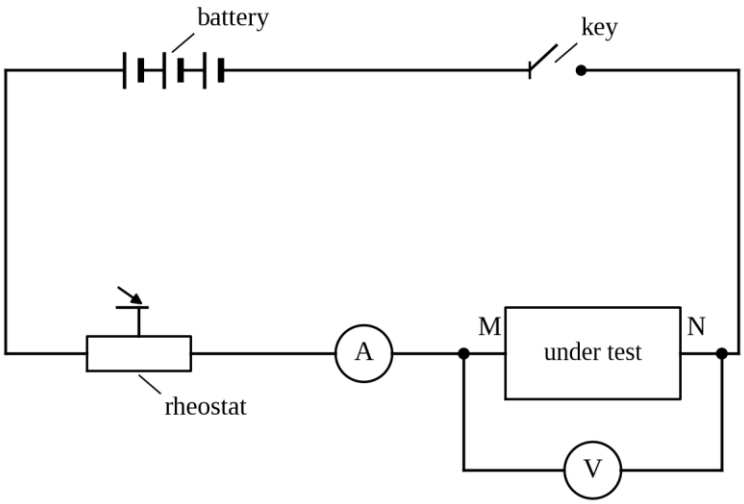
Examiner's eye

- Write out both comparisons, predicted against measured, with the difference as a percentage. A prediction with no verdict written under it leaves the verdict to the examiner, and the verdict is the answer.
- Which rule goes with which arrangement is worth a moment of thought rather than a moment of memory. Two resistors in series carry one current and their potential differences add, so the resistances add; two in parallel share one potential difference and their currents add, so the conductances add. Reconstructed that way the rules cannot be swapped.
- Every parallel pair is smaller than either of its members, and that is the arithmetic check to make before you write anything down. If your parallel figure comes out between R_1 and R_2 you have added the resistances somewhere you should have added the reciprocals.
- Four lines through one origin fan out from a single crowded corner, and the corner is where a line drawn by eye goes wrong. Fix each line by its far end, where the points are well apart, and let the ruler find the origin rather than starting from it.

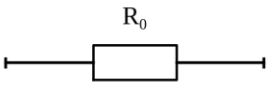
Practice 17.4

An electric iron came apart in the store and the element inside it, a flat coil the size of a hand, was still whole. **Kipanga** put it straight into the circuit from the last lesson to find out what it was worth, and the ammeter needle came to rest between two rulings near the bottom of the scale, where it stayed for the rest of the afternoon while the two of them argued about it.

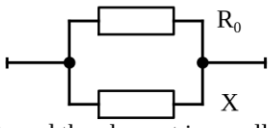
He read it as 0.06A and worked out an answer. **Kipute**, reading the same needle from the other side of the bench, read it as 0.07A and worked out an answer a seventh smaller. **Mr. Akilikubwa** looked at the two figures, said that both boys had read the meter honestly and that the fault was in where on the scale they had been made to read it. Then he took a 9.1Ω resistor out of the drawer and put it on the bench beside the element without saying what it was for.



what goes between M and N



R_0 alone



R_0 and the element in parallel

Procedure

Proceed as follows:

(a) Connect the circuit as drawn, with R_0 alone between M and N, and check it by tracing the loop before you close the key.

(b) Close the key, move the slider of the rheostat until the ammeter reads exactly 0.10A, read the voltmeter, and open the key again.

(c) Repeat step (b) at 0.14A, 0.18A, 0.22A, 0.26A and 0.30A.

(d) Join the element across R_0, so that the two are in parallel between M and N, and repeat steps (b) and (c).

(e) Record all your readings in a suitable table.

Questions

(i) Tabulate your readings for the two runs.

(ii) On the same axes plot a graph of V against I for both.

(iii) Determine the resistance of R_0 alone and of the parallel combination R_c.

(iv) Hence determine the resistance of the element.

(v) The ammeter reads to 0.01A on a range of 1A. State the largest current three cells can drive through the element on its own, and explain what R_0 beside it does about that.

(vi) The two resistances in (iii) are each uncertain by about 0.5%. Estimate the uncertainty in your answer to (iv).

(vii) Explain why the current the voltmeter takes for itself does not affect the answer to (iv).

(viii) Suggest a value of R_0 that would have given a more certain answer, and give the reason.

Chapter 18

E.M.F. AND INTERNAL RESISTANCE

A dry cell is sold with a number printed on it, and the number is not what a voltmeter reads once the cell is doing any work. The difference is the subject of this chapter. A cell drives charge round the outside of a circuit, but it must also drive that same charge through its own paste and plates, and the resistance it meets inside itself takes a share of the electromotive force before anything reaches the terminals. That share grows with the current, which is why a torch that is bright with one bulb is dim with two.

Six methods are worked here, and all six come from one equation. They differ in what is measured, in which way round the axes are set, and in one other thing that matters more than it looks: how much of the circuit the graph is looking at. A meter in the circuit has resistance of its own, and so do the leads, the plugs of a resistance box and the contacts of a switch. Some of these methods count that resistance as part of the cell and some of them do not, and the difference between two answers for one cell is not carelessness but arithmetic.

Experiments 18.1, 18.3 and 18.4 are run on the same dry cell on the same afternoon, so that the three answers can be set side by side. The second is the same rig on a cell that has been used. Of the last two, one puts a bare wire in place of the box, and the other lays a potential difference along a wire and reaches the cells only at the end.

The six varieties at a glance

	What you do	What comes out of it
18.1	A resistance box in series with the cell and an ammeter; the current is read as the box is stepped	the e.m.f. from the slope of R against 1/I, and the resistance of everything outside the box from the intercept
18.2	The same rig, plotted the other way about	the same two quantities again, from a slope of 1/E and an intercept of r/E
18.3	A rheostat sets the current and a voltmeter reads the terminal p.d. of the cell itself	the e.m.f. from the V-intercept and the internal resistance alone from the slope
18.4	A voltmeter across the box, with no ammeter in the circuit at all	the e.m.f. and the resistance outside the box from a graph of 1/V against 1/R
18.5	A bare wire as the load, its length varied	the resistivity of the wire and the resistance outside the wire from one line
18.6	The potential difference along a uniform wire, tapped at increasing lengths	the potential gradient, the current in the wire, and the internal resistance of the cells that drive it

All six are worked below in full. The practice questions change the cell and the wire, and the last of them puts two cells together and asks whether the rules for combining them survive a measurement.

Experiment 18.1

The torch that lives on the shelf by the door takes two cells and has a switch with two positions, one bulb and two. With one bulb it throws a circle of light as far as the gate. With two it throws two circles, and neither of them reaches the gate.

The obvious explanation is that two bulbs share out what one bulb had. It is the wrong explanation, and the right one can be got from the same torch: put a fresh pair of cells in, and the two bulbs together now reach the gate quite comfortably. Whatever is being shared out, there is more of it in a new cell than in an old one, and it is not the light.

What the second bulb does is halve the resistance the cells are working into, and so roughly double the current. A cell asked for twice the current gives up twice as much of its e.m.f. inside itself. The quantity that decides how much is lost is a resistance, and it belongs to the cell rather than to the circuit, which is the awkward part: it cannot be taken out and put on a bridge. It has to be inferred from what the cell does when the load is changed.

Aim

To determine the electromotive force of a dry cell and the resistance of the circuit outside the resistance box.

Apparatus

A dry cell of size D in a holder; a resistance box reading from 0 to 10Ω in steps of 0.1Ω; an ammeter reading 0 to 1A in divisions of 0.05A; a switch; six connecting leads with crocodile clips.

Setup

The cell, the switch, the ammeter and the resistance box are joined in one series loop, so that the whole of the current the cell delivers passes through the meter and through the box. The switch is closed only long enough to take each reading. Nothing is connected across the cell.

Procedure

Proceed as follows:

(a) Connect the dry cell, the switch, the ammeter and the resistance box in series, taking care that the ammeter is the right way round.

(b) Set the resistance box to 6Ω, close the switch, read the ammeter and open the switch again.

(c) Record the resistance R and the current I.

(d) Repeat the procedures in 18.1 (b) and (c) for the values of R equal to 5Ω, 4Ω, 2Ω and 1Ω.

Questions

(i) Draw a labelled diagram of the circuit you connected.

(ii) Tabulate your results, including the values of $1/I$ in A^{-1}.

(iii) Plot a graph of R (Ω) against $1/I$ (A^{-1}).

(iv) Determine the slope of your graph and state what it represents.

(v) Read the intercept your line makes on the R axis and state its physical meaning.

(vi) Explain why the switch is opened between one reading and the next.

(vii) Use your graph to predict the current the cell would drive through a box set to 3Ω.

(viii) A student says the intercept in (v) is the internal resistance of the cell and nothing else. Say whether that is right, and what the intercept contains.

Table of results

The values of R were set on the box and the currents were read to the nearest 0.01A by estimating fifths of a division.

R (Ω)	I (A)	1/I (A^{-1})
1	0.89	1.124
2	0.56	1.786
4	0.33	3.030
5	0.27	3.704
6	0.23	4.348

Making the measurement count

Practical tip 1: *One equation, and every graph in this chapter comes out of it.*

The cell drives a current I round a loop whose resistance is the box R together with everything else in the loop, so

$$E=I(R+r)$$

Divide by I and the equation is already a straight line, provided you are willing to treat $1/I$ as the quantity you plot:

$$R=E\times\frac{1}{I}-r$$

That is y = mx + c with R for y and 1/I for x. Part (iv) asks for the slope and part (v) for the intercept, and both come out of that one rearrangement. Write the rearrangement down before you plot anything; a graph drawn without it is a graph you cannot read.

Practical tip 2: *Read the ammeter to a fifth of a division, and read it while the switch is held down.*

The divisions on this meter are 0.05A apart and the eye can place a pointer inside one of them to about a fifth, which is 0.01A. That last figure is worth having, because the column you plot is not I but 1/I, and a reciprocal magnifies a small error into a large one when the current is small. At the foot of this table a hundredth of an ampere is nearly a twentieth of the reading.

Practical tip 3: *Count what the box leaves out.*

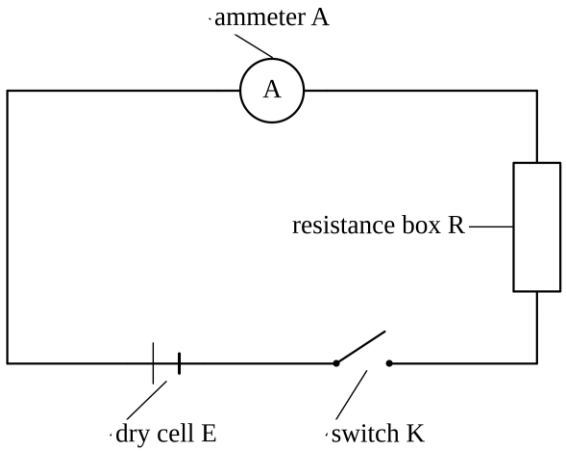
The box is one part of a loop that has several, and every part of it has some resistance. Walk round the loop before you plot anything and write down what you meet, then ask which of those things the box can be stepped through and which it cannot. Part (v) asks what the intercept means and part (viii) asks whether one name covers it.

Practical tip 4: *Step the box downwards, not upwards.*

Take the largest resistance first and the smallest last, and the heaviest current is drawn once, at the end. Taken the other way about, every later reading comes from a cell that has just been worked hard. Part (vi) asks what the switch is protecting you from, and the order of the readings is the same question asked once.

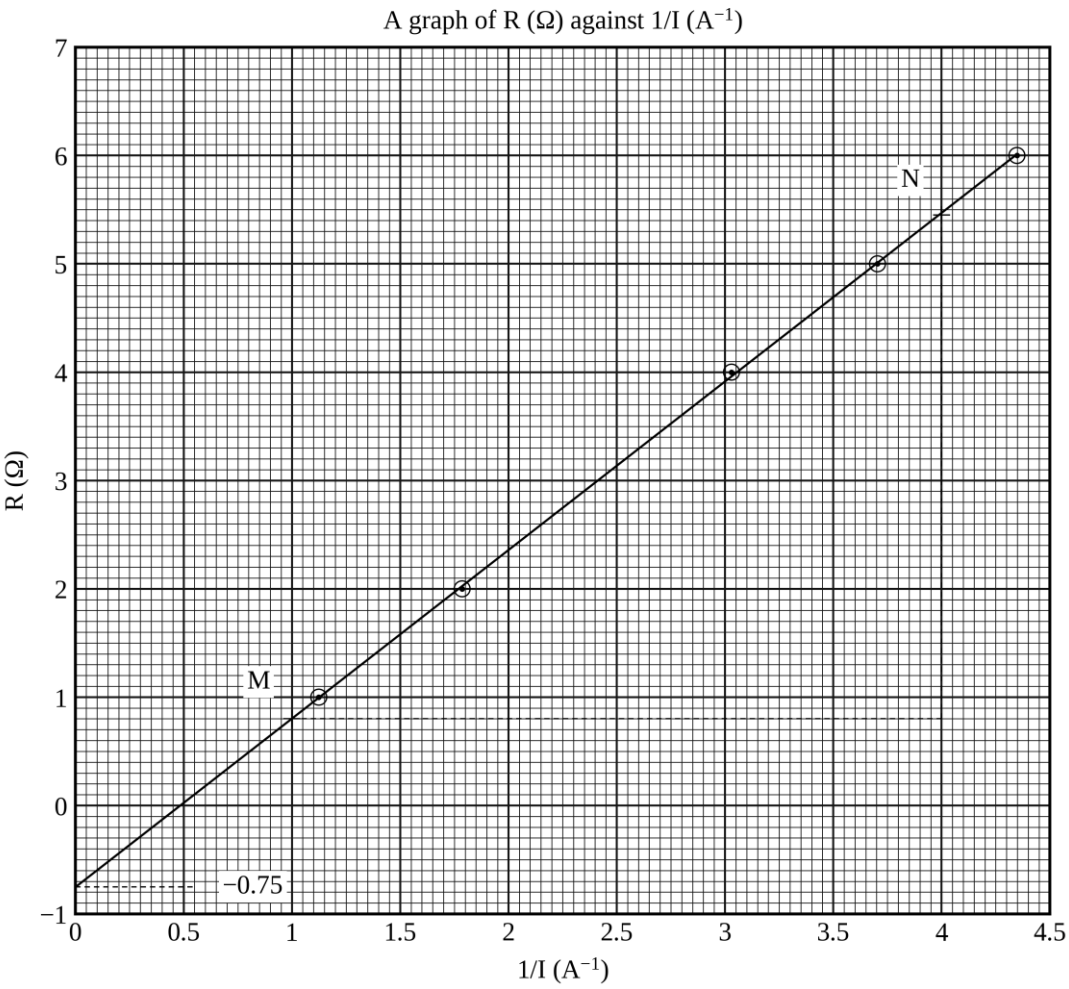
Answers to the questions

(i)



(ii) The readings are those in the table above, with the third column obtained by dividing 1 by each current.

(iii) Take 1/I at 2cm to $0.5A^{-1}$, running from 0 to $4.5A^{-1}$, and R at 2cm to 1Ω, running from -1Ω to 7Ω. That is 18cm by 16cm, which fits an A4 sheet upright. Give the graph the title A graph of R (Ω) against 1/I (A^{-1}). The vertical scale must run below zero, because the intercept the question wants is negative.



(iv) Taking two points on the line, M (1.0, 0.80) and N (4.0, 5.45),

$$\text{slope}=\frac{5.45\Omega\text{-}0.80\Omega}{4.0\text{A}^{-1}\text{-}1.0\text{A}^{-1}}=\frac{4.65\Omega}{3.0\text{A}^{-1}}=1.55\text{V}$$

An ohm divided by a reciprocal ampere is an ohm times an ampere, which is a volt. The slope is the electromotive force of the cell, 1.55V.

(v) The line cuts the R axis at -0.75Ω, so the intercept is 0.75Ω below zero. Put 1/I = 0 in the equation of the line and R = −r, so the intercept is minus the resistance of the whole circuit apart from the box: the cell's own internal resistance together with the ammeter, the leads, their clips and the switch. It is the resistance the box cannot get rid of.

(vi) Because a dry cell delivering a current warms up and polarises, and both of those lower the e.m.f. and raise the internal resistance. Left closed between readings the circuit would drain the cell steadily, so the cell that gave the last reading would not be the cell that gave the first, and the points would not lie on one line.

(vii) The line has slope E and intercept −r, so at R = 3Ω it gives

$$\frac{1}{\text{I}}=\frac{3.00\Omega+0.75\Omega}{1.55\text{V}}=2.419\text{A}^{-1}$$

so I = 0.41A. The prediction sits between the measured currents at 2Ω and 4Ω, as it must.

(viii) It is not right. The intercept is an upper limit on the internal resistance and not a measurement of it, because the ammeter, the leads, the clips and the switch are all in the same loop and none of them is in the box. Any of them could be reduced without touching the cell. Experiment 18.3 measures the cell's own share by putting the voltmeter across the cell alone, and the difference between the two answers is the size of what is being blamed on the cell here.

Examiner's eye

- Draw the vertical scale so that it reaches below zero. A graph of R against $1/I$ whose axis starts at the origin has no intercept on it, and no amount of producing the line afterwards recovers a scale that was never drawn.
- Slope carries units, and here they are not obvious. Write the ohms over the reciprocal amperes and simplify; an answer of 1.55Ω where $1.55V$ was meant loses the mark that the whole graph was drawn for.
- Quote the intercept as a resistance, not as a coordinate. The line cuts the axis at -0.75Ω and the resistance it stands for is 0.75Ω; a minus sign left in the final answer says the circuit has negative resistance in it.
- Three significant figures in the answers and no more. The currents carry two and the reciprocal column carries four for plotting, but a slope read across the whole line is good to three, and nothing afterwards can add a fourth that was never measured.

Practice 18.1

A card is propped against the batteries on a stall in the market, in the stallholder's own hand:

TESTED CELLS

EVERY ONE 1.5 VOLTS

TESTED THIS MORNING

The stallholder tests them with a small meter held across the two ends, and every cell he has ever tested has passed, including the ones customers bring back. He regards this as evidence that his stock is good. It is evidence of something else, and the experiment below is how you tell which.

Procedure

Proceed as follows:

(a) Connect the dry cell, the switch, the ammeter and the resistance box in series.

(b) Set the resistance box to 6Ω, close the switch, read the ammeter and open the switch again.

(c) Record the resistance R and the current I.

(d) Repeat the procedures in P18.1 (b) and (c) for the values of R equal to 4Ω, 3Ω, 2Ω and 1Ω.

Questions

(i) Tabulate your results, including the values of $1/I$ in A^{-1}.

(ii) Plot a graph of R (Ω) against $1/I$ (A^{-1}).

(iii) Determine the e.m.f. of the cell from your graph.

(iv) Determine the resistance of the circuit outside the box.

(v) The cell is marked $1.5V$. Comment on your answer to (iii).

(vi) The stallholder's meter is a voltmeter of very high resistance held across the two ends of a cell. Explain why every cell passes his test.

(vii) A torch bulb of resistance 4Ω is to be run from this cell. Use your graph to find the current it would take, and state what fraction of the energy the cell supplies would be lost before reaching the bulb.

(viii) Suggest one change to the procedure that would make the intercept more reliable, and say why it would.

Experiment 18.2

Kipute keeps two cells of the same make in the same drawer. One came out of the packet this morning. The other has been in the front of a torch since the beginning of term and has been left switched on twice overnight.

He held a voltmeter across each of them in turn and announced that there was nothing whatever the matter with the old one: 1.4V against 1.5V, which is a difference of about nothing. He then put the old one back in the torch and the new one in his pocket, and walked home in the dark.

The voltmeter was telling the truth and it was answering a question nobody had asked. A cell on open circuit is a cell doing no work, and a term of use is a term of work. Put the old cell on the rig of Experiment 18.1, make it work for its living, and the readings will say what the meter would not.

Aim

To determine the electromotive force of a used dry cell, and the resistance of the circuit outside the box, from a graph of the reciprocal of the current against the resistance.

Apparatus

The apparatus of Experiment 18.1, with the used cell in place of the new one.

Setup

The arrangement is the one drawn in Experiment 18.1 and is connected in the same way. One thing is different in the reading: the quantity plotted upwards is now the reciprocal of the current, and the resistance set on the box runs along the bottom.

Procedure

Proceed as follows:

(a) Connect the used dry cell, the switch, the ammeter and the resistance box in series.

(b) Set the resistance box to 5Ω, close the switch, read the ammeter and open the switch again.

(c) Record the resistance R and the current I.

(d) Repeat the procedures in 18.2 (b) and (c) for the values of R equal to 4Ω, 3Ω, 2Ω and 1Ω.

Questions

(i) Tabulate your results, including the values of $1/I$ in A^{-1}.

(ii) Show that $1/I$ and R are related by a straight line, and state what the slope and the intercept of that line stand for.

(iii) Plot a graph of $1/I$ (A^{-1}) against R (Ω).

(iv) Determine the electromotive force of the cell.

(v) Determine the resistance of the circuit outside the box.

(vi) The same rig with a new cell of the same make gave an electromotive force of 1.55V and an intercept of 0.75Ω. State which of the two quantities a term of use has changed the more, and by what factor.

(vii) A torch bulb takes 0.30A when it is working properly. Use your results to say whether this cell can still light it, and what the terminal p.d. would be.

(viii) In this method the resistance outside the box is found from the intercept divided by the slope. Explain why that makes it a less reliable answer than the electromotive force.

Table of results

The currents were read to the nearest 0.01A, as before.

R (Ω)	I (A)	1/I (A^{-1})
1	0.60	1.667
2	0.42	2.381
3	0.32	3.125
4	0.26	3.846
5	0.22	4.545

Making the measurement count

Practical tip 1: *Which unknown sits on the slope is a choice, and it has a price.*

Experiment 18.1 put R upwards and got the electromotive force straight off the slope and the resistance straight off the intercept. Turn the axes round and that no longer holds: neither unknown comes out on its own, and one of them cannot be reached until the other has been. Part (ii) asks you to work out what each of the two readings now carries, and part (viii) is about what the change costs.

Practical tip 2: *Take the reciprocal before you round, and round once.*

Work out $1/I$ from the current as you read it and round the answer to three figures. Round the current first to two figures and then take the reciprocal, and you throw the last figure away twice over; on this table the smallest current carries a twentieth of itself in its last digit.

Practical tip 3: *A used cell is not a faulty cell.*

Nothing in this experiment is going wrong. The readings are smaller than Experiment 18.1's because the cell is older, and the line is steeper for the same reason. Both graphs are honest measurements of two different cells, and part (vi) asks you to say what a term of use has done to each of the two quantities.

Practical tip 4: *The intercept is read where R is zero, and R is never zero in the table.*

The smallest resistance used is 1Ω, so the line has to be produced backwards to reach the vertical axis. Draw the axis from R = 0 whatever the smallest reading is, and produce the line with a ruler long enough to reach it without being lifted; a line produced in two pieces bends at the join, and the bend lands in the answer.

Answers to the questions

(i) The readings are those in the table above.

(ii) The cell drives the current round a loop of total resistance R + r, so

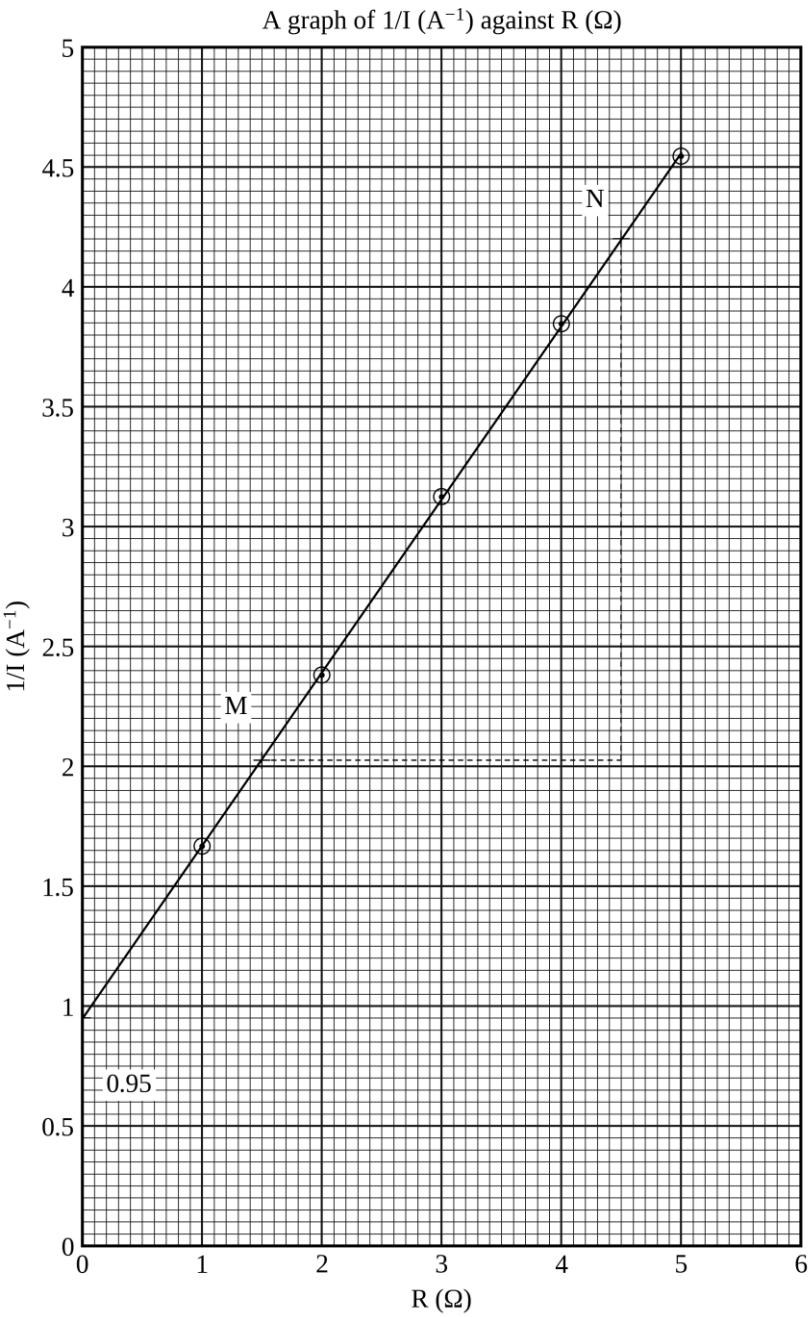
$$E=I(R+r)$$

Dividing throughout by IE,

$$\frac{1}{I}=\frac{R}{E}+\frac{r}{E}$$

which is a straight line of $1/I$ against R with slope $1/E$ and intercept r/E.

(iii) Take R at 2cm to 1Ω, running from 0 to 6Ω, and $1/I$ at 2cm to $0.5A^{-1}$, running from 0 to $5A^{-1}$. That is 12cm by 20cm, which fits an A4 sheet upright. Give the graph the title A graph of 1/I (A^{-1}) against R (Ω).



(iv) Taking two points on the line, M (1.5, 2.025) and N (4.5, 4.200),

$$\text{slope}=\frac{4.200\text{A}^{-1}-2.025\text{A}^{-1}}{4.5\Omega-1.5\Omega}=0.7250\text{A}^{-1}\Omega^{-1}$$

and the slope is $1/E$, so

$$E=\frac{1}{0.7250}=1.38\text{V}$$

(v) The line meets the vertical axis at 0.950A^{-1}, and that intercept is r/E, so

$$r=0.950\text{A}^{-1}\times1.38\text{V}=1.31\Omega$$

This is again the resistance of everything in the loop outside the box, so it carries the ammeter, the leads and the switch as well as the cell.

(vi) The electromotive force has fallen from 1.55V to 1.38V, which is a drop of 11%. The resistance outside the box has risen from 0.75Ω to 1.31Ω, which is a factor of 1.7. The meter and the leads are the same in the two experiments, and Experiment 18.3 will put their share at 0.33Ω in each of the two; the cell's own share has gone

from about 0.42Ω to about 0.98Ω, which is a factor of 2.3. The resistance is what a term of use has changed, and it has changed by a great deal more than the electromotive force.

(vii) Reading the graph at $1/I = 3.33A^{-1}$ gives R = 3.3Ω, so the bulb will take 0.30A only if the rest of the circuit adds up to that. A torch bulb meant for one cell has a resistance of a few ohms, so the cell can still light it. Part (vi) put the cell's own share at 0.98Ω, so the terminal p.d. would be

$$V=1.38V-0.30A\times0.98\Omega=1.09V$$

which is well below the 1.4V the voltmeter showed on open circuit. The bulb will light, and it will be dim.

(viii) Because the electromotive force comes from one reading of the graph and the resistance comes from two. The slope is measured over the whole length of the line, so a small error in either corner is divided by a long base; the intercept is a single point read where the line meets the axis, and it is found by producing the line beyond every measurement made. Multiply the two together and both errors land in the same answer, and the error in the intercept was the larger of them to begin with.

Examiner's eye

- This slope is not the electromotive force, and 0.7250V written beside it says the axes were never read. Take the reciprocal, and check the size of the answer against the number printed on the cell before going any further.
- Units survive a reciprocal. The slope here is measured in A^{-1} per ohm and its reciprocal is therefore in volts; the intercept is in A^{-1} and multiplying it by volts gives ohms. Units carried through cannot be multiplied the wrong way up.
- Both axes start at zero here, and both have to. The slope is safe either way but the intercept is not, and an axis begun at 1Ω because the readings begin there leaves nothing to read the intercept from.
- Part (vi) wants a comparison, which means a factor and not two numbers side by side. Say which quantity changed more and by how much, and take the meter's share out before you do it, or the factor comes out smaller than the truth.

Practice 18.2

The radio in the prefects' room takes two cells and plays perfectly well for about an hour after a new pair goes in. After that it begins to lose the station at the loud passages and hold it again at the quiet ones, which the owner attributes to the broadcasting.

Switched off for ten minutes it recovers and plays loudly again for a short while. Nothing has been broadcast differently during those ten minutes.

Procedure

Proceed as follows:

(a) Connect the dry cell, the switch, the ammeter and the resistance box in series.

(b) Set the resistance box to 5Ω, close the switch, read the ammeter and open the switch again.

(c) Record the resistance R and the current I.

(d) Repeat the procedures in P18.2 (b) and (c) for the values of R equal to 4Ω, 3Ω, 2Ω and 1Ω.

Questions

(i) Tabulate your results, including the values of $1/I$ in A^{-1}.

(ii) Plot a graph of $1/I$ (A^{-1}) against R (Ω).

(iii) Determine the electromotive force of the cell.

(iv) Determine the resistance of the circuit outside the box.

(v) Determine the current that would flow if a short thick wire were put in place of the box, and say why the answer should not be tested.

(vi) The radio draws 0.25A at the quiet passages and 0.60A at the loud ones. Determine the p.d. one cell delivers to the set at each of those two currents.

(vii) Use your answer to (vi) to explain the behaviour described in the opening, including the recovery after ten minutes at rest.

(viii) State one advantage this arrangement of the axes has over the arrangement used in Experiment 18.1.

Experiment 18.3

The lorry that brings maize to the mill starts badly on cold mornings. For the second or two while the starter is turning, the headlamps go from white to a dull orange, and they come back the instant the engine fires. The driver has twice had the lorry rewired, and has twice had it back with a bill and the same headlamps.

Nothing is loose. The starter is taking a very large current through the same battery, and the battery has to drive that current through itself before any of it reaches the lamps. What the lamps are showing is not their own trouble but the battery's, and it lasts exactly as long as the starter turns.

A voltmeter clipped to the battery terminals sees the same thing the lamps see: what the battery is offering the outside world at that moment. Set the current where you please and read that offer off, and five pairs of readings will say what the two men with bills could not.

Aim

To determine the electromotive force and the internal resistance of a dry cell from its terminal potential difference.

Apparatus

The dry cell of Experiment 18.1; an ammeter reading 0 to 2A in divisions of 0.05A; a voltmeter reading 0 to 3V in divisions of 0.1V; a rheostat of 0 to 15Ω; a switch; connecting leads with crocodile clips.

Setup

The cell, the switch, the ammeter and the rheostat make one series loop. The voltmeter is clipped directly to the two terminals of the cell itself and not to the ends of the leads that run away from it, so that what the voltmeter reads is what the cell offers and not what survives the journey. The rheostat is used with one end and the slider, so that moving the slider changes the resistance in the loop and therefore the current.

Procedure

Proceed as follows:

(a) Connect the circuit as drawn, with the voltmeter across the cell.

(b) Close the switch and move the slider until the ammeter reads 0.40A. Read the voltmeter, record both readings and open the switch.

(c) Repeat the procedure in 18.3 (b) for currents of 0.60A, 0.80A, 1.20A and 1.60A.

Questions

(i) Draw a labelled diagram of the circuit you connected.

(ii) Tabulate your results.

(iii) Write down the equation relating the terminal potential difference to the current.

(iv) Plot a graph of V (V) against I (A).

(v) Determine the electromotive force and the internal resistance of the cell.

(vi) Experiment 18.1 was done on this same cell and gave an intercept of 0.75Ω. Account for the difference between that figure and your answer to (v).

(vii) Determine the current at which the terminal potential difference would fall to half the electromotive force, and state the external resistance at that current.

(viii) Give a reason why the voltmeter must be clipped to the cell itself rather than to the far ends of the leads.

Table of results

The currents were set on the ammeter and the voltmeter was read to the nearest 0.02V by estimating fifths of a division.

I (A)	V (V)
0.40	1.38
0.60	1.30
0.80	1.22
1.20	1.04
1.60	0.88

Making the measurement count

Practical tip 1: *The voltmeter sees the cell; the ammeter does not.*

Everything between the two voltmeter clips is the cell and the cell alone. Before you plot anything, write down what those two clips have left outside them, and set that list beside the one Experiment 18.1 was working with. Part (vi) is about the size of the difference.

Practical tip 2: *Set the current, then read the voltage.*

The rheostat is what you adjust and the ammeter is what tells you when to stop. Bring the pointer onto the mark, hold it there and read the voltmeter; do not read both meters and then hope the pair belongs together. The current is the quantity plotted along the bottom, and a value set exactly is a point placed exactly.

Practical tip 3: *Work quickly at the large currents.*

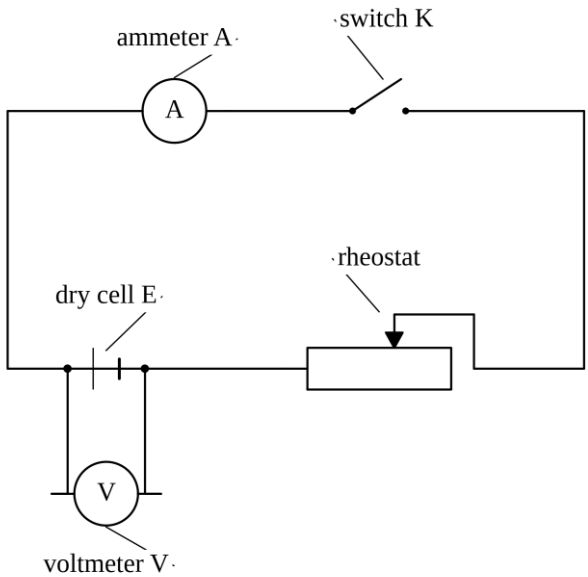
At 1.60A a cell of this size is being asked for more than it can hold steady. Leave the switch closed and the reading drifts downwards as you watch it, which is the cell polarising rather than the meter misbehaving. Take the reading in the first few seconds and open the switch, and take the largest current last so that the earlier points are not taken from a tired cell.

Practical tip 4: *Carry the sign through the working and leave it there.*

This line falls, so its slope comes out negative, and negative is what you should write at every step of the arithmetic. Say in words at the end which quantity the magnitude belongs to. A resistance quoted as a negative number is a number copied across without being read.

Answers to the questions

(i) The loop is cell, switch, ammeter and rheostat, with the voltmeter connected across the terminals of the cell only.



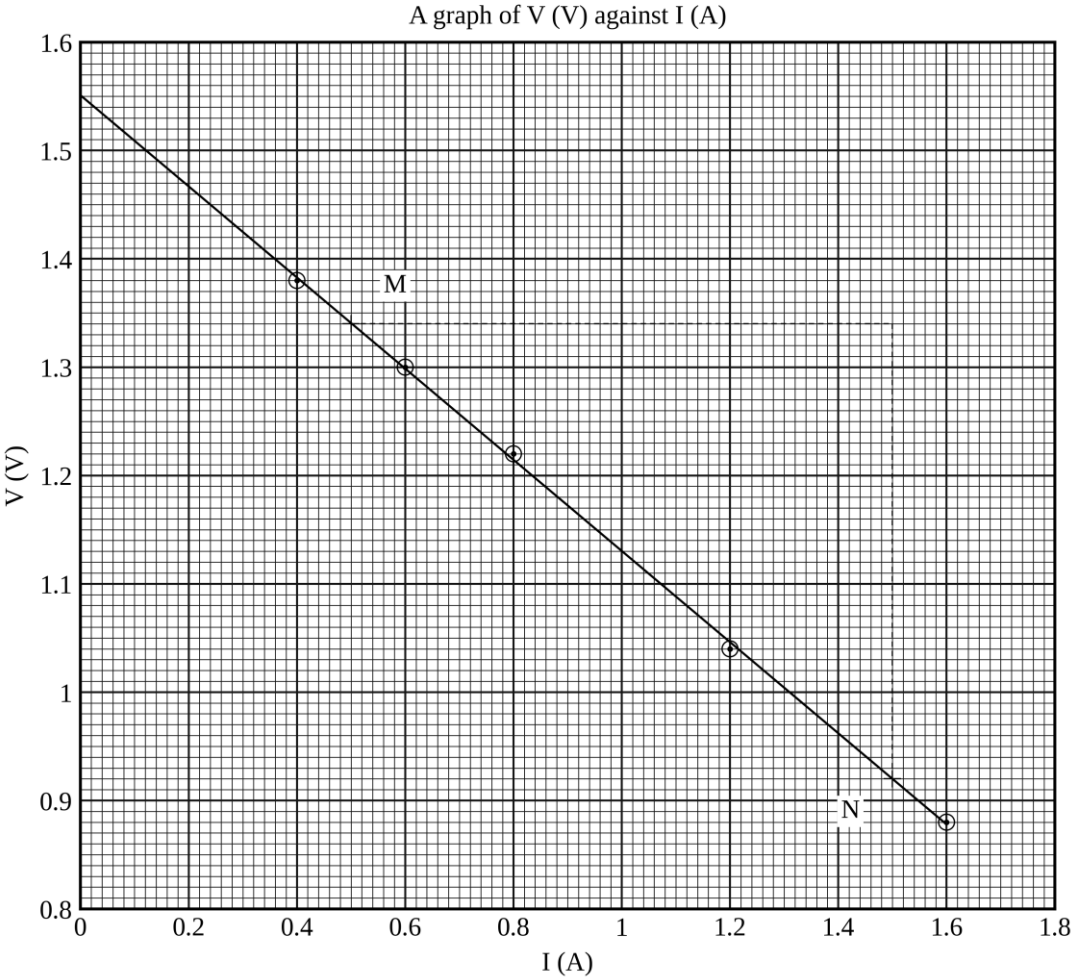
(ii) The readings are those in the table above.

(iii) The electromotive force is shared between the outside of the circuit and the inside of the cell, and the share lost inside is Ir, so

$$V=E-Ir$$

which is a straight line of V against I with intercept E and slope −r.

(iv) Take I at 2cm to 0.2A, running from 0 to 1.8A, and V at 2cm to 0.1V, running from 0.8V to 1.6V. That is 18cm by 16cm, which fits an A4 sheet upright. Give the graph the title A graph of V (V) against I (A). The current axis must start at zero, because the electromotive force is read where the line reaches I = 0.



(v) Taking two points on the line, M (0.50, 1.340) and N (1.50, 0.920),

$$\text{slope}=\frac{0.920\text{V}-1.340\text{V}}{1.50\text{A}-0.50\text{A}}=\frac{-0.420\text{V}}{1.00\text{A}}=-0.42\Omega$$

so the internal resistance is 0.42Ω. The line meets the V axis at 1.55V, which is the electromotive force, because at zero current nothing is lost inside the cell.

(vi) Experiment 18.1 measured the resistance of everything in its loop that was not in the box, and that loop had an ammeter, the leads, their clips and a switch in it as well as the cell. Here the voltmeter is across the cell alone, so the slope is the cell's own internal resistance. The difference,

$$0.75\Omega-0.42\Omega=0.33\Omega$$

is the ammeter together with the leads, the clips and the switch. It is not an error in either experiment: each measured exactly what its own connections enclosed.

(vii) Half the electromotive force reaches the terminals when the other half is lost inside, so Ir = E/2 and

$$I=\frac{1.55\text{V}}{2\times0.42\Omega}=1.85\text{A}$$

The external resistance carries the other half at that same current, so it equals the internal resistance, 0.42Ω. That current lies beyond the last reading and beyond the edge of the sheet as well, so it comes from the equation of the line and not from anything the graph shows.

(viii) Because the leads have resistance of their own, and a voltmeter clipped to their far ends reads the p.d. across the cell and the leads together. The reading would then fall faster with current than the cell alone does, the slope would be too steep and the internal resistance would come out too large by the resistance of the leads. Clip to the terminals themselves and the slope belongs to the cell alone.

Examiner's eye

- The intercept is on the V axis, and the V axis stands only at I = 0. Begin the horizontal scale at 0.4A because that is the first reading, and the place where the answer is read is not on the sheet at all.
- Half the electromotive force is not half the largest reading. Work part (vii) from E, which is the intercept; an answer that halves 1.38V has started from a terminal p.d. that already had something lost out of it.
- One cell can give two different values of internal resistance without any contradiction, and one sentence saying so is worth more than an apology. Name what each pair of connections enclosed and the two numbers explain each other.
- One value of r out of five pairs of readings is worth more than five values averaged. Each pair on its own needs the electromotive force you have not yet found; the line is what separates the two unknowns, and that is why a graph is asked for.

Practice 18.3

Two arguments were going on at once in the workshop. **Kipute** and **Kipanga** had the first, which was whether the welding set could be run off the same supply as the lights. The apprentices had the second, which was whether a battery has a voltage at all when nothing is connected to it.

A voltmeter settled the second argument in about four seconds, and everybody came back to help with the first, which is the harder one and is what the experiment below is about.

Procedure

Proceed as follows:

(a) Connect the cell, the switch, the ammeter and the rheostat in series, with the voltmeter across the cell.

(b) Close the switch and move the slider until the ammeter reads 0.20A. Read the voltmeter, record both readings and open the switch.

(c) Repeat the procedure in P18.3 (b) for currents of 0.50A, 0.80A, 1.10A and 1.40A.

Questions

(i) Tabulate your results.

(ii) Plot a graph of V (V) against I (A).

(iii) Determine the electromotive force of the cell.

(iv) Determine the internal resistance of the cell.

(v) Determine the power delivered to the outside of the circuit at 1.40A, and the power wasted inside the cell at the same current.

(vi) A lamp of resistance 1.2Ω is connected across this cell. Determine the current it takes and the p.d. across it.

(vii) A second lamp of the same kind is connected beside the first. State, with a reason and without further measurement, whether the pair together give more light than the single lamp did.

(viii) Suggest one change to the apparatus that would let the internal resistance be measured more precisely, and say why it would.

Experiment 18.4

Mr. Akilikubwa put a lamp marked 1.5V across a cell marked 1.5V, clipped a voltmeter across the lamp and invited the class to read it. The voltmeter said 1.2V.

Kipanga proposed that the missing 0.3V was in the leads. The leads were replaced with two short thick ones and the voltmeter said 1.2V again, at which point he proposed that the voltmeter was out by 0.3V. Mr. Akilikubwa said that a voltmeter which is out by exactly the amount you need is a lucky voltmeter, and asked what else was in the circuit.

Nobody in the class named it, and the lamp went on burning at four fifths of what it was sold for. The circuit had three things in it and the class had accounted for two. Put a resistance box where the lamp was, step it through a range of values, and the missing volts move as it goes; where they go is the whole of the experiment below, and it wants no ammeter at all.

Aim

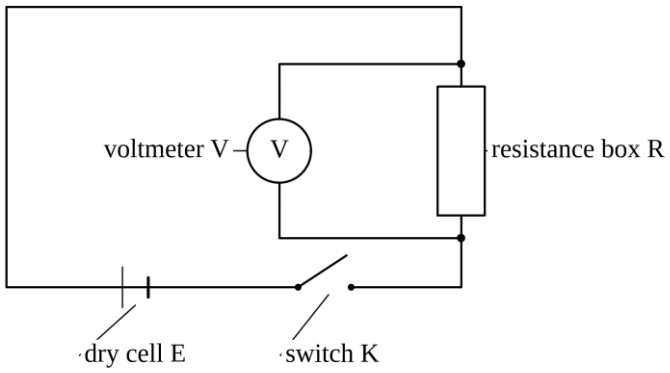
To determine the electromotive force of a dry cell, and the resistance of the circuit apart from the box, from the potential difference across a known resistance.

Apparatus

The dry cell of Experiment 18.1; the resistance box; a voltmeter reading 0 to 3V in divisions of 0.1V and of resistance not less than 2000Ω; a switch; connecting leads with crocodile clips.

Setup

The cell, the switch and the resistance box make one loop with no meter in it. The voltmeter is clipped across the box alone. Because the voltmeter is in parallel with the box it takes a small current of its own, and the box values used here are chosen small enough beside the voltmeter's resistance for that current to be neglected.



Procedure

Proceed as follows:

(a) Connect the cell, the switch and the resistance box in series, and clip the voltmeter across the box.

(b) Set the box to 10Ω, close the switch, read the voltmeter and open the switch.

(c) Record the resistance R and the potential difference V.

(d) Repeat the procedures in 18.4 (b) and (c) for the values of R equal to 5Ω, 3Ω, 2Ω and 1Ω.

Questions

(i) Tabulate your results, including the values of 1/R in Ω^{-1} and 1/V in V^{-1}.

(ii) Show that 1/V and 1/R are related by a straight line.

(iii) Plot a graph of 1/V (V^{-1}) against 1/R (Ω^{-1}).

(iv) Determine the electromotive force of the cell from the intercept.

(v) Determine the resistance in the circuit apart from the box.

(vi) Experiment 18.3, on this same cell, gave an internal resistance of 0.42Ω. Explain why that figure and your answer to (v) differ, and say what lies between them.

(vii) State the assumption you make about the voltmeter, and say what would happen to your value of the electromotive force if the assumption failed.

(viii) Determine the value of R at which the box and the rest of the circuit would take equal shares of the electromotive force.

Table of results

The voltmeter was read to the nearest 0.02V.

R (Ω)	V (V)	1/R (Ω^{-1})	1/V (V^{-1})
1	0.98	1.000	1.0204
2	1.20	0.500	0.8333
3	1.30	0.333	0.7692
5	1.38	0.200	0.7246
10	1.46	0.100	0.6849

Making the measurement count

Practical tip 1: *Space the reciprocals, not the resistances.*

The quantity plotted along the bottom is 1/R. Step the box 1, 2, 3, 5, 10 and the reciprocals fall at 1.000, 0.500, 0.333, 0.200 and 0.100, which reach both ends of the axis; step it 6, 7, 8, 9, 10 and every point crowds into a tenth of the sheet, leaving the intercept to a line produced from a huddle.

Practical tip 2: *No ammeter here, and one fewer thing in the loop because of it.*

This circuit has no meter carrying the main current, so what the graph finds outside the box is not the same list of parts that Experiment 18.1 was weighing. Part (vi) sets it against the figure Experiment 18.3 obtained with the voltmeter on the cell alone, and asks you to name what lies between the two.

Practical tip 3: *Four figures in a reciprocal column, and no apology for them.*

A voltmeter read to 0.02V gives V to three figures at most, but 1/V is what you plot and the whole column spans less than half a unit. The fourth figure is a guard digit, carried so that no point is moved on the paper by the rounding; it is dropped again in the answer, which can never be better than the reading it came from.

Practical tip 4: *Spend your best reading nearest the axis.*

The largest resistance gives the smallest 1/R, and it is that point which sits nearest the axis where the intercept is read. Put the largest value of the box first, while the cell is freshest, and if you can spare one more reading, spend it at that same setting and take the mean of the two.

Answers to the questions

(i) The readings are those in the table above.

(ii) The box and the rest of the circuit share the electromotive force in proportion to their resistances, so

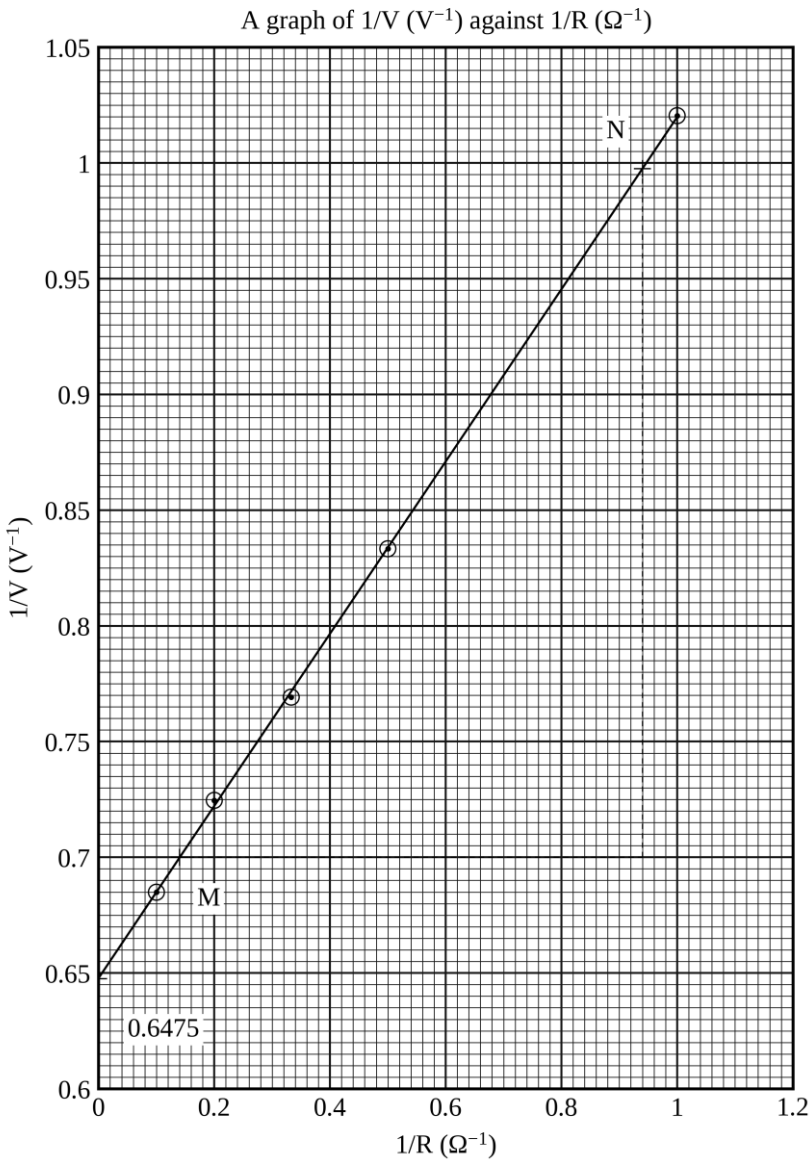
$$V=\frac{ER}{R+r}$$

Turning both sides upside down and separating the two terms,

$$\frac{1}{V}=\frac{1}{E}+\frac{r}{E}\times\frac{1}{R}$$

which is a straight line of 1/V against 1/R with intercept 1/E and slope r/E.

(iii) Take 1/R at 2cm to $0.2\Omega^{-1}$, running from 0 to $1.2\Omega^{-1}$, and 1/V at 2cm to $0.05V^{-1}$, running from $0.60V^{-1}$ to $1.05V^{-1}$. That is 12cm by 18cm, which fits an A4 sheet upright. Give the graph the title A graph of 1/V (V^{-1}) against 1/R (Ω^{-1}).



(iv) The line meets the vertical axis at $0.6475V^{-1}$, and that intercept is $1/E$, so

$$E=\frac{1}{0.6475}=1.54V$$

(v) Taking two points on the line, M (0.14, 0.7000) and N (0.94, 0.9975),

$$\text{slope}=\frac{0.9975V^{-1}-0.7000V^{-1}}{0.94\Omega^{-1}-0.14\Omega^{-1}}=0.3719\Omega V^{-1}$$

and the slope is r/E, so

$$r=0.3719\Omega V^{-1}\times 1.54V=0.57\Omega$$

(vi) Experiment 18.3 put the voltmeter across the cell alone, so its slope was the cell's own internal resistance. Here the voltmeter is across the box, so everything between the box and the cell is counted with the cell: the leads, their clips and the switch. The difference,

$$0.57\Omega-0.42\Omega=0.15\Omega$$

is what those connections are worth. Experiment 18.1 had the same connections and an ammeter besides, and its figure was 0.75Ω; the ammeter is therefore the remaining 0.18Ω. Three experiments on one cell have taken the loop apart.

(vii) The assumption is that the voltmeter takes a current small enough to be neglected beside the current in the box. With 2000Ω across a box of at most 10Ω the voltmeter carries about one part in two hundred of the total,

which is well below the reading error. If the assumption failed, the voltmeter would act as a second resistance in parallel with the box. Every reading would then be smaller than the box alone would give and every value of $1/V$ larger, so the intercept would come out too large and the electromotive force, being its reciprocal, too small.

(viii) Equal shares mean equal resistances, so R equals the resistance outside the box, 0.57Ω. On the graph that is the point where V is half the electromotive force, so $1/V = 1.299V^{-1}$ and $1/R = 1.75\Omega^{-1}$, which lies off the sheet as well as beyond the last plotted point, so it is a figure the equation of the line gives rather than one the graph shows.

Examiner's eye

- Two reciprocals in one question is where signs and inversions go astray. Write the equation of the line out in full before reading anything off it, and label which of the intercept and the slope carries which unknown.
- Units survive two reciprocals. An inverse volt divided by an inverse ohm is an ohm per volt, so the slope here is in ΩV^{-1} and the intercept in V^{-1}; multiply the first by volts and the ohms the question wants come out.
- An assumption is worth stating only if you can say which way it would push the answer. Part (vii) wants the direction: name which way the readings move, follow it to the intercept, and then through the reciprocal to the electromotive force.
- Neither experiment in part (vi) is the right one and the other wrong. Both are right about what they enclosed; write down the connections that lie between the two answers and the disagreement stops being one.

Practice 18.4

A letter arrived at the shop that had sold the parts, and was pinned up behind the counter for the general instruction of everybody:

> Sir, I bought the lamp and the bell from you on the same morning and ran them from the one battery as your man said I might. The lamp goes dim whenever the bell rings and the bell is faint whenever the lamp is on, and when I take the lamp out altogether the bell is as loud as it was in the shop. I have checked every wire and they are the wires you sold me. Kindly send somebody.

Nobody needs to be sent. The writer has bought two loads and one battery, and the experiment below is about the part of that purchase he has not thought about.

Procedure

Proceed as follows:

(a) Connect the cell, the switch and the resistance box in series, and clip the voltmeter across the box.

(b) Set the box to 10Ω, close the switch, read the voltmeter and open the switch.

(c) Record the resistance R and the potential difference V.

(d) Repeat the procedures in P18.4 (b) and (c) for the values of R equal to 5Ω, 3Ω, 2Ω and 1Ω.

Questions

(i) Tabulate your results, including $1/R$ and $1/V$.

(ii) Plot a graph of $1/V$ (V^{-1}) against $1/R$ (Ω^{-1}).

(iii) Determine the electromotive force of the cell.

(iv) Determine the resistance in the circuit apart from the box.

(v) Determine the potential difference the box would show if it were set to 0.5Ω.

(vi) The lamp has a resistance of 3Ω and the bell takes the same current as a resistance of 6Ω. Determine the p.d. across the lamp with the bell silent and with it ringing.

(vii) Use your answers to (vi) to explain what the writer of the letter describes, and state what he would have to change to cure it.

(viii) Explain why this method cannot be used with a box set to very large resistances.

Experiment 18.5

A poultry house up the road is warmed at night by a length of bare wire strung between two porcelain insulators and run from a battery. The man who built it cut the wire by eye, found the shed too cold, cut the wire shorter and found it too hot, and has been shortening and lengthening it ever since.

There is nothing wrong with his method except that it costs a wire each time. What he is really looking for is the length that gives a chosen current, and that can be had from five readings and a graph. The same five readings give two things he did not ask for: what the wire is made of, and how much of his battery is being spent inside the battery.

Aim

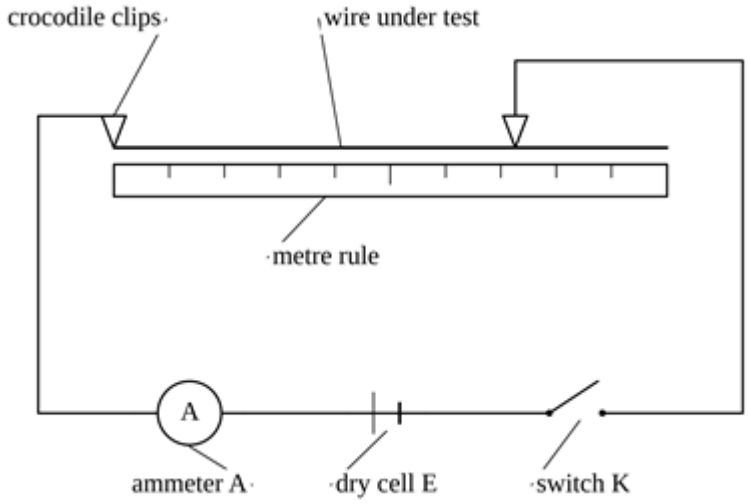
To determine the resistivity of a wire and the resistance of the circuit outside the wire.

Apparatus

A dry cell of size D in a holder; a bare wire about 120cm long stretched along a metre rule; an ammeter reading 0 to 1A in divisions of 0.05A; a voltmeter reading 0 to 3V; a micrometer screw gauge; a switch; two crocodile clips; connecting leads.

Setup

The wire is laid straight along the rule and held at each end. One crocodile clip is fixed at the zero of the rule and the other is moved along it, so that the length of wire in the circuit is read off the rule directly. The cell, the switch and the ammeter complete the loop through the two clips. The wire is bare, so nothing may be allowed to touch it between the clips.



Procedure

Proceed as follows:

(a) Measure the diameter of the wire with the micrometer screw gauge at three places along it and record the mean.

(b) With the switch open, clip the voltmeter to the two terminals of the cell and record its reading as the electromotive force E.

(c) Connect the cell, the switch and the ammeter to the two crocodile clips, with the fixed clip at the zero of the rule and the movable clip at 100cm.

(d) Close the switch, read the ammeter and open the switch. Record the length L and the current I.

(e) Repeat the procedures in 18.5 (c) and (d) for lengths of 80cm, 60cm, 40cm and 20cm.

Questions

(i) Tabulate your results, including the values of $1/I$ in A^{-1}.

(ii) Show that $1/I$ and L are related by a straight line, and state what its slope and intercept contain.

(iii) Plot a graph of $1/I$ (A^{-1}) against L (cm).

(iv) Determine the slope of your graph.

(v) Determine the resistivity of the wire.

(vi) Determine the resistance of the circuit outside the wire.

(vii) The wire is either constantan, of resistivity $4.9\times10^{-7}\Omega$m, or manganin, of resistivity $4.4\times10^{-7}\Omega$m. State which it is, and say whether your measurement is good enough to decide.

(viii) A heater is to be made from this wire to take 0.50A from the same cell. Determine the length required.

Table of results

The mean diameter of the wire was 0.40mm and the electromotive force read on open circuit was 1.52V.

L (cm)	I (A)	1/I (A^{-1})
20	0.98	1.020
40	0.66	1.515
60	0.49	2.041
80	0.39	2.564
100	0.33	3.030

Making the measurement count

Practical tip 1: *Read the cell on open circuit first, and write the figure down.*

Neither the slope nor the intercept of this graph can be untangled until E is known from somewhere else, and the graph will not give it to you. A voltmeter of high resistance across the cell with the switch open draws so little current that almost nothing is lost inside the cell, and what it reads is the electromotive force. Take that reading first, before the wire has had a chance to tire the cell.

Practical tip 2: *A diameter is squared, so its error is doubled.*

The area enters the resistivity directly and the area goes as the square of the diameter, so whatever share you are out by in the diameter you are out by twice over in the answer. Measure at three places, take the mean, and check the zero of the gauge before you start; then work out your own share before you attempt part (vii).

Practical tip 3: *Lengths in centimetres on the rule, metres in the answer.*

The rule reads centimetres and the resistivity is wanted in ohm metres, so the slope comes off the graph in A^{-1} per centimetre and has to be multiplied by 100 before it goes into the formula. Do that on a line of its own with both units written down. A factor of 100 lost here shows up as a resistivity a hundred times too small, which is lower than any metal has, and the question would then have no sensible answer at all.

Practical tip 4: *Move the clip, do not stretch the wire.*

The wire is the resistance being measured, so anything that changes it between readings is fatal. Do not pull it tighter between one length and the next, do not let the clip bite in a new place each time without cleaning it, and keep the switch open except while the pointer is being read: a bare wire carrying 1A warms noticeably, and a warm wire has a larger resistance than the cold one you started with.

Answers to the questions

(i) The readings are those in the table above.

(ii) The wire of length L and cross-sectional area A has resistance $\rho L/A$, and the cell drives its current through that and through everything else in the loop, so

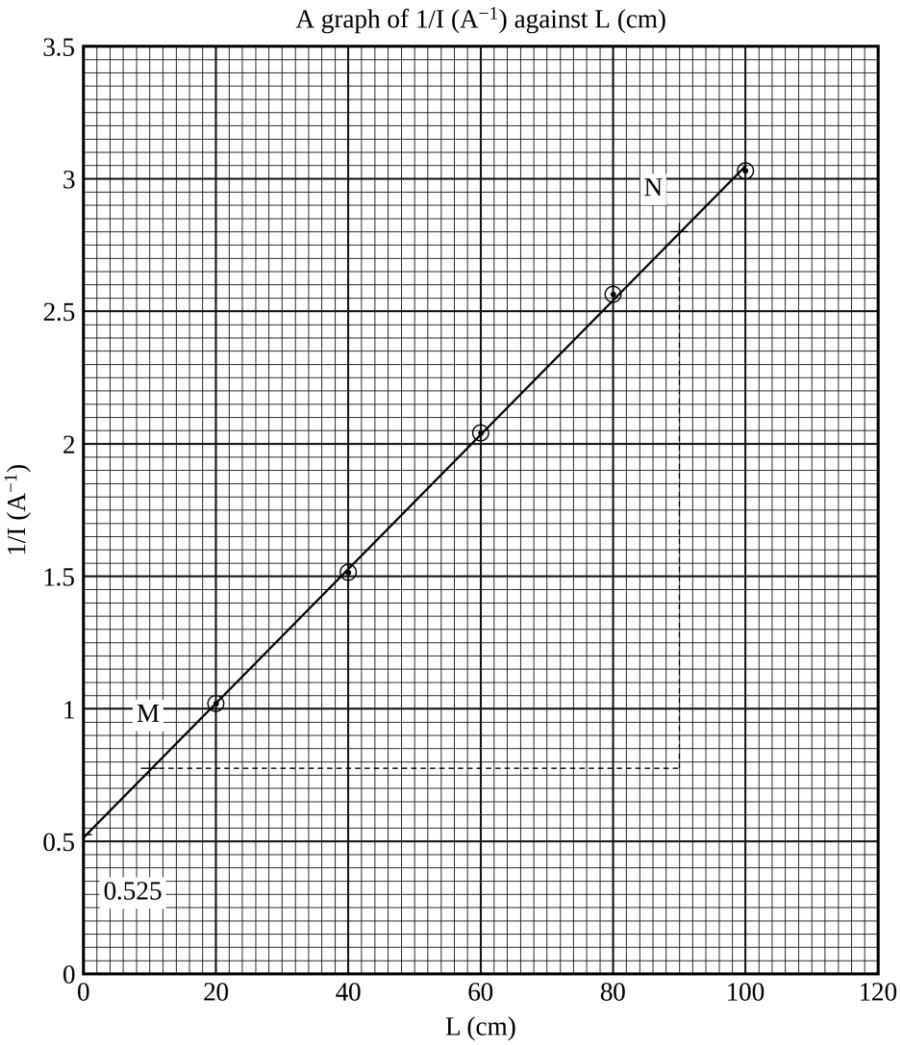
$$E=I\left(\frac{\rho L}{A}+r\right)$$

Dividing by E and by I,

$$\frac{1}{I}=\frac{\rho}{EA}L+\frac{r}{E}$$

which is a straight line of $1/I$ against L with slope ρ/EA and intercept r/E.

(iii) Take L at 2cm to 20cm, running from 0 to 120cm, and 1/I at 2cm to $0.5A^{-1}$, running from 0 to $3.5A^{-1}$. That is 12cm by 14cm, which fits an A4 sheet upright. Give the graph the title A graph of 1/I (A^{-1}) against L (cm).



(iv) Taking two points on the line, M (10, 0.775) and N (90, 2.800),

$$\text{slope}=\frac{2.800A^{-1}-0.775A^{-1}}{90cm-10cm}=0.02531A^{-1}cm^{-1}$$

In metres that is

$$\text{slope}=0.02531A^{-1}cm^{-1}\times100=2.531A^{-1}m^{-1}$$

(v) The area of the wire is

$$A=\frac{\pi d^2}{4}=\frac{3.142\times\left(0.40\times10^{-3}m\right)^2}{4}=1.257\times10^{-7}m^2$$

and the slope is ρ/EA, so

$$\rho=2.531A^{-1}m^{-1}\times1.52V\times1.257\times10^{-7}m^2=4.8\times10^{-7}\Omega m$$

(vi) The line meets the vertical axis at $0.525A^{-1}$, and that intercept is r/E, so

$$r=0.525A^{-1}\times1.52V=0.80\Omega$$

The cell is not the whole of that: the ammeter, the leads, their clips and the switch are all in the same loop and outside the wire.

(vii) The measured $4.8\times10^{-7}\Omega m$ is within about 2% of the constantan figure and 9% from the manganin one, so the wire is constantan. The two accepted values differ by about 11%, and the largest doubt in this

measurement is the diameter, which at 0.01mm is worth about 5% of the resistivity. The measurement is therefore just good enough to decide, and it would not be if the wire were any finer.

(viii) Read the graph at $1/I = 2.0A^{-1}$:

$$L=\frac{2.000A^{-1}-0.525A^{-1}}{0.02531A^{-1}cm^{-1}}=58cm$$

which lies between the readings at 40cm and 60cm on the graph and is therefore read off it rather than predicted beyond it.

Examiner's eye

- This intercept is not a resistance. It is a resistance divided by an electromotive force, measured in A^{-1}; multiply by E to get ohms, and quote ohms. 0.525Ω on its own has skipped a step.
- Record the micrometer reading in the table with everything else you measured. It carries a mark of its own, separate from the mark for the resistivity it goes into.
- Quote a resistivity with its power of ten and its unit. Ohm metres, not ohms per metre; the two differ by a factor of a square metre and only one of them is a property of the material.
- Every decision needs a reason for trusting it. Name the wire and you have half of part (vii); the other half is the gap between the two figures set against the doubt in your own.

Practice 18.5

Two reels of bare wire are kept in the store, both about the same thickness, one wound on wood and one on plastic. The labels came off years ago. Somebody has written on the shelf, in pencil:

ONE OF THESE IS CONSTANTAN

THE OTHER IS NICHROME

DO NOT MIX THEM

The advice is sound and useless, since the two reels look alike and the writer did not say which was which. The wire below is from one of the reels.

Procedure

Proceed as follows:

(a) Measure the diameter of the wire with the micrometer screw gauge at three places and record the mean.

(b) With the switch open, clip the voltmeter to the two terminals of the cell and record its reading as the electromotive force E.

(c) Connect the cell, the switch and the ammeter to the two clips, with the fixed clip at the zero of the rule and the movable clip at 50cm.

(d) Close the switch, read the ammeter and open the switch. Record the length L and the current I.

(e) Repeat the procedures in P18.5 (c) and (d) for lengths of 40cm, 30cm, 20cm and 10cm.

Questions

(i) Tabulate your results, including 1/I.

(ii) Plot a graph of 1/I (A^{-1}) against L (cm).

(iii) Determine the slope of your graph in A^{-1} per metre.

(iv) Determine the resistivity of the wire.

(v) Determine the resistance of the circuit outside the wire.

(vi) Constantan has a resistivity of $4.9\times10^{-7}\Omega m$ and nichrome of $1.1\times10^{-6}\Omega m$. State which reel this wire came from.

(vii) Determine the resistance of 1m of this wire, and hence the length that would have a resistance of 5Ω.

(viii) The lengths in this experiment were kept short. Give a reason why, in terms of the readings you would have obtained from a longer wire.

Experiment 18.6

The dispensary has one bell and one push, and the push is at the gate, 40m from the bell. When the bell was fitted the fitter ran two long thin wires out to the gate and the bell sounded like a bell. A year later somebody added a second push halfway along, spliced into the same pair, and now the far push produces a noise like a fly in a bottle.

Nothing has been broken and nobody has touched the far push. The second push revealed something that was always true of the long pair, and the way to see it is to leave the bells alone, put a voltmeter on a wire and move one of its contacts along it.

Aim

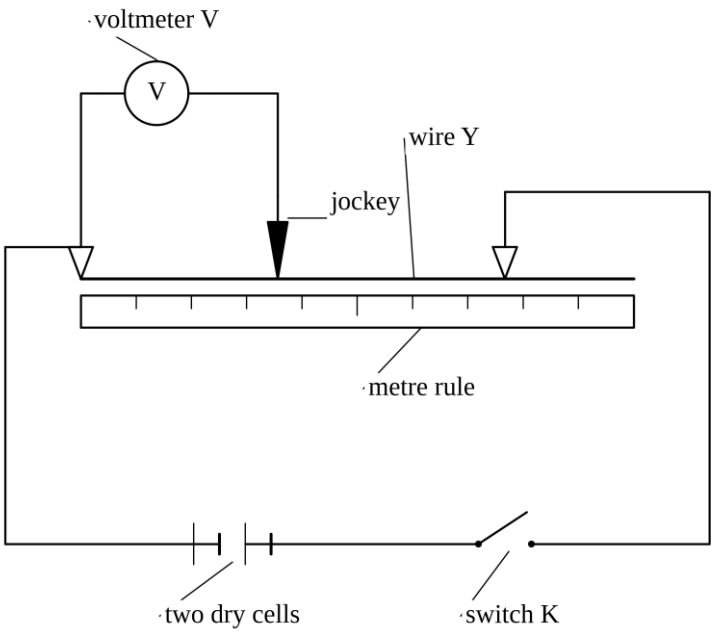
To determine the potential gradient along a uniform wire and hence the internal resistance of the cells driving it.

Apparatus

Two dry cells of size D in a holder; a uniform bare wire Y of resistance $8.0\Omega m^{-1}$ taped along a metre rule; a voltmeter reading 0 to 5V in divisions of 0.1V; a jockey; a switch; two crocodile clips; connecting leads.

Setup

The wire is taped along the rule and the two crocodile clips are fixed to it at the zero of the rule and at the 50cm mark, so that the length of wire carrying the current is 50cm and stays that length throughout. The two cells and the switch complete that loop. One terminal of the voltmeter is joined to the clip at the zero and the other to a jockey, which is pressed onto the wire at the length being measured and lifted again between readings.



Procedure

Proceed as follows:

(a) Connect the two cells, the switch and the wire so that 50cm of the wire carries the current.

(b) With the switch open, clip the voltmeter across the two cells and record its reading.

(c) Close the switch, press the jockey onto the wire at 10cm, read the voltmeter and lift the jockey.

(d) Record the length l and the potential difference V.

(e) Repeat the procedures in 18.6 (c) and (d) for lengths of 20cm, 30cm, 40cm and 50cm.

Questions

(i) Tabulate your results.

(ii) Plot a graph of V (V) against l (cm).

(iii) Determine the slope of your graph and state what it represents.

(iv) State what the graph shows about the wire itself.

(v) Use your graph to find the length of wire across which the voltmeter would read 2V.

(vi) Determine the current in the wire.

(vii) Determine the internal resistance of the two cells together with the leads.

(viii) Explain why the jockey is lifted between one reading and the next.

Table of results

The voltmeter read 3.06V across the two cells with the switch open, and was read to the nearest 0.02V at each length.

l (cm)	V (V)
10	0.50
20	1.00
30	1.48
40	1.98
50	2.48

Making the measurement count

Practical tip 1: *Give the long axis to the quantity you have to read finely.*

The lengths are set on a rule and are known to a millimetre without any help from the graph; the potential differences are read on a meter and are the ones that scatter. Give the potential axis the long scale and the length axis the short one, and the sheet is spent where the doubt is. A line drawn steeply is also easier to read across, which is what part (v) will ask you to do.

Practical tip 2: *Draw both axes from zero and let the line settle the matter.*

At zero length there is zero wire between the two voltmeter contacts, so you know what the p.d. must be there without measuring it. That makes the origin a test the line has to pass, and a line that misses it by more than the reading error is telling you something: a dirty clip at the zero end, or a jockey that is not making contact where you think it is.

Practical tip 3: *Move the jockey, and nothing else moves.*

The length of wire in the driving circuit is fixed by the two crocodile clips and not by the jockey, and the voltmeter takes almost nothing. So the current is the same for every reading in the table, and that is the reason the potential rises in proportion to the length. Move a clip instead of the jockey and the current changes at every reading, and nothing in this analysis survives.

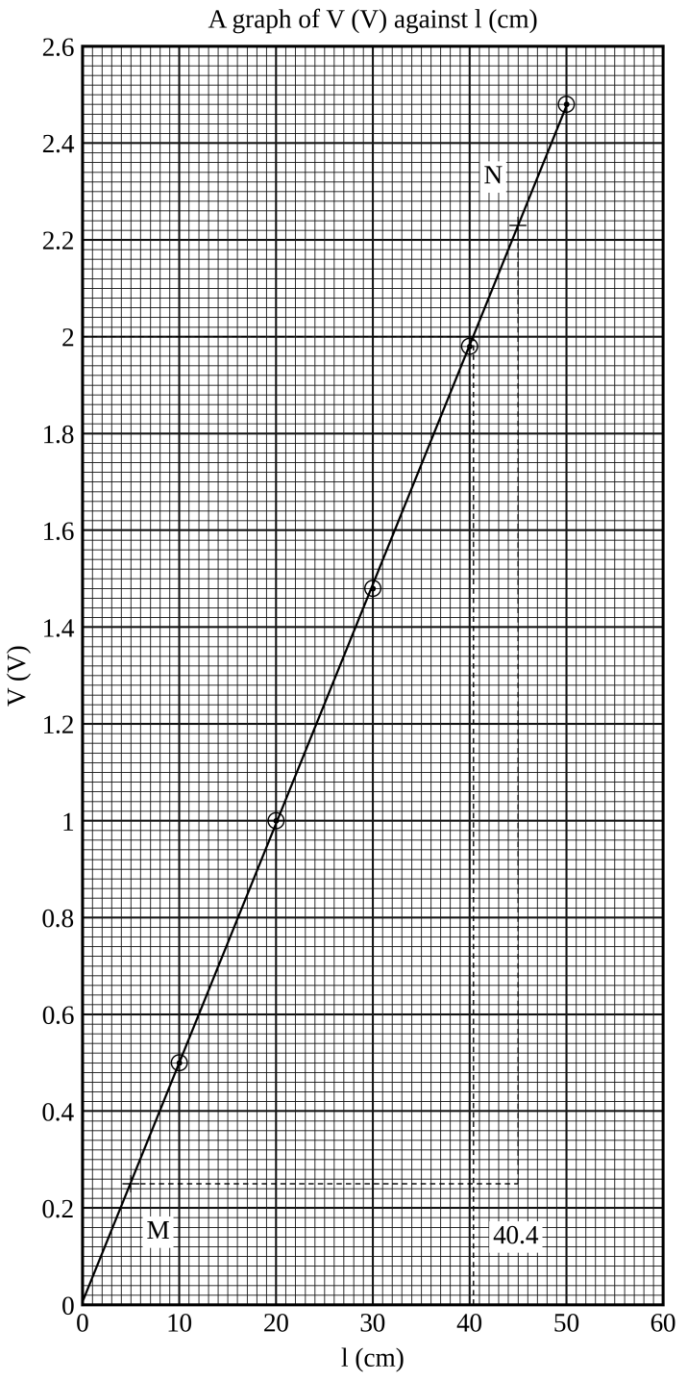
Practical tip 4: *Do not carry this slope to any other setting.*

What the graph measures is what one centimetre of this wire is worth while this current is flowing. Change the cells, the length in circuit, or the wire, and it changes with them. Part (iii) asks you to name the quantity; part (vi) asks you to turn it into something else, and for that you will need the resistance of a metre of the wire, which the apparatus list gives.

Answers to the questions

(i) The readings are those in the table above.

(ii) Take l at 2cm to 10cm, running from 0 to 60cm, and V at 2cm to 0.2V, running from 0 to 2.6V. That is 12cm by 26cm, which fits an A4 sheet upright. Give the graph the title A graph of V (V) against l (cm).



(iii) Taking two points on the line, M (5, 0.25) and N (45, 2.23),

$$\text{slope}=\frac{2.23\text{V}-0.25\text{V}}{45\text{cm}-5\text{cm}}=0.0495\text{Vcm}^{-1}$$

which in volts per metre is 4.95Vm^{-1}. It is the potential gradient along the wire: the potential difference between any two points of it divided by the distance between them.

(iv) That the line is straight and passes through the origin shows the wire is uniform: every centimetre of it has the same resistance as every other, so every centimetre carries the same share of the potential. A wire that had been drawn thinner at one end would give a curve.

(v) Read across from V = 2V to the line and down to the length axis. Worked from the gradient, that length is

$$l=\frac{2V}{0.0495Vcm^{-1}}=40.4cm$$

which lies inside the 50cm actually in the circuit, so it is a length that could be tapped off with the jockey and the reading checked.

(vi) The potential gradient is the current multiplied by the resistance of unit length, and the wire is specified per metre, so the gradient in volts per metre is the one to use:

$$I=\frac{4.95Vm^{-1}}{8.0\Omega m^{-1}}=0.62A$$

(vii) The whole 50cm in circuit carries

$$V=0.0495Vcm^{-1}\times 50cm=2.48V$$

and the two cells offered 3.06V on open circuit, so the p.d. lost inside the cells and the leads is 3.06V − 2.48V = 0.58V at 0.62A:

$$r=\frac{0.58V}{0.62A}=0.94\Omega$$

That is the pair together with the leads, so about 0.47Ω belongs to each cell and its share of the connections. Experiment 18.3 found 0.42Ω for one fresh cell with nothing else inside its clips, so the two figures are of the size they ought to be.

(viii) Because the jockey presses a metal edge onto a bare wire, and left down it scrapes a groove as it slides and heats the wire at the point of contact. A groove changes the wire's cross-section, and a hot patch changes its resistance, so the wire would no longer be the uniform wire the analysis assumes. Press and lift, and the contact stays clean as well, which is what the reading depends on.

Examiner's eye

- The gradient is not the electromotive force of the cells and not the p.d. across the wire. It is a p.d. per unit length, and an answer quoted in volts has lost the unit that makes it useful.
- Watch the centimetres. The slope comes off the graph in Vcm^{-1} and the wire is specified in Ωm^{-1}; one of the two has to be converted before they are divided, and the conversion belongs on its own line.
- Nothing on the graph will finish part (vii). It needs the open-circuit reading, which was taken in the procedure before the switch was ever closed; an answer sheet without that figure cannot start the question at all.
- Say what the straight line through the origin proves. "The graph is a straight line" describes the paper; "the wire is uniform" answers the question.

Practice 18.6

The workshop is lit from an accumulator at one end of the room through a bare strip of wire that has been extended, a metre at a time, as each new bench was added. The lamp over the first bench was bright enough to work under when it was the only lamp.

It is not bright now, and the man who works under it has taken to saying so on the day each new metre goes up. Nobody has touched his lamp, his tapping or his bench, and he has been told as much three times.

Procedure

Proceed as follows:

(a) Connect the two cells, the switch and the wire so that 50cm of the wire carries the current.

(b) With the switch open, clip the voltmeter across the two cells and record its reading.

(c) Close the switch, press the jockey onto the wire at 10cm, read the voltmeter and lift the jockey.

(d) Record the length l and the potential difference V.

(e) Repeat the procedures in P18.6 (c) and (d) for lengths of 20cm, 30cm, 40cm and 50cm.

Questions

(i) Tabulate your results.

(ii) Plot a graph of V (V) against l (cm).

(iii) Determine the potential gradient along the wire.

(iv) The wire has a resistance of $2.70\Omega m^{-1}$. Determine the current in it.

(v) Determine the internal resistance of the two cells together with the leads.

(vi) Use your graph to find the length of wire across which the voltmeter would read 2V.

(vii) Explain why the reading of 2V cannot in fact be obtained by moving the jockey further along this wire.

(viii) Determine the length of wire that would have to be put in the circuit for the whole of it to carry 2V, and comment on the complaint in the opening.

Practice 18.7

Kipanga had two cells and a torch that takes two, and he put them in the way the torch is built, one behind the other. **Kipute** asked why they should not go side by side instead, since two cells sharing the work ought to last twice as long, and was told that side by side is how you make no difference at all, since both cells push equally and neither gets anywhere. Kipute observed that the torch was not getting anywhere either.

Both arrangements are in the books and both are used, and the books say what each does to the electromotive force and to the internal resistance. Neither boy had opened a book. This question asks whether a measurement agrees with one.

Procedure

Proceed as follows:

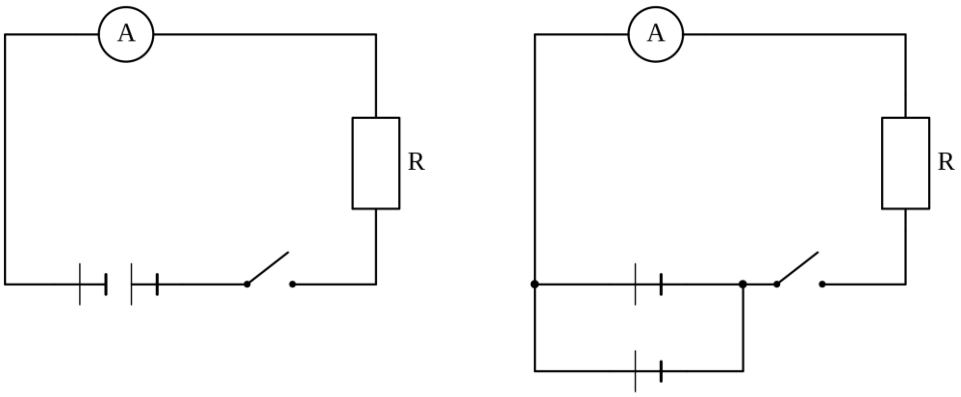
(a) Connect the two cells in series with the switch, the ammeter and the resistance box, as in the left-hand diagram.

(b) Set the box to 6Ω, close the switch, read the ammeter and open the switch.

(c) Repeat the procedure in P18.7 (b) for the values of R equal to 5Ω, 4Ω, 3Ω and 2Ω.

(d) Rearrange the two cells in parallel, as in the right-hand diagram, leaving the rest of the circuit untouched.

(e) Read the ammeter in the same way at each of the values of R equal to 6Ω, 4Ω, 3Ω, 2Ω and 1Ω, closing the switch only while each reading is taken.



(a) the two cells in series (b) the two cells in parallel

Questions

(i) Tabulate both sets of results, including 1/I.

(ii) Plot both graphs of R (Ω) against 1/I (A^{-1}) on the same axes.

(iii) Determine the electromotive force of each arrangement.

(iv) Determine the intercept of each line.

(v) The ammeter and the leads used here were found in Experiment 18.3 to add 0.33Ω to the circuit. Use this to obtain the internal resistance of each arrangement.

(vi) The rules give 2E and 2r for two identical cells in series and E and r/2 for the same two in parallel. State whether your results agree.

(vii) Determine which arrangement drives the larger current through a bulb of resistance 4Ω, and which through a bulb of resistance 0.2Ω.

(viii) Say whether the torch is better served by the arrangement Kipanga chose, and under what condition the other one would beat it.

Chapter 19

THE METRE BRIDGE FAMILY

Half of everything the practical paper has ever asked in Current Electricity has been a metre bridge, and it is the question that goes wrong most often. That is not because the physics is hard. It is because the apparatus has to be connected before anything can be measured, and because the one equation that governs all of it looks so simple that it is the line most often left off the page.

Here it is. A metre bridge is a wire exactly 1m long with a uniform cross-section, and two gaps above it holding two resistances. Call the resistance in the left-hand gap R_1 and the one in the right-hand gap R_2. Slide the jockey along the wire until the galvanometer reads zero, and let L_1 be the length of wire to the left of that point and L_2 the length to the right. Then

$$\frac{R_1}{R_2} = \frac{L_1}{L_2}$$

and every one of the thirteen questions in this chapter starts from that one line. What each of them adds to it is what you are being asked to find. What changes is which of the four quantities you know, which you vary, and what you have put in the gaps: a single resistor, a resistor in series with a length of wire you can clip anywhere along, a coil beside a standard resistor in parallel, a coil by itself, a clipped length of wire against a clamped balance point, or no bridge board at all.

Two habits pay for themselves before you meet any of them. Tap the jockey on the wire rather than drag it: drag it and you wear a groove, and a worn wire is no longer uniform. And look at where on the wire each balance lands, rather than accepting the first ladder of known resistances you think of. The first experiment asks you why that matters.

The six varieties at a glance

	What is in the gaps	What the measurements give you
19.1	A resistance box and one unknown resistor	the unknown resistance
19.2	A fixed resistor in series with a clipped length of wire	the fixed resistance and the resistivity of the wire
19.3	An unknown coil in parallel with a standard resistor	the resistance of the coil alone
19.4	A coil of known length, against a resistance box	the resistivity of the wire the coil is wound from
19.5	A standard resistor against a clipped length of wire	the length of that wire worth a stated resistance
19.6	A bridge built from a metre rule and two bare wires	the resistivity of one of them

All six are worked in full. The seven practice questions change the apparatus, the ladder of known resistances and the material, and the last of them takes the fourth variety somewhere the others do not go.

Experiment 19.1

The component that started the argument had come out of a radio, and it had no marking left on it at all. Somebody had cleaned it with petrol.

Kipanga turned it over twice and announced that the first band was brown. **Mr. Akilikubwa** pointed out that there were no bands. **Kipanga** said in that case the first band was brown and had worn off, which he seemed to feel was nearly as good.

Kipute asked whether they could not simply measure it. **Mr. Akilikubwa** said they could, and that measuring it would take about 8 minutes, which was 5 minutes less than the argument had already taken. He put the component on the bench between them.

"So," he said, *"what is it worth?"*

Aim

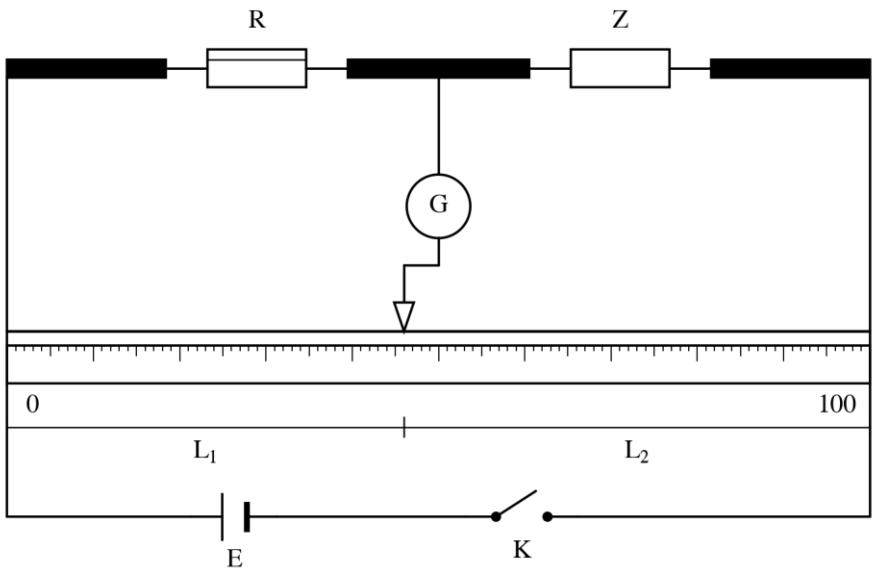
To determine the resistance of an unmarked resistor using a metre bridge and a resistance box.

Apparatus

A metre bridge with its jockey, a resistance box reading from 0.1Ω to 10Ω, an unmarked resistor labelled Z, a galvanometer, one dry cell, a switch and six connecting wires.

Setup

The resistance box is connected across the left-hand gap of the bridge and the unmarked resistor across the right-hand gap. The cell and the switch are joined in series between the two ends of the bridge wire. One terminal of the galvanometer goes to the copper strip between the two gaps and the other to the jockey. Every connection is made under a terminal screw and pulled gently to check it before the switch is closed.



Procedure

Proceed as follows:

(a) Connect the circuit as shown, leaving the switch open.

(b) Set the resistance box to 2Ω.

(c) Close the switch, tap the jockey along the bridge wire until the galvanometer shows no deflection, and read the balance length L_1 from the left-hand end of the wire. Open the switch.

(d) Repeat steps (b) and (c) with the box set to 3Ω, 4Ω, 5Ω, 6Ω and 7Ω.

Questions

(i) Tabulate your results, including the values of L_2 and L_1/L_2.

(ii) Plot a graph of L_1/L_2 against R and draw the best straight line through your points.

(iii) Determine the slope M of your line and state its unit.

(iv) Derive the equation governing this experiment.

(v) Evaluate $N=1/M$ and state what N is.

(vi) State, with a reason, what your line does at the origin.

(vii) One of your six balance points lies further from the middle of the bridge wire than any of the others. Say which it is, and explain why a balance near the middle of the wire can be trusted further than one near an end.

Table of results

The bridge wire was 100cm long and the balance length L_1 was read from its left-hand end.

R (Ω)	L_1 (cm)	L_2 (cm)	L_1/L_2
2	30.1	69.9	0.431
3	39.3	60.7	0.647
4	45.8	54.2	0.845
5	51.3	48.7	1.053
6	55.8	44.2	1.262
7	60.2	39.8	1.513

Making the measurement count

Practical tip 1: Finding the balance without hunting for it. *Tap first at one end of the wire and then at the other.*

The galvanometer kicks one way at one end and the other way at the other, and the balance point lies between them; if it kicks the same way at both ends, something is not connected and no amount of sliding will help. Then halve the interval, and halve it again. Four or five taps put you within 1cm of the balance and you can read the last part of it slowly. Tap, do not drag: drag the jockey and you wear a flat on the wire, and a wire that is thinner in one place is no longer the uniform wire the whole method depends on.

Practical tip 2: What the resistance box is for. *The box is the known resistance, and its dial is the one thing you set before each balance.*

The ladder used above was chosen after one trial reading, not before. Open the switch between readings while you are choosing: a box left carrying current warms, and a box that has warmed is no longer the resistance printed on its dial.

Practical tip 3: The arms do not begin at the graduations. *The scale runs from 0 to 100, but the resistance in an arm does not begin at the graduation. It begins at the terminal screw.*

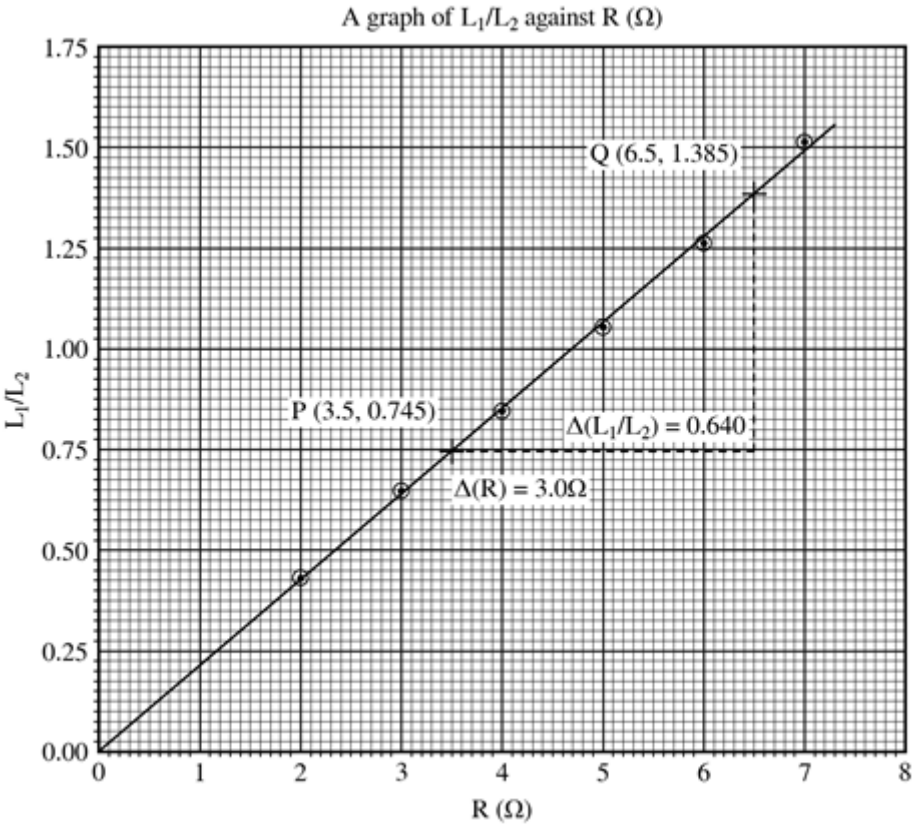
The copper strip, the screw and the soldered joint at each end add a few millimetres of effective wire to whichever arm they sit in, and so do the last centimetres of the connecting leads. These are the end corrections, and on a school bridge they are somewhere between 0.2cm and 0.5cm at each end. They are the same two lengths in every row, so they do not scatter your points. They are the honest reason a bridge measurement made this way is uncertain by about 1% and not by a tenth of one. Part (vii) asks you what follows.

Practical tip 4: Six settings rather than one. *One setting of the box and one balance length would give you the unknown resistance in two readings and about 90 seconds.*

Six settings give the same answer from the slope of a line, and a line drawn through six points is worth about half as much again as the best single reading, because the scatter of the individual balance points partly averages out. Six readings also tell you something a single reading cannot: whether the bridge wire is uniform over the stretch your balance points cover. A wire with a thin patch in it gives six points that do not lie on one straight line, and no single reading would ever have shown it.

Answers to the questions

(i) and (ii). Take R at 2cm to 1Ω, running from 0Ω to 8Ω, and L_1/L_2 at 2cm to 0.25, running from 0 to 1.75. That is 16cm by 14cm, which fits an A4 sheet upright. Give the graph the title A graph of L_1/L_2 against R (Ω).



(iii) Taking the two marked corners P (3.5, 0.745) and Q (6.5, 1.385),

$$M=\frac{1.385-0.745}{6.5\Omega-3.5\Omega}=\frac{0.640}{3.0\Omega}=0.213\Omega^{-1}$$

The slope is $0.213\Omega^{-1}$. The ratio L_1/L_2 is a length divided by a length and has no unit, so the unit of the slope is the reciprocal of an ohm.

(iv) At balance the bridge gives $R_1/R_2=L_1/L_2$. The box is in the left-hand gap and the unknown in the right, so R_1 is the box reading R and R_2 is the unknown resistance Z. Hence

$$\frac{L_1}{L_2}=\frac{R}{Z}=\frac{1}{Z}\times R$$

which is a straight line of L_1/L_2 against R through the origin, of slope $1/Z$.

(v) From the slope,

$$N=\frac{1}{M}=\frac{1}{0.213\Omega^{-1}}=4.69\Omega$$

N is the resistance of the unmarked resistor, and the resistor is worth 4.69Ω.

(vi) It passes through the origin. With the box set to zero the left-hand gap is shorted out, so the bridge can only balance where the length of wire to the left of the jockey also carries no resistance, at the left-hand end. The point 0, 0 is therefore fixed by the way the bridge is built rather than by your eye on a galvanometer, and the line is drawn through it.

(vii) The reading at 2Ω, which balanced at 30.1cm and is the furthest of the six from the middle. Two things go wrong towards an end, and both of them are least at 50cm. Take the reading error first. A millimetre of error in L_1 shifts L_2 by a millimetre the other way, so the fractional error in the ratio is 0.1cm divided by L_1 added to 0.1cm divided by L_2. At 30.1cm that is 0.48%; at the middle of the wire it is 0.40%. The sum is least when the two lengths are equal. The end corrections of Practical tip 3 are the larger effect and they work the same way round. Each end adds a small length to its own arm, so the two contributions enter the ratio as that length divided by L_1 and by L_2 with opposite signs; if the two ends are alike they cancel exactly at the middle of the wire, and at 30.1cm, taking 0.35cm as the middle of the range in that tip, they leave the reading 0.7% out. That one is systematic, so unlike the reading error it does not average down over six points. It tilts the line.

Examiner's eye

- Give the slope its unit. L_1/L_2 is a pure number and R is in ohms, so the slope is in Ω^{-1} and the answer to part (v) comes out in ohms. A slope quoted as 0.213 with nothing after it is a slope that could belong to anything.
- Take the balance length from the end the question names. The bridge wire has two ends and the scale runs from one of them; measure from the other and every L_1 in your table is 100cm minus what it should be. The one mercy is that the ratio then falls as R rises and the points bend away from any straight line, so the sheet tells you before the examiner does.
- Tabulate L_2 as well as L_1. It costs one subtraction a row and it is where a slip shows up: if L_1 and L_2 do not add to 100cm, something has been copied down wrongly.
- Draw the line through the origin, and put the reason on the sheet. The origin is not an act of faith here; it follows from the fact that a gap shorted out balances at the end of the wire. The reason earns more than the line does.

Practice 19.1

The two coils came out of the same carton and looked identical. One of them was the right one.

The storekeeper had a system, and the system was a ledger. Stock went into it by the size of the carton it arrived in, because that was what he could see from the door, and both coils had arrived in the same carton. The ledger therefore said that there was one line of stock and there were two of it.

Kipute asked what the form had actually said. The storekeeper looked at it for a while and reported that it said a number and a symbol he did not recognise, and that this was exactly the sort of thing that made his job difficult. **Mr. Akilikubwa**, who had written the form, said that the symbol was the whole of the message and that he would have drawn it larger if he had known. Both coils went on the counter, and somebody had to decide.

Procedure

Proceed as follows:

(a) Connect the circuit as in Experiment 19.1, with the resistance box in the left-hand gap and the coil in the right-hand gap, leaving the switch open.

(b) Set the resistance box to 3Ω.

(c) Close the switch, tap the jockey along the bridge wire until the galvanometer shows no deflection, and read the balance length L_1 from the left-hand end of the wire. Open the switch.

(d) Repeat steps (b) and (c) with the box set to 4Ω, 5Ω, 6Ω, 7Ω and 8Ω.

Questions

(i) Tabulate your results, including the values of L_2 and L_1/L_2.

(ii) Plot a graph of L_1/L_2 against R and draw the best straight line through your points.

(iii) Determine the slope of your line and state its unit.

(iv) Derive the equation governing this experiment, and hence determine the resistance of the coil.

(v) The requisition asked for a coil of 5.0Ω. State, with a reason, whether the coil you have tested is the one that was asked for.

(vi) The storekeeper says he could tell the two coils apart by weighing them. Say what a difference in mass would and would not tell him about the resistance.

(vii) Give one reason why a resistance box is a better partner for this measurement than a single fixed resistor.

Experiment 19.2

The welder on the corner buys resistance wire by the kilogram, because that is how it is sold, and uses it by the arm-span, because that is how he cuts it. His rule is fixed: one arm-span makes one element, two arm-spans make an element that lasts longer, and if it glows red he has cut it too short.

Kipanga said this was a perfectly good system as long as the welder never changed his arms. **Mr. Akilikubwa** said the arms were not the trouble. The trouble was that the shop had two reels on the shelf, one bright and one dull, and the welder had been cutting from whichever was nearer.

An arm-span of one of those reels and an arm-span of the other are not the same thing at all. The reel on the bench will have to say for itself what it is.

Aim

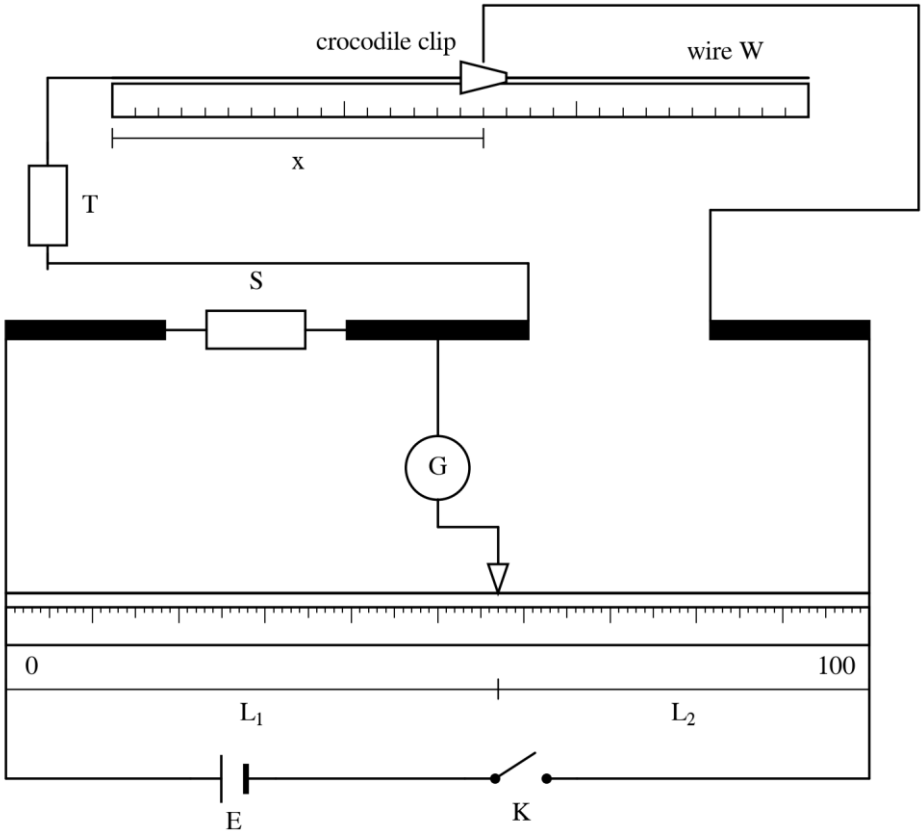
To determine the resistivity of a resistance wire, and the value of a sealed resistor connected in series with it, using a metre bridge.

Apparatus

A metre bridge with its jockey, a standard resistor of 5.00Ω, a sealed resistor labelled T whose value is not marked, 150cm of resistance wire labelled W stretched along a wooden rule, a crocodile clip, a micrometer screw gauge, a galvanometer, one dry cell, a switch and eight connecting wires.

Setup

The standard resistor is connected across the left-hand gap. The sealed resistor T and the wire W are joined in series and connected across the right-hand gap: one terminal of the gap goes to T and on to the zero end of the wire, and the other terminal of the gap goes to the crocodile clip. The wire is pulled straight along the rule and taped at each end, with the scale under it. The clip is squeezed on at a marked graduation and not between two.



Procedure

Proceed as follows:

(a) Connect the circuit as shown, leaving the switch open.

(b) Clip the crocodile clip on the wire W at 20cm from the zero end, so that x is 20cm.

(c) Close the switch, tap the jockey along the bridge wire until the galvanometer shows no deflection, and read the balance length L_1 from the left-hand end of the wire. Open the switch.

(d) Repeat steps (b) and (c) for x equal to 40cm, 60cm, 80cm, 100cm and 120cm.

(e) Measure the diameter d of the wire W with the micrometer screw gauge at three places along its length.

Questions

(i) Tabulate your results, including the values of L_2 and of the resistance R_2 in the right-hand gap.

(ii) Plot a graph of R_2 against x and draw the best straight line through your points.

(iii) Determine the slope of your line and state its unit.

(iv) Derive the equation governing this experiment, and state what the slope and the intercept of your graph each represent.

(v) Determine the value of the sealed resistor T.

(vi) Determine the cross-sectional area of the wire W, and hence its resistivity.

(vii) The crocodile clip grips the wire over a few millimetres and the length x is read to the middle of its jaws. Explain why an error of this kind, the same at every reading, changes one of your two answers and not the other, and say which of the two is therefore the safer.

Table of results

The standard resistor in the left-hand gap was 5.00Ω, and the micrometer read 0.38mm at each of the three places along the wire.

x (cm)	L_1 (cm)	L_2 (cm)	R_2 (Ω)
20	70.7	29.3	2.07
40	63.0	37.0	2.94
60	56.3	43.7	3.88
80	51.3	48.7	4.75
100	46.9	53.1	5.66
120	43.7	56.3	6.44

Making the measurement count

Practical tip 1: From the soldered joint to the jaws. *The length in the circuit runs from the soldered joint at the zero end of the wire to the jaws of the clip, and nowhere else.*

The tail of wire beyond the clip carries no current and does not count, however much of it there is. Read x to the middle of the jaws every time, and do not let the tail sag back and touch the live part of the wire, which short-circuits a length you have counted. Part (vii) asks what a mistake of this kind does to each of your two answers.

Practical tip 2: Why a resistor sits in series with the wire. *The shortest clipped length of W is worth well under an ohm on its own, and the other gap holds* 5.00Ω.

A gap holding a fraction of an ohm against 5.00Ω still balances, but it balances within a hand's breadth of one end of the wire, where a millimetre of error costs two or three times what the same millimetre costs in the middle. Put a resistor in series and the whole ladder lifts into the readable part of the wire. It does not spoil the measurement, because a fixed resistance and a length of wire enter the total in different ways, and the graph separates them again.

Practical tip 3: Reading a micrometer on a thin wire. *Close the gauge on nothing first and read the zero error, then close it on the wire with the ratchet and not with the thimble.*

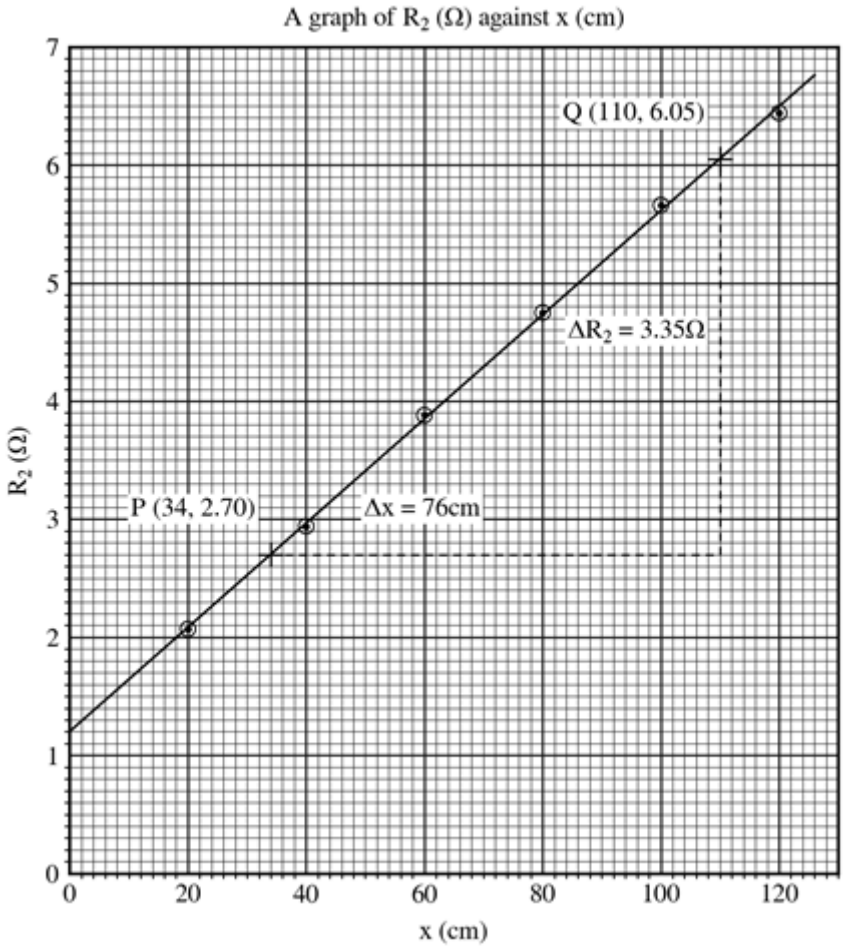
A wire squeezed by a thumb and a thimble reads thinner than it is, and the area goes as the square of the diameter, so the gauge is the weakest link in the whole resistivity and deserves more care than anything else on the bench. Take the reading at three places along the wire, because drawn wire is not perfectly round and not perfectly even. Quote the diameter to the 0.01mm the gauge can resolve and no finer.

Practical tip 4: Six clipped lengths rather than two. *Two lengths and two balance points would give the slope and the intercept in four readings.*

Six give both from a line, and the line does something the four readings cannot: it shows that the resistance really does grow in proportion to the length. A wire with a kink, a thin patch or a strand of another metal soldered into it gives six points that lie on no straight line at all, and two readings would have hidden it perfectly.

Answers to the questions

(i) and (ii) Take x at 2cm to 20cm, running from 0cm to 130cm, and R_2 at 2cm to 1Ω, running from 0Ω to 7Ω. That is 13cm by 14cm, which fits an A4 sheet upright. Give the graph the title A graph of R_2 (Ω) against x (cm).



(iii) Taking the two marked corners P (34, 2.70) and Q (110, 6.05),

$$\text{slope}=\frac{6.05\Omega\text{-}2.70\Omega}{110\text{cm-}34\text{cm}}=\frac{3.35\Omega}{76\text{cm}}=0.0441\Omega\text{cm}^{-1}$$

The slope is $0.0441\Omega\text{cm}^{-1}$, which is $4.41\Omega\text{m}^{-1}$.

(iv) At balance $R_1/R_2=L_1/L_2$. The standard resistor is in the left-hand gap, so $R_1=S$, and the right-hand gap holds T in series with a length x of the wire. A uniform wire of resistivity ρ and cross-sectional area A has a resistance ρ/A for every unit of its length, so

$$R_2=\frac{\rho}{A}x+T$$

which is a straight line of R_2 against x. The slope is the resistance of 1cm of wire, ρ/A, and the intercept is T, the resistance the gap still has when no wire at all is in it.

(v) The line cuts the R_2 axis at 1.20Ω, so the sealed resistor is worth 1.20Ω.

(vi) The micrometer read 0.38mm, which is 3.8×10^{-4}m, so

$$A=\frac{\pi d^2}{4}=\frac{3.14\times\left(3.8\times10^{-4}\text{m}\right)^2}{4}=1.13\times10^{-7}\text{m}^2$$

The slope is the resistance per unit length, so

$$\rho=\frac{\rho}{A}\times A=4.41\Omega\text{m}^{-1}\times1.13\times10^{-7}\text{m}^2=4.98\times10^{-7}\Omega\text{m}$$

The accepted resistivity of constantan is about $4.9\times10^{-7}\Omega m$, so the value found agrees with it to 2%, which is well inside the 5% that a slope good to 2% and a diameter read to the nearest 0.01mm allow between them.

(vii) It changes the intercept and leaves the slope alone. If every clipped length is short by the same amount, every point on the graph slides the same distance to the left; a set of points shifted bodily sideways lies on a line of exactly the same steepness, so the slope, and with it the resistivity, is untouched. What moves is where that line crosses the axis, and the intercept is the value of T. A clip set 5mm too far along at every reading would put T out by 0.02Ω. The resistivity is therefore the safer of the two answers, and it is safe against exactly the error the method is most likely to make.

Examiner's eye

- Show the unit conversion. The slope comes off the graph in Ωcm^{-1} because the axis is in centimetres, and the resistivity needs it in Ωm^{-1}. That factor of 100 is one line of working and it is the commonest single loss of marks in this experiment.
- Square the diameter and then divide by four. πd^2 is four times too big and gives a resistivity nobody could defend.
- Start the x axis at zero. The intercept is one of the two answers here, and an axis that begins at 20cm because the readings do puts it off the sheet.
- Measure only what is in the circuit. The wire runs to 150cm and the clip stops at 120cm at most; the part beyond the jaws carries no current, so a table that records the whole length is a table of the wrong quantity.

Practice 19.2

Nailed to the post of a hardware stall, on a board that had been painted white a long time ago, was this:

RESISTANCE WIRE

SOLD BY THE METRE

THICK 500/= THIN 700/=

THIN IS STRONGER

Underneath, in a different hand and a different paint, somebody had added the word STRONGER with a question mark after it, and somebody else had crossed out the question mark. The stallholder said he had been selling the thin wire for eleven years, which he seemed to think was the same kind of evidence.

Kipute bought a metre of each. **Kipanga** asked what he was going to do with them and was told that this was exactly the sort of question the board could not answer.

Procedure

Proceed as follows:

(a) Connect the circuit as in Experiment 19.2, with the standard resistor of 5.00Ω in the left-hand gap and a coil of unknown resistance in series with the wire Y in the right-hand gap, leaving the switch open.

(b) Clip the crocodile clip on the wire Y at 5cm from the zero end, so that x is 5cm.

(c) Close the switch, tap the jockey along the bridge wire until the galvanometer shows no deflection, and read the balance length L_1 from the left-hand end of the wire. Open the switch.

(d) Repeat steps (b) and (c) for x equal to 10cm, 15cm, 20cm, 25cm and 30cm.

(e) Measure the diameter d of the wire Y with the micrometer screw gauge at three places along its length.

Questions

(i) Tabulate your results, including the values of L_2 and of the resistance R_2 in the right-hand gap.

(ii) Plot a graph of R_2 against x.

(iii) Determine the slope and the intercept of your line, with their units.

(iv) State the physical meaning of the intercept.

(v) Determine the resistivity of the wire Y.

(vi) Determine the length of the wire Y whose resistance would equal that of the coil.

(vii) The stall board claims that the thinner wire is the stronger. Say what quantity the board is confusing with strength, and what your measurement of the wire Y can and cannot settle about the claim.

Experiment 19.3

At the charging stall by the bus stand, four thin wires run side by side from the battery to the row of sockets, twisted together at both ends. They did not go up together. The first is three years old. The second went up when a nephew said the phones were charging slowly, the third when a customer said the same thing, and the fourth after a wet week when everything was slow. Nobody wrote anything down and nobody kept the reel. The owner is sure the fourth wire helped, because the queue was shorter the week it went on, which it was, because the buses were on strike.

Mr. Akilikubwa stood in front of it for a while with **Kipute**, who wanted to know whether the fourth wire had made any difference at all. Nothing at the stall could tell them. Four wires twisted together at both ends are four resistances side by side, and what the bundle is worth as one conductor is a number nobody there has ever had.

Aim

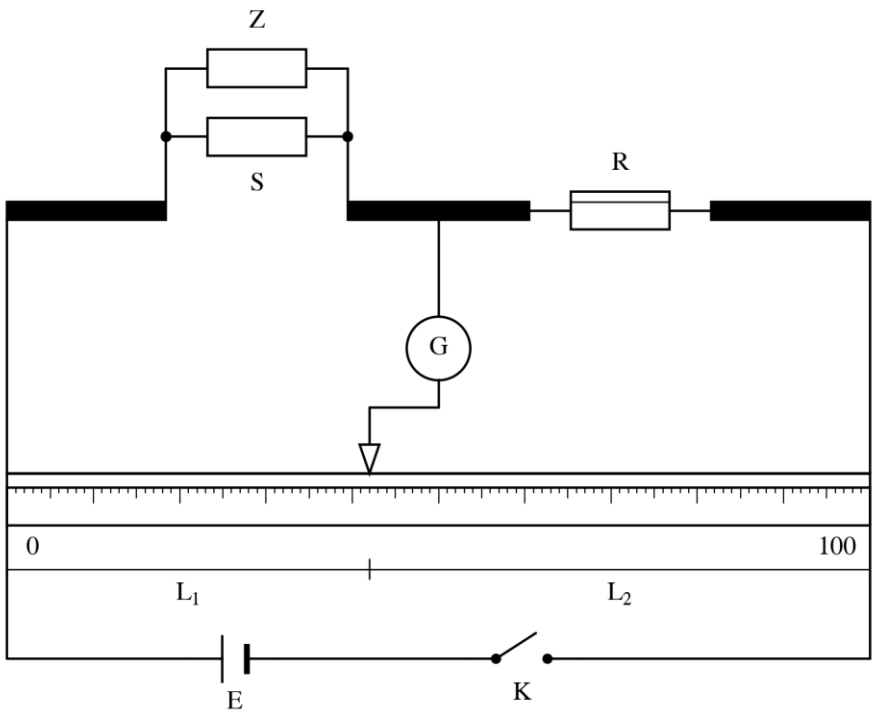
To determine the resistance of a coil by measuring it in parallel with a standard resistor on a metre bridge.

Apparatus

A metre bridge with its jockey, a resistance box reading from 0.1Ω to 10Ω, a standard resistor of 2.00Ω, a coil of unknown resistance labelled Z, a galvanometer, one dry cell, a switch and eight connecting wires.

Setup

The coil and the standard resistor are connected side by side across the left-hand gap, so that the two of them together bridge it, and the resistance box is connected across the right-hand gap. Both ends of the coil and both ends of the standard resistor meet at the two terminals of the left-hand gap and nowhere else. Join them at one end only, or in series by mistake, and the bridge still balances; it balances in the wrong place and gives back a confident wrong answer.



Procedure

Proceed as follows:

(a) Connect the circuit as shown, leaving the switch open.

(b) Set the resistance box to 1Ω.

(c) Close the switch, tap the jockey along the bridge wire until the galvanometer shows no deflection, and read the balance length L_1 from the left-hand end of the wire. Open the switch.

(d) Repeat steps (b) and (c) with the box set to 2Ω, 3Ω, 4Ω, 5Ω and 6Ω.

(e) Record the length L_0 of the bridge wire.

Questions

(i) Tabulate your results, including L_1 in metres and the values of $1/L_1$ in m^{-1}.

(ii) Plot a graph of $1/L_1$ against R and draw the best straight line through your points.

(iii) Determine the slope and the intercept of your line, with their units.

(iv) Derive the equation governing this experiment.

(v) Your intercept should be the reciprocal of a length you have measured. State which length, and compare the two.

(vi) Determine the combined resistance of the left-hand gap, and hence the resistance of the coil Z alone.

(vii) Your combined resistance is good to about 1%. Show that your value for the coil is not, and state what you would change about the apparatus to improve it.

Table of results

The bridge wire was 1.00m long and the standard resistor in parallel with the coil was 2.00Ω.

R (Ω)	L_1 (cm)	L_1 (m)	$1/L_1$ (m^{-1})
1	60.9	0.609	1.642
2	44.0	0.440	2.273
3	34.2	0.342	2.924
4	28.6	0.286	3.497
5	24.3	0.243	4.115
6	20.6	0.206	4.854

Making the measurement count

Practical tip 1: A parallel pair pulls the balance towards one end. *Two resistances in parallel are always worth less than the smaller of them, so putting the coil beside a standard resistor pulls the left-hand gap down and drives the balance towards the left-hand end of the wire.*

That is why the box has to be run over small values here, and why the last reading of the table is the one nearest the end of the wire. If the balance runs off the readable part of the wire at the left-hand end, the box is set too high; if it runs off at the right-hand end, too low.

Practical tip 2: Why this arrangement cannot measure the coil directly. *The whole difficulty of it is that the bridge never sees the coil by itself; it sees the pair, and the coil has to be got out of the pair by arithmetic.*

That arithmetic takes one reciprocal from another. Part (vii) is where the cost of it is counted.

Practical tip 3: Plotting a reciprocal. *Convert each balance length to metres before taking its reciprocal, and keep four figures in the column while you work.*

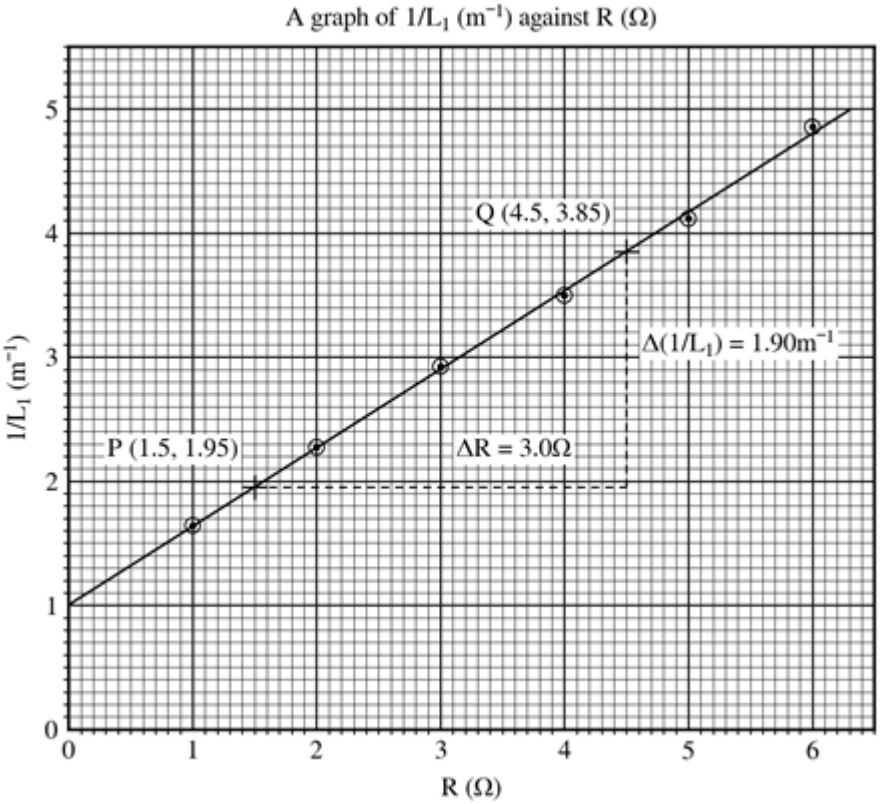
A length of 60.9cm is 0.609m and its reciprocal is $1.642m^{-1}$. Round that length to the nearest centimetre before dividing and the reciprocal barely moves; do the same at the short end of the table and it moves by nearly a whole small square on the graph, because the reciprocal of a short length changes fast. Keep the millimetre.

Practical tip 4: A check the graph gives you free. *This is one of the few experiments in the book whose intercept is known before any reading is taken.*

Say which length it is, then check it: that is part (v). A graph whose intercept comes out far from the predicted value has something wrong with it that no amount of care with the slope will repair, and you would rather find that out before you leave the laboratory than afterwards.

Answers to the questions

(i) and (ii) Take R at 2cm to 1Ω, running from 0Ω to 6.5Ω, and $1/L_1$ at 2cm to $1m^{-1}$, running from $0m^{-1}$ to $5.5m^{-1}$. That is 13cm by 11cm, which fits an A4 sheet upright. Give the graph the title A graph of $1/L_1$ (m^{-1}) against R (Ω).



(iii) Taking the two marked corners P (1.5, 1.95) and Q (4.5, 3.85),

$$\text{slope}=\frac{3.85m^{-1}-1.95m^{-1}}{4.5\Omega-1.5\Omega}=\frac{1.90m^{-1}}{3.0\Omega}=0.633m^{-1}\Omega^{-1}$$

and the line cuts the vertical axis at $1.00m^{-1}$.

(iv) Let R_e be the combined resistance of the coil and the standard resistor in parallel. It is in the left-hand gap and the box is in the right, so at balance $R_e/R=L_1/L_2$. The two lengths make up the whole wire, so $L_2=L_0-L_1$ and

$$\frac{R_e}{R}=\frac{L_1}{L_0-L_1}$$

Cross-multiplying, $R_eL_0-R_eL_1=RL_1$, so $L_1(R+R_e)=R_eL_0$, and dividing through by $R_eL_0L_1$ gives

$$\frac{1}{L_1}=\frac{1}{R_eL_0}R+\frac{1}{L_0}$$

a straight line of $1/L_1$ against R of slope $1/R_eL_0$ and intercept $1/L_0$.

(v) The length of the bridge wire. The intercept is $1/L_0$, and the wire is 1.00m long, so it should be $1.00m^{-1}$. The graph gives $1.00m^{-1}$, which agrees. This costs nothing to check and it tests the plotting, the reciprocals and the line all at once.

(vi) The slope is $1/R_eL_0$ and L_0 is 1.00m, so $1/R_e$ is $0.633\Omega^{-1}$ and

$$R_e=\frac{1}{0.633\Omega^{-1}}=1.58\Omega$$

For two resistances in parallel $1/R_e=1/Z+1/S$, so

$$\frac{1}{Z}=0.633\Omega^{-1}-\frac{1}{2.00\Omega}=0.633\Omega^{-1}-0.500\Omega^{-1}=0.133\Omega^{-1}$$

so the coil is worth 7.52Ω. Part (vii) shows that only two of those figures mean anything, so quote it as 7.5Ω. Accept 7.1Ω to 8.0Ω.

(vii) A 1% error in R_e is an error of 0.016Ω, and the same error appears in $1/R_e$ as $0.006\Omega^{-1}$. Since $1/S$ is known far better than $1/R_e$, almost the whole of that lands on $1/Z$, which is only $0.133\Omega^{-1}$ to begin with, so $1/Z$ is uncertain by nearly 5% and so is Z. The 1% has been multiplied by Z divided by R_e, which is close to 5 here. The cure is to choose a standard resistor comparable with the coil instead of much smaller than it: with a standard of 8Ω beside a coil of about 7.5Ω the multiplying factor falls to about 2, and the answer is twice as good for the same reading.

Examiner's eye

- Convert to metres before taking the reciprocal, and put the unit in your table heading. A column of $1/L_1$ worked out in centimetres is a hundred times smaller, the graph is perfectly straight, and the intercept has stopped meaning anything you can check.
- Subtract the reciprocals, not the resistances. Take 2.00Ω from 1.58Ω and you get a negative resistance, which should stop the pen; the parallel formula adds reciprocals and it has to be undone the same way.
- Quote the coil to two figures. That is all the subtraction in part (vi) leaves, and an answer written to five says the opposite of what part (vii) says two lines later.
- Choose the partner resistor before you start. The one thing that would have improved this experiment cost nothing and had to be decided before the first balance point was found.

Practice 19.3

The improvised water heater in the boarding house runs off the 12V winding of a dead transformer. It is a slab of hard paper with resistance wire laid back and forth across it, every crossing a separate strand running from one end of the slab to the other, and all the ends gathered into two twisted bunches, so the slab is a bundle of identical wires joined side by side. It has been rewound at least twice by people who did not write anything down.

Kipanga offered to count them by eye and got a different number the second time. **Mr. Akilikubwa** said that if the slab were a person it would be entitled to refuse to answer, but that it was not, and that a bundle of identical wires joined at both ends has a resistance that depends on how many there are.

The store book says the wire it was rewound from is worth 6.0Ω for each strand across the slab. Rewind it once more without a count and the next one is as likely to burn out as to run cool.

Procedure

Proceed as follows:

(a) Connect the slab across the left-hand gap and the resistance box across the right-hand gap, and complete the bridge in the usual way with the cell, the switch, the galvanometer and the jockey, leaving the switch open.

(b) Set the resistance box to 0.5Ω.

(c) Close the switch, tap the jockey along the bridge wire until the galvanometer shows no deflection, and read the balance length L_1 from the left-hand end of the wire. Open the switch.

(d) Repeat steps (b) and (c) with the box set to 1.0Ω, 1.5Ω, 2.0Ω, 2.5Ω and 3.0Ω.

Questions

(i) Tabulate your results, including the values of L_2 and L_2/L_1.

(ii) Plot a graph of L_2/L_1 against R.

(iii) Determine the slope of your line and state its unit.

(iv) Develop the relation that governs this experiment.

(v) Determine the resistance of the slab.

(vi) Each strand across the slab is worth 6.0Ω. Determine the number of strands the slab contains.

(vii) Your answer to part (vi) must be a whole number. Explain why, and state what you would conclude if it had come out at 4.3.

Experiment 19.4

The man who rewinds burnt-out motors behind the market has never owned an instrument of any kind. He judges wire between his thumb and forefinger, says a number, and is usually within a gauge of it, which after thirty years is not surprising and is still not a measurement.

Kipute said that if the man could do it by feel then the thickness must be what decides everything. **Kipanga** asked him whether he could tell copper from nichrome by feel, and **Kipute** said that anybody could, because one of them is orange.

Mr. Akilikubwa said that between two grey wires of the same gauge the eye and the thumb have nothing left to go on at all, and asked the two of them what, in that case, was left.

Aim

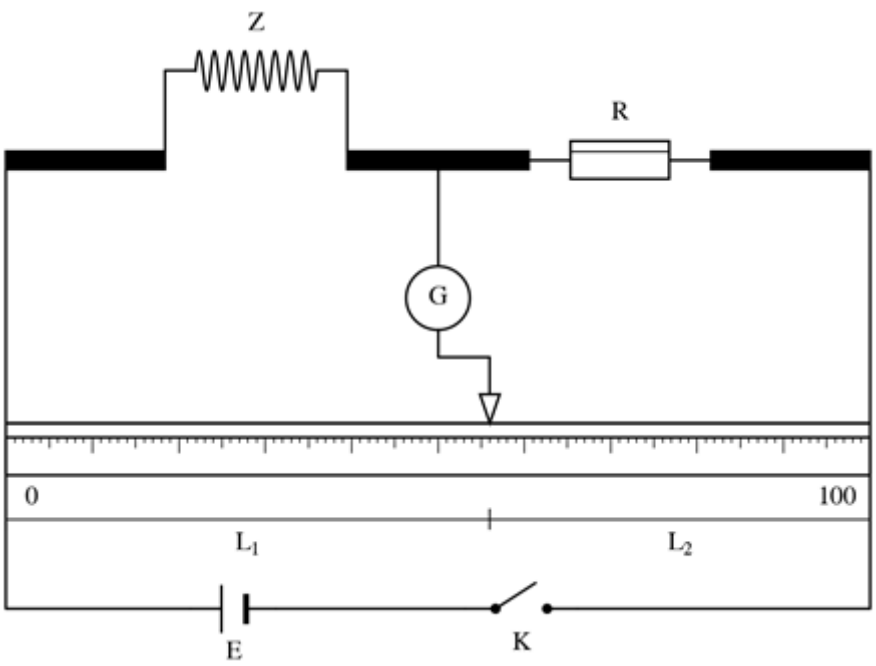
To determine the resistivity of the wire a coil is wound from, using a metre bridge and a resistance box.

Apparatus

A metre bridge with its jockey, a resistance box reading from 1Ω to 20Ω, a coil labelled Z wound from 75.0cm of resistance wire, a micrometer screw gauge, a galvanometer, one dry cell, a switch and six connecting wires.

Setup

The coil is connected across the left-hand gap and the resistance box across the right-hand gap, which is the other way round from Experiment 19.1. The coil is wound on a plastic former and its two free ends are scraped bright before they go under the terminal screws. The cell, the switch and the galvanometer are connected as before.



Procedure

Proceed as follows:

(a) Connect the circuit as shown, leaving the switch open.

(b) Set the resistance box to 2Ω.

(c) Close the switch, tap the jockey along the bridge wire until the galvanometer shows no deflection, and read the balance length L_1 from the left-hand end of the wire. Open the switch.

(d) Repeat steps (b) and (c) with the box set to 4Ω, 6Ω, 8Ω, 10Ω and 12Ω.

(e) Measure the diameter d of the wire the coil is wound from, with the micrometer screw gauge, at three places along one of the free ends.

Questions

(i) Tabulate your results, including the values of L_2 and L_2/L_1.

(ii) Plot a graph of L_2/L_1 against R and draw the best straight line through your points.

(iii) Determine the slope of your line and state its unit.

(iv) Establish a formula relating L_1, L_2, R and the resistance of the coil.

(v) Determine the resistance of the coil.

(vi) The coil is wound from a length L_c of wire equal to 75.0cm. Determine the cross-sectional area of that wire and hence its resistivity.

(vii) State, with a reason, whether winding the wire into a coil rather than leaving it straight changes the resistance you have measured.

Table of results

The coil was wound from 75.0cm of wire and the micrometer read 0.27mm at each of the three places along the free end.

R (Ω)	L_1 (cm)	L_2 (cm)	L_2/L_1
2	74.1	25.9	0.350
4	58.8	41.2	0.701
6	48.7	51.3	1.053
8	42.0	58.0	1.381
10	36.3	63.7	1.755
12	32.6	67.4	2.067

Making the measurement count

Practical tip 1: Where the length in the answer comes from. *The length of wire in a coil cannot be got by measuring the coil, and if you unwind it to find out you usually ruin it. That is why the length is given to you here.*

A coil wound from a measured length is how such an experiment is set up in practice, and the figure on the label is a reading like any other. If you ever have to find the length yourself, count the turns and multiply by the mean circumference of the winding, taking the mean at the middle of the wire rather than at the former.

Practical tip 2: The ladder runs the other way. *The unknown is in the left-hand gap here, so the box has to be run up past the coil rather than down to it.*

A box set well below the coil balances near the right-hand end, and a box set well above it balances near the left. The ladder used above straddles the coil, which is what puts balance points on both sides of the middle of the wire instead of crowding all six into one half of it. The 2Ω reading is the one furthest out, and a ladder that began at 3Ω would have been better still.

Practical tip 3: Reading a micrometer on a coiled wire. *Measure the diameter on a free end that has been straightened, never on the winding itself.*

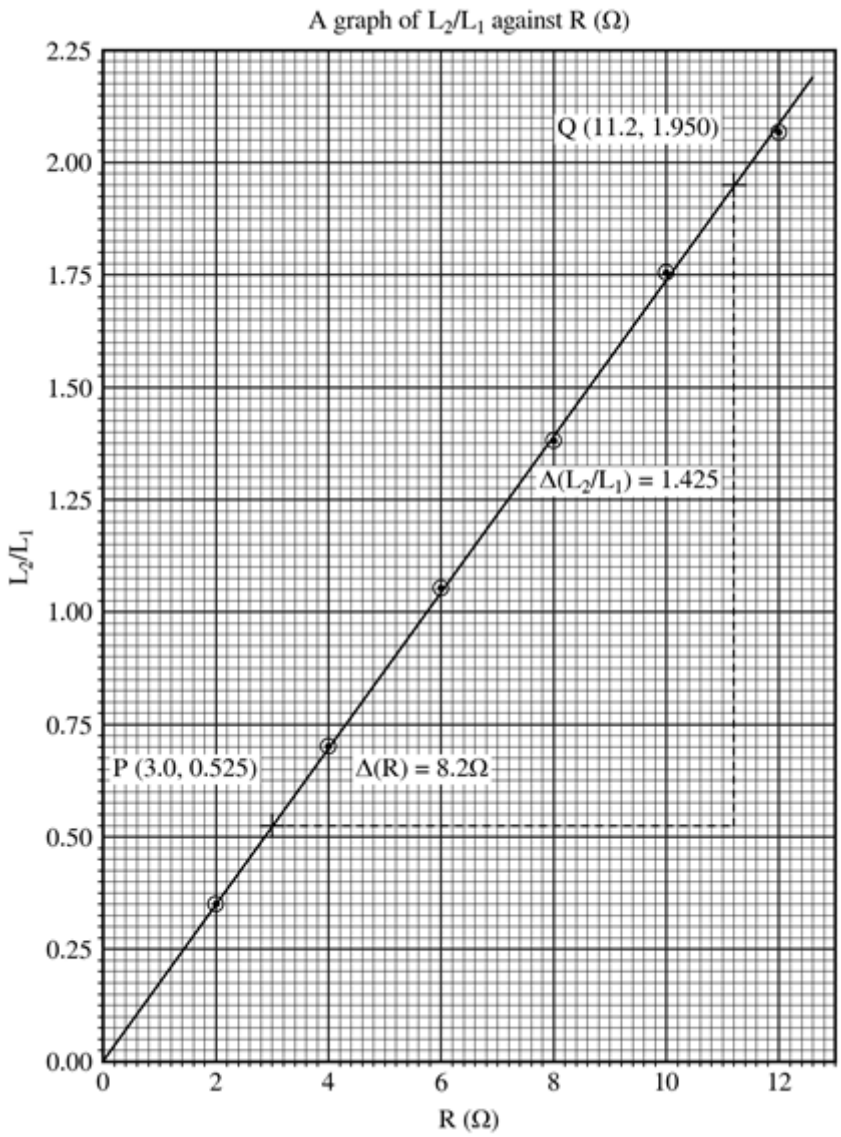
A wire bent round a former is slightly flattened where it crosses the corner of the turn, and the gauge closes on the flat. On the straight end the three readings should agree to the 0.01mm the gauge resolves; if they do not, the wire has been drawn unevenly and the mean of the three is the best you can do.

Practical tip 4: The terminal screws are part of the gap. *Everything between the two terminals of a gap counts as the resistance in that gap, including the screws, the scraped ends and the last centimetre of the connecting lead.*

A dirty terminal can add a few hundredths of an ohm, and a corroded one a couple of tenths. It is always an addition and never a subtraction, but where it lands depends on which gap it is in. Added to the coil it goes into the slope, and the coil measures high by the whole of it. Added to the box it is the same at every setting, so it slides all six points sideways: the slope survives untouched and what moves is the intercept, which ought to sit at the origin. A few hundredths lift the line by less than a millimetre on this sheet and you will never see them; a couple of tenths lift it clear, and that is the sign to go back to the box terminals. Scrape the ends bright, tighten the screws, and do not run the leads under the bridge where they can touch the copper strips.

Answers to the questions

(i) and (ii) Take R at 2cm to 2Ω, running from 0Ω to 13Ω, and L_2/L_1 at 2cm to 0.25, running from 0 to 2.25. That is 13cm by 18cm, which fits an A4 sheet upright. Give the graph the title A graph of L_2/L_1 against R (Ω).



(iii) Taking the two marked corners P (3.0, 0.525) and Q (11.2, 1.950),

$$\text{slope}=\frac{1.950\text{-}0.525}{11.2\Omega\text{-}3.0\Omega}=\frac{1.425}{8.2\Omega}=0.174\Omega^{-1}$$

(iv) At balance $R_1/R_2=L_1/L_2$. The coil is in the left-hand gap and the box in the right, so $R_1=Z$ and $R_2=R$, giving $Z/R=L_1/L_2$, or

$$ZL_2=RL_1$$

Written as a straight line with the box reading on the horizontal axis this is

$$\frac{L_2}{L_1}=\frac{1}{Z}\times R$$

so a graph of L_2/L_1 against R passes through the origin with slope $1/Z$.

(v) From the slope,

$$Z=\frac{1}{0.174\Omega^{-1}}=5.75\Omega$$

(vi) The micrometer read 0.27mm, which is 2.7×10^{-4}m, so

$$A=\frac{3.14\times\left(2.7\times10^{-4}\text{m}\right)^2}{4}=5.72\times10^{-8}\text{m}^2$$

The coil is 75.0cm of this wire, which is 0.750m, and a uniform wire has $Z=\rho L_c/A$, so

$$\rho=\frac{ZA}{L_c}=\frac{5.75\Omega\times5.72\times10^{-8}\text{m}^2}{0.750\text{m}}=4.39\times10^{-7}\Omega\text{m}$$

The accepted resistivity of manganin is about $4.4\times10^{-7}\Omega$m, and the two agree to within 1%.

(vii) No, not for the steady current a bridge reads. The resistance of a wire is fixed by its length, its cross-sectional area and its resistivity, and winding it changes none of the three. What winding does change is the magnetic field the current makes, and that matters only while the current is changing; once the switch is closed the current through the coil is steady, and the jockey makes and breaks only the galvanometer branch. Winding would change the answer if the insulation between neighbouring turns were broken, because two turns touching bare would short out the length between them and the coil would measure low.

Examiner's eye

- Notice which gap holds the box. Experiment 19.1 had it on the left; this experiment and Practice 19.3 have it on the right, and the whole difference between the graphs is which of the two lengths goes on top. Check yours against the gaps before you plot.
- Put the length of the wire, not the length of the coil, into the resistivity. The coil is a couple of centimetres long and the wire in it is 75.0cm; put the former's length in and the resistivity comes out about forty times too large, which is off the end of every table of materials there is.
- Convert the length to metres and the diameter to metres before either goes into the formula. Resistivity in Ωm needs both. Leave the length in centimetres and the answer is out by 100; leave the diameter in millimetres and it is out by a million.
- Scrape the ends and say that you did. Enamelled wire looks like bare wire under a terminal screw and conducts like a piece of string; a bridge that will not balance anywhere is usually a coil whose ends have never been cleaned.

Practice 19.4

The order had been placed months before and the reels came up from the coast on an open lorry in the rains. The invoice arrived dry, in an envelope, and it is perfectly clear: one reel of manganin, one of constantan, one of nichrome. The labels on the reels are pulp.

Mr. Akilikubwa said the school would have to send them back and wait another term. **Kipanga** said that seemed a long way to go for a question the invoice had already half answered, since it says what the materials are and it is only which reel is which that the rain has taken.

Procedure

Proceed as follows:

(a) Connect the circuit as in Experiment 19.4, with the coil across the left-hand gap and the resistance box across the right-hand gap, leaving the switch open.

(b) Set the resistance box to 2Ω.

(c) Close the switch, tap the jockey along the bridge wire until the galvanometer shows no deflection, and read the balance length L_1 from the left-hand end of the wire. Open the switch.

(d) Repeat steps (b) and (c) with the box set to 3Ω, 4Ω, 5Ω, 6Ω and 7Ω.

(e) Measure the diameter d of the wire the coil is wound from, with the micrometer screw gauge, at three places along one of the free ends.

Questions

(i) Tabulate your results, including the values of L_2 and L_2/L_1.

(ii) Plot a graph of L_2/L_1 against R.

(iii) Determine the slope of your line and state its unit.

(iv) Determine the resistance of the coil.

(v) The coil is wound from 60.0cm of wire. Determine the resistivity of that wire.

(vi) The coil is known to be wound from one of three materials: manganin, of resistivity $4.4\times10^{-7}\Omega$m, constantan, of $4.9\times10^{-7}\Omega$m, or nichrome, of $1.1\times10^{-6}\Omega$m. State which it is.

(vii) Your diameter is read to the nearest 0.01mm and your slope to about 2%. Estimate the uncertainty in your resistivity, and say whether the measurement is good enough to separate the two materials it lies between.

Experiment 19.5

The letter came from a village up the escarpment and was addressed to the school rather than to anybody in it. It asked whether the physics laboratory could say how long a piece of a certain wire should be cut to make an element for a water heater, and it enclosed a specimen of the wire wound round a piece of card.

> Dear Sir or Madam,
>
> The heater we have needs a coil of 50Ω. We have plenty of the wire. We do not know how long to cut it and we have already spoiled two coils by guessing. Please tell us the length and we will send a boy to fetch the answer.
>
> Yours faithfully,
>
> P.S. The boy left on Tuesday and should be with you by now.

Kipute looked up and said that there was a boy in the corridor. **Mr. Akilikubwa** said that the specimen wound round the card was the only thing in the envelope that could settle anything, and that they had until the boy got bored.

Aim

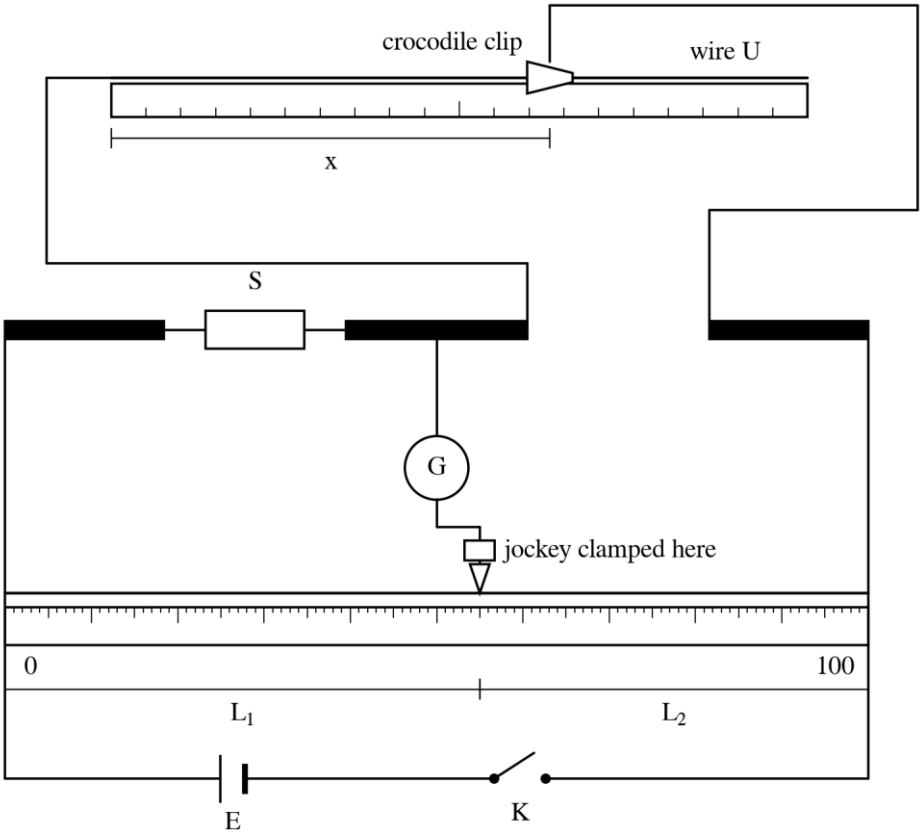
To determine the length of a given resistance wire that has a stated resistance, using a metre bridge.

Apparatus

A metre bridge whose jockey can be clamped at a chosen graduation, a standard resistor of 4.00Ω, 100cm of resistance wire labelled U stretched along a wooden rule, a crocodile clip, a galvanometer, one dry cell, a switch and eight connecting wires.

Setup

The standard resistor is connected across the left-hand gap. The wire U is connected across the right-hand gap, one terminal going to the zero end of the wire and the other to the crocodile clip, so that the length in the circuit is whatever the clip decides. The jockey is clamped on the bridge wire at a chosen graduation and left there, and the clip is then slid along the wire until the galvanometer reads zero.



Procedure

Proceed as follows:

(a) Connect the circuit as shown, leaving the switch open.

(b) Clamp the jockey on the bridge wire at 45.0cm from the left-hand end, so that L_1 is 45.0cm.

(c) Close the switch and slide the crocodile clip slowly along the wire U until the galvanometer shows no deflection. Read the length x from the zero end of U to the jaws of the clip. Open the switch.

(d) Repeat steps (b) and (c) with the jockey clamped at 50.0cm, 55.0cm, 60.0cm, 65.0cm and 70.0cm.

Questions

(i) Tabulate your results, including the values of L_2 and L_2/L_1.

(ii) Plot a graph of x against L_2/L_1 and draw the best straight line through your points.

(iii) Determine the slope of your line and state its unit.

(iv) Derive the equation governing this experiment.

(v) Determine the resistance of 1m of the wire U.

(vi) Determine the length of U that would have a resistance of 50.0Ω.

(vii) In this experiment the balance point is fixed and the resistance is adjusted, which is the opposite of the way the earlier experiments in this chapter work. Give one advantage of working this way.

Table of results

The standard resistor in the left-hand gap was 4.00Ω and the jockey was clamped in turn at each of the six graduations below.

L_1 (cm)	L_2 (cm)	L_2/L_1	x (cm)
45.0	55.0	1.222	77.3
50.0	50.0	1.000	64.0
55.0	45.0	0.818	52.2
60.0	40.0	0.667	42.0
65.0	35.0	0.538	34.7
70.0	30.0	0.429	27.1

Making the measurement count

Practical tip 1: Which way to move the clip. *Note which way the galvanometer needle points before you move anything, then move the clip a hand's breadth and look again.*

If the needle has swung further the same way, you are going the wrong way along the wire; if it has crossed over, the balance lies between where you were and where you are. Two moves settle the direction and four more settle the length. Do not slide the clip with the switch closed for longer than it takes, because a clip dragged along a live wire scrapes it bright in a line and thins it where it goes.

Practical tip 2: Clamping the jockey. *The clamp must hold the jockey on a marked graduation and hold it still while the clip is being moved.*

If the jockey creeps by a millimetre between one reading and the next, the abscissa of that point is wrong and no amount of care with the wire will repair it. Check the graduation again after each balance is found, before the switch is opened; it costs a second, and it is the only reading in this table you cannot recover once the clamp has moved.

Practical tip 3: Work the longest length out first. *The clamped position nearest the left-hand end is the one that calls for the most wire, and how much it calls for depends on the standard resistor you are using.*

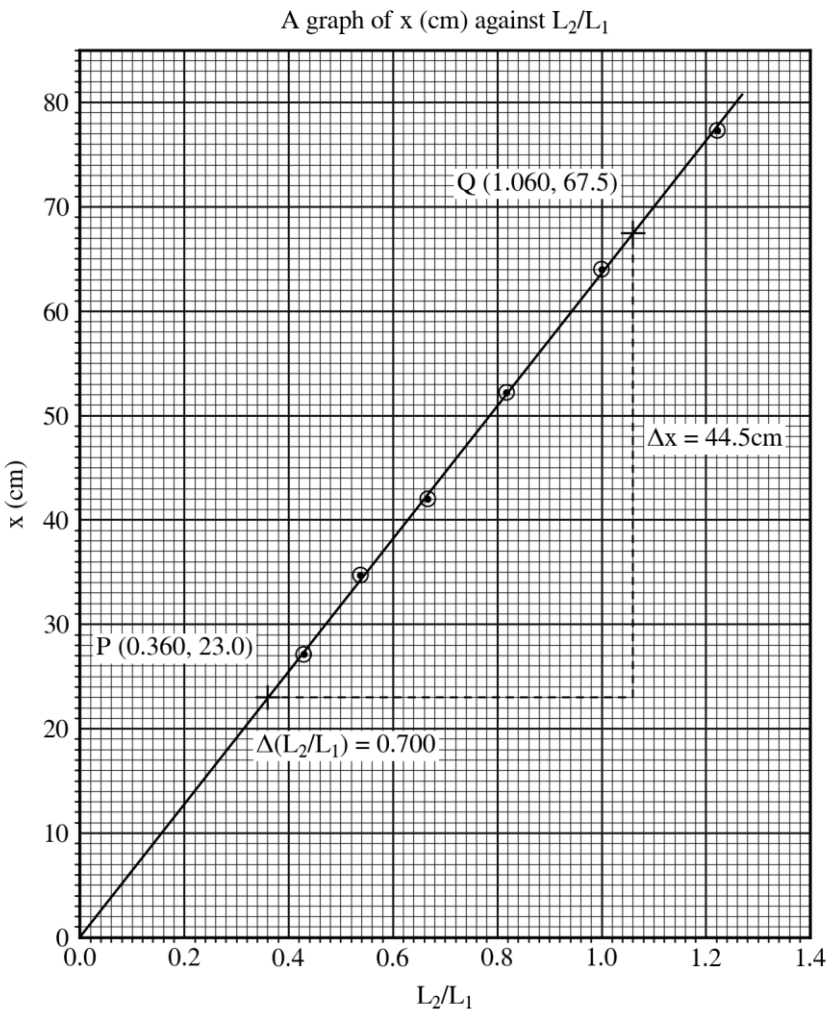
The wire you have is 100cm long, and the first position in the ladder above already calls for a good part of it. Take one trial balance, scale it up by a proportion sum, and check the answer against the wire you actually have before you clamp anything, rather than discovering it with the clip in your hand.

Practical tip 4: A ladder spread across the middle of the wire. *The six clamped positions above are spread over* 25cm *of the middle of the bridge wire, and none of them is near an end.*

A ladder crowded into a narrow range gives a line whose slope is fixed by hardly anything; one that reaches for the ends of the wire buys its spread with reading error. The six above are a compromise between the two, and the lengths of U they call for run from about a quarter of the wire to about three quarters of it.

Answers to the questions

(i) and (ii) Take L_2/L_1 at 2cm to 0.2, running from 0 to 1.4, and x at 2cm to 10cm, running from 0cm to 85cm. That is 14cm by 17cm, which fits an A4 sheet upright. Give the graph the title A graph of x (cm) against L_2/L_1.



(iii) Taking the two marked corners P (0.360, 23.0) and Q (1.060, 67.5),

$$\text{slope}=\frac{67.5\text{cm}-23.0\text{cm}}{1.060-0.360}=\frac{44.5\text{cm}}{0.700}=63.6\text{cm}$$

The abscissa is a ratio of two lengths and has no unit, so the slope is a length, in centimetres.

(iv) At balance $R_1/R_2=L_1/L_2$. The standard resistor is in the left-hand gap and the length x of U in the right, so the resistance of that length is SL_2/L_1. A uniform wire of resistance r for every unit of its length has rx in a length x, so $rx=SL_2/L_1$ and

$$x=\frac{S}{r}\times\frac{L_2}{L_1}$$

a straight line of x against L_2/L_1 through the origin, of slope S/r.

(v) The slope is S/r, which is 63.6cm or 0.636m, so

$$r=\frac{4.00\Omega}{0.636\text{m}}=6.29\Omega\text{m}^{-1}$$

A metre of U has a resistance of 6.29Ω.

(vi) A length x_t has a resistance rx_t, so

$$x_t=\frac{50.0\Omega}{6.29\Omega\text{m}^{-1}}=7.95\text{m}$$

The element wants 7.95m of this wire. Accept 7.80m to 8.10m.

(vii) The abscissa is set rather than found. The ratio L_2/L_1 is worked out from two lengths you clamp against a printed scale rather than from a balance point you had to hunt for, so it carries no balance-finding error at

all, only the error of setting and holding the clamp, which is far the smaller of the two. What scatter is left lies in one direction, up and down the page, and that makes the best line easier to draw and easier to defend. It also lets you choose in advance where on the bridge wire every reading will be taken.

Examiner's eye

- Plot the two axes the way the question names them: x vertically and the ratio horizontally. Turn the graph round and the slope becomes r/S, so the answer to part (v) comes out upside down with nothing on the sheet to warn you.
- Give the slope its unit. Here it is a length, because a length has been divided by a pure number, and a slope quoted as 63.6 with nothing after it will not do.
- Answer part (vi) in the units the letter used. It asked for a length to cut, so 7.95m or 795cm will both do and 0.00795km will not; write the answer as somebody with a pair of pliers would want to read it.
- Do not slide the jockey. It is clamped in this experiment and the clip is what moves; a table in which both have moved is a table of one equation with two unknowns in it.

Practice 19.5

The technician who looks after the boarding-house heaters says he can cut a 100Ω element by eye, and offered to bet a soda on it. The terms of the bet took some settling, because he wanted to be allowed to test his own piece by whether the water boiled.

Kipute worked it out on the back of his hand. Taking a metre of the wire to be worth 20Ω, he announced that 5m would do it, and that the bet was therefore settled without anybody having to measure anything. **Kipanga** asked where the 20Ω had come from. **Kipute** said it was a reasonable sort of number for a wire.

The technician, who had been listening, said that this was exactly his own method and that he would like his soda now.

Procedure

Proceed as follows:

(a) Connect the circuit as in Experiment 19.5, with a standard resistor of 3.00Ω in the left-hand gap and the wire V in the right-hand gap, leaving the switch open.

(b) Clamp the jockey on the bridge wire at 40.0cm from the left-hand end, so that L_1 is 40.0cm.

(c) Close the switch and slide the crocodile clip slowly along the wire V until the galvanometer shows no deflection. Read the length x from the zero end of V to the jaws of the clip. Open the switch.

(d) Repeat steps (b) and (c) with the jockey clamped at 45.0cm, 50.0cm, 55.0cm, 60.0cm and 65.0cm.

Questions

(i) Tabulate your results, including the values of L_2 and L_2/L_1.

(ii) Plot a graph of x against L_2/L_1.

(iii) Determine the slope of your line and state its unit.

(iv) Derive the equation governing this experiment.

(v) Determine the resistance of 1m of the wire V.

(vi) Determine the length of V that would have a resistance of 100Ω.

(vii) The reel the wire V was cut from holds 10m. State what your answer to part (vi) means for anybody who has to make the element, and give one way of obtaining 100Ω.

Experiment 19.6

The evening class arrived to find the laboratory cupboard locked and the man with the key at a funeral in another district. Behind the glass, three of the school's metre bridges could be seen quite clearly, which somebody said made it worse.

Kipanga proposed breaking the glass and offered to pay for it, which was generous, and to explain it to the headmaster, which was not. **Kipute** said there was no point, because a bridge is a precision instrument and you cannot make one out of a drawer.

"Name a part of it," said **Mr. Akilikubwa**, *"that is not in that drawer."*

Kipute named the scale, and the drawer produced a metre rule. He named the wire, and it produced two reels of it. It went on producing: a roll of masking tape, a standard resistor, a cell, a key, a galvanometer, a jockey. **Mr. Akilikubwa** laid them out in a row, said that this was now the apparatus, and asked whether a bridge built from it could be trusted to measure anything at all.

Aim

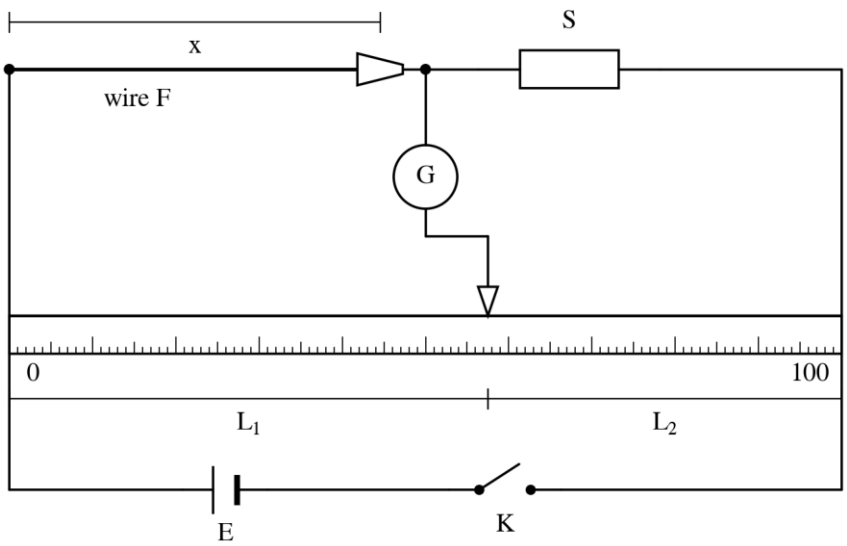
To determine the resistivity of a bare wire using a bridge built from a metre rule, without a bridge board.

Apparatus

A wooden metre rule, 110cm of bare resistance wire to be taped along it, 80cm of a second bare wire labelled F, a standard resistor of 2.00Ω, a crocodile clip, a micrometer screw gauge, masking tape, a galvanometer, one dry cell, a switch, a jockey and six connecting wires.

Setup

The first wire is pulled straight and taped down at the wire's two ends only, outside the zero and hundred marks, so that the whole of it over the graduations is bare and the jockey can be tapped anywhere along it. That wire takes the place of the bridge wire, and the scale printed on the rule takes the place of the bridge scale. A length x of the wire F runs from the zero end of the rule to a junction, and the standard resistor runs from that junction to the far end of the rule, so that the two of them together form a loop with the taped wire. The cell and the switch are joined between the two ends of the rule, and the galvanometer between the junction and the jockey. The length x is set by a crocodile clip at the junction end.



Procedure

Proceed as follows:

(a) Tape the first wire along the rule and connect the circuit as shown, leaving the switch open.

(b) Clip the crocodile clip on the wire F at 20cm from the zero end, so that x is 20cm.

(c) Close the switch, tap the jockey along the taped wire until the galvanometer shows no deflection, and read the balance length L_1 from the zero end of the rule. Open the switch.

(d) Repeat steps (b) and (c) for x equal to 30cm, 40cm, 50cm, 60cm and 70cm.

(e) Measure the diameter d of the wire F with the micrometer screw gauge at three places along its length.

Questions

(i) Tabulate your results, including the values of L_2 and L_1/L_2.

(ii) Plot a graph of x against L_1/L_2 and draw the best straight line through your points.

(iii) Determine the slope of your line and state its unit.

(iv) Derive the equation governing this experiment.

(v) Determine the resistance of 1m of the wire F.

(vi) Determine the cross-sectional area of F and hence its resistivity.

(vii) The wire taped to the rule was never made to be a bridge wire, and the method needs it to be uniform along the part of it that lies over the graduations. Describe a test you could make with the apparatus already in front of you, say what result would satisfy you, and name one kind of fault the test would let through.

Table of results

The standard resistor in the loop was 2.00Ω and the micrometer read 0.46mm at each of the three places along the wire F.

x (cm)	L_1 (cm)	L_2 (cm)	L_1/L_2
20	39.9	60.1	0.664
30	50.3	49.7	1.012
40	57.5	42.5	1.353
50	62.8	37.2	1.688
60	67.0	33.0	2.030
70	69.9	30.1	2.322

Making the measurement count

Practical tip 1: Tape at the ends and nowhere else. *The jockey has to reach bare metal at every point over the graduations, so the tape goes outside the zero and hundred marks and not between them.*

A strip of tape laid across the middle of the wire to keep it straight is exactly the piece of wire you will want to tap on later. Stretch the wire tight before the second piece of tape goes down, because a wire that sags takes a path longer than the scale beneath it and the two stop agreeing at the very place where you are reading them.

Practical tip 2: What the galvanometer must be connected to. *One terminal of the galvanometer goes to the junction between the wire F and the standard resistor, and to nothing else.*

That junction does on this rule what the copper strip between the gaps does on a bridge board, and part (iv) is where that matters. Connect the galvanometer to either end of the loop instead and it will read zero at one end of the rule and nowhere useful in between.

Practical tip 3: Cutting and clipping the specimen wire. *Lay the wire F straight against the rule to set each length before the clip goes on, and read the length to the jaws of the clip.*

A bare wire that has been on a reel keeps a curl in it, and a curled wire measured against a rule is always shorter than the wire you actually have in the circuit. Run it between finger and thumb once or twice to straighten it. Do not let the tail beyond the clip fall across the taped wire, which shorts out part of the bridge.

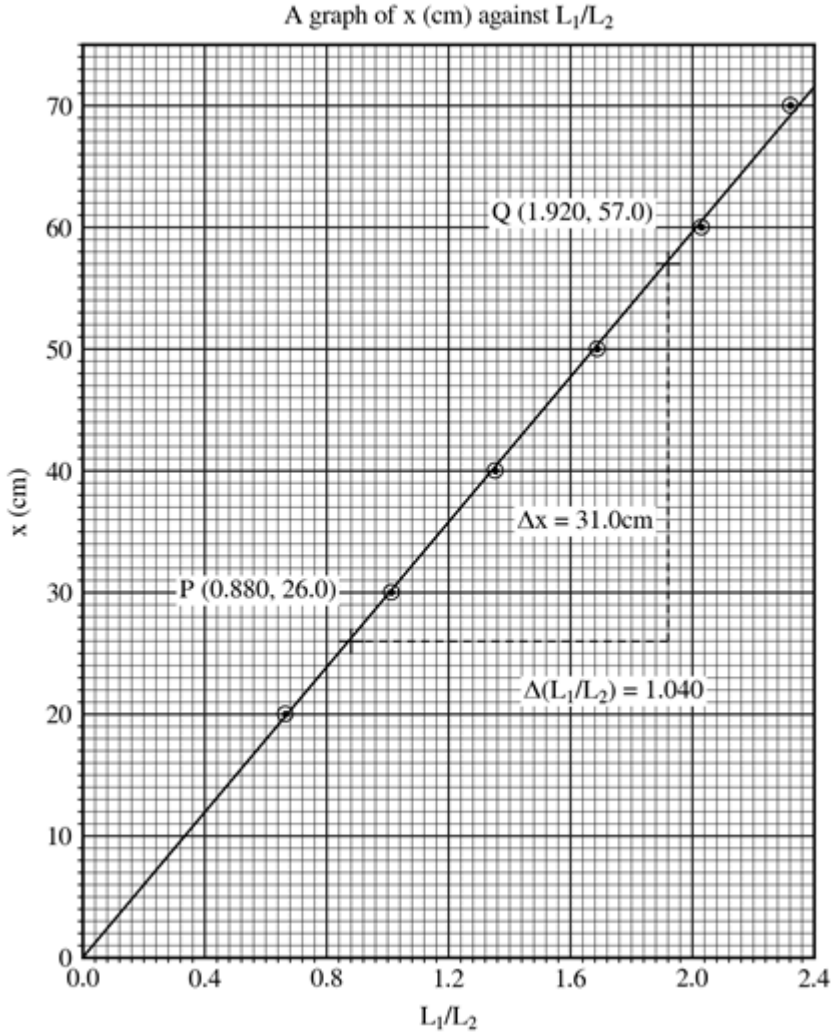
Practical tip 4: Where this bridge keeps its joints. *A bridge board hides its joints under terminal screws; this one has them in the open, at the clip, at the junction and at the two ends of the rule.*

Each of them adds a little resistance to whichever arm it sits in, and unlike the scatter of the readings it never averages away. Twist the bare ends tightly, scrape them first, and keep the same joint from one reading to the

next so that whatever it is worth it is worth the same in every row. Part (vii) is about a different defect, and about how you would find it.

Answers to the questions

(i) and (ii) Take L_1/L_2 at 2cm to 0.4, running from 0 to 2.4, and x at 2cm to 10cm, running from 0cm to 75cm. That is 12cm by 15cm, which fits an A4 sheet upright. Give the graph the title A graph of x (cm) against L_1/L_2.



(iii) Taking the two marked corners P (0.880, 26.0) and Q (1.920, 57.0),

$$\text{slope}=\frac{57.0\text{cm}-26.0\text{cm}}{1.920-0.880}=\frac{31.0\text{cm}}{1.040}=29.8\text{cm}$$

The abscissa is a ratio of two lengths and has no unit, so the slope is a length, in centimetres.

(iv) The loop is a Wheatstone bridge with no board under it. The length x of F is one arm and the standard resistor S is the other; the taped wire is the bridge wire, and the jockey divides it into L_1 on the side of F and L_2 on the side of S. At balance the two ratios are equal, so if the wire F has a resistance r for every unit of its length,

$$\frac{rx}{S}=\frac{L_1}{L_2}$$

and therefore

$$x=\frac{S}{r}\times\frac{L_1}{L_2}$$

a straight line of x against L_1/L_2 through the origin, of slope S/r.

(v) The slope is S/r, which is 29.8cm or 0.298m, so

$$r=\frac{2.00\Omega}{0.298\text{m}}=6.71\Omega\text{m}^{-1}$$

(vi) The micrometer read 0.46mm, which is 4.6×10^{-4}m, so

$$A=\frac{3.14\times\left(4.6\times10^{-4}\text{m}\right)^2}{4}=1.66\times10^{-7}\text{m}^2$$

The resistance per unit length is ρ/A, so

$$\rho=6.71\Omega\text{m}^{-1}\times1.66\times10^{-7}\text{m}^2=1.11\times10^{-6}\Omega\text{m}$$

The accepted resistivity of nichrome is about $1.1\times10^{-6}\Omega$m, and the two agree to 1%.

(vii) Find the balance for one clipped length of F, then interchange the two arms of the loop, putting the standard resistor on the zero side of the rule and F on the far side, and balance again. On a uniform wire, and with the joints at the two ends of the rule alike, the second balance length is the first one subtracted from 100cm; anything more than a millimetre or two out of that, and either the wire or those two joints is unfit to be read to a millimetre. The fault it lets through is a wire whose departure from uniformity is the same on both sides of the 50cm mark, because the two balances then move by equal and opposite amounts and still add to 100cm. What catches that is the line itself: six points that do not lie straight, as in Experiment 19.1.

Examiner's eye

- State what the taped wire is doing. It is not being measured and its resistance never enters the answer; it is there to divide in a known ratio. Do not treat it as a third resistance.
- Set the ratio by the way the loop is built, not by the last experiment you did. Here the length of wire whose resistance is unknown sits on the zero side of the rule, so L_1 belongs to it and goes on top; the standard was on the left in Experiment 19.5 and the ratio there is the other way up.
- Tape outside the graduations. It is the mistake here that costs most, because the reading you want next is the one you have just covered.
- Check an improvised instrument before you trust it. A test that can be done with what is already on the bench earns more than a list of instruments you were not given.

Practice 19.6

The science club has decided to build a metre bridge for the junior laboratory, on the grounds that the senior laboratory never lets them touch its own. The plan on the noticeboard is a metre rule, a length of wire, four terminals and a tin of varnish, and the estimated cost is less than a week of the club's subscriptions.

Kipute said the difficulty would be the wire, because the club could not afford a proper bridge wire and would have to use whatever the market had. **Mr. Akilikubwa** agreed, and said that before the club spent anything it should find out what the market wire actually was, using the bridge it had not built yet.

There was a pause while the club worked out whether that was a joke.

Procedure

Proceed as follows:

(a) Tape a wire along the metre rule and connect the circuit as in Experiment 19.6, with a standard resistor of 1.00Ω in the loop, leaving the switch open.

(b) Clip the crocodile clip on the wire B at 20cm from the zero end, so that x is 20cm.

(c) Close the switch, tap the jockey along the taped wire until the galvanometer shows no deflection, and read the balance length L_1 from the zero end of the rule. Open the switch.

(d) Repeat steps (b) and (c) for x equal to 30cm, 40cm, 50cm, 60cm and 70cm.

(e) Measure the diameter d of the wire B with the micrometer screw gauge at three places along its length.

Questions

(i) Tabulate your results, including the values of L_2 and L_1/L_2.

(ii) Plot a graph of x against L_1/L_2.

(iii) Determine the slope of your line and state its unit.

(iv) Derive the equation governing this experiment.

(v) Determine the resistance of 1m of the wire B.

(vi) Determine the resistivity of B.

(vii) The club wants a bridge wire whose resistance is large enough for the galvanometer to see a difference between one end of the rule and the other, but small enough not to flatten the cell. Say whether B would serve, and give the reason in terms of the quantity you have measured.

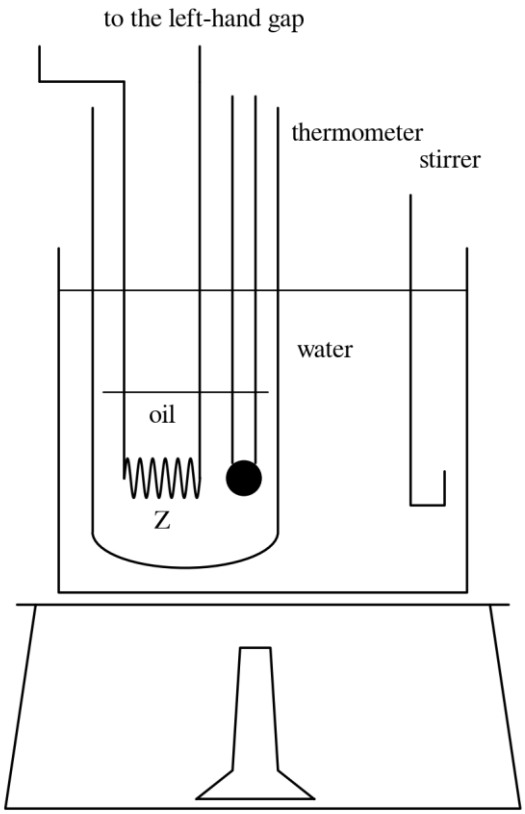
Practice 19.7

The transformer on the pole outside the school hums louder in the afternoon than it does at dawn, and the man who services it says this is because the windings grow when they are angry. Pressed, he explained that a hot winding fights the current harder than a cold one.

Kipanga said that if that were true then every measurement of resistance anybody had ever made in the laboratory was only good for the temperature of the laboratory. **Mr. Akilikubwa** said that if Kipanga were right, then a resistance written down without a temperature beside it was half a number, and asked him how much of one it was. Kipanga did not know, and neither did the man who services the transformer, and that is how Kipanga came to spend an afternoon with a beaker and a burner.

Procedure

A coil of enamelled copper wire labelled Z is sealed in a boiling tube of oil with a thermometer beside it, and the tube stands in a beaker of water on a tripod and gauze over a burner. A stirrer goes in the water. The two ends of the coil come out of the top of the tube and go to the left-hand gap of a metre bridge; a resistance box set at 5.00Ω and left there occupies the right-hand gap, and the cell, the switch, the galvanometer and the jockey are connected as in Experiment 19.4, whose circuit this is. The switch is left open until a balance is wanted. The coil now sits inside the boiling tube shown below. Proceed as follows:



(a) Stir the water and read the thermometer in the oil. When it is steady at 15°C, close the switch, tap the jockey along the bridge wire until the galvanometer shows no deflection, and read the balance length L_1 from the left-hand end of the wire. Open the switch.

(b) Warm the water a little past 30°C with the burner, remove the burner, and go on stirring until the thermometer in the oil is steady at 30°C. Repeat the balance.

(c) Repeat step (b) at 45°C, 60°C, 75°C and 90°C, taking each balance only when the thermometer has been steady for 30s.

Questions

(i) Tabulate your results, including the values of L_2 and of the resistance Z of the coil.

(ii) Plot a graph of Z against θ, with both axes starting at zero.

(iii) Determine the slope and the intercept of your line, with their units.

(iv) The resistance of a metal wire varies with temperature as $Z=Z_0(1+\alpha\theta)$, where Z_0 is its resistance at 0°C. Show that your graph is consistent with this, and say what Z_0 is.

(v) Determine α for the material of the coil, and state its unit.

(vi) Determine the temperature at which your line predicts that the coil would have no resistance at all, and comment on the value.

(vii) Suppose you plotted the same readings with the resistance axis running from 4.2Ω to 5.6Ω so that the points filled the sheet. Explain what that would do to each of the two answers taken from the graph.

Chapter 20

POTENTIOMETER

Every measurement in this chapter is made at the moment when a galvanometer does nothing at all. A steady current is sent along a uniform wire a metre long, so that the potential falls evenly from one end of it to the other. The source whose electromotive force you want is then joined to one end of that wire and to a sliding contact, and the contact is moved until the galvanometer between them stops deflecting. At that instant the wire is offering exactly what the source offers, and the length of wire it took becomes the measurement.

Chapter 18 measured a cell by loading it and watching what the loading cost. The instrument in this chapter is built the other way about, and the difference between the two runs through every page that follows. It is also why a length taken off a metre rule can beat an instrument with a scale on it, which is not a thing students expect and not a thing they forget.

One number turns a length into a volt. It is the potential difference per unit length of the wire, and it depends on the driving battery, on the rheostat in series with it and on the wire itself, so it has to be established afresh whenever any of those is touched. The third of the methods below exists to measure it.

The three varieties at a glance

	What you do	What comes out of it
20.1	A standard cell and an unknown cell are balanced in turn, at six settings of the rheostat	the electromotive force of the unknown cell, from the slope of one balance length against the other
20.2	A resistance box is joined across the cell under test and stepped through six values	the internal resistance of the cell, from the slope and the intercept of one line, and its electromotive force with one reading of a standard cell added
20.3	A potential divider is set to six readings of a voltmeter, and each of them is balanced on the wire	the potential gradient along the wire, and what the readings of that voltmeter are really worth

All three are worked in full. The practice questions put two cells into the tested branch at once, take the internal resistance again on a different pair of axes, and turn the wire on an ammeter.

Experiment 20.1

The tug of war at the sports day was the last event, and the rope had a strip of red cloth tied at its middle, over a line scratched in the dust. Two lines of boys took the strain. The cloth moved a hand's breadth one way, a hand's breadth back, and then it stopped, and for 30 seconds nothing happened at all except a great deal of shouting.

Kipanga complained that there was nothing to award a prize for. *"Nobody is winning,"* he said. *"Nobody is even moving."*

Mr. Akilikubwa was standing over the scratched line with his hands behind his back, looking extremely pleased. *"That cloth,"* he said, *"is the most exact instrument on this field. Every watch here is wrong and every rule here is worn. But while that cloth stays still, the two sides are equal, and they are equal to as many figures as you care to write down."*

Kipute asked how anybody was supposed to get a number out of it. Nobody answered, because at that moment the cloth went 4m to the left and the event was over. You are about to get a number out of it, with a wire, a rule and a needle that is asked to stay still.

Aim

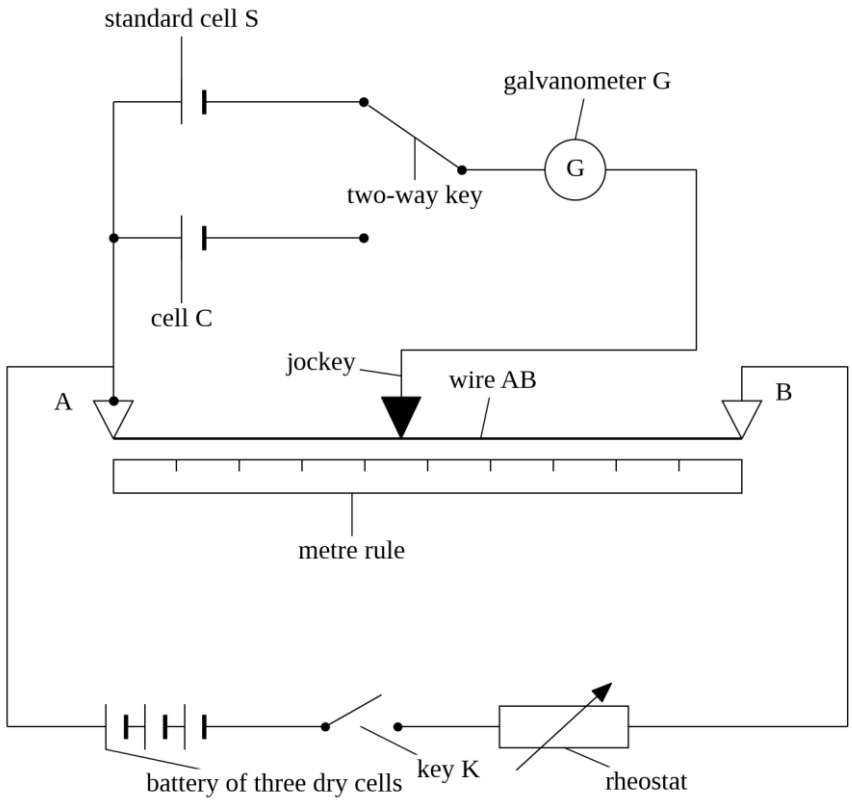
To determine the electromotive force of a dry cell by comparing it with that of a standard cell.

Apparatus

A uniform resistance wire AB of length 100cm stretched along a metre rule; a battery of three dry cells; a rheostat reading 0 to 50Ω; a key; a centre-zero galvanometer; a jockey; a two-way key; a standard cell S of electromotive force 1.08V; a dry cell C whose electromotive force is to be found; connecting leads with crocodile clips.

Setup

The battery, the key and the rheostat are joined in one series loop with the whole of the wire AB, so that the same current passes along every centimetre of it. The positive terminal of each of the two cells to be compared is joined to the end A, and their other terminals are taken to the two studs of the two-way key; the common terminal of that key goes through the galvanometer to the jockey. The jockey is pressed onto the wire and lifted again between readings, and never slid along it.



Procedure

Proceed as follows:

(a) Connect the battery, the key and the rheostat in series with the whole length of the wire AB.

(b) Join the positive terminal of the standard cell S and the positive terminal of the cell C to the end A, and take their other terminals to the two studs of the two-way key.

(c) Join the common terminal of the two-way key through the galvanometer to the jockey, and close the key.

(d) Put the two-way key over to the standard cell S, find the point on the wire at which the galvanometer shows no deflection, and record the balance length L_1 from A.

(e) Put the two-way key over to the cell C, find the balance point again, and record the balance length L_2 from A.

(f) Repeat the procedures in 20.1 (d) and (e) for five further settings of the rheostat.

Questions

(i) Draw a labelled diagram of the circuit you connected.

(ii) Tabulate your results.

(iii) Plot a graph of L_2 (cm) against L_1 (cm).

(iv) Determine the slope of your graph and state what it represents.

(v) Determine the electromotive force of the cell C.

(vi) Explain why the line passes through the origin.

(vii) Explain why your answer to (v) does not depend on the setting of the rheostat.

(viii) Explain why the balance length gives the electromotive force of the cell C and not the potential difference between its terminals.

Table of results

The rheostat was moved to a new setting for each pair of readings, and both balance lengths of a pair were taken before it was touched again. Each length was read from the end A against the millimetre divisions of the rule.

L_1 (cm)	L_2 (cm)
70.9	94.6
63.3	85.2
56.7	75.9
50.6	67.5
44.3	60.1
39.5	52.6

Making the measurement count

Practical tip 1: *Press the jockey and lift it.*

The jockey carries a sharp edge and the wire is bare and soft. Slide it and it ploughs a groove, which thins the wire where it runs and leaves a bright track that will not make clean contact tomorrow. Press it down at the place you want, read the galvanometer, and lift it. Work towards the balance point by pressing near A, then near B, and then between: the deflection changes sign across the balance point, so two presses at opposite ends tell you which half of the wire to search. Keep those first presses brief. Far from the balance point the tested cell is driving a current through the galvanometer, and if the galvanometer is a sensitive one it is worth putting a high resistance in series with it and shorting that out only for the final setting.

Practical tip 2: *Take both readings of a pair at one setting of the rheostat.*

The two cells have to be balanced against the same potential gradient, or the pair is worthless. Take L_1 and then L_2 without touching anything else, and move the rheostat only when the pair is written down. If the key has to be opened between the two, open it, but leave the slider where it stands.

Practical tip 3: *Positive terminals all go to the same end of the wire.*

The positive terminal of each cell goes to the end A, which is the end the positive terminal of the driving battery reaches. Put one cell in the other way about and the galvanometer deflects the same way everywhere along the wire, with no balance point anywhere on it. That is the commonest reason a potentiometer refuses to balance, and it costs nothing to check before blaming the apparatus.

Practical tip 4: *The null is wider than the rule you read it against.*

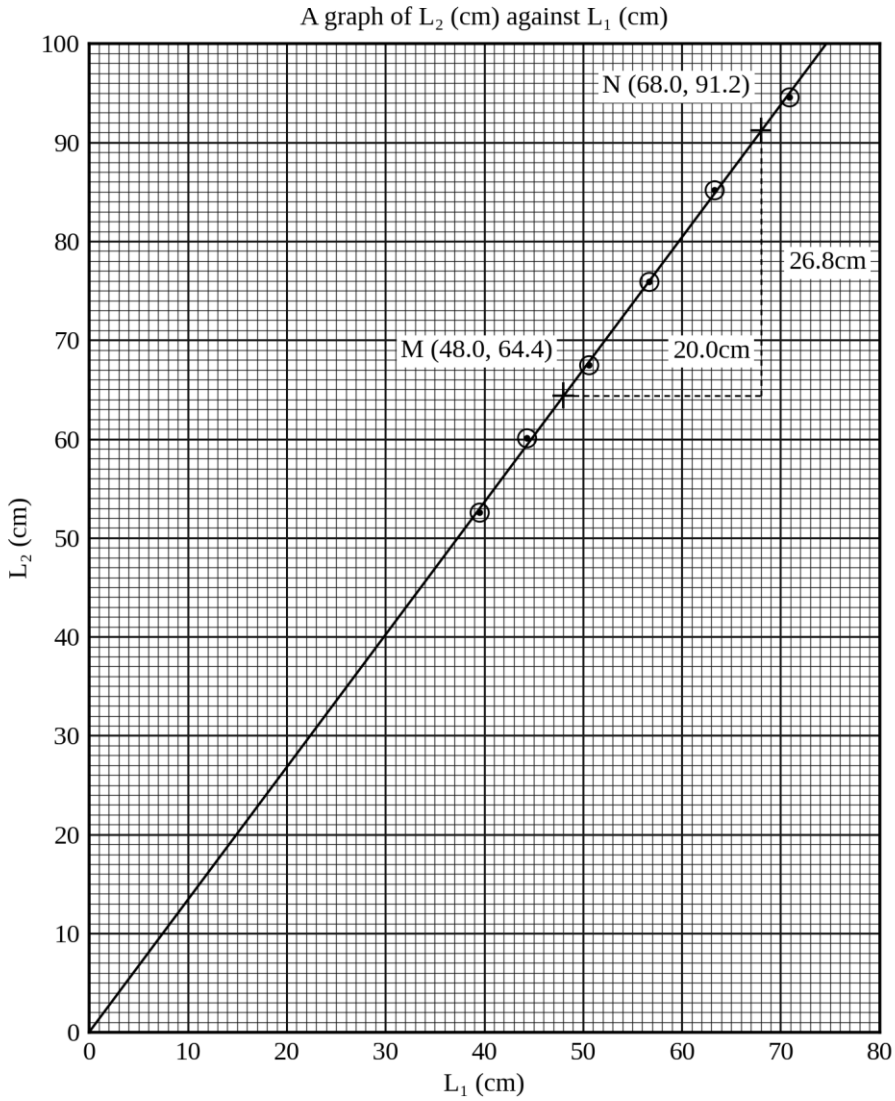
A metre rule is divided to a millimetre, but the balance point is not a point: it is the stretch of wire over which the needle does not visibly move, and how wide that stretch is depends on the galvanometer rather than on the rule. Press either side of it and take the middle of the range over which the needle stays put. Expect the readings to scatter by rather more than the rule can resolve, and let the line through all six settle the answer.

Answers to the questions

(i) The circuit is the one drawn above: the battery, the key and the rheostat in one loop with the whole of AB; the positive terminal of each tested cell at A; their other terminals at the two studs of the two-way key; and the common terminal of that key through the galvanometer to the jockey.

(ii) The readings are those in the table above.

(iii) Take L_1 at 2cm to 10cm, running from 0 to 80cm, and L_2 at 2cm to 10cm, running from 0 to 100cm. That is 16cm by 20cm, which fits an A4 sheet upright. Give the graph the title A graph of L_2 (cm) against L_1 (cm).



(iv) Taking M (48.0, 64.4) and N (68.0, 91.2) on the line,

$$\text{slope}=\frac{91.2\text{cm}-64.4\text{cm}}{68.0\text{cm}-48.0\text{cm}}=\frac{26.8\text{cm}}{20.0\text{cm}}=1.34$$

Both quantities are lengths, so the slope is a pure number. Every pair of readings was taken at one potential gradient k, and at that setting $E_S=kL_1$ and $E_C=kL_2$. Divide the second of those by the first: the potential gradient cancels, and the slope is the ratio of the two electromotive forces.

(v) The standard cell offers 1.08V, so

$$E_C=1.08\text{V}\times1.34=1.45\text{V}$$

(vi) The relation between the two lengths is a proportionality and not merely a straight line. At every setting the potential gradient is the same for both cells, so L_1 is E_S divided by k and L_2 is E_C divided by k, and the second is a fixed multiple of the first however k is chosen. A relation of that shape has no constant term to add, so the line has nowhere to cut either axis except at the origin. A line displaced from the origin says something is being added to both lengths alike, such as a rule whose zero is not where the tested branch meets the wire; a current that drifted within a pair would show as scatter about the line instead.

(vii) The potential gradient cancels out of the ratio. Move the rheostat and both balance lengths lengthen or shorten together, which carries the point further along the same line instead of off it. Six settings therefore give one answer and not six, and the setting that produced any particular pair need never be recorded.

(viii) Because at balance the galvanometer carries no current, so the cell C is driving no current either. Nothing is then lost across its internal resistance or in its leads, and the potential difference the wire is matching is the electromotive force itself. A voltmeter put across the cell has to take a current to move its pointer,

and unless its own resistance is very large beside the cell's, what it reads falls short of the electromotive force by whatever that current costs inside.

Examiner's eye

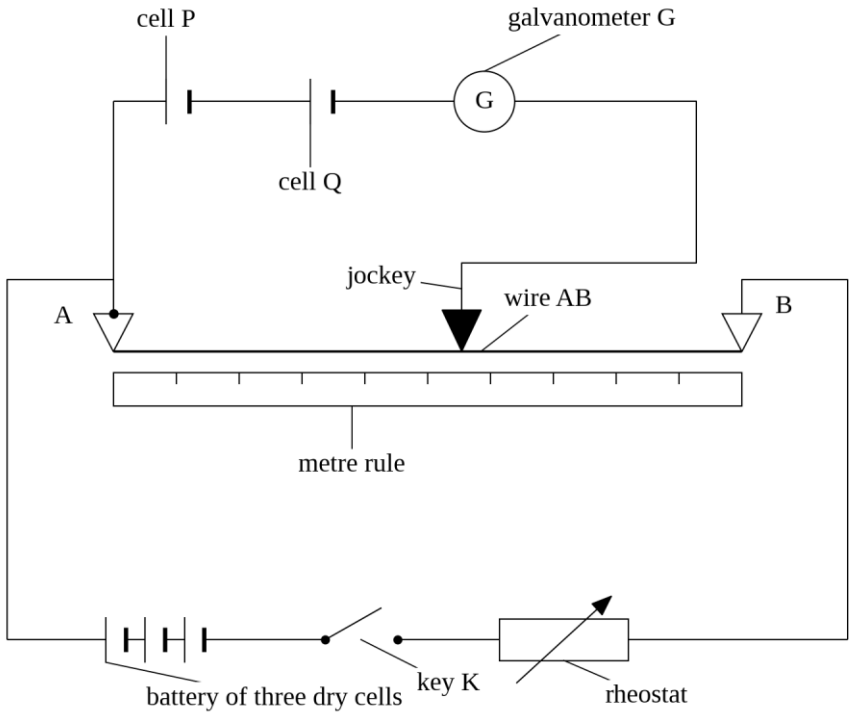
- Two lengths divided by one another leave no unit behind, so a slope quoted as 1.34cm makes the line after it dimensional nonsense. The volt has to enter this experiment from somewhere, and the standard cell is the only door.
- Say which cell is on which axis before you plot anything. The unknown cell here has the larger electromotive force, so its balance lengths are the longer ones, and the axis they need is the one with room for them.
- Nothing in this experiment measures the standard cell. It is a standard because its electromotive force is known and stays known, not because it is large or new, and an answer that sets out to check it has misread what the 1.08V is for.
- Where there is a line to draw, draw it. Six pairs divided one row at a time and averaged will land close enough, but only the line shows whether the six points fall together, and the straightness of it is half of what the paper is marked on.

Practice 20.1

Mr. Akilikubwa grew up on a river with a ferry on it, and the ferry took 15 minutes to cross going over and the better part of an hour coming back. The passengers had a low opinion of the ferryman on the return leg. He was, they said, a different man in the afternoon.

Kipanga wanted to know why anybody had put up with it, and suggested timing the man with a watch and paying him accordingly. **Kipute** pointed out that the river was going somewhere, and that no watch in the world would tell you how much of the crossing belonged to the ferryman and how much to the water.

"He is a very poor ferryman either way," said **Mr. Akilikubwa**, *"but he is an excellent experiment."* Join two cells into a circuit pulling together, then join them pulling against each other, and read the wire twice.



Procedure

Proceed as follows:

(a) Connect a battery of three dry cells, a key and a rheostat in series with the whole length of a potentiometer wire AB 100cm long.

(b) Join the cell P and the cell Q in series so that they act together, take the free positive terminal to the end A, and take the free negative terminal through a centre-zero galvanometer to the jockey.

(c) Close the key, find the balance point, and record the balance length L_1 from A.

(d) Interchange the two leads to the cell Q, so that the two cells now act against each other, find the balance point again, and record the balance length L_2 from A.

(e) Repeat the procedures in P20.1 (c) and (d) for five further settings of the rheostat.

Questions

(i) Tabulate your results, including for each setting the value of $(L_1 + L_2)/(L_1 - L_2)$.

(ii) Determine the mean value of that quantity.

(iii) The cell Q is a standard cell of electromotive force 1.08V. Show that the quantity in (i) is the ratio of the two electromotive forces.

(iv) Determine the electromotive force of the cell P.

(v) Explain why the second balance point of each pair lies so close to the end A.

(vi) One of the two lengths in a pair carries far more of the uncertainty in the ratio than the other. State which, and support your answer with a calculation.

(vii) State why the rheostat is moved between one pair of readings and the next.

(viii) State one advantage and one disadvantage of this method compared with the method of Experiment 20.1.

Experiment 20.2

The maize mill at the top of the hill is driven by a small engine with a governor on it, and the miller feeds the hopper by hand. With a thin trickle of grain the engine runs high and clean. As he opens the hopper the note drops, and with the hopper wide open it drops far enough that he has to close it again to save the belt.

Kipute said the engine was finished and wanted replacing. **Kipanga** asked him to say what the engine did wrong, exactly. It started, it ran, it held its speed under a light load, and it had done all of that for 11 years. What it would not do was hold its speed under a heavy one.

Mr. Akilikubwa had been waiting at the door with a sack under each arm, and put in a word for the miller. The man had bought that engine with the governor set for a lighter hopper, he said, and had been feeding it like a hungry brother ever since. *"He does not want a new engine,"* he said. *"He wants a smaller appetite."* A dry cell sags under load in the same way, and what follows puts a number on the sag.

Aim

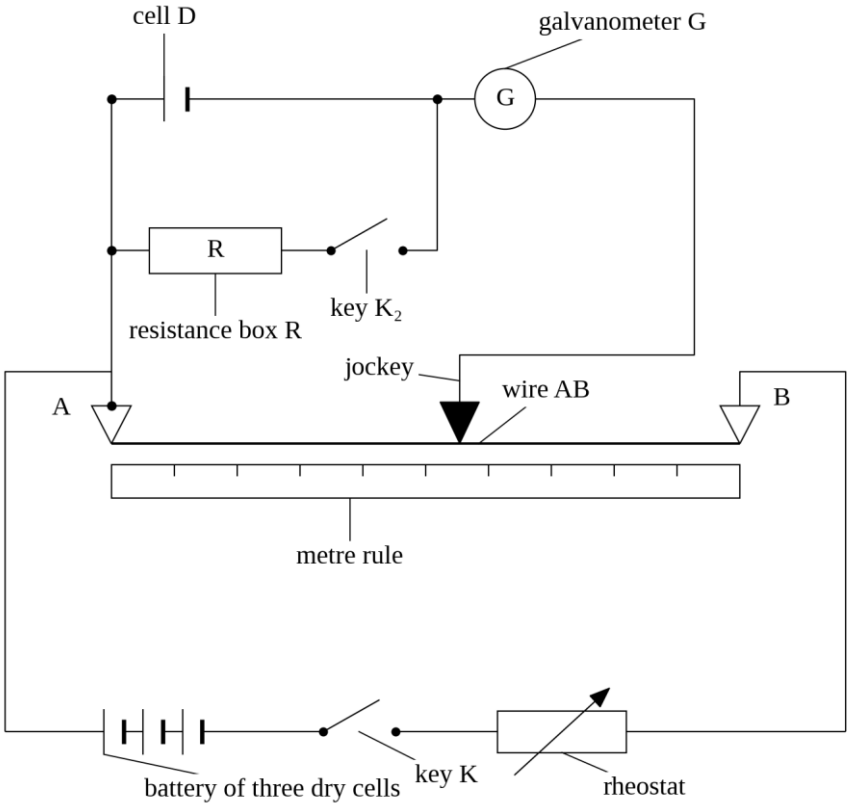
To determine the internal resistance and the electromotive force of a dry cell by a null method.

Apparatus

A uniform resistance wire AB of length 100cm stretched along a metre rule; a battery of three dry cells; a rheostat reading 0 to 50Ω; a key; a centre-zero galvanometer; a jockey; a standard cell of electromotive force 1.08V; the dry cell D under test; a resistance box reading 0 to 10Ω in steps of 0.1Ω; a key K_2 for the box branch; short thick connecting leads with crocodile clips.

Setup

The driving circuit is the one used throughout this chapter: the battery, the key and the rheostat in series with the whole of AB. The cell D has its positive terminal at A and its negative terminal through the galvanometer to the jockey. The resistance box and the key K_2 are joined in one branch straight across the terminals of the cell D, so that closing K_2 puts the box in parallel with the cell. The rheostat is set once, before the first reading, and is not touched again.



Procedure

Proceed as follows:

(a) Connect the battery, the key and the rheostat in series with the whole length of the wire AB.

(b) Balance the standard cell of 1.08V against the wire and record its balance length from A.

(c) Replace the standard cell by the cell D, positive terminal to A, and join the resistance box and the key K_2 in one branch across the terminals of the cell D.

(d) Set the box to 8Ω, close K_2 and find the balance point.

(e) Open K_2 at once, and record the resistance R and the balance length L from A.

(f) Repeat the procedures in 20.2 (d) and (e) for the values of R equal to 6Ω, 5Ω, 4Ω, 3Ω and 2Ω.

Questions

(i) Draw a labelled diagram of the circuit you connected.

(ii) Tabulate your results, including the values of R/L in Ωcm^{-1}.

(iii) Plot a graph of R/L (Ωcm^{-1}) against R (Ω).

(iv) Determine the slope of your graph.

(v) Read the intercept your line makes on the R/L axis.

(vi) Use the slope and the intercept to determine the internal resistance of the cell D.

(vii) The standard cell balanced at 62.5cm. Determine the electromotive force of the cell D.

(viii) Explain why K_2 is opened immediately after each balance point has been found.

Table of results

The standard cell balanced at 62.5cm with the box branch open, and the rheostat was not moved after that. Each balance length was read from the end A, and the third column is the second column divided into the first.

R (Ω)	L (cm)	R/L (Ωcm^{-1})
8	76.1	0.1051
6	73.5	0.0816
5	70.7	0.0707
4	67.9	0.0589
3	62.6	0.0479
2	54.7	0.0366

Making the measurement count

Practical tip 1: *Invert the relation and it is already a straight line.*

With the box branch open the cell drives no current and the wire matches its electromotive force, so $E = kL_0$ for some length L_0. With the box closed the cell drives a current round the box, the potential difference between its terminals falls, and the wire now matches that instead:

$$kL = \frac{ER}{R+r} = \frac{kL_0R}{R+r}$$

The potential gradient cancels, leaving $L = L_0R/(R + r)$. Turn both sides upside down and multiply by R, and the relation is already a straight line:

$$\frac{R}{L} = \frac{R}{L_0} + \frac{r}{L_0}$$

That is y = mx + c with R/L for y and R for x, and parts (iv) and (v) ask for the two constants of that line. Get the rearrangement onto paper before you plot a single point.

Practical tip 2: *Close the box branch only while the balance takes.*

At the smallest setting this cell is asked for something near 0.5A. Find the balance with short presses of the jockey and open K_2 the moment you have it. Part (viii) asks what a cell left working for the length of an experiment does to the readings that come after, and the answer is easier to write if you have watched the balance point creep while the key was down.

Practical tip 3: *Balance the standard cell first, then leave the slider alone.*

Every balance length in this experiment is a length on one wire carrying one current, and lengths taken at two different currents do not belong on one sheet. Balance the standard cell before the box is connected, write the length down, and do not move the slider again. Part (vii) will send you back to it.

Practical tip 4: *Work the third column to four decimal places.*

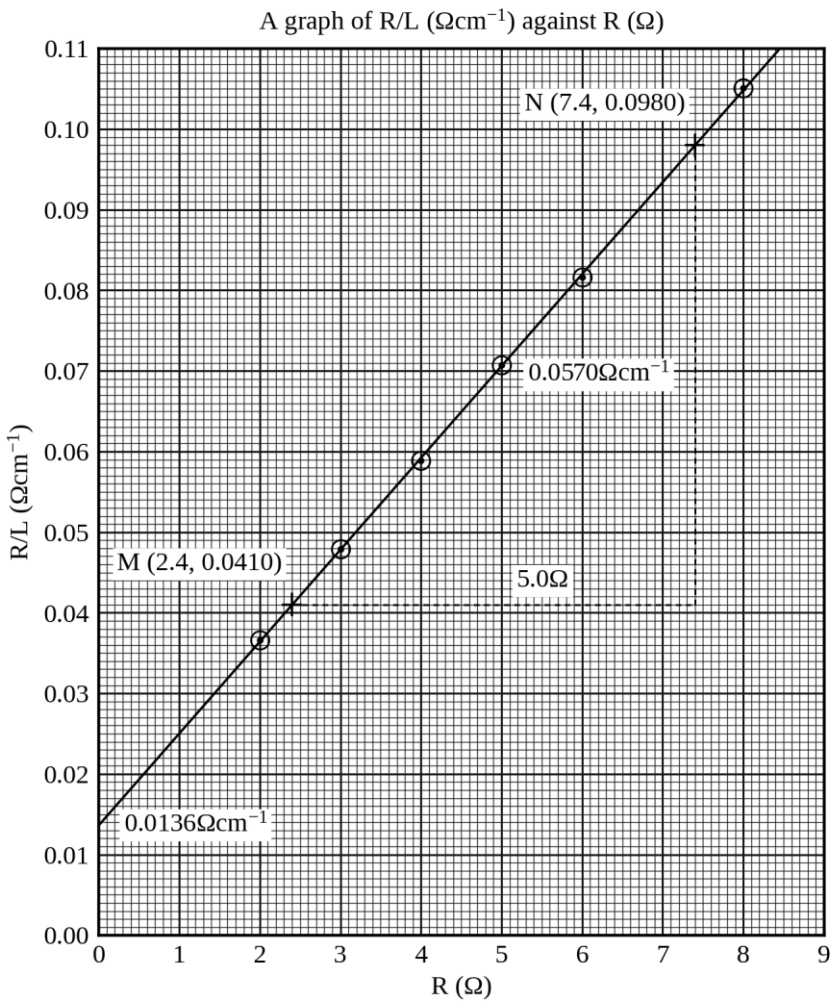
The quantities in that column are small, and three significant figures in them need four places after the point. Round to three places and the six points collapse onto a coarse ladder, the line can be laid through them several ways, and the intercept moves by more than the answer can afford.

Answers to the questions

(i) The circuit is the one drawn above: the battery, the key and the rheostat in one loop with the whole of AB; the cell D with its positive terminal at A and its negative terminal through the galvanometer to the jockey; and the resistance box with its own key in a branch straight across the two terminals of the cell D.

(ii) The readings are those in the table above, with the third column obtained by dividing each resistance by the balance length beside it.

(iii) Take R at 2cm to 1Ω, running from 0 to 9Ω, and R/L at 2cm to $0.01\Omega cm^{-1}$, running from 0 to $0.11\Omega cm^{-1}$. That is 18cm by 22cm, which fits an A4 sheet upright. Give the graph the title A graph of R/L (Ωcm^{-1}) against R (Ω).



(iv) Taking M (2.4, 0.0410) and N (7.4, 0.0980) on the line,

$$\text{slope}=\frac{0.0980\Omega\text{cm}^{-1}-0.0410\Omega\text{cm}^{-1}}{7.4\Omega-2.4\Omega}=\frac{0.0570\Omega\text{cm}^{-1}}{5.0\Omega}=0.0114\text{cm}^{-1}$$

(v) The line meets the R/L axis at $0.0136\Omega\text{cm}^{-1}$.

(vi) The slope is $1/L_0$ and the intercept is r/L_0, so the length the cell would balance at on open circuit is

$$L_0=\frac{1}{0.0114\text{cm}^{-1}}=87.7\text{cm}$$

and the internal resistance is the intercept divided by the slope:

$$r=\frac{0.0136\Omega\text{cm}^{-1}}{0.0114\text{cm}^{-1}}=1.19\Omega$$

(vii) The standard cell of 1.08V balanced at 62.5cm, which is 0.625m, so

$$k=\frac{1.08\text{V}}{0.625\text{m}}=1.73\text{Vm}^{-1}$$

and the cell D would balance at 87.7cm, which is 0.877m, so

$$E=1.73\text{Vm}^{-1}\times0.877\text{m}=1.52\text{V}$$

(viii) With it closed the cell is delivering a current throughout, and a working dry cell warms and polarises. Both of those lower the electromotive force and raise the internal resistance, so the cell of the sixth row would no longer be the cell of the first and the six points would fall on a curve. Open the key and the box stays cool as well, which matters because the box carries the quantity the graph is plotted against.

Examiner's eye

- Quote the intercept in the unit of the axis it sits on, which here is Ωcm^{-1} and not the ohm. Divide it by the slope and the ohms appear where they belong; an intercept written as 0.0136Ω has thrown away the point of the third column.
- Set the box out from the largest value to the smallest and the cell is asked for its heaviest current once, at the end. Taken the other way about, every later row comes from a cell that has just been worked hard, and the drift enters the readings that still matter most.
- One figure on the box and four decimal places beside it is not an inconsistency. The resistances are settings rather than readings, and the quotients are working numbers carried for plotting; the answers come back to three significant figures because that is what a line through six points supports.
- Watch which way the balance point moves as the box is opened out. A larger resistance takes a smaller current, so less is lost inside the cell and the balance length grows. A table in which the lengths fall as the resistances rise has the box somewhere other than across the cell. Keep the leads in that branch short and thick as well: their resistance sits with the box rather than with the cell, and it makes the internal resistance come out a shade small.

Practice 20.2

A stranger stopped at the school gate and asked the way to the dispensary.

Kipute told him to take the second turning past the mango tree, keep the football pitch on his left, and stop at the blue door. **Kipanga** told him it was four hundred paces to the corner, two hundred more to the pitch, and another hundred after that. Both of them were describing one road. The stranger, who now had two sets of instructions and no map, thanked them and set off in the wrong direction.

Mr. Akilikubwa watched him go. *"Two descriptions of one road,"* he said, *"and between them they have sent him to the market. Still, he will get a fine mango."*

Here is Experiment 20.2 again, on a cell that has spent a year in a radio, with the readings taken to a different pair of axes.

Procedure

Proceed as follows:

(a) Connect a battery of three dry cells, a key and a rheostat in series with the whole length of a potentiometer wire AB $100cm$ long.

(b) Balance a standard cell of electromotive force $1.08V$ against the wire and record its balance length from A.

(c) Replace the standard cell by the cell under test, positive terminal to A, and join a resistance box and a second key in one branch across the terminals of that cell.

(d) Set the box to 8Ω, close the second key and find the balance point.

(e) Open the second key at once, and record the resistance R and the balance length L from A.

(f) Repeat the procedures in P20.2 (d) and (e) for the values of R equal to 6Ω, 4Ω, 3Ω, 2Ω and 1Ω.

Questions

(i) Tabulate your results, including the values of L/R in $cm\Omega^{-1}$.

(ii) Plot a graph of L (cm) against L/R ($cm\Omega^{-1}$).

(iii) Determine the slope of your graph.

(iv) Read the intercept your line makes on the L axis.

(v) State what each of those two readings represents.

(vi) The standard cell balanced at $69.0cm$. Determine the electromotive force of the cell.

(vii) The balance length appears on both axes of your graph. Explain why the method is not circular.

(viii) Compare the internal resistance of this cell with the 1.19Ω of Experiment 20.2, and state what the comparison does and does not establish.

Experiment 20.3

A sheet of foolscap appeared on the noticeboard outside the laboratory, in the handwriting of the head of science, and it stayed there for the rest of the term:

NOTICE TO ALL CLASSES

THE SCHOOL KEEPS ONE STANDARD OF EACH KIND.

ONE METRE RULE. ONE SET OF MASSES. ONE CELL.

THEY LIVE IN THE CUPBOARD AND THEY ARE NOT TO BE USED FOR WORK.

EVERY OTHER INSTRUMENT IN THIS ROOM IS TO BE CHECKED AGAINST THEM.

Kipute read it out to the queue at the door and asked what use an instrument you are forbidden to use could possibly be. **Kipanga** asked the harder question, which was what the school proposed to check the standards against.

Mr. Akilikubwa said that measurement is an arrangement of instruments in an order of trust, and that the cupboard was the top of theirs. **Kipute** wanted to know who was at the top of everybody's. *"A man in a laboratory in Europe,"* said **Mr. Akilikubwa**, *"and he keeps his in a cupboard, and he is not allowed to use it either."* A voltmeter is about to be put one place down that order.

Aim

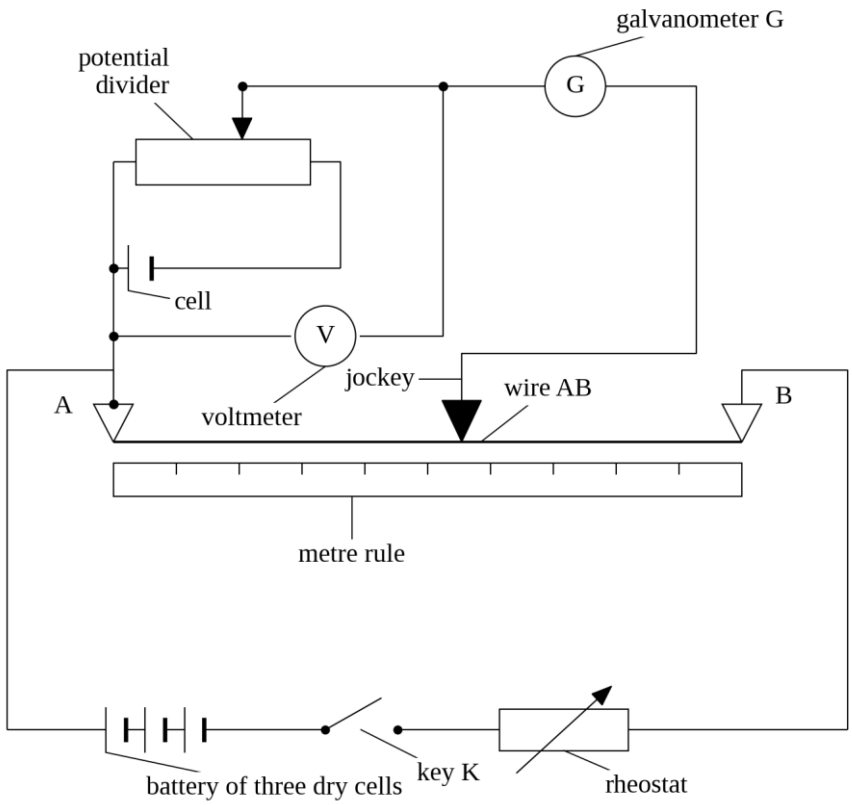
To determine the potential gradient along a potentiometer wire, and to calibrate a voltmeter against it.

Apparatus

A uniform resistance wire AB of length 100cm stretched along a metre rule; a battery of three dry cells; a rheostat reading 0 to 50Ω; a key; a centre-zero galvanometer; a jockey; a standard cell of electromotive force 1.08V; a second dry cell; a rheostat joined as a potential divider; a voltmeter reading 0 to 3V in divisions of 0.1V; connecting leads with crocodile clips.

Setup

The driving circuit is again the battery, the key and the rheostat in series with the whole of AB. The second dry cell is joined across the whole winding of the divider, and the potential difference between its sliding contact and one end of the winding is the quantity to be measured. The voltmeter is put across those two points, and the same two points are taken to the potentiometer: the end of the winding to A, and the sliding contact through the galvanometer to the jockey. The rheostat in the driving circuit is set once and left alone.



Procedure

Proceed as follows:

(a) Connect the battery, the key and the rheostat in series with the whole length of the wire AB.

(b) Balance the standard cell of 1.08V against the wire and record its balance length from A.

(c) Connect the second cell across the whole winding of the divider, and put the voltmeter between the sliding contact and the end of the winding that is joined to A.

(d) Move the sliding contact until the voltmeter reads exactly 0.20V.

(e) Find the balance point and record the voltmeter reading V and the balance length L from A.

(f) Repeat the procedures in 20.3 (d) and (e) for voltmeter readings of 0.40V, 0.60V, 0.80V, 1.00V and 1.20V.

Questions

(i) Draw a labelled diagram of the circuit you connected.

(ii) Tabulate your results.

(iii) Plot a graph of L (cm) against V (V).

(iv) Determine the slope of your graph, and hence the potential gradient along AB.

(v) The standard cell balanced at 79.4cm. Obtain the potential gradient a second time from that reading, and compare the two values.

(vi) Produce your line backwards and read the value of V at which it crosses the V axis.

(vii) State what the reading in (vi) measures.

(viii) Determine the true potential difference when the voltmeter reads 1.00V.

Table of results

The standard cell balanced at 79.4cm, and the rheostat in the driving circuit was not moved after that reading was taken. Each voltmeter reading was set exactly on a division of the scale, and each balance length was read from the end A.

V (V)	L (cm)
0.20	11.5
0.40	26.5
0.60	41.4
0.80	56.2
1.00	70.4
1.20	85.1

Making the measurement count

Practical tip 1: *Choose the long scale for the quantity that scatters.*

The voltmeter reading is not measured here; it is chosen, by moving the sliding contact until the pointer sits on a division of the scale. The balance length is the quantity that is measured and the one that scatters, so it earns the vertical axis and the longer of the two scales. The horizontal axis then has to be drawn from zero and not from the lowest reading, because part (vi) sends you back along it.

Practical tip 2: *A divider reaches zero and a rheostat in series does not.*

Put a rheostat in series with the voltmeter and the two of them share the cell in the ratio of their resistances. A voltmeter is built to have a very large resistance, so almost all of the cell stays on the meter whatever the rheostat is set to, and every reading in the table would be out of reach. Joined as a divider, with the cell across the whole winding and the tapping taken between the slider and one end, it goes down to zero.

Practical tip 3: *The potential gradient can be had two ways.*

One of them uses the standard cell and a single length. The other uses six readings of the instrument under test. Part (v) sets them side by side; work out what each is good to before you decide what their agreement, or their disagreement, is telling you.

Practical tip 4: *Produce the line no further than the readings will carry it.*

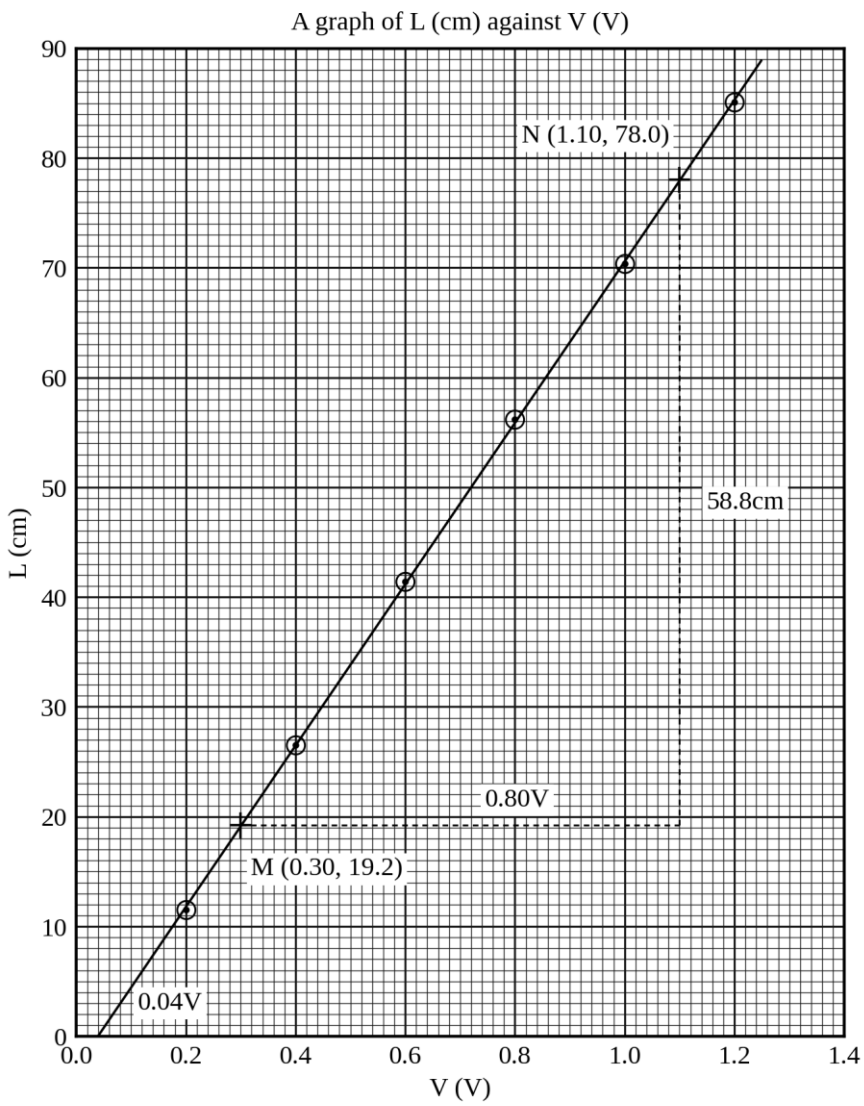
The lowest reading is at 0.20V and the line has to be carried back beyond it. That is a short journey and an honest one, because six points fix the direction of the line firmly. Carry the same line back from a scale that began at 0.20V instead of at zero and the reading part (vi) asks for is not on the sheet at all.

Answers to the questions

(i) The circuit is the one drawn above: the battery, the key and the rheostat in one loop with the whole of AB; the second cell across the whole winding of the second rheostat; the voltmeter between the sliding contact and the end of the winding joined to A; and that same sliding contact taken through the galvanometer to the jockey.

(ii) The readings are those in the table above.

(iii) Take V at 2cm to 0.2V, running from 0 to 1.4V, and L at 2cm to 10cm, running from 0 to 90cm. That is 14cm by 18cm, which fits an A4 sheet upright. Give the graph the title A graph of L (cm) against V (V).



(iv) Taking M (0.30, 19.2) and N (1.10, 78.0) on the line,

$$\text{slope}=\frac{78.0\text{cm}-19.2\text{cm}}{1.10\text{V}-0.30\text{V}}=\frac{58.8\text{cm}}{0.80\text{V}}=73.5\text{cmV}^{-1}$$

The slope is a length per volt, so the potential gradient is the volt per length that it turns into:

$$k=\frac{1}{73.5\text{cmV}^{-1}}=0.0136\text{Vcm}^{-1}$$

and 1cm is 0.01m, so

$$k=\frac{0.0136\text{V}}{0.01\text{m}}=1.36\text{Vm}^{-1}$$

(v) The standard cell offers 1.08V and balanced at 79.4cm, which is 0.794m, so

$$k=\frac{1.08\text{V}}{0.794\text{m}}=1.36\text{Vm}^{-1}$$

The two agree to three figures, and neither is better than about 0.5%. The slope is read across a line whose ends can each be judged to about a fifth of a small square; the single balance length is read far more finely than that, but it is then divided into 1.08V, which is itself quoted to three figures and so carries about the same 0.5%. What their agreement establishes is that the voltmeter's readings rise in step with the true potential differences, even where they are not equal to them.

(vi) The line produced backwards crosses the V axis at 0.04V.

(vii) It is what the voltmeter reads when the potential difference across it is nothing at all. At that crossing the balance length is zero, so the wire is matching no potential difference whatever, while the pointer stands at 0.04V. One further reading settles whose fault that is. Disconnect the meter and look at the pointer: if it rests at 0.04V there, the crossing is the zero error of the voltmeter and is added to everything the instrument gives. If it rests at zero, the 0.04V belongs to the wire, because a resistance at the clip that joins the wire to A takes a fixed amount off every balance length and displaces the line in exactly the same direction. Here the pointer rested at 0.04V.

(viii) Take the zero error off the reading:

$$1.00V-0.04V=0.96V$$

Over the range these six readings cover the voltmeter stands 0.04V high wherever it is set, which at this reading is 4% of the reading and 4.2% of the true value. What it does above 1.2V is not shown here, because a wire at $1.36Vm^{-1}$ cannot match more than 1.36V over its whole metre. On divisions of 0.1V the offset is under half a division, which is how a meter can carry one for a term with nobody troubling to look.

Examiner's eye

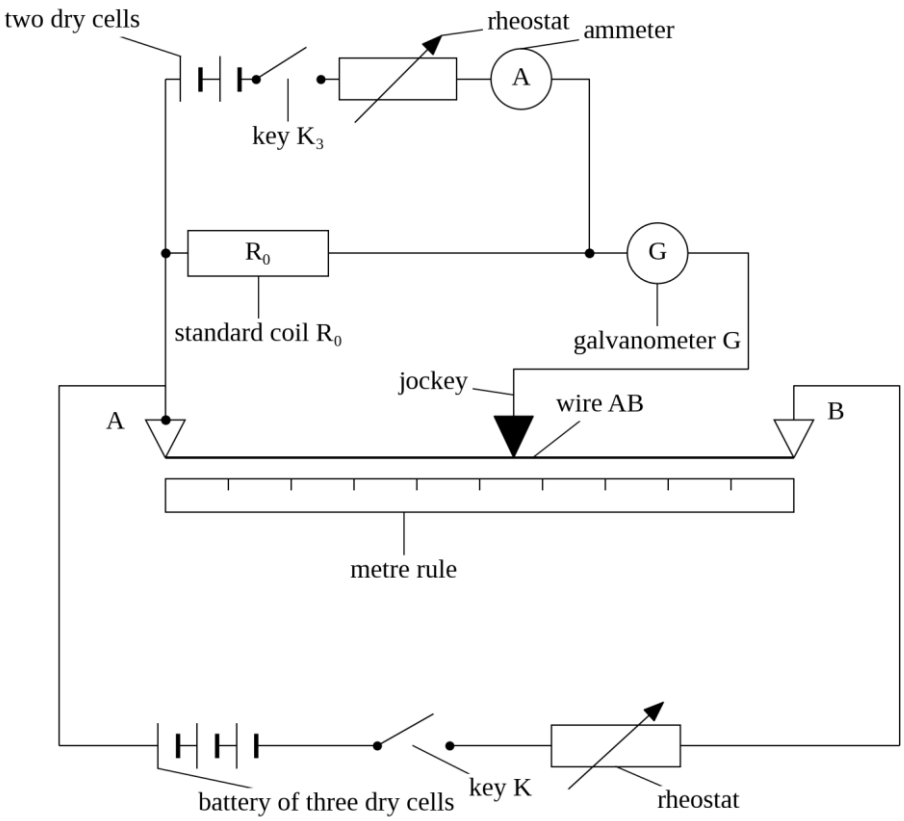
- Turn the slope over before you call it a potential gradient. This graph was drawn with the length upwards, so what comes off it is centimetres per volt, and the conversion into metres belongs on a line of its own.
- Two faults push this line off the origin in the same direction, and only one extra reading tells them apart. Name the reading and say what it showed; an answer that jumps straight to the meter has assumed the wire is perfect on no evidence at all.
- The straightness of this plot is a result and not a given. The wire's divisions are even and the null method adds no error of its own, so a straight line says the meter's divisions are even too. A curved one would mean the scale itself is at fault, and no single correction would put that right.
- Say which way the correction runs. This meter reads high, so every potential difference taken with it is smaller than it looks, and the correction is a subtraction. Add when you should subtract and the error is doubled instead of removed.

Practice 20.3

A builder in the town has made himself a reputation for generous rooms. His sitting rooms are wider than the plan says, his corridors are longer, and every client who has paced out a finished house has come away pleased. He works from a steel tape that has been dropped, trodden on and mended twice.

Kipute said the man was adding a little on purpose to please people. Kipanga said that nobody has ever been pleased by a large store, and that this builder's stores were large too. **Mr. Akilikubwa** said neither of them had asked the question he would have asked, which was when the tape had last been laid beside another one.

The client found the answer in the end, when the tiles he had ordered for the sitting room covered rather less of it than the floor required. Ammeters are checked the same way, against something better, and the wire is what an ammeter is checked against.



Procedure

Proceed as follows:

(a) Connect a battery of three dry cells, a key and a rheostat in series with the whole length of a potentiometer wire AB 100cm long.

(b) Balance a standard cell of electromotive force 1.08V against the wire and record its balance length from A.

(c) In a separate loop, join two dry cells, a key K_3, a rheostat, an ammeter reading 0 to 1A in divisions of 0.05A and a standard coil R_0 of resistance 2.00Ω in series.

(d) Join one end of the coil R_0 to the end A, and the other end through the galvanometer to the jockey.

(e) Close K_3, adjust the rheostat until the ammeter reads exactly 0.10A, find the balance point, record the ammeter reading I and the balance length L from A, and open K_3 again.

(f) Repeat the procedure in P20.3 (e) for ammeter readings of 0.20A, 0.30A, 0.40A, 0.50A and 0.60A.

Questions

(i) Tabulate your results.

(ii) Plot a graph of L (cm) against I (A).

(iii) Determine the slope of your graph.

(iv) The standard cell balanced at 83.1cm. Determine the potential gradient along AB.

(v) Determine the true current in the coil when the ammeter reads 0.500A.

(vi) Express the difference between the two as a percentage of the ammeter's reading.

(vii) State what the passage of the line through the origin tells you about the ammeter.

(viii) Explain why the potentiometer, rather than a second ammeter, is used as the standard here.

PART FIVE
GUIDE-LISTED BUT RARELY EXAMINED EXPERIMENTS

Chapter 21

VIBRATIONS AND WAVES: RESONANCE

Everything measured so far in this book has been measured by watching or by touching: a length against a rule, a time on a watch, a temperature on a thermometer, a current on a meter. In two of the three varieties that follow, the instrument is your ear, and in the third it is a scrap of paper. That is the difficulty the chapter opens with, and it is better said at the start than discovered at the bench.

What is being used throughout is resonance. A column of air closed at one end, or a wire stretched between two bridges, will answer a note strongly when the note is the right one for its length, and will hardly answer at all when it is not. Every question below is the same move made over and over: fix a frequency, hunt for the length that answers it, and read the length. The physics is in what the answering lengths do when something else is moved, which is the frequency through most of the chapter and the tension at the end of it.

For the air column the condition for the first resonance is

$$L+e=\frac{v}{4f}$$

with f the frequency sounded, L the length of the air column, v the speed of sound in air and e a small extra length called the end correction. The air just beyond the mouth of the tube is dragged along with the air inside it, so the column behaves as though it reached a little past the glass; e is how much. For the stretched wire the relation is

$$f=\frac{1}{2L}\sqrt{\frac{T}{\mu}}$$

with T the tension in the wire and μ its mass for every metre of its length. Both relations are supplied again in the questions that need them.

The apparatus is a resonance tube of glass with a reservoir of water joined to its foot, a burette with a tap, a sonometer with its masses and paper riders, a micrometer, and a set of tuning forks stamped with their frequencies. Every length in the chapter is read on a rule to the nearest millimetre, save the diameter of the sonometer wire, which is read on the micrometer; and frequencies are read off the forks themselves. In the first half of the chapter the quantity nobody measures directly is the speed of sound, and finding it is the point.

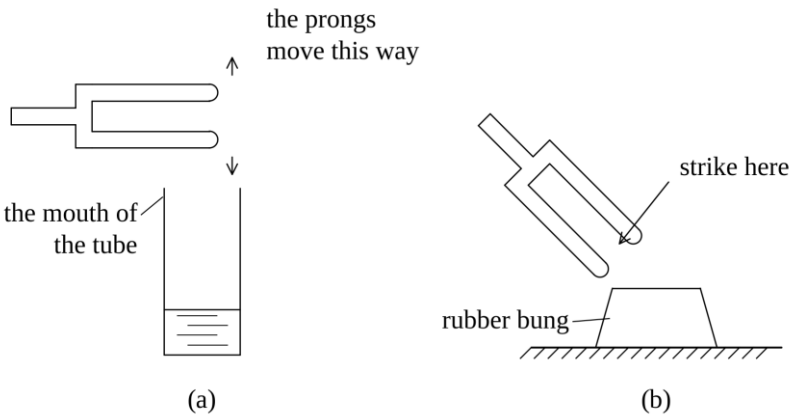
The three varieties at a glance

	What you do	What comes out of it
21.1	Five forks are sounded in turn over one tube and the first resonance is found for each of them	the speed of sound in air and the end correction of the tube, from the slope and the intercept of one line
21.2	The first and the second resonance are found for each of the same five forks	the speed of sound from one line on the sheet, and the end correction from the other
21.3	A sonometer wire under a fixed tension is brought to resonance by one fork after another	the mass per unit length of the wire, the density of its material, and the frequency of a fork whose stamp has worn away

All three are worked below in full. The practice questions do the first of them with a burette and a lower set of forks, take one pair of resonances five times over with a single fork, and put the sonometer under five different tensions.

Finding the resonance

Every length in the first four question sets is found by judging when a sound is loudest, and the ear is not a meter. Six habits turn that judgement into a measurement, and they repay practice on any tube before the readings begin to matter.



1. **Strike the fork on a rubber bung.** Never on the bench, on the tap, or on the edge of the sonometer box. A fork struck against something hard sounds its overtones as well as its own note, and the stamped frequency is then not the only frequency over the tube.
2. **Hold the fork horizontally.** One prong should be directly over the mouth, about 1cm above it, with the two prongs one above the other, so that they drive the air up and down the tube. Hold it the other way about and the prongs push the air across the mouth instead of into it, and the resonance is faint.
3. **Do not let the fork touch the glass.** A fork resting on the rim rattles, and gives up its energy to the tube in a second or two.
4. **Go past the maximum and come back.** The loudness rises to a peak and falls again, and the peak is broad. Find the two places on either side where you can just tell the sound is falling away, and settle between them; do not stop at the first level you like the sound of.
5. **Restrike the fork as often as you need.** A fork that has been sounding for 10s is quieter than the room, and a resonance you cannot hear is a resonance you will place wrongly.
6. **Read the water surface at eye level.** Read the flat middle of the surface, not the edge where it rides up the glass, and read it against a rule fixed beside the tube rather than held against it.

Experiment 21.1

Kipute's work on a Saturday is to fill the drum behind the kitchen from the standpipe, and he has never once stood over it. Through the wall he can hear the note the drum makes as it fills, climbing all the while and climbing fastest at the end; when it has climbed past a pitch he knows, he walks out and turns off the tap.

Kipanga heard about this and did not believe a word of it. *"The tap runs harder when nobody is watching it,"* he said. *"That is all you can hear."*

Kipute offered him the job. **Kipanga** took it, and stood over the drum for the whole of one filling to make sure of his ground. He came back to report that the tap had run at one speed from beginning to end, that the note had climbed anyway, and that he would be obliged if the matter were not raised again.

Mr. Akilikubwa had the whole story by Monday. On Thursday there was a metre of glass tubing clamped upright on the front bench with a can of water beside it, and a wooden box of tuning forks that nobody in the class had seen opened before.

Aim

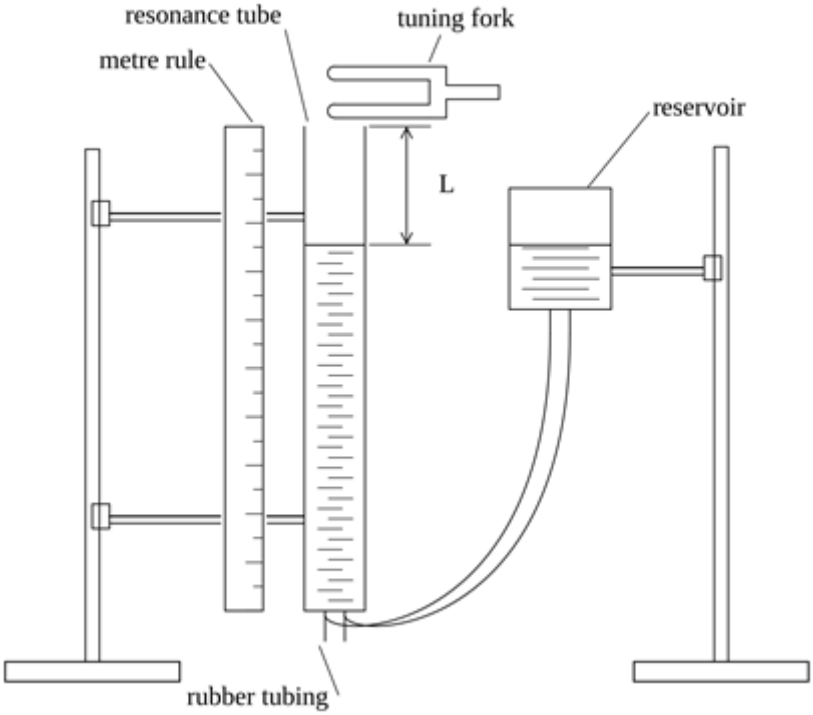
To determine the speed of sound in air and the end correction of a resonance tube.

Apparatus

A glass resonance tube 1.00m long and of 3.00cm internal diameter; a reservoir joined to the foot of the tube by rubber tubing; two retort stands with clamps; a metre rule; a thermometer; five tuning forks stamped 341Hz, 384Hz, 426Hz, 480Hz and 512Hz; a rubber bung to strike them on; water.

Setup

The arrangement is the one drawn below. The tube is clamped upright and the metre rule is clamped beside it with its zero level with the mouth of the tube, so that the water surface is read off directly as the length of the air column. Raising and lowering the reservoir raises and lowers the water in the tube; the two stand at the same level throughout, because the rubber tubing joins them. The room is at 26.0°C.



Procedure

Proceed as follows:

(a) Clamp the tube upright and clamp the metre rule beside it with its zero level with the mouth of the tube.

(b) Raise the reservoir until the water in the tube stands close to the mouth.

(c) Strike the fork stamped 341Hz on a rubber bung and hold it horizontally about 1cm above the mouth of the tube, with the two prongs one above the other.

(d) Lower the reservoir slowly, keeping the fork sounding, until the sound from the tube is loudest.

(e) Read the position of the water surface; then lower the reservoir past the resonance, bring it back up on to it, and take the mean of the two settings as L.

(f) Repeat steps (b) to (e) with each of the other four forks.

(g) Record the temperature of the room and all your readings in a suitable table.

Questions

(i) Tabulate your readings, including a column for $1/f$ in units of 10^{-3}s.

(ii) Plot a graph of $1/f$ against L.

(iii) Determine the slope of your graph.

(iv) Read from your graph the intercept on the $1/f$ axis.

(v) The length of the air column at resonance is related to the frequency by $L+e=v/4f$, where v is the speed of sound in air and e is the end correction of the tube. Use your slope to determine v.

(vi) Use your intercept, with your slope, to determine e.

(vii) The internal diameter of the tube is 3.00cm. Compare your value of e with 0.6 times the radius of the tube.

(viii) The speed of sound in air at θ°C is given by $v=331+0.6\theta$ in ms^{-1}. Compare your answer to (v) with the value this gives for the room, and comment on the agreement.

Table of results

The five forks were sounded in the order in which they are stamped, the lowest first, and each length is the mean of a setting made with the column shortening and one made with it lengthening.

f (Hz)	1/f (10^{-3}s)	L (cm)
341	2.933	24.5
384	2.604	21.7
426	2.347	19.7
480	2.083	17.0
512	1.953	16.1

Making the measurement count

Practical tip 1: *Loudness rises to a broad peak, and the width of that peak is what limits the reading, not the millimetres on the rule. Two people at one tube will differ by more than a millimetre and both be listening properly. Bracket the peak from both sides, take the middle, and quote the length to the nearest millimetre without believing the last figure of it too fondly.*

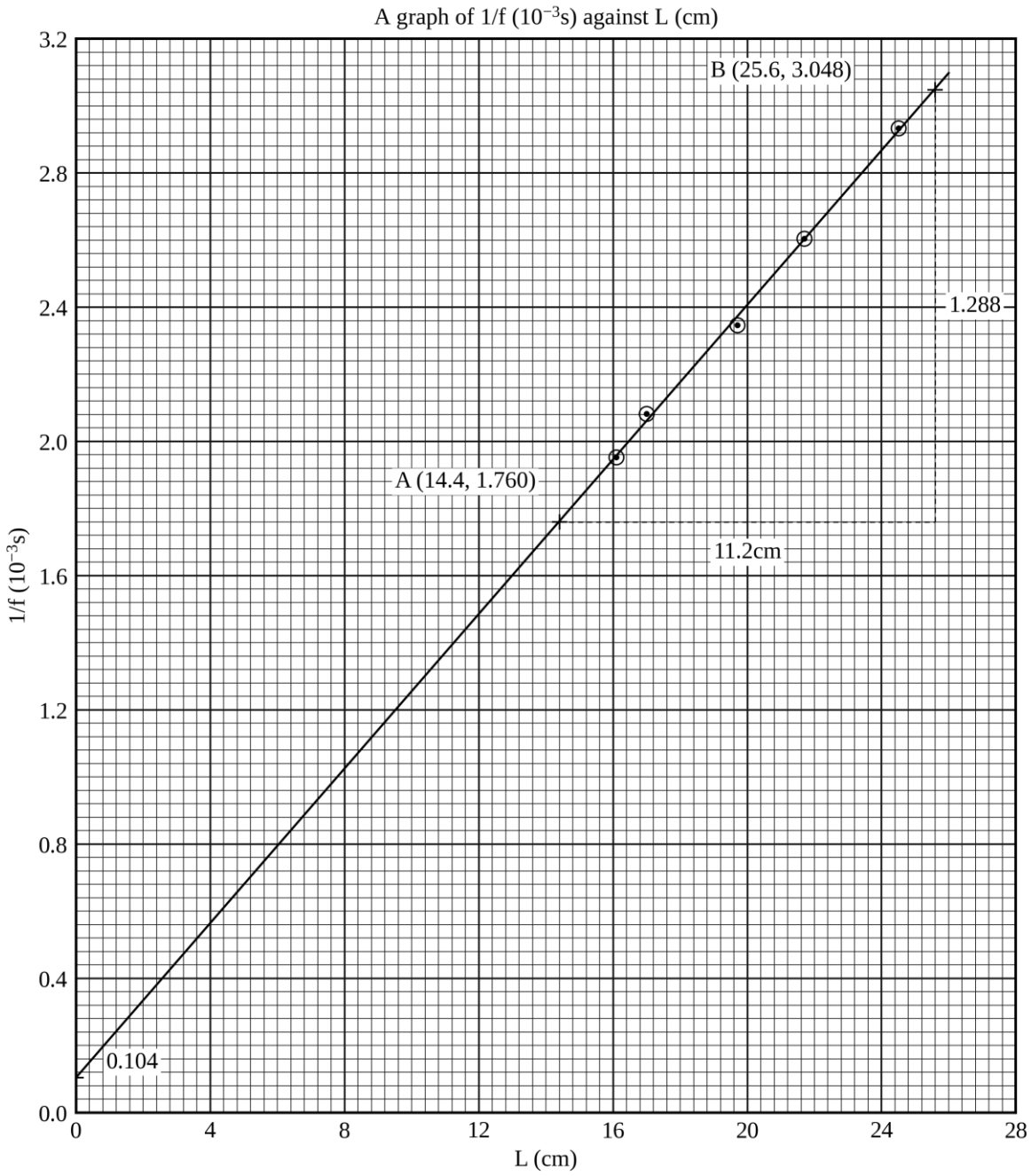
Practical tip 2: *Both axes have to start at zero, because the intercept is one of the two answers. The consequence is that the five points sit in one corner of the sheet and most of the paper is empty, which feels wrong and is right. Take the slope triangle over as much of the drawn line as you can rather than over the points alone; the line is what carries the slope. The smallest square on the paper is 2mm, and a point can be put on it, and read off it again, to about a fifth of that.*

Practical tip 3: *Reciprocals of hundreds are small numbers, and this plotted quantity is a reciprocal. Working in units of* 10^{-3}s *keeps the column tidy, but the factor of 1000 has to be put back before the slope means anything in metres and seconds. Write the unit at the head of the column and again beside the slope.*

Practical tip 4: *The speed of sound climbs by about* $0.6ms^{-1}$ *for every kelvin the room warms, so a laboratory that gains 5K during a lesson has moved the quantity being measured by* $3ms^{-1}$*. Sound all five forks in one sitting, and record the temperature once, at the middle of it.*

Answers to the questions

(i) and (ii) Take L along the horizontal axis at 2cm to 4cm, running from 0 to 28cm, and $1/f$ up the vertical axis at 2cm to 0.4×10^{-3}s, running from 0 to 3.2×10^{-3}s. That is 14cm by 16cm, which fits an A4 sheet upright. Give the graph the title A graph of 1/f (10^{-3}s) against L (cm).



(iii) Read two points off the drawn line as far apart as it will allow, A (14.4, 1.760) and B (25.6, 3.048). With the ordinates back in seconds and the abscissae in metres,

$$\text{slope}=\frac{0.003048\text{s}-0.001760\text{s}}{0.256\text{m}-0.144\text{m}}=\frac{0.001288\text{s}}{0.112\text{m}}=0.0115\text{s/m}$$

(iv) The line meets the vertical axis at 0.104×10^{-3}s, that is at 0.000104s.

(v) Making $1/f$ the subject of the relation given,

$$\frac{1}{f}=\frac{4L}{v}+\frac{4e}{v}$$

so a graph of $1/f$ against L is a straight line whose slope is $4/v$. Hence

$$v=\frac{4}{0.0115\text{s/m}}=348\text{ms}^{-1}$$

The speed of sound in air is 348ms^{-1}. Accept 341ms^{-1} to 355ms^{-1}.

(vi) The same rearrangement makes the intercept $4e/v$, so the intercept divided by the slope is e itself:

$$e=\frac{0.000104s}{0.0115s/m}=0.00904m=0.904cm$$

The end correction of the tube is 0.90cm. Accept 0.80cm to 1.00cm. That is a marking tolerance, not the precision of one carefully drawn graph.

(vii) The radius of the tube is half of 3.00cm, and

$$0.6\times1.50cm=0.90cm$$

which is what the graph gives. The agreement should not be read as a confirmation to three figures: the coefficient 0.6 is a rule of thumb good to about 5%, and the intercept it is compared with is worth about a tenth of itself. It is not the 0.6 of part (viii), which is a temperature coefficient and carries units.

(viii) At 26.0°C the relation gives

$$v=331+0.6\times26.0=346.6ms^{-1}$$

or $347ms^{-1}$ to three figures, against the $348ms^{-1}$ the graph gives; unrounded, the slope stands at $347.8ms^{-1}$, so the two are $1.2ms^{-1}$ apart. The two ordinates behind the slope were each placed to about 0.008×10^{-3}s, a fifth of a small square, which is 1% of the slope and some $3ms^{-1}$ on the speed. That is more than the gap, and no cause need be looked for.

Examiner's eye

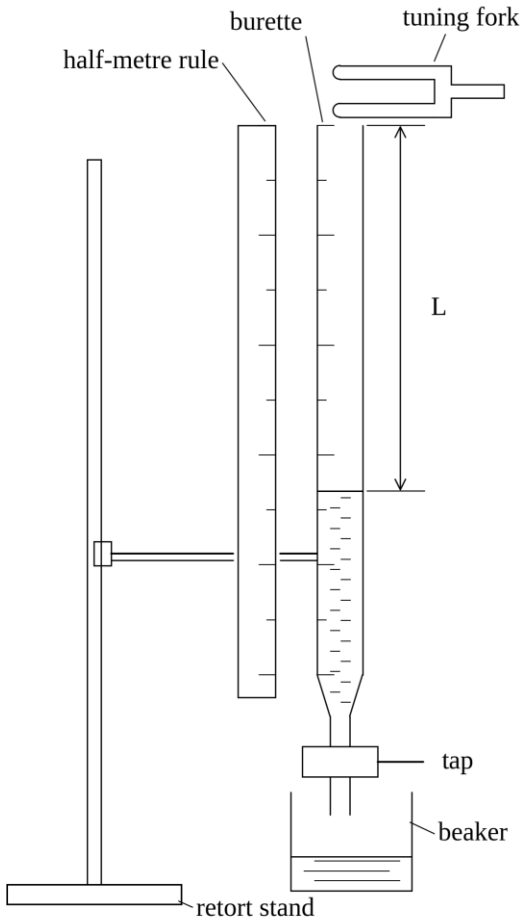
- Resist the axes you would rather have. The five points sit in one corner of the sheet, and starting the vertical axis at 1.9 and the horizontal one at 15 would spread them beautifully and throw away parts (iv) and (vi) with them: an axis that does not reach zero has no intercept on it to read.
- Watch the point at the top of the line. The 341Hz fork has the largest reciprocal and the longest column, so it plots highest and furthest out, where a misplaced millimetre tilts the drawn line most. Its water surface stands a quarter of a metre down the glass; take your eye down to it.
- Read the intercept against the axis, not against the lowest plotted point. Produce the line back until it crosses, and read it at the crossing. Your last reading stops 16cm short of the axis, and the whole of part (iv) happens inside those 16cm.
- Three quarters of the glass is water doing nothing, the longest column being a quarter of the tube. Lift the reservoir and see the surface come up to the mouth before a fork is struck.

Practice 21.1

The school band was promised six bamboo pipes for the parade, and the carpenter at the end of the road undertook to cut them. He works by ear. He cuts a length he thinks is near, blows across it, holds a fork against his bench, listens, takes a little more off, and blows again.

By the Thursday he had six pipes that satisfied him and a bundle of eleven others standing in the corner, which he described as the ones that had been nearly right. **Kipanga** said that the six were the ones that mattered and that the eleven were firewood. The carpenter agreed, and added that he had paid for the firewood twice: once for the bamboo and once for the afternoon.

Kipute thought the first pipe could have been measured and the other five cut to match it. The carpenter agreed that it could have, if anybody had ever told him what to measure.



Procedure

Proceed as follows:

(a) Clamp a burette upright with a half-metre rule beside it, its zero level with the mouth of the burette, and stand a beaker under the tap.

(b) Fill the burette with water to the brim.

(c) Strike the fork stamped 256Hz on a rubber bung and hold it horizontally about 1cm above the mouth of the burette, with the two prongs one above the other.

(d) Open the tap and let the water run out slowly, keeping the fork sounding, until the sound from the burette is loudest, and close the tap at that instant.

(e) Read the position of the water surface; then fill the burette again and come down on to the resonance a second time, and take the mean of the two settings as L.

(f) Repeat steps (b) to (e) with the forks stamped 288Hz, 320Hz, 341Hz and 384Hz.

(g) Record the temperature of the room and all your readings in a suitable table.

Questions

(i) Tabulate your readings, including a column for $1/f$ in units of 10^{-3}s.

(ii) Plot a graph of $1/f$ against L.

(iii) Determine the slope of your graph, and read the intercept on the $1/f$ axis.

(iv) The length of the air column at resonance is related to the frequency by $L+e=v/4f$, where v is the speed of sound in air and e is the end correction of the burette. Determine v.

(v) Determine e.

(vi) The internal diameter of the burette is 1.14cm. Compare your value of e with 0.6 times its radius, and state which of your two answers, v or e, this graph gives the better.

(vii) The room was at 23.0°C. Taking the speed of sound in air at θ°C to be $v=331+0.6\theta$ in ms^{-1}, compare your answer to (iv) with the value this gives.

(viii) The graduated part of the burette is 50.0cm long. Predict the length of air column at which a fork stamped 480Hz would resonate in it, and find the lowest frequency of fork the burette could be used with at all.

Experiment 21.2

The tube was still clamped on the bench on Friday, and **Kipute** had it to himself for the 20 minutes before the bell. He found the resonance for the fork stamped 341Hz within a few millimetres of where he had found it the day before. Then, out of idleness, he went on letting the water down, and half a metre lower the tube answered the same fork a second time, fainter than before and not to be mistaken for anything else.

He reported this at once as a fault in the apparatus. **Kipanga**, who had been reading ahead, said that it was not a fault, and that a longer tube would have done it a third time, or a fork stamped higher in the tube they had. **Kipute** asked what the third one would be for, and was not answered.

Neither of them could say what the second one was for. The box of forks was on the bench again on Thursday, with a note propped against it in **Mr. Akilikubwa**'s hand: both resonances, every fork, and the reason before the bell.

Aim

To determine the speed of sound in air and the end correction of a resonance tube from the first and the second resonance of one air column.

Apparatus

The apparatus of Experiment 21.1: the glass resonance tube 1.00m long and of 3.00cm internal diameter, its reservoir and rubber tubing, two retort stands with clamps, a metre rule, the five tuning forks stamped 341Hz, 384Hz, 426Hz, 480Hz and 512Hz, a thermometer, a rubber bung and water.

Setup

The arrangement is the one drawn in Experiment 21.1, set up in the same way, with the zero of the metre rule level with the mouth of the tube. The reservoir has to travel further in this experiment than in the last, so clamp it where it can be brought down to the bench without unclamping. The room is at 30.5°C.

Procedure

Proceed as follows:

(a) Set up the apparatus as in Experiment 21.1, with the zero of the metre rule level with the mouth of the tube.

(b) Raise the reservoir until the water in the tube stands close to the mouth.

(c) Strike the fork stamped 341Hz on a rubber bung and hold it horizontally about 1cm above the mouth of the tube, with the two prongs one above the other.

(d) Lower the reservoir slowly, keeping the fork sounding, until the sound from the tube is loudest, and record the length L_1 of the air column.

(e) Go on lowering the reservoir, sounding the fork all the while, until the sound is loudest a second time, and record the length L_2 of the air column.

(f) Repeat steps (b) to (e) with each of the other four forks.

(g) Record the temperature of the room and all your readings in a suitable table.

Questions

(i) Tabulate your readings, including a column for $1/f$ in units of 10^{-3}s and a column for $\frac{1}{2}(L_2 - L_1)$.

(ii) On the same axes plot graphs of $\frac{1}{2}(L_2 - L_1)$ and of L_1 against $1/f$, and draw the best straight line through each set of points.

(iii) Determine the slope of the line through the $\frac{1}{2}(L_2 - L_1)$ points.

(iv) State what the quantity $\frac{1}{2}(L_2 - L_1)$ is, and hence use your slope to determine the speed of sound in air.

(v) Read, on the vertical axis, the intercept of the line through the L_1 points on the vertical axis, and give the end correction of the tube.

(vi) Explain why the end correction does not enter your answer to (iv).

(vii) The tube is replaced by a wider one and the whole experiment repeated with the same five forks in the same room. State what happens to each of your two lines.

(viii) The room was at 30.5°C. Taking the speed of sound in air at θ°C to be $v=331+0.6\theta$ in ms^{-1}, compare your answer to (iv) with the value this gives.

Table of results

Each fork was taken through both resonances in one run, before the reservoir was raised again, so that the two lengths in a row belong to one filling of the tube.

f (Hz)	1/f (10^{-3}s)	L_1 (cm)	L_2 (cm)	½($L_2 - L_1$) (cm)
341	2.933	24.9	75.9	25.50
384	2.604	21.7	67.5	22.90
426	2.347	19.3	60.3	20.50
480	2.083	17.5	53.8	18.15
512	1.953	16.2	50.5	17.15

Making the measurement count

Practical tip 1: *The second resonance is the fainter of the two and it is a long way down. Keep the fork sounding all the way, restriking it whenever it fades, and do not let the water run down in one rush while you are restriking: the sound you are listening for can pass in a second.*

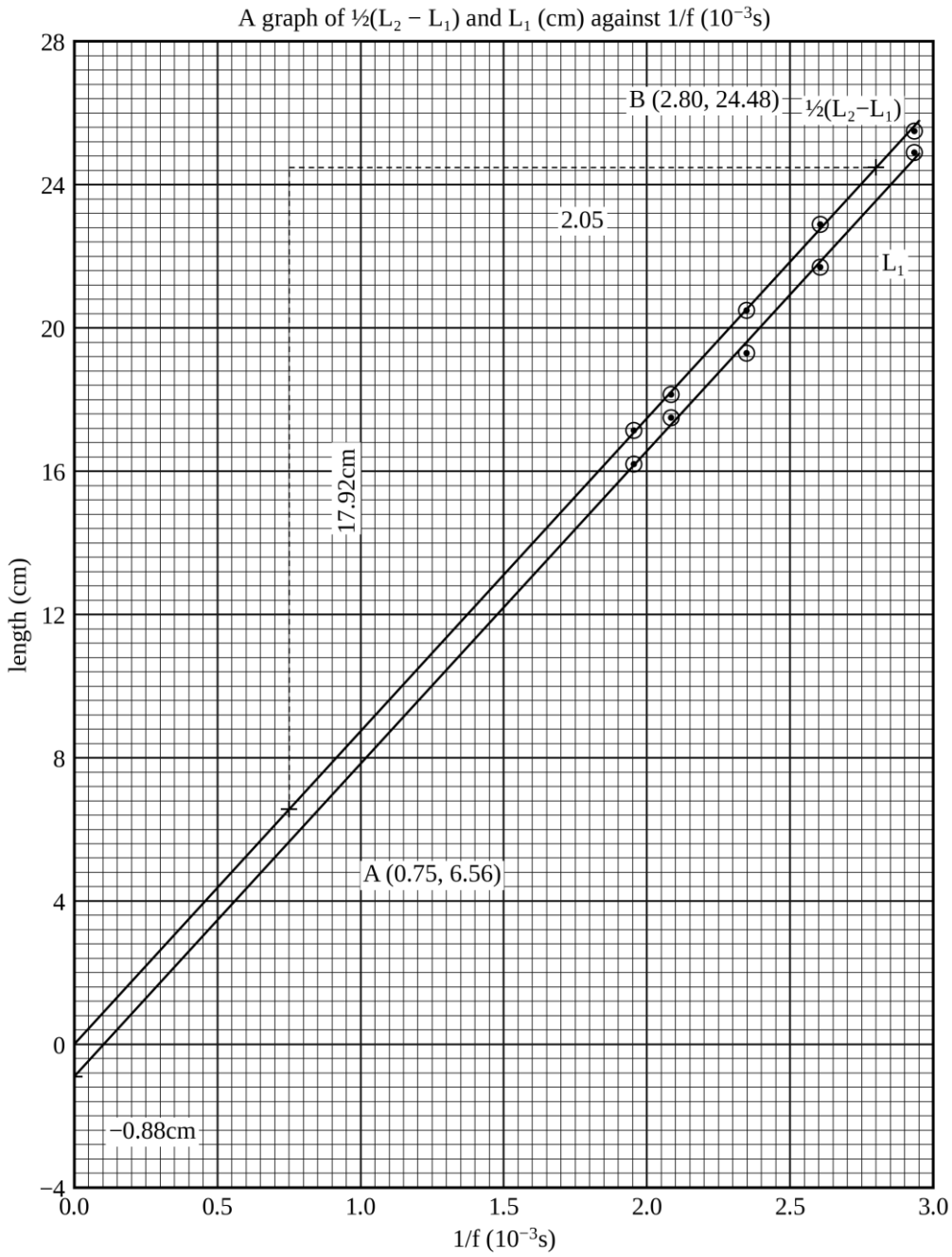
Practical tip 2: *Draw each of the two lines through its own points and do not force them parallel. Whether they come out parallel is a result of the experiment and worth a sentence in your account; a pair of lines ruled parallel because they ought to be has been assumed rather than measured.*

Practical tip 3: *A difference of two readings can be better than either of them. Each length is placed to a millimetre or two, and their difference runs to about 40cm, which is twice the shorter of the two. Part (iv) is built on the difference and not on the lengths, so work ½($L_2 - L_1$) out row by row and carry it to two decimal places.*

Practical tip 4: *Try the lowest fork first. It is the one that needs most room, and if the tube cannot show its second resonance nothing done later will mend the set. The four forks above it will then certainly fit.*

Answers to the questions

(i) and (ii). Take 1/f along the horizontal axis at 2cm to 0.5×10^{-3}s, running from 0 to 3.0×10^{-3}s, and length up the vertical axis at 2cm to 4cm, running from −4cm to 28cm. That is 12cm by 16cm, which fits an A4 sheet upright. Give the graph the title A graph of ½($L_2 - L_1$) and L_1 (cm) against 1/f (10^{-3}s).



(iii) The upper line passes through A (0.75, 6.56) and B (2.80, 24.48). In metres and seconds,

$$\text{slope}=\frac{0.2448\text{m}-0.0656\text{m}}{0.00280\text{s}-0.00075\text{s}}=\frac{0.1792\text{m}}{0.00205\text{s}}=87.4\text{ms}^{-1}$$

(iv) The first resonance stands at a quarter of a wavelength from the mouth and the second at three quarters of one, each of them short by the end correction. The difference between the two lengths is therefore half a wavelength, and half that difference is a quarter of the wavelength of the note:

$$1/4\,\lambda=\frac{v}{4f}$$

Half the difference between the two lengths is therefore the whole of the right-hand side, so the line through those points has a slope of $v/4$, and

$$v=4\times87.4\text{ms}^{-1}=349.6\text{ms}^{-1}$$

The speed of sound in air is 350ms^{-1}. Accept 343ms^{-1} to 357ms^{-1}.

(v) The lower line cuts the vertical axis at −0.88cm. Since $L_1 = v/4f - e$, that intercept is -e, so the end correction is 0.88cm. The rule of thumb, 0.6 times the radius of this tube, gives 0.90cm, and the two differ by less than the 0.08cm that one fifth of a small square comes to on this axis. Accept 0.78cm to 1.02cm.

(vi) Write the two resonance conditions out:

$$L_1 + e = 1/4\,\lambda \quad \text{and} \quad L_2 + e = 3/4\,\lambda$$

Subtract the first from the second and the end correction goes with it, leaving $L_2 - L_1 = 1/2\,\lambda$. Nothing that was plotted on the upper line ever contained e, so no error in e can reach the speed. The end correction survives only in the lower line, where it is the whole of the intercept.

(vii) The end correction grows with the bore, so a wider tube has a larger one. The upper line is unmoved: neither its slope nor its position depends on e at all, and the speed of sound is the same in the same room. The lower line keeps its slope, which is also v/4, but every first resonance is shorter by the increase in the end correction, so the line slides bodily downwards and the gap between the two lines opens out.

(viii) Put 30.5 into the relation and it gives

$$v = 331 + 0.6 \times 30.5 = 349.3\,\text{ms}^{-1}$$

which is $349\,\text{ms}^{-1}$ beside the $350\,\text{ms}^{-1}$ of (iv). Before either was rounded the two stood at $349.3\,\text{ms}^{-1}$ and $349.6\,\text{ms}^{-1}$, which differ by $0.3\,\text{ms}^{-1}$, or 0.09%. The two ordinates behind the slope were each placed to one fifth of a small square, which is 0.5% of it. A difference that small asks for no accounting.

Examiner's eye

- Label both lines, and say in your account which of them carried your triangle. The two run parallel and less than a centimetre apart the whole way across, with no sign on the sheet of which is which; write ½($L_2 - L_1$) against the upper and L_1 against the lower, out beyond the last point of each, where there is clear paper to write on.
- Rule the vertical axis from −4cm before you plot anything. The lower line has a negative intercept, and a sheet begun at zero has nowhere to put the answer to part (v); redrawing a graph at that stage costs more time than the whole question earns.
- Compare your two slopes before you leave the sheet. They are two measurements of one quantity, taken in one afternoon on one tube, and they should agree to within the thickness of a pencil line. Where they do not, one set of points has a reading in it that was found in a hurry.
- Count the maxima as they pass. There is a third resonance, at five quarters of a wavelength, and for the two highest forks it lies inside this tube; let the water run down without counting and it goes into the table as the second, with a difference twice the size it should be.

Practice 21.2

A length of galvanised pipe stands upright on the ground at the corner of the new classroom block, left over from the guttering and never carried away. On windy afternoons it hums. It hums on one note, always near enough the same note, and it can be heard from the far end of the field.

Kipute said that the wind was shaking the metal, and offered to wrap the pipe in a sack. The caretaker said that he had wrapped it in a sack in March and in a blanket in April, that neither had made the least difference to it, and that what had silenced it, for one week, was half a bucket of sand poured down the middle. **Kipanga** wanted to know what had happened after the week. It had rained.

Mr. Akilikubwa heard the account on Monday and remarked that the pipe had been measuring something all term, and that what it had been measuring could be had indoors from one fork and a quiet afternoon, with a good deal more precision than the wind manages. The caretaker said that he would sooner have the sand.

Procedure

Proceed as follows:

(a) Set up the resonance tube, its reservoir and the metre rule as in Experiment 21.2, with the zero of the rule level with the mouth of the tube.

(b) Strike the fork stamped 426Hz on a rubber bung and hold it horizontally about 1cm above the mouth of the tube, with the two prongs one above the other.

(c) Lower the reservoir slowly, keeping the fork sounding, until the sound from the tube is loudest, and record the length L_1 of the air column.

(d) Go on lowering the reservoir until the sound is loudest a second time, and record the length L_2.

(e) Raise the reservoir to the mouth again and repeat steps (b) to (d) four more times, making every setting afresh and without looking at the readings already taken.

(f) Record the temperature of the room and all your readings in a suitable table.

Questions

(i) Tabulate your readings, including a column for L_2-L_1.

(ii) The wavelength of the note is twice the difference L_2-L_1. Calculate the speed of sound in air from each of your five trials.

(iii) Give the mean of your five values and the half-range, and quote the speed of sound with its uncertainty.

(iv) The room was at 27.0°C. Taking the speed of sound in air at θ°C to be $v=331+0.6\theta$ in ms^{-1}, compare your answer to (iii) with the value this gives.

(v) Show that the end correction of the tube satisfies

$$2e=L_2-3L_1$$

and calculate it from each of your five trials.

(vi) Compare the spread of your five values of e with the spread of your five values of v, and account for the difference.

(vii) State one advantage this method has over one that uses a single resonance.

(viii) Suggest one change to the apparatus, and not to the procedure, that would reduce the spread in (iii).

Experiment 21.3

There is a man who plays a *zeze* outside the fruit market on Saturdays, one string over a resonator made from a paraffin tin. He tunes it in two ways and never confuses them: for a small change he slides the bridge along the neck, and for a large one he turns the peg.

Kipanga watched this for a while and offered to save him the trouble by pulling the string sideways with a finger while he played. The man handed the instrument over. The note went up, and up again, and then the string came off the peg and the man took the instrument back.

Kipute asked which of the two ways of tuning was the better. **Mr. Akilikubwa**, who had stopped to listen, bought a bag of oranges and said nothing at all until they were past the fish stalls. Then he remarked that there was a box in the physics store with one wire on it and no tune to play, and that both of the man's two ways of tuning were in one relation, and that the relation could be had off that box in an afternoon.

Aim

To determine the mass per unit length of a sonometer wire, the density of the material it is made of, and the frequency of an unmarked fork.

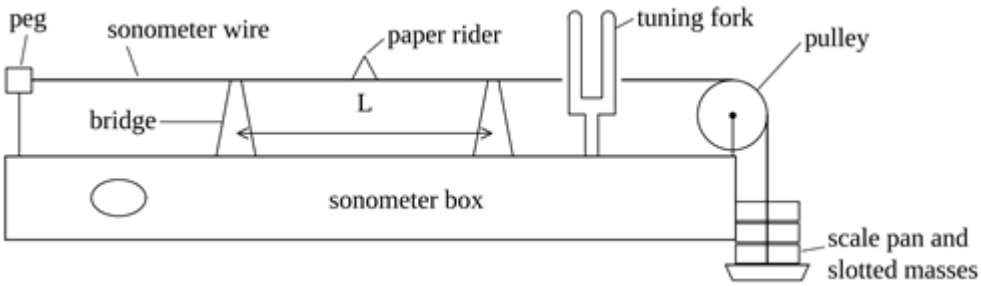
Apparatus

A sonometer with a drawn metal wire and two movable bridges; a scale pan and slotted masses making 3.00kg in all, pan included; five tuning forks stamped 341Hz, 384Hz, 426Hz, 480Hz and 512Hz, and a sixth fork, labelled X, whose stamp has worn away; a small paper rider; a metre rule; a micrometer screw gauge; a rubber bung.

Setup

The arrangement is the one drawn below. The wire runs from the peg at one end of the box, over both bridges, and over the pulley at the other end, where it carries the scale pan. The load is 3.00kg in all and stays on the

wire from the first reading to the last. The bridges slide along the box, and the distance L between the two of them is the length of wire that is free to vibrate.



Procedure

Proceed as follows:

(a) Set the load on the scale pan to 3.00kg in all and leave it hanging for 5 minutes before the first reading is taken.

(b) Set the two bridges about 35cm apart and lay a paper rider astride the wire between them.

(c) Strike the fork stamped 341Hz on a rubber bung and press its stem firmly on the box beside the wire.

(d) Move one bridge slowly inwards until the rider is thrown off the wire, and measure the distance L between the bridges, from the edge of one to the edge of the other where the wire leaves them.

(e) Repeat steps (b) to (d) with each of the other four stamped forks, and then with the unmarked fork X.

(f) Measure the diameter of the wire with the micrometer at three places along it and take the mean.

(g) Record all your readings in a suitable table.

Questions

(i) Tabulate your readings for the five stamped forks, including a column for $1/f$ in units of 10^{-3}s.

(ii) Plot a graph of L against $1/f$.

(iii) Determine the slope of your graph.

(iv) A wire of length L and mass per unit length μ, stretched by a tension T, sounds its fundamental note of frequency

$$f=\frac{1}{2L}\sqrt{\frac{T}{\mu}}$$

State what your slope represents.

(v) The load on the wire is 3.00kg. Determine μ.

(vi) Your micrometer readings give a mean diameter of 0.32mm. Determine the density of the material of the wire, and name a metal it could be.

(vii) The unmarked fork X threw the rider off at a length of 24.5cm. Use your graph to determine its frequency.

(viii) Give one reason why the paper rider is a better detector of resonance here than the ear, and one precaution you took in using it.

Table of results

The load stood at 3.00kg for the whole of the run and the wire was left under it for 5 minutes before the first reading. The unmarked fork X threw the rider off at 24.5cm and is kept out of the table below, since its frequency is not known.

f (Hz)	1/f (10^{-3}s)	L (cm)
341	2.933	31.4
384	2.604	28.2
426	2.347	25.6
480	2.083	22.3
512	1.953	21.3

Making the measurement count

Practical tip 1: *Press the stem of the fork on the box, not near it. The wire is thin and the fork is small, and neither of them can move enough air to be heard across a room; it is the box that does that, and it is also the box that carries the fork's vibration to the wire. A fork held in the air above the wire will not move the rider at all.*

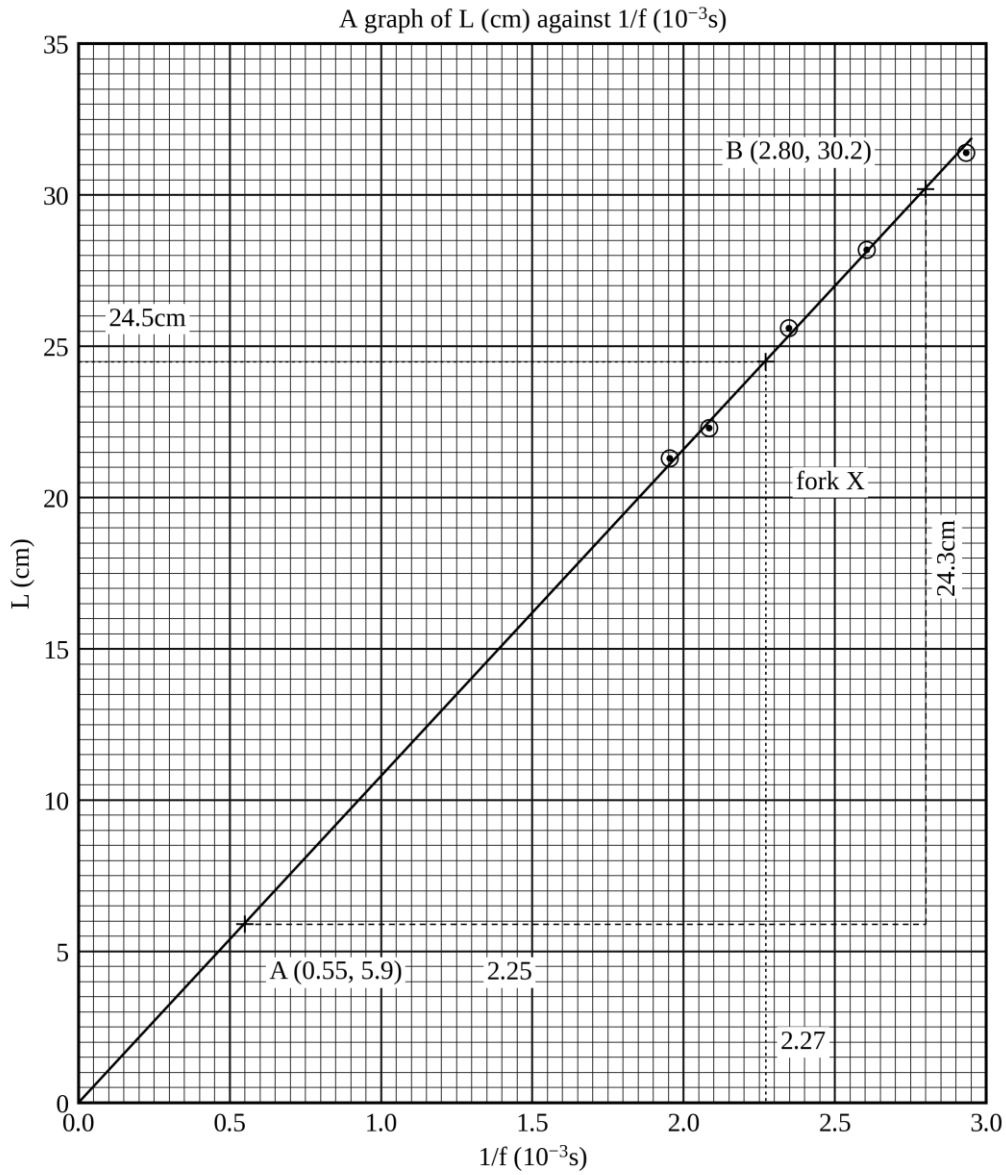
Practical tip 2: *Move one bridge only, and always in the same direction, so that whatever slipping there is at the bridges happens the same way in every reading. Check between forks that the pan is still clear of the floor and that nothing has been added to it or taken off.*

Practical tip 3: *A wire goes on stretching for some minutes after a new load is hung on it, and while it is stretching it is sliding over the two bridges and the pulley. Friction at those three places holds the sounding length back from the full weight of the load until the sliding stops. Hang the load, go and set out the forks, and come back to it.*

Practical tip 4: *Take the micrometer reading at three places along the wire and use the mean. The diameter enters part (vi) squared, so a 1% uncertainty in it propagates to 2% in the answer, and a wire that has been kinked or drawn unevenly is not the same thickness everywhere. Check the zero of the micrometer before you start and subtract it.*

Answers to the questions

(i) and (ii). Take 1/f along the horizontal axis at 2cm to 0.5×10^{-3}s, running from 0 to 3.0×10^{-3}s, and L up the vertical axis at 2cm to 5cm, running from 0 to 35cm. That is 12cm by 14cm, which fits an A4 sheet upright. Give the graph the title A graph of L (cm) against 1/f (10^{-3}s).



(iii) The points lie on a straight line through the origin. Taking A (0.55, 5.9) and B (2.80, 30.2) taken on it, in metres and seconds,

$$\text{slope}=\frac{0.302\text{m}-0.059\text{m}}{0.00280\text{s}-0.00055\text{s}}=\frac{0.243\text{m}}{0.00225\text{s}}=108\text{ms}^{-1}$$

(iv) Rearranged, the relation reads

$$L=\frac{1}{2}\sqrt{\frac{T}{\mu}}\times\frac{1}{f}$$

so a graph of L against $1/f$ is a straight line through the origin, and its slope is the whole of the factor in front of $1/f$. The square root in that factor is the speed at which a transverse wave runs along the wire, so the slope of the graph is half that speed.

(v) The tension is the weight of the load:

$$T=3.00\text{kg}\times9.81\text{ms}^{-2}=29.43\text{N}$$

The wave speed along the wire is therefore twice the slope, 216ms^{-1}, and squaring it gives T/μ, so

$$\mu=\frac{29.43\text{N}}{216\text{ms}^{-1}\times216\text{ms}^{-1}}=0.000631\text{kgm}^{-1}$$

The wire has a mass per unit length of 0.631g/m. Accept 0.600g/m to 0.665g/m.

(vi) The cross-section of the wire is

$$A=\frac{3.14\times0.32\text{mm}\times0.32\text{mm}}{4}=0.0804\text{mm}^2$$

A metre of the wire is 1000mm long, so its volume is

$$0.0804\text{mm}^2\times1000\text{mm}=80.4\text{mm}^3=0.0804\text{cm}^3$$

and that metre has a mass of 0.631g, so

$$\rho=\frac{0.631\text{g}}{0.0804\text{cm}^3}=7.85\text{gcm}^{-3}$$

Steel has a density of about 7.8gcm^{-3}, and the wire agrees with it: it could be steel. It could equally be iron, which stands at 7.87gcm^{-3}, and no sonometer will separate those two. What the measurement does settle is that the wire is not copper, which stands at 8.96gcm^{-3} and is far outside anything this reading allows. Brass, at about 8.4gcm^{-3}, lies outside it by very little, and a value at the top of the band would not exclude brass with any confidence. Accept 7.4gcm^{-3} to 8.3gcm^{-3}. That is the band on the mass per unit length carried straight through, because the same diameter goes into every answer. What widens it in the first place is the slope: the mass per unit length goes as the inverse square of the slope, so a slope 2% out moves the density by 4%.

(vii) Draw a horizontal line across from 24.5cm on the vertical axis to the drawn line, and drop from the crossing to the horizontal axis, where it meets it at 2.27×10^{-3}s. That is the reciprocal of the frequency, so

$$f=\frac{1}{0.00227\text{s}}=441\text{Hz}$$

Fork X sounds about 441Hz. The nearest fork made is the 440Hz one that instruments are tuned to, and the reading cannot separate the two: a fifth of a small square on the horizontal axis is 0.01×10^{-3}s, which is 2Hz here. Accept 430Hz to 452Hz.

(viii) The reason: the rider signals to the eye, and the eye has nothing to compete with, whereas the ear has to pick the wire's note out of the fork's own, which is sounding all the while. The precaution: the rider is a folded scrap no bigger than a fingernail, laid midway between the bridges. A heavy rider loads the wire, and one laid near a bridge sits where the wire hardly moves and is never thrown at all.

Examiner's eye

- Put the whole of the answer in the slope: the line goes through the origin, so the slope is the only thing to be taken off it. Take the triangle over most of the drawn line rather than between two neighbouring points, and leave both corners marked on the sheet where they can be found: somebody who was not there has to be able to check the two points your slope came from.
- Count the pan. The tension is the weight of everything hanging on the wire, and the 3.00kg given here is the pan and the slotted masses together, which is why it needs no addition. Where a question gives the pan separately, add it: 200g unaccounted for is 7% off the tension and 7% off the mass per unit length.
- Measure between the edges where the wire leaves the bridges and not between their middles. A bridge is a centimetre wide, so the wrong edge adds the same amount to all five lengths, and five points that should be heading for the origin then head for a point above it. Forced through the origin anyway, they give a slope that none of the readings supports.
- Invert at the end and never at the beginning: part (vii) wants a frequency and the graph gives a reciprocal. Read 24.5cm against the drawn line and not against the plotted points, which are five readings of which the line is the best.

Practice 21.3

A letter came to the school in the middle of the term, addressed to the physics room.

> Dear Mr. Akilikubwa,
>
> I mend instruments in the town and I buy my wire from the hardware shop by the bus stand. The reels there have lost their papers long ago and the man sells by the look of the wire. I have strung three zeze this month that would not come up to pitch, and a fourth that broke while I was tightening it.
>
> Is there a way of telling what a wire is before it goes on an instrument, using only what a school has on its shelves?
>
> Yours faithfully,
>
> A. Chonde
>
> P.S. I enclose a length of the wire in question. It is the only piece of it that has been of any use to me.

Kipute read the postscript out twice. The coil went on the front bench beside the sonometer, and **Kipanga** turned it over in his hand and said that it looked like every other wire he had ever seen, which was exactly what A. Chonde had written.

Procedure

Proceed as follows:

(a) String the sonometer with the wire A. Chonde sent and hang a scale pan and slotted masses making 1.00kg in all on it, leaving it for 5 minutes before the first reading is taken.

(b) Set the two bridges about 15cm apart and lay a paper rider astride the wire between them.

(c) Strike the fork stamped 384Hz on a rubber bung and press its stem firmly on the box beside the wire.

(d) Move one bridge slowly outwards until the rider is thrown off the wire, and measure the distance L between the bridges.

(e) Change the load to 2.00kg, leave it for 5 minutes, and repeat steps (b) to (d). Do the same for 3.00kg, 4.00kg and 5.00kg.

(f) Measure the diameter of the wire with the micrometer at three places along it and take the mean.

(g) Record all your readings in a suitable table.

Questions

(i) Tabulate your readings, including a column for the tension T in the wire and a column for L^2.

(ii) Plot a graph of L^2 against T.

(iii) Determine the slope of your graph.

(iv) A wire of length L and mass per unit length μ, stretched by a tension T, sounds a note of frequency

$$f=\frac{1}{2L}\sqrt{\frac{T}{\mu}}$$

Show that L^2 is proportional to T when the frequency is fixed, and state what your slope represents.

(v) Determine μ for this wire.

(vi) The density of steel is $7.80gcm^{-3}$. Find the diameter this wire would have if it were steel, and compare it with the 0.26mm your micrometer readings give.

(vii) Use your graph to find the tension that would bring this wire to resonance at a length of 30.0cm with the same fork.

(viii) State, with a reason, whether a wire of the same material and twice the diameter would give a larger or a smaller slope.

Chapter 12

LIGHT: REFLECTION AND REFRACTION

Light travels in straight lines, and that one fact is what makes this chapter possible. You cannot see a ray, and you cannot put a ruler against one. What you can do is plant two pins a long way apart on the path it took, and afterwards, with the light gone and the apparatus put away, join the two holes with a ruler. Every angle in this chapter is measured on a sheet of paper after the event.

Two things can happen to a ray when it arrives at the surface of something. It can come back into the medium it came from, or it can go on into the new medium with its direction changed. The first is reflection and the second is refraction, and both are measured by the angle the ray makes with the normal. Experiment 22.1 is about reflection at a plane mirror; the other three are about refraction, at a prism, at the surface of a liquid and at a block of glass.

The prism of Experiment 22.2 and the block of Experiment 22.4 are cut from the same glass. The water of Experiment 22.3 is a different material and stands on its own.

The instrument that sets the uncertainty of three of the four is the protractor. It is divided in degrees, so a reading is good to 0.5°, and only if the lines it is laid against were drawn sharply. The answer you quote is no better than the pencil work that carried it.

The four varieties at a glance

	What you do	What comes out of it
22.1	A ray is sent at a plane mirror and the reflected ray is recovered with pins	how the direction in which a ray leaves a mirror is fixed by the direction in which it arrives
22.2	A ray is traced through a triangular prism, and the angle through which the prism turns it is measured at a series of angles of incidence	the refractive index of the glass of the prism
22.3	A pin at the bottom of a tall jar is located through the liquid above it as the depth of that liquid is increased	the refractive index of water
22.4	A ray is traced through a rectangular block at a series of angles of incidence	the refractive index of the glass, the critical angle and the speed of light inside it

All four are worked in full. The practice questions turn the mirror into a measurement of the angle it turns a ray through, put a second prism on the bench, fill the jar with a liquid nobody can name, and ask whether a sheet sold as glass is glass.

Tracing a ray with pins

The technique is the same in Experiments 22.1, 22.2 and 22.4 and it repays getting right once. Six rules cover it.

(a) Pin the paper down at all four corners before you draw anything. A sheet that shifts by a millimetre between the first pair of pins and the second has spoiled the reading and will not look as though it has.

(b) Stand every pin upright. A pin leaning towards you marks the board in one place and the paper in another, and the hole you ring is the one in the paper.

(c) Put the two pins of a ray as far apart as the sheet allows, and never closer than 8cm.

(d) Get your eye down to the level of the paper, so that you are looking along the sheet and not down on to it, and move it slowly across until the two images and the two pins line up one behind the other. Set the far pin first and the near one afterwards.

(e) Take the pins out and ring each hole with a small circle before you move the paper. Four holes ringed are four points; four holes not ringed are a sheet of paper with holes in it.

(f) Draw with a sharp pencil against a ruler that reaches the whole length of the line, and read the protractor with its centre on the point of the angle and its baseline along one of the two lines.

Experiment 22.1

The notice board in the hall stands in the one corner the morning sun never reaches, and always has.

Kipanga propped a mirror on the windowsill to throw the light on to it, and lit up the ceiling.

"Tilt it down," said **Kipute**.

"It is down."

"Tilt it down more."

They tilted it down more and lit the floor, then the ceiling again, then the back of **Mr. Akilikubwa**, who had come in to find out what the argument was. He said that the mirror was doing exactly what mirrors do, that neither of them could say what that was, and that this was why they were tilting it by guesswork. There is a rule that settles it, and it is one line long.

Aim

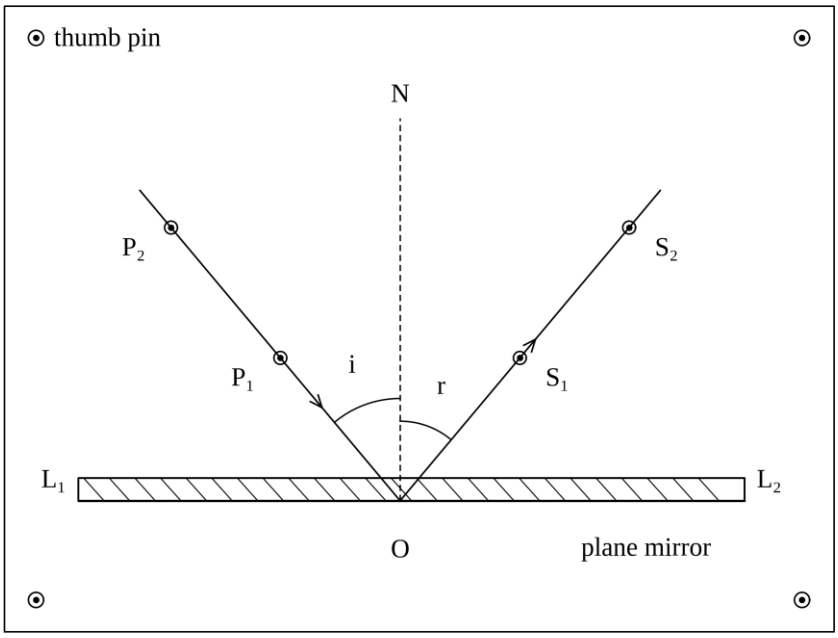
To examine, for a plane mirror, how the angle at which a ray leaves the surface is related to the angle at which it arrives.

Apparatus

A soft board; a plane mirror about 10cm long; four optical pins; four thumb pins; six sheets of plain paper; a ruler; a protractor; a sharp pencil.

Setup

The paper is pinned flat to the board and a line is drawn across it with the mirror standing on the line. A ray is sent in at a measured angle to the normal and is marked by two pins; the reflected ray is found by looking into the mirror from the other side of the normal and marked by two more. The arrangement is the one drawn below.



Procedure

Proceed as follows:

(a) Pin a sheet of plain paper to the board and draw a line L_1L_2 across the middle of it. Mark a point O at the middle of that line and draw the normal ON at right angles to the line at O.

(b) Stand the plane mirror on the line, with its silvered surface along the line.

(c) Draw a line through O on the left of the normal, making an angle i of 20° with ON, and fix the pins P_1 and P_2 upright on it as far apart as the sheet allows.

(d) Look into the mirror from the right of the normal and fix the pins S_1 and S_2 upright so that they are in line with the images of P_1 and P_2.

(e) Take out the four pins, ring the four holes and remove the mirror. Join S_1 and S_2 and produce the line back to O. Measure the angle r between that line and ON.

(f) Repeat the procedures in 22.1 (a) to (e) on a fresh sheet for each of the values of i equal to 30°, 40°, 50°, 60° and 70°.

Questions

(i) Tabulate your results.

(ii) Plot a graph of i (°) against r (°).

(iii) Determine the slope of your graph.

(iv) State the law your graph verifies.

(v) Another student obtains a line of the same slope as yours, but his line cuts the r axis at 2°. State what that shows about the way he did the experiment.

(vi) Explain why the two pins of each ray are put as far apart as the sheet allows.

(vii) State how the arrangement shows that the incident ray, the reflected ray and the normal at the point of incidence all lie in one plane.

(viii) A student says that one pair of readings settles the relation between i and r, and that the other five are a waste of paper. Use your graph to answer him.

Table of results

The angles of incidence were set with the protractor and the angles of reflection were read from the protractor to the nearest 0.5°.

i (°)	r (°)
20	20.5
30	29.5
40	39.0
50	50.0
60	60.5
70	70.0

Making the measurement count

Practical tip 1: *The silvered surface goes on the line, not the front of the glass.*

A mirror is a sheet of glass with the silver on the back of it, and the surface that reflects is the silver. Stand the mirror with its front face on the line and the light is turned round by a plane 2mm or 3mm behind the one you drew. The direction of the reflected ray is unaffected, which is why the readings survive it, but the line you produce back will not pass through O, and that looks like a mistake when it is not.

Practical tip 2: *Lay the protractor along the normal, not along the mirror line.*

Put the centre of the protractor exactly on O and its baseline along ON, and the scale then gives you the angle you want without any arithmetic. Laid along the mirror line instead it gives you the complement, and a table in which one row out of six has been subtracted from 90° has that one row wrong and the rest of it right.

Practical tip 3: *One sheet to each angle of incidence.*

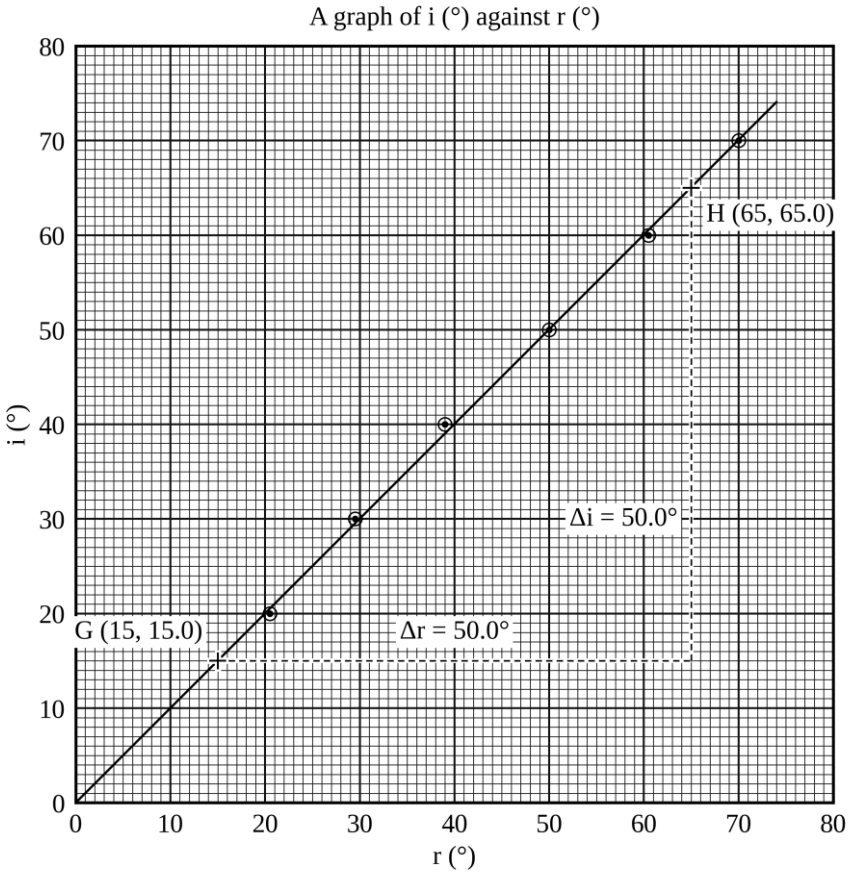
A sheet that has carried three settings has twelve holes in it and no way of telling which four belong together. Ring the four holes as the pins come out, write the angle of incidence in the corner, and start the next setting on clean paper.

Practical tip 4: *Why the readings stop where they do.*

Above about 75° the front of the glass reflects nearly as strongly as the silver behind it, and a second image appears beside the true one with little to choose between them; below about 5° the incident and the reflected rays lie so close together that four pins will not separate them. The table above keeps every setting well inside both limits.

Answers to the questions

(i) and (ii). The readings are those in the table above. Take r at 2cm to 10°, running from 0° to 80°, and i at 2cm to 10°, running from 0° to 80°. That is 16cm by 16cm, which fits an A4 sheet upright. Give the graph the title A graph of i (°) against r (°).



(iii) Taking G (15, 15.0) and H (65, 65.0) on the line,

$$\text{slope}=\frac{65.0°\text{-}15.0°}{65.0°\text{-}15.0°}=\frac{50.0°}{50.0°}=1.00$$

The two legs of the triangle are the same length, and a slope of 1.00 is what that means.

(iv) That the angle of reflection is equal to the angle of incidence, i=r. This is the first law of reflection, and the graph shows it holding over the whole range from 20° to 70° and not merely at one setting.

(v) That every one of his angles of reflection was 2° larger than it should have been, by the same amount at every setting. A fault that shifts every reading by the same amount and leaves the slope alone is not a fault in the reading; it is a fault in the thing the readings were measured from. Either his normal was not at right angles to the mirror line, or the mirror was not standing on the line but at a small angle to it. Both would tilt the reference and leave the relation between the two angles untouched.

(vi) Because the two pin holes are what fix the direction of the ray, and each of them can be out by about 0.5mm. Two holes 8cm apart, each out by that much, put the line out by about 0.5°; move them to 16cm apart and the same two put it out by half of that. Doubling the separation halves the doubt in the direction, and the separation is the part of it most under your control.

(vii) Because everything in the experiment happens on the sheet of paper. The first two pins stand on the paper and so fix the incident ray in the plane of the paper; the search pins are set in line with the images of the first two while the eye is held level with the paper, so the reflected ray lies in that plane too; and the normal ON is drawn on the paper. A reflected ray that left the plane of the paper could not be picked up by pins standing in it, and at no setting was there any difficulty in finding one.

(viii) One pair of readings gives one point, and a single point establishes nothing at all: any line whatever can be drawn through it. Two points give a slope but no check on it. Six points give a slope, and they also show that

the relation holds at 20° and at 70° and everywhere between, which is a good deal more than the relation at one angle. His own reading could have been out by 0.5° and he would never know; the line drawn through six averages those errors away, and the largest of them here is about 1°, which is plain on the graph and would be invisible in a single pair.

Examiner's eye

- Both scales here are angles. Use the same scale on the two axes and the correct line lies at 45° to them, which is a check you can make with the corner of a set square; use different scales and it does not, and a correct graph then looks wrong.
- Quote the slope as a number and nothing else. A slope written as 1.00° carries a unit that cancelled two steps earlier, and it says the axes were never read.
- Name the law, and name it fully. It carries a mark on its own and is thrown away by answering "they are equal." Say which angle equals which, and say that both are measured from the normal, because an angle measured from the mirror is a different number.
- Six rows in which i and r agree exactly have been copied rather than measured. Half a degree of disagreement is what careful work with pins gives, and it is better evidence than a column of perfect agreement, because it shows the second column was read and not assumed.

Practice 22.1

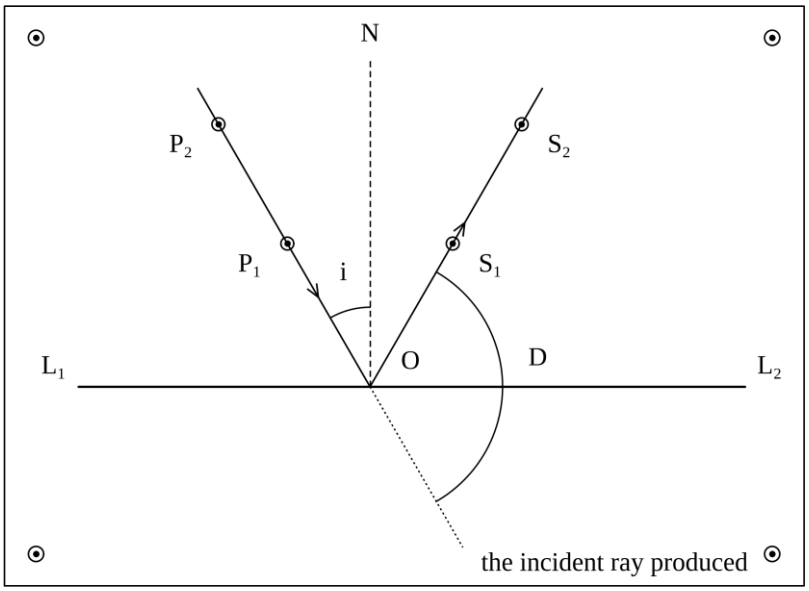
Kipute built a periscope out of a shoebox and two mirrors so that he could see over the heads at the parade. He cut two holes, one at the top of one side and one at the bottom of the other, and glued the mirrors in at what he judged to be a comfortable slope.

What he saw through it was the inside of the box.

Mr. Akilikubwa asked what angle the mirrors had been set at, and **Kipute** said comfortable.

Kipanga took the box off him, held it up, turned it round and said that the light was reaching the lower mirror perfectly well and leaving it in a direction no use to anybody. He said it as though the mirror had chosen. It has no choice: where the light goes is settled the moment it arrives, and the slope that felt comfortable was sending the parade into the lid.

The sheet below is the one from Experiment 22.1 with the mirror taken off it and one line added, the incident ray produced through O. The angle D is between that produced line and the reflected ray.



Procedure

Proceed as follows:

(a) Pin a sheet of plain paper to the board and draw a line L_1L_2 across the middle of it. Mark a point O at the middle of that line and draw the normal ON at right angles to the line at O.

(b) Stand the plane mirror on the line, with its silvered surface along the line.

(c) Draw a line through O on the left of the normal, making an angle i of 10° with ON, and fix the pins P_1 and P_2 upright on it as far apart as the sheet allows.

(d) Look into the mirror from the right of the normal and fix the pins S_1 and S_2 upright so that they are in line with the images of P_1 and P_2.

(e) Take out the four pins, ring the four holes and remove the mirror. Join S_1 and S_2 and produce the line back to O. Produce the incident ray through O with a broken line, and measure the angle D between that broken line and the reflected ray.

(f) Repeat the procedures in P22.1 (a) to (e) on a fresh sheet for each of the values of i equal to 20°, 30°, 40°, 50°, 60° and 70°.

Questions

(i) Tabulate your results.

(ii) Plot a graph of D (°) against i (°).

(iii) Determine the slope of your graph.

(iv) State the physical meaning of the slope.

(v) Read the intercept on the D axis and state its physical meaning.

(vi) Find from your graph the angle of incidence at which the mirror turns light through a right angle.

(vii) Show that the slope and the intercept together give the law you obtained in Experiment 22.1.

(viii) The mirror is turned through 10° about a vertical axis, the incident ray staying where it is. State through what angle the reflected ray turns and how your graph supports you.

Experiment 22.2

The prism has sat on the laboratory windowsill for years, and its one recognised use is throwing a stripe of colour across the register in the middle of the morning.

Kipute picked it up, stood it on a sheet of paper with a pin upright beyond it, and looked through. The pin was not where he had put it. He moved his head, and it stayed where it was not. He put the prism down, and the pin was where he had put it. He picked the prism up again, and the pin moved again.

"Somebody is moving that pin," he said, which was untrue and, in a room containing **Kipanga**, not unreasonable.

Mr. Akilikubwa said that the pin had not moved and the light had, and asked by how much. **Kipute** turned the prism a little and the pin shifted again, further this time, so the answer was not one amount but depended on how the prism was held.

Aim

To determine the refractive index of the glass of a triangular prism by tracing rays through it.

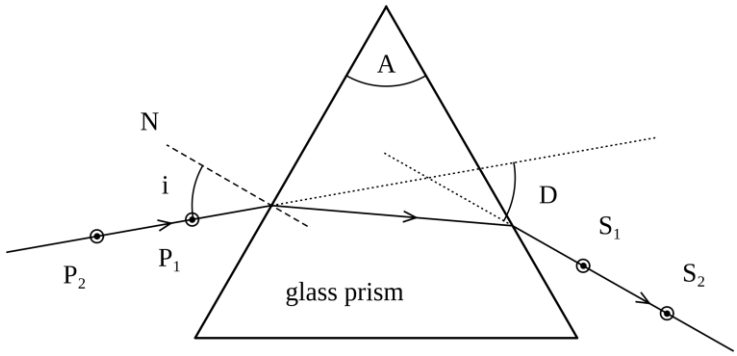
Apparatus

A soft board; a triangular glass prism; four optical pins; four thumb pins; nine sheets of plain paper; a ruler; a protractor; a sharp pencil.

Setup

The outline of the prism is traced on the paper and a ray is sent in at one refracting face at a measured angle to the normal there. The ray that comes out of the second face is found with two more pins, and the two rays are

produced until they cross. The angle between them at the crossing is the angle through which the prism has turned the light. The arrangement is the one drawn below.



Procedure

Proceed as follows:

(a) Pin a sheet of plain paper to the board, stand the prism on it and trace its outline with a sharp pencil.

(b) Mark a point O about halfway along one of the two refracting faces and draw the normal to that face at O.

(c) Draw the line making an angle i of 35° with that normal, and fix the pins P_1 and P_2 upright on it as far apart as the sheet allows.

(d) Put the prism back inside its outline, look through the other refracting face and fix the pins S_1 and S_2 upright so that they are in line with the images of P_1 and P_2.

(e) Take out the four pins, ring the four holes and remove the prism. Join S_1 and S_2, produce the incident ray forwards and the emergent ray backwards until the two lines cross, and measure the angle D between them at the crossing. Measure the refracting angle A of the prism from your traced outline.

(f) Repeat the procedures in 22.2 (a) to (e) on a fresh sheet for each of the values of i equal to 40°, 45°, 50°, 55°, 60°, 65°, 70° and 75°.

Questions

(i) Tabulate your results.

(ii) Plot a graph of D (°) against i (°) and draw a single smooth curve through your points.

(iii) Read from your curve the least value of D.

(iv) Read from your curve the angle of incidence at which that least value occurs.

(v) The refractive index of the glass is given by

$$n=\frac{\sin\frac{A+D_m}{2}}{\sin\frac{A}{2}}$$

where A is the refracting angle of the prism and D_m is the least angle of deviation. Determine the refractive index of the glass of this prism.

(vi) Your curve is nearly flat on either side of its lowest point. State what advantage that gives to this method of measurement.

(vii) Every angle in your table was read to the nearest 0.5°. Estimate the uncertainty this puts on your value of the refractive index.

(viii) A student takes one reading only, at i equal to 40°, and puts his value of D straight into the formula. Determine the refractive index he obtains, and state whether it is too large or too small.

Table of results

The refracting angle of the prism was measured from the traced outline as 60.0°. The angles of deviation were read from the protractor to the nearest 0.5°.

i (°)	D (°)	i (°)	D (°)
35	42.0	60	40.0
40	39.5	65	41.5
45	38.0	70	44.0
50	38.0	75	46.5
55	38.5		

Making the measurement count

Practical tip 1: *Two lines that were never rays give you the angle.*

Neither the incident ray produced forwards nor the emergent ray produced backwards is a path light took: both are ruled lines on paper. On most settings they cross inside the outline of the prism. That is where the crossing belongs and it is not a sign that anything has gone wrong.

Practical tip 2: *Measure the refracting angle from your own outline.*

Do not take it from the label on the box or from the fact that the prism looks equilateral. It appears twice in the formula of part (v), once with the least deviation added to it and once on its own, so an error in it does not cancel. Lay the protractor on the two traced faces and read the angle between them.

Practical tip 3: *A smooth curve is not a dot-to-dot.*

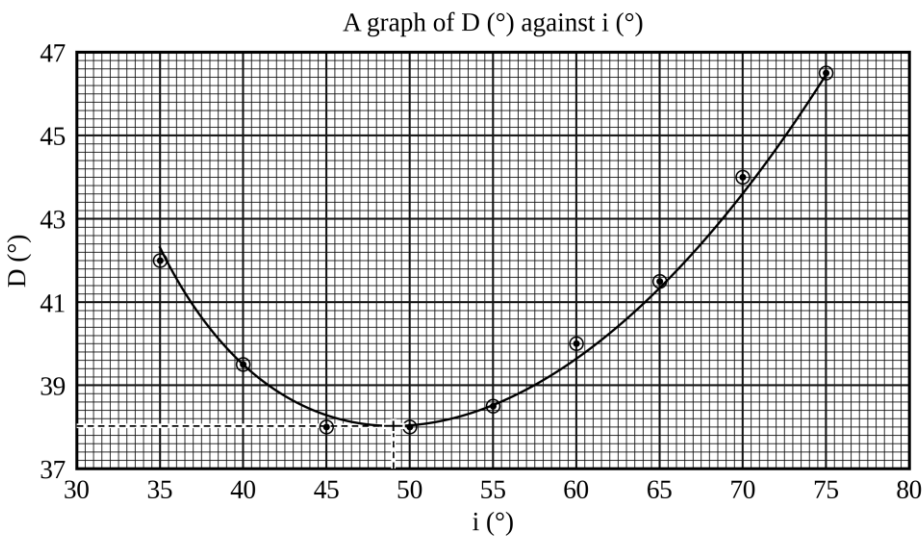
Draw one curve that runs evenly among the points and let two or three of them sit a little to one side of it. A curve bent to pass through every plotted circle has a kink at every circle, and a kinked curve cannot be read anywhere along its length. Turn the paper so that your hand pulls the pencil towards you and take the whole curve in one movement.

Practical tip 4: *What the finest reading on your protractor is worth.*

The scale is divided in degrees and the eye can place a line inside one of those divisions to about a half. That is the smallest change any angle in this table can show, and part (vii) asks you to turn it into a figure for the refractive index.

Answers to the questions

(i) and (ii). The readings are those in the table above. Take i at 2cm to 5°, running from 30° to 80°, and D at 2cm to 2°, running from 37° to 47°. That is 20cm by 10cm, which fits an A4 sheet on its side. Give the graph the title A graph of D (°) against i (°).



(iii) The lowest point of the curve lies at D = 38.0°.

(iv) The vertical through the lowest point of the curve meets the i axis at about 49°. That figure cannot be read at all closely. Over the 8° from 45° to 53° the whole fall and rise of the curve is a quarter of a degree, half of the half degree every angle in the table was read to, so a curve drawn by hand fixes the position of its lowest point only to within about 4° either way.

(v) With A = 60.0° and D_m = 38.0°,

$$n=\frac{\sin\frac{60.0°+38.0°}{2}}{\sin\frac{60.0°}{2}}=\frac{\sin 49.0°}{\sin 30.0°}=\frac{0.7547}{0.5000}=1.51$$

The refractive index has no unit: it is a ratio of two sines.

(vi) That the value read off does not depend on hitting the right angle of incidence. The quarter of a degree of part (iv) is spread over 8° of incidence, so any setting inside that stretch gives the least deviation to within the reading uncertainty. The quantity the formula needs is the deviation at the lowest point and not the incidence at which it happens, and the flatness protects the one it needs while spoiling the one it does not.

(vii) Work the formula again with the least deviation half a degree larger. At D_m = 38.5° it gives $0.7576/0.5000=1.515$ against 1.509 at 38.0°, so that half degree on the deviation is worth 0.006. Now do the same with the refracting angle. At A = 59.5° with the same deviation the formula gives $0.7518/0.4962=1.515$, so the same half degree on the prism angle is worth another 0.006. The two are independent, so they combine as the square root of the sum of their squares, which is 0.008. Quote the refractive index as 1.51 and the uncertainty as 0.01, which is 0.7%; the third figure is the uncertain one.

(viii) His reading at i = 40° is D = 39.5°, so the formula gives

$$n=\frac{\sin\frac{60.0°+39.5°}{2}}{\sin 30.0°}=\frac{0.7632}{0.5000}=1.53$$

That is 1.3% above the answer from the curve, and it is too large. It has to be too large: the least deviation is the smallest value D takes, so any single reading is at least as big as D_m, and the formula grows as the deviation grows. Apart from the 0.5° of reading uncertainty, the bias is one-sided: a single reading can only push the answer up.

Examiner's eye

- Draw the curve first and read it afterwards. The smallest number in the table need not be the least value of the curve, and it is only the same number here because two readings happen to sit at the bottom; move either of them and the two part company.
- An axis begun at 0° throws this curve away. The whole variation in D is 8.5°, and the scale in the answer spreads that across the sheet; drawn from zero it becomes a horizontal line with a wobble in it and there is no lowest point to read.
- The sine of half a sum is not the sum of two sines. The formula wants the sine of half of $A+D_m$: add the two angles, halve the total, and take the sine of what is left. Keep four figures in the sine until the division is done.
- No reading taken in this experiment is the angle of refraction inside the glass, and no mark asks for one. A table with a third column of angles that were never measured has an invented column in it.

Practice 22.2

Mr. Akilikubwa put two prisms on the bench, the one off the windowsill and one out of the new box, and asked which of them would bend a ray of light the more.

Kipanga had the answer before the question was finished. *"The bigger one,"* he said. They were the same size. He had another go. *"The new one."*

Kipute asked whether you could tell by looking through them. **Mr. Akilikubwa** said that you could, provided you were willing to say by how much. **Kipanga** changed his mind and said the old one, on the ground that it had been at it longer.

Procedure

Proceed as follows:

(a) Pin a sheet of plain paper to the board, stand the prism from the new box on it and trace its outline with a sharp pencil.

(b) Mark a point O about halfway along one of the two refracting faces and draw the normal to that face at O.

(c) Draw the line making an angle i of 40° with that normal, and fix the pins P_1 and P_2 upright on it as far apart as the sheet allows.

(d) Put the prism back inside its outline, look through the other refracting face and fix the pins S_1 and S_2 upright so that they are in line with the images of P_1 and P_2.

(e) Take out the four pins, ring the four holes and remove the prism. Join S_1 and S_2, produce the incident ray forwards and the emergent ray backwards until the two lines cross, and measure the angle D between them at the crossing. Measure the refracting angle A of the prism from your traced outline.

(f) Repeat the procedures in P22.2 (a) to (e) on a fresh sheet for each of the values of i equal to 45°, 50°, 55°, 60°, 65° and 70°.

Questions

(i) Tabulate your results.

(ii) Plot a graph of D (°) against i (°) and draw a single smooth curve through your points.

(iii) Read from your curve the least value of D.

(iv) The refracting angle of this prism was measured from its outline as 60.0°, and the refractive index of its glass is given by the formula of Experiment 22.2 (v). Determine that refractive index.

(v) Determine the critical angle for this glass.

(vi) Below a certain angle of incidence no ray leaves the second face of this prism at all. Determine that angle of incidence.

(vii) The prism of Experiment 22.2 has the same refracting angle as this one. State which of the two prisms turns a ray of light through the larger angle, and by how much at the least deviation.

(viii) A prism of refracting angle 80° is made of the same glass as this one. Show that no ray can pass through it and leave by the second face, whatever the angle of incidence.

Experiment 22.3

A notice is nailed to the post at the deep end of the school pool, and it has been repainted twice:

THE WATER AT THIS END

IS 1.4 METRES DEEP

Underneath it somebody has written in pencil, in a hand that has been rubbed out and put back more than once: *it is not.*

Mr. Akilikubwa measured that corner with a metre rule and a length of string on the day the pool was filled, and it is. What is interesting is that everybody who leans over the edge and looks down agrees with the pencil.

Kipute puts it at a good deal less than the notice says. **Kipanga** agrees with him, and then takes off a little more.

Neither of them is being careless. What each of them sees is a bottom that is not where the bottom is, and the water is what moved it.

Aim

To determine the refractive index of water by locating the image of a pin seen through it.

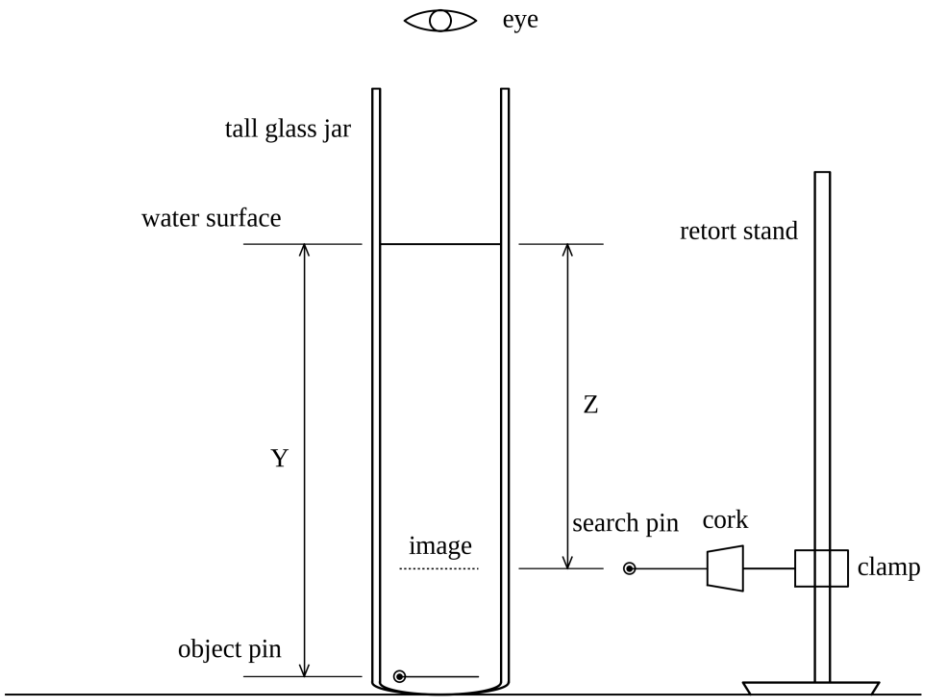
Apparatus

A tall glass jar of internal diameter 4.0cm and at least 26cm deep; a measuring cylinder; two optical pins; a cork; a retort stand with a clamp; a metre rule; water.

Setup

The object pin lies flat on the bottom of the jar and water is added to it in measured amounts. The second pin is held in a cork clamped beside the jar and is raised or lowered until it is level with the image of the pin at the

bottom, both being viewed from above. Two depths are then measured from the surface: that of the object pin and that of the search pin. The arrangement is the one drawn below.



Procedure

Proceed as follows:

(a) Stand the jar on the bench and lay the object pin flat on the bottom of it, against the wall.

(b) Pour $100cm^3$ of water in from the measuring cylinder.

(c) Clamp the search pin horizontally beside the jar and move it up and down until there is no parallax between it and the image of the object pin, both being looked at from above.

(d) Measure the depth Y of the object pin below the surface of the water and the depth Z of the search pin below that same surface.

(e) Add a further $30cm^3$ and repeat the procedures in 22.3 (c) and (d), and go on doing so until the jar has held $130cm^3$, $160cm^3$, $190cm^3$, $220cm^3$ and $250cm^3$ in turn.

Questions

(i) Tabulate your results.

(ii) Plot a graph of Y (cm) against Z (cm).

(iii) Determine the slope of your graph.

(iv) State the physical meaning of the slope.

(v) Describe the test for no parallax, and explain why it puts the search pin level with the image.

(vi) Use your graph to determine the depth of water that would make the image appear 10.0cm below the surface.

(vii) State one source of error in the measurement of Z and one precaution against it.

(viii) The same water is now poured into a jar of twice the diameter and the readings are taken again. State what difference this makes to the graph, and why.

Table of results

The volumes were measured out with a cylinder graduated in $5cm^3$ and the two depths were read to 0.1cm against a metre rule held against the outside of the jar.

V (cm^3)	Y (cm)	Z (cm)
100	7.9	5.7
130	10.4	7.7
160	12.5	9.5
190	14.9	11.1
220	17.4	13.1
250	19.9	14.9

Making the measurement count

Practical tip 1: *Both depths are measured from the same surface, and the surface moves.*

Every time water goes in, the surface rises and both depths change. Take them one after the other before you touch the jar again, and take them from the water and not from the rim.

Practical tip 2: *The volume sets the depth and then drops out.*

Measure it out in the cylinder, pour it in, and then leave it alone. The two numbers the analysis wants are read afterwards against the metre rule, so a volume poured a little short changes the depth and no more, and the depth is measured.

Practical tip 3: *Keep the object pin flat and still.*

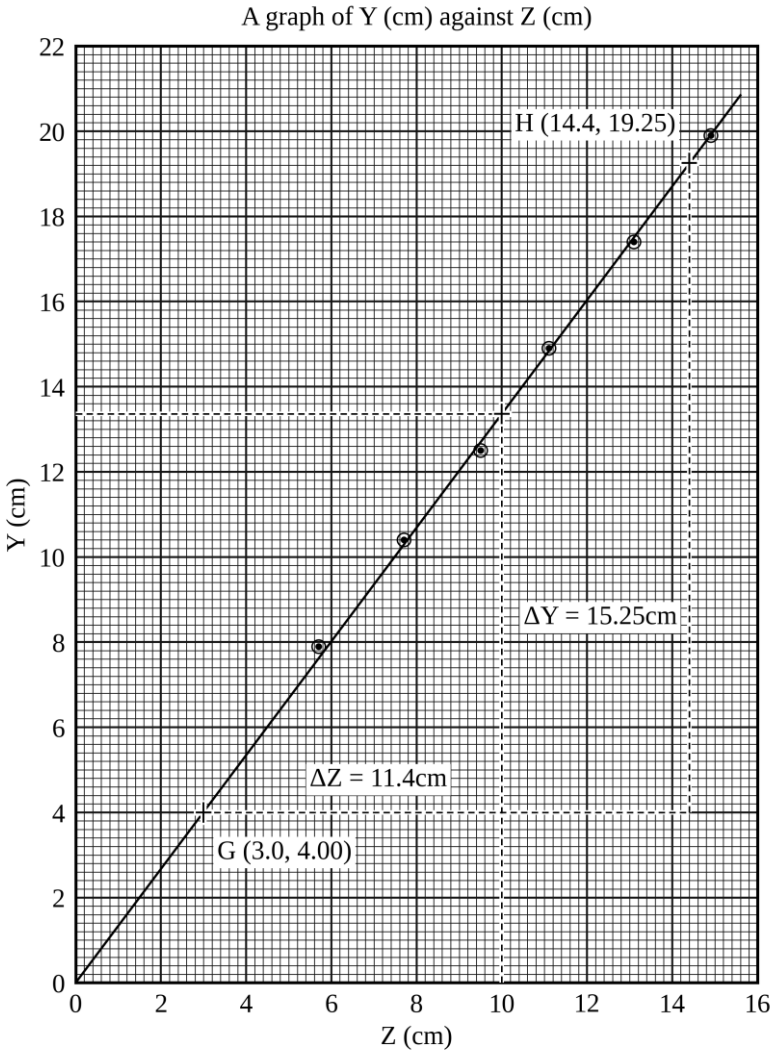
Pour the water down the wall of the jar and not on to the pin. A pin that rolls or lifts between one row and the next changes the bottom of the column, and the change shows up as a point a long way off the line with no cause you can point to.

Practical tip 4: *Both axes of this graph carry a measurement.*

Both axes carry a length read to the nearest millimetre, so the scatter you see is the two doubts together and not one of them. Part (vii) asks about one of the two only. Watch how that reading is taken while you are taking it, because a fault seen at the bench is a better answer than one remembered afterwards.

Answers to the questions

(i) and (ii). The readings are those in the table above. Take Z at 2cm to 2cm, running from 0 to 16cm, and Y at 2cm to 2cm, running from 0 to 22cm. That is 16cm by 22cm, which fits an A4 sheet upright. Give the graph the title A graph of Y (cm) against Z (cm).



(iii) The line goes through the origin. Taking G (3.0, 4.00) and H (14.4, 19.25) on the line,

$$\text{slope}=\frac{19.25\text{cm}-4.00\text{cm}}{14.4\text{cm}-3.0\text{cm}}=\frac{15.25\text{cm}}{11.4\text{cm}}=1.34$$

(iv) It is the refractive index of the water. The real depth and the apparent depth are in the ratio

$$Y=nZ$$

so a graph of one against the other has the refractive index for its slope. The accepted value for water is 1.33 and this graph gives 1.34, which is 0.01 above it. A slope taken from six points that scatter as these do, with corners read to half a millimetre over a rise of 15cm, cannot do better than 0.01 either. The two agree, and the difference needs no cause found for it.

(v) Look down at the search pin and at the image of the object pin together, and move your eye steadily from side to side. If the two are at different levels they separate as the eye moves, and the nearer of the two appears to travel against the direction the eye goes. Raise or lower the search pin until they stay together over a small sweep either side of the vertical. Two things that do not separate as the eye moves are the same distance away, and since both are being looked at from directly above, the same distance means the same level. Keep the sweep small: a view taken from well to one side raises the image further, and the relation the graph tests no longer holds.

(vi) Read up from Z = 10.0cm to the line and across to the Y axis, which gives Y = 13.4cm. The slope gives the same answer, since

$$1.34\times10.0\text{cm}=13.4\text{cm}$$

(vii) The metre rule is held against the outside of the jar while the level being read is inside it, so a line of sight that is not square to the rule puts the reading out; that is parallax in the reading of the rule, and it is a different

fault from the one the search pin was set by. The precaution is to hold the rule flat against the glass and bring your eye down until it is level with the mark before you read it.

(viii) The slope is unchanged. The refractive index belongs to the water and not to the vessel it is in. Both Y and Z are depths measured downwards from the surface, and neither of them knows how wide the jar is. What does change is the volume needed for a given depth, which goes up four times when the diameter is doubled, so the same six volumes would now cover a quarter of the range of depths and the points would be crowded into the bottom corner of the sheet. The graph would be worse without being wrong.

Examiner's eye

- A line that misses the origin here is telling you something, and it is not that the water has changed. A metre rule whose zero is set a little way in from its end shifts both depths by the same amount, which lifts the line off the origin and leaves the slope alone. An intercept means the same amount is in every reading, and the cure is to find it, not to force the line through the corner.
- Neither axis carries the volume. A graph of Y against V answers a question about the jar and not about the water.
- Write the slope as a bare number. Centimetres divided by centimetres leave nothing behind, and cm/cm written after the answer is a unit invented to fill a space.
- Check the size of the answer before you write it down. The apparent depth is the smaller of the two, so the slope of this graph is greater than 1, and a slope of 0.75 means the axes were interchanged and water made to look deeper than it is.

Practice 22.3

Water got into the chemical store during the long rains and took the labels off eleven bottles, leaving them on the floor in a heap like a hand of cards.

Mr. Akilikubwa dried the labels, read them and put them in a drawer, where they are of no use to anybody.

Four of the eleven bottles hold clear liquids, and none of the four smells of anything in particular. **Kipute** suggested tasting them, and was not serious. **Mr. Akilikubwa** said no anyway, at some length. **Kipanga** suggested weighing them, which is a good idea and needs a balance the school has not had since the beginning of term. What the school does have is a tall jar, a measuring cylinder, two pins in a clamp and a metre rule.

Procedure

Proceed as follows:

(a) Stand the jar on the bench and lay the object pin flat on the bottom of it, against the wall.

(b) Pour $100cm^3$ of the liquid in from the measuring cylinder.

(c) Clamp the search pin horizontally beside the jar and move it up and down until there is no parallax between it and the image of the object pin, both being looked at from above.

(d) Measure the depth Y of the object pin below the surface of the liquid and the depth Z of the search pin below that same surface.

(e) Add a further $30cm^3$ and repeat the procedures in P22.3 (c) and (d), and go on doing so until the jar has held $130cm^3$, $160cm^3$, $190cm^3$, $220cm^3$ and $250cm^3$ in turn.

Questions

(i) Tabulate your results, including the values of Y-Z in cm.

(ii) Plot a graph of Y-Z (cm) against Y (cm).

(iii) Determine the slope of your graph.

(iv) Show that the slope of this graph is $1-1/n$, where n is the refractive index of the liquid.

(v) Determine the refractive index of the liquid.

(vi) The four clear liquids in the store are water, methylated spirit, glycerine and carbon disulphide, whose refractive indices are 1.33, 1.36, 1.47 and 1.63 in that order. State which of them is in the jar.

(vii) The quantity plotted upwards is the difference of two measured lengths. State how that affects the reliability of your slope compared with a graph of Y against Z.

(viii) Suggest one change to the apparatus that would make the difference easier to measure, and say why it would.

Experiment 22.4

A letter came to the school from a glazier in town, and **Mr. Akilikubwa** read it out because he thought it was funny and because he wanted somebody to answer it.

> Dear Sir,
>
> I fit windows. A customer says the glass I have fitted is not glass, because when he stands at the side of the window and looks through it at his gatepost, the gatepost is not in line with itself. He says good glass does not do this. I have been fitting the same glass for eighteen years. Please advise.
>
> Yours faithfully,
>
> P.S. He will not pay.

Kipanga said that the customer was right about what he had seen and wrong about what it meant, which was the most useful thing anybody said that morning. **Kipute** wanted to know whether the glazier could be given a number to put in his reply.

He can be given one, and the eighteen years are neither here nor there. What the customer saw at the side of that window is what every sheet of glass in the man's van does, and the gatepost is evidence for the glass rather than against it.

Aim

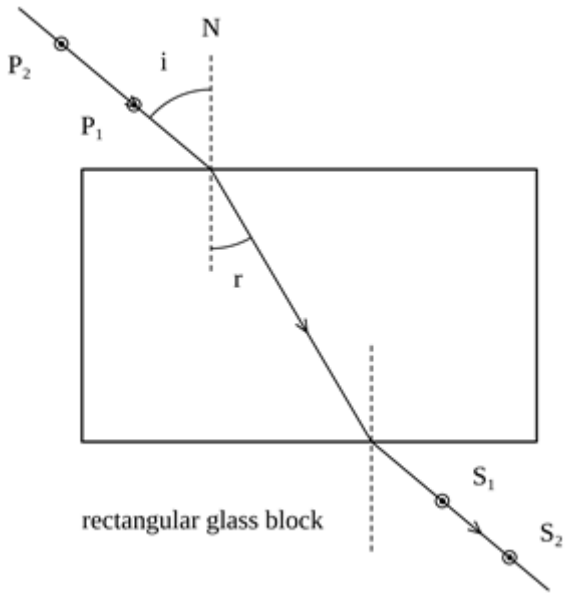
To determine the refractive index of the glass of a rectangular block by tracing rays through it.

Apparatus

A soft board; a rectangular glass block about 10cm by 6cm; four optical pins; four thumb pins; six sheets of plain paper; a ruler; a protractor; a sharp pencil.

Setup

The outline of the block is traced on the paper and a ray is sent into one of the long faces at a measured angle to the normal there. The ray that leaves the opposite long face is found with two more pins, and the point where it leaves is joined to the point where the light went in. That join is the path of the light inside the glass, and the angle it makes with the normal is the angle of refraction. The arrangement is the one drawn below.



Procedure

Proceed as follows:

(a) Pin a sheet of plain paper to the board, lay the block on it and trace its outline with a sharp pencil.

(b) Mark a point O about a third of the way along one of the long faces and draw the normal to that face at O.

(c) Draw the line making an angle i of $20°$ with that normal, and fix the pins P_1 and P_2 upright on it as far apart as the sheet allows.

(d) Put the block back inside its outline, look through the opposite long face and fix the pins S_1 and S_2 upright so that they are in line with the images of P_1 and P_2.

(e) Take out the four pins, ring the four holes and remove the block. Join S_1 and S_2 and produce the line back to the far side of the outline. Join that point to O and measure the angle r between the join and the normal at O.

(f) Repeat the procedures in 22.4 (a) to (e) on a fresh sheet for each of the values of i equal to $30°$, $40°$, $50°$, $60°$ and $70°$.

Questions

(i) Tabulate your results, including the values of sini and sinr.

(ii) Plot a graph of sini against sinr.

(iii) Determine the slope of your graph and state what it represents.

(iv) Determine the critical angle for this glass.

(v) The speed of light in air is $3.00\times10^8 m/s$. Determine the speed of light in this glass.

(vi) The two faces of the block are parallel, so the normals to them are parallel too. Use that to show that the light leaves the block at the same angle to the normal as the angle at which it entered.

(vii) The block is replaced by one of the same glass but twice as thick, and the light is sent in at the same angle. State what changes in the path of the light and what does not.

(viii) The prism of Experiment 22.2 is cut from the same glass as this block. Compare the two values obtained for the refractive index, and state whether the difference between them is larger than the two measurements can settle.

Table of results

The angles of incidence were set with the protractor and the angles of refraction were read from the protractor to the nearest $0.5°$. The sines are given to three decimal places.

i (°)	r (°)	sin i	sin r
20	13.0	0.342	0.225
30	19.5	0.500	0.334
40	24.5	0.643	0.415
50	30.0	0.766	0.500
60	35.5	0.866	0.581
70	38.0	0.940	0.616

Making the measurement count

Practical tip 1: *You never see the ray inside the block.*

No part of this experiment shows you the path through the glass, and it does not have to. The straight line from the hole where the light went in to the hole where it came out is the ray, and that is the one thing the method takes on trust. Everything in the second column of the table is measured off that one join.

Practical tip 2: *Trace the outline once and put the block back inside it.*

The pencil line is a third of a millimetre wide and the block has to go back within it, not beside it. A block replaced 1mm out moves the point where the light leaves by about the same amount, and on a join 6cm long that comes to a degree in the angle you are about to read.

Practical tip 3: *Where the emergent ray produced backwards stops.*

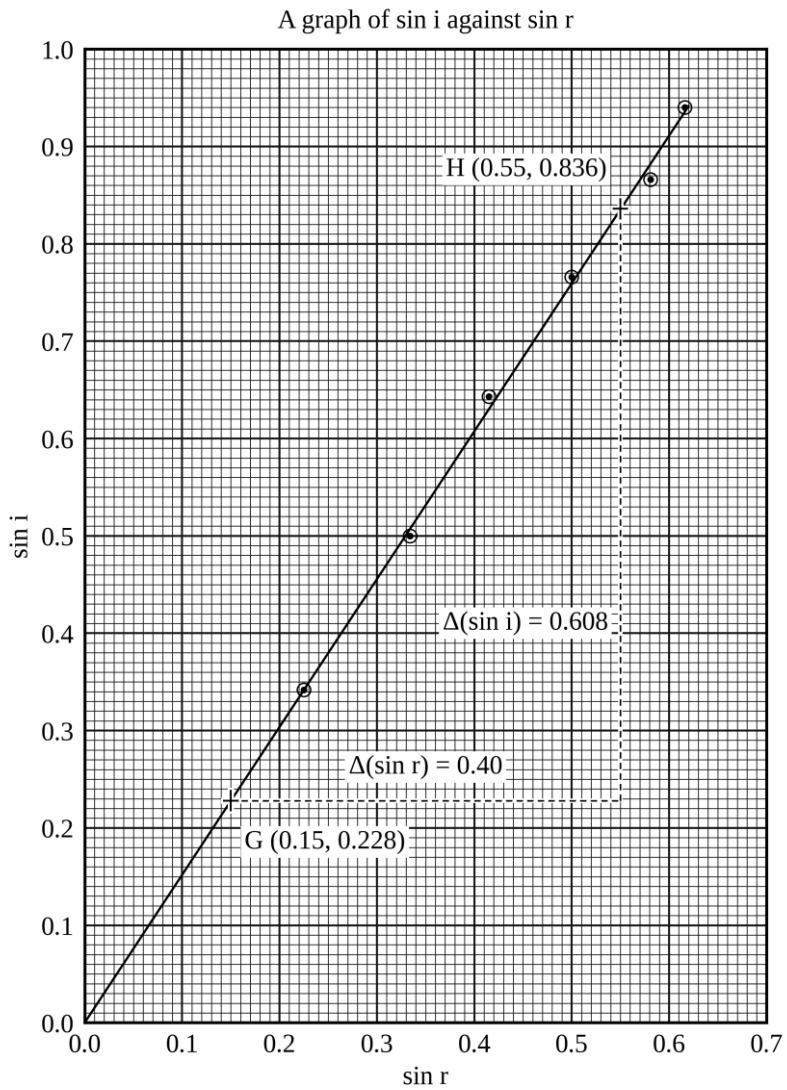
It runs parallel to the incident ray and a little to one side of it, so it does not pass through O and is not meant to. It stops where it meets the far face, at the point the light left by, and that point is joined to O afterwards. A line ruled straight from S_1 to O is a different line, and at the larger angles it is several degrees out.

Practical tip 4: *Take the sine of the angle you wrote down.*

Work from the angle in your table and not from the angle you meant to set, and carry three decimal places. A sine taken from a rounded angle and then rounded again has lost the last figure twice, and the last figure is the one the slope is built out of.

Answers to the questions

(i) and (ii). The readings are those in the table above. Take sinr at 2cm to 0.1, running from 0 to 0.7, and sini at 2cm to 0.1, running from 0 to 1.0. That is 14cm by 20cm, which fits an A4 sheet upright. Give the graph the title A graph of sini against sinr.



(iii) The line goes through the origin. Taking G (0.15, 0.228) and H (0.55, 0.836) on the line,

$$\text{slope}=\frac{0.836-0.228}{0.55-0.15}=\frac{0.608}{0.40}=1.52$$

Snell's law says that the sine of the angle of incidence and the sine of the angle of refraction are in a fixed ratio,

$$\frac{\sin i}{\sin r}=n$$

so the slope is the refractive index of the glass, and it has no unit because a sine has none.

(iv) At the critical angle the light inside the glass leaves along the surface, so

$$\sin C=\frac{1}{n}=\frac{1}{1.52}=0.6579$$

and C = 41.1°. Any light inside this glass that reaches a surface at more than that angle to the normal does not get out at all.

(v) The refractive index is the ratio of the speed in air to the speed in the glass, so

$$v=\frac{3.00\times10^{8}\,\mathrm{m/s}}{1.52}=1.97\times10^{8}\,\mathrm{m/s}$$

(vi) Inside the glass the light travels in one straight line, and the two normals are parallel, so that line makes the same angle with each of them: the angle at which it arrives at the far face is r, the angle it made with the normal at the near one. At the far face the light passes from glass into air, and it always can: $\sin r$ is $\sin i$ divided by n and so can never exceed $1/n$, which is the sine of the critical angle of part (iv). The angle of emergence e therefore obeys $\sin e=n\sin r$, and at the near face the same law gave $\sin i=n\sin r$. The right-hand sides are one number, so $\sin e=\sin i$ and the light leaves at the angle at which it entered, whatever that angle was.

(vii) The angle of refraction does not change, and neither does the direction in which the light finally travels. Snell's law fixes the angle of refraction from the angle of incidence and the two materials, and the thickness appears nowhere in it; part (vi) then gives the emergent direction from that angle alone. What changes is the path inside the glass, which doubles in length, and with it the sideways displacement of the emergent ray from the line the light would have taken had the block not been there. That displacement is what the glazier's customer saw, and a thicker pane would have made him crosser.

(viii) It is not. The prism gives 1.51 and the block gives 1.52, a difference of 0.01. Experiment 22.2 (vii) put the prism method at about 0.01, and this one is worth about the same: 0.5° in r moves $\sin r$ by about 0.008, which is 1.8% of the average plotted value of $\sin r$, and six independent readings share that out to 1.8% divided by the square root of 6, or 0.7%. On 1.52 that is 0.01 as well. The difference between the two answers is exactly one of those, so the two methods agree and neither is entitled to a third figure the other would dispute.

Examiner's eye

- Head the columns with the sines, and work them out before you plot. A sheet with the angles themselves on the axes gives a line that looks straight over a short range and is not.
- Sines run from 0 to 1, so a scale of 2cm to 0.1 fills the sheet. A scale of 2cm to 0.5 puts the work into a corner and throws away the third figure of every reading.
- Three significant figures in the answer and no more. The line is drawn through six points set by hand and read by eye, and a fourth figure after it is one the experiment never measured.
- Compare like with like in part (viii). Two values of a refractive index are compared by the difference between them set against what each method is worth, and a comparison that does not say what either is worth has not compared anything.

Practice 22.4

The frame-maker on the corner keeps two piles of clear sheet behind him and sells from whichever pile is nearer.

Mr. Akilikubwa asked which of the two he was being sold. The frame-maker said glass, and then, after a moment, said that both piles were glass and that one of them was better glass, and did not say which.

Kipanga tapped both piles with a knuckle and reported that they sounded different, which they did, and that he could not say which was which, which he could not.

Kipute said that if the two were the same to look at and the same to lift, the way to tell them apart was to measure something about them that is not the same. They went away with a piece cut off the nearer pile.

Procedure

Proceed as follows:

(a) Pin a sheet of plain paper to the board, lay the block cut from the frame-maker's sheet on it and trace its outline with a sharp pencil.

(b) Mark a point O about a third of the way along one of the long faces and draw the normal to that face at O.

(c) Draw the line making an angle i of 20° with that normal, and fix the pins P_1 and P_2 upright on it as far apart as the sheet allows.

(d) Put the block back inside its outline, look through the opposite long face and fix the pins S_1 and S_2 upright so that they are in line with the images of P_1 and P_2.

(e) Take out the four pins, ring the four holes and remove the block. Join S_1 and S_2 and produce the line back to the far side of the outline. Join that point to O and measure the angle r between the join and the normal at O.

(f) Repeat the procedures in P22.4 (a) to (e) on a fresh sheet for each of the values of i equal to 30°, 40°, 50°, 60° and 70°.

Questions

(i) Tabulate your results, including the values of sini and sinr.

(ii) Plot a graph of sini against sinr.

(iii) Determine the slope of your graph and state what it represents.

(iv) The block is made either of window glass, whose refractive index is 1.52, or of perspex, whose refractive index is 1.49. State which of the two it is, and say how far your measurement can be trusted to settle it.

(v) Suggest one change to the procedure that would put the answer beyond argument, and say why it would.

(vi) A slab of the same material 4.8cm thick is laid on a printed page and the print is viewed from directly above. Determine how far the print appears to be raised.

(vii) Use two of your own readings to show that i and r are not in a constant ratio.

(viii) Explain why your eye must be brought down level with the paper when the pins S_1 and S_2 are being set.

Chapter 23

FLUID DYNAMICS: VISCOSITY

Tip a jug of water and it is empty before you have thought about it. Tip a tin of honey and you can watch the stream leave the rim, hang, and take its time. Gravity is pulling on both of them the same way. What differs is how hard each liquid resists being made to slide over itself, and that resistance is what this chapter measures. The quantity is the **coefficient of viscosity** of the liquid, written η: the tangential force per unit area between two layers of the liquid, divided by the velocity gradient across them. It is measured in Nsm^{-2}.

There are two ways of getting at it with the apparatus a school has, and this chapter works one experiment of each kind. Either you make a body move through the liquid and watch how fast it settles down to travel, or you make the liquid move through a narrow opening and measure how much comes out. The first variety lets steel balls fall through glycerine in a tall jar. The second drives water and then a second liquid through one fine tube under one head.

Both of the relations these methods rest on are proportionalities, with no constant term in either of them, so every line drawn in this chapter passes through the origin, and the origin is a point you own before the first reading is taken. Rule through it and let the plotted points fix the direction; a set of points that plainly will not have the origin is telling you something about the apparatus rather than about the theory.

One thing has to be said before any of it. Viscosity depends on temperature far more sharply than most of the quantities this book has measured: a length does not care what the room is doing, and a resistance cares only a little, but a viscosity cares a great deal. A value of η with no temperature beside it is not a measurement. Every question here therefore records the temperature as carefully as it records a mass or a time, and every answer carries it.

The heart of it is small. A tall measuring cylinder of glycerine, five ball bearings, a micrometer screw gauge and a stopwatch carry the first variety; a wide vessel, a fine tube, a measuring cylinder and the same stopwatch carry the second, and each experiment lists what else it wants. Each question prints the constants it needs, the acceleration due to gravity as $9.81ms^{-2}$ among them.

The two varieties at a glance

	What you do	What comes out of it
23.1	Five ball bearings of different sizes are timed as they fall through glycerine between two marks on the jar	the coefficient of viscosity of the glycerine
23.2	Water and a second liquid are delivered in turn through one fine tube under one head, and the volume collected is read against the clock	the coefficient of viscosity of the second liquid, by comparison with that of water

Both are worked below in full. The practice questions put balls of an unmarked metal into the same jar and ask what the metal is, and change the fine tube for five others of different lengths.

Experiment 23.1

There is a shop at the bus stand that sells cooking oil out of a drum, and the man who owns it fills your bottle with a ladle because the tap on the drum takes too long about it. **Mr. Akilikubwa** was waiting his turn with **Kipute** and **Kipanga** when the ladle came up and the oil went back into the drum in one slow rope.

"Heavy," said **Kipanga**, with the certainty of a boy who has carried things. *"That is why it will not run. Water would have been down and gone."*

The shopkeeper took the cup of water that stood by his stool, let one drop of oil fall into it, and held it up. The drop sat on the top and stayed there.

"All these years they have been telling me my oil is heavy," he said, *"and not once have I charged anybody extra for it."*

Kipute wanted to know what did make it slow, then, if not the weight of it. **Mr. Akilikubwa** did not answer him in the queue. He answered him on Thursday, with a jar of glycerine and a handful of ball bearings.

Aim

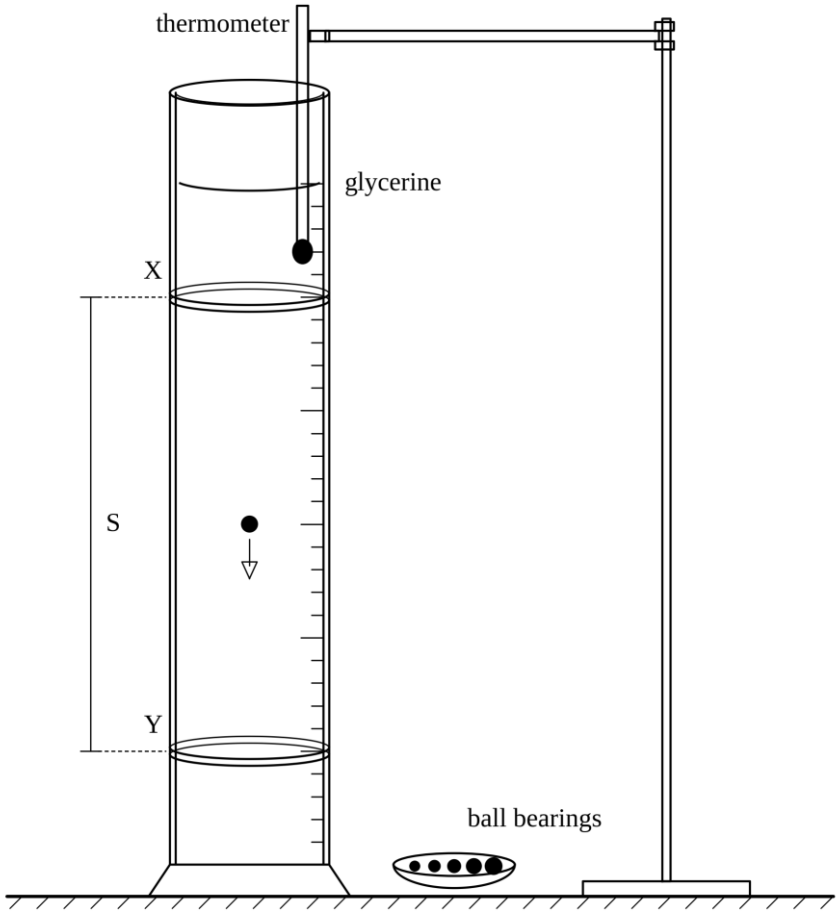
To determine the coefficient of viscosity of glycerine by timing the fall of spheres of different sizes through it.

Apparatus

A measuring cylinder of 1000cm^3, about 6.5cm across, and about 1500cm^3 of glycerine; five steel ball bearings of different sizes, none of them larger than 3.5mm across; a micrometer screw gauge; a stopwatch reading to 0.1s; a metre rule; two rubber bands; a pair of forceps; a small dish; a bar magnet; a thermometer reading to 0.5K; a retort stand with a clamp; a balance; a 100cm^3 measuring cylinder and a small beaker.

Setup

The arrangement is the one drawn below. The measuring cylinder, which the rest of the chapter calls the jar, stands on the bench and is filled with glycerine to the 1000cm^3 mark. One rubber band is slipped round it at X, 5.0cm below the surface of the glycerine, and a second at Y, 20.0cm below X, and the distance S between them is measured with the metre rule. The thermometer is clamped with its bulb in the glycerine above the upper band and close to the wall, clear of the path the balls take. The glycerine has been standing in the room and is at $21.0°\text{C}$.



Procedure

Proceed as follows:

(a) Find the mass of the dry beaker. Pour 100cm^3 of glycerine into it from the small measuring cylinder, find the mass again, and record both.

(b) Measure the diameter of each ball twice with the micrometer screw gauge, the second time at right angles to the first, and record both readings. Close the gauge fully and record the zero it shows.

(c) Wet all five balls by leaving them for a minute in the dish of glycerine.

(d) Hold the smallest ball in the forceps at the surface of the glycerine, at the middle of the cylinder, and let it go without pushing it. Start the stopwatch as the ball passes X and stop it as the ball passes Y, reading each band at eye level.

(e) Recover the ball with the bar magnet, wait until the glycerine is still, and repeat step (d) with the same ball.

(f) Repeat steps (d) and (e) with the other four balls in order of size.

(g) Record the temperature of the glycerine.

(h) Record all your readings in a suitable table.

Questions

(i) Tabulate your readings, including columns for the mean diameter of each ball, its radius r in centimetres, r^2 in cm^2, the mean time and the terminal velocity v.

(ii) Plot a graph of r^2 against v.

(iii) Determine the slope of your graph and state its unit.

(iv) Calculate the density σ of the glycerine.

(v) The terminal velocity of a sphere of radius r falling through a liquid is given by

$$9\eta v=2gr^2(\rho-\sigma)$$

for a sphere falling slowly through a liquid of unlimited extent, in which ρ is the density of the material of the sphere. Taking ρ as $7.80\times10^3 kgm^{-3}$ and g as $9.81ms^{-2}$, determine the coefficient of viscosity η of the glycerine.

(vi) State the physical meaning of the slope of your graph.

(vii) Use your graph to find the terminal velocity of a ball 2.60mm in diameter, and hence the time it would take to fall from X to Y.

(viii) Give one reason why the value found in (v) is likely to be an over-estimate, and suggest one change to the apparatus that would reduce that error.

Table of results

The dry beaker had a mass of 96.5g, and with $100cm^3$ of glycerine in it 222.5g. The glycerine held 21.0°C from the first ball to the last, the distance S from X to Y was 20.0cm, and the micrometer showed no zero error. The third column is the mean of the two diameters and the eighth the mean of the two times.

d_1 (mm)	d_2 (mm)	d (mm)	r (cm)	r^2 (cm^2)	t_1 (s)	t_2 (s)	t (s)	v (cms^{-1})
1.62	1.62	1.620	0.0810	0.00656	28.1	27.7	27.90	0.717
2.00	2.02	2.010	0.1005	0.01010	18.2	18.4	18.30	1.09
2.38	2.38	2.380	0.1190	0.01416	13.0	13.1	13.05	1.53
2.79	2.79	2.790	0.1395	0.01946	9.5	9.2	9.35	2.14
3.22	3.20	3.210	0.1605	0.02576	6.8	7.3	7.05	2.84

Making the measurement count

Practical tip 1: *Read the micrometer twice, and turn the ball between the readings.* A ball bearing is round to 0.001mm, so two readings that differ by more than a division or two of your gauge are telling you about the gauge and not about the ball. Close the anvils on nothing before you begin and record what the gauge then reads, because that zero comes off every diameter you take afterwards.

Practical tip 2: *Release the ball at the surface, and leave the upper band well below it.* A ball let go at the surface starts from rest and gathers speed until the viscous force has grown to match what is left of its weight after the upthrust; in glycerine that takes a fraction of a millimetre and is over before you have found the stopwatch. What the centimetres between the surface and X are really for is the disturbance. A ball put in at the surface sets the glycerine near the top moving, and the column is slower to settle than the ball is to reach its steady speed. Time it lower down, where the liquid ahead of it is still.

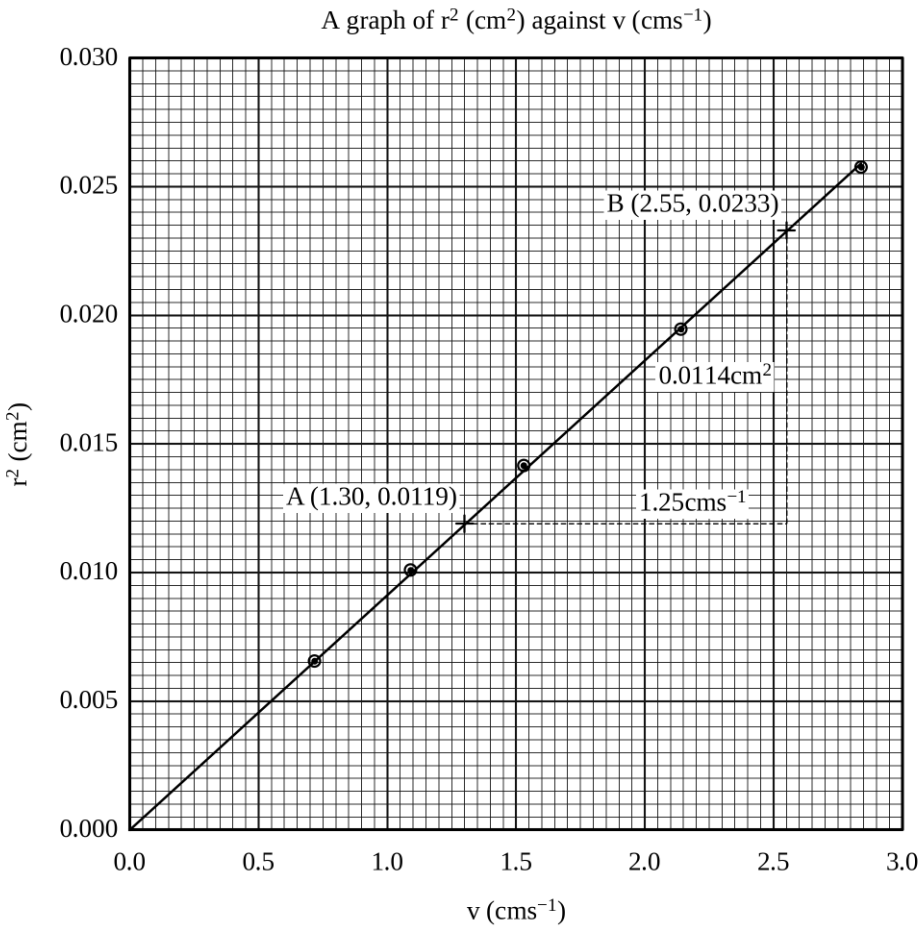
Practical tip 3: *A band on a glass jar looks different from above and from below.* It is a ring, and from above it hides the ball while from below it shows the ball early. A centimetre out at a band is a twentieth of the distance and so a twentieth of the time, whichever ball it is: 1.4s on the slowest and 0.4s on the fastest. Read both bands

at eye level, and time every ball twice: the two runs with one ball measure your own hands, and the fastest ball is the one whose two times will disagree.

Practical tip 4: *The jar must have been standing in the room, and the temperature belongs in the table.* Glycerine carried in from a cold store is a different liquid from the same glycerine an hour later. Near room temperature its viscosity falls by about 8% for every kelvin the temperature rises, which is 20 times as fast as the resistance of a copper coil changes. Leave the jar until the thermometer in it has stopped moving, and write the temperature beside the answer: a thermometer reading to 0.5K is itself worth 2% of the viscosity, and no part of this experiment is more easily got right.

Answers to the questions

(i) and (ii). Take v along the horizontal axis at 2cm to 0.5cms^{-1}, running from 0 to 3.0cms^{-1}, and r^2 up the vertical axis at 2cm to 0.005cm^2, running from 0 to 0.030cm^2. That is 12cm by 12cm, which fits an A4 sheet upright. Give the graph the title A graph of r^2 (cm^2) against v (cms^{-1}).



(iii) The line is ruled through the origin and through the five points, with the scatter falling on both sides of it. Taking A (1.30, 0.0119) and B (2.55, 0.0233) on the line,

$$\text{slope}=\frac{0.0233\text{cm}^2-0.0119\text{cm}^2}{2.55\text{cms}^{-1}-1.30\text{cms}^{-1}}=\frac{0.0114\text{cm}^2}{1.25\text{cms}^{-1}}=0.00912\text{cms}$$

The unit is the centimetre second, which is what you get when cm^2 is divided by cms^{-1}. It is not a velocity, however much it looks like one. Accept 0.00890cms to 0.00935cms.

(iv) The glycerine in the beaker had a volume of 100cm^3, so

$$\sigma=\frac{222.5\text{g}-96.5\text{g}}{100\text{cm}^3}=\frac{126.0\text{g}}{100\text{cm}^3}=1.26\text{gcm}^{-3}$$

which is $1.26\times10^3\text{kgm}^{-3}$.

(v) The relation rearranges to

$$r^2=\frac{9\eta}{2g(\rho-\sigma)}v$$

so the slope of the line is the whole of the factor that multiplies v. The slope carries the centimetre second and 1cm is 10^{-2}m, so 0.00912cms is 9.12×10^{-5}ms, which is a metre second and not a millisecond. The densities differ by

$$\rho-\sigma=7.80\times10^3\text{kgm}^{-3}-1.26\times10^3\text{kgm}^{-3}=6.54\times10^3\text{kgm}^{-3}$$

and therefore

$$\eta=\frac{2\times9.81\text{ms}^{-2}\times6.54\times10^3\text{kgm}^{-3}\times9.12\times10^{-5}\text{ms}}{9}=1.30\text{Nsm}^{-2}$$

The glycerine has a coefficient of viscosity of 1.30Nsm^{-2} at 21.0°C. Accept 1.27Nsm^{-2} to 1.33Nsm^{-2}.

(vi) The slope stands for the group of quantities that multiplies v in the relation, and every one of them except the viscosity is fixed for this jar and this box of balls: the acceleration due to gravity does not change, the glycerine keeps its density and the steel keeps its. So the slope is a measure of the viscosity alone, and a thicker liquid of about the same density in the same jar would give a steeper line.

(vii) A ball 2.60mm in diameter has a radius of 0.130cm, so r^2 is 0.0169cm^2. Read across from that value on the vertical axis to the line, then down to the horizontal axis: v is 1.85cms^{-1}, and the time to fall from X to Y is

$$t=\frac{20.0\text{cm}}{1.85\text{cms}^{-1}}=10.8\text{s}$$

Accept 10.5s to 11.1s.

(viii) The walls of the jar hold the ball back. Glycerine pushed aside by the ball has to return past it, and in a jar a few centimetres wide that return is squeezed between the ball and the glass, so every ball falls a little more slowly than it would in a liquid without walls. Every velocity in the table is therefore too low, every point lies to the left of where it belongs, the line stands steeper than it should, and the viscosity that comes out of the slope is too large. The retardation is roughly twice the ratio of the radius of the ball to the radius of the jar, which for the largest ball in a jar 6.5cm across is of the order of 10% and for the smallest about half of that. Because it grows with the ball, it does not shift the line bodily but tilts it, and the largest ball is the reading least entitled to its place. Use a wider jar: one twice as wide halves the error, though it wants four times the glycerine to fill it.

Examiner's eye

- Both quantities plotted are measured, so a point off the line may be the micrometer's fault and not the stopwatch's. The two are not the same size on the paper: one division of the micrometer moves r^2 by less than a millimetre, while a tenth of a second on the fastest ball moves v by nearly two.
- A dry ball carries a film of air down with it, and a bubble clinging to a ball 2mm across takes a real slice off the weight that is driving it while the arithmetic sees no change at all. That is why the Procedure leaves the balls in the dish for a minute before any of them is used, and why the minute is not wasted.
- Give the glycerine longer after a recovery than after a fall. The magnet brings the ball up along the glass, against the drift the ball itself left behind, and the two take a while to cancel. A ball released into liquid that has not finished moving gives a worthless time.
- Plot the graph the question names. Either way about leads to the viscosity in the end, but the two slopes are different numbers with different units, and the arithmetic that follows them is not the same arithmetic. A sheet that answers a question the paper did not ask earns nothing for the plot.

Practice 23.1

The school's water pump was stripped in the workshop over the holidays, and when it went back together there was a tin left over with four dozen small metal balls in it. The fundi said they were steel, on the grounds that everything inside a pump is steel, and **Kipute** agreed with him.

Kipanga poured the lot from one hand to the other and said they did not feel like steel, and that a handful of steel that size ought to pull harder than that. The fundi told him that a boy who can weigh with his hand can weigh the school's maize as well, and would he like the job.

Mr. Akilikubwa took five of them, one of each size he could find in the tin, and told the fundi he would have an answer for him by Friday.

Procedure

Proceed as follows:

(a) Measure the diameter of each ball twice with the micrometer screw gauge, the second time at right angles to the first, and record both readings. Close the gauge fully and record the zero it shows.

(b) Wet all five balls by leaving them for a minute in the dish of glycerine.

(c) Hold the smallest ball in the forceps at the surface of the glycerine, at the middle of the cylinder, and let it go without pushing it. Start the stopwatch as the ball passes X and stop it as the ball passes Y, reading each band at eye level. The bands are 20.0cm apart and the upper one is well below the surface.

(d) Recover the ball with a length of stiff wire bent into a loop at the end, wait until the glycerine is still, and repeat step (c) with the same ball.

(e) Repeat steps (c) and (d) with the other four balls in order of size.

(f) Record the temperature of the glycerine.

(g) Record all your readings in a suitable table.

Questions

(i) Tabulate your readings, including columns for the mean diameter of each ball, its radius r in centimetres, r^2 in cm^2, the mean time and the terminal velocity v.

(ii) Plot a graph of v against r^2.

(iii) Determine the slope of your graph and state its unit.

(iv) The terminal velocity of a sphere of radius r falling through a liquid is given by

$$9\eta v=2gr^2(\rho-\sigma)$$

for a sphere falling slowly through a liquid of unlimited extent. The glycerine in the jar has a coefficient of viscosity η of 1.30Nsm^{-2} and a density σ of $1.26\times10^3\text{kgm}^{-3}$ at the temperature of the room, and g is 9.81ms^{-2}. Determine the density ρ of the metal of the balls.

(v) The metals kept in the workshop are aluminium of density $2.70\times10^3\text{kgm}^{-3}$, zinc of $7.14\times10^3\text{kgm}^{-3}$ and brass of $8.50\times10^3\text{kgm}^{-3}$. State which of them the balls are made of, and say how safely your result decides it.

(vi) State the physical meaning of the slope of your graph.

(vii) Suppose the value of η you were given were 5% too large. State which way your answer to (iv) would move, and by how much.

(viii) Suggest a check on your answer that uses the balance and readings you have already taken, and state what the balance would have to be able to read.

Experiment 23.2

There is a noticeboard in the school workshop, and in the middle of it, where the timetable ought to have been, somebody had pinned a letter. It was from the driver of the school lorry to the head teacher, and it had been there long enough to go brown at the corners. **Mr. Akilikubwa** read it out while the class waited for the bell.

> Dear Sir,
>
> I am returning the four tins of oil that came last month. This is not the oil we have always had. It goes into the engine like water and it comes out of the engine like water, and an engine is not a well.
>
> Yours faithfully,
>
> Mzee Salum, driver

P.S. The tins are the right colour. It is what is inside them that is wrong.

Kipute wanted to know how the driver could be so certain, when one tin of oil looks very like another. **Kipanga** wanted to know what the head teacher was supposed to do about it, since neither of them could put a number on *like water*. **Mr. Akilikubwa** said that was the interesting part, took the letter down, and put it at the front of his file.

Aim

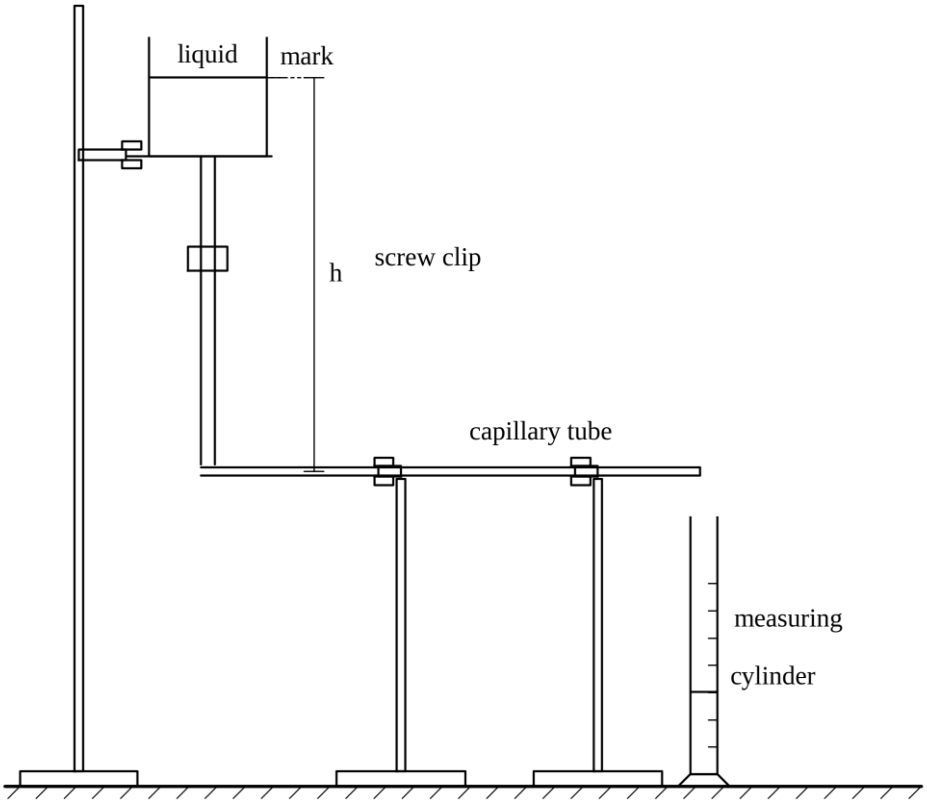
To compare the coefficient of viscosity of a liquid with that of water by the rate at which each flows through a fine tube.

Apparatus

A wide vessel about 12cm across, with a mark scratched on its side; a capillary tube 50.0cm long; a short length of rubber tubing with a screw clip; three retort stands with clamps; a $100cm^3$ measuring cylinder; a stopwatch reading to 0.1s; a metre rule; a thermometer reading to 0.5K; a balance; a small beaker; water, and about $1500cm^3$ of a liquid labelled Z.

Setup

The arrangement is the one drawn below. The wide vessel is carried on a ring clamped to a retort stand well above the bench, and is filled to the mark. The rubber tubing takes the liquid from the bottom of it, through the screw clip, to the capillary tube, which is clamped horizontal in two places. The head h from the mark to the axis of the capillary tube is 40.0cm and is measured with the metre rule. The measuring cylinder stands under the far end of the tube. The room is at 21.0°C.



Procedure

Proceed as follows:

(a) Find the mass of the dry beaker. Pour $100cm^3$ of liquid Z into it from the measuring cylinder, find the mass again, and record both.

(b) Fill the vessel with water to the mark and open the screw clip until water leaves the far end of the capillary tube steadily.

(c) Close the clip. Empty and dry the measuring cylinder, stand it under the end of the tube, and top the vessel up to the mark.

(d) Open the clip fully and start the stopwatch at the same instant.

(e) Read the volume in the measuring cylinder every 60s for 6min, reading the bottom of the meniscus with the eye level with it.

(f) Close the clip. Empty the vessel, the tubing and the cylinder, rinse all three with liquid Z, and repeat steps (b) to (e) with liquid Z in place of the water.

(g) Record the temperature of the room.

(h) Record all your readings in a suitable table.

Questions

(i) Tabulate your readings for the two runs.

(ii) On the same axes, plot a graph of the volume V collected against the time t for both liquids, labelling each line with the liquid it belongs to.

(iii) Determine the slope of each line.

(iv) Calculate the density of liquid Z.

(v) The volume of liquid delivered in unit time by a tube of radius r and length L under a pressure difference P is

$$Q=\frac{\pi P r^4}{8\eta L}$$

for steady flow in an unbroken laminar stream. Taking the pressure difference across the tube as ρgh, show that for one tube under one head the coefficient of viscosity of a liquid is proportional to its density divided by the volume it delivers in unit time.

(vi) The coefficient of viscosity of water at the temperature of the room is $9.8\times10^{-4}\text{Nsm}^{-2}$. Determine the coefficient of viscosity of liquid Z.

(vii) State why neither the radius nor the length of the tube enters your answer.

(viii) The two runs were made at the same temperature. Had the room warmed by 2K between them, state what your answer to (vi) would then be the viscosity of, and estimate the error in quoting it at 21.0°C, taking the viscosity of a liquid of this kind to fall by 2% for each kelvin.

Table of results

The dry beaker had a mass of 118.0g, and with 100cm^3 of liquid Z in it 197.0g. The thermometer read 21.0°C for both runs, and the head h was 40.0cm at the start of each.

t (s)	V, water (cm^3)	V, liquid Z (cm^3)
60	10.5	7.5
120	22.5	14.5
180	33.5	21.5
240	45.0	29.0
300	56.0	36.5
360	66.5	42.5

Making the measurement count

Practical tip 1: *The head must not move while the liquid runs, and that is why the vessel is wide.* The rate of flow follows the head exactly, so a head that falls during the run gives a line that bends. Take 70cm^3 out of a vessel 12cm across and the level drops rather more than 0.5cm, which is about 1.5% of a head of 40.0cm and cannot be seen on the paper. The same volume out of a bottle 4cm across drops the level by nearly 6cm.

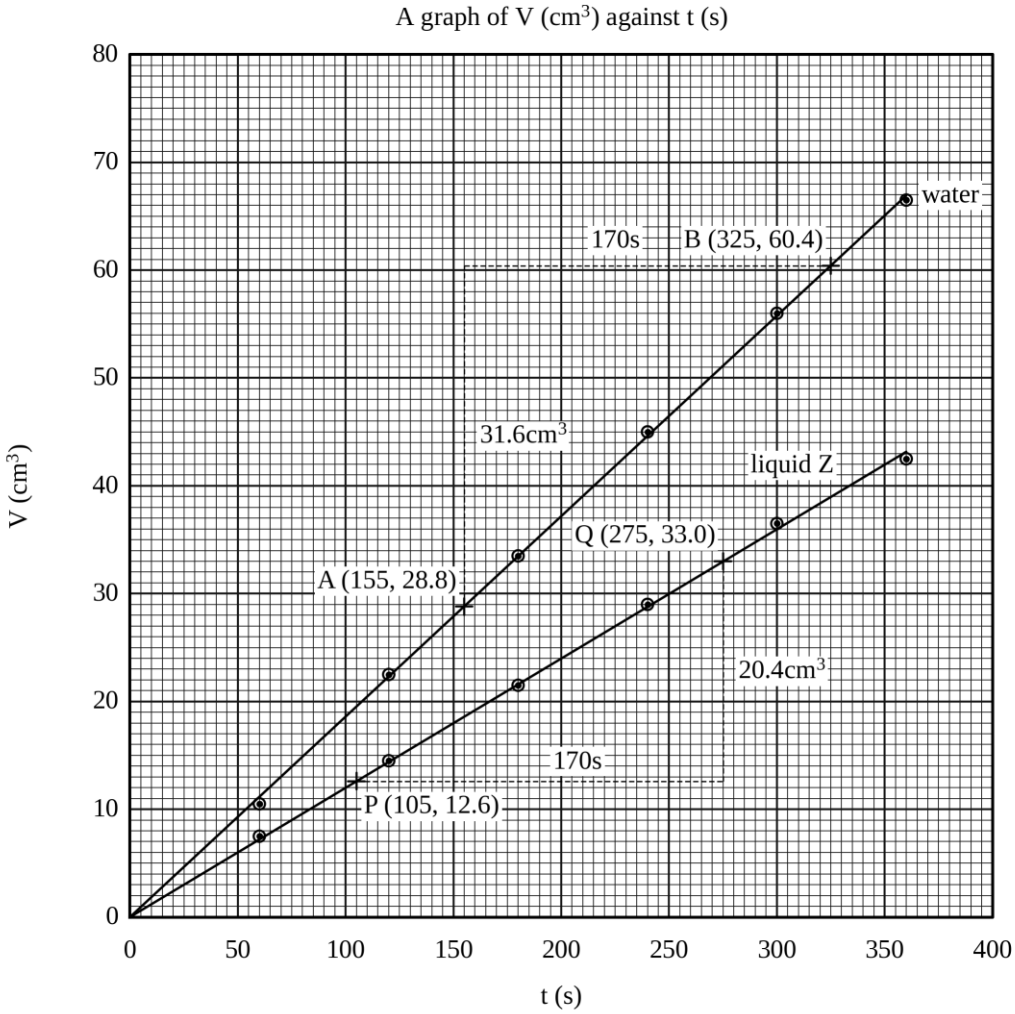
Practical tip 2: *Prime the tube before the clock starts.* Open the clip and let the liquid run until what leaves the far end is leaving steadily, then close it, empty the cylinder and begin. A tube that is still filling while you time it delivers nothing for the first few seconds, and the line you draw will then miss the origin through no fault of the liquid.

Practical tip 3: *Read the volume at the stroke of the minute, not the minute at a chosen volume.* A measuring cylinder graduated in whole cubic centimetres can be read to half of one, and a stopwatch can be read far more closely than that. Fix the times, let the volumes fall where they will, and keep the eye level with the bottom of the meniscus at every reading.

Practical tip 4: *Length and slowness are both deliberate here.* The relation printed in part (v) counts only the pressure spent against viscosity along the inside of the tube. A little is always spent setting the liquid moving as it enters. That amount grows as the square of the rate of flow, so the share it takes of the head grows in step with the rate itself, and a tube 50.0cm long run slowly holds it under about 2%. A tube 5cm long could not run ten times as fast: long before that the entrance would be asking more of the head than the head has to give.

Answers to the questions

(i) and (ii). Take t along the horizontal axis at 2cm to 50s, running from 0 to 400s, and V up the vertical axis at 2cm to $10cm^3$, running from 0 to $80cm^3$. That is 16cm by 16cm, which fits an A4 sheet upright. Give the graph the title A graph of V (cm^3) against t (s).



(iii) Both lines are ruled through the origin. The water line passes through A (155, 28.8) and B (325, 60.4), so

$$\text{slope}=\frac{60.4cm^3-28.8cm^3}{325s-155s}=\frac{31.6cm^3}{170s}=0.186cm^3s^{-1}$$

and the line for liquid Z through P (105, 12.6) and Q (275, 33.0), so

$$\text{slope}=\frac{33.0cm^3-12.6cm^3}{275s-105s}=\frac{20.4cm^3}{170s}=0.120cm^3s^{-1}$$

Accept $0.182\text{cm}^3\text{s}^{-1}$ to $0.190\text{cm}^3\text{s}^{-1}$ for the water and $0.118\text{cm}^3\text{s}^{-1}$ to $0.122\text{cm}^3\text{s}^{-1}$ for liquid Z.

(iv) The liquid in the beaker had a volume of 100cm^3, so

$$\rho_Z=\frac{197.0\text{g}-118.0\text{g}}{100\text{cm}^3}=\frac{79.0\text{g}}{100\text{cm}^3}=0.790\text{gcm}^{-3}$$

which is 790kgm^{-3}.

(v) The pressure difference across the tube is the head of liquid standing above it, so that

$$Q=\frac{\pi\rho ghr^4}{8\eta L}$$

Rearranged for the viscosity, this is

$$\eta=\frac{\pi ghr^4}{8L}\times\frac{\rho}{Q}$$

The first factor holds the radius and the length of the tube, the head and the acceleration due to gravity, and every one of them is the same for the two runs because the same tube was used at the same head. So within one pair of runs the viscosity is proportional to the second factor alone, the density of the liquid divided by the volume it delivers in unit time.

(vi) The volume delivered in unit time is the slope of each line, so

$$\eta_Z=9.8\times10^{-4}\text{Nsm}^{-2}\times\frac{790\text{kgm}^{-3}}{1000\text{kgm}^{-3}}\times\frac{0.186\text{cm}^3\text{s}^{-1}}{0.120\text{cm}^3\text{s}^{-1}}=1.20\times10^{-3}\text{Nsm}^{-2}$$

Liquid Z has a coefficient of viscosity of $1.20\times10^{-3}\text{Nsm}^{-2}$ at 21.0°C, about a quarter as much again as water, although it flows through the tube only two thirds as fast, because it is the lighter liquid and has less weight behind it. Accept $1.15\times10^{-3}\text{Nsm}^{-2}$ to $1.25\times10^{-3}\text{Nsm}^{-2}$.

(vii) They sit in the factor that is common to the two runs, and a common factor cancels when one run is divided by the other. That is the whole point of making the comparison: a tube whose bore nobody knows will do. The two would have to be known only if a value were wanted from one run standing by itself.

(viii) It would be the viscosity of liquid Z at 23.0°C, because that is the temperature liquid Z was at when its own run was made, while the figure quoted for the water belongs to 21.0°C. For a thin liquid of this kind the viscosity falls by roughly 2% for each kelvin, so the value found would sit about 4% below the value at 21.0°C. That is four times the scatter of the readings, and on its own it fills the whole of the tolerance this answer is marked to, so the temperature written beside a viscosity is not a decoration.

Examiner's eye

- A single reading will very nearly do, and very nearly is the trap. The last volume divided by the last time gives $0.185\text{cm}^3\text{s}^{-1}$ for the water, which is within 1% of the slope and worth no marks at all: the question asked for a line through six points, and it is the line that averages the six.
- Both runs must go through the same tube, and not merely through two tubes of the same nominal bore. The rate of flow follows the fourth power of the radius, so a bore 2% wider passes 8% more liquid, and a comparison made with two tubes would be measuring the tubes rather than the liquids.
- The liquid left in the tubing between the runs wrecks more of these experiments than any other single thing. Water still in the tube when liquid Z goes in makes the first minute of the second run a mixture, and a mixture has a viscosity of its own. Rinse everything, run the tube clear before you time anything, and begin.
- Watch the rate and not only the clock. Liquid leaving a fine tube this slowly leaves in drops, and drops are nothing to worry about; a change in how fast they come is. If the flow slackens part way through a run, an air lock has formed or the level has fallen below the mark, and every reading after that is from a different experiment. Stop where it slackened, refill, and treat what follows as a fresh run.

Practice 23.2

The tank behind the dining hall feeds two taps, one at the corner of the hall and one out by the gate, and the pipe to the gate is the longer of the two by a good many metres. At the corner tap a bucket fills while you are still talking to whoever is next in the queue. At the gate you can put the bucket down and go away.

The boys waiting at the gate had settled the matter among themselves: the tank was nearly empty, and what water there was went to the near tap first. **Kipanga** thought that sounded likely enough. **Kipute** pointed out that both taps hang from the same tank at the same height, and that the only thing the gate tap has more of is pipe.

Mr. Akilikubwa came past with a bucket of his own and listened to the end of it. The rest of the queue would not have it that pipe alone could be the whole of the difference, so the next lesson began with five lengths of capillary tubing cut to different sizes.

Procedure

The vessel, the rubber tubing and the screw clip are those of Experiment 23.2, and the head h is the same 40.0cm. Five capillary tubes are cut from one length of tubing, so that all five have the same bore. Proceed as follows:

(a) Measure the length of each tube with the metre rule and record it.

(b) Clamp one of the tubes horizontal in place of the one used before, with its far end over the measuring cylinder, and fill the vessel with water to the mark.

(c) Open the screw clip until water leaves the far end of the capillary tube steadily, then close it, empty and dry the measuring cylinder, and top the vessel up to the mark.

(d) Open the clip fully and start the stopwatch at the same instant.

(e) Stop the stopwatch as the water reaches the $50cm^3$ graduation of the measuring cylinder, reading the bottom of the meniscus with the eye level with it, and record the time.

(f) Repeat steps (b) to (e) with each of the other four tubes, taking them in no particular order of length.

(g) Record the temperature of the room.

(h) Record all your readings in a suitable table.

Questions

(i) Tabulate your readings.

(ii) Plot a graph of t against L.

(iii) Determine the slope of your graph and state its unit.

(iv) The volume of liquid delivered in unit time by a tube of radius r and length L under a pressure difference P is

$$Q=\frac{\pi Pr^4}{8\eta L}$$

for steady flow in an unbroken laminar stream. Taking the pressure difference as ρgh, show that the time to collect a fixed volume V is proportional to L, and write down what the slope of your graph stands for.

(v) The head h was 40.0cm, the volume collected $50cm^3$, the density of water $1000kgm^{-3}$ and its coefficient of viscosity $9.8\times10^{-4}Nsm^{-2}$ at the temperature of the room, and g is $9.81ms^{-2}$. Determine the radius of the bore of the tubing.

(vi) Use the slope of your graph to find the time the same volume of water would take through a tube of the same bore 100cm long.

(vii) Another student's slope is 1% larger than yours. By what percentage would the radius found from it differ from yours?

(viii) State what further measurement would be needed if these readings were to give the coefficient of viscosity of the water instead, and suggest how it could be made.

Chapter 24

SURFACE TENSION

Most of the quantities this book has measured can be pointed at. A period is a time you can count; an extension is a length you can watch grow. The quantity measured in this chapter cannot be pointed at, and that is the whole difficulty of it. Surface tension lies in the surface of a liquid itself. Draw any line in that surface, real or imagined, and the liquid on one side of the line pulls the liquid on the other side towards it, along the surface and at right angles to the line. The size of that pull is γ multiplied by the length of the line, and γ is what this chapter determines. Its unit is therefore the newton per metre, Nm^{-1}, and not the newton.

Two everyday things follow from that pull, and the chapter measures γ from each of them. A liquid running very slowly out of a tube does not run: it gathers into a drop, hangs, grows, and falls only when it has become too heavy to be held. What holds it is the pull of the surface round the neck where the drop is still joined to the tube, so the size of the drop that finally falls is a measure of γ. And a liquid standing in a very narrow tube does not stand level with the liquid outside it: it climbs, and the narrower the tube the higher it climbs. What lifts it is the same pull, acting now round the circle where the surface meets the wall of the tube, so the height it reaches is a measure of γ as well.

One sentence covers both. A length of contact multiplied by γ is a force, $F=\gamma L$ with L the length of the line along which the surface is pulling, and in each of these experiments that force is holding up a weight: the weight of the drop in the first, the weight of the raised column in the second. Read the sentence forwards and it tells you what a surface can carry. Read it backwards and a weight you have measured becomes a value of γ. Both experiments here are that sentence read backwards, and the whole of the work is in measuring the weight and in knowing the length of the line it hangs from.

The apparatus is as plain as anything in the book. A glass funnel, a rubber tube with a clamp on it, a glass jet, a measuring cylinder and a balance do the first; six capillary tubes of known bore, a trough and a millimetre scale do the second. Water is the liquid in both worked experiments, and a cooking oil appears in both practice questions, so that each method is tried twice on liquids that behave quite differently. Where a question needs the acceleration of free fall it supplies $9.81ms^{-2}$, and where it needs π it supplies 3.14. A density is supplied where it is wanted and not measured, and Experiment 24.1 shows how to weigh one where it is.

The two varieties at a glance

	What you do	What comes out of it
24.1	Drops of the liquid are counted into a measuring cylinder, and the volume collected is plotted against the number of drops	the surface tension, from the graph and two weighings
24.2	The liquid climbs of its own accord in tubes of different bore, and the rise is plotted against the reciprocal of the bore	the surface tension, from the slope of the line

Both are worked below in full. The practice questions put the same two methods on a cooking oil: the first sets its drops beside the drops of water from one and the same jet, and the second stands it in the same six tubes and asks what bore a tube would need if it were to do a wick's work.

Experiment 24.1

Mr. Akilikubwa had been asking for a fortnight to have the tap by the kitchen door mended, and had got nowhere, so on Tuesday he took the class out to it instead and told them to find out what it was costing. **Kipanga** was ready before anybody had a watch out.

"It drips about once a second," said **Kipanga**. *"That is a bucketful in a week. Perhaps two."*

Kipute said nothing, and looked at the bucket under the tap, which had been standing there since the term began and was not full.

The caretaker, who had been listening from the doorway, walked over, gave the tap half a turn, and stopped it. Everybody looked at the tap. **Mr. Akilikubwa** thanked him gravely, told the class that they had just measured

a leak of nothing at all with excellent precision, and said that on Friday they would begin again with the question nobody in the yard had thought to ask: how much liquid is there in one drop?

Aim

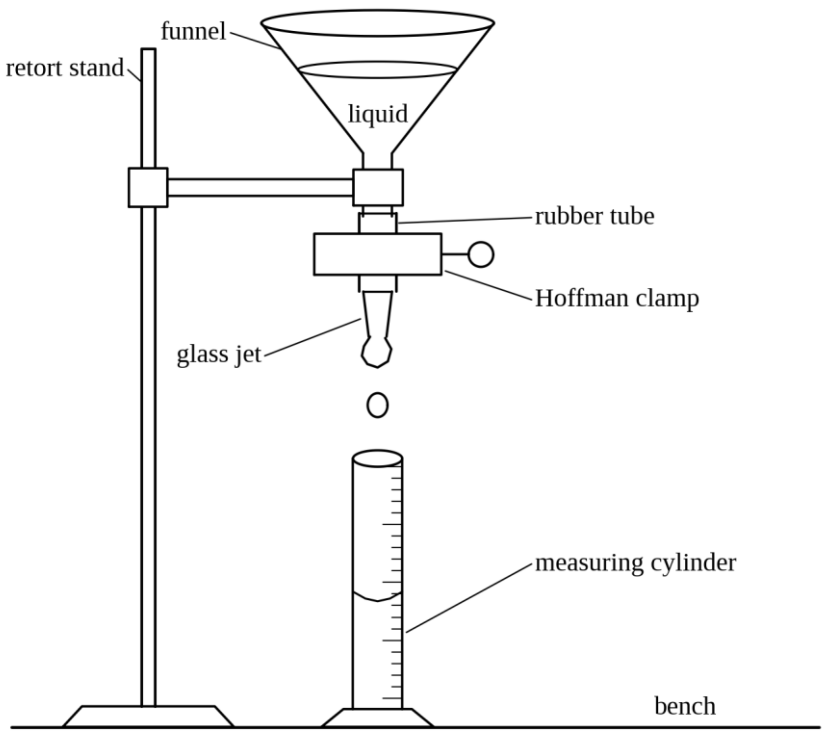
To determine the surface tension of water by the drop method.

Apparatus

A glass funnel of about $250cm^3$ with a stem; a short rubber tube and a Hoffman clamp; a glass jet of about $5mm$ bore; a measuring cylinder of $20cm^3$ graduated in $0.2cm^3$; a balance reading to $0.01g$; a thermometer; a retort stand with a clamp; about $500cm^3$ of clean water.

Setup

The arrangement is the one drawn below. The funnel is clamped upright well above the bench, the rubber tube is slipped over its stem, the Hoffman clamp is put on the tube, and the glass jet is fitted below the clamp so that every drop forms on glass and not on rubber. The measuring cylinder stands on the bench directly under the jet. The drawing shows the run in progress, with the cylinder partly filled.



Procedure

Proceed as follows:

(a) Find the mass of the dry measuring cylinder, and read the temperature of the room.

(b) Clamp the funnel upright, fit the rubber tube, the Hoffman clamp and the jet, and stand the measuring cylinder on the bench under the jet.

(c) With the clamp closed, fill the funnel with the water, then open the Hoffman clamp until the liquid leaves the jet in separate drops slow enough to be counted. Do not touch the clamp again for the rest of the run.

(d) Read the volume V_0 of liquid in the cylinder.

(e) Count 100 drops into the cylinder and read the new volume V.

(f) Count a further 25 drops and read V again. Repeat until six values of V have been recorded in all.

(g) Find the mass of the cylinder together with the whole of its contents.

(h) Record all your readings in a suitable table.

Questions

(i) Tabulate your readings, including the values of $V-V_0$.

(ii) Plot a graph of $V-V_0$ against N, where N is the number of drops collected.

(iii) Determine the slope S of your graph, and state what it represents.

(iv) Determine the density ρ of the liquid.

(v) The surface tension of the liquid is given by

$$\gamma=\frac{\sqrt[3]{\frac{\pi}{6}}\ \rho g S^{\frac{2}{3}}}{1.9}$$

where ρ is its density and g the acceleration of free fall. With ρ as you have found it, π = 3.14 and g = 9.81ms^{-2}, compute γ.

(vi) The accepted value of the surface tension of water at the temperature of the room is 0.072Nm^{-1}. Compare your value with it and comment on the agreement.

(vii) A few drops fell into the cylinder, uncounted, after V_0 had been read. State what your graph would then show, and whether the value of S would be affected.

(viii) The run is repeated with a jet of wider bore, so that the drops are larger. State what happens to the slope of the graph, and say what that implies about the relation used in (v).

Table of results

The dry cylinder had a mass of 61.24g and the cylinder with the whole of the collected liquid had a mass of 78.26g. The first reading of the cylinder, taken before the counting began, was $V_0 = 0.6\text{cm}^3$. The room was at 25°C.

N	V (cm^3)	$V - V_0$ (cm^3)
100	7.9	7.3
125	9.6	9.0
150	11.6	11.0
175	13.3	12.7
200	15.1	14.5
225	17.0	16.4

Making the measurement count

Practical tip 1: *Set the clamp once and then leave it alone.* A drop hurried off the jet by a fast flow is smaller than one allowed to grow until the surface can no longer hold it, and the whole method rests on every drop being the same size as every other. Open the clamp until the drops come slowly enough to be counted without hurrying, and keep your hands off it until the last reading has been taken.

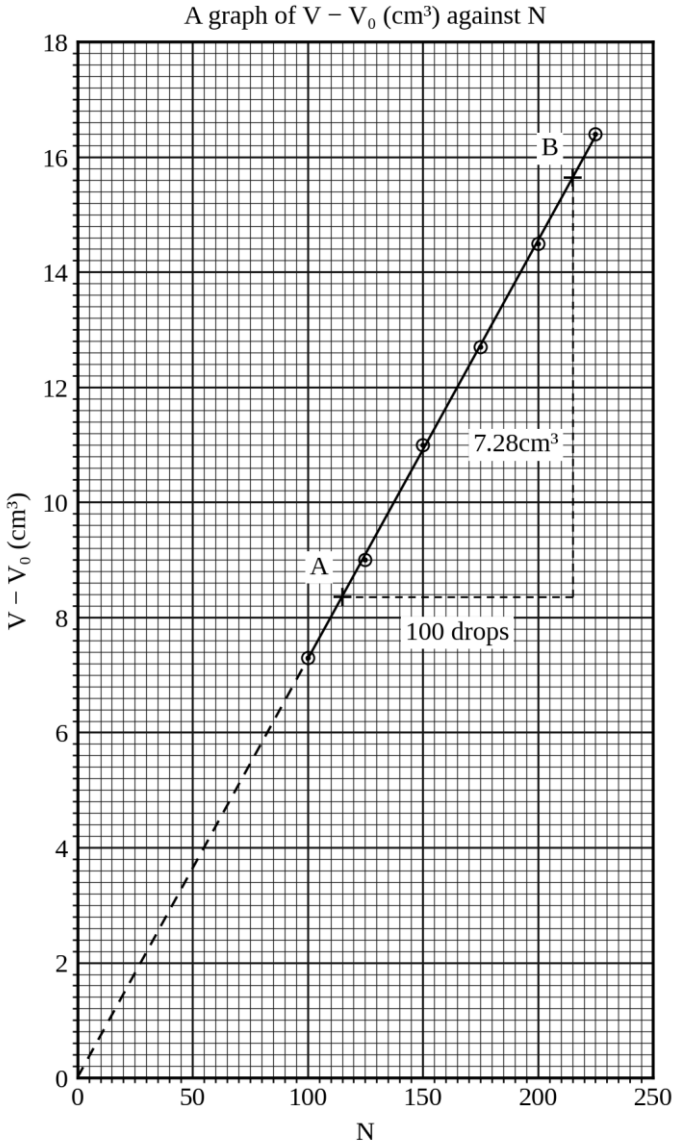
Practical tip 2: *Find the weight of one drop and you have measured the surface tension.* A drop hangs at the jet until its weight can no longer be carried by the pull of the surface round the neck where it is still joined to the glass, and that pull is the surface tension multiplied by the length of the neck. Two things have been done to that statement to reach the relation printed in the question. The neck, which is never measured, has been written in terms of the size of the drop; and the divisor 1.9 is π multiplied by about 0.6, the π turning the neck across into the length of the circle round it and the 0.6 being the share of the hanging drop that actually falls. Part (viii) asks what becomes of the relation when the jet is changed.

Practical tip 3: *Read the first volume as carefully as the last.* It enters every row of the table. Take it at the bottom of the meniscus with your eye level with it, and take every later volume the same way. Part (vii) asks what an error made once, before the counting begins, does to the graph.

Practical tip 4: *The last reading of the run is the most valuable one on the sheet.* The volume collected grows as the drops are counted, but the uncertainty of reading the cylinder stays where it was, so each reading tells you more than the one before it. Finish the run: stopping early throws away the best points you have.

Answers to the questions

(i) and (ii). Take N along the horizontal axis at 2cm to 50 drops, running from 0 to 250, and $V\text{-}V_0$ up the vertical axis at 2cm to $2cm^3$, running from 0 to $18cm^3$. That is 10cm by 18cm, which fits an A4 sheet upright. Give the graph the title A graph of $V\text{-}V_0$ (cm^3) against N.



(iii) Draw the line through the six points and produce it back, broken, to the origin. Taking A (115, 8.36) and B (215, 15.64) on it,

$$S=\frac{15.64cm^3\text{-}8.36cm^3}{215\text{-}115}=\frac{7.28cm^3}{100}=0.0728cm^3$$

The slope is the volume collected for each drop counted, that is the volume of one drop.

(iv) The liquid collected had a mass of

$$78.26g\text{-}61.24g=17.02g$$

and the cylinder then held $17.0cm^3$ in all, the $0.6cm^3$ that was there before the counting began included. That whole $17.0cm^3$ is what was weighed, so

$$\rho=\frac{17.02g}{17.0cm^3}=1.00gcm^{-3}=1000kgm^{-3}$$

(v) The volume of one drop must go into the relation in cubic metres, so that $S=0.0728\text{cm}^3=7.28\times10^{-8}\text{m}^3$ and

$$S^{\frac{2}{3}}=1.743\times10^{-5}\text{m}^2$$

With 3.14 for π, the cube root in front is $\sqrt[3]{0.5233}=0.8059$, and putting these into the relation,

$$\gamma=\frac{0.8059\times1000\text{kgm}^{-3}\times9.81\text{ms}^{-2}\times1.743\times10^{-5}\text{m}^2}{1.9}=0.0725\text{Nm}^{-1}$$

(vi) That is 0.7% above the accepted 0.072Nm^{-1}. Before deciding whether 0.7% means anything, work out what the determination is worth. Each corner of the triangle is read to a fifth of a small square, and the six points scatter a little about the line as well; together they leave the slope good to about 0.4%, and γ goes as its two-thirds power, so that becomes 0.3%. The density is the worse determined of the two: the weighings account for a fiftieth of a gram between them, which is 0.1%, but the volume is one reading of the cylinder, good to 0.1cm^3 in 17.0cm^3, or 0.6%. Add them and the value is worth about 1%. The difference of 0.7% is inside that, so there is nothing in it to explain, and no cause should be invented for it.

(vii) The uncounted drops are in the cylinder for the whole of the run, so every reading of V is too large by the same amount and so is every value of $V-V_0$. The six points are lifted bodily off the line they should lie on, and the line through them is parallel to the true one but cuts the vertical axis above the origin. The slope is untouched, and so is the value of γ that follows from it. A single row divided out on its own would have given a value of S too large, with nothing to show for it; the graph shows it at a glance, in the intercept.

(viii) A wider jet has a longer neck and can hold a heavier drop, so each drop carries more liquid: the line rises more steeply and the slope is larger. Put that larger slope into the relation of part (v) and it returns a larger surface tension for the same water at the same temperature, which cannot be right. The reason is plain once the relation is looked at again: nothing in it describes the jet at all. It is not a law but an empirical relation, fitted for a jet whose bore is about the size of the drop it makes. Use one jet from the first reading to the last, and do not carry a value obtained with one jet across to another: a jet 15% wider in the bore gives drops 15% larger in volume and returns a value about 10% high.

Examiner's eye

- Only the count has no instrument standing behind it, and it is the quantity most often got wrong. Lose your place in the middle of a run and nothing on the sheet will say so: the point sits off the line, where it reads as a bad volume rather than a bad count. Count in fives, and write the running total beside each volume as you record it.
- A drop that strikes the wall of the cylinder and runs down is still in the cylinder and still counted, and does no harm. A drop that strikes the rim and goes outside has been counted and is not there, and it costs about a third of a division of the scale. Set the cylinder square under the jet before the run begins and do not move it afterwards.
- Newtons per metre come out of the relation only if the volume goes in as cubic metres and the density as kilograms per cubic metre. A cubic centimetre is 10^{-6} of a cubic metre, not 10^{-3}, and a volume left in cubic centimetres makes the answer too large by a factor of 10^4. Convert first, and check that what the units leave behind is the kilogram per second squared, which is the newton per metre.
- Dust from the air and grease from the glass both lower the surface tension of water, and so does a thumb on the jet. That is why the funnel, the tube and the jet are rinsed in clean water before the run, and why the jet is picked up by its clean end; it is also why contamination cannot be offered as the reason a value comes out above the accepted one, since it can only push the answer the other way.

Practice 24.1

Pinned above the bench in the school workshop, in handwriting older than anybody who works there, is a card:

TWO DROPS ON EACH BEARING

NOT THREE

THE OIL IS NOT MINE AND IT IS NOT YOURS

The rule has held for eleven years and nothing in the workshop has ever seized. The tin under the card ran dry in the holidays and has been filled from the kitchen since, because cooking oil is what there is, and it has held for that too. **Mr. Akilikubwa** came in for a screwdriver, read the card twice, and said that it was the only measuring instrument in the school that nobody had ever calibrated. **Kipanga** wanted to know what there was to calibrate about a drop. **Kipute** said that if a drop of the kitchen oil and a drop of the old oil were not the same amount, the card had been telling two different stories all term and nobody had noticed. The man said that a drop was a drop and that he had work to do; then he looked at the card, and at the ceiling, and gave them a cupful to take away.

Procedure

The apparatus is the one drawn in Experiment 24.1, and the jet is the jet of that experiment, delivering both liquids in turn. Proceed as follows:

(a) Clamp the funnel upright, fit the rubber tube, the Hoffman clamp and the jet, and stand the measuring cylinder on the bench under the jet.

(b) With the clamp closed, fill the funnel with the water, then open the Hoffman clamp until the liquid leaves the jet in separate drops slow enough to be counted. Mark the position of the clamp on the tube.

(c) Read the volume V_0 of liquid in the cylinder.

(d) Count 100 drops into the cylinder and read the new volume V.

(e) Count a further 25 drops and read V again. Repeat until six values of V have been recorded in all.

(f) Close the clamp, empty and dry the cylinder and the funnel, fill the funnel with the cooking oil, open the clamp to the mark, and repeat steps (c), (d) and (e).

(g) Read the temperature of the room, and record all your readings in a suitable table.

Questions

(i) Tabulate your readings for the two runs, including the values of $V-V_0$.

(ii) On the same axes plot a graph of $V-V_0$ against N for both liquids.

(iii) Determine the slope of each of your two lines.

(iv) The density of water is 1000kgm^{-3} and that of the oil 920kgm^{-3}. Determine the mass of one drop of each liquid.

(v) When two liquids are delivered from one and the same jet, the weights of their drops are in the ratio of their surface tensions. Taking the surface tension of water as 0.072Nm^{-1}, determine the surface tension of the oil.

(vi) Experiment 24.1 obtained the surface tension of a single liquid from the volume of its drops and its density. Compute what that relation would give for the oil here, and say why it must not be used.

(vii) The oil is run again through a jet of half the bore. State, with a reason, how many drops would then be needed to collect 5.00cm^3 of it.

(viii) Give one source of error in this comparison, together with a precaution against it.

Experiment 24.2

The new block at the end of the field was finished in July, and in August the wall on its shaded side began to change colour. A dark band came up out of the ground, about as high as a hand is long, and stopped there. The fundi who had built it said it was rain splashing off the ground, and that it would dry.

Kipute agreed with him, and pointed out that the band was worst on the side the rain came from. **Mr. Akilikubwa** walked round to the other side, where the eaves came down almost to head height and no rain had ever reached the wall, and found the band there too, at exactly the same height. Rain that can climb to the same mark on a sheltered wall is a remarkable kind of rain. **Kipanga**, who had been feeling the blocks, said that the wet ones were the ones with the finest grain in them.

The wall, said **Mr. Akilikubwa**, was drinking. Nobody had put anything under the bottom course to stop it, so the water in the ground was climbing up through the blocks of its own accord, and it had stopped where it stopped for a reason. On Thursday the class would find out what it is that lifts water into a wall, and would put a number on it.

Aim

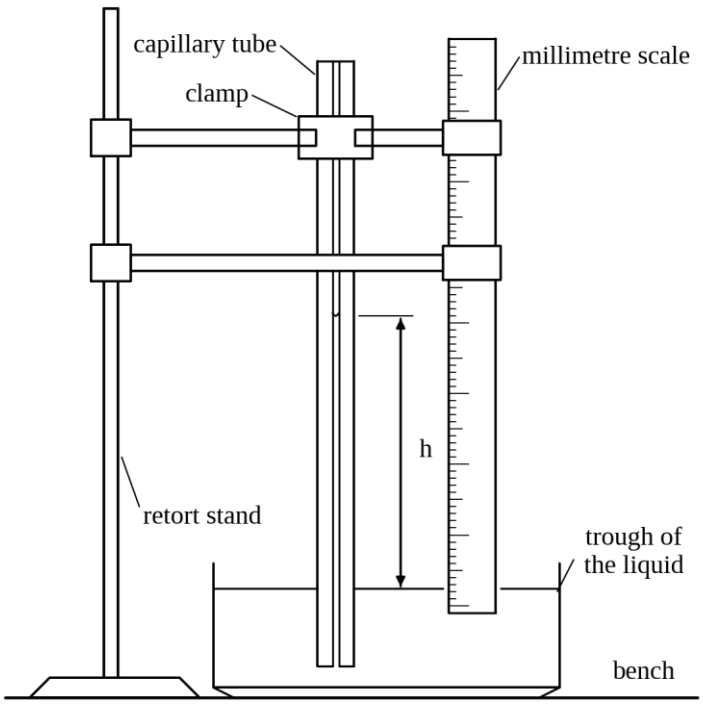
To determine the surface tension of water from the height to which it rises in tubes of different bore.

Apparatus

Six capillary tubes of the same length and of different bore, whose radii have been measured beforehand and are given with the readings; a shallow glass trough; a millimetre scale and a hand lens; a retort stand with two bosses and clamps; a thermometer; clean water; a beaker of warm sodium carbonate solution and a beaker of clean water for washing the tubes.

Setup

The arrangement is the one drawn below. One tube at a time is clamped upright with its lower end well below the surface of the liquid in the trough, and the millimetre scale is clamped beside it. The trough holds enough liquid that the lower end of the tube stays covered throughout.



Procedure

Proceed as follows:

(a) Wash all six tubes in warm sodium carbonate solution, rinse them in clean water, dry them, and stand them in a rack, touching each one only at the top.

(b) Fill the trough with clean water, and read the temperature of the room.

(c) Clamp the first tube upright with its lower end about 10mm below the surface, and clamp the millimetre scale beside it.

(d) Wait until the liquid in the tube has stopped rising, then read on the millimetre scale, with the hand lens, the position of the free surface of the liquid in the trough and the position of the bottom of the meniscus in the tube. Record h, the difference between the two positions.

(e) Repeat steps (c) and (d) for each of the remaining five tubes.

(f) Record all your readings in a suitable table.

Questions

(i) Tabulate your readings, including the values of $1/r$.

(ii) Plot a graph of h against $1/r$.

(iii) Determine the slope of your graph and give its unit.

(iv) The height to which a liquid rises in a tube of radius r is

$$h=\frac{2\gamma}{\rho g r}$$

where γ is the surface tension of the liquid, ρ its density and g the acceleration of free fall. With $\rho = 1000\text{kgm}^{-3}$ for water and $g = 9.81\text{ms}^{-2}$, determine γ.

(v) Experiment 24.1 obtained the surface tension of water by the drop method and got 0.0725Nm^{-1}. Compare the two values, and say which of the two determinations you would trust the more, and why.

(vi) Use your graph to find the height to which water would rise in a tube of radius 0.35mm.

(vii) Suppose that one of the tubes had not been properly cleaned, so that a film of grease was left on its bore. State how the point for that tube would lie with respect to your line, and explain why.

(viii) Give one source of error in the measurement of h, together with a precaution against it.

Table of results

The radii of the six tubes had been measured beforehand and are given in the first column. The room was at 25°C.

r (mm)	1/r (mm^{-1})	h (mm)
0.170	5.882	86.1
0.204	4.902	72.2
0.253	3.953	58.2
0.312	3.205	46.6
0.401	2.494	36.8
0.520	1.923	27.8

Making the measurement count

Practical tip 1: *Glass that has been handled is not clean glass.* Wash every tube in warm sodium carbonate solution, rinse it in clean water, and from that moment touch it only at the top. A tube picked up by the end that goes into the liquid will not give the rise the others give, and part (vii) asks what that does to the graph.

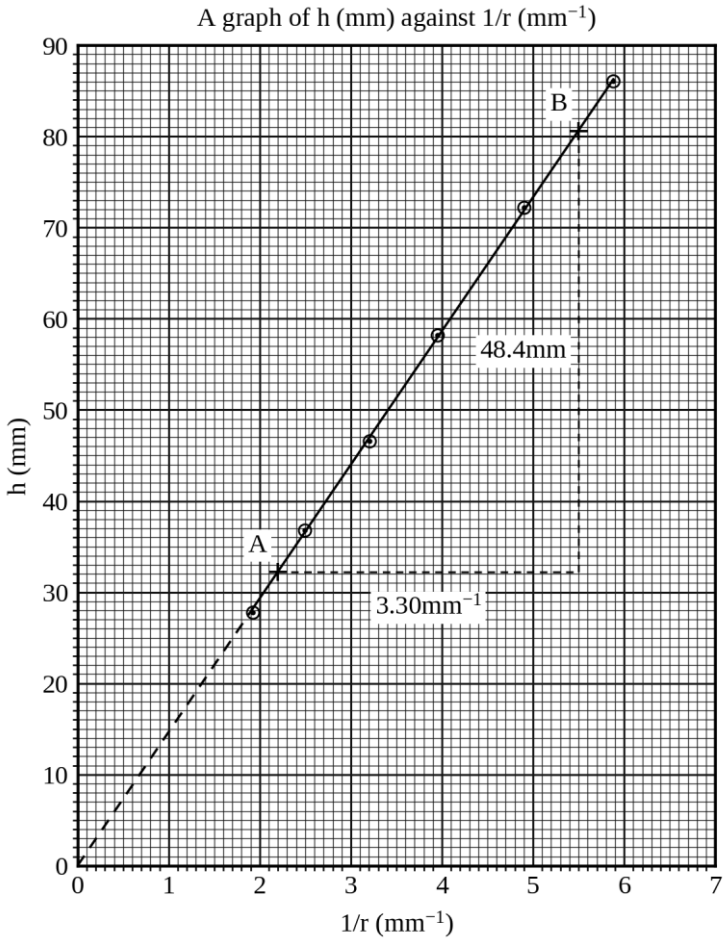
Practical tip 2: *The relation in the question is two lines of algebra, not a law to be learnt.* What holds the column of liquid up in the tube is the pull of the surface round the circle where it meets the wall of the glass. That circle has a length $2\pi r$, so the pull is $2\pi r\gamma$; the weight it carries is $\pi r^2 h\rho g$. Set the one equal to the other and it follows in a line.

Practical tip 3: *Give the liquid time.* A tube of fine bore fills slowly and the last millimetre of it can take a minute or more, and a reading taken while the column is still moving is simply a wrong reading. Watch the meniscus against the scale until it has not moved for half a minute. The finest tube of the six will keep you waiting longest.

Practical tip 4: *Some liquid stands above the point you read, and the relation ignores it.* What is held in the meniscus itself adds about a third of the radius to the rise: under 0.1% of it for the finest tube of the six, more than 0.5% for the widest, and growing steadily from the one to the other.

Answers to the questions

(i) and (ii). Take $1/r$ along the horizontal axis at 2cm to 1mm^{-1}, running from 0 to 7mm^{-1}, and h up the vertical axis at 2cm to 10mm, running from 0 to 90mm. That is 14cm by 18cm, which fits an A4 sheet upright. Give the graph the title A graph of h (mm) against $1/r$ (mm^{-1}).



(iii) The best line through the six points reaches the origin when it is produced back. Taking A (2.20, 32.2) and B (5.50, 80.6) on it,

$$\text{slope}=\frac{80.6\text{mm}-32.2\text{mm}}{5.50\text{mm}^{-1}-2.20\text{mm}^{-1}}=\frac{48.4\text{mm}}{3.30\text{mm}^{-1}}=14.7\text{mm}^2$$

A millimetre divided by a reciprocal millimetre is a square millimetre, so the slope is 14.7mm^2.

(iv) The relation can be written

$$h=\frac{2\gamma}{\rho g}\times\frac{1}{r}$$

so the slope of the line is $2\gamma/\rho g$. Converted to square metres it is $14.7\times10^{-6}\text{m}^2$, and then

$$\gamma=\frac{\rho g\times\text{slope}}{2}=\frac{1000\text{kgm}^{-3}\times9.81\text{ms}^{-2}\times14.7\times10^{-6}\text{m}^2}{2}=0.0721\text{Nm}^{-1}$$

(v) The tubes give 0.0721Nm^{-1} and the drops gave 0.0725Nm^{-1}, a difference of 0.0004Nm^{-1}, or 0.6% of either. Experiment 24.1 was worth about 1%, most of it in the single reading of the cylinder that fixes the

density; this one is worth about 0.4%. Put the two together and the comparison is worth about 1%, so a difference of 0.6% is inside it and there is nothing to explain.

Of the two, trust this one, because its relation is derived and the other is fitted. But keep in view what it hides: the meniscus of Practical tip 4, which has not been corrected for; the assumption that the water meets the glass at no angle at all, which part (vii) shows is a real one; and the radii, handed to you with no uncertainty although γ follows them exactly.

(vi) For a tube of radius 0.35mm, $1/r = 1/0.35\text{mm} = 2.86\text{mm}^{-1}$. Read up from 2.86mm^{-1} on the horizontal axis to the line and across to the vertical axis, and you get 42.0mm.

(vii) Water does not wet greasy glass as it wets clean glass. Against clean glass it runs up the wall and the whole of the pull is vertical; against a greasy wall the surface stands at an angle θ to the glass and only the part of the pull along the wall is left to carry the column, so the right-hand side of the relation gains a factor cosθ. That factor is less than one, so the rise is smaller than it should be and the point lies below the line: one point below a line of five good ones is the signature of a dirty tube.

(viii) The free surface in the trough and the meniscus in the tube are at different heights, so a scale sighted from one position is read wrongly for at least one of them. That is parallax, and here it enters h twice over, once at each end. Clamp the millimetre scale as close to the tube as it will go, and bring your eye level with each mark in turn.

Examiner's eye

- A graph of h against r would be a curve, and a curve through six points can be drawn a dozen ways. Work in reciprocals instead and the same six readings become a straight line whose slope can be read off with a ruler. Work them out to three decimal places before you plot anything.
- An air bubble drawn into the bore stops the liquid where it is and no reading afterwards is worth anything. It gets in when the lower end of the tube is knocked clear of the surface, and it will not come out by tapping; take the tube out, wash it, and start that reading again.
- Water that has stood in an open trough all afternoon is not the clean water the tubes were washed to meet. Dust settles on it, and what settles on a surface stays on the surface, where it does the most harm: it pulls the rise down in every tube at once, so the line is straight and its slope is too small, and not one of the six points betrays it. Draw the water fresh.
- Six tubes give six points and the graph needs all of them, but they do not carry equal weight: the finest tube sits furthest from the origin and does the most to fix the line, while the widest gives the smallest rise, read to the same tenth of a millimetre, and does the least. If one reading has to be taken again, take that one.

Practice 24.2

A letter came to the school addressed to nobody in particular, and **Mr. Akilikubwa** pinned it on the board outside the laboratory.

> Dear Teacher,
>
> I press oil at Mgeta and I have a trouble that nobody can explain to me. I filter the oil through a cloth into a drum. The oil does not stay in the drum. It goes up the cloth where the cloth hangs over the rim, and it comes down the outside, and by evening there is a ring of it on the floor. With a thicker cloth it is worse. My neighbour says I must buy a coarser weave. I do not want to buy the wrong cloth twice.
>
> Yours faithfully,
>
> Halima Mwenda
>
> P.S. My son says the oil climbs because it wants to get out. He is fourteen and he knows everything.

Kipute said that he would like to meet the son. **Kipanga** wanted to write back the same afternoon and had to be stopped. **Mr. Akilikubwa** said that nobody was writing anything until six glass tubes and a trough of oil had shown them what a weave has to do with how far a liquid can climb.

Procedure

The apparatus is the one drawn in Experiment 24.2, the six tubes are the same six, and the oil is the cooking oil of Practice 24.1. Proceed as follows:

(a) Wash all six tubes in warm sodium carbonate solution, rinse them in clean water, dry them, and stand them in a rack, touching each one only at the top.

(b) Fill the trough with the oil provided, and read the temperature of the room.

(c) Clamp the first tube upright with its lower end about 10mm below the surface, and clamp the millimetre scale beside it.

(d) Wait until the liquid in the tube has stopped rising, then read on the millimetre scale, with the hand lens, the position of the free surface of the liquid in the trough and the position of the bottom of the meniscus in the tube. Record h, the difference between the two positions.

(e) Repeat steps (c) and (d) for each of the remaining five tubes.

(f) Record all your readings in a suitable table.

Questions

(i) Tabulate your readings, including the values of $1/r$.

(ii) Plot a graph of h against $1/r$.

(iii) Determine the slope of your graph and give its unit.

(iv) The height to which a liquid rises in a tube of radius r is

$$h=\frac{2\gamma}{\rho g r}$$

where γ is the surface tension of the liquid and ρ its density. The density of the oil is 920kgm^{-3} and $g = 9.81\text{ms}^{-2}$. Determine the surface tension of the oil.

(v) Water rises to about twice the height in the same tubes. Taking the surface tension of water as 0.072Nm^{-1} and its density as 1000kgm^{-3}, show that this follows from the relation, and say which of the two quantities in it accounts for the greater part of the difference.

(vi) A wick is to be replaced by a glass tube in which the oil will stand 25.0mm above the free surface. Use your graph to find the radius of the bore required.

(vii) The surface tension of a liquid falls as its temperature rises. State what would happen to your graph if the whole experiment were repeated in a warmer room.

(viii) Give one source of error in this determination, together with a precaution against it.

SOLUTIONS TO PRACTICE EXPERIMENTS

Practice 7.1

L (cm)	t for 30 oscillations (s)	t^2 (s^2)
90	57.15	3266
80	53.92	2907
70	50.57	2557
60	46.77	2187
50	42.57	1812
40	38.07	1449

(ii) Plot t^2 against L. A vertical scale of 2cm to $200s^2$ running from 1400 to $3300s^2$, with a horizontal scale of 2cm to 10cm running from 0 to 100cm, spreads the six points across the sheet. They lie on one straight line.

(iii) Taking A (40cm, $1453s^2$) and B (90cm, $3273s^2$) on the best line:

$$\text{slope}=\frac{3273s^2-1453s^2}{90cm-40cm}=\frac{1820s^2}{50cm}=36.4s^2cm^{-1}$$

Accept any value from 35.5 to $36.8s^2cm^{-1}$.

(iv) Since t=nT and T = 2π times the square root of L over g, squaring both sides gives

$$t^2=\frac{4\pi^2n^2}{g}L$$

which is a straight line through the origin of slope $4\pi^2n^2$ over g. Rearranging and substituting the slope, with L in centimetres:

$$g=\frac{4\pi^2n^2}{\text{slope}}=\frac{4\times3.14^2\times30^2}{36.4s^2cm^{-1}}=\frac{35495}{36.4}cms^{-2}=975.1cms^{-2}$$

$$g=975.1cms^{-2}\times\frac{1m}{100cm}=9.75ms^{-2}$$

Accept any value from 9.6 to $10.0ms^{-2}$.

(v) At L = 55cm,

$$t^2=36.4s^2cm^{-1}\times55cm=2002s^2$$

$$t=\sqrt{2002s^2}=44.7s$$

Accept any value from 44.0 to 45.2s.

(vi) Her graph would be the same line, within experimental scatter. The period of a simple pendulum does not depend on the mass of the bob, because mass appears on both sides of the equation of motion and cancels. The same lengths therefore give the same times, whatever hangs on the rope.

(vii) For example, reaction time in starting and stopping the watch, which shortens every reading. Timing 30 oscillations rather than one, and repeating each timing and taking the mean, reduces its effect on the period.

Examiner's eye

- Every t here is the time for **30** oscillations, not 20. If your line shows a clear intercept when you extend it back to the origin, check this first.
- The slope carries units of s^2cm^{-1} because you tabulated L in centimetres, so the substitution returns g in cms^{-2}. An answer in the hundreds has simply not been converted; divide by 100 and give the unit you were asked for.
- The factor n^2, which is 900 here, is where this question is won or lost. Leave it out and your value of g comes out nine hundred times too small.

Practice 7.2

The specimen table below is what careful work gives. Check your own readings against it row by row before you check your graph.

L (m)	t for 20 oscillations (s)	T (s)	T^2 (s^2)
0.850	37.16	1.858	3.452
0.750	35.12	1.756	3.084
0.650	32.75	1.638	2.683
0.550	30.01	1.501	2.253
0.450	27.40	1.370	1.877
0.350	24.11	1.206	1.454

(iii) The slope is $4.01s^2m^{-1}$.

(iv) $T^2=4\pi^2(L+c)/g$ expands to $T^2=(4\pi^2/g)L+(4\pi^2/g)c$, so the slope is $4\pi^2/g$ and

$$g=\frac{4\pi^2}{4.01s^2m^{-1}}=9.84ms^{-2}$$

(v) The intercept is $0.060s^2$. It is not c, because it is a time squared and c is a length; it is c multiplied by the slope. Dividing removes the slope, and with it g:

$$c=\frac{0.060s^2}{4.01s^2m^{-1}}=0.0150m$$

so c is 1.50cm, which is exactly what the vernier calliper gives for the radius. Plotted the other way round, with L up the vertical axis, the intercept is the length itself and no division is needed. That is the only reason to prefer one axis order to the other here.

(vi) Stopping the watch a fraction early shortens every recorded time, which shortens T, which raises g, because g goes as 1 over T squared. It is a systematic error and repeating the timings will not remove it. Test for it by timing 40 oscillations instead of 20: a reaction-time bias is a fixed number of seconds, so it is halved when the count is doubled, and the recovered g should fall towards the accepted value.

Accept: a slope between 3.95 and $4.07s^2m^{-1}$; g between 9.69 and $9.98ms^{-2}$; an intercept between 0.04 and $0.08s^2$; c between 1.0 and 2.0cm; and a named systematic error that acts on the time and not on the length.

Practice 7.3

d (cm)	t for 20 oscillations (s)	t^2 (s^2)
5.0	37.30	1391.3
15.0	34.89	1217.3
25.0	32.52	1057.6
35.0	30.14	908.4
45.0	27.16	737.7
55.0	24.21	586.1

(iii) Taking the size of the slope, $0.0623cms^{-2}$, which is $0.000623ms^{-2}$, and with n = 20,

$$g=4\times3.14^2\times20^2\times0.000623ms^{-2}=9.83ms^{-2}$$

(iv) The intercept on the d axis is 91.3cm. It is the whole swinging length before the thread was drawn up, from the cork to the centre of the bob.

(v) The value is 0.2% above the accepted $9.81ms^{-2}$. A finite swing angle can only push g down, so it is not the explanation here; a watch stopped a fraction early shortens every time and pushes g up, and that fits both the direction and the size.

Accept: a slope between −0.0610 and $-0.0637cms^{-2}$; g between 9.62 and $10.05ms^{-2}$; an intercept between 89 and 93cm, identified as the original swinging length; and a systematic error whose direction matches the result.

Practice 7.4

x (cm)	t for 20 oscillations (s)	T (s)	T^2 (s^2)
90.0	37.51	1.876	3.519
80.0	35.31	1.766	3.119
70.0	33.14	1.657	2.746
60.0	30.61	1.531	2.344
50.0	27.68	1.384	1.915
40.0	24.62	1.231	1.515

(iii) Plotted this way round the slope is $g/4\pi^2$, and it is $24.9cms^{-2}$, which is $0.249ms^{-2}$, so

$$g=4\times3.14^2\times0.249ms^{-2}=9.82ms^{-2}$$

(iv) The intercept on the x axis is +2.0cm. Since $x=(g/4\pi^2)T^2-b$, the intercept is −b, so b=-2.0cm.

(v) A negative b means the swinging length is shorter than the distance you measured, so the point you measured to lies below the centre of the bob. You measured to the bottom of the bob, and the centre is above it by the radius, so b should be minus the radius: −2.00cm for the 4.00cm bob, which is what the graph gives. A negative length is not an absurdity here. It is a direction.

(vi) Measure x to the top of the bob instead of the bottom. The centre is then below the point measured to, the swinging length is longer than x, and b comes out positive and equal to the radius. Measuring to the centre itself would make b zero and the line pass through the origin.

Accept: a slope between 24.4 and $25.4cms^{-2}$; g between 9.62 and $10.02ms^{-2}$; an intercept between +1.5 and +2.5cm; b negative, between −1.5 and −2.5cm, explained as minus the radius of the bob with the sign reasoned about, not apologised for.

Practice 7.5

L (m)	t for 20 oscillations (s)	T (s)	log L	log T
0.900	37.90	1.895	-0.046	0.278
0.700	33.41	1.671	-0.155	0.223
0.550	29.79	1.490	-0.260	0.173
0.420	25.86	1.293	-0.377	0.112
0.320	22.74	1.137	-0.495	0.056
0.240	19.53	0.977	-0.620	-0.010

(ii) and (iii) The slope is 0.500 and the intercept on the logT axis is 0.302, so m = 0.500 and $k=10^{0.302}=2.004sm^{-1/2}$.

(iv) From $k=2\pi/\sqrt{g}$,

$$g=\left(\frac{2\times3.14}{2.004}\right)^2=9.82ms^{-2}$$

(v) Timing one length assumes the relation. It can produce a number for g, but it cannot tell you whether the thing on the end of the thread is behaving like a simple pendulum at all, because any single reading fits some value of g. Six lengths on logarithmic axes test the power as well as measuring the constant, and it is the power that says whether the law holds. A student who has only one reading has measured g on the assumption that g is measurable this way.

Accept: a slope between 0.47 and 0.53; an intercept between 0.296 and 0.309; k between 1.98 and $2.04sm^{-1/2}$, and the g that follows from it, between 9.5 and $10.1ms^{-2}$; and an answer to (v) that turns on testing the law rather than on precision.

Practice 7.6

L (cm)	t for 20 oscillations (s)	t^2 (s^2)
90.0	37.39	1398.0
80.0	35.36	1250.3
70.0	32.83	1077.8
60.0	30.29	917.5
50.0	27.70	767.3
40.0	24.44	597.3

(iii) The slope is 0.0623cms^{-2}, which is 0.000623ms^{-2}. With n = 20,

$$g=4\times3.14^2\times20^2\times0.000623\text{ms}^{-2}=9.83\text{ms}^{-2}$$

(iv) The intercept on the L axis is +2.6cm. Measuring from the top of the cork makes every length too long by the depth of the cork, so the swinging length is L-d and the intercept is +d. The cork is therefore about 2.6cm deep, which is the distance from the top of it, where the rule was laid, down to the underside, where the thread actually pivots. Compare Practice 7.4, where measuring to the bottom of the bob made every length too long in the same way and gave a positive intercept for the same reason.

(v) He is right that it does not affect g. A constant added to every length slides the line without turning it, so the slope, and therefore g, is untouched. He is wrong that it does not show: it shows as an intercept of exactly the size of his mistake, and if the question asks for a line through the origin, or for the swinging length, the cork is now his problem. Being consistently wrong is better than being randomly wrong, but it is not the same as being right.

Accept: a slope between 0.0611 and 0.0635cms^{-2}; g between 9.63 and 10.02ms^{-2}; an intercept between +2.0 and +3.2cm, positive, identified as the depth of the cork; and an answer to (v) that separates the slope from the intercept.

Practice 7.7

h (cm)	t for 20 oscillations (s)	t^2 (s^2)
25.0	58.85	3463.3
50.0	55.49	3079.1
75.0	51.54	2656.4
100.0	47.47	2253.4
125.0	43.19	1865.4
150.0	38.04	1447.0

(iii) Taking the size of the slope, 0.0619cms^{-2}, which is 0.000619ms^{-2}, and with n = 20,

$$g=4\times3.14^2\times20^2\times0.000619\text{ms}^{-2}=9.76\text{ms}^{-2}$$

(iv) The intercept on the h axis is 239.9cm, so the point of suspension is 2.40m above the ground.

(v) At h=200cm the line gives $t^2=(239.9\text{-}200)\div0.0619=645\text{s}^2$, so t is about 25.4s. It is less trustworthy because 200cm lies outside every reading taken, so the line is being extrapolated rather than read: the experiment says nothing about whether the relation still holds there, and a swinging length of only 0.40m with a bob of ordinary size is also getting close to where the bob's own size stops being negligible.

Accept: a slope between −0.0607 and -0.0631cms^{-2}; g between 9.57 and 9.95ms^{-2}; a height between 236 and 244cm; a predicted time between 24.5 and 26.5s; and extrapolation named as the reason for the caution.

Practice 7.8

θ (°)	θ^2 ($°^2$)	t for 20 oscillations (s)	T (s)
5	25	40.22	2.011
10	100	40.24	2.012
15	225	40.30	2.015
20	400	40.54	2.027
25	625	40.71	2.036
30	900	40.83	2.042

(iii) The intercept is 2.009s, which is 0.002s below the 5° reading. It is T_0, and it is the period the relation $T=2\pi\sqrt{L/g}$ describes. Every reading in the table is longer than it.

Two things about that graph need saying. The angles were tabulated in degrees and not in radians, and it did not matter, because changing the unit of θ multiplies every value along the horizontal axis by the same constant, and multiplying one axis by a constant tilts a line without moving the point where it meets the other. The intercept is therefore safe. The slope is not, and a slope read off $°^2$ axes is nothing like $T_0/16$ until it has been converted. Second, θ is the angle you released the bob from, and the swing dies a little while you count, so every recorded time is a shade short of what a truly constant angle would have given, and the widest rows lose the most.

(iv) With L=1.000m,

$$g=\frac{4\pi^2L}{T_0^2}=\frac{4\times3.14^2\times1.000\text{m}}{(2.009\text{s})^2}=9.77\text{ms}^{-2}$$

(v) Taking the 30° row on its own, T=2.042s, and

$$g=\frac{4\times3.14^2\times1.000\text{m}}{(2.042\text{s})^2}=9.46\text{ms}^{-2}$$

which is 3.6% below the accepted 9.81ms^{-2} and outside any band that would be accepted. Note the direction. A wide swing always lengthens the period, so wherever the period is used directly, as it is here, it lowers g and never raises it. A value of g too small by a few percent is therefore the signature of a swing that was not small, while one that is too large points somewhere else altogether.

(vi) The whole variation in T is 2.042s-2.011s=0.031s. On an axis that began at zero, that is about a millimetre of an eighty-millimetre axis: all six points would lie in one band and no line would be worth drawing. The T axis therefore starts at 2.000s. The precaution is this. The intercept wanted is the value of T at $\theta^2 = 0$, and the left-hand edge of the graph is that place only because the θ^2 axis was left with a true origin. Give both axes false origins and the value where the line meets the corner is not an intercept at all, and quoting it as one throws away every mark the graph carries. Experiment 7.1 put the rule as "when a question turns on where your line meets an axis, that axis must start at zero". Read it carefully. It is the axis you run the line along that must start at zero, and here that is θ^2, not the axis you read the answer off.

Accept: an intercept between 2.004 and 2.014s; g between 9.72 and 9.82ms^{-2}; a value from the 30° row between 9.35 and 9.55ms^{-2}; the intercept named as the period at zero swing angle; and the false origin explained with the θ^2 axis identified as a true one.

Practice 8.1

h (m)	t for 30 oscillations (s)	t^2 (s^2)	t^2h (s^2m)	h^2 (m^2)
0.400	45.29	2051.2	820.5	0.1600
0.350	44.26	1958.9	685.6	0.1225
0.300	43.66	1906.2	571.9	0.0900
0.250	43.19	1865.4	466.4	0.0625
0.200	44.19	1952.8	390.6	0.0400
0.150	46.40	2153.0	323.0	0.0225

(ii) A vertical scale of 2cm to $100\text{s}^2\text{m}$ running from 0 to $900\text{s}^2\text{m}$, with a horizontal scale of 2cm to 0.02m^2 running from 0 to 0.18m^2, puts the six points across the sheet and still shows the intercept.

(iii) Taking A (0.024m^2, $330.2\text{s}^2\text{m}$) and B (0.166m^2, $843.0\text{s}^2\text{m}$) on the best line:

$$\text{slope}=\frac{843.0\text{s}^2\text{m}-330.2\text{s}^2\text{m}}{0.166\text{m}^2-0.024\text{m}^2}=3610\text{s}^2\text{m}^{-1}$$

and the line meets the t^2h axis at $244\text{s}^2\text{m}$.

(iv) Since t=nT, squaring the governing equation and multiplying by h gives

$$t^2h=n^2T^2h=\frac{4\pi^2n^2}{g}h^2+\frac{4\pi^2n^2k^2}{g}$$

so the slope is $4\pi^2n^2/g$ and, with n=30,

$$g=\frac{4\pi^2n^2}{\text{slope}}=\frac{39.4384\times900}{3610\text{s}^2\text{m}^{-1}}=9.83\text{ms}^{-2}$$

Accept any value from 9.6ms^{-2} to 10.0ms^{-2}.

(v) The intercept divided by the slope is k^2, and the factor n^2 cancels between them:

$$k^2=\frac{244\text{s}^2\text{m}}{3610\text{s}^2\text{m}^{-1}}=0.0676\text{m}^2,\qquad k=0.260\text{m}$$

(vi) If $k=l/\sqrt{12}$ then $l=k\sqrt{12}$, so

$$l=0.260\text{m}\times3.464=0.901\text{m}$$

against a measured 0.900m, which agrees to about a part in a thousand.

Accept any length from 0.86m to 0.94m.

(vii) He is wrong, and the reason is in the derivation rather than in the graph. The mass appears in the moment of inertia and again in the restoring moment, and cancels between them, so the period of a compound pendulum does not depend on the mass at all. A bar of twice the mass and the same shape gives the same times, the same slope, and the same value of g. The quantity that would change the slope is g itself, and the quantity that would change the intercept is the shape.

Examiner's eye

- This question tabulates t and never asks for T, which saves a column and keeps the reading you actually took. The price is that n^2 now lives inside the slope. Forget it and your g comes out nine hundred times too small, and no amount of careful plotting will rescue it.
- Both n^2 factors cancel in part (v), so the radius of gyration is the same whether you plot t^2h or T^2h. If you divided one of them by n^2 and not the other, k comes out thirty times too large and looks absurd against the length of the bar. That comparison in part (vi) is your own check; use it.
- "It is heavier, so it must swing slower" is a sentence, not an answer. Part (vii) asks where in the derivation the mass goes, and the mark is for saying that it cancels between the moment of inertia and the restoring moment.

Practice 8.2

x (cm)	t for 50 oscillations (s)	T (s)
5	71.06	1.4212
10	69.19	1.3838
15	68.32	1.3664
20	68.36	1.3672
25	71.34	1.4268
55	71.01	1.4202
60	68.20	1.3640
65	67.96	1.3592
70	69.20	1.3840
75	70.81	1.4162

(ii) Put T on the vertical axis at 2cm to 0.010s, running from 1.350s to 1.430s, and x on the horizontal axis at 2cm to 10cm, running from 0 to 80cm. Two smooth curves, one for each group of five points, not joined across the middle.

(iii) The line at 1.400s cuts the curves at 7.5cm, 23.6cm, 56.4cm and 72.5cm. Accept each within 0.4cm.

(iv) For a fixed periodic time, L is fixed. Multiplying $L=(k^2+h^2)/h$ through by h and collecting terms,

$$h^2\text{-}Lh+k^2=0$$

a quadratic in h with two roots. The sum of the roots of $h^2\text{-}Lh+k^2=0$ is L and their product is k^2, so the two distances that give the same period satisfy $h_1+h_2=L$. They are on opposite sides of the centre of gravity, so their two pivots are a distance h_1+h_2, that is L, apart.

(v) Pair the outer reading on one curve with the inner reading on the other. Those two positions lie at h_1 and h_2 from the centre of gravity, on opposite sides of it, so the distance between them is h_1+h_2. The readings at 7.5cm and 56.4cm are such a pair, and their separation is

$$L=56.4\text{cm}-7.5\text{cm}=48.9\text{cm}$$

The other such pair, at 23.6cm and 72.5cm, gives 72.5cm-23.6cm=48.9cm, which is the same, and quoting both is the check. Pair the two outer readings instead, or the two inner ones, and you get $2h_1$ or $2h_2$ and a value of g that is out by a factor. Then, from $T=2\pi\sqrt{L/g}$,

$$g=\frac{4\pi^2L}{T^2}=\frac{39.4384\times0.489\text{m}}{(1.400\text{s})^2}=9.84\text{ms}^{-2}$$

Accept any value from 9.6ms^{-2} to 10.0ms^{-2}.

(vi) Because L is a difference between two positions read off the same scale. Slide the origin of x anywhere you like and the difference does not change. The centre of gravity would be needed to work out h_1 and h_2 separately, and the method never asks for them separately: it asks only for their sum, which the bar itself measures out between the two holes. That is the whole point of the notice on the door, and it is why the correction pencilled underneath it was wrong.

(vii) The two curves are lowest at about 16.9cm and 63.1cm, and those two positions are 2k apart, so

$$k=\frac{63.1\text{cm}-16.9\text{cm}}{2}=23.1\text{cm}$$

against $l/\sqrt{12}=80\text{cm}/3.464=23.1\text{cm}$ for a uniform bar. Accept any value from 21.5cm to 24.5cm. Notice that the equivalent length at the minimum would be 2k, or 46.2cm, which is shorter than the 48.9cm you found in part (v), as it must be.

Examiner's eye

- The four readings in part (iii) are the whole question. Read them to the nearest millimetre of your own scale and write all four down, even though the calculation uses two. The pair you do not use is your check, and a check that agrees is worth writing.
- Do not pair the two points that are symmetrical about the middle of the bar. Their separation is $2h_1$ or $2h_2$, not h_1+h_2, and it gives a value of g that is wrong by a factor you cannot spot from the arithmetic alone. Pair an outer reading with the inner reading on the *other* curve.
- 1.400s is a choice, not a given. Any horizontal line that cuts both curves twice will do. If you have time, do a second line at a different period; two agreeing answers are the strongest thing a practical script can show, and a few percent between them is what to expect.

Practice 8.3

h (m)	t for 20 oscillations (s)	T (s)	T^2 (s^2)	T^2h (s^2m)	h^2 (m^2)
0.060	35.66	1.783	3.179	0.1907	0.0036
0.100	29.25	1.463	2.140	0.2140	0.0100
0.140	27.05	1.353	1.831	0.2563	0.0196
0.180	25.98	1.299	1.687	0.3037	0.0324
0.220	25.95	1.298	1.685	0.3707	0.0484
0.260	26.28	1.314	1.727	0.4490	0.0676

(ii) A vertical scale of 2cm to $0.05\text{s}^2\text{m}$ running from 0 to $0.50\text{s}^2\text{m}$, with a horizontal scale of 2cm to 0.01m^2 running from 0 to 0.07m^2, uses the sheet and keeps both origins true.

(iii) Taking A (0.006m^2, $0.1993\text{s}^2\text{m}$) and B (0.070m^2, $0.4580\text{s}^2\text{m}$) on the best line:

$$\text{slope}=\frac{0.4580\text{s}^2\text{m}-0.1993\text{s}^2\text{m}}{0.070\text{m}^2\text{-}0.006\text{m}^2}=4.04\text{s}^2\text{m}^{-1}$$

and the intercept on the T^2h axis is $0.175\text{s}^2\text{m}$.

(iv) The slope is $4\pi^2/g$, so

$$g=\frac{4\pi^2}{\text{slope}}=\frac{39.4384}{4.04}\text{ms}^{-2}=9.76\text{ms}^{-2}$$

Accept any value from 9.6ms^{-2} to 10.0ms^{-2}.

(v) The intercept divided by the slope is k^2:

$$k^2=\frac{0.175\text{s}^2\text{m}}{4.04\text{s}^2\text{m}^{-1}}=0.04332\text{m}^2,\qquad k=0.208\text{m}$$

(vi) With a=0.600m and b=0.400m,

$$\sqrt{\frac{a^2+b^2}{12}}=\sqrt{\frac{0.360\text{m}^2+0.160\text{m}^2}{12}}=\sqrt{0.04333\text{m}^2}=0.208\text{m}$$

which is the value the graph gave. Accept any value of k from 0.195m to 0.220m.

(vii) When the holes lie 2cm to one side of G, the true distance from G to the pivot is $\sqrt{h^2+(0.02\text{m})^2}$, not h, and the shortfall differs in every row. His points therefore lie on a line that is slightly curved, and it is curved most where the 2cm matters most. At the outermost hole the departure is three parts in a thousand; at the innermost it is 5%, which on his own scale is a millimetre and a half of pencil. That is exactly why it escapes him. A mistimed reading jumps off the line and asks to be looked at, while a wrongly measured length quietly tilts the whole line, and a ruled line through gently curved points always looks like a good result. His slope would come out about 2.5% high, which puts g near 9.53ms^{-2} and k near 0.203m.

Examiner's eye

- The holes must lie on a line through the centre of gravity. If they do not, h is not the distance from the pivot to G and the governing equation does not apply, however carefully you time.
- For a lamina swinging in its own plane both sides enter the radius of gyration, through $\sqrt{(a^2+b^2)/12}$. Using only the side along which the holes were punched gives 0.173m here, which looks plausible and is wrong by seventeen percent.
- This graph and the one in Experiment 8.3 are the same relation drawn the two possible ways round. Write down the rearranged equation before you decide which quantity multiplies and which divides.

Practice 8.4

h **(m)**	t **for 20 oscillations (s)**	T **(s)**	T^2 **(s^2)**	T^2h **(s^2m)**	h^2 **(m^2)**
0.550	33.33	1.667	2.779	1.5285	0.3025
0.500	32.24	1.612	2.599	1.2995	0.2500
0.450	31.40	1.570	2.465	1.1093	0.2025
0.400	30.78	1.539	2.369	0.9476	0.1600
0.350	30.17	1.509	2.277	0.7970	0.1225
0.300	29.87	1.494	2.232	0.6696	0.0900

(ii) A vertical scale of 2cm to $0.05m^2$ running from $-0.10m^2$ to $0.35m^2$, with a horizontal scale of 2cm to $0.2s^2m$ running from 0 to $1.6s^2m$. The vertical axis must go below zero because the intercept is negative.

(iii) Taking A ($0.68s^2m$, $0.0937m^2$) and B ($1.56s^2m$, $0.3128m^2$):

$$\text{slope}=\frac{0.3128m^2-0.0937m^2}{1.56s^2m-0.68s^2m}=0.249ms^{-2}$$

and the line meets the h^2 axis at $-0.0756m^2$.

(iv) The graph is $h^2=\frac{g}{4\pi^2}T^2h-k^2$, so the slope is $g/4\pi^2$ and

$$g=4\pi^2\times\text{slope}=39.4384\times0.249ms^{-2}=9.82ms^{-2}$$

Accept any value from $9.6ms^{-2}$ to $10.0ms^{-2}$.

(v) The intercept is $-k^2$, so

$$k^2=0.0756m^2, \qquad k=0.275m$$

(vi) Take moments about the new balance point, the 60.0cm mark. The weight of the rule acts at the 50.0cm mark, 10.0cm on one side of it, and the weight of the load acts at 10.0cm on one side of it, and the weight of the load acts at the 80.0cm mark, 20.0cm on the other. The g in each weight is the same and cancels, so the moments may be taken in masses directly:

$$m=\frac{100g\times10.0cm}{20.0cm}=50g$$

so the load is 50g, found without a balance and without any part of the graph. Accept any value from 45g to 55g.

(vii) Smaller, and the reason is worth setting out because the first guess usually goes the other way. Adding mass far from the centre of gravity raises the radius of gyration; adding it close lowers it. What decides the matter is not the distance from the old centre of gravity but the distance from the new one, set against what the rest of the body already has. The load sits 20.0cm from the new balance point. The rule itself, now centred 10.0cm from that same point, has a radius of gyration about it of

$$\sqrt{\frac{l^2}{12}+(0.100m)^2}=\sqrt{0.09333m^2}=0.306m$$

The load is the closer of the two, so it pulls the radius of gyration down. The measured 0.275m is indeed smaller than the 0.289m of the bare rule. Both figures are radii of gyration about each body's own centre of gravity, which is what makes them comparable at all.

Examiner's eye

- h is measured from the pivot to the centre of gravity of the *whole loaded system*, which is the balance point you found in step (b), not the 50cm mark. Every row of the table is wrong if this is wrong, and the graph will still look like a good straight line.
- Part (vi) uses no graph at all. Questions in this paper often carry one part that is pure moments or pure arithmetic; do it first, while you are fresh, rather than leaving it until the plotting has used up the time.
- "The load makes it heavier so it swings slower" is not an answer to part (vii) and it is not true either. The mass of a compound pendulum cancels out of its period. What the load changes is where the centre of gravity is and how the mass is spread about it, and the answer has to be in those terms.

Practice 9.1

m (kg)	**t for 30 oscillations (s)**	**T (s)**	**T^2 (s^2)**	**$1/T^2$ (s^{-2})**
0.020	37.27	1.242	1.543	0.648
0.040	34.61	1.154	1.332	0.751
0.060	32.26	1.075	1.156	0.865
0.080	30.62	1.021	1.042	0.960
0.100	28.97	0.966	0.933	1.072
0.120	27.52	0.917	0.841	1.189

(ii) A vertical scale of 2cm to $0.2s^{-2}$ running from 0 to $1.4s^{-2}$, with a horizontal scale of 2cm to 0.02kg running from 0 to 0.14kg, is 14cm by 14cm. Halving either scale to fill more of the sheet would give 28cm, which will not fit, so this is the largest square that can be drawn. Both axes must start at zero, because the intercept is half the answer.

(iii) Taking A (0.022kg, $0.656s^{-2}$) and B (0.122kg, $1.194s^{-2}$) on the best line:

$$\text{slope}=\frac{1.194s^{-2}-0.656s^{-2}}{0.122kg-0.022kg}=5.38s^{-2}kg^{-1}$$

and the line meets the $1/T^2$ axis at $0.538s^{-2}$.

(iv) The plasticine adds to the mass that gravity acts on but not to the moment of inertia, because it sits on the axis. So the moment of inertia stays $I=Mk^2$ while the restoring couple becomes $(M+m)gd^2/4L$, and

$$T^2=\frac{16\pi^2k^2L}{gd^2}\times\frac{M}{M+m}$$

Turning that upside down,

$$\frac{1}{T^2}=\frac{gd^2}{16\pi^2k^2L}\left(1+\frac{m}{M}\right)$$

which is a straight line against m. The intercept is the bracket with $m=0$, that is $gd^2/16\pi^2k^2L$, and the slope is that same quantity divided by M. Their ratio is therefore M itself.

$$M=\frac{\text{intercept}}{\text{slope}}$$

(v) From the graph,

$$M=\frac{0.538s^{-2}}{5.38s^{-2}kg^{-1}}=0.100kg=100g$$

Accept any value from 90g to 110g.

(vi) Because those three quantities appear in the intercept and in the slope in exactly the same combination, and the answer is the ratio of the two. Whatever $gd^2/16\pi^2k^2L$ happens to be, it divides out. That is worth more than a trick: an answer built from the ratio of a slope to an intercept on the same graph is free of every constant they share, which is why the ratio turns up again and again in this book.

(vii) Because the derivation needs the added mass to change the restoring couple without changing the moment of inertia, and only a mass on the axis does that. A mass a distance x from the centre adds mx^2 to the moment of inertia as well, in this case $m\times0.010m^2$. That is not a small addition: the bar itself has only $Mk^2=0.00833kgm^2$, so the misplaced mass increases it by nearly a seventh and the period of that one reading rises to match.

One misplaced mass spoils one reading, not the shape of the graph. The other five points are untouched, so what you would see is a single point sitting about $0.15s^{-2}$ below the line, which on the scale above is a centimetre and a half and impossible to miss. A line ruled to pass through it would be flatter than it should be, and since the mass of the bar is the intercept divided by the slope, the answer would come out too large. That is the reason for plotting rather than averaging: an averaged result would swallow the bad reading, and the graph puts it where you can see it.

Examiner's eye

- $1/T^2$ is the plotted quantity, so the table needs a T^2 column and then a reciprocal column. Compute the reciprocal from the T^2 you wrote down, not from an unrounded value in your calculator, or your points will not sit where your own table says they should.
- The intercept here is an answer, not a decoration. A vertical axis beginning at $0.6s^{-2}$ would give a handsome line and no mass at all.
- Put the added mass on the axis and say in your report that you did. An examiner reading "a mass was added to the bar" has no way of knowing whether you understood why the position mattered.

Practice 9.2

L (m)	√L (√m)	t for 20 oscillations (s)	T (s)
0.200	0.4472	20.67	1.034
0.300	0.5477	25.45	1.273
0.400	0.6325	29.27	1.464
0.500	0.7071	32.81	1.641
0.600	0.7746	36.07	1.804
0.700	0.8367	38.75	1.938

(ii) A vertical scale of 2cm to 0.2s running from 0 to 2.0s, with a horizontal scale of 2cm to $0.1\sqrt{m}$ running from 0 to $0.9\sqrt{m}$, is 20cm by 18cm. The six points lie on one straight line through the origin.

(iii) Taking A ($0.45\sqrt{m}$, 1.044s) and B ($0.85\sqrt{m}$, 1.972s):

$$\text{slope}=\frac{1.972s-1.044s}{0.85\sqrt{m}-0.45\sqrt{m}}=\frac{0.928s}{0.40\sqrt{m}}=2.32s/\sqrt{m}$$

(iv) Grouping the constants in the governing equation the other way,

$$T=\frac{4\pi k}{d}\sqrt{\frac{L}{g}}=\left(\frac{4\pi k}{d\sqrt{g}}\right)\sqrt{L}$$

so the slope is $4\pi k/(d\sqrt{g})$. Squaring it and putting $k^2=l^2/12$ in,

$$g=\frac{16\pi^2k^2}{d^2\times\text{slope}^2}=\frac{4\pi^2l^2}{3d^2\times\text{slope}^2}=9.77ms^{-2}$$

Accept any value from $9.6ms^{-2}$ to $10.0ms^{-2}$.

(v) At $L=0.450m$, $\sqrt{L}=0.671\sqrt{m}$, and reading the line at that value, or using the equation of the line,

$$T=2.32s/\sqrt{m}\times0.671\sqrt{m}=1.56s$$

Accept any value from 1.50s to 1.62s.

(vi) Its graph would be the same line, within experimental scatter. The mass of the bar appears in the moment of inertia and again in the restoring couple, and cancels between them, so a bar of the same length and shape gives the same period whatever it is made of. What would change the line is the length of the bar, which sets k.

(vii) His points would still lie on a straight line. Unequal threads do not unbalance the tensions, as Practical tip 4 of Experiment 9.1 shows; what they do is make the suspension behave as though both threads had the length halfway between the two. Every length he wrote down is therefore 0.010m shorter than the length the bar is actually using, and

$$T=\left(\frac{4\pi k}{d\sqrt{g}}\right)\sqrt{L+0.010m}$$

Over the range 0.200m to 0.700m that is still very nearly straight: the points bow away from a ruled line by about two thousandths of a second, which is smaller than his timing scatter and he would never see it. What he would see, if he looked, is that the line no longer starts at the origin. It meets the T axis about 0.04s above it, and its slope comes out about one percent low, so his g, which goes as the inverse square of the slope, is about two and a half percent too large.

The intercept is the whole of his warning. That is why both axes are started at zero here and why a line that misses the origin must never be forced through it: the miss is the measurement.

Examiner's eye

- $\sqrt{L}$ is a column of your table, not something to do on the graph paper. Give it a heading with a unit: the square root of a length is in $\sqrt{m}$, however odd that looks.
- Part (v) asks you to use the graph. Read it off the line, mark the reading on the graph with dashed lines, and say you have done so. Substituting into the equation instead is worth the same arithmetic and fewer marks.
- A straight line is not proof that the experiment was done properly; where the line sits is the other half of the evidence. Part (vii) is the whole lesson of this question, and it applies far beyond bifilar suspensions: the fault that hides completely in Experiment 9.1 shows here as an intercept, and only because both axes were started at zero.

Practice 9.3

L (m)	t for 20 oscillations (s)	T (s)	T^2 (s^2)
0.300	27.95	1.398	1.954
0.400	32.17	1.609	2.589
0.500	35.93	1.797	3.229
0.600	39.27	1.964	3.857
0.700	42.43	2.122	4.503
0.800	45.57	2.279	5.194

(ii) A vertical scale of 2cm to $0.5s^2$ running from 0 to $5.5s^2$, with a horizontal scale of 2cm to 0.1m running from 0 to 0.9m, is 22cm by 18cm. The six points lie on one straight line through the origin.

(iii) Taking A (0.31m, $2.003s^2$) and B (0.82m, $5.298s^2$):

$$\text{slope}=\frac{5.298s^2-2.003s^2}{0.82m-0.31m}=6.46s^2m^{-1}$$

(iv) Substituting the stiffness of the wire into the governing equation and squaring,

$$T^2=\frac{4\pi^2 I}{C}=\frac{4\pi^2 I\times 2L}{\pi\eta r^4}=\left(\frac{8\pi I}{\eta r^4}\right)L$$

so the slope is $8\pi I/\eta r^4$, and the line passes through the origin.

(v) With r=0.600mm, that is 6.00×10^{-4}m, and I equal to $1.20\times10^{-3}kgm^2$,

$$\eta=\frac{8\pi I}{r^4\times\text{slope}}=\frac{8\times3.14\times1.20\times10^{-3}kgm^2}{(6.00\times10^{-4}m)^4\times6.46s^2m^{-1}}$$

$$\eta=\frac{0.03014}{8.372\times10^{-13}}Nm^{-2}=3.60\times10^{10}Nm^{-2}$$

against an accepted value for brass of about $3.70\times10^{10}Nm^{-2}$. Accept any value from $3.30\times10^{10}Nm^{-2}$ to $4.10\times10^{10}Nm^{-2}$.

(vi) That the wire is doing all the twisting. If the chucks gripped the wire loosely, or if part of the twisting happened in the support, the effective length would be longer than the length you measured by a fixed amount, and the line would meet the T^2 axis above the origin rather than at it. A line through the origin is therefore not a formality here; it is the evidence that L is the length that matters.

(vii) A diameter of 1.25mm makes the radius 6.25×10^{-4}m, and

$$\eta=\frac{0.03014}{(6.25\times10^{-4}m)^4\times6.46s^2m^{-1}}=3.06\times10^{10}Nm^{-2}$$

which is fifteen percent lower, from a zero error of five hundredths of a millimetre. Check it against the rule of thumb: the diameter is wrong by 0.05mm in 1.20mm, four percent, and four times four is sixteen. That is the fourth power at work, and it is why the manager cannot settle his argument with an unchecked instrument. Two wires whose stiffness genuinely differs by a few percent cannot be told apart by an instrument whose zero error moves the answer by fifteen.

Examiner's eye

- The moment of inertia is given here because the rod cannot be taken apart. When a question hands you a constant, use it and say where it came from; do not try to derive it.
- r^4 means the arithmetic is unforgiving. Write the radius in metres in standard form before raising it to the fourth power, and count the powers of ten twice.
- Part (vii) is a calculation, not an opinion. "It would be less accurate" earns nothing; the number does.

Practice 9.4

n	Reading with no load (mm)	Reading with load (mm)	e (mm)
10	137.5	149.8	12.3
20	146.0	171.5	25.5
30	153.5	192.7	39.2
40	162.0	214.8	52.8
50	170.5	235.4	64.9
60	178.0	257.7	79.7

(ii) A vertical scale of 2cm to 10mm running from 0 to 90mm, with a horizontal scale of 2cm to 10 turns running from 0 to 70 turns, is 18cm by 14cm. The six points lie on one straight line through the origin.

(iii) Taking A (12, 15.8mm) and B (62, 81.4mm):

$$\text{slope}=\frac{81.4\text{mm}-15.8\text{mm}}{62-12}=\frac{65.6\text{mm}}{50}=1.31\text{mm}$$

The number of turns is a pure number, so the unit of the slope is millimetres and nothing else. Write it as 1.31mm per turn all the same: the words say what the number is millimetres of, and an answer that reads only 1.31mm looks like a length somebody forgot to finish.

(iv) The load F is the same for every spring, and so are D and d, so the whole of $8FD^3/Gd^4$ is a constant and

$$e=\left(\frac{8FD^3}{Gd^4}\right)n$$

which is e proportional to n: a straight line through the origin.

(v) The load is 0.200kg, so F is 1.962N; D is 1.40×10^{-2}m, d is 8.00×10^{-4}m, and the slope is 1.31mm per turn, that is 1.31×10^{-3}m per turn. Then

$$G=\frac{8FD^3}{d^4\times\text{slope}}=\frac{8\times1.962\text{N}\times\left(1.40\times10^{-2}\text{m}\right)^3}{(8.00\times10^{-4}\text{m})^4\times1.31\times10^{-3}\text{m}}$$

$$G=\frac{4.307\times10^{-5}}{5.366\times10^{-16}}\text{Nm}^{-2}=8.03\times10^{10}\text{Nm}^{-2}$$

against an accepted value for steel of about $8.00\times10^{10}\text{Nm}^{-2}$. Accept any value from $7.00\times10^{10}\text{Nm}^{-2}$ to $9.00\times10^{10}\text{Nm}^{-2}$.

(vi) That the hooks are not stretching. Every spring carries the same load and every hook is the same, so a hook that opened out would add the same extra extension to all six, and a fixed amount added to all six points lifts the line bodily off the origin. The line goes through the origin, so all the extension measured happened in the turns. Notice how much that argument owes to the hooks being alike: bent carelessly and differently, the same fault would have scattered the points rather than shifted the line, and the graph would have said nothing.

(vii) Dividing the extension wanted by the extension one turn provides:

$$n=\frac{100\text{mm}}{1.31\text{mm}}=76.3$$

so about 76 turns; turns come in whole numbers, so the decimal is not worth carrying. Treat it with caution because 100mm is past the end of the evidence: the largest extension measured was 79.7mm, on the 60-turn spring. A spring of 76 turns will follow the graph only if the wire is still elastic there and the turns still clear of one another, and both must be checked on the finished spring rather than assumed.

Examiner's eye

- Every spring in this question has the same load, the same wire and the same coil diameter. Say so before you claim the slope is a constant; the mark is for noticing what was held fixed, not for quoting the formula.
- The slope is in millimetres per turn and must be turned into metres per turn before it meets a diameter in metres: this is the one place in the question where a factor of a thousand can hide. Say the words as well, even though n carries no unit, because a bare 1.31mm in an answer about turns invites the reader to wonder what it is the length of.
- A number of turns is a counted quantity, so 76.3 turns is not an answer. Round it and say why you rounded it.
- Part (vii) is an extrapolation and the examiner knows it. The caution is worth as much as the number.

Practice 10.1

The pointer read 143.0mm with no load on the springs.

M (g)	F (N)	Pointer reading (mm)	e (mm)
20	0.196	169.4	26.4
40	0.392	196.5	53.5
60	0.589	222.0	79.0
80	0.785	248.8	105.8
100	0.981	274.6	131.6
120	1.177	302.6	159.6

(ii) A vertical scale of 2cm to 20mm running from 0 to 180mm, with a horizontal scale of 2cm to 0.2N running from 0 to 1.2N, is 18cm by 12cm. The six points lie on one straight line through the origin.

(iii) Taking A (0.3N, 40.5mm) and B (1.1N, 148.4mm) on the line:

$$\text{slope}=\frac{148.4\text{mm}-40.5\text{mm}}{1.1\text{N}-0.3\text{N}}=\frac{107.9\text{mm}}{0.8\text{N}}=135\text{mmN}^{-1}$$

(iv) Hang the load and it pulls on the lower spring; the lower spring pulls on the upper one with the same force, because it has nowhere else to put it. So the load adds the same force to both springs, and the force is not shared between them. Each stretches by that force divided by its own constant, and the extension of the pair is the sum of the two,

$$e=e_1+e_2=\frac{F}{k_1}+\frac{F}{k_2}=F\left(\frac{1}{k_1}+\frac{1}{k_2}\right)$$

and since the pair, considered as one spring of constant k, extends by F/k, the reciprocals add:

$$\frac{1}{k}=\frac{1}{k_1}+\frac{1}{k_2}$$

(v) The spring constant of the pair is the reciprocal of the slope, with the slope in metres per newton:

$$k=\frac{1}{0.135\text{mN}^{-1}}=7.41\text{Nm}^{-1}$$

Then, from part (iv),

$$\frac{1}{k_2}=\frac{1}{7.41\text{Nm}^{-1}}-\frac{1}{12.3\text{Nm}^{-1}}=0.0537\text{mN}^{-1}$$

$$k_2=18.6\text{Nm}^{-1}$$

Accept any value from 17.0Nm^{-1} to 20.0Nm^{-1}.

(vi) The first spring, the one whose constant is on the label. The load adds the same force to both springs and each extends by F/k, so the two extensions are in the inverse ratio of the constants: $\frac{18.6}{12.3}=1.51$, so the softer spring provides half again as much extension as the stiffer one, three fifths of the total against two fifths. The softer spring always takes the larger share, and it takes nearly all of it only when the other is very much stiffer: a spring half again as stiff, as here, still contributes two fifths of everything you measure.

(vii) Reciprocals add, and there are now two of the second spring:

$$\frac{1}{k}=\frac{1}{12.3\text{Nm}^{-1}}+\frac{2}{18.6\text{Nm}^{-1}}=0.1888\text{mN}^{-1}$$
$$k=5.30\text{Nm}^{-1}$$

and the slope of the new graph is that reciprocal written in millimetres per newton, 189mmN^{-1}. Notice how much the third spring costs. It is the stiffest of the three, and it still adds two fifths again to the extension the pair already had, taking them from 7.41Nm^{-1} down to 5.30Nm^{-1}. Every spring you add in series makes the arrangement softer, and the softest spring in it takes the largest single share of the extension: here the 12.3Nm^{-1} spring holds 43 percent of the total, against 57 for the two stiffer ones together.

Examiner's eye

- The load adds the same force to both springs. It is not halved, shared or divided in any way. Say that in one sentence and you have the whole of part (iv); the addition of the extensions follows from it in a line.
- A difference of two reciprocals carries fewer figures than the reciprocals do. Move the constant of the pair by one in its last figure and the answer for the second spring moves by about six hundredths of a newton per metre, which is already in its third figure. Do not quote it to four.
- The extension plotted is the extension of the pair. Nothing on this graph separates the two springs, and nothing needs to: the separation is done by arithmetic afterwards, using the constant that was on the label.
- Do not read the origin as evidence that the springs began unstretched. The upper spring is already carrying the lower one before you hang anything at all, and that extension sits inside your no-load reading, where it belongs. What a line through the origin does show is that the extension you measured is proportional to the force you added.

Practice 10.2

M (kg)	t for 30 oscillations (s)	T (s)	T^2 (s^2)
0.100	16.33	0.544	0.296
0.150	18.63	0.621	0.386
0.200	20.99	0.700	0.490
0.250	22.72	0.757	0.573
0.300	24.52	0.817	0.667
0.350	26.42	0.881	0.776

(ii) A vertical scale of 2cm to 0.1s^2 running from 0 to 0.9s^2, with a horizontal scale of 2cm to 0.05kg running from 0 to 0.40kg, is 18cm by 16cm. Start both axes at zero: the intercept is half of what this question asks for.

(iii) The cross-piece stays horizontal and the two springs are alike, so each of them carries half the load and each stretches by the same amount. If the load applies a force F altogether, each spring is stretched by $\frac{F}{2}$ and so extends by $\frac{F}{2k}$. The pair therefore behaves as a single spring of constant 2k: side by side, spring constants add. Each spring also contributes a third of its own mass to what is moving, so the two contribute $2m_s/3$ between them, and

$$T=2\pi\sqrt{\frac{M+2m_s/3}{2k}}$$

Squaring,

$$T^2=\frac{4\pi^2}{2k}M+\frac{4\pi^2}{2k}\times\frac{2m_s}{3}$$

so the slope is $4\pi^2/2k$ and the intercept on the T^2 axis is that slope multiplied by $2m_s/3$.

(iv) Taking A (0.110kg, 0.313s^2) and B (0.360kg, 0.788s^2) on the line:

$$\text{slope}=\frac{0.788\text{s}^2-0.313\text{s}^2}{0.360\text{kg}-0.110\text{kg}}=\frac{0.475\text{s}^2}{0.250\text{kg}}=1.90\text{s}^2\text{kg}^{-1}$$

and the intercept on the T^2 axis is 0.104s^2.

(v) The slope is $4\pi^2/2k$, so

$$2k=\frac{39.4384}{1.90\text{s}^2\text{kg}^{-1}}=20.8\text{Nm}^{-1}$$
$$k=10.4\text{Nm}^{-1}$$

Accept any value from 9.8Nm^{-1} to 11.0Nm^{-1}.

(vi) The intercept divided by the slope is $2m_s/3$, so

$$\frac{2m_s}{3}=\frac{0.104\text{s}^2}{1.90\text{s}^2\text{kg}^{-1}}=0.05474\text{kg}$$
$$m_s=\frac{3}{2}\times0.05474\text{kg}=0.0821\text{kg}=82\text{g}$$

The balance gives 83g, so the two are within a gram of each other, which is closer than they had any right to come. On this scale one small square is 0.01s^2, and an intercept of 0.104s^2 read to half a square carries about five percent, so 82g is worth two figures and no more. Quote it as two.

(vii) With one spring the stiffness is halved, which lengthens the period, but the springs' own contribution to the moving mass is halved too, from $2m_s/3$ to $m_s/3$, which shortens it. Using the values from this graph,

$$T=2\pi\sqrt{\frac{0.200\text{kg}+0.0274\text{kg}}{10.4\text{Nm}^{-1}}}=0.929\text{s}$$

against 0.700s measured with the two springs, a factor of 1.33 rather than $\sqrt{2}$. The square root of two would be right only if the moving mass were the same in both cases. It is not: taking one spring away removes 27.4g of effective mass along with half the stiffness, and the two effects pull the period in opposite directions.

Examiner's eye

- Two springs side by side make a stiffer arrangement, not a softer one, and their constants add. Say which way round it goes, and why, before you write 2k anywhere.
- The intercept holds $2m_s/3$: a third of each spring, and there are two of them. Multiply the ratio by three, as Experiment 10.2 does, and you report twice the spring; the factor here is three halves.
- Check that the springs really are alike before trusting the derivation: hang the same load on each in turn and compare the extensions. Two springs that differ take unequal shares of the load, the cross-piece tilts, and the load rocks as well as bounces.
- Part (vii) asks for a number and a reason. "It would be slower" earns nothing. The calculation earns everything, and the reason it is not $\sqrt{2}$ is worth as much as the calculation itself.

Practice 10.3

V (cm^3)	t for 30 oscillations (s)	T (s)	T^2 (s^2)
50	13.36	0.445	0.198
100	15.31	0.510	0.260
150	16.89	0.563	0.317
200	18.21	0.607	0.368
250	19.52	0.651	0.424
300	20.71	0.690	0.476

(ii) A vertical scale of 2cm to $50cm^3$ running from $-150cm^3$ to $350cm^3$, with a horizontal scale of 2cm to $0.1s^2$ running from 0 to $0.5s^2$, is 20cm by 10cm. The V axis must reach below zero, where the intercept lies.

(iii) Taking A ($0.22s^2$, $66cm^3$) and B ($0.46s^2$, $283cm^3$) on the line:

$$\text{slope}=\frac{283cm^3-66cm^3}{0.46s^2-0.22s^2}=\frac{217cm^3}{0.24s^2}=904cm^3s^{-2}$$

and the line meets the V axis at $-133cm^3$.

(iv) The load on the spring is the calorimeter, the liquid and a third of the spring, and a volume V of a liquid of density ρ has mass ρV. So

$$T^2=\frac{4\pi^2}{k}(m_c+\rho V+m_s/3)$$

and making V the subject gives

$$V=\frac{k}{4\pi^2\rho}T^2-\frac{m_c+m_s/3}{\rho}$$

whose slope is $k/4\pi^2\rho$, as required.

(v) The graph is in grams and cubic centimetres, so put the spring constant into those units first: $28.5Nm^{-1}$ is $28.5kgs^{-2}$, and a kilogram is a thousand grams, so k is $28500gs^{-2}$. Rearranging the slope for the density,

$$\rho=\frac{k}{4\pi^2\times\text{slope}}=\frac{28500gs^{-2}}{39.4384\times904cm^3s^{-2}}=0.799gcm^{-3}$$

The relative density is the same number without a unit, 0.799, because the density of water is $1.00gcm^{-3}$. Accept any value from $0.77gcm^{-3}$ to $0.83gcm^{-3}$: that is a marking tolerance, wide enough to cover the student's own plotting and line, and it is not the precision of a carefully drawn graph.

One carefully drawn graph does better. Two ordinates read to half a small square fix this slope, and with it the density, to about one and a half percent, which on 0.799 is about 0.012. So the answer is what kerosene should be, and the graph clears the kerosene of any water it could have found. It is worth asking how much that is. A tenth of the bottle as water would raise the relative density to about 0.82, which is nearly two of those steps and would have shown; a fortieth would move it by less than one step and would not.

What the graph cannot say is what is wrong instead. It has cleared the kerosene, and that is all it was asked to do.

(vi) The intercept multiplied by the density is the mass it stands for, so the calorimeter and a third of the spring come to $133cm^3$ times $0.799gcm^{-3}$, that is 106g. A third of 33g is 11g, so the calorimeter is 95g.

(vii) Water is denser than the liquid, so every volume would have hung a greater mass on the spring. The slope, which is $k/4\pi^2\rho$, would have been smaller in the ratio of the densities, about $722cm^3$ per s^2 instead of $904cm^3$ per s^2, so the line would have been less steep; and the intercept, which is $-(m_c+m_s/3)/\rho$, would have moved up towards the origin, to about $-106cm^3$, because the same mass of copper and steel is a smaller volume of water than it is of kerosene.

Examiner's eye

- The density comes out of the slope and the mass of the calorimeter out of the intercept. Two answers, two features, and neither can be got from the other: say which is which before you start substituting.
- A relative density has no unit and a density has one. Quote both and keep them apart; an answer of $0.799gcm^{-3}$ to a question about relative density has not read the question.
- The spring constant is given here because there is no way to get it from this graph without knowing the density first, and the density is what the question wants. When a question hands you a constant, that is usually the reason.
- Part (vii) asks about both features of the line. An answer that changes the slope and forgets the intercept has done half of it.

Practice 10.4

M (g)	Pointer reading x (mm)	t for 25 oscillations (s)	T (s)	T^2 (s^2)
100	172.5	12.67	0.507	0.257
200	227.2	17.17	0.687	0.472
300	281.6	20.76	0.830	0.689
400	335.0	23.94	0.958	0.918
500	389.2	26.67	1.067	1.138
600	444.4	28.96	1.158	1.341

(ii) A vertical scale of 2cm to 50mm running from 0 to 500mm, with a horizontal scale of 2cm to $0.2s^2$ running from 0 to $1.4s^2$, is 20cm by 14cm. Both scales run from zero, and this time the intercept sits above the origin rather than below it: the readings are measured down a rule whose zero is above the pointer, so the line meets the x axis at a positive reading.

(iii) The pointer reading is the no-load reading plus the extension, $x=x_0+e$, and Experiment 10.4 showed that $e=\frac{g}{4\pi^2}T^2-\frac{gm_s}{3k}$. Adding x_0 to both sides,

$$x=x_0+\frac{g}{4\pi^2}T^2-\frac{gm_s}{3k}$$

The unknown x_0 sits in the constant term with the other constants and nowhere else. It moves the whole line up or down and cannot change its steepness, so the slope is $g/4\pi^2$ whether you know x_0 or not.

(iv) Taking A ($0.50s^2$, 233mm) and B ($1.10s^2$, 382mm) on the line:

$$\text{slope}=\frac{382\text{mm}-233\text{mm}}{1.10\text{s}^2-0.50\text{s}^2}=\frac{149\text{mm}}{0.60\text{s}^2}=248\text{mms}^{-2}$$

and the line meets the x axis at 109mm.

(v) g is $4\pi^2$ times the slope:

$$g=39.4384\times0.248\text{ms}^{-2}=9.78\text{ms}^{-2}$$

Accept any value from 9.5ms^{-2} to 10.1ms^{-2}. One small square on this scale is 5mm, so the intercept is worth about 3mm either way and g two figures: the band is wide because the graph is coarse, not because the method is.

(vi) From part (iii) the intercept is $x_0-gm_s/3k$, so the correction to be added back is

$$\frac{gm_s}{3k}=\frac{9.78\text{ms}^{-2}\times0.048\text{kg}}{3\times18.0\text{Nm}^{-1}}=8.7\text{mm}$$

$$x_0=109\text{mm}+8.7\text{mm}=117.7\text{mm}$$

which is 118mm to the nearest millimetre, and part (v) has already said that this graph is not worth more than that.

(vii) Nothing that Practice 10.4 itself needed. The slope is untouched, so g is untouched, and the reading itself has come back out of the intercept. But look at what that recovery leaned on: the spring constant and the mass of the spring, both taken from the question. An extension column built from a no-load reading obtained that way could not then be used to test F=ke, because k has already been assumed in order to get it. So what has been lost is the evidence, and in Experiment 10.1 the evidence was the whole of parts (iii) and (vi): the line through the origin, which is what says the extension is proportional to the force, and the second no-load reading at the end, which is what says the spring came back. Both need a zero measured at the time. He can find g without one. He could not have verified Hooke's law without one.

Examiner's eye

- Plot what you have. The pointer reading is a perfectly good quantity to put on an axis, and inventing a no-load reading to subtract from it is worse than leaving it out: the invented number would go into every row and could not be checked.
- The intercept is not the no-load reading. It is the no-load reading less $gm_s/3k$, so the correction has to be added back, and the question gives you the two constants you need for it.
- Say how many oscillations you timed. Twenty-five is not the number used earlier in the chapter, and a reader who assumes thirty divides by the wrong number: every period comes out a sixth too small, every T^2 at about seven tenths of its value, and g nearly half again too large.
- A slope that survives a missing zero is not the same as an experiment that survives one. Part (vii) wants both halves of that: what is safe, and what is gone.

Practice 11.1

The projecting length was 55.0cm throughout, the breadth of the rule 25.6mm and its thickness 5.72mm.

M (g)	M (kg)	t for 50 vibrations (s)	T (s)	T^2 (s^2)
60	0.060	10.69	0.214	0.0458
100	0.100	13.03	0.261	0.0681
140	0.140	15.08	0.302	0.0912
180	0.180	17.07	0.341	0.1163
220	0.220	18.66	0.373	0.1391
260	0.260	20.37	0.407	0.1656

(i) and (ii). The table is printed above. A vertical scale of 2cm to $0.02s^2$, running from $0s^2$ to $0.18s^2$, with a horizontal scale of 2cm to 0.05kg, running from 0kg to 0.30kg, is 12cm by 18cm, which fits an A4 sheet upright. The vertical axis must start at zero, because part (iii) asks for the intercept and part (v) asks what it means.

(iii) Taking A (0.035kg, $0.0296s^2$) and B (0.245kg, $0.1552s^2$) on the line:

$$\text{slope}=\frac{0.1552\text{s}^2-0.0296\text{s}^2}{0.245\text{kg}-0.035\text{kg}}=\frac{0.1256\text{s}^2}{0.210\text{kg}}=0.598\text{s}^2\text{kg}^{-1}$$

and the line meets the T^2 axis at $0.0087s^2$.

(iv) With the projecting length fixed, $T^2=4\pi^2M/k$ and $k=Ebd^3/4L^3$, so the slope is $16\pi^2L^3/Ebd^3$ and

$$E=\frac{16\pi^2L^3}{\text{slope}\times bd^3}$$

With $L^3=(0.550\text{m})^3=0.1664\text{m}^3$ and $bd^3=0.0256\text{m}\times\left(5.72\times10^{-3}\text{m}\right)^3=4.791\times10^{-9}\text{m}^4$,

$$E=\frac{16\times3.14^2\times0.1664\text{m}^3}{0.598\text{s}^2\text{kg}^{-1}\times4.791\times10^{-9}\text{m}^4}=9.16\times10^9\text{Nm}^{-2}$$

Accept any value from $8.8\times10^9\text{Nm}^{-2}$ to $9.5\times10^9\text{Nm}^{-2}$. That is a marking tolerance, not the precision of one carefully drawn graph.

(v) The rule itself. The load is not the only thing that has to be accelerated: the projecting part of the rule moves too, fastest at the free end and not at all at the clamp, so some share of its mass is being carried up and down with every vibration. Call that share m_e. Then the mass in the period formula is $M+m_e$ and not M, so

$$T^2=\frac{16\pi^2L^3}{Ebd^3}(M+m_e)$$

The bracket is the only place the load appears, so the graph is still straight and the slope is untouched. What m_e does is lift the whole line: the intercept is the slope multiplied by m_e, and it is the one place in this experiment where the rule's own mass shows itself.

(vi) From part (v) the intercept is $\text{slope}\times m_e$, so

$$m_e=\frac{0.0087\text{s}^2}{0.598\text{s}^2\text{kg}^{-1}}=0.0145\text{kg}=14.5\text{g}$$

and the fraction of the projecting rule that rides with the load is

$$\frac{14.5\text{g}}{61.6\text{g}}=0.235$$

against $33/140=0.236$, which is the value theory gives for a cantilever loaded at its free end. The agreement is better than the reading deserves. The intercept here is about four small squares above the origin and can be read to perhaps a fifth of one, so m_e is good to about five percent, and anything from 0.22 to 0.25 should be read as agreement.

(vii) It would fall to a sixteenth of its present value. The intercept is $16\pi^2L^3m_e/Ebd^3$, and both of the quantities in it that depend on the length change together: the slope goes as L^3 and so is divided by eight, while m_e is a fixed fraction of the mass of the projecting part and so is halved. Eight times two is sixteen. The same halving divides the slope by eight, so the two together make the intercept very much harder to read, which is a good reason for choosing a long projecting length and light loads when the rule's own mass is what you are after.

Examiner's eye

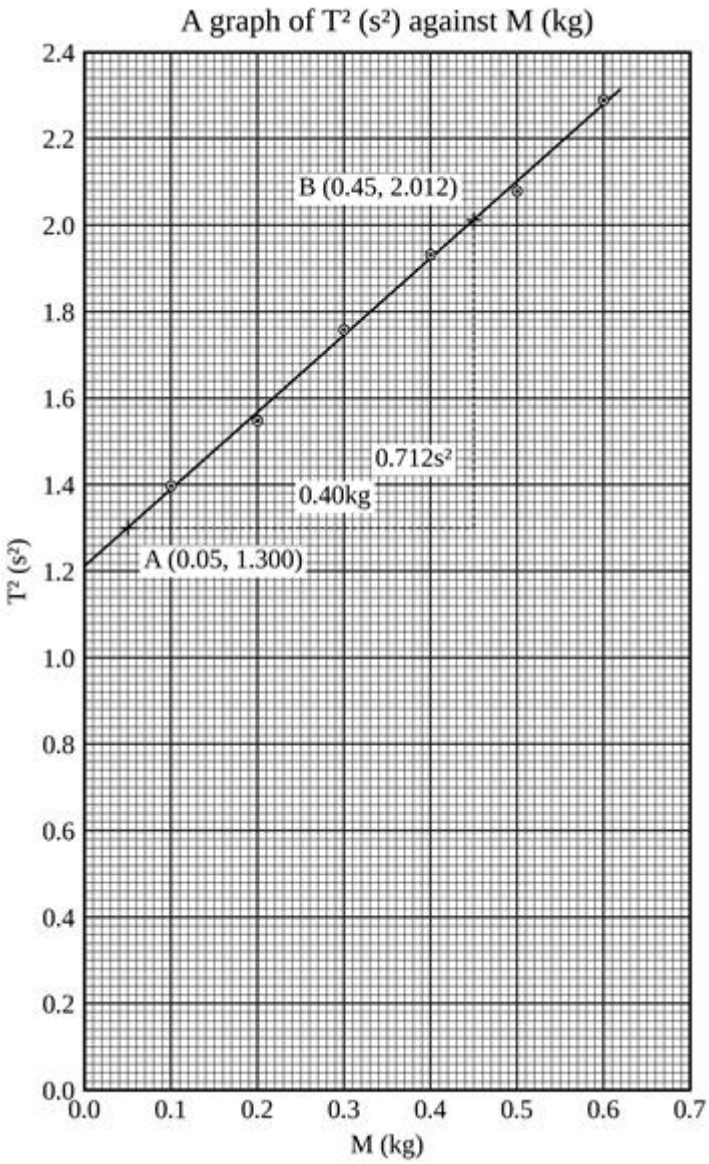
- Give the vertical axis a scale that reaches the axis it is to be read against. A vertical scale beginning at 0.04s^2 would fit these six points beautifully and would leave parts (v) and (vi) with nothing to work on.
- The slope of this graph does not contain the rule's own mass. That is worth saying in part (v), because it is the reason Young's modulus from part (iv) can be trusted even though the intercept shows that something has been left out of the load column.
- Do not add m_e into the mass column and replot. The whole method depends on m_e being unknown at the start and coming out of the intercept at the end; put it in by hand first and you have assumed the answer.
- A fraction has no unit and does not want one. Part (vi) ends in a pure number, and a pure number quoted as 0.235kg has stopped meaning anything.

Practice 11.2

The projecting length was 24.0cm and the thread length 30.0cm throughout.

M (g)	M (kg)	t for 25 oscillations (s)	T (s)	T^2 (s^2)
100	0.100	29.54	1.182	1.397
200	0.200	31.10	1.244	1.548
300	0.300	33.15	1.326	1.758
400	0.400	34.76	1.390	1.932
500	0.500	36.04	1.442	2.079
600	0.600	37.83	1.513	2.289

(i) and (ii). The table is printed above. A vertical scale of 2cm to 0.20s^2, running from 0s^2 to 2.40s^2, with a horizontal scale of 2cm to 0.10kg, running from 0kg to 0.70kg, is 14cm by 24cm, which fits an A4 sheet upright. The vertical axis has to start at zero and the points then sit in the upper half of it, which is the price of an experiment whose intercept is the answer to part (v).



(iii) Taking A (0.05kg, $1.300s^2$) and B (0.45kg, $2.012s^2$) on the line:

$$\text{slope}=\frac{2.012s^2-1.300s^2}{0.45kg-0.05kg}=\frac{0.712s^2}{0.40kg}=1.78s^2kg^{-1}$$

and the line meets the T^2 axis at $1.211s^2$.

(iv) With h fixed, the first term is a constant and the slope is $4\pi^2/k$, so

$$k=\frac{4\times3.14^2}{1.78s^2kg^{-1}}=22.2Nm^{-1}$$

and then

$$E=\frac{4kL^3}{bd^3}=\frac{4\times22.2Nm^{-1}\times0.013824m^3}{4.688\times10^{-10}m^4}=2.62\times10^9Nm^{-2}$$

using $bd^3=0.0300m\times\left(2.50\times10^{-3}m\right)^3$ and $L^3=(0.240m)^3$. Accept any value from $2.4\times10^9Nm^{-2}$ to $2.9\times10^9Nm^{-2}$. That is a marking tolerance, not the precision of one carefully drawn graph.

(v) The intercept is $\left(4\pi^2/g\right)h$, so

$$g=\frac{4\times3.14^2\times0.300m}{1.211s^2}=9.77ms^{-2}$$

That is four parts in a thousand below the accepted $9.81ms^{-2}$, and the intercept cannot be read to anything like that precision: half a small square on this scale is $0.01s^2$, which is one part in a hundred of the intercept and so one part in a hundred of g. Accept any value from $9.5ms^{-2}$ to $10.1ms^{-2}$. That is a marking tolerance, not the precision of one carefully drawn graph.

(vi) The slope, and with it k and E, both of which come out a little small. Part of the rule's own mass has to be set moving as well as the load, exactly as in Practice 11.1. But here the rule's free end does not travel as far as the load does: the load hangs on a thread, and the rule takes only the share of the movement its own softness allows it. That share is not the same in every row. A heavy load bends the rule further than a light one, so the rule's own mass counts for more at the heavy end of the table than at the light end, and an effect that grows with M goes into the slope and not into the intercept.

It is a small effect, a few parts in a thousand, and it takes the modulus down and not up. The intercept, and with it the acceleration due to gravity, is very nearly untouched by it, which is worth saying plainly: the value above sits below the accepted one because six points scatter, and not because the rule has a mass.

(vii) The slope would be an eighth of what it is and the intercept would be unchanged. The stiffness goes as $1/L^3$, so halving the projecting length multiplies k by eight, and the slope, which is $4\pi^2/k$, is divided by eight: about $0.223s^2kg^{-1}$. The intercept would be very nearly where it is: it is built of h and g, neither of which has been touched, and the small share belonging to the rule itself, which part (vi) is about, would shrink to a sixteenth and take its error with it. So the new line would start from almost the same place and climb far more slowly. It would be a worse experiment, because the six points would be crammed into a band an eighth as deep and the slope would have to be read across that.

Examiner's eye

- Look at what is being varied before you decide what the slope means. The same two-term equation gives $4\pi^2/g$ for a slope when the thread is varied and $4\pi^2/k$ when the load is varied, and only the question tells you which experiment you are in.
- A graph whose intercept is larger than its whole rise is telling you something, not failing. It means the pendulum is doing most of the work and the rule very little, and it is why the answer for k out of this graph deserves fewer figures than the answer for g.
- Keep the thread the same length from the first reading to the last. It is the constant that the intercept is made of, and a thread retied a centimetre shorter half way down the table moves the intercept without moving any single point far enough to look wrong.
- Every abscissa in this table is a mass you read off a piece of brass, hanger included. Leave the hanger out and all six are short by the same amount, which slides the line sideways: the slope survives it and the intercept does not, so the modulus lives and g dies.

Practice 11.3

The knife edges were 80.0cm apart throughout, the breadth of the rule was 25.2mm and its thickness 5.40mm. The pointer read 98.0mm with no load on the rule.

M (g)	M (kg)	Reading, loaded (mm)	s (mm)
400	0.400	112.5	14.5
600	0.600	120.0	22.0
800	0.800	126.0	28.0
1000	1.000	135.0	37.0
1200	1.200	140.5	42.5
1400	1.400	149.5	51.5

(i) and (ii). The table is printed above. A vertical scale of 2cm to 10mm, running from 0mm to 60mm, with a horizontal scale of 2cm to 0.20kg, running from 0kg to 1.60kg, is 16cm by 12cm, which fits an A4 sheet.

(iii) The points lie on a straight line through the origin, so $s \propto M$: the depression is proportional to the load, which is Hooke's law for a beam and the reason the plank is a weighing machine. Taking A (0.1kg, 3.6mm) and B (1.3kg, 47.4mm) on the line:

$$\text{slope}=\frac{47.4\text{mm}-3.6\text{mm}}{1.3\text{kg}-0.1\text{kg}}=\frac{43.8\text{mm}}{1.2\text{kg}}=36.5\text{mmkg}^{-1}$$

(iv) The slope of a graph of s against M is $gL^3/4Ebd^3$, so

$$E=\frac{gL^3}{4\times\text{slope}\times bd^3}=\frac{9.81\text{ms}^{-2}\times0.512\text{m}^3}{4\times0.0365\text{mkg}^{-1}\times3.968\times10^{-9}\text{m}^4}=8.67\times10^9\text{Nm}^{-2}$$

with the slope written as 0.0365mkg^{-1} and $bd^3=0.0252\text{m}\times\left(5.40\times10^{-3}\text{m}\right)^3=3.968\times10^{-9}\text{m}^4$. Accept any value from $8.2\times10^9\text{Nm}^{-2}$ to $9.2\times10^9\text{Nm}^{-2}$. That is a marking tolerance, not the precision of one carefully drawn graph.

(v) They do not. The load sits between the knife edges and the reactions are at the knife edges, so beyond them there is nothing left to bend the wood: the projecting ends carry no part of the load, whatever their length. What sag they have is their own, and they had it before the load was hung on and have it still, so it sits inside the no-load reading, where it does no harm. The length in the formula is the distance between the supports, and no other length of the rule enters it.

(vi) Because nothing that could change it was changed. The rule's own sag depends on the span and on the rule, and the span was set once and left alone, so the position of the pointer with no load was the same for every row and one reading of it does for all six. In Experiment 11.3 the projecting length was altered at every step and the rule's own sag altered with it, which is why a fresh no-load reading was needed each time.

(vii) From the slope,

$$M=\frac{25.0\text{mm}}{36.5\text{mmkg}^{-1}}=0.685\text{kg}$$

and the experiment supports it, because 0.685kg lies inside the range of loads actually used, between the 600g reading and the 800g one, and the two depressions on either side of it, 22.0mm and 28.0mm, straddle the wanted one. This is reading off the line rather than extending it, which is the safest prediction a single graph can make; Experiment 11.3 (vii) does the other kind and shows what it costs.

Examiner's eye

- The L in this formula is the distance between the supports. Use the length of the rule and the modulus comes out nearly twice as large, and the arithmetic will not warn you, because a metre and eighty centimetres are the same sort of number.
- A beam on two supports is not a cantilever with the numbers changed. The same rule, the same span and the same load give a depression a sixteenth as large, so a formula borrowed from Experiment 11.3 will hand you a modulus sixteen times out.
- Read the pointer with your eye level with the pin, and say in your precautions that you did. It is worth a mark, and on depressions of a few millimetres it is worth rather more than a mark.
- Check the load ladder before you trust the line. The heaviest load here bends the rule by about a sixteenth of the span, which is still inside the range where the depression is proportional to the load; take it much further and the line begins to bend and the slope stops meaning anything.

Practice 12.1

The known mass hung at a fixed distance b of 15.0cm from the knife edge throughout, and the rule balanced at the 49.7cm mark with nothing hanging on it.

m (g)	a (cm)
40	13.0
60	19.7
80	26.4
100	32.3
120	38.8
140	45.9

(i) and (ii). A horizontal scale of 2cm to 20g, running from 0g to 140g, with a vertical scale of 2cm to 5cm, running from 0cm to 50cm, is 14cm by 20cm, which fits an A4 sheet upright. Give the graph the title A graph of a (cm) against m (g).

(iii) The six points lie on a straight line through the origin, so $a \propto m$: the heavier the known mass, the further out the unknown has to go to balance it, and in the same proportion. The line passes through the origin because with no known mass on the rule there is nothing to balance and no distance to record.

(iv) Taking two well separated points on the line, A (10g, 3.3cm) and B (110g, 35.9cm),

$$\text{slope}=\frac{35.9\text{cm}-3.3\text{cm}}{110\text{g}-10\text{g}}=\frac{32.6\text{cm}}{100\text{g}}=0.326\text{cmg}^{-1}$$

(v) Taking moments about the knife edge, $Xa=mb$, so $a=bm/X$ and the slope is b/X. Hence

$$X=\frac{b}{\text{slope}}=\frac{15.0\text{cm}}{0.326\text{cmg}^{-1}}=46.0\text{g}$$

(vi) What sets the limit is the rule. The knife edge is close to the middle of a metre rule, so no loop can be hung more than about 50cm from it, and a grows in step with the known mass. Reading the line the other way round, $m=a/0.326\text{cmg}^{-1}$, so 50cm corresponds to about 153g. The heaviest reading taken, 140g, already puts the unknown at 45.9cm, and a known mass of 160g would want it off the end of the rule altogether. Notice that the limit is not a limit on how heavy an unknown body may be: a heavier body would simply be hung closer in, and Experiment 12.1 hangs one at 10.0cm.

Examiner's eye

- The question asks for a against m, so the known mass goes along the horizontal axis. Turn the graph the other way up and the slope becomes X/b, which is easy to forget by the time you come to divide.
- This slope does have a unit, and it is cmg^{-1}. The one in Experiment 12.1 does not. Work out the unit of every slope from the two axes rather than remembering it from the last question.
- The known mass is the one that stays where it is put, so the fixed distance b belongs to it and the unknown is the one that moves. Put b into the wrong side of the equation and the answer comes out as 4.89g, which is a mass you could balance on a fingernail and should stop the pen.
- Say where the balance point of the empty rule was. It is a reading like any other, it takes one line, and without it the account does not show that the rule's own mass was dealt with.

Practice 12.2

The load hung at the mark at which the scale begins throughout, and x was measured from that loop to the knife edge.

m (kg)	x (cm)	x (m)	1/x (per metre)
0.020	40.7	0.407	2.457
0.040	35.0	0.350	2.857
0.060	30.8	0.308	3.247
0.080	26.8	0.268	3.731
0.100	24.5	0.245	4.082
0.120	21.9	0.219	4.566

(i) and (ii) A horizontal scale of 2cm to 0.02kg, running from 0kg to 0.12kg, with a vertical scale of 2cm to 0.5m^{-1}, running from 0m^{-1} to 5.0m^{-1}, is 12cm by 20cm, which fits an A4 sheet upright. Give the graph the title A graph of 1/x (m^{-1}) against m (kg). The six points then sit in the upper half of the sheet with a great deal of empty paper below them, and that is correct: the vertical axis must start at zero, because part (iv) asks for the intercept on it. Start it at 2.4m^{-1} to spread the points out and the intercept is gone, and with it both answers.

(iii) $1/x$ increases linearly with m, and the line does not pass through the origin: it meets the vertical axis at a positive value.

(iv) Taking two well separated points on the line, A ($0.010\text{kg}, 2.23\text{m}^{-1}$) and B ($0.110\text{kg}, 4.33\text{m}^{-1}$),

$$\text{slope}=\frac{4.33\text{m}^{-1}-2.23\text{m}^{-1}}{0.110\text{kg}-0.010\text{kg}}=\frac{2.10\text{m}^{-1}}{0.100\text{kg}}=21.0\text{m}^{-1}\text{kg}^{-1}$$

and the line cuts the vertical axis at 2.02m^{-1}.

(v) Taking moments about the knife edge, $mx=M_r(x_G-x)$. Divide through by $M_r x_G x$ and

$$\frac{1}{x}=\frac{1}{M_r x_G}m+\frac{1}{x_G}$$

so the intercept is $1/x_G$ and the slope is $1/M_r x_G$. Hence

$$x_G=\frac{1}{2.02\text{m}^{-1}}=0.495\text{m}$$

$$M_r=\frac{1}{21.0\text{m}^{-1}\text{kg}^{-1}\times 0.495\text{m}}=0.0962\text{kg}$$

The rule has a mass of 96.2g and its centre of gravity is 49.5cm from the loop.

(vi) With x equal to 0.250m, $1/x$ is 4.00m^{-1}. Read across from 4.00m^{-1} on the vertical axis to the line and down to the horizontal axis, or use the equation of the line:

$$m=\frac{4.00\text{m}^{-1}-2.02\text{m}^{-1}}{21.0\text{m}^{-1}\text{kg}^{-1}}=0.0943\text{kg}$$

A load of about 94g. The reading is taken between the fourth and fifth points of the line rather than beyond either end of it, which is the safest prediction a single graph can make.

Examiner's eye

- Turn the centimetres into metres before taking the reciprocal, not after. $1/24.5\text{cm}$ is 0.0408cm^{-1} and 4.08m^{-1}, and a table with a column of the first and a heading claiming the second is the commonest single fault on this question.
- A reciprocal column needs one more figure than the reading it came from: x to three figures gives $1/x$ to four, and the whole of the answer lives in the differences between those numbers.
- The unit of a slope is the unit of the vertical axis divided by the unit of the horizontal one, whatever the algebra afterwards looks like. Here that is $\text{m}^{-1}\text{kg}^{-1}$.
- Both constants of the rule come out of this line, and x_G has to be found first because the slope contains both of them. Divide the slope into the intercept the wrong way round and the rule comes out at 10.4kg.

Practice 12.3

The known mass was 100g throughout, and the rule balanced at the 50.8cm mark with nothing hanging on it.

a (cm)	b in air (cm)	b in water (cm)
10.0	10.3	7.5
14.0	14.8	10.6
18.0	19.1	13.4
22.0	22.8	16.6
26.0	27.3	19.8
30.0	31.8	22.3

(i) and (ii). A horizontal scale of 2cm to 5cm, running from 0cm to 35cm, with a vertical scale of 2cm to 5cm, running from 0cm to 25cm, is 14cm by 10cm, which fits an A4 sheet upright. Give the graph the title A graph of b_w (cm) against b_a (cm). Notice that the distance a appears nowhere on this graph: it has been used to make the readings and then left behind.

(iii) The six points lie on a straight line through the origin, so $b_w \propto b_a$: the two balancing distances are in a fixed ratio, whatever distance the solid itself is hung at.

(iv) Taking two well separated points on the line, A (7.5cm, 5.4cm) and B (29.5cm, 21.1cm),

$$\text{slope}=\frac{21.1\text{cm}-5.4\text{cm}}{29.5\text{cm}-7.5\text{cm}}=\frac{15.7\text{cm}}{22.0\text{cm}}=0.714$$

a ratio of two lengths, with no unit.

(v) Taking moments in air and again in water and dividing one balance by the other, the known mass and the distance a both cancel and

$$\frac{b_w}{b_a}=\frac{m_s-\rho_w V}{m_s}$$

The slope is therefore $1-\rho_w V/m_s$, and the relative density of the solid is $m_s/\rho_w V$, which is one divided by what the slope has taken away from one:

$$\text{relative density}=\frac{1}{1-0.714}=\frac{1}{0.286}=3.50$$

Accept 3.3 to 3.7. That is a marking tolerance, not the precision of one carefully drawn graph.

(vi) The solid displaces its own volume of water, 30.0cm^3, which has a mass of 30.0g. Its own mass is that multiplied by the relative density:

$$m_s=3.50\times 30.0\text{g}=105\text{g}$$

Examiner's eye

- A slope of 0.714 is not a relative density and nothing on the page says it is. It is one minus the reciprocal of the relative density, and the arithmetic that turns it into 3.50 is the whole of part (v).
- The known mass never enters the answer here, because it cancels when one balance is divided by the other. That is worth a sentence in your account and it is worth checking against Experiment 12.3, where it does not cancel and 100g is used twice.
- Plot the water reading on the vertical axis, as the question asks. The other way up the slope is 1.40, the same arithmetic gives -2.5, and no solid has a negative relative density.
- Every point on this line came from two balances, so a point that sits badly is telling you that one of them was disturbed. Go back and take that pair again rather than drawing the line to please it.

Practice 12.4

The known mass was 100g throughout, and the rule balanced at the 50.2cm mark with nothing hanging on it.

a (cm)	b in air (cm)	b in water (cm)	b in liquid Q (cm)	air − water (cm)	air − liquid Q (cm)
10.0	12.8	8.5	7.7	4.3	5.1
14.0	18.2	12.1	11.2	6.1	7.0
18.0	22.8	16.1	14.7	6.7	8.1
22.0	28.3	19.4	17.4	8.9	10.9
26.0	33.0	23.0	20.7	10.0	12.3
30.0	38.3	26.4	24.3	11.9	14.0

(i) and (ii) Take both axes at 2cm to 2cm, running from 0cm to 16cm on each. That is 16cm by 16cm, which fits an A4 sheet upright. Give the graph the title A graph of b_a-b_Q (cm) against b_a-b_w (cm).

(iii) b_a-$b_Q \propto b_a$-b_w: the two differences keep a fixed ratio at every distance the solid is hung at, and both vanish together, because at zero distance nothing is balancing anything.

(iv) Taking two well separated points on the line, A (3.0cm, 3.6cm) and B (11.6cm, 13.9cm),

$$\text{slope}=\frac{13.9\text{cm}-3.6\text{cm}}{11.6\text{cm}-3.0\text{cm}}=\frac{10.3\text{cm}}{8.6\text{cm}}=1.20$$

A length divided by a length: no unit.

(v) Each balancing distance is proportional to the mass the thread is carrying, so a difference of two balancing distances is proportional to the upthrust that separates them. The numerator is proportional to the upthrust in the liquid Q and the denominator to the upthrust in water, and both upthrusts act on the same volume, so

$$\frac{b_a-b_Q}{b_a-b_w}=\frac{\rho_Q}{\rho_w}$$

The slope is the relative density of the liquid Q, read straight off the graph: 1.20. Accept 1.17 to 1.23. That is a marking tolerance, not the precision of one carefully drawn graph.

(vi) Both were disposed of by division. Write the balance for each of the three states, subtract one from another, and you are left with $b_a-b_Q=a\rho_Q V/m_k$ and $b_a-b_w=a\rho_w V/m_k$, and the two share the factor aV/m_k exactly. Divide one by the other and the known mass, the volume of the solid and the distance a all go together, which is why this graph needs neither a density for water nor a mass for the solid, and why the answer is a pure ratio.

Examiner's eye

- Subtract in the same order in both columns. One column taken as b_a-b_w and the other as b_Q-b_a gives a line of negative slope and a relative density of -1.20, which no liquid has.
- The differences are small numbers made out of large ones, so they need every figure the rule will give. A balancing distance read to the nearest half centimetre turns a difference of 4.1cm into one of 4.0cm or 4.5cm, and the answer moves with it.
- This plot gives the relative density and nothing else. If the question also asks for the mass or the volume of the solid, those come from the three original lines, and Experiment 12.4 shows how.
- Say what your method can and cannot settle. Milk is about 1.03 and watering it moves that by a few thousandths, while this graph is worth a few parts in a hundred; it would catch a churn of water with milk in it and would say nothing useful about a churn of milk with water in it. What the dairy wants is a hydrometer, and this experiment is how a hydrometer gets its scale.

Practice 13.1

The calorimeter and its stirrer had a mass of 74.0g, it held 160g of water, and the temperature of the room was 27.0°C.

t (min)	θ (°C)	t (min)	θ (°C)
0	85.0	14	62.0
2	81.5	16	60.0
4	77.5	18	58.0
6	74.5	20	55.0
8	70.5	22	53.5
10	67.5	24	52.0
12	65.0		

(i) and (ii) Take t at 2cm to 4 minutes, running from 0 to 24 minutes, and θ at 2cm to 5°C, running from 45°C to 90°C. That is 12cm by 18cm, which fits an A4 sheet upright. Give the graph the title A graph of θ (°C) against t (min).

(iii) The curve passes 60.0°C a little before 16 minutes. Taking two well-separated points on the tangent drawn there, A (3.2, 75.0) and B (23.2, 51.4),

$$\text{slope}=\frac{51.4°\text{C}-75.0°\text{C}}{23.2\text{min}-3.2\text{min}}=\frac{-23.6\text{K}}{20.0\text{min}}=-1.18\text{Kmin}^{-1}$$

The rate of cooling at 60.0°C is 1.18Kmin^{-1}. Accept 1.13Kmin^{-1} to 1.23Kmin^{-1}. That is a marking tolerance, not the precision of one carefully drawn graph.

(iv) The heat capacity of the calorimeter and its contents is

$$74.0\text{g}\times0.38\text{J/gK}+160\text{g}\times4.2\text{J/gK}=28.12\text{JK}^{-1}+672\text{JK}^{-1}=700.12\text{JK}^{-1}$$

so the rate of loss of heat at 60.0°C is

$$700.12\text{JK}^{-1}\times1.18\text{Kmin}^{-1}=826\text{Jmin}^{-1}$$

and turning minutes into seconds,

$$\frac{826\text{J}}{60\text{s}}=13.8\text{Js}^{-1}$$

(v) This one comes out of the table and not off the graph. The water was at 85.0°C at the start and at 70.5°C after 8 minutes, a fall of 14.5K, and every kelvin of that fall sent 700.12J to the room. So

$$700.12\text{JK}^{-1}\times14.5\text{K}=10152\text{J}$$

The fall of 14.5K is a difference of two readings each good to half a degree, so it is worth about 2% and no more. Quote the heat lost as 1.02×10^{4}J and stop there.

(vi) Because the rate of loss of heat is not the same throughout those 8 minutes. You may multiply a rate by a time only when the rate holds for the whole of the time, and this one does not: the answer to (iv) belongs to the instant the water passes 60.0°C, which is more than 15 minutes into the run, while during the first 8 minutes the water was between 85.0°C and 70.5°C and losing heat faster all the way. The product $826\text{Jmin}^{-1}\times8\text{min}=6608\text{J}$ is a third short of the true figure for exactly that reason.

(vii) Record every 2 minutes rather than every 5 and you have more points to fix the curve by, and a curve fixed by thirteen points can be drawn far more confidently through a bend than one fixed by 5, which matters because a tangent is only as good as the curve under it. Record every 30 seconds and you make the opposite mistake. At the end of this run the water falls by less than half a degree in 30 seconds, which is one division of the thermometer, so successive readings would repeat themselves and then jump; the points would scatter about the curve instead of defining it, and the sheet would be full of readings that carry no extra information at all.

Examiner's eye

- Parts (iv) and (v) look alike and are not. One is a rate, in joules per second, and comes from a tangent; the other is a quantity of heat, in joules, and comes from two rows of the table. Check the unit of your answer against the unit the question asks for before you move on.
- Read the fall of temperature in part (v) from the printed table and not from the smooth curve you drew. The curve is your estimate of what the readings mean; the readings are what happened.
- Spread the two points you read the tangent between as widely as the drawn line allows. Take them 4 minutes apart and the rise is only 4.7K, and one small square of uncertainty at each end of that is 20% of the answer before you have made any other mistake; the same square at the ends of the 20 minutes marked here is 4%.
- Keep the masses in grams when the specific heat capacities are given in joules per gram, and the answer comes out in joules without a single conversion. Turn the grams into kilograms, forget to turn the specific heat capacities with them, and the answer is out by a factor of 1000.

Practice 13.2

The calorimeter and its stirrer had a mass of 105g, it held 180g of water, and the temperature of the room was 24.5°C.

t (min)	θ (°C)	$\theta-\theta_r$ (K)	$\log_{10}(\theta-\theta_r)$
0	78.0	53.5	1.728
2	74.5	50.0	1.699
4	70.0	45.5	1.658
6	67.5	43.0	1.633
8	64.0	39.5	1.597
10	61.0	36.5	1.562
12	59.0	34.5	1.538
14	56.5	32.0	1.505
16	53.5	29.0	1.462
18	51.5	27.0	1.431
20	49.5	25.0	1.398
22	48.0	23.5	1.371

(i) and (ii) Take t at 2cm to 4 minutes, running from 0 to 24 minutes, and $\log_{10}(\theta-\theta_r)$ at 2cm to 0.05, running from 1.35 to 1.75. That is 12cm by 16cm, which fits an A4 sheet upright. Give the graph the title A graph of $\log_{10}(\theta-\theta_r)$ against t (min).

(iii) Read the line between P (2.8, 1.683) and Q (21.2, 1.381),

$$\text{slope}=\frac{1.381-1.683}{21.2\text{min}-2.8\text{min}}=\frac{-0.302}{18.4\text{min}}=-0.0164\text{min}^{-1}$$

so K is 0.0164min^{-1}. The intercept, read where the line crosses t=0, is 1.729. Accept 0.0159min^{-1} to 0.0169min^{-1} for K.

(iv) k=2.303K, so

$$k=2.303\times0.0164\text{min}^{-1}=0.0378\text{min}^{-1}$$

A minute is 60s, so one per minute is one sixtieth of one per second and the conversion divides where you might expect it to multiply: $0.0378/60=0.000630$, which is $6.30\times10^{-4}\text{s}^{-1}$.

(v) Half of the excess temperature at $t=0$ means a logarithm smaller by $\log_{10}2=0.301$. The line starts at 1.729 and is therefore at 1.428 when the excess has halved. Read across to the line and down to the time axis:

$$t=\frac{0.301}{0.0164\text{min}^{-1}}=18.4\text{min}$$

The excess temperature halves in about 18.4min. Accept 17.5 to 19.5min.

(vi) Start the watch at any temperature you please and the relation is still $\log_{10}(\theta-\theta_r)=A-Kt$, with a different A and the same K. To halve the excess you always drop the logarithm by $\log_{10}2$, so the time it takes is

$$t_{1/2}=\log_{10}2/K$$

in which A does not appear. The halving time belongs to the calorimeter and its contents, not to the temperature you happened to start at, so over the range in which the law holds a cooling curve keeps the same shape whether it begins hot or barely warm.

(vii) The points at the right-hand end, where the excess temperature is small. The resolution of the thermometer is half a degree everywhere, so every excess in the third column carries an absolute uncertainty of about 0.25K whatever its size, but a logarithm cares about the fraction and not about the number. At the start the excess is 53.5K and 0.25K of it is 1 part in 200, which moves the logarithm by 0.002. At the end the excess is 23.5K and the same 0.25K is 1 part in 90, which moves the logarithm by 0.005, a whole small square. The last points are entitled to sit twice as far off the line as the first ones, and a line drawn to please them at the expense of the others is a line drawn the wrong way round.

Examiner's eye

- Answer the halving-time question from the graph: drop 0.301 down the logarithm axis and read the time across. Hunt through the temperature column instead, for a value that happens to be half of another one, and you have answered a different question and will probably find no such value.
- A reciprocal minute divided into seconds goes the other way from a minute divided into seconds. Write the unit down at every line of the conversion and the factor of 60 cannot end up upstairs.
- Three significant figures in K, and no more. The fourth figure would be a claim that twelve points and a ruler can place a line to 1 part in 1000, which they cannot.
- A logarithm has no unit, so neither has the intercept A. Write kelvin after it, or degrees, and you have made the same mistake as writing degrees after a ratio.

Practice 13.3

The calorimeter and its stirrer had a mass of 82.0g and it held 150g of water. No reading of the room was taken.

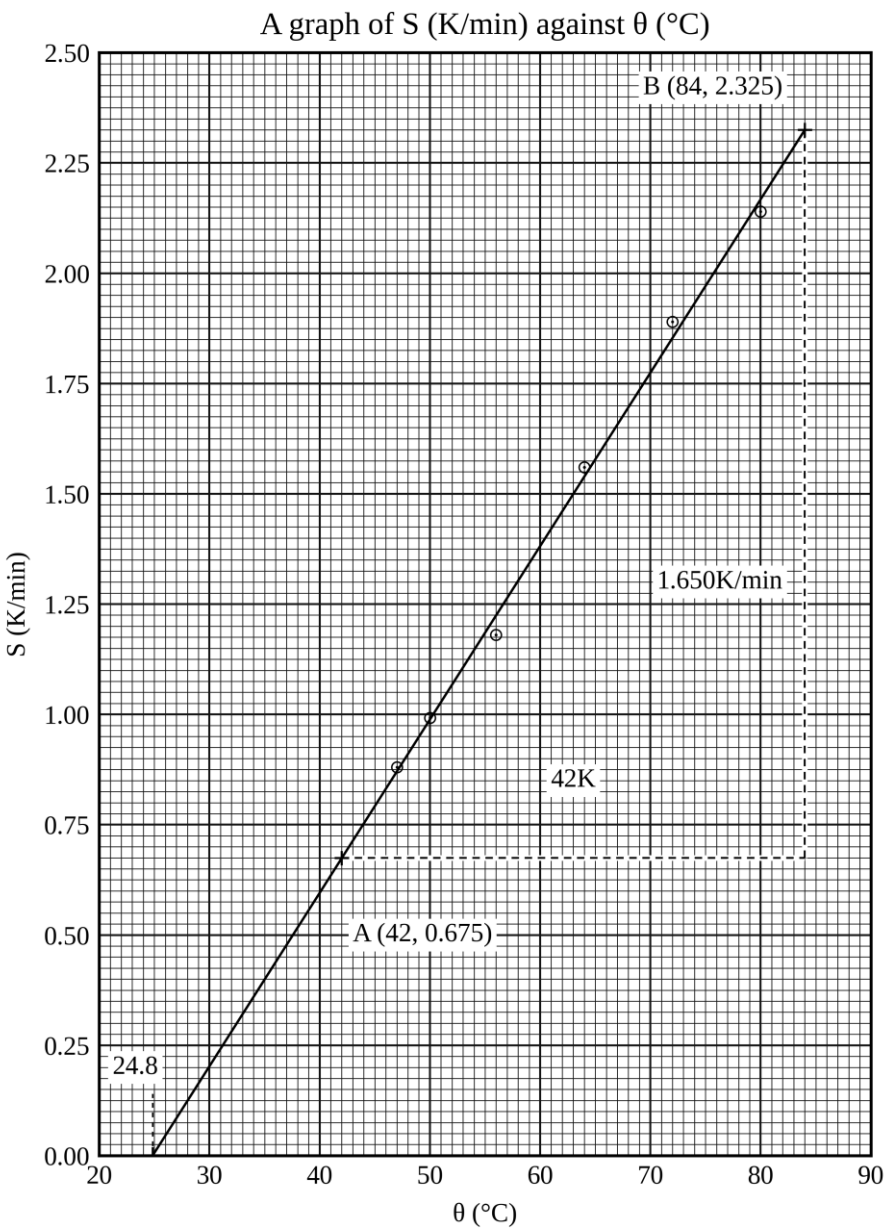
t (min)	θ (°C)	t (min)	θ (°C)
0	88.0	18	56.0
3	81.5	21	52.5
6	75.0	24	49.5
9	69.0	27	47.5
12	65.0	30	44.5
15	60.5	33	42.5

(i) and (ii) Take t at 2cm to 5 minutes, running from 0 to 35 minutes, and θ at 2cm to 5°C, running from 40°C to 90°C. That is 14cm by 20cm, which fits an A4 sheet upright. Give the graph the title A graph of θ (°C) against t (min).

(iii) Read each tangent between two widely separated points on it, in the way Experiment 13.3 shows in full:

θ (°C)	S (K/min)
80.0	2.14
72.0	1.89
64.0	1.56
56.0	1.18
50.0	0.992
47.0	0.880

(iv) Take θ at 2cm to 10°C, running from 20°C to 90°C, and S at 2cm to 0.25K/min, running from 0 to 2.50K/min. That is 14cm by 20cm, which fits an A4 sheet upright. Give the graph the title A graph of S (K/min) against θ (°C).



(v) The line runs from A (42, 0.675) to B (84, 2.325), so

$$\text{slope}=\frac{2.325\text{K/min}-0.675\text{K/min}}{84°\text{C}-42°\text{C}}=\frac{1.650\text{K/min}}{42\text{K}}=0.0393\text{min}^{-1}$$

A difference of two temperatures is an interval, so the denominator is in kelvin and the slope is 0.0393min^{-1}, the cooling constant of this calorimeter and its contents. Accept 0.0375min^{-1} to 0.0410min^{-1}.

(vi) Produce the line backwards until it crosses the temperature axis. It does so at 24.8°C, and that is the temperature of the surroundings: the point at which the rate of cooling has fallen to zero is the point at which the water has nothing left to be hotter than. The room was at about 25°C. Accept 23 to 27°C.

Do not claim more than that. The nearest tangent was drawn at 47.0°C, so this answer is an extrapolation of more than 20K beyond the last point; a slope uncertain by 2% swings the crossing point by three quarters of a degree on its own, and the scatter of the six points makes it a degree or so altogether. A thermometer left on the bench would have done better in one reading than the whole afternoon's work does here. What the graph gives that a thermometer cannot is the check: if the crossing point comes out at 40°C in a room that is plainly cooler than that, the fault is in the tangents and not in the room.

(vii) Because the rate of cooling is fixed by the excess temperature and not by the temperature. Write $S=k(\theta-\theta_r)$ and multiply it out:

$$S=k\theta-k\theta_r$$

which is a straight line against θ of slope k and intercept $-k\theta_r$ on the S axis. That intercept is zero only when θ_r is zero, so a line through the origin would be a claim that the room was at 0°C. Plotted against the excess temperature instead, as in Experiment 13.3, the same six readings do give a line through the origin, and the two graphs are the same physics with the axis shifted along by θ_r.

Examiner's eye

- Fit the line to the points, and then take whatever it does at the axes as the result. Force it through the origin because lines usually go there and you have thrown away the one thing this graph had to say.

- Two intercepts sit on this sheet and only one of them is wanted. Read the temperature where the line crosses S=0, and see that the temperature axis reaches far enough back for the crossing to be on the paper before you plot a single point.
- Plot the six points before you fix the scale, in rough if you must. A sheet ruled from 40°C upwards takes every point comfortably and has nowhere at all to put the answer.
- Work the unit out from the two axes rather than carrying it over from the last question. It comes out the same as Experiment 13.3's, because a difference of two temperatures and an excess temperature are both intervals, but that is a reason and not a shortcut.

Practice 14.1

Each calorimeter and its stirrer had a mass of 84.0g, calorimeter 1 held 240g of water, and the temperature of the room was 27.0°C.

θ (°C)	t_1 (min)	t_2 (min)
76.0	3.7	2.2
71.0	7.4	4.5
66.0	11.4	6.5
61.0	15.7	9.1
56.0	20.7	12.5
51.0	26.9	16.1

(i) and (ii) Take t_2 at 2cm to 2 minutes, running from 0 to 18 minutes, and t_1 at 2cm to 4 minutes, running from 0 to 28 minutes. That is 18cm by 14cm, which fits an A4 sheet upright. Give the graph the title A graph of t_1 (min) against t_2 (min).

(iii) Taking two well-separated points on the line, A (2.8, 4.7) and B (15.0, 25.2),

$$\text{slope}=\frac{25.2\text{min}-4.7\text{min}}{15.0\text{min}-2.8\text{min}}=\frac{20.5\text{min}}{12.2\text{min}}=1.68$$

The slope is 1.68 and it has no unit. Accept 1.63 to 1.73.

(iv) C_1 is the copper and the water together:

$$C_1=84.0\text{g}\times0.38\text{J/gK}+240\text{g}\times4.2\text{J/gK}=31.92\text{JK}^{-1}+1008\text{JK}^{-1}=1039.92\text{JK}^{-1}$$

(v) The slope is C_1/C_2, so

$$C_2=\frac{1039.92\text{JK}^{-1}}{1.68}=619.00\text{JK}^{-1}$$

and taking the copper out of it, which is the same 31.92JK^{-1} as before,

$$m_w=\frac{619.00\text{JK}^{-1}-31.92\text{JK}^{-1}}{4.2\text{J/gK}}=\frac{587.08\text{JK}^{-1}}{4.2\text{J/gK}}=139.8\text{g}$$

Calorimeter 2 held 139.8g of water. Accept 135g to 145g.

(vi) That watch was started a little after it should have been, so every reading in its column is short by the time the calorimeter needed to fall the extra 1.0K, and it is short by the same amount in every row. The points still lie on a straight line of the same slope, but the line is displaced sideways and no longer heads for the origin: it cuts one of the axes at a fixed distance from it. The slope is unharmed and the mass found from it stands, which is why the slope was asked for and not one pair of times.

(vii) The water never leaves the calorimeter, so none of it is lost on the way to the balance and none evaporates while the transfer is made. Nor is anything hot put on a balance pan, where it sets up a draught of its own that the balance reads as a change of mass.

Examiner's eye

- Two identical calorimeters mean one figure for the copper, used twice. Work it out once and subtract it once at the end; that subtraction is the step most often left out altogether.
- Quote the slope to three figures even though it has no unit. A slope of 1.7 carries the mass of water to 4g and no closer.
- Part (vi) is a question about the intercept and not about the slope. A mistake made in the same way at every reading moves a line sideways; it does not tilt it.
- Tabulate the temperature first and the two times beside it, because the temperature is the quantity you chose and the times are what you measured.

Practice 14.2

The temperature of the room was 24.0°C throughout, calorimeter 1 was the bare one and calorimeter 2 the one wrapped in foil, and each held 160g of water.

t (min)	θ_1 (°C)	$\theta_1-\theta_r$ (K)	$\log_{10}(\theta_1-\theta_r)$	θ_2 (°C)	$\theta_2-\theta_r$ (K)	$\log_{10}(\theta_2-\theta_r)$
0	76.0	52.0	1.716	76.0	52.0	1.716
2	71.5	47.5	1.677	72.0	48.0	1.681
4	69.0	45.0	1.653	69.5	45.5	1.658
6	65.5	41.5	1.618	66.5	42.5	1.628
8	61.5	37.5	1.574	64.0	40.0	1.602
10	59.5	35.5	1.550	60.5	36.5	1.562
12	56.0	32.0	1.505	58.0	34.0	1.531
14	53.5	29.5	1.470	56.5	32.5	1.512
16	51.5	27.5	1.439	54.0	30.0	1.477
18	49.5	25.5	1.407	52.0	28.0	1.447
20	47.5	23.5	1.371	50.0	26.0	1.415
22	46.0	22.0	1.342	48.5	24.5	1.389

(i) and (ii) Take t at 2cm to 4 minutes, running from 0 to 24 minutes, and $\log_{10}(\theta-\theta_r)$ at 2cm to 0.05, running from 1.25 to 1.75. That is 12cm by 20cm, which fits an A4 sheet upright. Give the graph the title A graph of $\log_{10}(\theta-\theta_r)$ against t (min).

(iii) On the bare calorimeter's line, with corners P (2.8, 1.668) and Q (20.8, 1.358),

$$\text{slope}=\frac{1.358-1.668}{20.8\text{min}-2.8\text{min}}=\frac{-0.310}{18.0\text{min}}=-0.0172\text{min}^{-1}$$

and on the foil-wrapped calorimeter's line, with R (6.8, 1.614) and S (19.2, 1.429),

$$\text{slope}=\frac{1.429-1.614}{19.2\text{min}-6.8\text{min}}=\frac{-0.185}{12.4\text{min}}=-0.0149\text{min}^{-1}$$

So $K=0.0172\text{min}^{-1}$ for the bare calorimeter and 0.0149min^{-1} for the wrapped one.

(iv) Multiplying each by 2.303,

$$k=2.303\times0.0172\text{min}^{-1}=0.0396\text{min}^{-1}$$
$$k=2.303\times0.0149\text{min}^{-1}=0.0343\text{min}^{-1}$$

(v) The ratio of the two cooling constants is

$$\frac{0.0396\text{min}^{-1}}{0.0343\text{min}^{-1}}=1.15$$

Accept 1.10 to 1.21.

(vi) The rate of loss at any stated temperature is proportional to k, so the foil takes away

$$\frac{0.0396\text{min}^{-1}-0.0343\text{min}^{-1}}{0.0396\text{min}^{-1}}=\frac{0.0053\text{min}^{-1}}{0.0396\text{min}^{-1}}=0.134$$

which is 13.4% on the arithmetic and 13% in the answer. Each cooling constant comes from a ruler laid through twelve points and is good to about 1.5%, which is 0.0006min^{-1} on the first and 0.0005min^{-1} on the second. Their difference, 0.0053min^{-1}, therefore carries about 0.0011min^{-1}, or 21% of itself. So the reduction is 13% give or take 3 points, and the second figure of 13.4% is not there. The closer a ratio sits to 1.00, the less of a difference survives the subtraction.

(vii) The foil is a second layer with a film of air trapped under it, so it changes the path the heat takes as well as the surface it leaves from. Polish the outside instead and you change the surface and nothing else.

Examiner's eye

- The bright surface here is not the copper; it is a sheet wrapped round the copper, and part (vii) turns on exactly that difference.
- Two things are asked in part (vi) and both carry marks: a percentage, and how much of it to believe. Work the percentage, then work what the two cooling constants are good to, and let the second decide how you write the first.
- Take the difference of the two constants before dividing, not after. $1-1/1.15$ and $(k_1-k_2)/k_1$ are the same quantity, but the first invites you to work with a ratio quoted to three figures and then subtract 1, which throws two of them away.
- Suppress the origin, which is not the same as leaving the axis out. Nothing in this question is read below an excess of 22K, and a logarithm axis started at zero throws four fifths of the paper away on readings that never happened.

Practice 14.3

The calorimeter and its stirrer had a mass of 104g, it held 200g of water in both runs, and the temperature of the room was 26.0°C on both afternoons.

t (min)	θ bare (°C)	θ lagged (°C)
0	84.0	84.0
5	73.5	79.0
10	64.5	74.0
15	57.5	70.0
20	51.5	66.0
25	46.5	62.0
30	43.0	59.0
35	39.5	56.0
40	37.5	53.5

(i) and (ii). Take t at 2cm to 5 minutes, running from 0 to 40 minutes, and θ at 2cm to 5°C, running from 35°C to 85°C. That is 16cm by 20cm, which fits an A4 sheet upright. Give the graph the title A graph of θ (°C) against t (min).

(iii) The bare run takes 2.7min to reach 78.0°C and 14.5min to reach 58.0°C, a fall lasting 11.8min; the lagged run takes 5.9min and 31.7min, a fall lasting 25.8min. So

$$\frac{25.8\text{min}}{11.8\text{min}}=2.19$$

Accept 2.10 to 2.30.

(iv) The excess falls from 52.0K to 32.0K in both runs, so

$$kt=2.303\times\log_{10}\frac{52.0\text{K}}{32.0\text{K}}=2.303\times0.2109=0.4857$$
$$k_{\text{bare}}=\frac{0.4857}{11.8\text{min}}=0.0412\text{min}^{-1}$$
$$k_{\text{lagged}}=\frac{0.4857}{25.8\text{min}}=0.0188\text{min}^{-1}$$

(v) The heat capacity of the calorimeter and its contents is

$$C=104\text{g}\times0.38\text{J/gK}+200\text{g}\times4.2\text{J/gK}=39.52\text{JK}^{-1}+840\text{JK}^{-1}=879.52\text{JK}^{-1}$$

At 68.0°C the excess over the room is 42.0K, so

$$879.52\text{JK}^{-1}\times0.0412\text{min}^{-1}\times42.0\text{K}=1522\text{Jmin}^{-1}$$
$$879.52\text{JK}^{-1}\times0.0188\text{min}^{-1}\times42.0\text{K}=694\text{Jmin}^{-1}$$

The lagging therefore takes away

$$\frac{1522\text{Jmin}^{-1}-694\text{Jmin}^{-1}}{1522\text{Jmin}^{-1}}=\frac{828\text{Jmin}^{-1}}{1522\text{Jmin}^{-1}}=0.544$$

which is 54% of what the bare calorimeter was losing.

(vi) A different mass of water would leave the ratio where it is. The cooling constant is $k=hA/C$, and the ratio of the two constants is the ratio of the two values of h: the heat capacity C is the same in both runs whatever it is, so it cancels. A calorimeter of a different size is another matter, because it changes the thickness of wool the same bale provides, so the two values of h need not stand in the same ratio at all.

(vii) The temperature of the room has to be the same for both runs, and here that is a matter of choosing the two afternoons rather than of arrangement. Two calorimeters side by side share the room by construction.

Examiner's eye

- Answer part (vi) with the formula and not with the graph, which has nothing more to give by then. The two halves of it have different answers, which is why it is asked in two halves.
- Notice where the fourth read-off falls. The lagged run reaches 58.0°C on the flattest part of its curve, where a small slip in temperature costs a large one in time. It is the weakest of the four readings.
- Convert nothing in part (v). A constant in min^{-1} multiplied by a heat capacity in JK^{-1} and an excess in K gives $Jmin^{-1}$, which is what was asked for.
- Percentages come from the two rates or from the two constants, and both give the same figure here. Show whichever you use; the working is two lines.

Practice 14.4

The calorimeter and its stirrer had a mass of 88.0g, it held 170g of water in both runs, and the temperature of the room was 27.0°C on both afternoons.

t (min)	θ dry (°C)	θ wet cloth (°C)
0	82.0	82.0
2	78.0	75.5
4	74.0	69.5
6	70.5	64.5
8	66.5	60.0
10	63.5	56.0
12	61.0	52.0
14	58.0	49.0
16	55.5	47.0
18	53.0	44.5
20	51.5	42.0

(i) and (ii) Take t at 2cm to 4 minutes, running from 0 to 20 minutes, and θ at 2cm to 5°C, running from 35°C to 85°C. That is 10cm by 20cm, which fits an A4 sheet upright. Give the graph the title A graph of θ (°C) against t (min).

(iii) The dry run takes 2.4min and 14.8min to reach the two temperatures, a fall lasting 12.4min; the wrapped run takes 1.5min and 9.4min, a fall lasting 7.9min. So

$$\frac{12.4\text{min}}{7.9\text{min}}=1.57$$

Accept 1.50 to 1.64.

(iv) The excess falls from 50.0K to 30.0K in both runs, so

$$kt=2.303\times\log_{10}\frac{50.0\text{K}}{30.0\text{K}}=2.303\times0.2218=0.5108$$

$$k_{dry}=\frac{0.5108}{12.4\text{min}}=0.0412\text{min}^{-1}$$

$$k_{wet}=\frac{0.5108}{7.9\text{min}}=0.0647\text{min}^{-1}$$

(v) The heat capacity of the calorimeter and its contents is

$$C=88.0\text{g}\times0.38\text{J/gK}+170\text{g}\times4.2\text{J/gK}=33.44\text{JK}^{-1}+714\text{JK}^{-1}=747.44\text{JK}^{-1}$$

The two constants differ by 0.0235min^{-1}, and at 62.0°C the excess over the room is 35.0K, so

$$747.44\text{JK}^{-1}\times0.0235\text{min}^{-1}\times35.0\text{K}=615\text{Jmin}^{-1}$$

(vi) Dividing by the latent heat,

$$\frac{615\text{Jmin}^{-1}}{2260\text{J/g}}=0.27\text{g/min}$$

Accept 0.24g/min to 0.31g/min.

(vii) A milk can is not a calorimeter scaled up. The rate at which water leaves wet sacking is fixed by the area of sacking, by how far above the room the surface stands and by how dry the air is, and none of those three scales with the mass inside in the same way. A can of 40kg holds about 240 times the contents of this calorimeter, and surface area grows as the two-thirds power of volume, so it carries about 40 times the sacking. It will use roughly 40 times the water, and because the cooling time follows the contents divided by the area it will take about 6 times as long to cool.

Examiner's eye

- Print both readings of the room with your table. A question that gives you one figure for it is telling you the two runs were comparable, and if you measured it twice you can say so yourself.
- Both curves start at the same temperature and separate at once, so the sheet has to be laid out for the wet run, which reaches furthest down. Fix the scale after plotting both, not after plotting the first.
- Keep the latent heat out of part (v), which is a rate of loss of heat and stops at $Jmin^{-1}$. It belongs in part (vi), where it turns that rate into a mass. Mix the two parts and neither of them is answered.

- Scaling is what part (vii) is about, not apparatus. The quantities that matter grow at different rates when the vessel grows, and naming which of them follow the area and which the mass is the whole of the answer.

Practice 14.5

Each calorimeter and its stirrer had a mass of 88.0g, calorimeter 1 held 160g of liquid B, calorimeter 2 held 160g of water, and the temperature of the room was 26.0°C.

θ (°C)	t_1 (min)	t_2 (min)
77.0	1.4	2.6
72.0	2.9	5.1
67.0	4.2	8.7
62.0	6.4	11.8
57.0	8.2	16.1
52.0	11.1	20.9

(i) and (ii) Take t_2 at 2cm to 4 minutes, running from 0 to 22 minutes, and t_1 at 2cm to 2 minutes, running from 0 to 12 minutes. That is 11cm by 12cm, which fits an A4 sheet upright. Give the graph the title A graph of t_1 (min) against t_2 (min).

(iii) Taking two well-separated points on the line, A (7.2, 3.8) and B (19.6, 10.3),

$$\text{slope}=\frac{10.3\text{min}-3.8\text{min}}{19.6\text{min}-7.2\text{min}}=\frac{6.5\text{min}}{12.4\text{min}}=0.524$$

Accept 0.51 to 0.54.

(iv) For calorimeter 2,

$$C_2=88.0\text{g}\times0.38\text{J/gK}+160\text{g}\times4.2\text{J/gK}=33.44\text{JK}^{-1}+672\text{JK}^{-1}=705.44\text{JK}^{-1}$$

(v) The slope is C_1/C_2, so

$$C_1=0.524\times705.44\text{JK}^{-1}=369.65\text{JK}^{-1}$$

and taking out the copper, which is the same on both sides,

$$c_B=\frac{369.65\text{JK}^{-1}-33.44\text{JK}^{-1}}{160\text{g}}=\frac{336.21\text{JK}^{-1}}{160\text{g}}=2.10\text{J/gK}$$

Accept 2.02J/gK to 2.18J/gK.

(vi) A dull outside loses heat faster than a bright one, so calorimeter 1 reaches every temperature sooner than it should, every value of t_1 is too small, and the slope comes out too small with them. The heat capacity of calorimeter 1 and its contents is then under-stated, and so is the specific heat capacity of liquid B. The experiment blames the liquid for what the surface did, which is exactly the fault the whole chapter is arranged to avoid.

(vii) Use a smaller mass of liquid B and leave the water as it is; the two masses never had to be equal, because each side of the comparison is worked out on its own. Two things are given up for it. The first is the matched depth: less liquid stands lower in the vessel, and Practical tip 1 wants the two levels close together. The second is precision. The copper contributes 33.44JK^{-1} whatever you do, so the smaller the mass of liquid, the larger a fraction of calorimeter 1 the copper is, and the more of the answer goes into the subtraction at the end. Halve the liquid and the uncertainty in the specific heat capacity goes from about 2.8% to about 3.0%, so the depth is the greater of the two losses.

Examiner's eye

- Plot the liquid on the vertical axis because that is what the question asked, and let the slope come out below 1.00 without worrying about it. A slope under 1.00 is not a mistake; it is the answer.
- Say which way in part (vi), and say it in one direction only. Half the marks in a question of that shape go to the word "smaller" or the word "larger" appearing beside the right quantity, and a paragraph that argues both ways scores neither.
- Stop at three figures in the specific heat capacity. The slope carries three, the balance carries more, and a fourth figure claims a precision the slope never had.
- Every mass in this question is in grams and every specific heat capacity is per gram, so nothing needs converting anywhere, and a factor of 1000 arriving from somewhere is a factor of 1000 too many.

Practice 14.6

Each calorimeter and its stirrer had a mass of 100g, calorimeter 2 held 150g of water, and the temperature of the room was 26.0°C.

t (min)	θ_1 (°C)	θ_2 (°C)
0	80.0	80.0
3	76.0	74.0
6	72.5	68.5
9	69.0	64.0
12	66.0	59.5
15	63.0	56.0
18	60.5	52.5
21	58.0	49.5
24	55.5	47.0

(i) and (ii) Take t at 2cm to 4 minutes, running from 0 to 24 minutes, and θ at 2cm to 5°C, running from 40°C to 85°C. That is 12cm by 18cm, which fits an A4 sheet upright. Give the graph the title A graph of θ (°C) against t (min).

(iii) Calorimeter 1 passes the two temperatures at 3.9min and 18.5min, a fall lasting 14.6min; calorimeter 2 passes them at 2.5min and 11.7min, a fall lasting 9.2min. So

$$\frac{14.6\text{min}}{9.2\text{min}}=1.59$$

(iv) The two calorimeters are identical outside and stand in the same room, so each is losing heat at the same rate as the other whenever the two are at the same temperature. Both runs cover the same fall, from 75.0°C to 60.0°C. Take any small step of that fall: calorimeter 1 must give up $C_1\delta\theta$ to make it and calorimeter 2 must give up $C_2\delta\theta$, and both are losing heat at the same rate while they do it, so the two step times are in the ratio $C_1:C_2$. Adding the steps,

$$\frac{t_1}{t_2}=\frac{C_1}{C_2}$$

(v) Calorimeter 2 is the one whose contents are known:

$$C_2=100g\times0.38J/gK+150g\times4.2J/gK=38.00JK^{-1}+630JK^{-1}=668.00JK^{-1}$$

From the ratio,

$$C_1=1.59\times668.00JK^{-1}=1062.12JK^{-1}$$

and taking out the copper,

$$m_w=\frac{1062.12JK^{-1}-38.00JK^{-1}}{4.2J/gK}=\frac{1024.12JK^{-1}}{4.2J/gK}=244g$$

Calorimeter 1 held about 244g of water. Accept 230g to 255g. That is a marking tolerance, not the precision of one carefully drawn graph.

(vi) Experiment 14.1 gives the better answer. Its ratio comes from a line drawn through six pairs of times, so the scatter of the individual readings averages down and the origin fixes one end of the line for nothing. This method rests on four read-offs, two of them on the shorter curve where 0.1min of uncertainty is over 1%, and no line is fitted to anything. Against that, it hands you the two cooling curves themselves, which is why it is the one an examiner sets when the shape of the cooling is part of what is being marked.

(vii) The curve is drawn through nine readings, so a single misread temperature moves it hardly at all, and the time read off it is a time the whole run agrees on. A watch stopped by hand at the instant a thermometer passes 75.0°C carries the reaction time of the person holding it and the lag of the thermometer itself, and neither of those shows up anywhere afterwards to warn you.

Examiner's eye

- Nine readings on each curve, and both curves on one sheet. The two start together and separate slowly, so the first three points nearly coincide; plot them carefully and let the curves do the separating.
- Divide the longer time by the shorter one, and check that the answer exceeds 1.00 before going on. The curve that lies above the other all the way along belongs to the calorimeter holding more water, and its time is the longer of the two.
- Show the step argument in part (iv) rather than asserting the result. Two sentences and a ratio of two heat capacities will do it, and the marks are for the argument, not for the formula it ends at.
- Round the mass to three figures. Four read-offs at 0.1min apiece leave the ratio good to about 4%, which is nearly 10g on this answer, so a mass written to 0.1g claims a balance the experiment never used.

Practice 15.1

The calorimeter and its stirrer had a mass of 110.0g, the calorimeter with the stirrer and the oil had a mass of 260.0g, the brass cylinder had a mass of 248.0g, the oil began at 26.0°C, the boiling water stood at 98.0°C, the highest temperature of the mixture was 41.0°C and the room was at 26.5°C.

(i) The readings above, set out in two columns with the quantity named on the left and the value on the right. The mass of oil is the difference of the two weighings,

$$m_L=260.0g-110.0g=150.0g$$

(ii) The brass falls 98.0°C − 41.0°C = 57.0K, so

$$Q_{lost}=248.0g\times0.380J/gK\times57.0K=5371.7J$$

(iii) The calorimeter rises 41.0°C − 26.0°C = 15.0K, so

$$Q_{copper}=110.0g\times0.38J/gK\times15.0K=627.0J$$

(iv) What is left over went into the oil:

$$5371.7J-627.0J=4744.7J$$

and that heat raised 150.0g of oil through 15.0K, so

$$c=\frac{4744.7J}{150.0g\times15.0K}=\frac{4744.7J}{2250.0gK}=2.11J/gK$$

Accept 2.05J/gK to 2.17J/gK.

(v) It is above the published figure by

$$\frac{2.11-2.00}{2.00}\times100=5.5\%$$

(vi) They move it up, because the unknown has changed sides. In Experiment 15.1 the unknown belonged to the body giving heat up, and what escaped was heat the water never received, so the brass appeared to have less in it than it had. Here the unknown belongs to the body taking heat in. The escaped heat lowers the highest reading just as before, but that now lowers the rise the oil is credited with and raises the fall the brass is charged with, so the oil appears to have taken in more heat per kelvin than it did. A neglected loss flatters whichever of the two was the colder, so long as the vessel spends most of the transfer above the room.

(vii) It supports the claim and it measures it: 2.11J/gK against 4.2J/gK for water is about half, so gram for gram this oil stores half the heat water does. What it does not say is anything about equal volumes, which is what a bottle and a beaker actually hold. Oil is less dense than water, so the ratio by volume is smaller still, and the card would need a density before it could be turned into advice about filling a bath.

(viii) Stir so that the reading at the thermometer is the temperature of the whole of the oil. A liquid as thick as this one will not carry its own heat about: stir badly and the oil against the brass stays hotter than the oil against the bulb, so the highest reading comes later and lower than the true one. A low highest reading acts on the arithmetic exactly as a neglected loss does in part (vi), and the value comes out high for the same reason.

Examiner's eye

- Take the copper out first, and then divide. The brass gives its heat to the oil and to the vessel holding it, so divide the whole of it by the mass of oil and you have credited the copper's share to the liquid too: 627J in 5372J, which puts the answer up by 13%.
- Watch the unit of the answer. A specific heat capacity of 2.11J/gK is 2110J/kgK, and a table that gives the figure for oil in the second form is not disagreeing with you.
- Oil wets glass more stubbornly than water, so a measuring cylinder keeps back a film that never reaches the calorimeter. Weigh the oil in the calorimeter; never measure it out first.
- Read the card again before you write part (vii). It gives an instruction and a reason for it, and your result speaks only to the reason.

Practice 15.2

The calorimeter and its stirrer had a mass of 88.0g, the calorimeter with the stirrer and the liquid had a mass of 248.0g, so it held 160.0g of liquid F, and the temperature of the room was 24.0°C.

t (min)	θ (°C)	t (min)	θ (°C)
0.0	85.0	14.0	60.2
2.0	80.7	16.0	57.7
4.0	76.3	18.0	55.3
6.0	72.7	20.0	52.6
8.0	69.4	22.0	50.5
10.0	65.9	24.0	48.6
12.0	63.1		

(i) and (ii) Take t at 2cm to 4 minutes, running from 0 to 24 minutes, and θ at 2cm to 5K, running from 45°C to 90°C. That is 12cm by 18cm, which fits an A4 sheet upright. Give the graph the title A graph of θ (°C) against t (min).

(iii) The curve crosses 60.0°C near t = 14.1 minutes. Taking two corners on the drawn tangent, M (4.8, 72.5) and N (14.8, 59.0),

$$S=\frac{72.5°C-59.0°C}{14.8min-4.8min}=\frac{13.5K}{10.0min}=1.35K/min$$

Accept 1.28K/min to 1.42K/min.

(iv) Over the modest excess temperatures worked in here the rate of loss is proportional to the excess temperature. At 80.0°C the excess is 56.0K and at 60.0°C it is 36.0K, so

$$P=15.4Js^{-1}\times\frac{36.0K}{56.0K}=9.90Js^{-1}$$

(v) The slope in kelvin per second is

$$S=\frac{1.35K}{60s}=0.0225K/s$$

and therefore

$$C=\frac{9.90Js^{-1}}{0.0225K/s}=440JK^{-1}$$

Accept $418JK^{-1}$ to $464JK^{-1}$, which is the band the slope carries.

(vi) Taking the copper out,

$$m_c c_c=88.0g\times0.38J/gK=33.44JK^{-1}$$

$$c=\frac{440JK^{-1}-33.44JK^{-1}}{160.0g}=\frac{406.56JK^{-1}}{160.0g}=2.54J/gK$$

Accept 2.40J/gK to 2.69J/gK. That is a marking tolerance, not the precision of one carefully drawn graph.

(vii) Larger, provided the water stays in the liquid. Water has a specific heat capacity of 4.2J/gK, well above the value found here, so a mixture of the two takes in more heat for every kelvin than the liquid alone does, and the answer rises. Above 80°C the water will not stay for long, and what boils off leaves by a route the supplied rate does not cover, which pushes the answer back down again.

(viii) It narrows the field without closing it. A specific heat capacity of 2.54J/gK rules out water and rules out any of the metals. It stands well above the 2.0J/gK the vegetable oils sit at and lands among the alcohols instead, so whatever the shop has been selling for hinges is not an oil. It cannot name the liquid, because a great many liquids sit inside the band a single tangent can set. To name it you would need a property that separates one liquid sharply from the next, such as the boiling point or the density, and neither of those was measured here.

Examiner's eye

- Scale the rate of loss before you use it. The figure supplied here is for 80.0°C and the tangent is drawn at 60.0°C; put the two together untouched and the heat capacity comes out more than half as large again as it should.
- Excess temperature is measured from the room, never from zero. The ratio wanted in part (iv) is 36 to 56; use 60 to 80 instead and part (iv) comes out 17% high.
- Turn the stirrer slowly and steadily all the way through. Liquid that barely flows lets the wall sit several kelvin cooler than the middle, which is where the thermometer is.
- Look at the width of the accept band above and then at the gap between one liquid and the next. Two specific heat capacities that both fall inside that band are one liquid as far as this bench is concerned, and part (viii) is marked on saying so.

Practice 15.3

The temperature of the room was 24.0°C throughout. The calorimeter and its stirrer had a mass of 88.0g, the calorimeter with the stirrer and the water of the first run had a mass of 208.0g, solid Y had a mass of 535.5g, and the calorimeter with the stirrer, the solid and the water of the second run had a mass of 670.8g.

t (min)	θ_A (°C)	θ_B (°C)	$\theta_A - \theta_B$ (K)
0	78.0	78.0	0.0
2	71.5	70.0	1.5
4	65.8	63.5	2.3
6	60.7	57.7	3.0
8	55.9	52.5	3.4
10	52.1	48.4	3.7
12	48.5	44.7	3.8
14	45.6	41.7	3.9
16	42.9	39.1	3.8
18	40.6	36.8	3.8
20	38.5	34.8	3.7
22	36.8	33.2	3.6
24	35.2	32.2	3.0

(i) and (ii). Take t at 2cm to 4 minutes, running from 0 to 24 minutes, and θ at 2cm to 5K, running from 30°C to 80°C. That is 12cm by 20cm, which fits an A4 sheet upright. Give the graph the title A graph of θ (°C) against t (min).

(iii) By subtraction,

$$m_A = 208.0\text{g} - 88.0\text{g} = 120.0\text{g}$$

and the calorimeter with the solid weighs 88.0g + 535.5g = 623.5g, so

$$m_B = 670.8\text{g} - 623.5\text{g} = 47.3\text{g}$$

(iv) The difference is greatest at t = 14 minutes, where it reaches 3.9K. Curve A reads 45.6°C there and curve B reads 41.7°C. Accept 12 to 16 minutes for the time.

(v) The heat capacity of the first run is

$$C_1 = 88.0\text{g} \times 0.38\text{J/gK} + 120.0\text{g} \times 4.2\text{J/gK} = 33.44\text{JK}^{-1} + 504.0\text{JK}^{-1} = 537.4\text{JK}^{-1}$$

$$(m_A - m_B)c_w = 72.7\text{g} \times 4.2\text{J/gK} = 305.3\text{JK}^{-1}$$

$$\frac{C_1 \Delta\theta}{\theta_A - \theta_r} = \frac{537.4\text{JK}^{-1} \times 3.9\text{K}}{45.6°\text{C} - 24.0°\text{C}} = \frac{537.4\text{JK}^{-1} \times 3.9\text{K}}{21.6\text{K}} = 97.0\text{JK}^{-1}$$

$$m_s c = 305.3\text{JK}^{-1} - 97.0\text{JK}^{-1} = 208.3\text{JK}^{-1}$$

$$c = \frac{208.3\text{JK}^{-1}}{535.5\text{g}} = 0.389\text{J/gK}$$

Accept 0.35J/gK to 0.43J/gK. The band is that wide because the time in part (iv) can be misjudged by 2 minutes without the graph looking any different.

(vi) The block pushed out 120.0g − 47.3g = 72.7g of water, so

$$V = \frac{72.7\text{g}}{0.97\text{gcm}^{-3}} = 74.9\text{cm}^3$$

$$\rho = \frac{535.5\text{g}}{74.9\text{cm}^3} = 7.15\text{gcm}^{-3}$$

(vii) A specific heat capacity of 0.389J/gK rules out aluminium at 0.90J/gK outright and leaves iron at 0.46J/gK at the far edge of what a band of ±10% can exclude. It cannot separate zinc, brass and copper, which lie within 0.01J/gK of each other, and it does not have to: the density does that. Brass is about 8.5gcm^{-3}, copper is 8.9gcm^{-3} and zinc is 7.14gcm^{-3}, against the 7.15gcm^{-3} found here. The block is zinc, and it is the density that says so. That also answers the cupboard: this block takes 208J for every kelvin it is warmed through, and until the lighter one has been through the same measurement the balance can say which is heavier and nothing more, which was Kipute's point.

(viii) Because the rate at which the vessel loses heat has to be the same in both runs at any given temperature, and it depends on the wetted height of the copper. The same mass of water with the block in it would stand higher up the calorimeter than the same mass without, so the second run would present a larger surface to the room, and the difference between the two curves would then be partly the block and partly the surface, with nothing to say which was which.

Examiner's eye

- Divide by the density, do not multiply. Water at the filling temperature is less dense than cold water, so 72.7g of it takes up rather more than 72.7cm^3; multiply instead of dividing and 7.15gcm^{-3} becomes 7.59gcm^{-3}, which is no longer zinc's figure and is not anything else's either.
- Quote the two figures you are putting side by side. 7.15gcm^{-3} beside 7.14gcm^{-3} is an argument; "the density fits zinc" is an assertion, and the two are marked differently.
- Lower the heaviest block into the calorimeter; do not drop it. Check too that the balance is reading zero before the last weighing, which puts 670.8g on the pan: 1g of zero error there runs straight into the difference of the two masses of water and moves the answer by 2%.
- Run the two halves on the same afternoon. The room enters the working once, and a change in it between the two runs cancels nowhere: warm the laboratory by a kelvin between run A and run B and curve B has spent its whole run falling towards a room that never appears in your arithmetic.

Practice 15.4

The temperature of the room was 26.0°C throughout. The calorimeter and its stirrer had a mass of 90.0g, the calorimeter with the stirrer and the water had a mass of 190.0g, the steel block had a mass of 300.0g and the boiling water stood at 98.0°C. The block was transferred at t = 2.0 minutes.

t (min)	θ (°C)	t (min)	θ (°C)
0.0	22.4	4.0	39.5
0.5	22.5	4.5	39.5
1.0	22.5	5.0	39.3
1.5	22.5	5.5	39.2
2.0	22.6	6.0	38.9
2.5	38.5	6.5	38.9
3.0	39.7	7.0	38.6
3.5	39.6		

(i) and (ii) Take t at 2cm to 1 minute, running from 0 to 7 minutes, and θ at 2cm to 2K, running from 20°C to 42°C. That is 14cm by 22cm, which fits an A4 sheet upright. Give the graph the title A graph of θ (°C) against t (min).

(iii) The five readings before the transfer rise by 0.2K in all, and a ruler goes through them without argument. Produced to t = 2.0 minutes it gives θ_1 = 22.6°C. Accept 22.5°C to 22.7°C.

(iv) From t = 3.5 minutes the readings drop away from 39.6°C to 38.6°C on a curve that is still bending. Carried back to t = 2.0 minutes it reaches θ_f = 40.1°C. Accept 40.0°C to 40.2°C.

(v) The heat capacity of the calorimeter and its water is

$$C=90.0g\times0.38J/gK+100.0g\times4.2J/gK=34.2JK^{-1}+420.0JK^{-1}=454.2JK^{-1}$$

The corrected rise is 40.1°C − 22.6°C = 17.5K, the fall of the block is 98.0°C − 40.1°C = 57.9K, and so

$$c=\frac{454.2JK^{-1}\times17.5K}{300.0g\times57.9K}=\frac{7948.5J}{17370.0gK}=0.458J/gK$$

Accept 0.450J/gK to 0.468J/gK. That is a marking tolerance, not the precision of one carefully drawn graph. It stops short of the uncorrected figure below, deliberately.

(vi) The highest reading was 39.7°C, giving a rise of 17.1K and a fall of 58.3K:

$$c=\frac{454.2JK^{-1}\times17.1K}{300.0g\times58.3K}=0.444J/gK$$

so the correction raised the answer by

$$\frac{0.458-0.444}{0.444}\times100=3.2\%$$

(vii) The corrected value is 0.4% below the accepted one and the uncorrected value is 3.5% below it. Both are low, and the correction has removed the greater part of the shortfall; what is left is the heat the block lost in the air between the bath and the water, which this construction cannot reach.

(viii) It works on the two branches in opposite directions. Before the transfer the water stands below the room and drifts up towards it, so a warmer room widens the gap it is climbing and steepens that drift; after the transfer the mixture stands above the room and falls towards it, so a warmer room shortens the fall and flattens the cooling branch. Neither shows here, because 0.5K on a gap of some 3K before the transfer, and on some 14K after it, moves either branch by less than the thermometer can read. It changes nothing at all in the working of part (v), where the temperature of the room does not appear: the specific heat capacity comes from a rise, a fall and three masses. So Kipute is right about the metal and Kipanga is right about the uncorrected answer, which does lean on the room, and the correction is the step that takes the room back out again. Between them they have told the metal from the answer; neither has done it alone, so Mr. Akilikubwa keeps the bread.

Examiner's eye

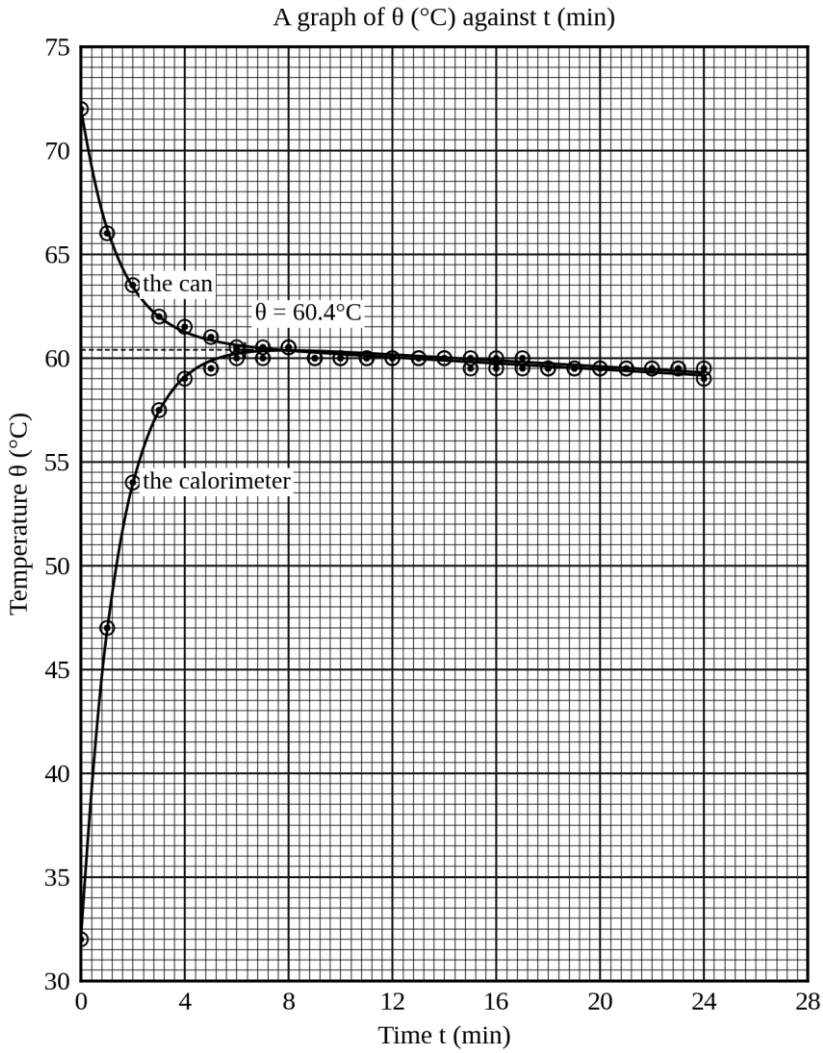
- Steel and iron are not the same word, but this measurement cannot hear the difference. Anything between 0.45J/gK and 0.47J/gK covers mild steel and pure iron alike, and this method cannot tell them apart.
- The percentage in part (vi) is of your own uncorrected answer and not of the accepted value. Divide by the accepted value instead and you get 3.0% here, close enough to the right figure to pass unnoticed and still the wrong quantity.
- Put both values on the page in part (vii). An answer that quotes only the corrected figure has thrown away the point of the whole experiment, which is the size of what was recovered.
- Half fill the calorimeter and no more. A block of 300g displaces enough water to raise the level noticeably, and water that goes over the rim when the block enters takes its mass out of your arithmetic and leaves no trace on the balance.

Practice 16.1

The calorimeter and its stirrer had a mass of 78.0g and held 150g of cold water, the large can and its stirrer had a mass of 140g, and the temperature of the room was 26.0°C.

t (min)	can (°C)	calorimeter (°C)	t (min)	can (°C)	calorimeter (°C)
0	72.0	32.0	13	60.0	60.0
1	66.0	47.0	14	60.0	60.0
2	63.5	54.0	15	59.5	60.0
3	62.0	57.5	16	59.5	60.0
4	61.5	59.0	17	59.5	60.0
5	61.0	59.5	18	59.5	59.5
6	60.5	60.0	19	59.5	59.5
7	60.5	60.0	20	59.5	59.5
8	60.5	60.5	21	59.5	59.5
9	60.0	60.0	22	59.5	59.5
10	60.0	60.0	23	59.5	59.5
11	60.0	60.0	24	59.0	59.5
12	60.0	60.0			

(i) and (ii) Take t along the horizontal axis at 2cm to 4min, running from 0 to 28min, and θ up the vertical axis at 2cm to 5K, running from 30°C to 75°C. That is 14cm by 18cm, which fits an A4 sheet upright. Give the graph the title A graph of θ (°C) against t (min) and label each curve with the vessel it belongs to.



(iii) The falling curve and the rising curve come together at 60.4°C, and they first do so at the eighth minute, which is the first reading at which the two thermometers give the same number.

(iv) The calorimeter and its water have between them a thermal capacity of

$$C_1 = 78.0g \times 0.38J/gK + 150g \times 4.2J/gK = 29.6JK^{-1} + 630JK^{-1} = 659.6JK^{-1}$$

and they have risen from 32.0°C to 60.4°C, a rise of 28.4K, so the heat they have gained is

$$659.6JK^{-1} \times 28.4K = 18733J$$

(v) If none of that heat came from anywhere but the can, the can and its water gave out 18733J while falling from 72.0°C to 60.4°C, a fall of 11.6K. Their thermal capacity is therefore

$$C_2 = 18733J/11.6K = 1615JK^{-1}$$

The copper of the can accounts for 140g at 0.38J/gK, which is $53.2JK^{-1}$. What is left belongs to the water:

$$m=\frac{1615JK^{-1}-53.2JK^{-1}}{4.2J/gK}=\frac{1561.8JK^{-1}}{4.2J/gK}=372g$$

The can held about 370g of hot water. Accept 355g to 390g.

(vi) Once the two vessels are level, the exchange between them nearly stops, and neither can raise or lower the other by more than a fraction of a division. The whole arrangement is still 34K above the room, however, and lagging slows that loss without stopping it. Everything inside therefore cools together, which is why the two curves become one line and why that line slopes gently downwards, falling about 1.0K over the last 16min. The calorimeter stays the marginally hotter of the two throughout that tail, because its only road to the room runs through the can's water, and that is why the two columns part by half a division here and there instead of agreeing exactly.

(vii) Below the true mass. The loss to the room was going on during the first 8min as well as after them, so the temperature at which the curves met is lower than it would have been in a perfectly lagged can. A lower junction makes the rise of the calorimeter smaller and the fall of the can larger, and (v) divides the first of those by the second. Both changes push the quotient down, and the mass with it.

(viii) Most of the loss is from the open mouth of the can, which the wool jacket does not cover. Lay a pad of cotton wool over the mouth, with holes only for the two thermometers and the two stirrers.

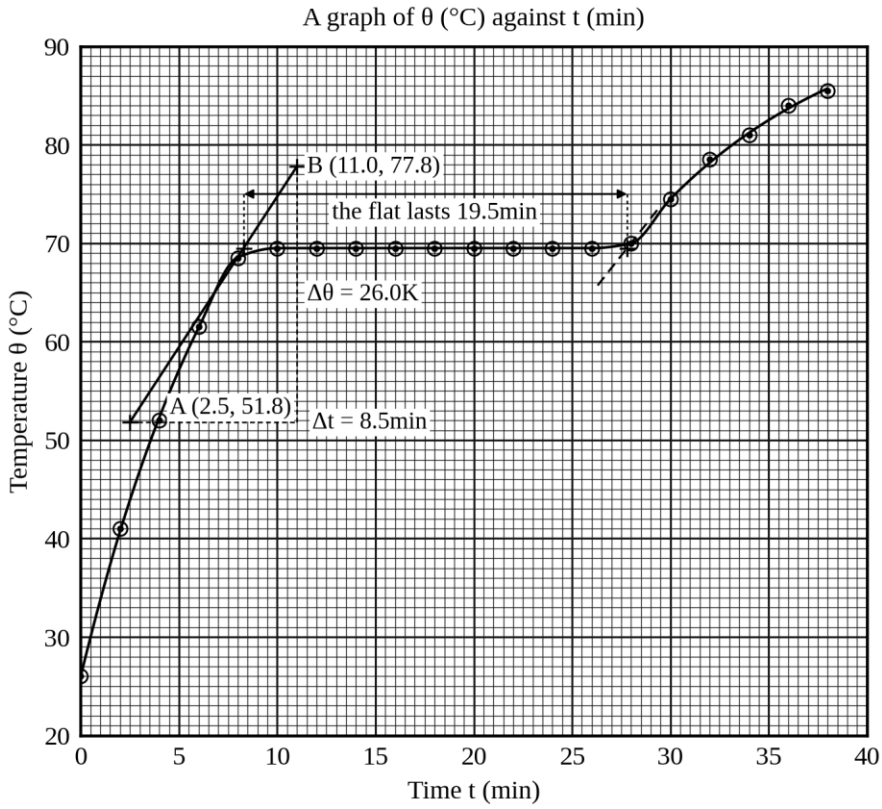
Examiner's eye

- Read the junction off the curves, not out of the table. Two thermometers that both read 60.5°C at the eighth minute have told you that the junction is near that, not that it is exactly that, because each of them was rounded to the nearest half kelvin before you saw it. The curves you drew through the points are the better instrument.
- Heat gained is set equal to heat lost, and that is a statement about a closed pair of vessels. Write the equation out with the words in it, "heat gained by the calorimeter and its water = heat lost by the can and its water", before you put a single number in. Half the marks in this question hang on the two sides being the right way round.
- Both thermometers have to be read at the same instant, and with one pair of eyes that is harder than it sounds. Read the can first every time, so that whatever lag there is falls the same way in every row; a junction found from two columns read half a minute apart is a junction in the wrong place.
- Every minute of the tail is evidence. A record that stops at the junction cannot show whether the system was leaking heat, and part (vii) is then a guess. The flat that is not quite flat is the measurement that tells you which way your answer is wrong.

Practice 16.2

t (min)	θ (°C)	t (min)	θ (°C)
0	26.0	20	69.5
2	41.0	22	69.5
4	52.0	24	69.5
6	61.5	26	69.5
8	68.5	28	70.0
10	69.5	30	74.5
12	69.5	32	78.5
14	69.5	34	81.0
16	69.5	36	84.0
18	69.5	38	85.5

(i) and (ii). Take t along the horizontal axis at 2cm to 5min, running from 0 to 40min, and θ up the vertical axis at 2cm to 10K, running from 20°C to 90°C. That is 16cm by 14cm, which fits an A4 sheet upright. Give the graph the title A graph of θ (°C) against t (min).



(iii) The flat lies at 69.5°C, so that is the melting point of solid X. Produce the rising part forwards and the flat backwards; they cross at 8.3min; at the far end the crossing is at 27.8min. The temperature therefore held steady for 19.5min. Accept 18.5min to 20.5min.

(iv) The tangent drawn where the rising curve meets the flat passes through (2.5, 51.8) and (11.0, 77.8), so the rate of rise just below the melting point is

$$S=\frac{26.0K}{8.5min}=3.06Kmin^{-1}$$

The thermal capacity of the tube with the solid in it is

$$C_s=15.0g\times1.60J/gK+26.0JK^{-1}=24.0JK^{-1}+26.0JK^{-1}=50.0JK^{-1}$$

so by $S=P/C$ the heat entering the tube at 69.5°C is

$$P=C_sS=50.0JK^{-1}\times3.06Kmin^{-1}=153Jmin^{-1}$$

(v) While the flat lasts, the bath is at 95.0°C and the tube is at 69.5°C, exactly as it was at the instant the tangent was drawn, so heat goes on entering at $153Jmin^{-1}$ and every joule of it goes into melting the solid. Over 19.5min the heat that entered is

$$153Jmin^{-1}\times19.5min=2984J$$

and that melted 15.0g of solid X, so

$$L=\frac{2984J}{15.0g}=199J/g$$

The specific latent heat of fusion of solid X is 199J/g. Accept 185J/g to 213J/g, which is the 7% that part (vii) arrives at by a different road.

(vi) The temperature is the same on the two sides of the flat, so the bath is 25.5K hotter than the tube in both cases and heat is entering at the same $153Jmin^{-1}$. What has changed is what that heat has to warm. Liquid X has a larger specific heat capacity than solid X, so the same P divided by a larger C gives a smaller S.

(vii) Two read quantities go into the answer and their percentage uncertainties propagate by addition. The tangent is drawn at a knee and is worth no better than $0.10Kmin^{-1}$ in $3.06Kmin^{-1}$, which is 3%. Neither end of the flat is a plotted point. At each end one reading lies within a division of the flat and the next is several divisions clear of it, so each crossing has to be constructed from two produced lines and neither is worth better than 0.4min. Together the length is uncertain by 0.8min in 19.5min, which is 4%. The two make about 7% in all, or 14J/g, and the dominant term is the length of the flat.

(viii) A cooling curve of the same substance would dip below the flat before settling on it, and this heating curve does not. A liquid needs a first crystal before the rest can freeze on to it, and until one appears the liquid goes on cooling below its freezing point. A solid needs nothing of the kind: as soon as any part of it reaches the melting point that part melts, so the curve arrives at the flat and stays there.

Examiner's eye

- Solid X melts at one temperature and a candle does not. Wax is a mixture of many substances, each with its own melting point, so it softens over a range and its curve has no flat at all. If your own curve has a shoulder where it should have a flat, the sample is not pure, and that is a result to report rather than an accident to hide.
- Only the rate just below the flat will do. An average rate over the whole rise from 26.0°C comes to $5.2Kmin^{-1}$, because the bath was 69K hotter than the tube at the start and the heat was pouring in far faster then. That average carries the latent heat up to about 340J/g, some 70% above the figure in part (v).

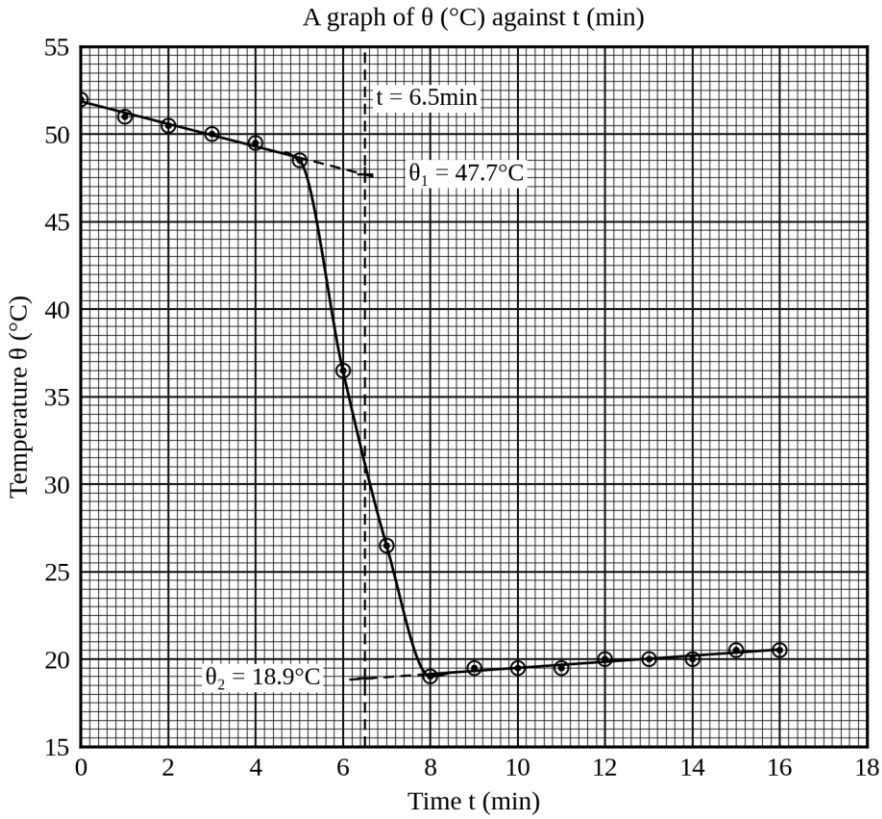
- Solid X has a label and no name, so there is no table value to check it against. That is deliberate on the examiner's part: every mark in the question is for the method and none for recognising a number.
- Quote the unit. A specific latent heat in J/g and the same quantity in Jkg^{-1} differ by a factor of 1000, and an answer of 199 with no unit beside it can be marked right only by guesswork.

Practice 16.3

Quantity	Reading
mass of calorimeter and stirrer	78.0g
mass of calorimeter, stirrer and warm water	228.0g
mass of calorimeter, stirrer and all water at the end	273.0g
temperature of the room	26.0°C

t (min)	θ (°C)	t (min)	θ (°C)
0	52.0	9	19.5
1	51.0	10	19.5
2	50.5	11	19.5
3	50.0	12	20.0
4	49.5	13	20.0
5	48.5	14	20.0
6	36.5	15	20.5
7	26.5	16	20.5
8	19.0		

(i) and (ii). Take t along the horizontal axis at 2cm to 2min, running from 0 to 18min, and θ up the vertical axis at 2cm to 5K, running from 15°C to 55°C. That is 18cm by 16cm, which fits an A4 sheet upright. Give the graph the title A graph of θ (°C) against t (min).



(iii) The ice went in at the fifth minute and the lowest reading came at the eighth, so the vertical line goes at 6.5min. The line through the first six readings, produced forwards, meets it at θ_1 = 47.7°C. The line through the lowest reading and the eight after it, produced backwards, meets it at θ_2 = 18.9°C.

(iv) The mass of warm water is 228.0g less 78.0g, which is 150.0g, and the mass of ice is 273.0g less 228.0g, which is 45.0g. The thermal capacity of the calorimeter with its water is

$$C=78.0g\times0.38J/gK+150.0g\times4.2J/gK=29.6JK^{-1}+630JK^{-1}=659.6JK^{-1}$$

and the corrected fall is 47.7°C to 18.9°C, which is 28.8K, so

$$659.6JK^{-1}\times28.8K=18996J$$

(v) The melted ice is warmed from 0°C to 18.9°C, which takes

$$45.0g\times4.2J/gK\times18.9K=3572J$$

so the heat that went into melting is 18996J less 3572J, and

$$L=\frac{18996J-3572J}{45.0g}=\frac{15424J}{45.0g}=343J/g$$

Accept 332J/g to 351J/g. The band is shorter above the answer than below it on purpose; part (vi) shows what sits just outside its upper end.

(vi) Without the correction the two temperatures used are the last reading before the ice went in, 48.5°C, and the lowest reading after it, 19.0°C. The fall is then 29.5K, so the calorimeter gives out

$$659.6\text{JK}^{-1}\times 29.5\text{K}=19458\text{J}$$

and the heat needed to warm the melted ice to 19.0°C is

$$45.0\text{g}\times 4.2\text{J/gK}\times 19.0\text{K}=3591\text{J}$$

so that

$$\text{L}=\frac{19458\text{J}-3591\text{J}}{45.0\text{g}}=\frac{15867\text{J}}{45.0\text{g}}=353\text{J/g}$$

The uncorrected value is 10J/g higher, which is about 3% of the answer. It lies outside the band quoted in (v), and it is meant to: an answer worked without the correction cannot land inside that band.

(vii) The calorimeter was hotter than the room for the whole of the first 5min and was losing heat all that time, at a rate the first straight line measures directly: the water fell 3.5K before any ice was added. That loss did not stop while the ice was melting, and the uncorrected arithmetic charges every kelvin of it to the ice. Produce the two slow lines to a common instant and you are asking what the temperatures would have been had the mixing been instantaneous, and it is only for an instantaneous mixing that heat given out equals heat taken in.

(viii) The corrected value is 343J/g against an accepted 334J/g, an excess of 9J/g. Ice below zero takes 2.1J/gK to warm, so an excess of that size means the ice was about 4.3K below zero when it went in. The construction has removed the loss to the room, which was worth 10J/g; what is left is the ice itself, and no graph can remove it. The remedy is the one given at the end of Experiment 16.3.

Examiner's eye

- The 5min of readings taken before anything happens look like a waste of a stopwatch and are the most valuable readings on the sheet. They turn "the calorimeter loses heat to the room" from a remark into a number, and part (vii) has nothing to stand on without them.
- Watch the sign of each correction. Both produced lines land below the readings that generated them, so both θ_1 and θ_2 come out below their uncorrected counterparts. The answer still moves, for two reasons: the two shift by different amounts, so the fall changes, and θ_2 appears a second time on its own in the term that warms the melted ice.
- Here the same quantity is asked for twice and both answers are right. Part (v) is the corrected value and part (vi) is what the same readings give without the construction; an answer that quotes only one of them has answered half the question.
- An accepted value is a target, not a verdict. A determination that lands 3% above 334J/g after a correction has been made honestly is a better piece of work than one that lands on it by leaving two errors to cancel, and the account you write should make clear which of the two you did.

Practice 17.1

box (Ω)	I (A)	V (V)
14	0.24	1.05
11	0.28	1.20
8	0.34	1.45
6	0.40	1.75
4	0.49	2.10
2	0.62	2.70

(i) and (ii). Take I along the horizontal axis at 2cm to 0.1A, running from 0 to 0.7A, and V up the vertical axis at 2cm to 0.5V, running from 0 to 3.0V. That is 14cm by 12cm, which fits an A4 sheet upright. Title the graph "A graph of V (V) against I (A)".

(iii) The points lie on a straight line through the origin. Taking A (0.31, 1.34) and B (0.58, 2.51) on that line,

$$\text{slope}=\frac{2.51\text{V}-1.34\text{V}}{0.58\text{A}-0.31\text{A}}=\frac{1.17\text{V}}{0.27\text{A}}=4.33\Omega$$

(iv) The voltmeter is across the ammeter and the coil together, and the same current passes through both, so what the slope measures is the two resistances added:

$$\text{slope}=\text{S}+\text{R}_\text{A} \quad \text{so} \quad \text{S}=4.33\Omega-0.40\Omega=3.93\Omega$$

The coil has a resistance of 3.93Ω. Accept 3.70Ω to 4.10Ω.

(v) The correction is

$$\frac{0.40\Omega}{4.33\Omega}\times 100=9.2\%$$

so a little under a tenth of what the graph gave has nothing to do with the coil at all.

(vi) This one. The ammeter adds 0.40Ω to the slope whatever the coil is, so on 500Ω the correction would be under 0.1% and could be made exactly in any case. The other connection loses a share of the current to the voltmeter instead, and that share grows with the coil: against 500Ω a voltmeter of a few thousand ohms takes a seventh of what the ammeter reads, or more. Put the ammeter inside the voltmeter's leads for a large resistance and outside them for a small one.

(vii) The origin says only that with no current there is no potential difference anywhere, and that is as true of the ammeter and the coil together as it is of the coil alone. What the connection changes is how much resistance sits between the voltmeter's leads, which is the slope, and the slope of a line is settled quite separately from whether it goes through the origin.

(viii) Replace the coil with a short thick wire and repeat the run, keeping the box at its four largest settings so that the ammeter is not driven past the end of its scale. The voltmeter is then reading across the ammeter and a few thousandths of an ohm, so the slope is the resistance of the ammeter on its own. The potential differences run from about two divisions of the voltmeter to about five, so a slope taken from those four points is worth something like 7%; compare it with the figure the question supplied and expect no more than that.

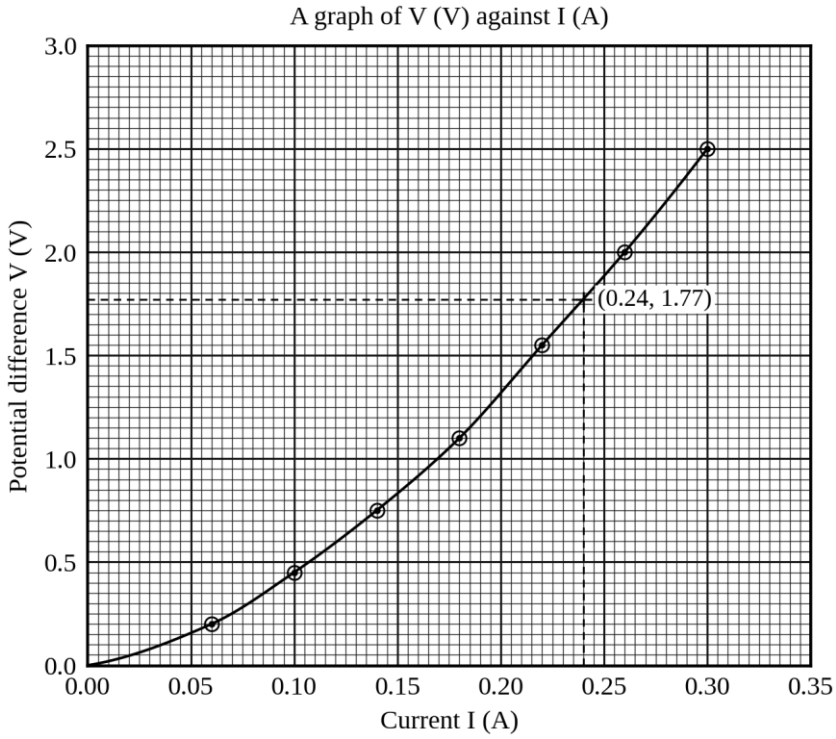
Examiner's eye

- Draw the circuit before you write anything else, even when the question does not ask for one. This whole question turns on which side of the ammeter a voltmeter lead is clipped, and that is invisible in a table of numbers and obvious in a diagram.
- Do not skip a correction because it makes the answer smaller. Marks are given for recognising that the slope is not the answer, for the subtraction, and for the sentence saying why, and a script that quotes the slope as the resistance loses all three while getting the graph completely right.
- Notice what the two connections do *not* differ in. Both give a straight line, both pass through the origin, both have the same scatter, and neither graph shows in its shape which of the two you drew. The only record of it is your circuit diagram.
- No connection of two meters to one coil measures the coil by itself. One of the two leaves a correction you can make exactly and the other leaves a share of the current you can only estimate, which is what part (vi) turns on. Say which you chose and why, and the examiner has what the question asked for.

Practice 17.2

I (A)	V (V)	R = V/I (Ω)
0.06	0.20	3.33
0.10	0.45	4.50
0.14	0.75	5.36
0.18	1.10	6.11
0.22	1.55	7.05
0.26	2.00	7.69
0.30	2.50	8.33

(i) and (ii). Take I along the horizontal axis at 2cm to 0.05A, running from 0 to 0.35A, and V up the vertical axis at 2cm to 0.5V, running from 0 to 3.0V. That is 14cm by 12cm, which fits an A4 sheet upright. Title the graph "A graph of V (V) against I (A)".



(iii) The curve passes through (0.24, 1.77), so at 0.24A the potential difference across the bulb is 1.77V. Accept 1.70V to 1.85V.

(iv) The third column of the table above. The quotient rises from 3.33Ω at the smallest current to 8.33Ω at the largest.

(v) No. The graph is a curve and not a straight line, so V is not proportional to I, and the quotient R=V/I is not the same at every current. The current warms the filament, its resistance climbs with its temperature, and a conductor whose resistance depends on the current in it is outside the law.

(vi) At 0.10A the resistance is 4.50Ω and at 0.30A it is 8.33Ω, so

$$\frac{8.33\Omega}{4.50\Omega}=1.85$$

The filament is nearly twice the resistance at the larger current. Accept 1.70 to 2.00.

(vii) The equation V=IR is a definition of resistance and holds at every point of the graph: at each current there is a potential difference, and their quotient is the resistance of the filament at that current. What fails is the law, which is the further claim that the quotient is a constant. Ohm's law asks for one temperature, and a filament that goes from a dull red to white is not at one temperature.

(viii) With the currents taken in rising order, each reading is made while the filament settles towards a higher temperature from below, so the seven readings are reached the same way and any lag affects them alike. In the other direction the filament arrives at each reading still hot from the one before, the resistances come out high, and a run taken downwards will not lie on the curve a run taken upwards gives.

Examiner's eye

- There is no slope triangle anywhere in this answer, and that is the point of the question. What the examiner is watching for is a straight line ruled through a curve because a straight line is what every graph before this one wanted.
- Your curve should reach the bulb's marked potential difference at very nearly its marked current, which is a check on the whole run worth quoting. If your curve gets there at 0.20A the ammeter is reading low, and if it never gets there at all the bulb is not the one on the label.
- The commonest wrong reason offered is that the filament "does not obey Ohm's law because it is not a metal." It is a metal, and a purer one than most of the wire in the laboratory. What puts it outside the law is that it is not held at one temperature, and the law says nothing about conductors that are not.
- Any question that asks whether a law holds wants evidence in both directions. Say what the graph would have looked like had the filament obeyed the law, say what it looks like instead, and the answer is complete in two sentences.

Practice 17.3

R (Ω)	V (V)	1/V (per volt)
500	2.45	0.408
900	2.15	0.465
1300	1.85	0.541
1700	1.65	0.606
2100	1.50	0.667
2500	1.35	0.741

(i) and (ii). Take R along the horizontal axis at 2cm to 500Ω, running from 0 to 3000Ω, and 1/V up the vertical axis at 2cm to 0.1 per volt, running from 0 to 0.8 per volt. That is 12cm by 16cm, which fits an A4 sheet upright. Title the graph "A graph of 1/V (per volt) against R (Ω)".

(iii) The points lie on a straight line. Taking A (750, 0.446) and B (2250, 0.696) on it, the slope is 0.250 per volt divided by 1500Ω, which is 1.67×10^{-4} per volt per ohm, and the line cuts the vertical axis at 0.320 per volt.

(iv) The intercept is 1/E, so E is 1 divided by 0.320 per volt, which is 3.13V. Accept 3.00V to 3.25V.

(v) The resistance of the voltmeter is the intercept divided by the slope:

$$R_V=\frac{0.320\text{ per volt}}{1.67\times10^{-4}\text{ per volt per ohm}}=1920\Omega$$

Accept 1750Ω to 2150Ω.

(vi) The voltmeter across one 2000Ω resistor makes a parallel pair of

$$\frac{2000\Omega\times1920\Omega}{2000\Omega+1920\Omega}=980\Omega$$

and that pair is in series with the second resistor, which is still a plain 2000Ω. The meter therefore reads

$$V=3.13V\times\frac{980\Omega}{2000\Omega+980\Omega}=1.03V$$

against a true 1.57V with no meter connected, which is what **Kipanga** saw. Accept 0.95V to 1.15V.

(vii) The error is

$$\frac{1.57V-1.03V}{1.57V}\times100=34\%$$

The reading is low by more than a third of the true value. Accept 31% to 37%.

(viii) Build the divider out of much smaller resistors. What decides the disturbance is the resistance of the meter set against the resistance it is put across, so two resistors of 20Ω in place of the two of 2000Ω would bring this same meter down to about 0.5% instead of 34%. The other move, the largest range of the meter, helps far less than it looks: a moving coil voltmeter is a fixed movement with a resistor in series with it and the resistor is chosen for the range, so the 10V range has exactly twice the resistance of the 5V range and the error falls only to about 21%.

Examiner's eye

- An instrument that changes what it measures is not the same thing as an instrument that is wrong. This distinction is worth a sentence in any account, and part (vii) is where the marks for it sit; a script that writes "the voltmeter is inaccurate" has described the symptom and missed the cause.
- Work the true value out first and the disturbed value second. A percentage error needs a denominator, and the denominator here is the potential difference the divider has before the meter is connected, which no instrument in the room can show you. It has to be calculated, and a script that divides by the reading instead has quietly used the wrong one of the two.
- Notice how much of part (vi) is circuit reasoning and how little is arithmetic. Redraw the divider with the voltmeter in place, mark the parallel pair, and the sum takes two lines. Attempt it without the redrawing and the commonest slip is to put the voltmeter in series with the resistor it is measuring.
- Check the direction of every column you invert. 1/V grows as V falls, so the largest box setting gives the highest point on the graph; a reciprocal column plotted the other way up gives a line of negative slope and an intercept that means nothing.

Practice 17.4

I (A)	V for R_0 alone (V)	V for the pair (V)
0.10	0.90	0.70
0.14	1.30	0.95
0.18	1.65	1.25
0.22	2.00	1.55
0.26	2.35	1.80
0.30	2.75	2.10

(i) and (ii). Take I along the horizontal axis at 2cm to 0.05A, running from 0 to 0.35A, and V up the vertical axis at 2cm to 0.5V, running from 0 to 3.0V. That is 14cm by 12cm, which fits an A4 sheet upright. Title the graph "A graph of V (V) against I (A)" and label each line.

(iii) Both sets of points lie on straight lines through the origin. For R_0 alone the line passes through A (0.13, 1.19) and B (0.28, 2.55), so its resistance is 1.36V divided by 0.15A, which is 9.07Ω. For the pair the line passes through C (0.16, 1.11) and D (0.28, 1.95), so its resistance is 0.84V divided by 0.12A, which is 7.00Ω. Accept 8.85Ω to 9.30Ω for R_0 and 6.83Ω to 7.15Ω for the pair.

(iv) The element and R_0 are in parallel, so their conductances add and the element carries what is left:

$$\frac{1}{X}=\frac{1}{R_c}-\frac{1}{R_0}=0.1429 \text{ per ohm}-0.1103 \text{ per ohm}=0.0326 \text{ per ohm}$$

and turning it up the other way,

$$X=\frac{1}{0.0326 \text{ per ohm}}=30.7\Omega$$

The element has a resistance of about 30.7Ω. Accept 25Ω to 38Ω. That is a marking tolerance, not the precision of one carefully drawn graph, and it is as wide as it is because the two bands above it are subtracted.

(v) About 0.14A, with the rheostat at its smallest, and the rheostat can only reduce it from there. One division of this ammeter is 0.01A, so the whole run would sit in the bottom seventh of the scale, and the 0.06A the needle showed **Kipanga** is good to no better than 1 part in 6, which is why he and **Kipute** could read the same needle a division apart and both be honest. Put 9.1Ω beside the element and the same battery drives up to 0.54A, so the run can be taken where one division is a small fraction of the reading.

(vi) 0.5% of 7.00Ω is 0.035Ω and 0.5% of 9.07Ω is 0.045Ω. Those move the two reciprocals by

$$\frac{0.035\Omega}{7.00\Omega^2}+\frac{0.045\Omega}{9.07\Omega^2}=0.00126 \text{ per ohm}$$

and the difference of the reciprocals is only 0.0326 per ohm, so the uncertainty is 3.9% and the element is 30.7Ω with an uncertainty of 1.2Ω. Two measurements good to 0.5% have produced one good to about 4%, and the whole of the loss happened in the subtraction. The parallel arrangement buys a current the ammeter can read and pays for it in precision; part (viii) is about buying some of that back.

(vii) The voltmeter is across M and N in both runs, so in both runs the ammeter reads the current in the voltmeter as well as the current in whatever is between the terminals. What each line measures is therefore the combination in parallel with the voltmeter, and the conductance of the voltmeter is added to both of the reciprocals in part (iv). Subtract one from the other and it goes out exactly.

(viii) A larger one, of about 30Ω. The difference between the two reciprocals is about a quarter of the larger of them here; with R_0 near the value of the element itself the difference would be a half, so the same 0.5% on each would leave about 1.5% on the element instead of 3.9%. The limit on how large R_0 may be is the ammeter again: the pair must still pass a current the meter can read across a useful part of its scale.

Examiner's eye

- Work out the magnifying factor of a subtraction while you are still choosing the method. Two quantities each known to a fraction f give a difference known to f times the sum of the two divided by the difference of the two, which comes to about 8 here and would come to 3 with a better choice of R_0. That factor decides whether a method of this kind is worth setting up at all, and the arithmetic that follows cannot mend it.
- The run on the known resistor is what makes the two lines comparable. 20 minutes saved by taking R_0 from the label throws away every cancellation the method offers, including the one part (vii) is about, and the label was never measured with your leads and your joints in any case.
- This is the second question in the chapter where a meter is disposed of, and it is disposed of the other way. In Practice 17.1 the ammeter's resistance was known and taken away; here the voltmeter's current is never known at all and is subtracted from itself. Learn to ask, of every systematic effect, whether it cancels, whether it can be corrected, or whether it can only be admitted.
- An iron element is meant to get hot, and hot metal has a larger resistance. What you have measured is its resistance cold, at about a twentieth of the current it was built to carry, and the account should say so rather than offering the figure as the resistance of the iron in use.

Practice 18.1

R (Ω)	I (A)	1/I (A^{-1})
1	0.75	1.333
2	0.49	2.041
3	0.37	2.703
4	0.30	3.333
6	0.21	4.762

(i) and (ii). Take 1/I at 2cm to $0.5A^{-1}$, running from 0 to $5A^{-1}$, and R at 2cm to 1Ω, running from -1Ω to 7Ω. That is 20cm by 16cm, which fits an A4 sheet on its side. Give the graph the title A graph of R (Ω) against 1/I (A^{-1}).

(iii) Taking M (1.5, 1.25) and N (4.5, 5.65) on the line,

$$E=\frac{5.65\Omega-1.25\Omega}{4.5A^{-1}-1.5A^{-1}}=1.47V$$

Accept 1.42V to 1.52V.

(iv) The line cuts the R axis at -0.95Ω, so the resistance outside the box is 0.95Ω. Accept 0.85Ω to 1.05Ω.

(v) It is 2.0% below the marked figure, which is not a fault in the measurement. The figure on the case is a nominal value for a cell that has done no work; this one has, and a cell of this size settles to a little under its marked e.m.f. within the first few hours of use.

(vi) A voltmeter of very high resistance draws almost no current, and with almost no current there is almost no potential difference lost inside the cell. The meter therefore reads the e.m.f. whatever the internal resistance is, and the internal resistance is exactly what makes a tired cell useless. The test separates cells that are dead from cells that are alive, and nothing else.

(vii) With the box at 4Ω the graph gives $1/I = 3.375A^{-1}$, so

$$I=\frac{1.47V}{4.00\Omega+0.95\Omega}=0.30A$$

The share lost before the bulb is the resistance outside the box over the whole resistance of the loop, so

$$\frac{0.95\Omega}{4.00\Omega+0.95\Omega}\times100=19\%$$

so about a fifth of the energy never reaches the bulb, and some of that fifth is spent in the meter and the leads rather than in the cell itself.

(viii) Take readings at smaller resistances, with an ammeter of higher range, so that the plotted points reach closer to the $1/I$ axis. The intercept is found by producing the line back to $1/I = 0$, and the further the nearest point stands from that axis the more a small error in the slope moves the intercept. Every reading here lies beyond $1.3A^{-1}$, so the line is produced through more than a quarter of its own length before it reaches the axis.

Examiner's eye

- More marks go astray on the wrong axis here than anywhere else. The intercept the equation gives is on the axis of R, where $1/I$ is zero, and that is the left-hand edge of the sheet, not the bottom.
- Part (v) asks for a comment, which means a number and a reason. "It is lower than the marked value" is the observation; the comment is why a marked value is not a measurement.
- Keep the units of the third column. A^{-1} is not a typing accident and it is what makes the slope come out in volts; a column headed 1/I with no unit costs a mark that the arithmetic has already earned.
- Do not average the five values of E worked out reading by reading. Each of those needs the internal resistance you do not yet know, so each is a different guess; the graph is what separates the two unknowns, and it is why the question asks for a graph.

Practice 18.2

R (Ω)	I (A)	1/I (A^{-1})
1	0.81	1.235
2	0.53	1.887
3	0.39	2.564
4	0.31	3.226
5	0.26	3.846

(i) and (ii). Take R at 2cm to 1Ω, running from 0 to 6Ω, and $1/I$ at 2cm to $0.5A^{-1}$, running from 0 to $4A^{-1}$. That is 12cm by 16cm, which fits an A4 sheet upright. Give the graph the title A graph of 1/I (A^{-1}) against R (Ω).

(iii) Taking M (1.5, 1.575) and N (4.5, 3.525) on the line, the slope is $0.6500A^{-1}$ per ohm, and the electromotive force is its reciprocal:

$$E=\frac{1}{0.6500}=1.54V$$

Accept 1.48V to 1.60V.

(iv) The intercept is $0.575A^{-1}$, so

$$r=0.575A^{-1}\times1.54V=0.89\Omega$$

Accept 0.82Ω to 0.96Ω.

(v) A short thick wire adds nothing to the loop, so R = 0 and the current is fixed by the resistance outside the box alone:

$$I=\frac{1.54V}{0.89\Omega}=1.7A$$

It should not be tested because with nothing outside to take a share, the whole of the power is developed in the cell and in the connections that carry its current, and most of it in the cell. The cell heats, the case swells and the contents leak, and the answer is destroyed in the measuring of it.

(vi) What the set receives is what is left of the electromotive force once the loop outside the box has taken its share, so with the values found above,

$$V=1.54V-0.25A\times0.89\Omega=1.32V$$
$$V=1.54V-0.60A\times0.89\Omega=1.01V$$

(vii) The loud passages draw the larger current, and the larger current costs the larger share of the electromotive force inside the cell, so the supply voltage falls just when the set needs it most and the receiver loses the station. At the quiet passages the current falls again, the supply voltage recovers and the station comes back. Switch the set off and the products of the reaction disperse from the plates, which lowers the internal resistance again, so the set plays loudly for a short while until they build up once more. The transmitter has been steady throughout.

(viii) The resistance set on the box is the quantity known exactly, and here it lies along the horizontal axis where the plotted positions are exact. All of the reading error is then carried by the vertical axis, which is where a best line is judged. It also spaces the points evenly, because R is stepped in equal amounts while $1/I$ is not, and evenly spaced points weight the line evenly.

Examiner's eye

- Both halves of part (v) are marked: the calculation and the warning. A number with no warning answers half the question; a warning with no number answers the easier half.
- Watch which cell part (vi) is about. The set takes two cells and the graph was drawn for one, so what is worked out here is what one cell delivers; doubling it is a separate step and the question did not ask for it.
- Say what recovers and why in part (vii). An answer that stops at "the battery is weak" has described the symptom; the mark is for naming the current as the thing that changes and the internal resistance as the thing that turns it into a lost voltage.
- Read the accept bands as marking tolerance, not as the precision of the method. Yours should be far better than the band printed here, which is set wide enough to cover a line drawn by hand through five points on a poor afternoon.

Practice 18.3

I (A)	V (V)
0.20	1.50
0.50	1.38
0.80	1.28
1.10	1.16
1.40	1.04

(i) and (ii). Take I at 2cm to 0.2A, running from 0 to 1.6A, and V at 2cm to 0.1V, running from 1.0V to 1.7V. That is 16cm by 14cm, which fits an A4 sheet upright. Give the graph the title A graph of V (V) against I (A).

(iii) The line meets the V axis at 1.575V, so the electromotive force is 1.58V. Accept 1.54V to 1.62V.

(iv) Taking M (0.30, 1.460) and N (1.30, 1.080) on the line,

$$\text{slope}=\frac{1.080\text{V}-1.460\text{V}}{1.30\text{A}-0.30\text{A}}=-0.38\Omega$$

so the internal resistance is 0.38Ω. Accept 0.34Ω to 0.42Ω.

(v) At 1.40A the terminal p.d. is 1.04V, so the power delivered outside is

$$P=1.04\text{V}\times1.40\text{A}=1.46\text{W}$$

and the power wasted inside is

$$P=(1.40\text{A})^2\times0.38\Omega=0.74\text{W}$$

(vi) The lamp and the cell make a loop of 1.2Ω + 0.38Ω, so

$$I=\frac{1.58\text{V}}{1.2\Omega+0.38\Omega}=1.00\text{A}$$

$$V=1.00\text{A}\times1.2\Omega=1.20\text{V}$$

(vii) Two lamps side by side halve the external resistance to 0.60Ω, which is now under twice the internal resistance, so

$$I=\frac{1.58\text{V}}{0.60\Omega+0.38\Omega}=1.61\text{A}$$

The total current has risen but not doubled, so each lamp now takes 0.81A against the 1.00A the single lamp took, and each is dimmer than the single lamp was. The pair together still give more light, because the power delivered outside the cell has risen from 1.20W to 1.56W, but the gain is far short of double.

(viii) Take readings over a wider range of current, or use a voltmeter with finer divisions. The internal resistance is a slope, and a slope is measured by the fall in V divided by the rise in I: over the range used here the terminal p.d. falls by less than half a volt, so a reading error of 0.02V at each end is worth several per cent of the answer. Double the range of current and that share is halved.

Examiner's eye

- Power wasted inside the cell is I^2r and not V^2/r, because V is the p.d. across the outside and not across the internal resistance. The two differ by nearly a factor of four here.
- No new readings are offered for the comparison in part (vii), so the argument has to come from the graph you already have. Halve the external resistance, work the current out again, and set the two numbers side by side.
- Set a rheostat to zero and the cell is across the ammeter alone. Start with the slider at the far end and bring it down; the first current you set should be the smallest, not the largest.
- Power outside and power inside are different questions, and part (v) asks both. Give the two separately, each with its own formula beside it; a single figure answers neither, however carefully it was worked.

Practice 18.4

R (Ω)	V (V)	1/R (Ω^{-1})	1/V (V^{-1})
1	0.86	1.000	1.1628
2	1.08	0.500	0.9259
3	1.18	0.333	0.8475
5	1.28	0.200	0.7813
10	1.36	0.100	0.7353

(i) and (ii). Take 1/R at 2cm to $0.2\Omega^{-1}$, running from 0 to $1.2\Omega^{-1}$, and 1/V at 2cm to $0.05V^{-1}$, running from $0.65V^{-1}$ to $1.20V^{-1}$. That is 12cm by 22cm, which fits an A4 sheet upright.

(iii) The intercept is $0.6875V^{-1}$, so

$$E=\frac{1}{0.6875}=1.45\text{V}$$

Accept 1.40V to 1.50V.

(iv) Taking M (0.14, 0.7550) and N (0.94, 1.1350) on the line, the slope is $0.4750\Omega V^{-1}$, so

$$r=0.4750\Omega V^{-1}\times1.45\text{V}=0.69\Omega$$

Accept 0.62Ω to 0.76Ω.

(v) At 0.5Ω the box takes the share

$$V=\frac{1.45\text{V}\times0.5\Omega}{0.5\Omega+0.69\Omega}=0.61\text{V}$$

which is less than half the electromotive force, because at that setting the box is the smaller of the two resistances.

(vi) With the bell silent the load is the lamp alone, 3Ω:

$$V=\frac{1.45\text{V}\times3\Omega}{3\Omega+0.69\Omega}=1.18\text{V}$$

With the bell ringing, 3Ω and 6Ω in parallel make 2Ω, so

$$V=\frac{1.45V\times 2\Omega}{2\Omega+0.69\Omega}=1.08V$$

(vii) Ring the bell and the resistance the cell works into falls, so the current rises, so more of the electromotive force is spent inside the cell and less reaches either device. The lamp and the bell then share a smaller p.d. than the lamp had to itself, and both are worse off. No wire is at fault. The cure is a battery of lower internal resistance, or two cells in parallel, which halves it.

(viii) Because the p.d. across the box climbs towards the electromotive force as R is raised, so the values of $1/V$ crowd together and the whole column shrinks towards one number. The resistance is carried by the slope, and a slope taken from points that lie within the reading error of one another is not a measurement. The voltmeter makes it no better: it stands across the box at every setting, so what carries the current is always a little less than the resistance you set, and at large settings that difference is no longer small.

Examiner's eye

- Work out the parallel resistance before the sharing, not after. Two loads across one cell do not each get their own share of the electromotive force; they make one load between them and that load gets one share.
- An intercept read as $0.7V^{-1}$ instead of $0.6875V^{-1}$ moves the electromotive force by more than 0.02V. Read the intercept to the half-ruling and write down all four figures before inverting.
- No wire is what part (vii) asks the writer to change. Name the quantity, say which way it has to move, and give one way of moving it.
- Nothing that can be measured is named by "the battery is too small". Internal resistance can be, and this whole experiment has just measured it.

Practice 18.5

The mean diameter of the wire was 0.40mm and the electromotive force read on open circuit was 1.50V.

L (cm)	I (A)	1/I (A^{-1})
10	0.90	1.111
20	0.59	1.695
30	0.44	2.273
40	0.35	2.857
50	0.29	3.448

(i) and (ii). Take L at 2cm to 10cm, running from 0 to 60cm, and $1/I$ at 2cm to $0.5A^{-1}$, running from 0 to $3.5A^{-1}$. That is 12cm by 14cm, which fits an A4 sheet upright.

(iii) Taking M (5, 0.825) and N (45, 3.150) on the line, the slope is $0.05813A^{-1}cm^{-1}$, which in metres is $5.813A^{-1}m^{-1}$.

(iv) The area is $1.257\times10^{-7}m^2$, so

$$\rho=5.813A^{-1}m^{-1}\times1.50V\times1.257\times10^{-7}m^2=1.1\times10^{-6}\Omega m$$

Accept $1.05\times10^{-6}\Omega m$ to $1.15\times10^{-6}\Omega m$.

(v) The intercept is $0.525A^{-1}$, so

$$r=0.525A^{-1}\times1.50V=0.79\Omega$$

(vi) The nichrome reel. The measured resistivity is more than twice the constantan figure and within a few per cent of the nichrome one, and no error in the diameter of the size a micrometer can make would close a gap of that width.

(vii) The resistance of 1m is

$$\frac{\rho}{A}=\frac{1.1\times10^{-6}\Omega m}{1.257\times10^{-7}m^2}=8.8\Omega m^{-1}$$

so the length with a resistance of 5Ω is

$$l=\frac{5\Omega}{8.8\Omega m^{-1}}=0.57m$$

(viii) This wire has more than twice the resistance per metre of the one in Experiment 18.5, so a metre of it would have brought the current down to about 0.16A. A hundredth of an ampere is a sixteenth of that, and the reciprocal carries the same share, so the last points would have scattered far more widely than the first and the line would have been decided by its worst readings.

Examiner's eye

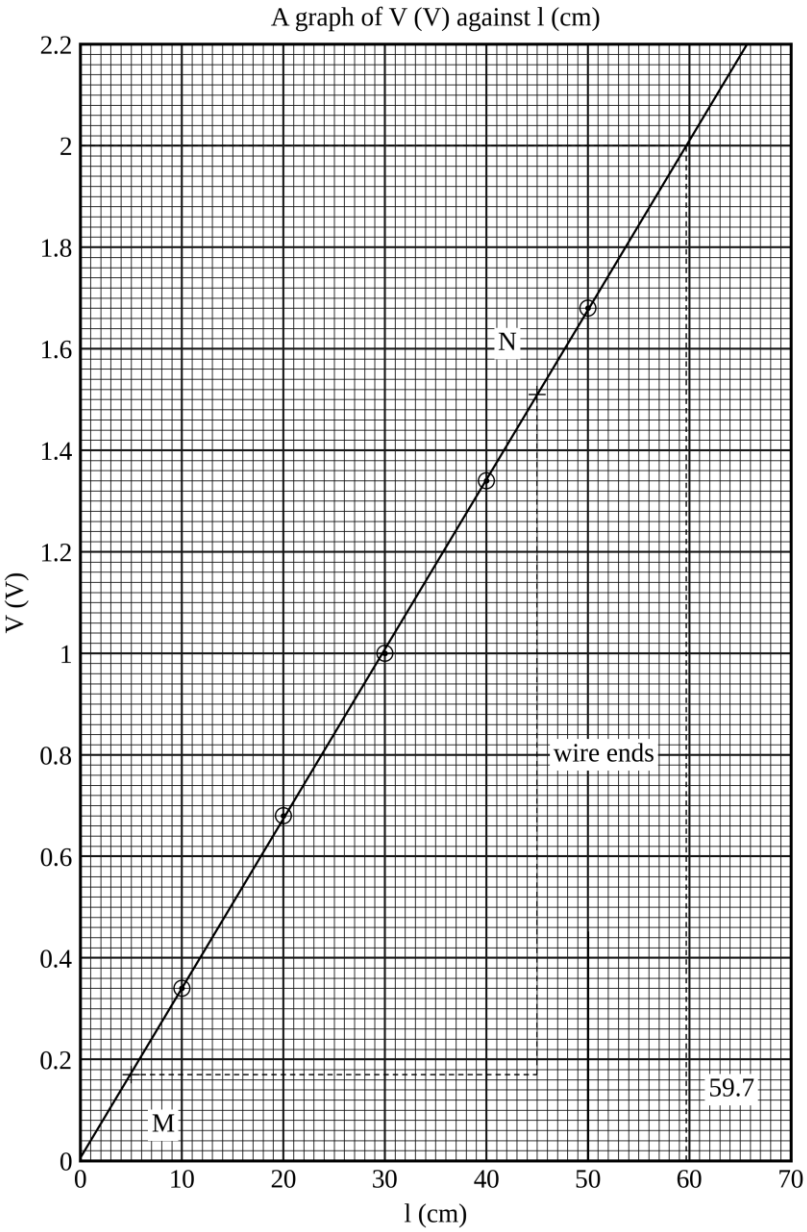
- Compare resistivities in the same power of ten before deciding anything. $1.1\times10^{-6}\Omega m$ and $4.9\times10^{-7}\Omega m$ differ by a factor of more than two, and a comparison of 1.1 with 4.9 picks the wrong wire.
- Resistance per metre is ρ/A and not ρA. Check it against the wire itself: a thin wire has more resistance per metre than a thick one, so the area belongs underneath.
- Give the length in part (vii) to the same number of figures as the resistivity it came from, and no more. Three figures in a length that rests on a micrometer reading of two is an answer pretending to be better than its worst measurement.
- Readings, not the wire, are the subject of part (viii). Say what would have happened to the currents, and to the column you plot, before saying anything about the line.

Practice 18.6

The voltmeter read 3.02V across the two cells with the switch open.

l (cm)	V (V)
10	0.34
20	0.68
30	1.00
40	1.34
50	1.68

(i) and (ii). Take l at 2cm to 10cm, running from 0 to 70cm, and V at 2cm to 0.2V, running from 0 to 2.2V. That is 14cm by 22cm, which fits an A4 sheet upright. The length axis is carried past the last reading because part (vi) asks for a value the readings do not reach.



(iii) Taking M (5, 0.17) and N (45, 1.51) on the line, the gradient is 0.0335Vcm^{-1}, which is 3.35Vm^{-1}.

(iv) The gradient is the current times the resistance of a metre of the wire, so

$$I=\frac{3.35\text{Vm}^{-1}}{2.70\Omega\text{m}^{-1}}=1.24\text{A}$$

(v) The 50cm in circuit carries 1.68V, the cells offered 3.02V on open circuit, and the difference is lost inside them:

$$r=\frac{3.02\text{V}-1.68\text{V}}{1.24\text{A}}=1.08\Omega$$

(vi) Produced beyond the last reading, the line reaches V = 2V at

$$l=\frac{2\text{V}}{0.0335\text{Vcm}^{-1}}=59.7\text{cm}$$

(vii) Only 50cm of the wire is in the circuit, and the jockey cannot be pressed onto wire that is carrying no current. Beyond the second crocodile clip there is no potential gradient at all, so the voltmeter reading stops rising at 1.68V however far the jockey is moved. The graph was produced past the last reading on the assumption that the gradient continues, and past 50cm that assumption is false.

(viii) Lengthen the wire in the circuit and its resistance rises, which lowers the current, which lowers the gradient, so the answer is not the 59.7cm of part (vi). Write L for the length in metres; the wire has resistance 2.70L and takes the share

$$2.00\text{V}=\frac{3.02\text{V}\times2.70\text{L}}{2.70\text{L}+1.08\Omega}$$

$$2.70\text{L}\times(3.02-2.00)=2.00\times1.08$$

$$\text{L}=\frac{2.16}{2.70\times1.02}=0.78\text{m}$$

so about 78cm of wire, not the 59.7cm the graph gave. The man under the first lamp has the better of the argument. His own tapping is where it always was, but every metre added to the run raises its resistance, lowers the current and lowers the gradient with it, so the share his own stretch of wire carries falls with every metre added.

Examiner's eye

- Produce a line as far as the physics holds and no further. Part (vii) is there to make you notice where the wire stops, and an answer that gives (vi) and falls silent has answered only the arithmetic.
- Keep the two lengths in parts (vi) and (viii) apart. One is read off a graph drawn for a fixed current; the other allows the current to change, and they are different numbers for that reason and not because one of them is wrong.
- Write the open-circuit reading into the table of results with everything else you measured. Part (v) cannot be started without it.
- Comment means judgement, and a comment that only says who is right has not made one. Name the quantity that changes when the run is lengthened, and say which way it moves.

Practice 18.7

R (Ω)	I (A) series	1/I (A^{-1})	R (Ω)	I (A) parallel	1/I (A^{-1})
2	0.90	1.111	1	0.96	1.042
3	0.69	1.449	2	0.59	1.695
4	0.56	1.786	3	0.42	2.381
5	0.47	2.128	4	0.33	3.030
6	0.41	2.439	6	0.23	4.348

(i) and (ii) Take 1/I at 2cm to $0.5A^{-1}$, running from 0 to $4.5A^{-1}$, and R at 2cm to 1Ω, running from -2Ω to 7Ω. That is 18cm by 18cm, which fits an A4 sheet upright. The two lines share one pair of axes, and each is labelled where it leaves the plotted points.

(iii) For the cells in series, taking M (1.25, 2.40) and N (2.30, 5.55), the slope is 3.00V. For the cells in parallel, taking M (1.25, 1.30) and N (4.00, 5.45), the slope is 1.51V.

(iv) The series line cuts the R axis at -1.35Ω and the parallel line at -0.60Ω, so the intercepts stand for 1.35Ω and 0.60Ω.

(v) Taking the meter and the leads out of each,

$$r_{series}=1.35\Omega-0.33\Omega=1.02\Omega$$
$$r_{parallel}=0.60\Omega-0.33\Omega=0.27\Omega$$

(vi) The two electromotive forces stand in the ratio

$$\frac{3.00V}{1.51V}=1.99$$

against the 2 the rule predicts, and the two internal resistances in the ratio

$$\frac{1.02\Omega}{0.27\Omega}=3.8$$

against the 4 the rules predict. The first agrees closely. The second is 5% short, and the reason is that it is a ratio of two small numbers each obtained by subtracting one measured quantity from another: 0.27Ω is what is left after taking 0.33Ω from 0.60Ω, so an error of half a hundredth of an ohm in that intercept is worth a fiftieth of the answer. Two intercepts cannot settle a ratio any more closely than this, and within that limit the results agree with the rules.

(vii) With a bulb of 4Ω,

$$I_{series}=\frac{3.00V}{4\Omega+1.02\Omega}=0.60A$$
$$I_{parallel}=\frac{1.51V}{4\Omega+0.27\Omega}=0.35A$$

so the series arrangement wins easily. With a bulb of 0.2Ω,

$$I_{series}=\frac{3.00V}{0.2\Omega+1.02\Omega}=2.46A$$
$$I_{parallel}=\frac{1.51V}{0.2\Omega+0.27\Omega}=3.21A$$

and the parallel arrangement now wins.

(viii) Kipanga chose well for the torch, and for anything else whose resistance is large beside the internal resistance of a cell: there the series arrangement doubles the electromotive force and the doubled internal resistance hardly matters. Side by side wins when the load is small, because then it is the internal resistance that decides the current and halving it beats doubling the electromotive force. The condition is the size of the load compared with r, and the two arrangements change places at 0.49Ω, where the two currents are equal. What Kipanga added, that cells side by side make no difference at all, is wrong: they are joined like terminal to like terminal, so neither drives current through the other, and the pair behaves as one cell of the same electromotive force and half the internal resistance. Kipute's reason for asking was sound as well, since two cells that share a current between them last longer than one carrying the whole of it.

Examiner's eye

- Two lines on one sheet need two labels on the sheet. Describe the steeper line in words and leave the graph unlabelled, and the reader is left to guess which is which.
- Take the meter and the leads out before comparing the two internal resistances. They are the same in both arrangements and they do not halve, so a ratio taken from the raw intercepts comes out well short of 4 for a reason that has nothing to do with the cells.
- A ratio of two small differences is the least reliable number in any of these questions. Say so when you quote it, and quote it to two figures at most.
- A condition is what part (viii) wants, not a winner. The two arrangements each win somewhere, and the mark is for naming the comparison between the load and the internal resistance that decides it.

Practice 19.1

The bridge wire was 100cm long and the balance length L_1 was read from its left-hand end.

R (Ω)	L_1 (cm)	L_2 (cm)	L_1/L_2
3	32.4	67.6	0.479
4	38.5	61.5	0.626
5	43.8	56.2	0.779
6	49.0	51.0	0.961
7	52.8	47.2	1.119
8	55.5	44.5	1.247

(i) and (ii) Take R at 2cm to 1Ω, running from 0Ω to 9Ω, and L_1/L_2 at 2cm to 0.2, running from 0 to 1.4. That is 18cm by 14cm, which fits an A4 sheet upright. Give the graph the title A graph of L_1/L_2 against R (Ω).

(iii) Taking corners at P (2.4, 0.380) and Q (7.6, 1.200) on the line,

$$\text{slope}=\frac{1.200\text{-}0.380}{7.6\Omega\text{-}2.4\Omega}=\frac{0.820}{5.2\Omega}=0.158\Omega^{-1}$$

(iv) At balance $R_1/R_2=L_1/L_2$. The box R is in the left-hand gap and the coil Z in the right, so $L_1/L_2=R/Z$, a line through the origin of slope $1/Z$. Hence

$$Z=\frac{1}{0.158\Omega^{-1}}=6.33\Omega$$

Accept 6.20Ω to 6.45Ω.

(v) No. The coil is worth 6.33Ω, which is more than a quarter above the 5.0Ω asked for. The band above is 0.25Ω wide, and the discrepancy is more than five times that, so the measurement settles the question.

(vi) It would tell him that the two coils are not identical, which is more than he had before. It would not tell him what either of them is worth. Resistance depends on the length of the wire, on its cross-section and on the material, and mass depends on the length, the cross-section and the density; a long thin winding and a short thick one of the same mass differ in resistance by a large factor. Only a measurement of resistance answers the requisition.

(vii) The box lets you choose where the balance point falls. With a single fixed resistor you get one balance length and have to take whatever part of the wire it lands on, which may be a poor part; with the box you can put six balance points across the middle of the wire and get the answer from a line instead of a point.

Examiner's eye

- Answer part (v) with a comparison, not a value. The question asks whether this is the coil that was asked for; a reply that stops at 6.33Ω has done the physics and not answered the question.
- Hold the ratio the right way up. With the box on the left it is L_1/L_2 that is proportional to R; plot L_2/L_1 instead and the points bend away from the line.
- Six points, one line, one slope. Work R divided by L_1/L_2 out row by row, average the six answers, and you are not wrong; but you have thrown away the check that the points really do lie on a straight line, and that check is what tells you the bridge wire is sound.
- Say what the mass argument in part (vi) can reach. It distinguishes the coils; it does not measure either of them. An answer that simply calls the storekeeper wrong has missed the half of him that was right.

Practice 19.2

The standard resistor in the left-hand gap was 5.00Ω, and the micrometer read 0.31mm at each of the three places along the wire.

x (cm)	L_1 (cm)	L_2 (cm)	R_2 (Ω)
5	61.8	38.2	3.09
10	56.6	43.4	3.83
15	52.3	47.7	4.56
20	49.1	50.9	5.18
25	46.0	54.0	5.87
30	42.8	57.2	6.68

(i) and (ii) Take x at 2cm to 5cm, running from 0cm to 35cm, and R_2 at 2cm to 1Ω, running from 0Ω to 7.5Ω. That is 14cm by 15cm, which fits an A4 sheet upright. Give the graph the title A graph of R_2 (Ω) against x (cm).

(iii) Taking corners at P (6.5, 3.32) and Q (28.5, 6.42),

$$\text{slope}=\frac{6.42\Omega\text{-}3.32\Omega}{28.5\text{cm-}6.5\text{cm}}=\frac{3.10\Omega}{22.0\text{cm}}=0.141\Omega\text{cm}^{-1}$$

and the line cuts the R_2 axis at 2.40Ω. Accept $0.138\Omega\text{cm}^{-1}$ to $0.144\Omega\text{cm}^{-1}$ for the slope and 2.34Ω to 2.46Ω for the intercept.

(iv) It is the resistance of the coil: what the gap still holds when the clipped wire is nothing.

(v) The slope is $0.141\Omega\text{cm}^{-1}$, which is $14.1\Omega\text{m}^{-1}$, and the micrometer read 0.31mm, which is 3.1×10^{-4}m. So

$$A=\frac{3.14\times(3.1\times10^{-4}\text{m})^2}{4}=7.54\times10^{-8}\text{m}^2$$

$$\rho=14.1\Omega\text{m}^{-1}\times7.54\times10^{-8}\text{m}^2=1.06\times10^{-6}\Omega\text{m}$$

The slope contributes about 2%, and the diameter, uncertain by 0.005mm in 0.31mm, which is 1.6%, contributes 3.2% through the square, so accept $1.00\times10^{-6}\Omega$m to $1.12\times10^{-6}\Omega$m, against an accepted resistivity for nichrome of about $1.1\times10^{-6}\Omega$m. That is a marking tolerance, not the precision of one carefully drawn graph.

(vi) A length x of the wire is worth $0.141\Omega cm^{-1}$ times x, so

$$x=\frac{2.40\Omega}{0.141\Omega cm^{-1}}=17.0cm$$

(vii) The board is confusing running temperature with strength. Cut two wires of the same material to the same resistance and the thinner one is much the shorter, so it has far less surface to lose heat from and runs hotter on the same power. That is what the stallholder has noticed; mechanically the thin wire is the weaker. Your measurement of Y settles its resistivity, and so narrows what it can be made of; it settles nothing about how hard it is to break.

Examiner's eye

- Take the resistivity from the slope, never from a single row. Divide one R_2 by its own x and you have put the whole coil into the answer.
- Part (vi) asks for a length: divide a resistance by a resistance per unit length. An answer in ohms means you multiplied.
- Hold the intercept in the units of the vertical axis. It is a resistance in ohms, read where the line crosses x=0, not a length.
- Grant the stallholder the half he has right. Part (vii) is not asking you to call him a fool; it is asking which quantity he has measured by eye and which one he has named.

Practice 19.3

The bridge wire was 100cm long and the balance length L_1 was read from its left-hand end.

R (Ω)	L_1 (cm)	L_2 (cm)	L_2/L_1
0.5	75.1	24.9	0.332
1.0	59.8	40.2	0.672
1.5	50.2	49.8	0.992
2.0	43.0	57.0	1.326
2.5	37.7	62.3	1.653
3.0	33.1	66.9	2.021

(i) and (ii) Take R at 2cm to 0.5Ω, running from 0Ω to 3.5Ω, and L_2/L_1 at 2cm to 0.25, running from 0 to 2.25. That is 14cm by 18cm, which fits an A4 sheet upright. Give the graph the title A graph of L_2/L_1 against R (Ω).

(iii) Taking corners at P (0.6, 0.400) and Q (2.9, 1.935) on the line,

$$\text{slope}=\frac{1.935-0.400}{2.9\Omega-0.6\Omega}=\frac{1.535}{2.3\Omega}=0.667\Omega^{-1}$$

(iv) The slab is in the left-hand gap and the box in the right, so at balance $Z/R=L_1/L_2$, which turned up the other way is

$$\frac{L_2}{L_1}=\frac{1}{Z}\times R$$

a straight line through the origin of slope 1/Z.

(v) From the slope,

$$Z=\frac{1}{0.667\Omega^{-1}}=1.50\Omega$$

Accept 1.46Ω to 1.54Ω.

(vi) The strands run side by side between the same two twisted ends, so they are in parallel and n of them, each of 6.0Ω, are together worth 6.0Ω divided by n. Hence

$$n=\frac{6.0\Omega}{1.50\Omega}=4.00$$

The slab contains 4 strands. Accept 3.9 to 4.1, rounded to 4.

(vii) Because a strand is a physical piece of wire and there is no such thing as three tenths of one. A result of 4.3 would not mean that the slab held four strands and a fragment; it would mean that one of the assumptions behind the arithmetic is wrong. The likeliest explanations are that the strands are not all the same length, that the twisted joints at the ends carry a resistance of their own, or that the figure of 6.0Ω in the store book belongs to a different reel from the one somebody used. An answer that comes out close to a whole number is a check on the method, and 4.3 would be the method telling you to look again.

Examiner's eye

- Put the length belonging to the box on top. The slab is on the left this time, not the box, so it is L_2/L_1 that grows with R.
- Parallel strands divide the resistance; they do not multiply it. Four wires side by side give four times as many paths for the current, so the bundle is worth a quarter of one strand and not four times it.
- Round the number of strands at the very end and nowhere earlier. Here the slab comes out at 1.50Ω and rounding it costs nothing, but carry the three figures through as a habit: a division that lands near the halfway mark between two whole numbers can be pushed either side of it by a rounding done too soon.
- A count that lands on a whole number is evidence. Say so in your account: it tells the examiner you knew what the check was for, and it is the one line in this question that cannot be got by accident.

Practice 19.4

The coil was wound from 60.0cm of wire and the micrometer read 0.31mm at each of the three places along the free end.

R (Ω)	L_1 (cm)	L_2 (cm)	L_2/L_1
2	66.2	33.8	0.511
3	56.5	43.5	0.770
4	49.0	51.0	1.041
5	43.9	56.1	1.278
6	39.4	60.6	1.538
7	35.4	64.6	1.825

(i) and (ii) Take R at 2cm to 1Ω, running from 0Ω to 8Ω, and L_2/L_1 at 2cm to 0.25, running from 0 to 2.0. That is 16cm by 16cm, which fits an A4 sheet upright. Give the graph the title A graph of L_2/L_1 against R (Ω).

(iii) Taking corners at P (2.6, 0.670) and Q (7.4, 1.910) on the line,

$$\text{slope}=\frac{1.910\text{-}0.670}{7.4\Omega\text{-}2.6\Omega}=\frac{1.240}{4.8\Omega}=0.258\Omega^{-1}$$

(iv) The coil is in the left-hand gap, so L_2/L_1 is R/Z and the slope is 1/Z. Hence

$$Z=\frac{1}{0.258\Omega^{-1}}=3.88\Omega$$

Accept 3.80Ω to 3.96Ω.

(v) The micrometer read 0.31mm, which is 3.1×10^{-4}m, and the coil is 60.0cm of this wire, which is 0.600m. So

$$A=\frac{3.14\times(3.1\times10^{-4}\text{m})^2}{4}=7.54\times10^{-8}\text{m}^2$$

$$\rho=\frac{3.88\Omega\times7.54\times10^{-8}\text{m}^2}{0.600\text{m}}=4.88\times10^{-7}\Omega\text{m}$$

Accept $4.6\times10^{-7}\Omega$m to $5.1\times10^{-7}\Omega$m.

(vi) Constantan. The value found is $4.88\times10^{-7}\Omega$m, which is within 1% of the constantan figure and 11% above the manganin one. Nichrome is more than twice as large and is out of the question.

(vii) The area goes as the square of the diameter, so a diameter uncertain by 0.005mm in 0.31mm, which is 1.6%, makes the resistivity uncertain by 3.2%. Add the 2% the slope contributes and the total is about 5%, so the resistivity lies between about $4.6\times10^{-7}\Omega$m and $5.1\times10^{-7}\Omega$m. Manganin, at $4.4\times10^{-7}\Omega$m, lies outside that band, so the measurement does separate the two, but only just; a better gauge would settle it more comfortably.

Examiner's eye

- Name the material and say why. Part (vi) is answered by a comparison of numbers, not by the colour of the reel or the card tied to it, and an answer with no comparison in it is an answer that could have been guessed.
- Do not quote the resistivity to four figures after a diameter read to two. The precision of the weakest measurement in a chain governs the precision of everything downstream of it, and here that is the micrometer.
- Watch the direction of the diameter error. A gauge closed too hard reads the wire thinner than it is, the area comes out small and the resistivity comes out low; the same wire, with the gauge closed too hard, comes back reported as manganin.
- Work the estimate out; do not guess it. Part (vii) wants a percentage got from a stated reading error, and one line of arithmetic does it.

Practice 19.5

The standard resistor in the left-hand gap was 3.00Ω and the jockey was clamped in turn at each of the six graduations below.

L_1 (cm)	L_2 (cm)	L_2/L_1	x (cm)
40.0	60.0	1.500	54.6
45.0	55.0	1.222	43.7
50.0	50.0	1.000	35.9
55.0	45.0	0.818	29.4
60.0	40.0	0.667	24.5
65.0	35.0	0.538	19.0

(i) and (ii) Take L_2/L_1 at 2cm to 0.2, running from 0 to 1.6, and x at 2cm to 10cm, running from 0cm to 60cm. That is 16cm by 12cm, which fits an A4 sheet upright. Give the graph the title A graph of x (cm) against L_2/L_1.

(iii) Taking corners at P (0.300, 10.8) and Q (1.380, 49.8) on the line,

$$\text{slope}=\frac{49.8\text{cm}\text{-}10.8\text{cm}}{1.380\text{-}0.300}=\frac{39.0\text{cm}}{1.080}=36.1\text{cm}$$

(iv) At balance $R_1/R_2=L_1/L_2$. With the standard resistor in the left-hand gap and the length x of V in the right, the resistance of that length is SL_2/L_1. If the wire has a resistance r for every unit of its length then $rx=SL_2/L_1$, so

$$x=\frac{S}{r}\times\frac{L_2}{L_1}$$

a line through the origin of slope S/r.

(v) The slope is 36.1cm, which is 0.361m, so

$$r=\frac{3.00\Omega}{0.361\text{m}}=8.31\Omega\text{m}^{-1}$$

Accept $8.10\Omega\text{m}^{-1}$ to $8.55\Omega\text{m}^{-1}$.

(vi) The length wanted is

$$x_t=\frac{100\Omega}{8.31\Omega m^{-1}}=12.0m$$

Accept 11.7m to 12.4m.

(vii) It cannot be done from the reel he has, because 12.0m is longer than the 10m on it; the whole reel of this wire reaches only 83.1Ω. Either join a second piece in series with the first, or use a thinner wire of the same material, which has a larger resistance for every metre and needs less of it: a wire of half the cross-sectional area would give 100Ω in about 6m.

Examiner's eye

- Get the length from the slope, not from a single balance. One clamped position and one clipped length give a value for the resistance per metre in two readings, and the line through six is worth about half as much again as the best of them.
- State what the answer to part (vi) is for. A bare 12.0m leaves the technician cutting a length he has not got.
- Keep the clamped lengths in the table even though they are not plotted. They are the readings the abscissa is worked out from, and a table that shows only the ratio has hidden where it came from.
- A proportion sum needs a measured number to start from. The arithmetic in the hook is faultless and the answer it gives is less than half the right length, because the figure it started from was invented; the whole of this question is the business of replacing that figure with a measured one.

Practice 19.6

The standard resistor in the loop was 1.00Ω and the micrometer read 0.46mm at each of the three places along the wire B.

x (cm)	L_1 (cm)	L_2 (cm)	L_1/L_2
20	37.2	62.8	0.592
30	47.5	52.5	0.905
40	54.3	45.7	1.188
50	60.1	39.9	1.506
60	64.4	35.6	1.809
70	67.4	32.6	2.067

(i) and (ii) Take L_1/L_2 at 2cm to 0.4, running from 0 to 2.2, and x at 2cm to 10cm, running from 0cm to 75cm. That is 11cm by 15cm, which fits an A4 sheet upright. Give the graph the title A graph of x (cm) against L_1/L_2.

(iii) Taking corners at P (0.360, 12.0) and Q (1.960, 65.6) on the line,

$$\text{slope}=\frac{65.6\text{cm}-12.0\text{cm}}{1.960-0.360}=\frac{53.6\text{cm}}{1.600}=33.5\text{cm}$$

(iv) The length x of B is one arm of the loop and the standard resistor S is the other, so at balance $rx/S=L_1/L_2$ and

$$x=\frac{S}{r}\times\frac{L_1}{L_2}$$

(v) The slope is S/r, which is 33.5cm or 0.335m, so

$$r=\frac{1.00\Omega}{0.335m}=2.99\Omega m^{-1}$$

Accept $2.90\Omega m^{-1}$ to $3.08\Omega m^{-1}$.

(vi) The micrometer read 0.46mm, which is 4.6×10^{-4}m, so

$$A=\frac{3.14\times(4.6\times10^{-4}m)^2}{4}=1.66\times10^{-7}m^2$$

$$\rho=2.99\Omega m^{-1}\times1.66\times10^{-7}m^2=4.96\times10^{-7}\Omega m$$

The slope contributes about 3%, and the diameter, uncertain by 0.005mm in 0.46mm, which is 1.1%, another 2% through the square, so accept $4.7\times10^{-7}\Omega m$ to $5.2\times10^{-7}\Omega m$. The accepted resistivity of constantan is about $4.9\times10^{-7}\Omega m$, and the value found is within 2% of it.

(vii) Yes. A metre of B is worth 2.99Ω, so a bridge wire cut from it puts about 3Ω across the cell. That keeps the current down to a few tenths of an ampere, which a dry cell supplies while the jockey is down without collapsing, and it is large beside the cell's own internal resistance, so most of the cell's voltage appears along the wire and moving the jockey from end to end swings the potential there through a large fraction of it. A wire of a tenth of that resistance would draw several amperes and flatten the cell in an afternoon. The objection to a much larger one is not electrical but mechanical: to reach ten times the resistance in a metre the wire has to be very much finer, and a fine bare wire stretched along a rule bends, sags and wears under the jockey.

Examiner's eye

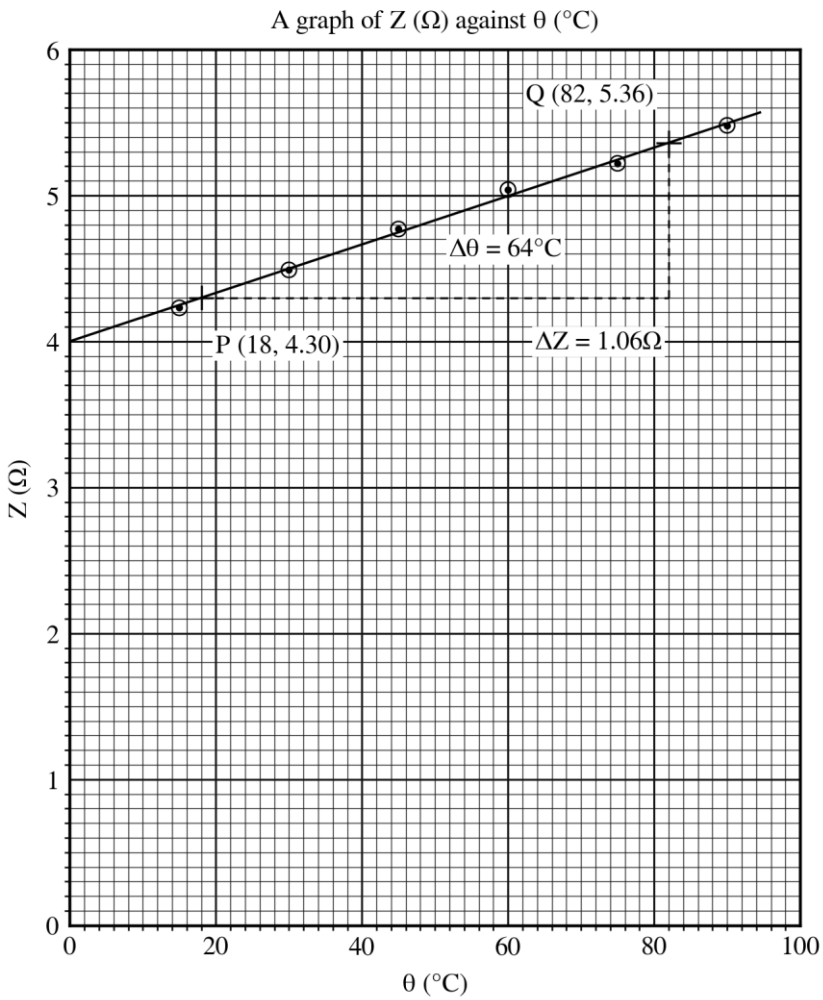
- The resistivity identifies the material; the resistance per metre decides whether the wire will do the job. Parts (vi) and (vii) ask different questions, and one number given twice will not answer both.
- Separate the wire being measured from the wire being taped down. Both are in the circuit, only one of them is on the graph, and it is the one on the graph that the club is proposing to buy.
- Work this wire out from this wire's readings. B has the same diameter as the wire in Experiment 19.6 and a different resistance for every metre of it, so an answer that arrives at nichrome has been carried over rather than measured.
- Read part (vii) as an engineering question. It has a yes or a no in it and a reason attached to a measured number, and a paragraph about bridges in general will not earn the mark.

Practice 19.7

The resistance box in the right-hand gap was left at 5.00Ω throughout, and the resistance of the coil was worked out from $Z=RL_1/L_2$.

θ (°C)	L_1 (cm)	L_2 (cm)	Z (Ω)
15	45.8	54.2	4.23
30	47.3	52.7	4.49
45	48.8	51.2	4.77
60	50.2	49.8	5.04
75	51.1	48.9	5.22
90	52.3	47.7	5.48

(i) and (ii) Take θ at 2cm to 20°C, running from 0°C to 100°C, and Z at 2cm to 1Ω, running from 0Ω to 6Ω. That is 10cm by 12cm, which fits an A4 sheet upright. Give the graph the title A graph of Z (Ω) against θ (°C).



(iii) Taking the two marked corners P (18, 4.30) and Q (82, 5.36),

$$\text{slope}=\frac{5.36\Omega-4.30\Omega}{82°\text{C}-18°\text{C}}=\frac{1.06\Omega}{64°\text{C}}=0.0166\Omega\text{K}^{-1}$$

and the line cuts the Z axis at 4.00Ω. A temperature difference is the same number of kelvin as it is of degrees Celsius, so the slope may be written in either.

(iv) Multiplying out, $Z=Z_0+Z_0\alpha\theta$, which is a straight line of Z against θ of slope $Z_0\alpha$ and intercept Z_0. The six points do lie on a straight line, so the readings are consistent with it over this range, and Z_0 is the intercept, 4.00Ω: the resistance the coil would have at 0°C.

(v) The slope is $Z_0\alpha$ and the intercept is Z_0, so

$$\alpha=\frac{0.0166\Omega\text{K}^{-1}}{4.00\Omega}=4.15\times10^{-3}\text{K}^{-1}$$

Accept $3.9\times10^{-3}\text{K}^{-1}$ to $4.4\times10^{-3}\text{K}^{-1}$. That is a marking tolerance, not the precision of one carefully drawn graph. The accepted value for pure copper referred to 0°C is about $4.3\times10^{-3}\text{K}^{-1}$, and the value found is 3.5% below it.

(vi) The line reaches 0Ω where $1+\alpha\theta=0$, that is at

$$\theta=-\frac{1}{4.15\times10^{-3}\text{K}^{-1}}=-241°\text{C}$$

The accepted value of α gives -233°C the same way. The two agree as closely as the two values of α do, and they must, because the temperature is only α restated. It is not a temperature at which copper has ever been found to lose its resistance. It lies about 32K above absolute zero. That is a temperature which can be reached, and copper at it is known to keep a resistance of its own; the straight line the readings support over a range of 75°C has been produced backwards through more than three times that range to

get there, into a region where the law it assumes has stopped holding. What the number is good for is as a compact way of stating α itself, since it is the negative of its reciprocal.

(vii) The slope survives and the intercept is destroyed, and since α is the slope divided by the intercept, the intercept half of part (iii) goes, and with it the Z_0 of part (iv) and the α of part (v). Stretch the vertical axis and the line gets steeper on the paper, but the number of ohms per degree it represents does not change, so the slope half of part (iii) is safe. The intercept, though, is read where the line crosses $\theta=0$ at a resistance of zero, and an axis that begins at 4.2Ω has no such point on it. Read the crossing on that sheet and you get nothing; assume the bottom of the axis is zero and you get 0Ω for Z_0 and an infinite α. A scale chosen to fill the sheet costs an intercept in several places in this chapter; here it costs the whole of one answer.

Examiner's eye

- Start both axes at zero and say why. The whole of α comes from a slope divided by an intercept, and an intercept that is not on the sheet cannot be read from it.
- Attach the unit to α. It is the fractional change in resistance for every kelvin, so the ohms cancel and what is left is K^{-1}. A bare 0.00415 has lost the mark for the unit and invited the question of what it is a fraction of.
- Let the thermometer settle before the jockey is touched. The oil in the tube lags behind the water in the beaker, and a balance taken while the two are still coming together belongs to a temperature you have not recorded.
- Treat the temperature in part (vi) as an extrapolation and nothing more. It is the point where a line fitted over 75°C of a curve would cross the axis if it went on being straight, and copper stops obeying the law long before it gets there.

Practice 20.1

L_1 (cm)	L_2 (cm)	$(L_1 + L_2)/(L_1 - L_2)$
95.5	15.7	1.393
91.5	15.1	1.395
87.3	14.2	1.389
83.4	13.6	1.390
80.4	12.9	1.382
77.0	12.4	1.384

(i) and (ii) The readings are those in the table above, and the mean of the six values in the third column is

$$\frac{8.333}{6}=1.389$$

Accept 1.379 to 1.399.

(iii) With the cells acting together the wire matches their sum, and with them acting against each other it matches their difference, so at one potential gradient k,

$$kL_1=E_P+E_Q \quad \text{and} \quad kL_2=E_P-E_Q$$

Add the two and then subtract them:

$$k(L_1+L_2)=2E_P \quad \text{and} \quad k(L_1-L_2)=2E_Q$$

Divide one by the other. Both the 2 and the wire's own setting go, leaving the third column equal to the electromotive force of the cell P divided by that of the cell Q.

(iv) The cell Q offers 1.08V, so

$$E_P=1.08\text{V}\times1.389=1.50\text{V}$$

Accept 1.49V to 1.51V.

(v) The two cells very nearly cancel. Their sum is about 2.6V and their difference is about 0.4V, so the second balance needs about a sixth of the length the first one needed, and a sixth of a metre is close to A. The closer the two electromotive forces are to each other, the shorter that second length becomes.

(vi) It is L_2. Take the last row and add 0.2cm to each length in turn. With L_2 raised,

$$\frac{77.0\text{cm}+12.6\text{cm}}{77.0\text{cm}-12.6\text{cm}}=1.391$$

and with L_1 raised instead,

$$\frac{77.2\text{cm}+12.4\text{cm}}{77.2\text{cm}-12.4\text{cm}}=1.383$$

The first shifts the ratio by 0.007 and the second by 0.001, so almost all of the uncertainty in a row is carried by the shorter length, which is also the length on which a fixed slip of the jockey is the larger fraction.

(vii) A ratio that comes out the same at six different settings is evidence that it is a property of the two cells rather than of the apparatus that produced it. Six readings at one setting would be six copies of a single measurement, and they would agree with each other whether the reasoning behind them was sound or not.

(viii) The advantage is that both readings pass through one galvanometer branch and one pair of contacts, with no two-way key in the circuit, so a dirty contact or a faulty lead cannot favour one cell over the other. The disadvantage is the short second length: the answer leans on readings near 14cm, whereas Experiment 20.1 leans on two lengths that are both long.

Examiner's eye

- Check which way up the ratio goes. L_1 came from the cells acting together and is the longer of the pair, so the sum belongs on top. Put the shorter length there instead and the answer for P comes out below the standard cell, which the readings themselves rule out.
- Reverse one cell, not the pair. Interchange both sets of leads and the two cells are still acting together, but the pair now opposes the wire, and there is then no balance point anywhere on it: the galvanometer deflects the same way from A to B.
- Average the six ratios rather than the two columns. The two routes happen to give the same three figures here, but a mean of lengths cannot show you a row that has gone astray. Nothing is drawn in this question, so the column of ratios is doing the work a line does elsewhere in the chapter.

- Take the second length twice, once creeping up on it from A and once from beyond it. It is short, the needle swings through zero within a couple of centimetres of it, and a jockey that is consistently a shade late shows up at once as a difference between the two.

Practice 20.2

R (Ω)	L (cm)	L/R (cmΩ^{-1})
8	72.4	9.05
6	67.2	11.20
4	59.2	14.80
3	52.9	17.63
2	44.1	22.05
1	28.7	28.70

(i) and (ii) Take L/R at 2cm to 5cmΩ^{-1}, running from 0 to 30cmΩ^{-1}, and L at 2cm to 10cm, running from 0 to 100cm. That is 12cm by 20cm, which fits an A4 sheet upright. Give the graph the title A graph of L (cm) against L/R (cmΩ^{-1}).

(iii) Taking M (13.0, 63.4) and N (26.0, 34.8) on the line,

$$\text{slope}=\frac{34.8\text{cm}-63.4\text{cm}}{26.0\text{cm}\Omega^{-1}-13.0\text{cm}\Omega^{-1}}=\frac{-28.6\text{cm}}{13.0\text{cm}\Omega^{-1}}=-2.20\Omega$$

Accept -2.26Ω to -2.14Ω.

(iv) The line meets the L axis at 92.0cm. Accept 91.0cm to 93.0cm.

(v) Rearranged for this pair of axes the relation reads $L = L_0 - rL/R$, in which L_0 is the balance length the cell would give with nothing across it. So the intercept is that open-circuit length, 92.0cm, and the slope is minus the internal resistance, which is therefore 2.20Ω.

(vi) The standard cell of 1.08V balanced at 69.0cm, which is 0.690m, so

$$k=\frac{1.08\text{V}}{0.690\text{m}}=1.57\text{Vm}^{-1}$$

and the cell would balance at 92.0cm, which is 0.920m, so

$$E=1.57\text{Vm}^{-1}\times0.920\text{m}=1.44\text{V}$$

Accept 1.42V to 1.46V.

(vii) The relation plotted is an exact rearrangement of $L = L_0R/(R + r)$, so nothing has been assumed that the line then claims to prove. What the line does is separate two unknowns out of six pairs of readings, and it would do that just as well with the axes labelled any other way the algebra allows. A misread length does move both coordinates of its own point together, along a direction fixed by the R of that row, but the six rows carry six different values of R, so no common tilt reaches the line.

(viii) This cell has an internal resistance of 2.20Ω against 1.19Ω for the cell of Experiment 20.2, so it is nearly twice as large. That is as far as the comparison goes. Each figure belongs to one cell measured once; neither says what a year in a radio does to a cell in general, and neither can: one cell of each kind is one cell, and the two were not alike to begin with.

Examiner's eye

- Put the minus sign into words rather than into the answer. A resistance carried through the working with a minus in front of it is a statement about the cell that no cell obeys; the sign belongs to the direction the line runs in.
- Read the unit of the slope and the unit of the intercept off different axes, although both come from one line. That is what the awkward cmΩ^{-1} on the horizontal axis pays for.
- Plot the row at 1Ω with particular care. The two coordinates of that point are numerically equal, it sits furthest from the intercept, and it therefore has the most say in which way the line leans.
- A falling line is easy to lay through five points and a stray. Sight along the pencil from the paper's edge before you commit to it: a slight tilt costs most at the intercept, which is the part of the line furthest from any reading, and the intercept is where the volts come from.

Practice 20.3

I (A)	L (cm)
0.10	15.7
0.20	31.8
0.30	48.1
0.40	63.8
0.50	79.8
0.60	96.3

(i) and (ii) Take I at 2cm to 0.1A, running from 0 to 0.7A, and L at 2cm to 10cm, running from 0 to 100cm. That is 14cm by 20cm, which fits an A4 sheet upright. Give the graph the title A graph of L (cm) against I (A).

(iii) Taking M (0.150, 24.0) and N (0.550, 88.0) on the line,

$$\text{slope}=\frac{88.0\text{cm}-24.0\text{cm}}{0.550\text{A}-0.150\text{A}}=\frac{64.0\text{cm}}{0.400\text{A}}=160\text{cmA}^{-1}$$

Accept 159cmA^{-1} to 161cmA^{-1}.

(iv) The standard cell of 1.08V balanced at 83.1cm, which is 0.831m, so

$$k=\frac{1.08\text{V}}{0.831\text{m}}=1.30\text{Vm}^{-1}$$

Accept 1.29Vm^{-1} to 1.31Vm^{-1}.

(v) At a reading of 0.500A the line gives

$$L=160\text{cmA}^{-1}\times0.500\text{A}=80.0\text{cm}$$

which is 0.800m, so the potential difference across the coil is

$$V=1.30\text{Vm}^{-1}\times0.800\text{m}=1.04\text{V}$$

and the current that produced it is

$$\frac{1.04\text{V}}{2.00\Omega}=0.520\text{A}$$

Accept 0.51A to 0.53A.

(vi) The meter reads low, and the shortfall is

$$\frac{0.520\text{A}-0.500\text{A}}{0.500\text{A}}=0.040=4.0\%$$

so every reading has to be raised by 4.0% of itself. Accept 2% to 6%. That is a marking tolerance, not the precision of one carefully drawn graph.

(vii) That the ammeter has no zero error. A pointer resting off the zero would add the same potential difference to every reading, which on these axes is a fixed length and would show as an intercept. There is none, so what is wrong with the meter is the size of its divisions and not the place they begin, and one factor corrects the whole scale.

(viii) Because at balance the potentiometer takes no current, so it neither alters the current it is measuring nor depends on its own resistance being known. What it rests on instead is a length read off a metre rule, a coil of stated resistance and a cell of stated electromotive force. A second ammeter would have to be trusted for exactly the reason the first is being distrusted, and two instruments of one kind that disagree cannot say which of them is at fault.

Examiner's eye

- Notice how much worse the percentage is known than the current it came from. The 0.520A is good to about 1%, which is 0.005A, and all of that lands on the 0.020A difference, a quarter of it. Quote the percentage to two figures at most and expect the second of them to move.
- The coil turns a length into a current, so its resistance has to be right. A coil marked 2Ω and worth 2.1Ω would put 5% into the answer at once, which is larger than the fault being hunted; the coil is itself a standard, measured elsewhere before it is trusted here.
- Keep the two currents apart in the working. One is what the pointer says and the other is what the wire says; give them one letter between them and the subtraction at the end is 4% of nothing in particular.
- A percentage needs a base. This one is taken on the ammeter's reading, because that is the number in front of whoever is using the meter; taken on the true current instead it is 3.8%, and an answer that does not say which base it used cannot be marked.

Practice 21.1

The burette has an internal diameter of 1.14cm and its graduated part is 50.0cm long, and the room stood at 23.0°C throughout.

f (Hz)	1/f (10^{-3}s)	L (cm)
256	3.906	33.1
288	3.472	29.6
320	3.125	26.8
341	2.933	25.1
384	2.604	21.8

(i) and (ii) Take L along the horizontal axis at 2cm to 5cm, running from 0 to 35cm, and 1/f up the vertical axis at 2cm to 0.4×10^{-3}s, running from 0 to 4.0×10^{-3}s. That is 14cm by 20cm, which fits an A4 sheet upright. Give the graph the title A graph of 1/f (10^{-3}s) against L (cm).

(iii) Two points on the drawn line are A (12.0, 1.432) and B (28.0, 3.288). In seconds and metres,

$$\text{slope}=\frac{0.003288\text{s}-0.001432\text{s}}{0.280\text{m}-0.120\text{m}}=\frac{0.001856\text{s}}{0.160\text{m}}=0.0116\text{s/m}$$

and the line meets the vertical axis at 0.040×10^{-3}s, that is at 0.000040s.

(iv) The relation rearranges to $1/f=4L/v+4e/v$, so the slope is $4/v$ and

$$v=\frac{4}{0.0116\text{s/m}}=345\text{ms}^{-1}$$

Accept 338ms^{-1} to 352ms^{-1}.

(v) The intercept is $4e/v$, so dividing it by the slope gives

$$e=\frac{0.000040\text{s}}{0.0116\text{s/m}}=0.0034\text{m}=0.34\text{cm}$$

The intercept carries two figures and no more, so the quotient is quoted to two as well. The end correction of the burette is 0.34cm. Accept 0.25cm to 0.45cm.

(vi) The radius of the burette is 0.57cm, and the rule of thumb, 0.6 times the radius, gives 0.342cm; the value in (v) sits on it, which at this precision is as much luck as agreement. The speed is much the better of the two answers. The slope behind it is read across 16cm of the horizontal axis, and it is worth about 1% for the reason given in 21.1 (viii). The intercept is one small square above the origin and is read to the same fifth of a square as everything else on the sheet, so it is worth about 20%, and the end correction with it. A narrow tube has little end correction to find, and this graph finds it badly.

(vii) At 23.0°C the relation gives

$$v=331+0.6\times23.0=344.8\text{ms}^{-1}$$

or 345ms^{-1} to three figures, which is the value in (iv). Agreement to the last figure the graph carries is not a claim that the method is good to 0.3%; the nearest slope the graph could have given instead of 0.0116s/m was 0.0115s/m, and that would have moved the speed by 3ms^{-1}.

(viii) For the fork stamped 480Hz,

$$L=\frac{345\text{ms}^{-1}}{4\times480\text{Hz}}-0.0034\text{m}=0.1797\text{m}-0.0034\text{m}=0.1763\text{m}$$

so it would answer at about 17.6cm. The lowest fork the burette can take is the one whose column just fills it, L=50.0cm:

$$f=\frac{345\text{ms}^{-1}}{4\times0.5034\text{m}}=171\text{Hz}$$

A fork stamped below about 171Hz could not be brought to its first resonance in this burette at all. Accept 165Hz to 178Hz.

Examiner's eye

- Every reading here is taken with the level falling, because a burette empties from the top. That is a virtue and a trap together: the ear lags the same way in every reading, so the five points keep their line, but every one of them may be a millimetre or two past the true resonance, and the graph cannot tell you so.
- Shut the tap on the maximum itself and think about it afterwards. Water that goes on running while you decide has moved the surface away from the reading you are about to take, and the faster it runs the further it moves.
- Take every reading on the rule and none of them on the burette. A burette is graduated for what has run out of it, in millilitres, counting downwards from a zero at the top; the length wanted here is measured from the mouth, on the half-metre rule clamped beside it with its own zero level with the mouth. Two scales run side by side down this rig and only the outer one is answering the question.
- Plot the reciprocal and not the frequency. A graph of L against f is a curve, and a curve has no one slope to quote; the reciprocal that nobody measured is the one that straightens it, and it is worked from the stamped frequencies alone.

Practice 21.2

The tube is the one of Experiment 21.2, 1.00m long and of 3.00cm internal diameter, and the fork is stamped 426Hz.

trial	L_1 (cm)	L_2 (cm)	$L_2 - L_1$ (cm)	v (ms^{-1})	e (cm)
1	19.3	60.4	41.1	350.2	1.25
2	19.6	60.1	40.5	345.1	0.65
3	19.7	60.0	40.3	343.4	0.45
4	19.5	60.3	40.8	347.6	0.90
5	19.4	60.0	40.6	345.9	0.90

(i) and (ii) The five differences are in the fourth column. For the first trial the wavelength is twice 41.1cm, and

$$v=f\lambda=426\text{Hz}\times0.822\text{m}=350.2\text{ms}^{-1}$$

The other four are worked the same way and are given in the fifth column.

(iii) The five values add to 1732.2ms^{-1}, so the mean is 346.4ms^{-1}. The largest is 350.2ms^{-1} and the smallest 343.4ms^{-1}, so the half-range is

$$\frac{350.2\text{ms}^{-1}-343.4\text{ms}^{-1}}{2}=3.4\text{ms}^{-1}$$

The speed of sound in air is 346ms^{-1} with an uncertainty of about 3ms^{-1}, which is 1%. Do not quote the mean as 346.4ms^{-1}: that claims 0.1ms^{-1} from readings that scatter over 7ms^{-1}. Accept 340ms^{-1} to 353ms^{-1}.

(iv) The relation puts the speed in that room at

$$v=331+0.6\times27.0=347.2\text{ms}^{-1}$$

or 347ms^{-1} to three figures. The five trials straddle it, three below and two above, and the mean is within the uncertainty of it. Nothing in the difference asks to be explained.

(v) The two resonance conditions are $L_1+e=1/4\lambda$ and $L_2+e=3/4\lambda$. Three quarters of a wavelength is three times a quarter of one, so the second condition is three times the first:

$$L_2+e=3L_1+3e$$

and taking 3e to the other side, $L_2-3L_1=2e$, which is the result asked for. For the first trial $3L_1$ is 57.9cm, so

$$2e=60.4\text{cm}-57.9\text{cm}=2.5\text{cm}$$

and the end correction from that trial is 1.25cm.

The five values are in the last column of the table. They average 0.83cm, against the 0.90cm that the rule of thumb, 0.6 times the radius, gives for this tube.

(vi) The five speeds run from 343.4ms^{-1} to 350.2ms^{-1}, a spread of 1% about their mean. The five end corrections run from 0.45cm to 1.25cm, a spread of about half of their own mean. The readings behind the two are the same readings. What differs is the combination they are put into. The speed comes from L_2-L_1, which is about 41cm, so a millimetre of uncertainty in each length is a small share of it. The end correction comes from L_2-3L_1, which is a centimetre or two and carries three times the uncertainty of L_1 as well as that of L_2. A small difference of large numbers is where precision is lost.

(vii) It needs no end correction. One resonance taken with one fork gives only $L_1+e=1/4\lambda$, and e has then to be found from somewhere before the speed can be had at all; here it cancels before the arithmetic begins, and one fork is enough.

(viii) Sound a fork of lower frequency. The whole answer rests on L_2-L_1, which is half a wavelength, and the ear places each maximum to a millimetre or two whatever fork is sounding. At 426Hz that difference is 41cm; at 288Hz it is 60cm, so the same slip of the ear takes two thirds of the share it had. The tube fixes the floor, because the second resonance has still to stand inside it: at 288Hz it comes 90cm down, and a fork much lower has nowhere to put it. A tube of larger bore is the change that suggests itself, and it is not the one. A wide tube does lose less at its wall for the air it holds, but the same width gives it a larger mouth to radiate from, and the maximum comes out no sharper than before.

Examiner's eye

- Five readings have a half-range, not a standard deviation. Take the largest and the smallest, halve the difference, and quote that to one figure. Then round the mean back to the place that figure reaches: 346ms^{-1} with an uncertainty of 3ms^{-1}, and not 346.4 with an uncertainty of 3.4.
- Keep the value of 350.2ms^{-1}. It is not an outlier: five readings scattered over 1% will always have one at each edge, and that is what five readings look like. A set from which the extremes have been removed is a set whose scatter has been invented.

- The wavelength is twice the difference between the two lengths. Four times the first length falls short of a wavelength by four end corrections, and using it throws away the one advantage the method has.
- Work every value in the last two columns from the two lengths printed beside it in its own row. A column of speeds all equal to one another was computed once and copied down, and it leaves part (iii) with no scatter to measure.

Practice 21.3

The fork is stamped 384Hz throughout, the tension is the weight of the load, and the density of steel is taken as 7.80gcm^{-3}.

load (kg)	T (N)	L (cm)	L^2 (cm^2)
1.00	9.81	20.1	404
2.00	19.62	28.3	801
3.00	29.43	35.2	1239
4.00	39.24	40.6	1648
5.00	49.05	44.9	2016

(i) and (ii) The tension in the first row is $1.00\text{kg}\times9.81\text{ms}^{-2}=9.81\text{N}$, and the others follow. Take T along the horizontal axis at 2cm to 10N, running from 0 to 60N, and L^2 up the vertical axis at 2cm to 400cm^2, running from 0 to 2400cm^2. That is 12cm by 12cm, which fits an A4 sheet upright. Give the graph the title A graph of L^2 (cm^2) against T (N).

(iii) The points lie on a straight line through the origin. Two points on it are A (17.0, 704) and B (54.0, 2240):

$$\text{slope}=\frac{2240\text{cm}^2\text{-}704\text{cm}^2}{54.0\text{N-}17.0\text{N}}=\frac{1536\text{cm}^2}{37.0\text{N}}=41.5\text{cm}^2/\text{N}$$

(iv) Square the relation given and rearrange it:

$$L^2=\frac{T}{4f^2\mu}$$

With f fixed by the fork and μ fixed by the wire, everything on the right except T is a constant, so L^2 is proportional to T and the line passes through the origin. The slope is the reciprocal of $4f^2\mu$.

(v) The slope is 41.5cm^2 for every newton, that is 0.00415m^2 for every newton, and by (iv) it is one divided by $4f^2\mu$. Here $4f^2$ is $4\times384\text{Hz}\times384\text{Hz}$, which is 589824Hz^2, so

$$\mu=\frac{1}{589824\text{Hz}^2\times0.00415\text{m}^2/\text{N}}=0.000409\text{kgm}^{-1}$$

The wire has a mass per unit length of 0.409g/m. Accept 0.390g/m to 0.430g/m.

(vi) A metre of the wire has a mass of 0.409g, so if it is steel its volume is

$$\frac{0.409\text{g}}{7.80\text{gcm}^{-3}}=0.0524\text{cm}^3=52.4\text{mm}^3$$

That volume runs along 1000mm, so the cross-section is

$$A=\frac{52.4\text{mm}^3}{1000\text{mm}}=0.0524\text{mm}^2$$

and the diameter follows from the area of a circle:

$$d=\sqrt{\frac{4\times0.0524\text{mm}^2}{3.14}}=0.258\text{mm}$$

The micrometer gives 0.26mm, which the calculation matches to within the 0.01mm resolution of the instrument. Accept 0.250mm to 0.270mm. The wire has the mass per unit length that steel of that gauge would have, which is as much as one sonometer can settle, and it is the part of A. Chonde's question that can be answered from the shelves of a school.

(vii) A length of 30.0cm is 900cm^2 on the vertical axis. Read across to the line and drop to the horizontal axis: the tension there is 21.6N, and the load that would give it is

$$\frac{21.6\text{N}}{9.81\text{ms}^{-2}}=2.20\text{kg}$$

Accept 20.5N to 23.0N, that is 2.09kg to 2.34kg.

(viii) Smaller, and smaller by a factor of four. The mass per unit length of a wire of given material goes as the square of its diameter, so doubling the diameter makes μ four times what it was; and the slope is $4f^2\mu$ inverted, which is inversely proportional to μ.

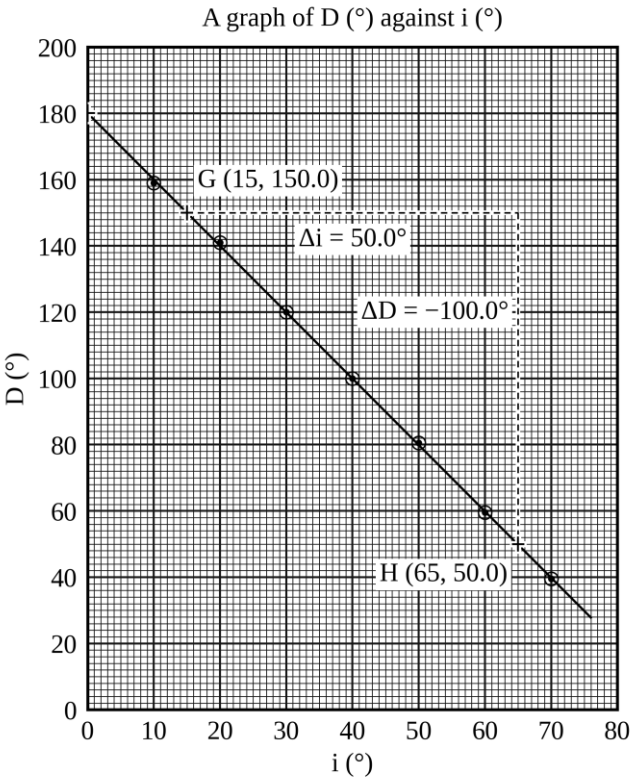
Examiner's eye

- Weighing a metre of the wire would seem to settle it in one step, and it is fair to ask why nobody did. The resolution of a school balance is 0.01g, and 0.01g on 0.409g is 2.4%; the metre has then to be cut to a millimetre, which is another 0.1%. The slope here is read to about 1.4%, and it comes from five readings instead of one.
- Square the length you wrote down, and not a length remeasured afterwards with the load already changed. Work each square from the length in your own table, to the nearest square centimetre, and do one of the five a second time as a check: a slip in squaring moves a point further than any reading error in this experiment can.
- Add the masses gently and one at a time, steadying the pan before you let go of it. A load of 5.00kg on a wire of 0.26mm is about half of what the wire will bear, and a wire taken past its elastic limit does not give the same length twice; the point that shows it will be the last one on the graph, where it is easiest to excuse.
- Record the position at which the rider first leaves the wire. It is thrown off over a small band of bridge positions rather than at one, and that band is much the same fraction of the resonating length at every load; since the lengths here run from 20cm to 45cm, the band in millimetres is more than twice as wide at the foot of the table as at its head.

Practice 22.1

i (°)	D (°)
10	159.0
20	141.0
30	120.0
40	100.0
50	80.5
60	59.5
70	39.5

(i) and (ii) The readings are those in the table above. Take i at 2cm to 10°, running from 0° to 80°, and D at 2cm to 20°, running from 0° to 200°. That is 16cm by 20cm, which fits an A4 sheet upright. Give the graph the title A graph of D (°) against i (°).



(iii) Taking G (15, 150.0) and H (65, 50.0) on the line,

$$\text{slope}=\frac{50.0°-150.0°}{65°-15°}=\frac{-100.0°}{50°}=-2.00$$

Accept -2.05 to -1.95.

(iv) That every degree added to the angle of incidence takes two degrees off the angle through which the mirror turns the light. The sign is part of the meaning: as the ray is brought round towards the surface the turning gets smaller, not larger.

(v) The line meets the D axis at 180°. That is the angle through which the mirror would turn a ray that arrived along the normal itself: such a ray goes back along its own path, and a ray sent back the way it came has been turned through 180°. Accept 178° to 182°.

(vi) Read across from D = 90° to the line and down: i = 45°. Accept 43° to 47°. That is the angle a periscope mirror is set at.

(vii) The line is

$$D=180°-2i$$

Now the ray arrives at O making i with the normal and leaves making r with it on the other side, so the angle between the incident ray produced and the reflected ray is what is left of a straight angle when both of those have been taken out of it:

$$D=180°-i-r$$

Compare that with the line and i+r=2i, which is i=r, the law of Experiment 22.1. The slope of -2 carries the equality of the two angles and the intercept of 180° carries the straight angle.

(viii) Through 20°. Turn the mirror through 10° with the incident ray fixed and the angle of incidence changes by 10°, and the graph says that a change of 10° in the angle of incidence changes the turning of the ray by 20°. The reflected ray therefore swings through twice the angle the mirror was turned through. Read the graph at two settings 10° apart, say 30° and 40°, and the two values of D differ by 20°. That is the same statement read off the sheet.

Examiner's eye

- Produce the incident ray. Leave that step out and there is no angle to measure at all, and the angle between the incident ray itself and the reflected ray is a different angle with a different graph.
- Carry the minus sign through to the answer. A slope quoted as 2.00 here describes a line that rises, and the line drawn on the sheet falls; the sign is not decoration.

- Scales that reach much further on one axis than on the other look unbalanced, and here they are correct. The intercept has to be on the sheet, so the vertical axis must be drawn long enough to reach past it.
- Part (viii) is about the mirror moving, and no other part of the question is. Answer it with the angle the reflected ray turns through and the reason, and resist the temptation to describe the whole experiment again.

Practice 22.2

i (°)	D (°)	i (°)	D (°)
40	48.0	60	45.5
45	46.0	65	47.0
50	44.5	70	49.5
55	44.5		

(i) and (ii). The readings are those in the table above. Take i at 2cm to 5°, running from 35° to 75°, and D at 2cm to 1°, running from 43° to 50°. That is 16cm by 14cm, which fits an A4 sheet upright. Give the graph the title A graph of D (°) against i (°).

(iii) The lowest point of the curve lies at D = 44.5°. Accept 44.0° to 45.0°.

(iv) With A = 60.0° and D_m = 44.5°,

$$n=\frac{\sin\frac{60.0°+44.5°}{2}}{\sin 30.0°}=\frac{0.7907}{0.5000}=1.58$$

Accept 1.57 to 1.59.

(v) At the critical angle the refracted ray leaves along the surface itself, so

$$\sin C=\frac{1}{n}=\frac{1}{1.58}=0.6329$$

and C = 39.3°. Accept 39.0° to 39.6°.

(vi) Inside the prism the ray meets the second face at an angle which, with the angle of refraction at the first face, adds up to the refracting angle. For the ray to leave, that angle may not exceed the critical angle, so at most it is 39.3° and the angle of refraction at the first face is then

$$60.0°-39.3°=20.7°$$

At the first face the light passes from air into glass, so

$$\sin i=n\sin r=1.58\times\sin 20.7°=1.58\times 0.3535=0.5585$$

and i = 34.0°. Below that angle of incidence the light is totally reflected inside the prism at the second face and none comes out. That is why the readings in the table begin at 40° and not lower. Accept 33° to 35°.

(vii) This one. At the least deviation it turns a ray through 44.5° against 38.0° for the prism of Experiment 22.2, which is 6.5° more. The two are the same shape, so the whole of the difference belongs to the glass: the larger the refractive index, the more it bends light.

(viii) The two angles inside the prism add up to the refracting angle, and neither of them may exceed the critical angle if the ray is to get out. The largest refracting angle that can be passed is therefore twice the critical angle,

$$2\times 39.3°=78.6°$$

and 80° is larger than that. Whatever the angle of incidence, at least one of the two internal angles must exceed 39.3°, and at that face the light is totally reflected.

Examiner's eye

- Two curves of this shape look alike on the page and differ by a few degrees at the bottom. Read the lowest point off the sheet in front of you and not off the memory of the last one you drew.
- Read the critical angle as an angle. What the equation gives you first is a sine, and 0.6329 written down as the answer has not been through the inverse function yet.
- Every angle inside a prism is measured from the normal to the face it is at, and the two faces have different normals. The subtraction in part (vi) rests on that, and it is the step most often written down the wrong way round.
- Where a question says show that, a number on its own earns no mark. Part (viii) wants the comparison between the refracting angle and twice the critical angle, and one line of reasoning to go with it.

Practice 22.3

V (cm^3)	Y (cm)	Z (cm)	Y − Z (cm)
100	7.8	5.3	2.5
130	10.2	7.0	3.2
160	12.9	9.0	3.9
190	15.3	10.5	4.8
220	17.5	11.7	5.8
250	19.8	13.4	6.4

(i) and (ii) The readings are those in the table above. Take Y at 2cm to 2cm, running from 0 to 22cm, and Y-Z at 2cm to 1cm, running from 0 to 7cm. That is 22cm by 14cm, which fits an A4 sheet on its side. Give the graph the title A graph of Y-Z (cm) against Y (cm).

(iii) The line goes through the origin. Taking G (3.0, 0.96) and H (19.0, 6.08) on the line,

$$\text{slope}=\frac{6.08\text{cm}-0.96\text{cm}}{19.0\text{cm}-3.0\text{cm}}=\frac{5.12\text{cm}}{16.0\text{cm}}=0.320$$

Accept 0.305 to 0.335.

(iv) The real depth and the apparent depth are in the ratio of the refractive index, Y=nZ, so the apparent depth is Z=Y/n and the difference between them is

$$Y-Z=Y\left(1-\frac{1}{n}\right)$$

which is a straight line through the origin of slope 1-1/n.

(v) From the slope,

$$n=\frac{1}{1\text{-}0.320}=\frac{1}{0.680}=1.47$$

Accept 1.44 to 1.50.

(vi) Glycerine. The band 1.44 to 1.50 takes in 1.47 and no other value in the list: methylated spirit at 1.36 is 0.08 below the bottom of it, water is further below still, and carbon disulphide at 1.63 is 0.13 above the top. Three of the four are ruled out by more than the whole width of the band, so glycerine is the only one it can be.

(vii) It does not affect it at all. Both Y and Z are read to 0.1cm, so their difference carries both uncertainties and is uncertain by about 0.14cm where either depth alone is uncertain by 0.10cm. Against that, a slope of 0.320 is pinned down by these points more tightly than a slope of 1.34 is by the direct plot. The two effects are the same size and opposite in sign, and they cancel: the same six readings give the refractive index to about 0.01 whichever way they are plotted. Neither plot is proof against a level taken from the wrong place: a constant added to both depths changes neither slope and pushes both lines away from the origin. On either sheet a line that will not lie on the origin is a reading to go back to the bench for.

(viii) Use a deeper jar and fill it further. The difference grows in proportion to the depth while the uncertainty in reading it stays at about 0.14cm, so at a depth of 40cm the difference would be near 13cm and that 0.14cm would count for half what it does at the deepest reading in this table. More readings would help as well, but only as the square root of their number, where depth helps in direct proportion.

Examiner's eye

- Plot the difference, not the two depths. The question asks for a graph of one quantity against another and the first of them is a column you have to work out; a sheet with Y and Z both plotted on it answers a different question.
- Subtract in the order the heading says. Y-Z is positive here because the image is nearer the surface than the pin, and a column of negative numbers means the subtraction went the wrong way round.
- An answer of 0.320 is not a refractive index. Turn the slope round through the relation in part (iv), and check the size of what comes out: it must be bigger than 1 where the slope was smaller.
- Say what the result rules out as well as what it allows. A band that excludes three of the four liquids is a stronger answer than a bare name, and it is the part of the reasoning that shows the measurement was understood.

Practice 22.4

i (°)	r (°)	sin i	sin r
20	13.0	0.342	0.225
30	19.0	0.500	0.326
40	25.0	0.643	0.423
50	31.5	0.766	0.522
60	35.0	0.866	0.574
70	39.0	0.940	0.629

(i) and (ii). The readings are those in the table above. Take sinr at 2cm to 0.1, running from 0 to 0.7, and sini at 2cm to 0.1, running from 0 to 1.0. That is 14cm by 20cm, which fits an A4 sheet upright. Give the graph the title A graph of sini against sinr.

(iii) The line goes through the origin. Taking G (0.15, 0.224) and H (0.55, 0.824) on the line,

$$\text{slope}=\frac{0.824\text{-}0.224}{0.55\text{-}0.15}=\frac{0.600}{0.40}=1.50$$

The slope is the refractive index of the material of the block.

Accept 1.49 to 1.51.

(iv) Perspex, and by one band. The half degree in r works through to the refractive index exactly as it does in Experiment 22.4 (viii), and it comes to 0.01. The answer is therefore 1.50 with a band running from 1.49 to 1.51. Perspex at 1.49 lies at the edge of that band and window glass at 1.52 lies a further band outside it, so the block is perspex. One band is a thin margin, and part (v) is about widening it.

(v) Take the whole set of six readings four times over, on fresh paper throughout, and average the four values of r at each angle of incidence. The band narrows as the square root of the number of readings behind it, so four sets halve it, from 0.01 to 0.005. Window glass and perspex are 0.03 apart and that band is a sixth of the gap between them, so wherever the average falls it falls on one of the two and six bands clear of the other, which decides the question instead of leaning towards an answer. Longer rays would help as well, because two pin holes fix a direction the better the further apart they are, but the sheet sets a limit to how far apart they will go. Repetition has no such limit, though it can do nothing about a fault that goes the same way on every sheet: a protractor read from the wrong end of its scale is read from the wrong end four times over.

(vi) The apparent depth is the real depth divided by the refractive index,

$$\frac{4.8\text{cm}}{1.50}=3.2\text{cm}$$

so the print appears to be raised by 4.8cm-3.2cm=1.6cm. Accept 1.5cm to 1.7cm. This is Experiment 22.3 done on a solid instead of a liquid.

(vii) At i = 20° the angle of refraction is 13.0°, so the ratio is 20°/13.0°=1.54. At i = 70° the angle of refraction is 39.0° and the ratio is 70°/39.0°=1.79. The two differ by 16%, which is many times the half degree either angle was read to, so the ratio of the angles is not a constant of the glass. The ratio of their sines is, and that is why the sines are plotted.

(viii) Because the ray whose direction you are recovering lies in the plane of the paper, and the line your eye judges is the line joining the pins as you see them. An eye above the paper lines the pins up by their heads, while the holes it leaves behind are made by their points. Two pins that look in line from above need not be in line along the paper unless both are perfectly upright, and a pin that leans

is the commonest fault in the whole method. With your eye down at the level of the paper the line you judge is the line you will draw, and any lean shows at once.

Examiner's eye

- Work the uncertainty out before you commit to an answer in part (iv). A name with no band behind it is a guess however lucky it is, and the band is the whole difference between a measurement that decides and one that only leans.
- Draw the line to the points and not to the ends of the table. A line laid on the first reading and the last throws away four of the six and the check they carry.
- Nothing in part (vi) needs the graph. It needs the number the graph gave and one division, and an answer that goes back to the plotted points has misread what was asked.
- Show the two ratios in part (vii) and the size of the difference between them. A statement that the ratio is not constant, with no numbers under it, repeats the question instead of answering it.

Practice 23.1

The glycerine held 21.0°C throughout, the distance S from X to Y was 20.0cm, and the micrometer showed no zero error. The third column is the mean of the two diameters and the eighth the mean of the two times.

d_1 (mm)	d_2 (mm)	d (mm)	r (cm)	r^2 (cm^2)	t_1 (s)	t_2 (s)	t (s)	v (cms^{-1})
3.01	3.03	3.020	0.1510	0.02280	36.1	36.0	36.05	0.555
3.50	3.48	3.490	0.1745	0.03045	26.9	26.8	26.85	0.745
4.02	4.00	4.010	0.2005	0.04020	20.7	20.8	20.75	0.964
4.97	4.99	4.980	0.2490	0.06200	13.0	13.2	13.10	1.53
6.03	6.03	6.030	0.3015	0.09090	9.3	8.9	9.10	2.20

(i) and (ii). Take r^2 along the horizontal axis at 2cm to $0.02cm^2$, running from 0 to $0.10cm^2$, and v up the vertical axis at 2cm to $0.25cms^{-1}$, running from 0 to $2.5cms^{-1}$. That is 10cm by 20cm, which fits an A4 sheet upright. Give the graph the title A graph of v (cms^{-1}) against r^2 (cm^2).

(iii) Rule the line from the origin so that the five points sit evenly about it. It passes through A (0.026, 0.63) and B (0.070, 1.70), so

$$\text{slope}=\frac{1.70cms^{-1}-0.63cms^{-1}}{0.070cm^2-0.026cm^2}=\frac{1.07cms^{-1}}{0.044cm^2}=24.3cm^{-1}s^{-1}$$

Accept $23.8cm^{-1}s^{-1}$ to $24.8cm^{-1}s^{-1}$.

(iv) The relation rearranges to

$$v=\frac{2g(\rho-\sigma)}{9\eta}r^2$$

so the slope of the line is the factor that stands in front of r^2. One reciprocal centimetre is 100 reciprocal metres, so $24.3cm^{-1}s^{-1}$ is $2430m^{-1}s^{-1}$, and

$$\rho-\sigma=\frac{9\times1.30Nsm^{-2}\times2430m^{-1}s^{-1}}{2\times9.81ms^{-2}}=1449kgm^{-3}$$

The glycerine has a density of $1260kgm^{-3}$, so

$$\rho=1260kgm^{-3}+1449kgm^{-3}=2709kgm^{-3}$$

The metal has a density of $2.71\times10^3kgm^{-3}$. Accept $2.68\times10^3kgm^{-3}$ to $2.74\times10^3kgm^{-3}$.

(v) The balls are aluminium. The three metals stand so far apart that no error this method can make would put another of them in the running: the nearest rival is heavier than the answer by more than the answer itself. What the method cannot do is tell one aluminium alloy from another, and it is not asked to.

(vi) The slope is the terminal velocity that goes with each square centimetre of r^2, and everything in it but the density of the metal is fixed by the jar. So what the line measures is not how dense the metal is but how much denser it is than the liquid it falls through: balls of the glycerine's own density would give a line lying along the horizontal axis, and the graph would have nothing to say at all.

(vii) The difference of the densities is directly proportional to η, so a value of η 5% too large makes that difference 5% too large: $1449kgm^{-3}$ becomes $1521kgm^{-3}$ and

$$\rho=1260kgm^{-3}+1521kgm^{-3}=2781kgm^{-3}$$

The density moves up, and by 2.7% rather than by 5%, because the density of the glycerine is added afterwards and takes no share of the error. That is the weakness of the method: it is a comparison with a number the last experiment measured in this same jar, and the walls that made that number a little large make this slope a little small, so the two errors work against each other and the metal survives what neither of them alone would settle.

(viii) The volume of the five balls can be had from the diameters already in the table, since the volume of a sphere is π times the cube of its diameter divided by 6. The cubes of the five mean diameters, in centimetres, add to $0.477cm^3$, so

$$V=\frac{\pi}{6}\times0.477cm^3=0.250cm^3$$

Weigh the five balls together on the balance and divide. At a density of $2.71\times10^3kgm^{-3}$ they come to 0.68g, so the balance must read to 0.01g if the check is to be worth 1%. One that reads only to 0.1g settles nothing here.

Examiner's eye

- The line goes through the origin, and it is worth knowing why before you rule it. A ball of no size at all cannot fall at a finite speed, so a line fitted with an intercept is making a claim about a ball that does not exist. Put the ruler on the origin first and swing it until the scatter is even.
- Aluminium is only just heavier than the glycerine it displaces. The difference of the densities here is a little under a quarter of what it is for steel, so a ball of a given size takes more than four times as long to fall, and that is why the sizes in this question reach

6mm where the earlier ones stopped at 3.5mm. Larger balls pay for it at the wall: the retardation Experiment 23.1 put at 10% for its largest ball runs to 20% for the largest here, which is why this method settles the metal and would not settle the alloy.

- These balls are not magnetic, and the trick that recovers steel ones will not touch them. A wire hook takes longer, disturbs more glycerine on the way up, and makes the wait before the next release worth keeping.
- Time the balls in order of size and the pattern shows itself before you plot anything: each step up in diameter should divide the time by the square of the step. But the order that shows the pattern is the order that hides a drift, and glycerine loses 8% of its viscosity in a kelvin. Time the first ball again at the end: if the two times agree the room held still, and if they do not, the sheet is not worth plotting.

Practice 23.2

The room stood at 21.0°C throughout, the head h was 40.0cm at the start of every run, and each run was stopped at the $50cm^3$ graduation.

L (cm)	t (s)
40.2	214.4
49.7	269.8
60.4	324.2
69.6	371.2
80.3	435.0

(i) and (ii). Take L along the horizontal axis at 2cm to 10cm, running from 0 to 90cm, and t up the vertical axis at 2cm to 50s, running from 0 to 450s. That is 18cm by 18cm, which fits an A4 sheet upright. Give the graph the title A graph of t (s) against L (cm).

(iii) Rule the line from the origin so that the five points sit evenly about it. It passes through A (25, 135) and B (75, 404), so

$$\text{slope}=\frac{404\text{s}-135\text{s}}{75\text{cm}-25\text{cm}}=\frac{269\text{s}}{50\text{cm}}=5.38\text{scm}^{-1}$$

Accept 5.27scm^{-1} to 5.49scm^{-1}.

(iv) The pressure difference across the tube is the head of water standing above it, and the time to collect a volume V at a steady rate is that volume divided by the rate, so

$$t=\frac{V}{Q}=\frac{8\eta VL}{\pi\rho ghr^4}$$

Everything in that expression except L is fixed while the five tubes are tried, so t is proportional to L and the graph is a straight line through the origin. The slope stands for $8\eta V$ divided by $\pi\rho ghr^4$.

(v) Rearrange for the fourth power of the radius and write the slope in seconds per metre, 5.38scm^{-1} being 538sm^{-1}:

$$r^4=\frac{8\times9.8\times10^{-4}\text{Nsm}^{-2}\times5.00\times10^{-5}\text{m}^3}{3.14\times1000\text{kgm}^{-3}\times9.81\text{ms}^{-2}\times0.400\text{m}\times538\text{sm}^{-1}}=5.91\times10^{-14}\text{m}^4$$

Two square roots give

$$r=4.93\times10^{-4}\text{m}$$

The bore of the tubing has a radius of 0.493mm. Accept 4.91×10^{-4}m to 4.96×10^{-4}m.

(vi) The line is straight through the origin, so the time is the slope times the length:

$$t=5.38\text{scm}^{-1}\times100\text{cm}=538\text{s}$$

Accept 527s to 549s.

(vii) The radius goes as the fourth root of one divided by the slope, so a slope 1% larger gives a radius smaller by 0.25%: 4.93×10^{-4}m becomes 4.92×10^{-4}m, so the two agree to two figures and differ by one unit in the third. The fourth power is being read the comfortable way round here: it makes the flow desperately sensitive to the bore, and the bore very forgiving of the flow.

(viii) The slope holds the viscosity and the fourth power of the radius together, and one line cannot separate them; the radius has to be measured some other way before the viscosity can be got out. Draw a short thread of mercury into one of the tubes, measure its length with the metre rule and find its mass on the balance; the mass divided by the density of mercury is the volume of the thread, and that volume divided by the length is the area of the bore. Where there is no mercury, weigh one tube dry and then full of water and use the density of water instead, remembering that the mass to be weighed is then well under a gram and the balance must be able to see it.

Examiner's eye

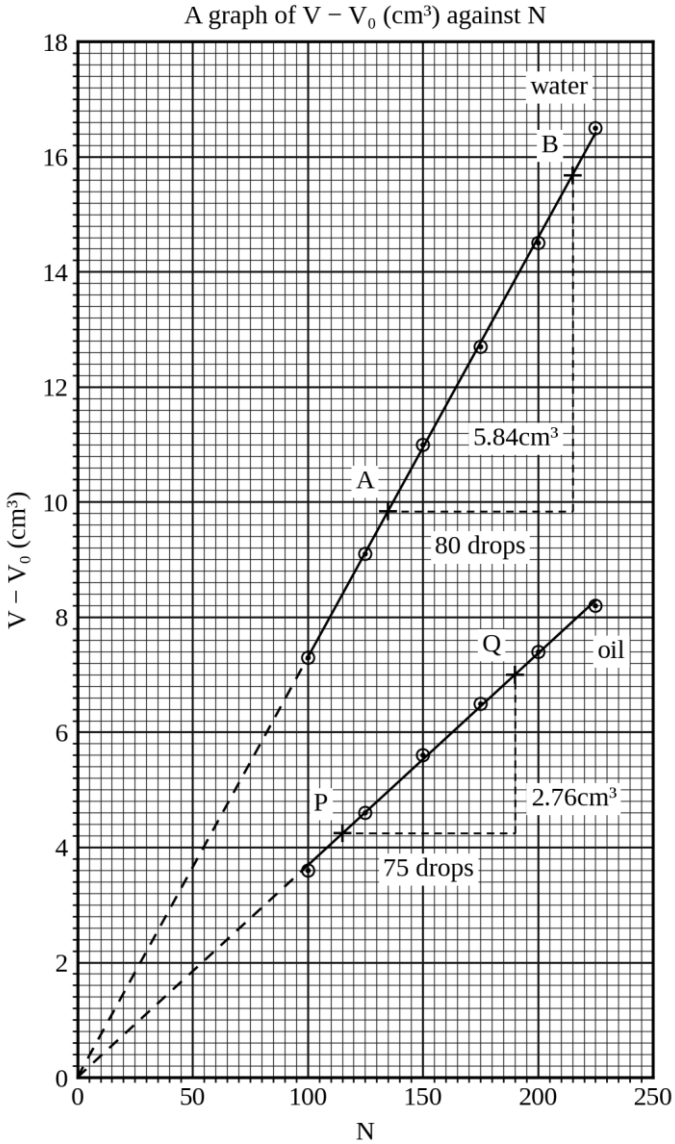
- An intercept here has a suspect list. Nearly all of the head is spent against viscosity along the bore, and that part is strictly proportional to the length; the little spent at the mouth of the tube does not shrink as the tube lengthens, so the shortest tubes sit a few seconds above the line and a small positive intercept is honest. More than a few seconds is something else outside the capillary resisting the flow, and the rubber tubing and the clip are the first two suspects.
- All five tubes come from one length because all five must share one bore. Take them from two different reels and each point carries its own fourth power; the line then means nothing, and no amount of care with the stopwatch will bring it back.
- Glass tubing is scratched and snapped, not sawn, so a tube meant to be 60cm can be 60.4cm. Measure every one and write down what you find: the horizontal axis of this graph is a row of measurements, not a row of labels.
- Five timed runs alone come to nearly 30min, and the whole business to twice that; a room that warms while you work will let the later tubes deliver faster than they should. Scramble the order of the tubes, and a drift shows itself as scatter, which is honest, instead of as a bend in the line, which is not.

Practice 24.1

One glass jet delivered both liquids. The density of water is 1000kgm^{-3}, that of the cooking oil 920kgm^{-3}, and the surface tension of water at the temperature of the room is 0.072Nm^{-1}. The first reading of the cylinder was $V_0 = 0.7\text{cm}^3$ in the water run and $V_0 = 0.3\text{cm}^3$ in the oil run. The room was at 25°C.

N	V (cm^3) water	V − V_0 (cm^3) water	V (cm^3) oil	V − V_0 (cm^3) oil
100	8.0	7.3	3.9	3.6
125	9.8	9.1	4.9	4.6
150	11.7	11.0	5.9	5.6
175	13.4	12.7	6.8	6.5
200	15.2	14.5	7.7	7.4
225	17.2	16.5	8.5	8.2

(i) and (ii) Take N along the horizontal axis at 2cm to 50 drops, running from 0 to 250, and V-V_0 up the vertical axis at 2cm to $2cm^3$, running from 0 to $18cm^3$. That is 10cm by 18cm, which fits an A4 sheet upright. Give the graph the title A graph of V-V_0 (cm^3) against N, and label each line with the liquid it belongs to.



(iii) Both lines pass through the origin when produced back. On the water line, taking A (135, 9.84) and B (215, 15.68),

$$S_w=\frac{15.68cm^3-9.84cm^3}{215-135}=\frac{5.84cm^3}{80}=0.0730cm^3$$

and on the oil line, taking P (115, 4.24) and Q (190, 7.00),

$$S_o=\frac{7.00cm^3-4.24cm^3}{190-115}=\frac{2.76cm^3}{75}=0.0368cm^3$$

Accept $0.0719cm^3$ to $0.0741cm^3$ for the water line and $0.0362cm^3$ to $0.0374cm^3$ for the oil. The water line agrees with the slope of Experiment 24.1 well inside the 0.4% either slope is worth, which is as much as such a comparison can show.

(iv) Each slope is the volume of one drop, so

$$m_w=1.000gcm^{-3}\times0.0730cm^3=0.0730g$$
$$m_o=0.920gcm^{-3}\times0.0368cm^3=0.0339g$$

Accept 0.0719g to 0.0741g for the water and 0.0333g to 0.0344g for the oil.

(v) The weights are in the ratio of the surface tensions, and the weights are in the ratio of the masses, so

$$\gamma_o=0.072Nm^{-1}\times\frac{0.0339g}{0.0730g}=0.072Nm^{-1}\times0.464=0.0334Nm^{-1}$$

Accept $0.0323Nm^{-1}$ to $0.0345Nm^{-1}$. That band is wider, in proportion, than either of the bands on the slopes, because the answer is a ratio of two quantities read off a graph and the two tolerances add.

(vi) The relation of Experiment 24.1 has no jet in it, and part (viii) of that experiment shows what follows: it is fitted for a jet whose bore is about the size of the drop it makes. Water on this jet is such a case, which is why it served there. The oil makes drops half the volume on the same jet, so the jet is now about a quarter wider across than the drops it is making, and the relation applied to the oil returns 0.0424Nm^{-1}, which is 27% above the value found here. The comparison used in (v) escapes that difficulty because the jet is the same for both liquids: whatever the length of the neck may be, it is the same length in the two runs and cancels out of the ratio. What does not quite cancel is the share of each drop that is left clinging behind, which is a little different for the two liquids, and that is why the answer is good to about 3% and no better.

(vii) The weight the neck can carry is the surface tension multiplied by its length, and that length goes with the bore of the jet. Halve the bore and each drop weighs half as much and carries half the volume, so twice as many drops are needed for the same volume. With the present jet the number of drops in 5.00cm^3 is

$$\frac{5.00\text{cm}^3}{0.0368\text{cm}^3}=136$$

so with a jet of half the bore it would take about 272. Accept 267 to 276.

(viii) The two runs have to be made with the same jet at the same setting, and the oil leaves a film on the glass that the water does not. Run the water first, and rinse the jet in warm water and dry it before any second run; better still, take the two runs one straight after the other on the same afternoon, so that the jet, the room and the clamp are all as they were.

Examiner's eye

- Two lines on one sheet need two labels on the sheet. The steeper line is the water and the shallower the oil, and saying so in the account is not the same as writing it on the graph, where the reader is looking.
- Oil drips more slowly than water through the same jet, and a run that took ten minutes with the water can take half an hour with the oil. Allow for that when the afternoon is planned: a run abandoned half way through is four points on a line that needs six.
- Ratios do not care what units the two masses are in, so long as both are in the same ones. They do care that each is the mass of one drop, and not of 100 drops in one run and 25 in the other. Take both from the slopes and the difficulty cannot arise.
- An oil is not a fixed substance the way water is. Two tins from different presses, or one tin left open for a month, will not give the same surface tension, and no book you are likely to have will carry a value to set beside yours. A second method on the same oil is the check that is open to you, and Practice 24.2 is that second method.

Practice 24.2

The six tubes are those of Experiment 24.2 and their radii are given again with the readings. The density of the oil is 920kgm^{-3}, that of water 1000kgm^{-3}, the surface tension of water at the temperature of the room is 0.072Nm^{-1}, and $g = 9.81\text{ms}^{-2}$. The room was at 25°C.

r (mm)	1/r (mm^{-1})	h (mm)
0.170	5.882	42.9
0.204	4.902	36.3
0.253	3.953	29.3
0.312	3.205	23.2
0.401	2.494	17.9
0.520	1.923	14.2

(i) and (ii) Take $1/r$ along the horizontal axis at 2cm to 1mm^{-1}, running from 0 to 7mm^{-1}, and h up the vertical axis at 2cm to 5mm, running from 0 to 45mm. That is 14cm by 18cm, which fits an A4 sheet upright. Give the graph the title A graph of h (mm) against $1/r$ (mm^{-1}).

(iii) The best line through the six points reaches the origin when it is produced back. Taking A (2.20, 16.1) and B (5.50, 40.3) on it,

$$\text{slope}=\frac{40.3\text{mm}-16.1\text{mm}}{5.50\text{mm}^{-1}-2.20\text{mm}^{-1}}=\frac{24.2\text{mm}}{3.30\text{mm}^{-1}}=7.33\text{mm}^2$$

Accept 7.22mm^2 to 7.44mm^2.

(iv) The slope is $2\gamma/\rho g$, and converted to square metres it is $7.33\times10^{-6}\text{m}^2$, so

$$\gamma=\frac{\rho g\times\text{slope}}{2}=\frac{920\text{kgm}^{-3}\times9.81\text{ms}^{-2}\times7.33\times10^{-6}\text{m}^2}{2}=0.0331\text{Nm}^{-1}$$

Accept 0.0326Nm^{-1} to 0.0336Nm^{-1}. Practice 24.1 measured the same oil by the drop method and got 0.0334Nm^{-1}; the two differ by 0.9%, which is less than either is worth.

(v) In the same tube the rise goes as γ/ρ, so

$$\frac{h_o}{h_w}=\frac{\gamma_o}{\gamma_w}\times\frac{\rho_w}{\rho_o}=\frac{0.0331}{0.072}\times\frac{1000}{920}=0.460\times1.087=0.500$$

The oil therefore rises to half the height water does, as stated. The surface tension does almost the whole of it: 0.0331Nm^{-1} against 0.072Nm^{-1} is a factor of 0.460, while the densities work the other way and only by a factor of 1.087. Accept 0.49 to 0.51.

(vi) Read across from 25.0mm on the vertical axis to the line and down to the horizontal axis: you get 3.41mm^{-1}, so

$$r=\frac{1}{3.41\text{mm}^{-1}}=0.293\text{mm}$$

The tube needs a bore of radius 0.293mm, that is 0.59mm across. Accept 0.288mm to 0.298mm.

(vii) Every rise would be smaller, so every point would drop, the line would be less steep, and the slope and the value of γ taken from it would both be smaller. The line would still reach the origin, because a tube of infinite bore lifts nothing at any temperature at all.

(viii) An oil wets glass so well that it creeps up the bore above the meniscus and leaves a film there, and in a tube that has been used before the film is already present and the meniscus is harder to place. Use tubes that have been washed and dried since they last held oil, and do not use the same tube twice in an afternoon without washing it.

Examiner's eye

- Every rise here is half its counterpart in Experiment 24.2 while the reading is no better than it was, so each of these six readings carries twice the percentage error of the one it matches. That is an argument for taking more care, not for taking fewer readings.
- Part (v) can be answered without either surface tension, from the two slopes alone, since the slope of each graph is $2\gamma/\rho g$ and g is common to both. That is the safer route: it needs no accepted value, and a ratio taken the wrong way up is caught at once, because the oil must be the one that rises less.
- The eye settling on the ring where the liquid meets the wall, instead of on the lowest point of the curve, reads every rise too high by about the radius of the bore. That lifts the widest tubes most and the finest hardly at all, so the line is flattened and the surface tension comes out too small. It is the one error here that grows as the tubes get wider, and the widest point is where to look for it.
- The letter that opens this question asks about a cloth, and six glass tubes cannot answer it outright: a weave is not a bore and the threads of a cloth are not smooth. What the tubes do settle is the direction of the thing. The finer the channel the higher the liquid climbs, so a coarser cloth will carry the oil less far over the rim, and the neighbour is right. Say that much, and say no more than that.

INDEX OF EXPERIMENTS

What the book determines, and by what method. A number such as 24.1 is Experiment 24.1; a number carrying a P, such as P24.1, is Practice 24.1. The figure after it is the page the set begins on. Apparatus is indexed separately, in the Index of Apparatus that follows this one.

Acceleration due to gravity, g
- by a bifilar suspension, the separation varied 9.1 p126
- by a bifilar suspension, the thread length varied 9.2 p132; P9.2 p135
- by a circular lamina swung on a pin 8.3 p117; P8.3 p120
- by a compound pendulum 8.1 p106; P8.1 p111; 8.2 p112; P8.2 p116
- by a pendulum from an inaccessible support 7.7 p97; P7.7 p100
- by a pendulum hung from a ring 7.6 p93; P7.6 p96
- by a pendulum measured to a knot 7.4 p87; P7.4 p90
- by a rule pivoted at a fixed point with a movable load 8.4 p121; P8.4 p125
- by a simple pendulum 7.1 p74; P7.1 p79; 7.2 p80; P7.2 p83
- by a spring loaded statically and set oscillating 10.4 p161; P10.4 p164
- by shortening a pendulum through measured steps 7.3 p83; P7.3 p86
- from a plot of time against the square root of length 7.8 p100
- from the constant k in T=kL raised to a power m 7.5 p90; P7.5 p93
- from the intercept of period against the square of the amplitude P7.8 p104
- from the oscillations of a loaded rule 11.2 p170; P11.2 p174

Ammeter, calibration of an, against a potentiometer wire P20.3 p366

Bifilar suspension *see Acceleration due to gravity, g*

Bore, the radius of a
- from the rate of flow through a fine tube P23.2 p414
- the bore a stated capillary rise needs P24.2 p425

Capillary rise *see Surface tension*

Centre of gravity, position of, of a metre rule, by the principle of moments 12.2 p186; P12.2 p189

Cooling constant
- from a logarithmic plot 13.2 p206; P13.2 p210
- from six tangents to one curve 13.3 p211; P13.3 p215
- of a bare and of a foil-wrapped calorimeter P14.2 p226
- of a blackened and of a polished calorimeter 14.2 p222
- of a lagged and of a bare calorimeter 14.3 p227; P14.3 p231
- of a wet-wrapped and of a dry calorimeter 14.4 p232; P14.4 p236

Critical angle
- from the refractive index of a block 22.4 p400
- from the refractive index of a prism P22.2 p394

Current in a wire, from the potential gradient and the resistance per metre 18.6 p319; P18.6 p322

Density
- of a liquid, by weighing a measured volume of it 23.1 p405; 23.2 p410
- of a liquid, by weighing what a cylinder collects 24.1 p416
- of a liquid, from the slope of a spring graph P10.3 p160
- of a solid, from its mass and the water it displaces P15.3 p256
- of the material of a sonometer wire 21.3 p380
- of the metal of falling balls, by Stokes' law P23.1 p410

Diameter of a wire, from its mass per unit length P21.3 p385

Drop method *see Surface tension*

Elasticity *see Young's modulus*

Electromotive force
- of a battery, from a graph of 1/V against R 17.3 p287; P17.3 p291
- of a dry cell, against a standard cell 20.1 p352; P20.1 p356
- of a dry cell, by a null method 20.2 p357; P20.2 p361
- of a dry cell, from 1/V against 1/R, with no ammeter 18.4 p311; P18.4 p314
- of a dry cell, from R against 1/I 18.1 p298; P18.1 p302
- of a dry cell, from V against I 18.3 p307; P18.3 p310
- of a used cell, from 1/I against R 18.2 p303; P18.2 p306
- of two cells in series and in parallel P18.7 p323

Emf *see Electromotive force*

End correction of a resonance tube
- from the intercept of 1/f against L 21.1 p370; P21.1 p374
- from the intercept of the first-resonance line 21.2 p376
- from the two resonances of one tube P21.2 p379

Evaporation, water lost by, from the extra rate of cooling of a wet cloth 14.4 p232; P14.4 p236

Frequency of a tuning fork
- of an unmarked fork, on a sonometer 21.3 p380
- the lowest a given tube can serve P21.1 p374

Gravity, acceleration due to *see Acceleration due to gravity, g*

Gyration, radius of *see Radius of gyration*

Heat capacity of a calorimeter
- and its contents, by comparison 14.1 p218; P14.1 p221; 14.5 p237; P14.5 p240
- and its contents, from a cooling curve 15.2 p246; P15.2 p251
- of a hot can and its contents, from a junction temperature P16.1 p266

Height of an inaccessible point, from the intercept of a pendulum graph 7.7 p97; P7.7 p100

Hooke's law *see Spring constant*

Internal resistance of a cell
by a null method 20.2 p357; P20.2 p361
from the slope of V against I 18.3 p307; P18.3 p310
of the cells driving a potential gradient 18.6 p319; P18.6 p322
of two cells in series and in parallel P18.7 p323

Latent heat *see Specific latent heat of fusion*

Length, an unmeasured
an offset in the length of a pendulum 7.4 p87; P7.4 p90
of a pendulum beating seconds, from the graph 7.1 p74
of wire having a stated resistance, by metre bridge 19.5 p341; P19.5 p345
the internal diameter of the ring a pendulum hangs from 7.6 p93
the length of a bar from its radius of gyration P8.1 p111
the original length of a shortened pendulum 7.3 p83; P7.3 p86
where a pendulum on a cork really pivots P7.6 p96

Mass
of a bar from its bifilar period P9.1 p131
of a body by the principle of moments 12.1 p182; P12.1 p185
of a calorimeter, by spring and measuring cylinder 10.3 p156; P10.3 p160
of a load carried on a swinging rule P8.4 p125
of a metre rule by the principle of moments 12.2 p186; P12.2 p189
of a solid weighed in air and in water 12.3 p190; P12.3 p193
of hot water, from a junction temperature P16.1 p266
of the spring itself, from the intercept 10.2 p152; P10.2 p155; 10.4 p161
of water, by comparing two calorimeters cooling 14.1 p218; P14.1 p221; P14.6 p241

Mass per unit length of a wire
from a sonometer at fixed frequency, the tension varied P21.3 p385
from a sonometer at fixed tension 21.3 p380

Melting point
of an unnamed solid, from the flat of a warming curve P16.2 p272
of naphthalene, from the flat of a cooling curve 16.2 p268

Method of mixtures *see Specific heat capacity; Specific latent heat of fusion*

Minimum deviation *see Refractive index*

Modulus of rigidity
of a brass wire by a torsion pendulum P9.3 p139
of a steel wire by a torsion pendulum 9.3 p135
of steel from a helical spring under load 9.4 p140; P9.4 p144

Moment of inertia, of a rod and chuck alone, from the intercept 9.3 p135

Newton's law of cooling *see Cooling constant*

Null method *see Electromotive force; Internal resistance of a cell*

Ohm's law, test of
on a torch bulb, where it fails P17.2 p286
on two coils cut from one reel 17.2 p283

Period and amplitude, the dependence of, one bob swung through six amplitudes P7.8 p104

Period and mass, the independence of, five bobs of different mass on one thread 7.8 p100

Potential gradient
along a potentiometer wire 20.3 p362; P20.3 p366
along a uniform wire 18.6 p319; P18.6 p322

Power law, the index of a, from a logarithmic plot of period against length 7.5 p90; P7.5 p93

Radius of a pendulum bob, from the intercept of a pendulum graph 7.2 p80; P7.2 p83

Radius of gyration
of a bar about its centre of gravity P8.1 p111; P8.2 p116
of a circular lamina 8.3 p117; P8.3 p120
of a loaded metre rule P8.4 p125
of a metre rule about its centre of gravity 8.1 p106; 8.2 p112; 8.4 p121

Rate of loss of heat
at a stated temperature, from the cooling constant P14.3 p231
from a tangent to a cooling curve 13.1 p200; P13.1 p205
the factor a cotton-wool jacket changes it by 14.3 p227
the percentage a wrapping saves P14.2 p226

Real and apparent depth *see Refractive index*

Reflection, laws of
tested with a plane mirror and pins 22.1 p387
the deviation of a ray by a plane mirror P22.1 p390

Refractive index
of a glass block, from sin i against sin r 22.4 p400; P22.4 p403
of a liquid, by real and apparent depth P22.3 p399
of a prism, from the angle of minimum deviation 22.2 p391; P22.2 p394
of water, by real and apparent depth 22.3 p395

Relative density
of a liquid by the principle of moments 12.4 p194; P12.4 p197
of a solid by the principle of moments 12.3 p190; P12.3 p193

Resistance
of a coil measured in parallel with a standard resistor 19.3 p333
of a coil, by metre bridge P19.1 p328; 19.4 p337; P19.4 p340
of a coil, from a graph of potential difference against current 17.1 p279
of a coil, the ammeter corrected for P17.1 p282
of a lamp filament, at two currents P17.2 p286
of a sealed resistor in series with a wire 19.2 p328
of a slab of parallel strands, by metre bridge P19.3 p336
of an element, measured in parallel with a known resistor P17.4 p296
of an unmarked resistor, by metre bridge 19.1 p324
of the circuit apart from the box 18.4 p311; P18.4 p314

of the circuit outside the resistance box 18.1 p298; P18.1 p302; 18.2 p303; P18.2 p306
of the circuit outside the wire 18.5 p315; P18.5 p318
of two coils cut from one reel, compared 17.2 p283

Resistance of a voltmeter, from a graph of 1/V against R 17.3 p287; P17.3 p291

Resistivity
of a bare wire on a metre rule 18.5 p315; P18.5 p318
of a bare wire, on a bridge built from a metre rule 19.6 p346; P19.6 p349
of a resistance wire, by metre bridge 19.2 p328; P19.2 p332
of the wire a coil is wound from 19.4 p337; P19.4 p340

Resistors in series and in parallel, test of the rules for, four lines on one sheet 17.4 p292

Rigidity, modulus of *see Modulus of rigidity*

Room temperature from a graph, from the intercept of a rate-against-temperature line 13.3 p211; P13.3 p215

Shear modulus *see Modulus of rigidity*

Sound, the speed of *see Speed of sound in air*

Specific heat capacity
of a liquid by the method of mixtures P15.1 p245
of a liquid from a tangent to its cooling curve 15.2 p246; P15.2 p251
of a liquid from the rate at which a bath warms it 16.1 p262
of a liquid, by cooling it beside water 14.5 p237; P14.5 p240
of a solid by mixtures, with the cooling corrected 15.4 p256; P15.4 p260
of a solid by the method of mixtures 15.1 p242
of a solid from two cooling runs on one calorimeter 15.3 p251; P15.3 p256
of a solid, from the tangents either side of its flat 16.2 p268

Specific latent heat of fusion
of an unnamed solid, from a warming curve P16.2 p272
of ice by the method of mixtures 16.3 p272
of ice by the method of mixtures, the cooling corrected P16.3 p275

Speed of light in glass, from the refractive index of a block 22.4 p400

Speed of sound in air
from the first and second resonances 21.2 p376; P21.2 p379
from the first resonance of a tube 21.1 p370; P21.1 p374

Spring constant
by static loading 10.1 p147
from the period of a loaded spring 10.2 p152; P10.2 p155
from the slope of extension against volume 10.3 p156
of two springs joined P10.1 p151

Stokes' law *see Viscosity, the coefficient of*

Strands in a slab, the number of, from the resistance of one strand P19.3 p336

Surface tension
of an oil, by capillary rise P24.2 p425
of an oil, by comparing its drops with water's from one jet P24.1 p421
of water, by capillary rise 24.2 p422
of water, by the drop method 24.1 p416

Temperature coefficient of resistance, of copper, from a coil warmed in oil P19.7 p350

Thermal capacity *see Heat capacity of a calorimeter*

Torsion pendulum *see Modulus of rigidity*

Torsional stiffness of a wire, from the slope of the period against the load 9.3 p135

Viscosity, the coefficient of
of a liquid, from the rate of flow through a fine tube 23.2 p410
of glycerine, by Stokes' law 23.1 p405

Voltmeter, calibration of a, against a potentiometer wire 20.3 p362

Volume of a solid, from the water it displaces 12.3 p190; 12.4 p194; P15.3 p256

Young's modulus
from the oscillations of a loaded rule 11.2 p170; P11.2 p174
from the static depression of a loaded cantilever 11.3 p175; P11.3 p179
from the vibrations of a loaded cantilever 11.1 p165; P11.1 p170

INDEX OF APPARATUS

Every worked experiment listed under each piece of apparatus it calls for, so that a school can see at once what it is equipped to set up. The list is taken from the Apparatus section of each experiment. Each practice question uses the apparatus of the experiment it accompanies, with any change stated in the question itself.

Ammeter 17.1 p279; 17.2 p283; 17.4 p292; 18.1 p298; 18.2 p303; 18.3 p307; 18.5 p315

Balance 7.8 p100; 8.4 p121; 13.1 p200; 13.2 p206; 13.3 p211; 14.1 p218; 14.2 p222; 14.3 p227; 14.4 p232; 14.5 p237; 15.1 p242; 15.2 p246; 15.3 p251; 15.4 p256; 16.1 p262; 16.3 p272; 23.1 p405; 23.2 p410; 24.1 p416

Ball bearings, steel 23.1 p405

Bar magnet 23.1 p405

Bath of hot water 14.5 p237; 15.1 p242; 15.2 p246; 15.3 p251; 15.4 p256; 16.1 p262; 16.2 p268

Beaker 12.3 p190; 12.4 p194; 13.1 p200; 13.2 p206; 13.3 p211; 14.1 p218; 14.2 p222; 14.3 p227; 14.4 p232; 15.1 p242; 15.2 p246; 15.3 p251; 15.4 p256; 16.1 p262; 16.2 p268; 23.1 p405; 23.2 p410; 24.2 p422

Bench with a straight edge 11.1 p165; 11.2 p170; 11.3 p175

Block, wooden 7.4 p87; 12.3 p190; 12.4 p194

Board, soft 22.1 p387; 22.2 p391; 22.4 p400

Bob, pendulum 7.1 p74; 7.2 p80; 7.3 p83; 7.4 p87; 7.5 p90; 7.6 p93; 7.7 p97; 7.8 p100

Boiling tube 16.2 p268

Brass cylinder 15.1 p242; 15.4 p256

Bung, rubber 21.1 p370; 21.2 p376; 21.3 p380

Burner, tripod and gauze 15.1 p242; 15.2 p246; 15.3 p251; 15.4 p256; 16.1 p262; 16.2 p268

Calorimeter, copper 10.3 p156; 13.1 p200; 13.2 p206; 13.3 p211; 14.1 p218; 14.2 p222; 14.3 p227; 14.4 p232; 14.5 p237; 15.1 p242; 15.2 p246; 15.3 p251; 15.4 p256; 16.1 p262; 16.3 p272

Candle 14.2 p222

Capillary tube 23.2 p410; 24.2 p422

Cell, standard 20.1 p352; 20.2 p357; 20.3 p362

Cells, dry 17.1 p279; 17.2 p283; 17.3 p287; 17.4 p292; 18.1 p298; 18.2 p303; 18.3 p307; 18.4 p311; 18.5 p315; 18.6 p319; 19.1 p324; 19.2 p328; 19.3 p333; 19.4 p337; 19.5 p341; 19.6 p346; 20.1 p352; 20.2 p357; 20.3 p362

Clamp, Hoffman 24.1 p416

Clip, crocodile 17.1 p279; 18.1 p298; 18.2 p303; 18.3 p307; 18.4 p311; 18.5 p315; 18.6 p319; 19.2 p328; 19.5 p341; 19.6 p346; 20.1 p352; 20.2 p357; 20.3 p362

Clip, screw 23.2 p410

Clock 15.1 p242; 15.4 p256

Cloth, cotton square 14.4 p232

Cloth, for polishing 14.2 p222

Coil of unknown resistance 17.1 p279; 17.2 p283; 19.3 p333; 19.4 p337

Cork or cork ring 16.1 p262; 16.2 p268; 22.3 p395

Cork, split 7.1 p74; 7.2 p80; 7.3 p83; 7.5 p90; 7.8 p100

Cotton wool 14.3 p227; 16.3 p272

Dish, small 23.1 p405

Filter paper 16.3 p272

Forceps 23.1 p405

Funnel, glass 24.1 p416

G-clamp 11.1 p165; 11.2 p170; 11.3 p175

Galvanometer 19.1 p324; 19.2 p328; 19.3 p333; 19.4 p337; 19.5 p341; 19.6 p346; 20.1 p352; 20.2 p357; 20.3 p362

Glass block, rectangular 22.4 p400

Glass jar, tall 22.3 p395

Glass jet 24.1 p416

Glass rod 15.1 p242; 15.4 p256

Glycerine 23.1 p405

Ice 16.3 p272

Jockey 18.6 p319; 19.1 p324; 19.2 p328; 19.3 p333; 19.4 p337; 19.5 p341; 19.6 p346; 20.1 p352; 20.2 p357; 20.3 p362

Key 17.1 p279; 17.2 p283; 17.3 p287; 17.4 p292; 20.1 p352; 20.2 p357; 20.3 p362

Knife edge 8.1 p106; 8.2 p112; 8.4 p121; 12.1 p182; 12.2 p186; 12.3 p190; 12.4 p194

Lamina, cardboard 8.3 p117

Lens, hand 24.2 p422

Lid for a calorimeter 15.1 p242; 15.2 p246; 15.3 p251; 15.4 p256

Liquid, unnamed sample 12.4 p194; 14.5 p237; 15.2 p246; 16.1 p262; 16.2 p268; 23.2 p410

Logarithm tables 13.2 p206

Masses, known 8.4 p121; 9.3 p135; 11.2 p170; 12.1 p182; 12.3 p190; 12.4 p194

Masses, slotted, and hanger 9.4 p140; 10.1 p147; 10.2 p152; 10.4 p161; 11.1 p165; 11.3 p175; 12.2 p186; 21.3 p380

Mathematical tables or a calculator 7.5 p90

Measuring cylinder 10.3 p156; 22.3 p395; 23.1 p405; 23.2 p410; 24.1 p416

Metre bridge 19.1 p324; 19.2 p328; 19.3 p333; 19.4 p337; 19.5 p341

Metre rule 7.1 p74; 7.2 p80; 7.3 p83; 7.4 p87; 7.5 p90; 7.6 p93; 7.7 p97; 7.8 p100; 8.1 p106; 8.2 p112; 8.3 p117; 8.4 p121; 9.1 p126; 9.2 p132; 9.3 p135; 9.4 p140; 10.1 p147; 10.4 p161; 11.1 p165; 11.2 p170; 11.3 p175; 12.1 p182; 12.2 p186; 12.3 p190; 12.4 p194; 18.5 p315; 18.6 p319; 19.2 p328; 19.5 p341; 19.6 p346; 20.1 p352; 20.2 p357; 20.3 p362; 21.1 p370; 21.2 p376; 21.3 p380; 22.3 p395; 23.1 p405; 23.2 p410

Micrometer screw gauge 9.3 p135; 9.4 p140; 11.1 p165; 11.2 p170; 11.3 p175; 18.5 p315; 19.2 p328; 19.4 p337; 19.6 p346; 21.3 p380; 23.1 p405

Millimetre scale 24.2 p422

Mirror, plane 22.1 p387

Nail 7.4 p87

Naphthalene 16.2 p268
Optical pins 8.3 p117; 22.1 p387; 22.2 p391; 22.3 p395; 22.4 p400
Paper, plain sheets of 22.1 p387; 22.2 p391; 22.4 p400
Pencil 22.1 p387; 22.2 p391; 22.4 p400
Plasticine 11.3 p175
Pointer, light 9.4 p140; 10.1 p147; 10.4 p161; 11.3 p175
Polish, metal 14.2 p222
Prism, triangular glass 22.2 p391
Protractor 22.1 p387; 22.2 p391; 22.4 p400
Resistance box 17.1 p279; 17.3 p287; 18.1 p298; 18.2 p303; 18.4 p311; 19.1 p324; 19.3 p333; 19.4 p337; 20.2 p357
Resistance wire 18.5 p315; 18.6 p319; 19.2 p328; 19.4 p337; 19.5 p341; 19.6 p346; 20.1 p352; 20.2 p357; 20.3 p362
Resistor, sealed or unmarked 17.4 p292; 19.1 p324; 19.2 p328
Resistor, standard 19.2 p328; 19.3 p333; 19.5 p341; 19.6 p346
Resonance tube 21.1 p370; 21.2 p376
Retort stand with boss and clamp 7.1 p74; 7.2 p80; 7.3 p83; 7.5 p90; 7.6 p93; 7.8 p100; 8.1 p106; 8.3 p117; 8.4 p121; 9.4 p140; 10.1 p147; 10.2 p152; 10.3 p156; 10.4 p161; 13.1 p200; 13.2 p206; 13.3 p211; 14.1 p218; 14.2 p222; 14.3 p227; 14.4 p232; 14.5 p237; 15.2 p246; 16.1 p262; 16.2 p268; 16.3 p272; 21.1 p370; 21.2 p376; 22.3 p395; 23.1 p405; 23.2 p410; 24.1 p416; 24.2 p422
Rheostat 17.2 p283; 17.4 p292; 18.3 p307; 20.1 p352; 20.2 p357; 20.3 p362
Rider, paper 21.3 p380
Ring, steel 7.6 p93
Rod, uniform 9.3 p135
Rubber bands 23.1 p405
Rubber tubing 21.1 p370; 21.2 p376; 23.2 p410; 24.1 p416
Ruler 22.1 p387; 22.2 p391; 22.4 p400
Ruler, transparent 13.1 p200; 13.2 p206; 13.3 p211; 14.1 p218; 14.2 p222; 14.3 p227; 14.4 p232; 14.5 p237
Slab, wooden 13.1 p200; 13.2 p206; 13.3 p211; 14.1 p218; 14.2 p222; 14.3 p227; 14.4 p232; 14.5 p237; 15.1 p242; 15.2 p246; 15.3 p251; 15.4 p256
Sodium carbonate solution 24.2 p422
Solid, unnamed sample 12.1 p182; 12.3 p190; 12.4 p194; 15.3 p251
Sonometer 21.3 p380
Spring wire, steel 9.4 p140
Spring, spiral 10.1 p147; 10.2 p152; 10.3 p156; 10.4 p161
Stirrer 13.1 p200; 13.2 p206; 13.3 p211; 14.1 p218; 14.2 p222; 14.3 p227; 14.4 p232; 14.5 p237; 15.1 p242; 15.2 p246; 15.3 p251; 15.4 p256; 16.1 p262; 16.2 p268; 16.3 p272
Stopwatch 7.1 p74; 7.2 p80; 7.3 p83; 7.4 p87; 7.5 p90; 7.6 p93; 7.7 p97; 7.8 p100; 8.1 p106; 8.2 p112; 8.3 p117; 8.4 p121; 9.1 p126; 9.2 p132; 9.3 p135; 10.2 p152; 10.3 p156; 10.4 p161; 11.1 p165; 11.2 p170; 13.1 p200; 13.2 p206; 13.3 p211; 14.1 p218; 14.2 p222; 14.3 p227; 14.4 p232; 14.5 p237; 15.2 p246; 15.3 p251; 15.4 p256; 16.1 p262; 16.2 p268; 23.1 p405; 23.2 p410
Support, fixed horizontal 9.1 p126; 9.2 p132
Switch 18.1 p298; 18.2 p303; 18.3 p307; 18.4 p311; 18.5 p315; 18.6 p319; 19.1 p324; 19.2 p328; 19.3 p333; 19.4 p337; 19.5 p341; 19.6 p346
Tape, masking 7.7 p97; 19.6 p346
Test tube 9.4 p140; 16.2 p268
Thermometer 13.1 p200; 13.2 p206; 13.3 p211; 14.1 p218; 14.2 p222; 14.3 p227; 14.4 p232; 14.5 p237; 15.1 p242; 15.2 p246; 15.3 p251; 15.4 p256; 16.1 p262; 16.2 p268; 16.3 p272; 21.1 p370; 21.2 p376; 23.1 p405; 23.2 p410; 24.1 p416; 24.2 p422
Thread 7.1 p74; 7.2 p80; 7.3 p83; 7.4 p87; 7.5 p90; 7.6 p93; 7.7 p97; 7.8 p100; 9.1 p126; 11.1 p165; 11.2 p170; 11.3 p175; 12.1 p182; 12.2 p186; 12.3 p190; 12.4 p194; 15.1 p242; 15.4 p256
Thumb pins 22.1 p387; 22.2 p391; 22.4 p400
Tongs 15.3 p251
Trough, glass 24.2 p422
Tuning forks 21.1 p370; 21.2 p376; 21.3 p380
Vernier callipers 7.2 p80; 7.6 p93; 7.8 p100; 9.4 p140; 11.1 p165; 11.2 p170; 11.3 p175
Vessel, outer, with lagging 16.3 p272
Vessel, wide, as a constant head 23.2 p410
Voltmeter 17.1 p279; 17.2 p283; 17.3 p287; 17.4 p292; 18.3 p307; 18.4 p311; 18.5 p315; 18.6 p319; 20.3 p362
Water 10.3 p156; 12.3 p190; 12.4 p194; 13.1 p200; 13.2 p206; 13.3 p211; 14.1 p218; 14.2 p222; 14.3 p227; 14.4 p232; 14.5 p237; 15.1 p242; 15.2 p246; 15.3 p251; 15.4 p256; 16.1 p262; 16.2 p268; 16.3 p272; 21.1 p370; 21.2 p376; 22.3 p395; 23.2 p410; 24.1 p416; 24.2 p422
Wire, chucks for 9.3 p135
Wire, connecting 17.1 p279; 17.2 p283; 17.3 p287; 17.4 p292; 18.1 p298; 18.2 p303; 18.3 p307; 18.4 p311; 18.5 p315; 18.6 p319; 19.1 p324; 19.2 p328; 19.3 p333; 19.4 p337; 19.5 p341; 19.6 p346; 20.1 p352; 20.2 p357; 20.3 p362
Wire, steel 9.3 p135