EXAMINATION QUESTIONS ON HEAT AND PROPERTIES OF MATTER

Useful constants: *Young's modulus of steel* $= 2.0 \times 10^{11} Pa$, *Young's modulus of copper* $= 1.1 \times 10^{11} Pa$, *Young's modulus of aluminium* $= 7.0 \times 10^{10} Pa$, $c_{water} = 4200 Jkg^{-1}K^{-1}$, $L_f(ice) = 3.36 \times 10^5 Jkg^{-1}$, $L_v(water) = 2.26 \times 10^6 Jkg^{-1}$, $R = 8.314 Jmol^{-1}K^{-1}$, *atmospheric pressure* $= 1.013 \times 10^5 Pa$, $\sigma = 5.67 \times 10^{-8} Wm^{-2}K^{-4}$, *Wien displacement constant* $b = 2.898 \times 10^{-3} mK$, $0°C = 273K$, *solar constant* $S = 1370 Wm^{-2} k_B = 1.381 \times 10^{-23} JK^{-1}, N_A = 6.022 \times 10^{23} mol^{-1}$

Question 1

(a) A student claims that the strain energy stored in a stretched wire is the kinetic energy of its atoms. Identify the error and account for the form in which the energy is actually stored.
(b) A daladala's rear leaf-spring suspension creaks under heavy passenger load but is silent when the vehicle is empty. Account for the difference, with reference to the elastic deformation of the leaf springs.
(c) A steel wire of length 2.0m and cross-sectional area $1.5 \times 10^{-6} \text{m}^2$ is stretched by an axial force of 300N. Take Young's modulus of steel as 2.0×10^{11}Pa. Calculate the extension of the wire and the elastic strain energy stored in it.

Question 2

(a) A student examining a stress-strain curve for a metal identifies the proportional limit and the elastic limit as the same point. Identify the error and account for the physical difference between the two limits.
(b) A bicycle pump used in a **Kigoma** shop, after months of daily use, develops a permanent slight bend in its piston rod even though no single push has snapped the rod. Account for the bend.
(c) A copper wire of yield stress 2.5×10^8Pa and breaking stress 4.0×10^8Pa has a circular cross-section of diameter 1.0mm. Calculate the maximum load the wire can support before plastic deformation begins, and the maximum load it can support before fracture.

Question 3

(a) Account for why the strain energy per unit volume of an elastic material is given by $\frac{1}{2} \times \text{stress} \times \text{strain}$, and not by $\text{stress} \times \text{strain}$ alone.
(b) A child in **Shinyanga** builds a slingshot using a thick rubber band. He notices that stretching the band to twice its initial extension launches the projectile to approximately four times the previous range, at the same angle. Account for this observation.
(c) A spring of force constant 200Nm^{-1} is stretched 0.30m from its natural length and then released, propelling a 0.20kg projectile horizontally from rest along a frictionless surface. Calculate the elastic potential energy stored in the spring and the launch velocity of the projectile.

Question 4

(a) For a fluid, distinguish between the isothermal bulk modulus and the adiabatic bulk modulus, and account for why the two values differ.
(b) The speed of sound in fresh water at room temperature is observed to be substantially higher than the speed of sound in air at the same temperature. Identify the physical property of water mainly responsible, and account for why this property is so much larger in water than in air.
(c) An ideal monatomic gas is at a pressure of 1.5×10^5Pa. Calculate (i) the isothermal bulk modulus of the gas, (ii) the adiabatic bulk modulus of the gas. Take $\gamma = 5/3$.

Question 5

(a) A bar of varying cross-section carries the same axial tensile force throughout its length. Account for why the strain in the bar is not the same everywhere, and identify where in the bar the strain is greatest.
(b) A glass tumbler dropped on a tiled floor in a kitchen typically shatters into many fragments, while a metal cup dropped from the same height suffers only a small dent. Account for this difference in behaviour.
(c) A steel rod of uniform cross-sectional area A_0 carries an axial tensile force F. A small circular hole drilled through the rod reduces its local cross-sectional area to $A_0/4$ at the hole. Treating the stress concentration

factor at the edge of the hole as 3, calculate the factor by which the peak local stress at the hole exceeds the stress in the unaffected part of the rod.

Question 6

(a) A cable made of n parallel wires of identical cross-section is loaded with a force F. A student claims that each wire carries the same load F because they share the load. Identify the error in the student's reasoning and state the correct sharing of the load among the n wires.

(b) The lift cables of a passenger lift in a **Dar es Salaam** office building consist of multiple parallel steel wire ropes rather than a single thicker rope. Account for two engineering reasons (beyond the simple multiplication of cross-section) why a multi-rope arrangement is preferred.

(c) A rigid horizontal beam is supported in equilibrium by three vertical wires of identical material and identical length, attached to the beam at equal spacing. The two outer wires each have cross-sectional area A; the middle wire has cross-sectional area 2A. A vertical load W is hung from the centre of the beam. Calculate the tension in each of the three wires.

Question 7

(a) A small mass attached to the lower end of a vertical elastic wire and pulled downward from its equilibrium position oscillates after release. Identify the type of motion that results, and account for why the period of this motion depends on the elastic properties of the wire as well as the mass.

(b) A bus driver in **Mtwara** notices a steady low-pitched hum from his bus's wheel-and-axle assembly at one particular cruising speed, but not at higher or lower speeds. Account for this resonance in terms of the elastic properties of the suspension.

(c) A 2.0kg mass hangs in equilibrium from a vertical steel wire of length 1.5m and cross-sectional area $5.0 \times 10^{-7}\text{m}^2$. Take Young's modulus of steel as $2.0 \times 10^{11}\text{Pa}$. The mass is pulled a small distance below its equilibrium position and released. Calculate the period of the resulting vertical oscillation.

Question 8

(a) A student plots stress against strain for a metal wire and finds that the curve becomes non-linear shortly after leaving the origin. Identify what physical limit has been passed, and account for why the slope of the straight portion alone gives the Young's modulus.

(b) A factory in **Mwanza** tensile-tests every batch of incoming steel wire before installing it in a suspension bridge. Account for why a tensile test alone is sufficient to detect a sample of below-specification steel, but is not sufficient to confirm an above-specification sample as fully usable.

(c) In a tensile test on a steel rod of original length 200mm and original diameter 10mm, the following data are recorded at the proportional limit: applied force 14kN; extension 0.180mm. Calculate the Young's modulus of the steel.

Question 9

(a) A student claims that a load suddenly released onto an elastic wire produces the same maximum extension as the same load placed onto the wire gradually. Identify the error and account for what happens to the wire in the moments after a sudden release.

(b) A worker on a **Sumbawanga** construction site lowers a sack of cement onto a wire-supported hook by carefully releasing the sack at the level of the hook, rather than dropping it from above his shoulders. Account for why the dropping method is more likely to snap the supporting wire even though the cement weight is the same in both cases.

(c) A 5.0kg load is released from rest at a height of 0.10m above the lower end of a vertical steel wire of original length 2.0m and cross-sectional area $1.0 \times 10^{-6}\text{m}^2$. Take Young's modulus of steel as $2.0 \times 10^{11}\text{Pa}$ and $g = 9.81\text{ms}^{-2}$. Calculate the maximum dynamic extension of the wire and the factor by which it exceeds the static extension produced by the same load.

Question 10

(a) The fundamental frequency of a stretched wire fixed at both ends depends on its tension, length, and linear mass density. Account for why a wire of higher Young's modulus does not, by that fact alone, have a higher fundamental frequency than a similar wire of lower Young's modulus.
(b) A guitarist tuning her instrument in **Zanzibar** finds that tightening the tuning peg of a string raises its pitch. Account for this behaviour by reference to the formula for the fundamental frequency of a stretched wire.
(c) A steel wire of length 0.75m and linear mass density $8.0 \times 10^{-3}\text{kgm}^{-1}$ is stretched between two fixed pegs and is observed to vibrate at its fundamental frequency of 220Hz. Calculate the tension in the wire.

Question 11

(a) A student claims that any physical property of a substance that varies with temperature can serve as the basis of a thermometer. Identify three additional requirements that the property must satisfy, and account for why each is needed.
(b) A laboratory in **Mpanda** maintains a working thermometer for routine use and a reference thermometer kept locked in a cupboard, used only to recalibrate the working thermometer. Account for why the reference thermometer is kept specifically for this purpose rather than simply rotating two identical thermometers.
(c) A platinum resistance thermometer has resistance 25.0Ω at the ice point and 35.5Ω at the steam point. In a constant-temperature bath, the resistance is 31.5Ω. Calculate the temperature of the bath on the platinum-resistance scale.

Question 12

(a) Account for why the constant-volume gas thermometer is regarded as the near-ideal thermometer over a wide range of temperatures.
(b) An industrial calibration laboratory in **Dar es Salaam** keeps a constant-volume gas thermometer as its primary reference standard, using it once or twice a year to recalibrate platinum resistance thermometers used for daily measurements. Account for why the gas thermometer, despite its bulk and inconvenience, is preferred as the primary reference.
(c) A constant-volume gas thermometer has a pressure of 0.800×10^5Pa when its bulb is at the triple point of water (273.16K). When the bulb is placed in a heated bath, the pressure becomes 1.099×10^5Pa. Calculate the temperature of the bath on the ideal-gas scale, in kelvin and in degrees Celsius.

Question 13

(a) A student claims that since a platinum resistance thermometer gives the same reading as the ideal-gas scale at the ice point and at the steam point, the two scales must agree at every temperature in between. Identify the error in this claim, and account for the physical origin of any disagreement.
(b) A power-station engineer uses a platinum resistance thermometer (rather than a mercury-in-glass thermometer) to monitor the temperature of superheated steam at about 400°C. Account for two reasons why the platinum thermometer is preferred for this application.
(c) A platinum resistance thermometer has resistance 25.0Ω at the ice point and 35.6Ω at the steam point. In a heated bath, its resistance is 41.6Ω. A constant-volume gas thermometer placed in the same bath gives a temperature reading of 158.0°C. Calculate the temperature of the bath on the platinum-resistance scale, and the discrepancy between this value and the ideal-gas scale value.

Question 14

(a) A thermocouple consists of two different metal wires joined at two junctions, one at a known reference temperature and the other at the unknown temperature to be measured. Account for the origin of the thermoelectric EMF generated between the two junctions, and identify why the cold junction is necessary.
(b) A meat-processing factory in **Iringa** uses a thermocouple probe (rather than a glass thermometer) to measure the temperature inside cuts of meat as they leave the oven. Account for two practical advantages of the thermocouple in this application.
(c) A copper-constantan thermocouple has its cold junction held in a bath of melting ice at 0°C. When the hot junction is held at the steam point of water (100°C), the EMF measured across the open ends of the thermocouple is 4.10mV. Treating the EMF as varying linearly with the temperature of the hot junction in

the working range 0 to 200°C, calculate the temperature of an oil bath that produces an EMF of 7.40mV in the same thermocouple.

Question 15

(a) Account for why the modern thermodynamic temperature scale is defined using the triple point of water as a single fixed reference point, rather than the ice point and the steam point used historically.
(b) A standards laboratory in **Dar es Salaam** realises a triple-point cell of water as part of its temperature-reference chain. Identify what is special about a triple-point cell that makes its temperature more reproducible than a simple ice-water bath.
(c) A constant-volume gas thermometer reads a pressure of 100kPa when its bulb is at the triple point of water (273.16K). The same thermometer, with its bulb placed in a vessel of liquid nitrogen at its boiling point, reads a pressure of 28.0kPa. Calculate the temperature of the boiling point of liquid nitrogen on the ideal-gas scale, in kelvin and in degrees Celsius.

Question 16

(a) Account for why a thermistor (a semiconductor temperature sensor) has an electrical resistance that decreases with rising temperature, the opposite of a metal resistance thermometer, and account for why this property makes thermistors particularly suited to detecting small changes in temperature near room temperature.
(b) A digital clinical thermometer used in a **Bagamoyo** health clinic has a thermistor at the tip of its probe rather than a platinum resistance element. Account for two practical reasons (beyond cost) why a thermistor is preferred in this application.
(c) A thermistor has resistance 10.0kΩ at 25°C and 3.30kΩ at 50°C. The resistance varies with absolute temperature according to $\mathrm{R} = \mathrm{R_0 e^{(B/T)}}$, where $\mathrm{R_0}$ and B are constants. Calculate the value of B and the resistance of the thermistor at body temperature 37°C.

Question 17

(a) Account, by reference to the physical structure of a clinical mercury-in-glass thermometer, for why the reading remains at the maximum reached during measurement, even when the thermometer has been removed from the patient and cooled to room temperature.
(b) A nurse at a **Chalinze** health clinic must shake the clinical thermometer vigorously after each patient's measurement before the next reading. Account for the physical reason this shaking is necessary.
(c) The bulb of a clinical thermometer contains 0.20cm³ of mercury at 0°C. The cubic expansion coefficient of mercury is $\gamma_{\mathrm{Hg}} = 1.82 \times 10^{-4}\mathrm{K^{-1}}$. Take the cubic expansion of the glass as negligible. Calculate the change in the volume of mercury when the thermometer reading rises from room temperature 24°C to the patient's body temperature 39.5°C.

Question 18

(a) Distinguish between the operating ranges and sensitivities of a mercury-in-glass thermometer and an alcohol-in-glass thermometer, and account for why each is preferred for its own range of applications.
(b) A weather station outside **Mbeya** in the Southern Highlands, where night-time temperatures occasionally fall below -20°C in the cooler months, uses an alcohol-in-glass thermometer (rather than a mercury-in-glass thermometer) for its outdoor minimum-temperature record. Account for the choice.
(c) A mercury-in-glass thermometer has a bulb containing 0.50cm³ of mercury at 0°C. It is graduated such that a 1.0mm rise of the mercury thread in its stem corresponds to a temperature rise of 1.0K. Take the cubic expansion coefficient of mercury as $\gamma_{\mathrm{Hg}} = 1.82 \times 10^{-4}\mathrm{K^{-1}}$ and assume the expansion of the glass is negligible. Calculate the cross-sectional area of the bore of the stem.

Question 19

(a) A radiation thermometer (pyrometer) measures the temperature of a body without making physical contact with it. Account for the physical principle that allows this measurement, and identify the upper temperature limit above which contact thermometers (mercury-in-glass, platinum resistance) become impractical or impossible.

(b) A steel-mill furnace operator at **Tanga** monitors the temperature of molten steel from outside the furnace using an optical pyrometer, while a poultry-incubator operator at a **Bagamoyo** farm uses a digital probe thermometer to monitor the temperature inside the incubator. Account for why each operator's choice of thermometer is appropriate to the application.

(c) An optical pyrometer is calibrated by viewing a black body at a series of known temperatures, and is found to give an output reading proportional to the fourth power of the absolute temperature, in agreement with the Stefan-Boltzmann law. When pointed at a black body at 1000K, the pyrometer reads 1.00 (in arbitrary units). When pointed at a different glowing body, treated as a black body, the pyrometer reads 4.00. Calculate the temperature of this body.

Question 20

(a) Distinguish between thermal conductivity and thermal conductance, and account for why the conductance of a wall containing a window of poorer thermal insulation than the wall itself depends not only on the conductivity of the two materials but also on the area of each.

(b) An office building in **Lindi** has a south-facing wall containing both a brick portion and a single glass window. The architect adds a second window of the same area on the same wall. Account for whether the heat lost through the wall doubles, less than doubles, or more than doubles, identifying the dominant heat-loss path.

(c) A wall of total area $10m^2$ has a brick portion of area $8.5m^2$ and thickness 0.10m, with thermal conductivity $1.0Wm^{-1}K^{-1}$. A glass window of area $1.5m^2$ and thickness 5.0mm fills the rest, with thermal conductivity $0.80Wm^{-1}K^{-1}$. When the inside is at 25°C and the outside at 15°C, calculate the rate of heat conduction through (i) the brick portion, (ii) the glass window, and (iii) the wall as a whole.

Question 21

(a) Account for why a cylindrical hot-water pipe loses heat at a rate per unit length that depends, for a given temperature difference between the pipe surface and the surroundings, on the natural logarithm of the ratio of the outer to the inner radius of any insulating layer wrapped around it, rather than on the layer thickness alone.

(b) A solar water heater installed on a **Mwanza** rooftop carries hot water from the rooftop tank to a tap on the ground floor through a copper pipe wrapped in two distinct layers of lagging: an inner layer of mineral wool (low thermal conductivity) and an outer layer of foil-faced foam (slightly higher conductivity but also low). Account for why the engineer uses two layers in series rather than the same total thickness of a single material.

(c) A copper hot-water pipe of inner radius 2.0cm carries water at 80°C through a building where the surrounding air is at 20°C. The pipe is lagged with two concentric layers: an inner layer of thickness 2.0cm and thermal conductivity $0.05Wm^{-1}K^{-1}$, surrounded by an outer layer of thickness 1.0cm and thermal conductivity $0.10Wm^{-1}K^{-1}$. Treating the pipe surface as held at 80°C and the outermost lagging surface as held at 20°C, calculate the rate of heat loss per unit length of the pipe.

Question 22

(a) Newton's law of cooling, in the form rate of cooling proportional to temperature difference between the body and its surroundings, holds rigorously for forced convection but only as an approximation for natural convection at small temperature differences. Account for why this distinction arises.

(b) A barbecue cook in **Bukoba** finds that a piece of grilled meat cools faster if she fans it with a banana leaf than if she leaves it on the table to cool in still air. Account for the difference, identifying which heat-transfer mechanism is enhanced by the fanning.

(c) A small body cools from 80°C to 60°C in still air at 20°C in 5.0 minutes. When the same body is placed in a steady draught at the same air temperature, it cools from 80°C to 60°C in 1.5 minutes. Calculate the ratio of the cooling-law constant under forced convection to the constant under free convection.

Question 23

(a) A blackbody is heated continuously from 1000K through 2000K to 5000K. Account for the systematic shift in the wavelength at which it radiates maximum energy, and account for the corresponding change in the colour at which the body appears to a human observer.
(b) Astronomers in a Tanzanian observatory classify stars by colour: red giants appear cool red, our Sun appears yellow-white, and certain hot young stars appear bluish white. Account for this colour-temperature relationship by reference to Wien's displacement law.
(c) Wien's displacement constant is $b = 2.898 \times 10^{-3}$mK. A particular star emits its maximum radiation at a wavelength of 480nm. Calculate the surface temperature of the star, and compare it with the surface temperature of the Sun (which emits its maximum radiation at approximately 500nm).

Question 24

(a) A grey body has the same emissivity at all wavelengths, but the emissivity is less than that of a perfect blackbody at the same temperature. Account, by reference to Kirchhoff's law of thermal radiation, for why a body that is a poor emitter at a particular wavelength must also be a poor absorber at the same wavelength.
(b) The roof of a corrugated-iron house in a **Dar es Salaam** neighbourhood is painted matt black to reduce the temperature of the interior on a hot afternoon. Account, by reference to the difference in dominant radiation wavelengths during the day and at night, for whether the matt black paint actually achieves the desired cooling.
(c) A grey-bodied object of emissivity 0.60 is held at a steady temperature of 400K in surroundings at 300K. Take the Stefan-Boltzmann constant as $\sigma = 5.67 \times 10^{-8}\text{Wm}^{-2}\text{K}^{-4}$. Calculate the net rate of radiation loss per unit area of the object's surface.

Question 25

(a) A vacuum flask is designed to minimise heat transfer between its contents and the surroundings by all three mechanisms (conduction, convection, and radiation). Identify the design feature that minimises each mechanism, and account for why the flask cannot completely eliminate any of them.
(b) A market trader in **Chakechake** fills a vacuum flask with hot tea at 90°C in the morning and finds that the tea is still pleasantly hot at 70°C when she drinks it eight hours later. Account for the tea's slow cooling, and identify the residual heat-loss mechanism that ultimately limits the flask's effectiveness.
(c) A vacuum flask of internal surface area 0.010m² holds tea at 80°C in a room at 20°C. The internal walls of the flask are silvered, with effective emissivity 0.05. Take the Stefan-Boltzmann constant as $\sigma = 5.67 \times 10^{-8}\text{Wm}^{-2}\text{K}^{-4}$. Calculate the rate of heat loss by radiation through the silvered vacuum gap, and compare it with the approximate rate of heat loss from an unlagged open cup of the same area by free convection alone, taking the convection coefficient as $h = 10\text{Wm}^{-2}\text{K}^{-1}$.

Question 26

(a) A solar absorber panel is painted matt black on its sun-facing side to absorb the maximum fraction of incident solar radiation, but the same blackness causes it to radiate strongly in the infrared at its operating temperature. Account for why this radiative loss is unavoidable, and identify a coating strategy (a 'selective surface') that allows a high absorption of solar radiation without the corresponding high emission of thermal radiation.
(b) A solar water heater installed on a rooftop has a flat absorber panel of area 2.0m². On a clear day at noon the panel receives a solar flux of 800Wm^{-2} and reaches an operating temperature of 70°C while the surroundings are at 30°C. Account for why the panel does not continue to heat indefinitely, and identify the limiting factor on its steady-state temperature.
(c) A solar absorber panel of area 2.0m² and emissivity 0.90 (assumed equal to its solar absorptivity) sits on a rooftop and receives solar radiation of flux 800Wm^{-2}. At its steady operating temperature of 70°C in surroundings at 30°C, calculate the rate at which the panel absorbs solar radiation, the rate at which it loses heat by infrared radiation to the surroundings, and the net useful heat-collection rate.

Question 27

(a) Account, by reference to thermal radiation to the sky, for why frost can form on the surface of a tin roof or on the leaves of a plant on a clear night even when the air temperature one metre above the surface is several degrees above 0°C.
(b) A farmer in **Mufindi** observes that on clear cold nights his crops in an open field develop frost on their leaves, while on cloudy nights with the same air temperature no frost appears. Account for the role of cloud cover in suppressing frost formation.
(c) On a clear night, the effective sky temperature for thermal radiation is approximately 250K, while on an overcast night with low clouds it is approximately 280K. The ground surface is at 5°C (278K) on both nights and emits as a blackbody. Calculate the net rate of radiative heat loss per unit area of ground surface for each case, and account for why one case can produce frost while the other cannot.

Question 28

(a) The Earth, treated as a perfect absorber of solar radiation and a perfect emitter of thermal radiation, would settle at a steady-state temperature determined by balancing the absorbed solar power with the thermal-infrared emission to space. Account for the geometric factor (involving the Earth's cross-section as seen from the Sun and its surface area for emission) that enters this balance.
(b) Climate scientists at a Tanzanian research institute observe that the average measured temperature at the Earth's surface (around 288K) is substantially higher than the calculated equilibrium temperature for a bare planet of the same albedo. Account for this discrepancy by reference to the Earth's atmosphere.
(c) Take the solar constant at the Earth's orbit as $S = 1370\text{Wm}^{-2}$ and the Earth's average albedo (the fraction of solar radiation reflected back to space) as 0.30. Treating the Earth as a sphere that absorbs the un-reflected solar radiation and re-emits as a blackbody, calculate the equilibrium effective temperature of the Earth on the Stefan-Boltzmann scale.

Question 29

(a) A typical horticultural greenhouse uses glass walls and roof to admit solar radiation while restricting the loss of heat at night. Account for the optical property of glass that allows it to act in this dual role, and identify the wavelength region in which glass is transparent and the region in which it is opaque.
(b) A market gardener in **Iringa** builds a polythene-sheet greenhouse to extend the growing season for tomatoes. The polythene is transparent to visible light but, unlike glass, is also fairly transparent to thermal infrared. Account for why the polythene greenhouse is less effective at retaining heat than a glass one, and account for the limited improvement that polythene provides over no enclosure at all.
(c) A greenhouse glass panel of area 4.0m^2 is at a steady temperature of 20°C while the air outside is at 5°C. The glass behaves as a blackbody for thermal infrared radiation. Calculate the rate at which the panel loses heat to the cooler outdoor air by net infrared radiation, comparing this with the rate at which an equivalent perfectly transparent (zero-emissivity) film of the same area would lose. (Take the air's effective radiation temperature as 5°C.)

Question 30

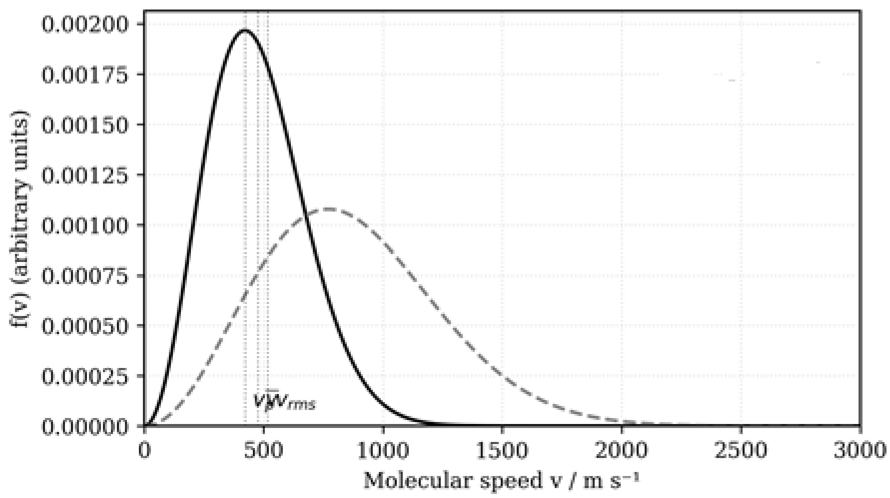
(a) Account, by reference to the kinetic theory of an ideal gas, for why the root-mean-square speed of the molecules of a gas at a given temperature depends on the molar mass of the gas, but the average translational kinetic energy per molecule does not.
(b) At a balloon-display event, two helium-filled balloons are released simultaneously with hydrogen-filled balloons. The hydrogen balloons are observed to deflate (lose gas through the rubber wall) noticeably faster than the helium balloons, even though both gases are at the same temperature and the rubber walls are identical. Account for this observation, identifying the molecular property responsible.
(c) Calculate the root-mean-square speeds of hydrogen and nitrogen molecules at 300K, given molar masses $M_{H_2} = 2.0 \times 10^{-3}\text{kgmol}^{-1}$ and $M_{N_2} = 28 \times 10^{-3}\text{kgmol}^{-1}$. Find the ratio of the two speeds and comment briefly on its physical significance. Take $R = 8.314\text{Jmol}^{-1}\text{K}^{-1}$.

Question 31

(a) Account for why the total translational kinetic energy of all the molecules in a sample of an ideal gas is far larger than the kinetic energy of a single sports ball, even though each individual molecule has only a tiny kinetic energy and even though the gas appears to be at rest.
(b) A football placed on a windless day at the centre of a **Benjamin Mkapa** pitch appears stationary. A nearby kicked football flies through the air. State, with reason from the kinetic theory of gases, whether the stationary football's mass-and-air system contains more, less, or comparable translational kinetic energy than the kicked football itself.
(c) Calculate the total translational kinetic energy of 2.0 mol of an ideal gas at 300K. Compare this with the kinetic energy of a 0.45kg football kicked at 25ms^{-1}. Take $\text{R} = 8.314\text{Jmol}^{-1}\text{K}^{-1}$.

Question 32

The figure below shows the Maxwell-Boltzmann speed distribution for molecules of a fixed-mass sample of nitrogen at two different temperatures.



(a) Identify the curve corresponding to the higher temperature, and account for the shift of the peak and the broadening of the distribution as the temperature rises.
(b) On a hot afternoon in Dodoma, a few molecules of perfume sprayed in one corner of a still room are smelt at the opposite corner only some seconds later, even though the mean speed of perfume molecules at 300K is comparable to the speed of a fast bicycle. Account, by reference to the Maxwell-Boltzmann distribution and to the molecules' frequent collisions, for the apparent slowness of perfume diffusion.
(c) From the figure, identify the most probable speed and estimate the rms speed of nitrogen molecules at 300K. Compare your estimate of the rms speed with the calculated value from $\text{v}_{\text{rms}} = \sqrt{3\text{RT}/\text{M}}$, taking $\text{M}_{\text{N}_2} = 28 \times 10^{-3}\text{kgmol}^{-1}$ and $\text{R} = 8.314\text{Jmol}^{-1}\text{K}^{-1}$.

Question 33

(a) Distinguish between effusion and diffusion as processes by which gas molecules escape from one region to another, and account for why Graham's law *(rate of effusion inversely proportional to the square root of molar mass*) holds for effusion but is only approximately applicable to diffusion.
(b) A factory in Tanzania uses a long set of porous-membrane stages to enrich uranium for nuclear-power use, where the natural mixture of UF_6 molecules contains both $^{235}UF_6$ and $^{238}UF_6$ at very similar molar masses (349 and 352 g/mol respectively). Account for why a single membrane stage gives only a tiny enrichment of the lighter isotope, and why many stages in series are required.
(c) Two identical containers each have a small hole in the wall. One contains helium gas, the other oxygen gas, both at the same pressure and temperature. Calculate the ratio of the rate at which helium effuses through its hole to the rate at which oxygen effuses through the identical hole. Take $\text{M}_{\text{He}} = 4.0 \times 10^{-3}\text{kgmol}^{-1}$, $\text{M}_{\text{O}_2} = 32 \times 10^{-3}\text{kgmol}^{-1}$.

Question 34

(a) The speed of sound in an ideal gas is given by $v = \sqrt{\gamma RT/M}$. Account for why a person who inhales helium and then speaks has an audibly higher-pitched voice than usual, even though the vocal cords themselves vibrate at the same frequency as before.
(b) A safety officer at a Tanga industrial gas plant warns workers against inhaling helium directly from a cylinder for amusement, despite the high-pitched-voice effect being apparently harmless. Account for the physical danger of inhaling pure helium, identifying the property of helium that the warning addresses.
(c) Calculate the speed of sound in dry air and in helium, both at 27°C, taking $\gamma_{air} = 1.40$, $M_{air} = 29 \times 10^{-3}\text{kgmol}^{-1}$, $\gamma_{He} = 5/3$, $M_{He} = 4.0 \times 10^{-3}\text{kgmol}^{-1}$, and $R = 8.314\text{Jmol}^{-1}\text{K}^{-1}$. Find the ratio of the two speeds.

Question 35

(a) A diatomic molecule at room temperature stores its thermal energy in five accessible degrees of freedom: three translational and two rotational. Account, by reference to the geometry of a dumbbell-shaped molecule, for why two and not three rotational modes are available.
(b) A factory in Mwanza compares the cost of filling its argon balloons and its nitrogen balloons. The technician notes that for the same volume and temperature, the gases contain the same number of moles, but the nitrogen balloon has more total internal energy than the argon balloon. Account for this observation in terms of degrees of freedom.
(c) Calculate the total internal energy of 1.0 mol of nitrogen (N_2) at 300K. Find the fraction of this internal energy that resides as translational kinetic energy and the fraction that resides as rotational kinetic energy. Take $R = 8.314\text{Jmol}^{-1}\text{K}^{-1}$.

Question 36

(a) The molar heat capacity at constant volume of an ideal gas with f accessible degrees of freedom is $C_v = (f/2)R$, and at constant pressure is $C_p = ((f+2)/2)R$. Account, by reference to the first law of thermodynamics applied at constant pressure, for the additional R in C_p relative to C_v.
(b) A welder in a workshop uses bottled argon gas as a shielding gas for arc welding because the arc stays cooler in argon than it would in nitrogen at the same heating power. Account for this observation in terms of the heat capacities of the two gases.
(c) For a diatomic ideal gas with $f = 5$ accessible degrees of freedom, calculate C_v, C_p, and the ratio of principal heat capacities $\gamma = C_p/C_v$. Take $R = 8.314\text{Jmol}^{-1}\text{K}^{-1}$.

Question 37

(a) The first law of thermodynamics states that the change in internal energy of a closed system equals the heat added to it minus the work done by it. Account for why, in any heating process at constant pressure, the heat supplied to an ideal gas always exceeds the change in its internal energy by exactly the work done by the gas during the expansion.
(b) A welder in a workshop fills a flexible balloon with helium from a cylinder, then leaves it on a sunny bench. The sun warms the helium and the balloon expands. Account, by reference to the first law of thermodynamics, for why the heat absorbed by the helium from the Sun exceeds the rise in the gas's internal energy by an amount equal to the work the helium has done in pushing back the surrounding atmosphere.
(c) 0.50 mol of an ideal monatomic gas at 300K is heated at the constant pressure $1.0 \times 10^5\text{Pa}$ until its temperature is 600K. Calculate the heat supplied, the work done by the gas, and the change in its internal energy. Take $R = 8.314\text{Jmol}^{-1}\text{K}^{-1}$.

Question 38

(a) The change in internal energy of an ideal gas between two given states depends only on the two states, not on the path joining them. Account for why the work done by the gas in moving between the same two states is different along different paths.
(b) A factory technician in Mwanza uses a piston compressor to raise a gas from atmospheric pressure to operating pressure. He notices that the compressor uses less work in cool weather (when the gas is allowed

to remain near atmospheric temperature during compression) than in hot weather (when the gas heats during the compression and the pressure rises more rapidly). Account for this observation.

(c) An ideal gas undergoes a transition from state A at (P_0, V_0) to state B at $(P_0/2, 2V_0)$ along two different paths. Path 1: isochoric drop from A to $(P_0/2, V_0)$, then isobaric expansion to B. Path 2: isobaric expansion from A to $(P_0, 2V_0)$, then isochoric drop to B. Calculate the work done by the gas along each path, expressed in terms of P_0V_0, and identify which path delivers more work.

Question 39

(a) The net work done by an ideal gas during one complete traversal of any closed cycle on a PV diagram equals the area enclosed by the cycle, with the sign of the area set by the direction of traversal. Account, by reference to the integral $W = \oint PdV$, for why this geometric area rule applies to cycles of every shape, not just rectangular ones.

(b) A small two-stroke engine in a motorbike has its indicator diagram (a real-time PV record of the gas in the cylinder) recorded by a diagnostic technician. The diagram is roughly triangular rather than perfectly rectangular. Account for why the technician can still infer the engine's net work output per cycle from the shape of the diagram alone.

(c) An ideal gas executes a triangular cycle on a PV diagram with corners at $A(V = 1.0 \times 10^{-3}m^3, P = 1.0 \times 10^5Pa)$, $B(V = 1.0 \times 10^{-3}m^3, P = 2.0 \times 10^5Pa)$, and $C(V = 2.0 \times 10^{-3}m^3, P = 1.0 \times 10^5Pa)$, traversed in the order $A \to B \to C \to A$. The path $B \to C$ is a straight line on the PV diagram. Calculate the net work done by the gas per cycle.

Question 40

(a) In an isothermal expansion of an ideal gas the temperature is held constant by allowing heat to flow in from the surroundings. Account, by reference to the first law and the property that internal energy of an ideal gas depends only on temperature, for why the heat absorbed during the expansion exactly equals the work done by the gas.

(b) A natural-gas pipeline in a Tanzanian gas-distribution network carries compressed methane from a wellhead at 80 atmospheres into a distribution network at 5 atmospheres, with the gas held at approximately ground temperature by long contact with the soil. Account for why the heat absorbed by the gas from the soil equals the work the gas does in expanding, even though the gas does not visibly heat up during the expansion.

(c) One mole of methane (treated as an ideal gas) at 290K is expanded isothermally from a pressure of 80 atmospheres to a pressure of 5 atmospheres. Calculate the work done by the gas during the expansion. Take $R = 8.314Jmol^{-1}K^{-1}$.

Question 41

(a) Carnot's theorem fixes the maximum efficiency of any heat engine operating between two reservoirs at temperatures T_{hot} and T_{cold}, in kelvin, as $\eta_{Carnot} = 1 - T_{cold}/T_{hot}$. Account for why this maximum cannot be approached except in the limit of a reversible (and therefore infinitely slow) cycle, and account for the consequences for real engines run at finite power.

(b) A geothermal power station built near the volcanic Lake Natron in northern Tanzania extracts heat from underground steam at 150°C and rejects waste heat to the atmosphere at 27°C. Account, with reason from Carnot's theorem, for why the engineer designing the plant should expect a thermal efficiency of at most about 30%, and identify two factors that drive the realised efficiency below this Carnot ceiling.

(c) A heat engine operates between an underground steam reservoir at 150°C and a cooling tower at 27°C. Calculate the Carnot efficiency for this engine, and the maximum work that could be extracted from each kilojoule of heat absorbed from the steam reservoir.

Question 42

(a) In a calorimetry experiment, the measured final temperature is always slightly lower than the true value if the system is losing heat to the surroundings during the mixing process. Explain which mechanism is primarily responsible and why the error increases with longer experimental times.

(b) A domestic refrigerator manufacturer in Mwanza advertises its product as having a coefficient of performance of 12 between an interior temperature of 4°C and a kitchen temperature of 28°C. State, with reason from Carnot's theorem applied in reverse, whether this claim is feasible.

(c) A domestic refrigerator maintains its interior at 4°C in a kitchen at 28°C. Its compressor consumes 200W of electrical power. Calculate the maximum possible coefficient of performance for this refrigerator, and the maximum rate at which it could extract heat from its interior.

Question 43

(a) A corrugated iron roof in Dar es Salaam becomes extremely hot in midday sun but cools rapidly after sunset. Explain the rapid cooling, identifying which heat-transfer mechanisms dominate during the night-time cooling and why the roof cools faster than the concrete walls of the same building.

(b) A gas-filled cylinder in an industrial laboratory is briefly connected through a valve to an adjacent evacuated cylinder. Both cylinders are well insulated. The gas rushes from the filled to the empty cylinder until pressure equalises. Account for why the final temperature of the gas (an ideal gas, well above its boiling point) is essentially the same as its initial temperature, even though the expansion is rapid and the system is thermally insulated.

(c) 0.50 mol of an ideal gas at 27°C and 1.0×10^5Pa fills one half of a thermally insulated 2.0×10^{-3}m^3 container, with the other half evacuated. The partition is suddenly removed. Calculate the final temperature, the final pressure, and the change in internal energy of the gas. Take $R = 8.314\text{Jmol}^{-1}\text{K}^{-1}$.

SOLUTIONS

Question 1

(a) The error: the atoms in a stretched wire are not moving any faster than at zero strain; the temperature is unchanged. The strain energy is stored as the increase in the interatomic potential energy of the atoms, which sit slightly farther apart than at the equilibrium spacing of the unstretched wire. The work done by the applied force is converted into this potential energy.

(b) Under heavy load each leaf of the spring deflects further, so adjacent leaves slide against each other through a larger distance and the surface friction between them produces the audible squeak. With no load the leaves are in their natural curved shape, the contact between them is essentially static, and no sliding (and therefore no squeak) occurs.

(c)

$$\Delta L = \frac{FL}{AE} = \frac{300\text{N} \times 2.0\text{m}}{1.5\times10^{-6}\text{m}^2 \times 2.0\times10^{11}\text{Pa}} = 2.0\times10^{-3}\text{m}$$

$$U = \frac{1}{2}F\Delta L = \frac{1}{2} \times 300\text{N} \times 2.0\times10^{-3}\text{m} = 0.30\text{J}$$

Extension is 2.0mm and strain energy stored is 0.30J.

Question 2

(a) The error: the proportional limit and the elastic limit need not coincide. The proportional limit is the highest stress at which Hooke's law (stress proportional to strain) holds. The elastic limit is the highest stress at which the wire still returns to its original length on unloading. Between these two limits the wire is still elastic but no longer linear in its response. For most metals the two limits are very close but not identical, and the elastic limit lies just above the proportional limit.

(b) Each downstroke loads the rod beyond its elastic limit by a small amount. The rod recovers most of its shape elastically after each stroke but retains a tiny plastic deformation. Over many cycles these tiny plastic deformations accumulate into the visible permanent bend, even though no single stroke applied a force large enough to fracture the rod.

(c)

$$A = \pi\left(\frac{d}{2}\right)^2 = \pi \times (0.5\times10^{-3}\text{m})^2 = 7.85\times10^{-7}\text{m}^2$$

$$F_{yield} = \sigma_y A = 2.5\times10^8\text{Pa} \times 7.85\times10^{-7}\text{m}^2 \approx 196\text{N}$$

$$F_{break} = \sigma_b A = 4.0\times10^8\text{Pa} \times 7.85\times10^{-7}\text{m}^2 \approx 314\text{N}$$

Plastic deformation begins at approximately 196N; fracture occurs at approximately 314N.

Question 3

(a) During an elastic stretch, the applied force builds up gradually from zero to its final value F. The work done is the area under the force-extension graph, which is the area of a triangle: $\frac{1}{2}F\Delta L$. Per unit volume, this becomes $\frac{1}{2}\sigma\varepsilon$ where σ is stress and ε is strain. Multiplying final stress by final strain would assume the force was at its full final value throughout the stretch, which overcounts the work by a factor of two.

(b) The strain energy stored in a Hookean rubber band is proportional to the square of the extension, so doubling the extension stores four times the energy. On release this energy is converted to projectile kinetic energy $\frac{1}{2}mv^2$, so the launch velocity doubles. With doubled launch velocity at the same angle, the projectile range (proportional to v^2) becomes four times its previous value.

(c)

$$U = \frac{1}{2}kx^2 = \frac{1}{2} \times 200\text{Nm}^{-1} \times (0.30\text{m})^2 = 9.0\text{J}$$

$$v = \sqrt{\frac{2U}{m}} = \sqrt{\frac{2 \times 9.0\text{J}}{0.20\text{kg}}} = 9.49\text{ms}^{-1}$$

Energy stored 9.0J; launch velocity approximately 9.5m/s.

Question 4

(a) The isothermal bulk modulus B_T measures the resistance of a fluid to compression when the temperature is held constant; for an ideal gas $B_T = P$. The adiabatic bulk modulus B_S measures the same resistance when no heat flows during the compression; for an ideal gas $B_S = \gamma P$. The two differ because in an adiabatic compression the gas heats up, raising the pressure further than the volume reduction alone produces; the gas appears stiffer when compressed adiabatically.

(b) The bulk modulus, since the speed of sound in a fluid is $v = \sqrt{B/\rho}$. The bulk modulus of water is about 2.2×10^9Pa, while that of air at atmospheric pressure is only about 1.4×10^5Pa, a ratio of about 10^4. Water molecules are tightly packed and resist compression strongly because the intermolecular separation is already close to the equilibrium value of the strong attractive forces. Air molecules are far apart and offer little resistance to compression beyond the kinetic-pressure term.

(c) (i) Isothermal bulk modulus:

$$B_T = P = 1.5 \times 10^5\text{Pa}$$

(ii) Adiabatic bulk modulus:

$$B_S = \gamma P = \frac{5}{3} \times 1.5 \times 10^5\text{Pa} = 2.5 \times 10^5\text{Pa}$$

Question 5

(a) At every cross-section the same force F is transmitted, but stress is force per unit area; a smaller cross-section therefore carries a larger stress, and the strain $\varepsilon = \sigma/E$ is correspondingly larger. The strain is greatest where the cross-section is narrowest, which is also where the bar is most likely to fail under load.

(b) Glass is a brittle material: under stress it stores elastic energy but cannot dissipate it through plastic deformation. When the elastic limit is reached, the stored energy is released suddenly through the propagation of cracks, shattering the tumbler. A metal cup is ductile: stress beyond the elastic limit causes plastic deformation, in which atoms slip past each other and absorb energy as heat through dislocation motion. The cup dents instead of shattering because plastic flow can absorb the impact energy without catastrophic crack propagation.

(c) Stress in main rod:

$$\sigma_0 = \frac{F}{A_0}$$

Nominal stress at the reduced section:

$$\sigma_{nom} = \frac{F}{A_0/4} = 4\sigma_0$$

Peak stress at the edge of the hole, with stress concentration factor 3:

$$\sigma_{peak} = 3\sigma_{nom} = 12\sigma_0$$

The peak local stress at the hole is 12 times the stress in the unaffected part of the rod.

Question 6

(a) The error: in a parallel-wire cable each wire carries only its share of the total load, not the entire load. Since the wires are identical and forced to extend by the same amount (they are joined at the ends), Hooke's law requires each to carry the same tension. The correct sharing is therefore F/n in each wire.

(b) First, redundancy: if one rope fails, the remaining ropes can still hold the lift (each rope is rated to carry the full load with a safety margin), avoiding a catastrophic fall. Second, fatigue and inspection: thinner ropes flex more easily over the drive sheave, reducing bending stresses and fatigue, and they can be inspected and replaced one at a time without taking the lift out of service for long. A single thick rope would be stiffer, would fatigue faster at the sheave, and would offer no redundancy.

(c) Since the beam is rigid and remains horizontal, all three wires extend by the same amount x. Each tension is $T = (AE/L)x$ for a wire of area A. The middle wire has area 2A, so its tension is twice that of an outer wire.

$$T_1 = T_3 = \frac{AEx}{L}, \quad T_2 = \frac{2AEx}{L}$$

Vertical equilibrium: $T_1 + T_2 + T_3 = W$:

$$\frac{AEx}{L}(1 + 2 + 1) = W \quad \Rightarrow \quad \frac{AEx}{L} = \frac{W}{4}$$

$$T_1 = T_3 = \frac{W}{4}, \quad T_2 = \frac{W}{2}$$

Tensions in the ratio $1:2:1$, with the middle (thicker) wire carrying half the load and each outer wire carrying a quarter.

Question 7

(a) Simple harmonic motion. The wire behaves as a Hookean spring of effective force constant $k = AE/L$, where A, L are the wire's cross-sectional area and natural length and E is its Young's modulus. The restoring force on the mass at displacement x from equilibrium is $-kx$, giving SHM with angular frequency $\omega = \sqrt{k/m}$ and period $T = 2\pi\sqrt{m/k}$, which depends on the wire's A, L, and E as well as on the mass.

(b) The suspension acts as a spring-mass system with its own natural frequency, set by the spring constant of the suspension and the sprung mass of the bus. At one particular cruising speed, the rotation frequency of the wheel (or some sub-multiple of it) coincides with this natural frequency, and small unbalanced forces from the wheel drive the suspension into resonance, producing the audible hum. Above and below that speed, the driving frequency is off-resonance and the response is too small to hear.

(c)

$$k = \frac{AE}{L} = \frac{5.0 \times 10^{-7}\text{m}^2 \times 2.0 \times 10^{11}\text{Pa}}{1.5\text{m}} = 6.67 \times 10^4\text{Nm}^{-1}$$

$$T = 2\pi\sqrt{\frac{m}{k}} = 2\pi\sqrt{\frac{2.0\text{kg}}{6.67 \times 10^4\text{Nm}^{-1}}} = 3.44 \times 10^{-2}\text{s}$$

Period of oscillation is approximately 34ms.

Question 8

(a) The proportional limit has been passed: beyond this point the stress is no longer linearly proportional to the strain. Young's modulus is defined for the linear (Hookean) regime, where $\sigma = E\varepsilon$, so the slope of the straight portion of the stress-strain curve gives E directly. The slope of the non-linear portion does not represent any single material constant and cannot be used.

(b) A tensile test loads the sample to failure and records its yield stress, breaking stress and Young's modulus. A below-specification sample fails one or more of these tests directly and is detected. An above-specification sample passes the tension tests, but a bridge cable also fatigues under cyclic loading, suffers stress concentration at fittings, and is exposed to salt corrosion; tensile testing measures only the static-load properties and cannot detect defects that show up only under cyclic, environmental, or geometric stress. Above-specification by tension alone is therefore necessary but not sufficient.

(c)

$$A = \pi\left(\frac{d}{2}\right)^2 = \pi \times (5.0 \times 10^{-3}\text{m})^2 = 7.85 \times 10^{-5}\text{m}^2$$

$$\sigma = \frac{F}{A} = \frac{14 \times 10^3\text{N}}{7.85 \times 10^{-5}\text{m}^2} = 1.78 \times 10^8\text{Pa}$$

$$\varepsilon = \frac{\Delta L}{L} = \frac{0.180 \times 10^{-3}\text{m}}{0.200\text{m}} = 9.00 \times 10^{-4}$$

$$E = \frac{\sigma}{\varepsilon} = \frac{1.78 \times 10^{8}\text{Pa}}{9.00 \times 10^{-4}} = 2.0 \times 10^{11}\text{Pa}$$

Young's modulus of the steel is approximately 2.0×10^{11}Pa.

Question 9

(a) The error: a load placed gradually does work $\frac{1}{2}F\Delta L$ on the wire as the force builds up; a load suddenly released does work $F(\Delta L + h)$ as it falls through the wire's stretching distance plus any height h of free fall. The sudden load therefore stores more elastic energy in the wire and causes a larger maximum extension than the equilibrium static extension. After a sudden release the wire oscillates about the static extension, with the first downswing reaching the maximum dynamic extension, which can be much larger than the static value.

(b) In the controlled lowering, the wire stretches by its static extension only and the maximum tension is just the cement weight. In the dropping method, the falling sack converts gravitational potential energy into both elastic strain energy and kinetic energy of the sack just before contact; the wire must then absorb the total energy by stretching beyond its static extension, raising the peak tension well above the static value. The peak tension can exceed the breaking stress of the wire even though the static load alone is well within the safe range.

(c) Effective spring constant of the wire and weight of the load:

$$k = \frac{AE}{L} = \frac{1.0 \times 10^{-6}\text{m}^2 \times 2.0 \times 10^{11}\text{Pa}}{2.0\text{m}} = 1.0 \times 10^{5}\text{Nm}^{-1}$$

$$mg = 5.0\text{kg} \times 9.81\text{ms}^{-2} = 49.05\text{N}$$

Static extension:

$$x_s = \frac{mg}{k} = \frac{49.05\text{N}}{1.0 \times 10^{5}\text{Nm}^{-1}} = 4.91 \times 10^{-4}\text{m} \approx 0.49\text{mm}$$

Energy conservation from release to maximum extension (gravitational PE lost = strain energy stored):

$$mg(h + x_{max}) = \frac{1}{2}k{x_{max}}^2$$

$$\frac{1}{2} \times 1.0 \times 10^{5}{x_{max}}^2 - 49.05\,x_{max} - 4.905 = 0$$

Solving the quadratic (taking the positive root):

$$x_{max} = 1.04 \times 10^{-2}\text{m} = 10.4\text{mm}$$

$$\frac{x_{max}}{x_s} = \frac{10.4\text{mm}}{0.49\text{mm}} \approx 21$$

Maximum dynamic extension is approximately 10.4mm, about 21 times the static extension.

Question 10

(a) The fundamental frequency of a stretched wire fixed at both ends is $f = \frac{1}{2L}\sqrt{T/\mu}$, which depends on the tension T, the length L, and the linear mass density μ, but does not contain Young's modulus explicitly. The Young's modulus enters the problem only through the relation between extension and tension; once a particular tension is established in the wire, the wire's vibration frequency is determined by T, L, and μ alone, regardless of which value of E produced that tension. A stiffer wire requires a larger applied force to reach a given tension, but the resulting frequency at that tension is the same as for a softer wire at the same tension.

(b) Tightening the tuning peg increases the tension T in the string. Since $f \propto \sqrt{T}$ at constant L and μ, raising T raises the fundamental frequency, which is heard as a rise in pitch.

(c)

$$f = \frac{1}{2L}\sqrt{\frac{T}{\mu}}$$

$$T = \mu(2Lf)^2 = 8.0 \times 10^{-3}\text{kgm}^{-1} \times (2 \times 0.75\text{m} \times 220\text{Hz})^2 \approx 871\text{N}$$

Tension in the wire is approximately 871N.

Question 11

(a) The property must (i) vary continuously and reproducibly with temperature, so that a single value of the property unambiguously identifies a single temperature; (ii) vary monotonically (single-valued in the working range), so that the same temperature does not correspond to two different values of the property; (iii) vary by an amount large enough to be measured precisely with available instruments, so that small temperature differences produce a distinguishable change in the property.

(b) The reference thermometer is used only at the ice point, the steam point, and the calibration laboratory, so it accumulates very little wear, drift, or chemical contamination. The working thermometer accumulates these effects rapidly through daily handling. By keeping the reference thermometer in a controlled environment, the laboratory preserves a stable standard against which the working thermometer's drift can be detected and corrected.

(c)

$$\theta_{Pt} = \frac{R - R_0}{R_{100} - R_0} \times 100°C$$

$$\theta_{Pt} = \frac{31.5\Omega - 25.0\Omega}{35.5\Omega - 25.0\Omega} \times 100°C = 61.9°C$$

Temperature on the platinum-resistance scale is approximately 62°C.

Question 12

(a) The pressure of an ideal gas at constant volume varies linearly with the absolute temperature, $P/T = \text{const}$, and this relation is independent of the gas used in the limit of low pressure. Different gases (helium, hydrogen, nitrogen) all give the same temperature reading when their thermometers are each calibrated at the triple point of water and used at low enough pressure that they behave ideally. The gas thermometer therefore defines a temperature scale that is essentially independent of the working substance, as required of a thermodynamic temperature scale.

(b) The gas thermometer's calibration is fixed by the laws of physics, not by the specific properties of any one gas, so it does not drift over time and does not need to be re-traced to a higher-tier reference. Platinum resistance thermometers are convenient and quick to read, but their resistance-temperature relation depends on the purity and prior heat-treatment of the platinum, both of which can change with use; they need periodic recalibration against an absolute reference. The gas thermometer fills that role.

(c)

$$T = T_{tp} \times \frac{P}{P_{tp}} = 273.16K \times \frac{1.099 \times 10^5 Pa}{0.800 \times 10^5 Pa} = 375.3K = 102.15°C$$

Temperature of the bath is approximately 375K, or about 102°C.

Question 13

(a) The error: agreement at two fixed points does not imply agreement at intermediate points. The platinum-resistance scale is constructed by linear interpolation between the ice and steam points, but the actual relationship between the resistance of platinum and its temperature is not exactly linear; the ideal-gas scale, by contrast, is linear in absolute temperature. Wherever the platinum's true response curve deviates from the straight line that joins the two calibration points, the platinum-resistance scale and the ideal-gas scale disagree, with the largest discrepancy typically at the midpoint of the calibration range.

(b) First, the temperature 400°C is well above the boiling point of mercury (357°C); a mercury-in-glass thermometer cannot operate above this limit without the mercury vaporising and breaking the column. Second, the platinum thermometer can be sealed inside a metal sheath connected to its electronics by long leads, allowing remote reading from outside the steam chamber, while a mercury-in-glass thermometer would have to be visible at the chamber, requiring a window and an operator at the hot location.

(c)

$$\theta_{Pt} = \frac{R - R_0}{R_{100} - R_0} \times 100°C = \frac{41.6 - 25.0}{35.6 - 25.0} \times 100°C = 156.6°C$$

On the ideal-gas scale, $T = 158.0°C$. The platinum-resistance scale reads about 1.4°C below the ideal-gas value at this temperature. The discrepancy is consistent with the slight non-linearity of platinum's resistance-temperature curve over the 0 to 158°C range.

Question 14

(a) At each junction between the two metals, an electric potential difference arises (the Seebeck effect), with magnitude that depends on the temperature of the junction and on the pair of metals. When the two junctions are at different temperatures the two potential differences do not cancel, and a net EMF appears around the loop, driving a current if the loop is closed. The cold junction is necessary because only the temperature difference between the two junctions can be inferred from the EMF; without a reference junction at a known temperature, the hot-junction temperature cannot be deduced from the EMF reading alone.

(b) First, the thermocouple junction is small and has very low thermal mass, so it reaches thermal equilibrium with the meat quickly and does not cool the meat noticeably. Second, the junction can be inserted on a long thin probe into the centre of a piece of meat, while the EMF is read on a meter at a convenient distance; a glass thermometer would have to be inserted along its full length and read at the meat itself.

(c) Linear calibration: $E = k\theta$, with $k = 4.10\text{mV}/100°\text{C} = 0.0410\text{mV}/°\text{C}$:

$$\theta = \frac{E}{k} = \frac{7.40\text{mV}}{0.0410\text{mV}/°\text{C}} = 180.5°\text{C}$$

Temperature of the oil bath is approximately 180.5°C.

Question 15

(a) The ice point and the steam point are both pressure-dependent: the boiling point of water depends on the surrounding atmospheric pressure, and the melting point of ice changes slightly with pressure. Realising either point to the precision required for primary metrology demands controlling the pressure to high accuracy. The triple point of water, by contrast, is the unique combination of pressure and temperature at which all three phases of water coexist; it is fixed by the substance itself and is independent of external conditions, so a single fixed point is sufficient and is more reproducible than either of the historical pair.

(b) In a triple-point cell, water is sealed inside a glass cell with three phases (ice, liquid water, and water vapour) coexisting at the triple-point temperature 273.16K and pressure 611Pa. So long as all three phases are present, the temperature is fixed by the physics of the substance, independent of the surrounding atmospheric pressure (which cannot reach the sealed water inside the cell). An open ice-water bath, by contrast, is at the ice point at the surrounding atmospheric pressure, which varies from day to day and from one altitude to another; its temperature is therefore less reproducible by a few millikelvins.

(c)

$$T = T_{tp} \times \frac{P}{P_{tp}} = 273.16\text{K} \times \frac{28.0\text{kPa}}{100\text{kPa}}$$

$$T = 273.16\text{K} \times 0.280 = 76.5\text{K}$$

$$\theta = T - 273.15 \approx -196.7°\text{C}$$

Boiling point of liquid nitrogen on the ideal-gas scale is approximately 76.5K or -196.7°C.

Question 16

(a) In a metal, the conduction electrons are essentially free at all temperatures, and the resistance arises mainly from electron scattering by lattice vibrations; rising temperature increases the lattice vibrations and so the resistance grows with temperature. In a semiconductor, the number of conduction electrons is itself temperature-dependent: thermal excitation liberates more charge carriers from the valence band as the temperature rises, and this rapid increase in carrier density dominates the resistance, which therefore falls steeply with temperature. The fall is exponential, so the fractional change in resistance per kelvin is large near room temperature, allowing thermistors to detect small temperature changes that a metal resistance thermometer would resolve only with difficulty.

(b) First, the thermistor's resistance changes more steeply with temperature than that of any practical metal element, so a small temperature change at the patient produces a large change in resistance and hence in the digital display, giving high sensitivity over the narrow clinical range 35 to 42°C. Second, thermistors can be made very small (sub-millimetre), so the probe reaches thermal equilibrium with the patient's tissue rapidly, giving a reading in a few seconds rather than the minute or more needed for a larger-thermal-mass platinum element.

(c) From $R = R_0 e^{(B/T)}$, taking the ratio at the two given temperatures:

$$\frac{R_1}{R_2} = e^{\left(\frac{B}{T_1} - \frac{B}{T_2}\right)} \Rightarrow \ln\frac{R_1}{R_2} = B\left(\frac{1}{T_1} - \frac{1}{T_2}\right)$$

$$B = \frac{\ln R_1/R_2}{1/T_1 - 1/T_2} = \frac{\ln 10.0/3.30}{1/298 - 1/323} = 4270\text{K}$$

At $T = 310\text{K}$ (37°C):

$$R(37°\text{C}) = R_1 e^{\left(B\left(\frac{1}{T} - \frac{1}{T_1}\right)\right)} = 10.0\text{k}\Omega \times e^{(-0.554)} = 5.74\text{k}\Omega$$

$B = 4270\text{K}$ and $R(37°\text{C}) = 5.74\text{k}\Omega$.

Question 17

(a) A clinical thermometer has a fine constriction in the capillary just above the bulb. As the bulb is heated, mercury expands and is forced through the constriction into the stem, where it rises to the appropriate temperature mark. When the thermometer is removed and cools, the mercury in the bulb contracts and pulls back; but the mercury thread in the stem cannot pass back through the narrow constriction by capillary attraction alone, so it remains in place at the maximum mark. The reading therefore stays at the maximum until the thermometer is shaken to force the mercury back through the constriction.

(b) Shaking accelerates the trapped mercury thread sharply along the stem; the resulting inertial force on the mercury column overcomes the capillary attraction holding it above the constriction, and the column is pushed back through the constriction into the bulb. Without this manual reset, the thermometer would still display the previous patient's maximum and would underread the next patient by however much the next temperature was below the previous reading.

(c)

$$\Delta V = \gamma_{Hg} V_0 \Delta T = 1.82 \times 10^{-4} K^{-1} \times 0.20 cm^3 \times (39.5 - 24.0) K = 5.64 \times 10^{-4} cm^3$$

The volume of mercury increases by approximately $5.6 \times 10^{-4} cm^3$ (about $0.56 mm^3$) over the 15.5K rise. The narrow stem bore converts this small volume change into a visible rise of the mercury thread.

Question 18

(a) Mercury freezes at -39°C and boils at 357°C, giving a working range of about -38 to +350°C; ethanol freezes at -114°C and boils at 78°C, giving a working range of about -110 to +75°C. Mercury therefore covers high temperatures up to several hundred degrees, while alcohol covers very low temperatures (below -38°C) where mercury would solidify. Mercury has a smaller cubic expansion coefficient ($1.82\times10^{-4} K^{-1}$) than alcohol ($1.1\times10^{-3} K^{-1}$), so for the same bore an alcohol thermometer is roughly six times more sensitive (larger thread movement per kelvin), but covers a narrower range. The choice between mercury and alcohol therefore trades high-temperature reach against low-temperature reach and sensitivity.

(b) -20°C is well above the freezing point of ethanol (-114°C) but already cold enough to be approaching mercury's freezing point (-39°C). On a colder than average night, a mercury thermometer can solidify and stop responding, failing to record the actual minimum. Alcohol remains a free-flowing liquid throughout the temperature range encountered in Mbeya, so an alcohol thermometer is the safe choice for low-temperature work.

(c) Volume change per kelvin of temperature rise:

$$\frac{\Delta V}{\Delta T} = \gamma_{Hg} V_0 = 1.82 \times 10^{-4} K^{-1} \times 0.50 cm^3 = 9.10 \times 10^{-5} cm^3/K$$

This volume change drives a 1.0mm = 0.10cm rise in the stem, occupying a length Δl of bore of cross-sectional area A:

$$\Delta V = A\Delta l \Rightarrow A = \frac{\Delta V}{\Delta l} = \frac{9.10 \times 10^{-5} cm^3}{0.10 cm} = 9.10 \times 10^{-4} cm^2$$

Cross-sectional area of the stem bore is approximately $9.1 \times 10^{-4} cm^2$ (about $0.091 mm^2$, corresponding to a bore diameter of about 0.34mm).

Question 19

(a) Every body at non-zero absolute temperature radiates electromagnetic energy, and for a hot body in the visible-and-infrared range the spectrum and total flux of this radiation depend on the body's temperature, by Planck's law and the Stefan-Boltzmann law. A pyrometer collects this thermal radiation through a lens and infers the temperature of the body from the radiation flux or its spectrum, without any physical contact. Above about 600 to 700°C, mercury-in-glass thermometers fail (mercury boils above 357°C) and platinum resistance thermometers, though usable to about 1000°C, suffer contamination from hot gases and are no longer accurate; for body temperatures of order 1000°C and above, the pyrometer is the only practical instrument.

(b) The molten steel furnace operates at temperatures of order 1500°C, where any contact probe would be destroyed and where the body emits enough visible-and-infrared radiation for a pyrometer to read its temperature easily; the non-contact method is therefore both necessary (for survival of the instrument) and sufficient (for accuracy). The poultry incubator operates at modest temperatures of about 38°C, where a contact probe is survivable, and where a pyrometer would have to be calibrated at low temperatures with much weaker radiation; a thermistor or platinum probe gives a more accurate reading at room-temperature scales.

(c) The pyrometer reading is proportional to T^4:

$$\frac{T^4}{1000^4 K^4} = \frac{4.00}{1.00} \Rightarrow T = 1000K \times 4^{1/4} \approx 1414K \approx 1141°C$$

Temperature of the glowing body is approximately 1414K, or about 1141°C.

Question 20

(a) Thermal conductivity k is a property of the material alone, with units $Wm^{-1}K^{-1}$. Thermal conductance is a property of a particular component (slab, wall, window), $C = kA/L$ in units WK^{-1}, depending on the material's k, the cross-sectional area A through which heat flows, and the path length L. A wall with a window has two parallel paths for heat: the conductances of the wall and the window add (parallel rule), so the area of each portion enters the total conductance directly. A material of poor insulation occupying a large fractional area can dominate the heat loss, even if its thickness alone is favourable.

(b) The dominant heat-loss path is the window, because the glass window has both a much smaller thickness (5mm vs 100mm of brick) and a small but comparable conductivity to the brick; per square metre, the window conducts much more heat than the brick. Doubling the window area roughly doubles the window contribution, while leaving the small brick contribution unchanged; the overall heat lost therefore increases by approximately the window's share of the total, which can be more than doubling if the original window already carries most of the heat. In practice the increase is close to a doubling because the brick contribution is small to begin with.

(c) (i) Heat conduction through the brick portion:

$$P_{brick} = \frac{k_b A_b \Delta T}{L_b} = \frac{1.0Wm^{-1}K^{-1} \times 8.5m^2 \times 10K}{0.10m} = 850W$$

(ii) Heat conduction through the glass window:

$$P_{glass} = \frac{k_g A_g \Delta T}{L_g} = \frac{0.80Wm^{-1}K^{-1} \times 1.5m^2 \times 10K}{5.0 \times 10^{-3}m} = 2400W$$

(iii) Total rate of heat conduction:

$$P_{total} = P_{brick} + P_{glass} = 850W + 2400W = 3250W$$

Total heat-loss rate is approximately 3250W, of which the small glass window carries 2400W (about 74%) despite occupying only 15% of the wall area.

Question 21

(a) Heat flowing radially outward through a cylindrical layer crosses successive shells of increasing area $A = 2\pi rL$, so the local temperature gradient dT/dr falls as r increases at the same heat-flow rate P. Integrating Fourier's law $P = -kA(dT/dr)$ between the inner radius r_i and outer radius r_o gives $P = 2\pi kL(T_i - T_o)/\ln(r_o/r_i)$. The relevant geometric factor is therefore the logarithm of the radius ratio, not the thickness $r_o - r_i$ alone; a thin layer wrapped on a large-radius pipe and the same thickness on a small-radius pipe contribute differently to the insulation.

(b) The mineral wool, although a better thermal insulator per unit thickness, is not robust against compression and outdoor weather; the outer foil-faced foam, with a higher conductivity but better mechanical and weather protection, shields the inner layer from damage. The two-layer arrangement therefore combines the high-thermal-resistance of the inner wool with the durability of the outer foam, an option not available with a single-material lagging.

(c) For two cylindrical layers in series carrying the same heat per unit length, the total thermal resistance per unit length is:

$$\frac{R}{L} = \frac{1}{2\pi}\left(\frac{\ln r_2/r_1}{k_1} + \frac{\ln r_3/r_2}{k_2}\right)$$

With $r_1 = 2.0cm$, $r_2 = 4.0cm$, $r_3 = 5.0cm$:

$$\frac{R}{L} = \frac{1}{2\pi}\left(\frac{\ln 2}{0.05} + \frac{\ln 1.25}{0.10}\right) = 2.56KmW^{-1}$$

$$\frac{P}{L} = \frac{\Delta T}{R/L} = \frac{60K}{2.56KmW^{-1}} \approx 23.4Wm^{-1}$$

Heat loss is approximately 23W per metre of pipe.

Question 22

(a) Newton's cooling law assumes the heat-transfer coefficient h between the body and the fluid is independent of the temperature difference, so that the cooling rate is strictly linear in the excess temperature. In forced convection (a fan or wind), the fluid speed past the body is fixed by external means, h is determined by that speed and is independent of the temperature difference; the law holds rigorously. In natural convection, the buoyancy-driven fluid speed itself depends on the temperature difference (warmer body drives faster updraft), so h increases with ΔT, and the cooling rate depends on ΔT to a power slightly greater than one. At small ΔT the variation in h is small and the law is approximately linear; at large ΔT it is no longer linear.

(b) Fanning forces room-temperature air to flow rapidly past the meat, replacing the warm air layer next to the surface with cool air far more rapidly than buoyancy alone could; the heat-transfer coefficient for forced convection is much larger than for natural convection, so the meat loses heat at a higher rate. Radiation and conduction to the table are essentially unchanged by the fanning, so convection is the dominant mechanism enhanced.

(c) For Newton's law $\theta - \theta_s = (\theta_0 - \theta_s)e^{-kt}$, the cooling constant satisfies $kt = \ln[(\theta_0 - \theta_s)/(\theta - \theta_s)]$.

In free air, $\theta_0 - \theta_s = 60K$, $\theta - \theta_s = 40K$, $t = 5.0min$:

$$k_{free} = \frac{1}{5.0min}\ln\frac{60}{40} = 0.0811min^{-1}$$

In the draught, same temperature change in $t = 1.5min$:

$$k_{forced} = \frac{\ln 1.5}{1.5min} = 0.270min^{-1}$$

$$\frac{k_{forced}}{k_{free}} = \frac{5.0}{1.5} = 3.3$$

Forced-convection cooling constant is approximately 3.3 times the free-convection cooling constant under these conditions.

Question 23

(a) As the temperature of a blackbody rises, the wavelength at which the emission spectrum peaks shifts towards shorter wavelengths according to Wien's displacement law $\lambda_{max}T = b$, where $b \approx 2.898 \times 10^{-3}mK$ is the Wien constant. At 1000K the peak lies in the infrared (around 3μm), invisible to the eye; at 2000K it has moved to around 1.5μm, still infrared, but the visible tail of the curve makes the body glow dull red. At 5000K the peak is at 580nm, in the yellow-orange region, and the body glows yellow-white as our Sun does. The colour shift from invisible-infrared through dull red, orange, yellow, and finally bluish-white as temperature rises is a direct visualisation of Wien's law.

(b) The surface temperature of a star sets the wavelength at which its thermal emission peaks, by $\lambda_{max} = b/T$. Cool red giants have surface temperatures of around 3000K; their peak wavelength is $b/3000 \approx 970nm$, in the near-infrared, and the visible-spectrum tail is dominated by long red wavelengths. The Sun, at about 5800K, peaks at $b/5800 \approx 500nm$, in the green-yellow, with a broad visible spectrum that appears whitish-yellow. Hot young stars at 10000K or above peak in the ultraviolet ($b/10000 = 290nm$), with the visible-spectrum tail weighted to short blue wavelengths, appearing bluish-white. The hierarchy red-yellow-blue corresponds directly to increasing surface temperature.

(c)

$$T = \frac{b}{\lambda_{max}} = \frac{2.898 \times 10^{-3}mK}{480 \times 10^{-9}m} \approx 6038K$$

For comparison, the Sun's surface temperature from $\lambda_{max} = 500nm$:

$$T_{sun} = \frac{2.898 \times 10^{-3}mK}{500 \times 10^{-9}m} \approx 5796K$$

The star is at approximately 6038K, which is 242K hotter than the Sun's surface.

Question 24

(a) At thermal equilibrium between a body and a cavity at temperature T, the body must emit and absorb radiation at every wavelength at exactly the same rate; otherwise its temperature would not stay equal to T. Since the cavity radiation is a perfect blackbody field, the body's emissivity ε_λ at each wavelength equals its absorptivity $\alpha lpha_\lambda$ at the same wavelength: this is Kirchhoff's law. A poor emitter at a particular wavelength is therefore necessarily a poor absorber at the same wavelength, and conversely a good emitter at a wavelength is also a good absorber at that wavelength.

(b) During the day, the dominant incoming radiation is solar (peak around 500nm, in the visible), and a matt black surface absorbs strongly at these short wavelengths, heating the roof and the interior more than a reflective white surface would. At night, the roof is itself the warm body radiating to the sky; the dominant outgoing radiation is in the thermal infrared (peak around 10μm at 300K), and the matt black surface emits strongly in the IR by Kirchhoff's law, cooling itself rapidly. The matt black paint therefore **heats** the roof during the day (bad) and **cools** it at night (good); on net, in tropical daytime climates, white or shiny aluminium-coated paint reduces interior temperatures more effectively. The matt black choice is therefore poorly matched to the daytime cooling goal.

(c)

$$\frac{P}{A} = \varepsilon\sigma(T^4 - {T_s}^4)$$

$$\frac{P}{A} = 0.60 \times 5.67 \times 10^{-8} Wm^{-2}K^{-4} \times (400^4 - 300^4)K^4 \approx 595 Wm^{-2}$$

Net radiation loss per unit area is approximately $595 Wm^{-2}$.

Question 25

(a) The flask consists of a double-walled glass vessel with an evacuated gap between the walls and silvered inner surfaces. The vacuum eliminates both conduction and convection across the gap, since both mechanisms require a material medium. The silvered surfaces have very low emissivity ($\varepsilon \approx 0.05$), reducing radiative heat exchange across the gap by about 20-fold compared with unsilvered glass. Conduction along the narrow neck of the flask cannot be fully eliminated (the lid is in contact with both the contents and the outer wall), and radiation across the silvered gap is reduced but not zero, so all three mechanisms contribute residual heat losses.

(b) Over the eight hours the contents lose only about 20K of temperature, corresponding to an average heat-loss rate of less than a watt for a half-litre of tea (heat capacity around 2.1kJ/K). The slow cooling reflects the success of the vacuum at suppressing conduction and convection across the gap, plus the silvered surfaces at suppressing radiation. The dominant residual loss is conduction along the neck of the flask through the stopper and the threaded fitting, paths that cannot be evacuated; at very long times this neck-conduction loss dominates and limits the flask's effectiveness.

(c) Radiation loss across the silvered vacuum gap (T = 353K, $T_s = 293K$):

$$P_{rad} = \varepsilon\sigma A(T^4 - T_s{}^4) = 0.05 \times 5.67 \times 10^{-8} \times 0.010 \times (353^4 - 293^4) = 0.23W$$

Free-convection loss from an unlagged cup of the same area:

$$P_{conv} = hA\Delta T = 10Wm^{-2}K^{-1} \times 0.010m^2 \times 60K = 6.0W$$

The vacuum-flask radiation loss (0.23W) is approximately 26 times smaller than the open-cup convection loss (6.0W), reflecting the design's effectiveness at suppressing the dominant convective mechanism.

Question 26

(a) A blackbody absorber and a blackbody emitter are necessarily the same surface, by Kirchhoff's law of thermal radiation: $\varepsilon_\lambda = \alpha\lambda$ at every wavelength. A surface that is black at visible-and-near-infrared wavelengths (where the Sun radiates) is automatically also black at the longer thermal-infrared wavelengths (where it radiates at its 350K operating temperature) unless the surface's optical properties differ between the two wavelength ranges. A 'selective surface' achieves this by being coated with a multi-layer film tuned to absorb strongly at solar wavelengths but reflect (and therefore emit poorly) at thermal infrared wavelengths; commercially this is achieved by sputtered metal-on-oxide layers (e.g. nickel-on-tin oxide) and brings emissivity at 10μ m down below 0.1 while keeping absorptivity at $0.5\mu m$ above 0.9.

(b) As the panel temperature rises, the rate at which it loses heat by infrared radiation increases as the fourth power of its absolute temperature, while the absorbed solar flux is fixed by the Sun's output. Steady state is reached when the radiative loss plus any convective and conductive losses just equal the absorbed solar power; any further temperature rise would cause net heat loss. The limiting factor on the steady-state temperature is therefore the balance between solar input (constant at noon) and combined infrared and convective output (rising rapidly with T). For a fixed-flux input of $800 Wm^{-2}$ and ordinary panel emissivity, the equilibrium temperature is typically around 70 to 90°C in tropical sunshine.

(c) Solar absorbed:

$$P_{abs} = \alpha SA = 0.90 \times 800Wm^{-2} \times 2.0m^2 = 1440W$$

Radiative loss to surroundings ($T = 343K$, $T_s = 303K$):

$$P_{rad} = \varepsilon\sigma A(T^4 - T_s{}^4) = 0.90 \times 5.67 \times 10^{-8} \times 2.0 \times (343^4 - 303^4) \approx 552W$$

Net useful collection rate:

$$P_{net} = P_{abs} - P_{rad} = 1440W - 552W = 888W$$

Net useful heat-collection rate is approximately 890W, an efficiency of about 62% relative to the absorbed solar input.

Question 27

(a) On a clear night, the surface radiates strongly in the thermal infrared to a sky whose effective radiative temperature is much lower than the surface temperature (the cold upper atmosphere and ultimately deep space radiate very little back). The radiative loss can exceed the slow conductive heat supply from the warmer air just above and the warmer ground beneath, allowing the surface temperature to fall to the dew point and then below freezing, even when the air at one metre is above 0°C. The local cooling of the surface is fast enough to deposit frost in a few hours of clear sky.

(b) Cloud cover reflects and re-emits thermal radiation back towards the ground, raising the effective sky temperature seen from the surface to a value close to the local air temperature. The net radiative loss $\sigma(T_{ground}{}^4 - T_{sky}{}^4)$ becomes small or even negative, and the surface stays warm enough that frost cannot form. The cloud layer acts as a radiation barrier blocking the surface's view of the cold upper atmosphere; under it, only the much weaker convective cooling operates.

(c)

Clear night, $T_{ground} = 278K$, $T_{sky} = 250K$:

$$\frac{P}{A} = \sigma(T_g^4 - T_{sky}^4) = 5.67 \times 10^{-8} \times (278^4 - 250^4) \approx 117Wm^{-2}$$

Cloudy night, $T_{ground} = 278K$, $T_{sky} = 280K$:

$$\frac{P}{A} = 5.67 \times 10^{-8} \times (278^4 - 280^4) \approx -10Wm^{-2}$$

(The negative sign means the ground gains a small amount of net radiation from the warmer cloud layer rather than losing it.)

Under the clear sky the surface loses about $117Wm^{-2}$ to space, easily enough to drop the surface below 0°C overnight and form frost. Under the cloudy sky the surface gains a small amount of radiation back from the warmer cloud and stays close to the air temperature; no frost forms.

Question 28

(a) The Earth presents a circular cross-section of area πR^2 to the incoming parallel sunlight, so the absorbed power is $(1 - A)S\pi R^2$, where A is the albedo and S the solar constant. The Earth radiates to space from its full spherical surface area $4\pi R^2$, isotropic in direction, with power $4\pi R^2 \sigma T^4$ if treated as a blackbody at temperature T. Equating absorbed and emitted powers in steady state gives $\sigma T^4 = (1 - A)S/4$, with the factor 1/4 reflecting the ratio of the cross-sectional intercepting area to the full spherical emitting area.

(b) The Earth's atmosphere is largely transparent to incoming solar radiation (concentrated in the visible-and-near-infrared) but strongly absorbing in the thermal infrared (around 10μm) where the surface radiates. The atmosphere intercepts the surface's outgoing infrared, warms slightly, and re-radiates partly back towards the surface, reducing the net rate of heat loss to space. Steady-state is therefore achieved at a higher surface temperature than for a bare planet without atmosphere; this is the natural greenhouse effect, which raises the surface temperature from the calculated effective value of about $-18°C$ to the observed average of about $+15°C$. Without it, the Earth's surface would be permanently frozen.

(c)

$$\sigma T^4 = \frac{(1 - A)S}{4} = \frac{(1 - 0.30) \times 1370Wm^{-2}}{4} = 240Wm^{-2}$$

$$T = \left(\frac{240}{5.67 \times 10^{-8}}\right)^{1/4} \approx 255K$$

Effective equilibrium temperature is approximately 255K, or about $-18°C$, well below the observed surface average of 288K and reflecting the absence of atmospheric greenhouse warming in this calculation.

Question 29

(a) Glass is highly transparent to visible and near-infrared light (roughly 0.4 to $2.5\mu m$), where solar radiation is concentrated, and admits the bulk of incoming sunlight to the greenhouse interior. At thermal-infrared wavelengths around $10\mu m$, where a body at about 300K emits its peak radiation, glass is essentially opaque and absorbs strongly. The glass therefore lets sunlight in (warming the interior) but blocks the escape of longer-wavelength thermal radiation from the warmed contents (preventing heat from leaving), a property known as spectral selectivity. The same trick works for a planetary atmosphere with selective greenhouse gases such as CO_2 and H_2O vapour.

(b) Polythene transmits both the incoming visible-and-near-IR radiation **and** the outgoing thermal IR, so it does not block the radiative loss as effectively as glass does. The polythene greenhouse retains heat mainly through its barrier against air-flow convection (it stops the warm interior air from escaping into the outside), while glass adds a radiative-loss barrier on top of the convective barrier. Polythene is therefore a partial improvement over no enclosure (it stops warm-air convection and provides shelter from rain), but its thermal performance is inferior to glass because it does not block the radiative heat-loss pathway.

(c)

$$P_{rad} = \sigma A(T_{glass}^4 - T_{air}^4) = 5.67 \times 10^{-8} \times 4.0 \times (293^4 - 278^4)W \approx 107W$$

A perfectly transparent zero-emissivity film loses zero heat by radiation; the glass panel loses approximately 107W. The radiative loss through actual glass is therefore non-negligible on cool nights, but is the price paid for being opaque to incoming long-wave radiation from a warm interior emitting back to a colder exterior.

Question 30

(a) For an ideal gas, $\frac{1}{2}m\bar{c}^2 = \frac{3}{2}k_BT$, where m is the mass of one molecule and $\bar{c}^2$ the mean square speed. The right-hand side depends only on temperature, so the average translational kinetic energy per molecule is fixed by T alone, independently of the mass of the molecule. Solving for the speed: $v_{rms} = \sqrt{3k_BT/m} = \sqrt{3RT/M}$, which depends inversely on the square root of the molar mass. A lighter molecule moves faster, but with the same average kinetic energy, as a heavier molecule at the same temperature.

(b) Both gases have the same average translational kinetic energy per molecule (set by the common temperature), but the lighter hydrogen molecules ($M = 2gmol^{-1}$) move much faster than the heavier helium molecules ($M = 4gmol^{-1}$). Faster molecules collide with the rubber wall more frequently and have a larger probability of penetrating the microscopic pores of the rubber. Hydrogen therefore effuses out of the balloon faster than helium, and the hydrogen balloon deflates first.

(c)

$$v_{rms,H_2} = \sqrt{\frac{3RT}{M_{H_2}}} = \sqrt{\frac{3 \times 8.314 \times 300}{2.0 \times 10^{-3}}}ms^{-1} \approx 1934ms^{-1}$$

$$v_{rms,N_2} = \sqrt{\frac{3RT}{M_{N_2}}} = \sqrt{\frac{3 \times 8.314 \times 300}{28 \times 10^{-3}}}ms^{-1} \approx 517ms^{-1}$$

$$\frac{v_{rms,H_2}}{v_{rms,N_2}} = \sqrt{\frac{M_{N_2}}{M_{H_2}}} = \sqrt{14} \approx 3.74$$

The rms speed of hydrogen at 300K (about $1934ms^{-1}$) is about 3.74 times that of nitrogen (about $517ms^{-1}$), reflecting the inverse square-root dependence on molar mass at the same temperature.

Question 31

(a) The total translational kinetic energy of a sample of n moles of an ideal gas is $\frac{3}{2}nRT$, which scales with the number of molecules present. A small flask of gas contains of order 10^{22} molecules; each carries a tiny kinetic energy of order $10^{-21}J$ at room temperature, but the product is of order kilojoules. The gas appears at rest because the molecular velocities are randomly directed, so the vector sum of momenta is zero; the **scalar** kinetic energy is not zero, because energy adds without regard to direction.

(b) The kicked football carries the kinetic energy of a single object of ordinary mass moving at an ordinary speed. The stationary football sits in air, which itself contains an enormous number of molecules per unit volume (of order Avogadro's number per mole, with several moles in even a small volume of surrounding air). Each molecule's translational kinetic energy is tiny, but their sum over the entire surrounding gas vastly exceeds the bulk kinetic energy of any one sports ball. The stationary-football-and-air system therefore contains **more** translational kinetic energy than the kicked football. The gas's energy is distributed isotropically across the molecules, with the vector sum of momenta close to zero; this is why it does not push the football in any particular direction, and the stationary football appears at rest despite being immersed in this large pool of molecular kinetic energy.

(c)

$$KE_{gas} = \frac{3}{2}nRT = \frac{3}{2} \times 2.0mol \times 8.314Jmol^{-1}K^{-1} \times 300K \approx 7483J$$

$$KE_{ball} = \frac{1}{2}mv^2 = \frac{1}{2} \times 0.45kg \times (25ms^{-1})^2 \approx 141J$$

$$\frac{KE_{gas}}{KE_{ball}} = \frac{7483J}{141J} \approx 53$$

Total translational KE of the gas is approximately 7483J, about 53 times the kinetic energy of the kicked football. The KE per molecule is tiny, but N_A molecules per mole make the total substantial.

Question 32

(a) **The dashed grey curve**, peaked at higher speed and broadened, corresponds to the higher temperature. As T rises, the average translational kinetic energy per molecule rises in proportion to T, so the distribution is centred on a larger speed: the peak shifts to the right (higher v_p). The total area under each curve must remain equal to the total number of molecules, which is fixed; the peak height therefore falls and the distribution spreads to a wider range of speeds, giving the broader profile at higher temperature.

(b) Although the typical speed of a perfume molecule is around $400ms^{-1}$ (comparable to a fast bicycle), the molecule does not travel in a straight line through the room: it suffers about 10^{10} collisions per second with surrounding air molecules, and after each collision its direction changes randomly. The net displacement therefore grows much more slowly than the straight-line speed would predict, scaling as $\sqrt{t}$ rather than t (Brownian-like diffusion). The perfume effectively performs a slow random walk, taking many seconds to cross a few metres.

(c) From the diagram, the most probable speed of nitrogen at 300K is approximately $400ms^{-1}$ (peak position), and the rms speed is approximately $500ms^{-1}$ (slightly to the right of the peak). From the kinetic-theory formula:

$$v_{rms} = \sqrt{\frac{3RT}{M}} = \sqrt{\frac{3 \times 8.314 \times 300}{28 \times 10^{-3}}} ms^{-1} \approx 517ms^{-1}$$

The graph estimate ($\sim 500ms^{-1}$) is in good agreement with the calculated value ($517ms^{-1}$).

Question 33

(a) Effusion is the escape of gas molecules through a hole much smaller than the **mean free path** (*the average distance a molecule travels between two successive collisions with other molecules*), so molecules pass one by one without collisions in the hole; the rate is therefore proportional to the mean speed of the molecules and obeys Graham's law strictly. Diffusion is the spreading of one gas through another by random walk, in which molecules undergo many collisions per centimetre travelled; the rate depends on the diffusion coefficient D, which involves both molecular speed and mean free path. Graham's law applies approximately to diffusion (because both processes scale roughly as the molecular speed), but exactly only to effusion.

(b) By Graham's law, the ratio of effusion rates of the two isotopes is $\sqrt{352/349} \approx 1.0043$, a 0.4% enrichment per pass. Starting from natural uranium (0.7% ^{235}U), reaching the 4% enrichment needed for reactor fuel requires the lighter isotope to pass through several hundred porous-membrane stages. Each stage adds only a tiny enrichment, but the cumulative effect of hundreds of stages in series compounds the small per-stage advantage into a useable enrichment.

(c)

$$\frac{r_{He}}{r_{O_2}} = \sqrt{\frac{M_{O_2}}{M_{He}}} = \sqrt{\frac{32}{4.0}} = 2.83$$

Helium effuses at approximately 2.83 times the rate of oxygen under identical conditions, reflecting the inverse-square-root dependence on molar mass.

Question 34

(a) The pitch of a human voice is set by the resonant frequencies of the vocal-tract cavities (mouth, throat), which scale as f_n πropto v_{sound}/L, where v_{sound} is the speed of sound in the gas filling the cavity and L is the cavity length. The vocal cords vibrate at the same fundamental frequency in either gas; what changes is the resonant-cavity filtering. Helium has both a higher γ and a much smaller molar mass than air, so v_{sound} in helium is about three times that in air, and all of the formant frequencies of the voice rise by the same factor of about three, producing the audibly higher-pitched effect.

(b) Helium contains no oxygen; inhaling pure helium from a cylinder replaces the oxygen-containing air in the lungs with an inert gas, and the body cannot extract oxygen from helium. Within seconds the oxygen partial pressure in the lung drops to a level too low to support consciousness, and the worker can pass out without warning and (if the pure helium is continued) suffocate. The high-pitched-voice effect is harmless in itself, but the means of producing it (direct inhalation of pure helium) is dangerous because helium does not trigger the body's CO_2-driven breathing reflex; the worker may not feel breathless before passing out.

(c)

$$v_{sound,air} = \sqrt{\frac{\gamma_{air}RT}{M_{air}}} = \sqrt{\frac{1.40 \times 8.314 \times 300}{29 \times 10^{-3}}} ms^{-1} \approx 347ms^{-1}$$

$$v_{sound,He} = \sqrt{\frac{\gamma_{He}RT}{M_{He}}} = \sqrt{\frac{(5/3) \times 8.314 \times 300}{4.0 \times 10^{-3}}} ms^{-1} \approx 1019ms^{-1}$$

$$\frac{v_{sound,He}}{v_{sound,air}} \approx \frac{1019}{347} \approx 2.94$$

Speed of sound in helium is approximately $1020ms^{-1}$, against approximately $347ms^{-1}$ in air; the helium voice's resonant frequencies are therefore about 2.9 times higher than in air.

Question 35

(a) A diatomic molecule has its two atoms joined by a chemical bond along a single axis. Rotation about an axis perpendicular to this bond carries the two atoms in circles of large radius and stores energy as rotational kinetic energy; rotation about the bond axis itself, however, carries each atom in a circle of essentially zero radius (the atoms sit on the axis), so the moment of inertia about that axis is much smaller than the other two and the rotational quantum is correspondingly larger than k_BT at room temperature. The bond-axis rotational mode is therefore frozen out, and only the two perpendicular rotational modes are accessible at room temperature, giving five active degrees of freedom in total (3 translational + 2 rotational).

(b) By equipartition, each accessible degree of freedom contributes $\frac{1}{2}k_BT$ of energy per molecule, so the total internal energy per molecule is $f \cdot \frac{1}{2}k_BT$. Argon, monatomic with $f = 3$, has total $U = \frac{3}{2}nRT$ per mole. Nitrogen, diatomic with $f = 5$, has total $U = \frac{5}{2}nRT$ per mole. For the same n and T, nitrogen contains 5/3 times as much internal energy as argon, and is correspondingly more capable of doing work or absorbing heat. The same number of molecules can store more energy in the diatomic gas because each molecule has more accessible energy stores.

(c)

$$U_{total} = \frac{f}{2}nRT = \frac{5}{2} \times 1.0 \times 8.314 \times 300J \approx 6236J$$

Translational contribution (f = 3):

$$U_{trans} = \frac{3}{2}nRT = \frac{3}{2} \times 1.0 \times 8.314 \times 300J \approx 3741J$$

Rotational contribution (f = 2):

$$U_{rot} = \frac{2}{2}nRT = nRT = 1.0 \times 8.314 \times 300J \approx 2494J$$

$$\frac{U_{trans}}{U_{total}} = \frac{3}{5} = 60\%, \quad \frac{U_{rot}}{U_{total}} = \frac{2}{5} = 40\%$$

Total internal energy is 6236J, of which 60% (3741J) is translational and 40% (2494J) is rotational.

Question 36

(a) At constant volume, the gas does no work on the surroundings, and all heat input goes into increasing the internal energy: $Q_V = \Delta U = (f/2)nR\Delta T$, hence $C_v = (f/2)R$ per mole. At constant pressure, the gas expands as it heats: the heat input raises both the internal energy AND does work $P\Delta V = nR\Delta T$ against the surroundings (using the equation of state). The heat input per mole per kelvin is therefore $C_p = (f/2)R + R = ((f+2)/2)R$, with the additional R corresponding to the work done by the expanding gas at constant pressure. The result $C_p - C_v = R$ is independent of f and is known as Mayer's relation.

(b) Argon is monatomic ($f = 3$) so its $C_v = (3/2)R$, while nitrogen is diatomic ($f = 5$) so its $C_v = (5/2)R$. For the same heating power supplied per mole, the heat-up rate $dT/dt = P_{heat}/(nC_v)$ is larger for argon than for nitrogen, so argon's temperature rises faster but the total energy stored at any temperature is less. In the welding context, this means the argon shielding-gas envelope reaches its design temperature quickly and stays there, with less thermal mass to drag the arc temperature down; the arc actually runs **hotter** in argon, contradicting the premise. The benefit of argon for welding is its inertness (non-reactive with hot metal) rather than thermal-management.

(c)

$$C_v = \frac{f}{2}R = \frac{5}{2} \times 8.314Jmol^{-1}K^{-1} = 20.79Jmol^{-1}K^{-1}$$

$$C_p = \frac{f+2}{2}R = \frac{7}{2} \times 8.314Jmol^{-1}K^{-1} = 29.10Jmol^{-1}K^{-1}$$

$$\gamma = \frac{C_p}{C_v} = \frac{f+2}{f} = \frac{7}{5} = 1.40$$

For a diatomic ideal gas at room temperature: $C_v = 20.79Jmol^{-1}K^{-1}$, $C_p = 29.10Jmol^{-1}K^{-1}$, $\gamma = 1.40$.

Question 37

(a) From the first law $Q = \Delta U + W$, the heat input goes both into raising the internal energy of the gas and into doing work on the surroundings. At constant pressure the gas expands as it heats; the work done by the gas $W = P\Delta V$ is non-zero, so Q exceeds ΔU by exactly W. The relationship is general for any gas-on-piston experiment at constant pressure.

(b) The Sun supplies heat Q to the helium. By the first law $Q = \Delta U + W$, this heat is partitioned between an internal-energy rise ΔU (the helium gets hotter) and the work W done by the gas on the surroundings (the gas expands and pushes back the atmosphere by an amount V at atmospheric pressure P_{atm}). Q exceeds ΔU by $W = P_{atm}\Delta V$, in agreement with the first law of thermodynamics.

(c)

$$\Delta U = nC_v\Delta T = 0.50 \times \frac{3}{2} \times 8.314 \times 300\,\text{J} \approx 1871\text{J}$$

$$W = nR\Delta T = 0.50 \times 8.314 \times 300\,\text{J} \approx 1247\text{J}$$

$$Q = nC_p\Delta T = 0.50 \times \frac{5}{2} \times 8.314 \times 300\,\text{J} \approx 3118\text{J}$$

Heat supplied is approximately 3118J, internal energy change is 1871J, and the work done by the gas is 1247J.

Check: $\Delta U + W = 1871 + 1247 = 3118\text{J} = Q$, in agreement with the first law.

Question 38

(a) Internal energy is a state function: it depends only on the current state of the gas (its T, P, V), not on how the gas reached that state. The work done by the gas, by contrast, is $W = \int PdV$ along the chosen path on the PV diagram, which is the area under the chosen path. Different paths between the same two states enclose different areas, so the work done is different. Heat exchange $Q = \Delta U + W$ is also path-dependent, since ΔU is fixed and W varies.

(b) In cool weather the compression is approximately isothermal: heat escapes to the surroundings as the gas is compressed, keeping the temperature close to ambient and the pressure rise more gradual. In hot weather (or when the compression is fast enough that heat cannot escape), the compression is closer to adiabatic: the gas heats during compression and the pressure rises more steeply for the same volume reduction. The work done on the gas equals the area under the compression curve on the PV diagram; the steeper adiabatic curve requires more work to reach the same final pressure than the shallower isothermal curve, so the compressor uses more work in hot weather.

(c) Path 1: isochoric $A \to (P_0/2, V_0)$ has $W_{1a} = 0$. Then isobaric expansion at $P_0/2$ from V_0 to $2V_0$:

$$W_{1b} = \frac{P_0}{2}(2V_0 - V_0) = \frac{1}{2}P_0V_0$$

$$W_{\text{path 1}} = 0 + \frac{1}{2}P_0V_0 = 0.5\,P_0V_0$$

Path 2: isobaric expansion at P_0 from V_0 to $2V_0$:

$$W_{2a} = P_0(2V_0 - V_0) = P_0V_0$$

Then isochoric drop at $2V_0$: $W_{2b} = 0$:

$$W_{\text{path 2}} = P_0V_0 + 0 = 1.0\,P_0V_0$$

Path 2 delivers P_0V_0 of work, twice as much as path 1's $0.5\,P_0V_0$, because path 2 expands at the higher pressure P_0 throughout while path 1 expands only at $P_0/2$.

Question 39

(a) The work done by the gas in any infinitesimal step is $dW = PdV$. Around a closed cycle, $W_{net} = \oint P\,dV$, which is the signed area enclosed by the path. The area-as-line-integral rule applies to any closed curve, regardless of shape: rectangular, triangular, oval, or any other. Clockwise traversal on the standard PV diagram gives a positive enclosed area (heat engine direction); counter-clockwise gives a negative area (refrigerator direction). The shape of the cycle determines the area; the direction of traversal determines the sign.

(b) The area enclosed by the indicator diagram on the PV plane equals the net work done by the gas per cycle. The shape of the diagram (triangular, rectangular, or rounded) does not matter for calculating the net work; only the enclosed area does. The technician measures the area (by counting squares on graph paper, by planimeter, or by digital integration) to obtain the work output per cycle, then multiplies by the engine's cycle frequency to get the power output.

(c) The triangle has base $V_C - V_A = 1.0 \times 10^{-3}\text{m}^3$ and height $P_B - P_A = 1.0 \times 10^5\text{Pa}$:

$$W_{net} = \frac{1}{2} \times \text{base} \times \text{height} = \frac{1}{2} \times 1.0 \times 10^{-3}\text{m}^3 \times 1.0 \times 10^5\text{Pa} = 50\text{J}$$

Net work done by the gas per cycle is 50J, equal to the area of the triangular cycle on the PV diagram, with positive sign because the cycle is traversed in the heat-engine direction.

Question 40

(a) For an ideal gas the internal energy depends only on temperature: $U = U(T)$. In an isothermal process, T is constant, so $\Delta U = 0$. The first law $\Delta U = Q - W$ then gives $Q = W$: every joule of heat absorbed from the surroundings is delivered as work done by the gas, with no fraction stored as internal energy. The gas acts as a perfect heat-to-work converter at the surrounding-temperature isotherm, with no temperature change.

(b) The pipeline gas, held by ground contact at the soil temperature, undergoes an essentially isothermal expansion as it flows from high to low pressure. By the first law for an isothermal expansion of an ideal gas, $\Delta U = 0$ and $Q = W$. The gas does positive work $W = nRT\ln(P_i/P_f)$ in expanding against the back pressure; to keep the temperature unchanged, an equal quantity of heat Q must flow into the gas from the surrounding soil. The gas does not visibly heat up because the inflow of heat exactly balances the work loss; the soil cools imperceptibly to compensate.

(c)

$$W = nRT\ln\frac{P_i}{P_f} = 1 \times 8.314 \times 290 \times \ln\frac{80}{5}\,\text{J} \approx 6685\text{J}$$

Work done by the gas during the isothermal expansion is approximately 6685J per mole.

Question 41

(a) Carnot's theorem requires the cycle to be reversible: every infinitesimal step must occur quasi-statically, with no friction, no temperature gradients across heat-exchange surfaces, and no non-equilibrium expansions. Quasi-static means the process is slow enough that the system stays in equilibrium throughout, which implies an infinitely long cycle time and zero power output. Real engines run at finite speed, with finite temperature drops across heat exchangers, friction in moving parts, and finite-rate combustion; all of these introduce irreversibility and reduce the efficiency below Carnot. Practical engines therefore trade some efficiency for finite power output, reaching perhaps 50 to 70% of the Carnot ceiling.

(b) The Carnot ceiling between 150°C (423K) and 27°C (300K) is $\eta_{Carnot} = 1 - 300/423 \approx 29\%$. Two factors drive the realised efficiency below this: (i) finite temperature drops across the heat-exchanger surfaces (the working fluid cannot be at exactly the reservoir temperature; some difference is needed to drive heat flow at the design rate), reducing the effective Carnot ceiling for the working fluid; and (ii) friction and viscous losses in the turbine, pump, and generator, dissipating some of the work output as heat. A realistic geothermal plant operates at 15 to 20% efficiency.

(c)

$$\eta_{Carnot} = 1 - \frac{T_{cold}}{T_{hot}} = 1 - \frac{300\text{K}}{423\text{K}} = 0.291 = 29.1\%$$

$$W_{max} = \eta_{Carnot} \times Q_{hot} = 0.291 \times 1000\text{J} = 291\text{J}$$

Carnot efficiency is approximately 29% for this temperature pair, with at most 291J of work extractable per kilojoule of heat absorbed.

Question 42

(a) The primary mechanism is convection: the warm calorimeter surface heats the air in contact with it, and the warm air rises and is replaced by cooler air. Radiation also contributes, but at the modest excess temperatures of a typical calorimetry experiment (10-30°C above room temperature), convection dominates. The error increases with longer experimental times because the heat loss accumulates: the system loses energy continuously to the surroundings, and the longer it takes for the water and metal to reach thermal equilibrium, the more energy escapes before the final temperature is read.

(b) $COP_{max} = T_{cold}/(T_{warm} - T_{cold}) = 277/(301 - 277) = 277/24 \approx 11.5$. The advertised value 12 slightly exceeds the Carnot ceiling and is therefore physically infeasible. Real refrigerators typically achieve 25 to 50% of the Carnot ceiling, so a realistic COP for these reservoir temperatures is about 3 to 6. The manufacturer's claim of 12 is too good to be true.

(c)

$$COP_{max} = \frac{T_{cold}}{T_{warm} - T_{cold}} = \frac{277\text{K}}{301\text{K} - 277\text{K}} = \frac{277}{24} \approx 11.5$$

$$\frac{Q_{cold}}{dt} = COP_{max} \times P = 11.5 \times 200\text{W} \approx 2308\text{W}$$

Maximum coefficient of performance is approximately 11.5, and the maximum rate of heat extraction is approximately 2300W.

Question 43

(a) After sunset, the iron roof loses heat by two dominant mechanisms. First, radiation: the roof, now at perhaps 50-60°C, radiates infrared energy upward toward the cold night sky (which can be as cold as −20°C in the upper atmosphere). The temperature difference between the roof and the sky is large, driving substantial radiative loss. Second, convection: any breeze passing over the roof carries heat away by forced convection. The roof cools faster than the concrete walls because iron has a much lower thermal mass per unit area than concrete: iron is thin (typically 0.5mm corrugated sheet) and has a relatively low specific heat capacity ($450 Jkg^{-1}K^{-1}$), while concrete walls are thick (150-200mm) and have a large thermal mass. The roof's small thermal reservoir is emptied quickly by the same rate of radiative and convective loss that would take much longer to cool the massive concrete walls.

(b) The gas does no work on the surroundings as it expands, since the adjacent cylinder is evacuated; there is nothing to push back. The thermal insulation prevents heat exchange with the surroundings. By the first law $\Delta U = 0$, and for an ideal gas this gives $\Delta T = 0$. The temperature of the gas in the combined cylinders is therefore essentially the same as the initial temperature, regardless of the speed at which the expansion occurred. (For a real gas, intermolecular forces produce a small temperature change in a free expansion: slight cooling for most gases at room temperature, the Joule-Thomson effect, but this effect vanishes for an ideal gas.)

(c) For an ideal gas, free expansion has $Q = 0$, $W = 0$, hence $\Delta U = 0$ and $\Delta T = 0$:

$$T_f = T_i = 300K$$

Initial volume $V_i = 1.0 \times 10^{-3} m^3$ (half the container); final volume $V_f = 2.0 \times 10^{-3} m^3$. Constant temperature means $PV = \text{const}$:

$$P_f = P_i \times \frac{V_i}{V_f} = 1.0 \times 10^5 Pa \times \frac{1}{2} = 5.0 \times 10^4 Pa$$

$$\Delta U = nC_v \Delta T = nC_v \times 0 = 0$$

Final temperature is 300K, final pressure is $5.0 \times 10^4 Pa$ (half the initial value), and the change in internal energy is zero, all in agreement with the qualitative argument from the first law of thermodynamics.