EXAMINATION QUESTIONS ON ELECTRICITY

Question 1

(a) Account for the fact that the electric field between two large, closely spaced, oppositely charged parallel plates is uniform, whereas the field around an isolated point charge grows weaker with distance from it.
(b) In a capacitor (condenser) microphone one plate is a light flexible diaphragm and the other is fixed, the pair being held at a constant charge. Account for how sound waves arriving at the diaphragm are turned into a varying electrical voltage.
(c) A parallel-plate capacitor has plates of area $150cm^2$ separated by an air gap of 2.0mm and is charged to a potential difference of 200V. Calculate (i) the capacitance, (ii) the charge stored, (iii) the electric field between the plates, and (iv) the energy stored.

Question 2

(a) A charge Q is delivered to a capacitor that finally rests at a potential difference V. Account for the fact that the energy stored is one half of QV, and not QV.
(b) When a negatively charged rod is brought close to the cap of an uncharged gold-leaf electroscope, without touching it, the leaves diverge; they fall again when the rod is taken away. Explain this behaviour.
(c) A 4.0μF capacitor charged to 300V is connected in parallel with an uncharged 2.0μF capacitor. Calculate (i) the common potential difference, (ii) the final charge on each capacitor, and (iii) the energy lost in the process.

Question 3

(a) Account for the choice of infinity, rather than the charge itself, as the place of zero electric potential for a point charge.
(b) Explain why a small capacitor is often connected across a pair of switch contacts, in order to reduce the sparking seen when the switch is opened.
(c) A 50μF capacitor is charged to 100V and then discharged through a 10kΩ resistor. Calculate (i) the time constant, (ii) the initial discharge current, (iii) the charge remaining after one time constant, and (iv) the total heat dissipated in the resistor.

Question 4

(a) Account for the fact that, although the charge on a capacitor is proportional to the voltage across it, the ratio of the two, namely its capacitance, stays the same whatever the voltage is.
(b) Explain why a charged metal sphere loses its charge to the surrounding air much more rapidly on a humid day than on a dry one.
(c) An isolated metal sphere of radius 15cm stands in air, whose dielectric strength is 3.0×10^6V/m. Calculate (i) the capacitance of the sphere, (ii) the maximum potential to which it can be raised before the air breaks down, (iii) the corresponding charge, and (iv) the energy then stored.

Question 5

(a) Account for the fact that at the midpoint of the line joining two equal positive point charges the electric field is zero, yet the electric potential there is not zero.
(b) Explain why sliding a sheet of glass into the air gap of a charged parallel-plate capacitor increases its capacitance.
(c) A parallel-plate capacitor of plate area $200cm^2$ and air gap 1.0mm is charged to 100V and then disconnected from the supply. A slab of mica of dielectric constant 5.0 is then slid in to fill the gap completely. Calculate (i) the capacitance before and after, (ii) the new potential difference, (iii) the field between the plates before and after, and (iv) the change in stored energy.

Question 6

(a) Account for the fact that electric field lines meet the surface of a charged conductor at right angles.

(b) A defibrillator stores energy in a capacitor and then discharges it through the patient. Explain why a capacitor is used for this, rather than connecting the patient straight to the charging supply.
(c) Three capacitors of 2.0μF, 3.0μF and 6.0μF are connected in series across a 90V supply. Calculate (i) the combined capacitance, (ii) the charge on each capacitor, and (iii) the voltage across each.

Question 7

(a) Account for the fact that the force between two point charges held a fixed distance apart is smaller when they are immersed in oil than when they are in air.
(b) Account for why an electrolytic capacitor can be made physically small yet have a very large capacitance compared with an air capacitor of the same size.
(c) Two small identical conducting spheres carry charges of +6.0nC and +2.0nC and are held 4.0cm apart in air. Calculate (i) the force between them; they are then momentarily joined by a fine wire and separated again; calculate (ii) the charge now on each and (iii) the new force between them.

Question 8

(a) Account for the fact that work must be done to pull the plates of a charged, isolated parallel-plate capacitor farther apart, and state what becomes of this work.
(b) Explain why the metal casing of a sensitive electronic instrument shields the circuit inside it from external electric fields.
(c) A parallel-plate capacitor has plates of area $100cm^2$ at a separation of 1.0mm; it is charged to 50V and then disconnected. The plates are now pulled apart to a separation of 3.0mm. Calculate (i) the original capacitance and charge, (ii) the new capacitance, (iii) the new potential difference, and (iv) the work done in separating the plates.

Question 9

(a) Account for the fact that the electric field outside a uniformly charged sphere is the same as if all its charge were concentrated at its centre.
(b) Explain why the capacitor in a camera flash can light the bulb far more brightly, for a brief instant, than the small battery that charged it could on its own.
(c) A 2.0μF capacitor is charged to 200V. Calculate (i) the energy it stores; (ii) if it discharges fully in 2.0ms, the average power delivered; and (iii) state how this compares with the 5.0W that the charging battery can supply.

Question 10

(a) Account for the fact that a capacitor in a d.c. circuit passes a brief current when first connected but no steady current once it is fully charged.
(b) The dome of a Van de Graaff generator charged to several hundred thousand volts gives only a harmless tingle, yet the 240V mains can be lethal. Account for this.
(c) A 2.0μF capacitor is charged through a 1.5MΩ resistor from a 9.0V supply. Calculate (i) the time constant, (ii) the maximum charge, (iii) the charge after 3.0s, and (iv) the charging current at 3.0s.

Question 11

(a) Explain why a charged particle moving through a uniform electric field follows a parabolic path, just as a projectile does under gravity.
(b) Give reasons why every capacitor is marked with a maximum working voltage, and why exceeding it can permanently destroy the capacitor.
(c) An electron enters mid-way between two horizontal parallel plates 4.0cm long and 1.0cm apart, with 200V across them, moving horizontally at 4.0×10^{7}m/s. Calculate (i) the field between the plates, (ii) the vertical acceleration of the electron, (iii) its vertical deflection on leaving the plates, and (iv) the vertical velocity it has gained. (electron charge 1.6×10^{-19}C, mass 9.11×10^{-31}kg.)

Question 12

(a) Account for the fact that the electric field around a long straight charged wire falls off as 1/r with distance, whereas that of a point charge falls off as $1/r^2$.
(b) Explain why a coaxial cable, such as a television aerial lead, has capacitance between its inner and outer conductors, and why this capacitance matters when the cable carries high-frequency signals.
(c) A coaxial cable has an inner conductor of radius 1.0mm and an outer conductor of inner radius 4.0mm, with air between them, and is 50m long. Determine (i) its capacitance, and (ii) the charge stored when 200V is applied between the conductors.

Question 13

(a) Justify the view that the energy of a charged capacitor is stored in the electric field between its plates, rather than on the plates themselves.
(b) In electrostatic paint spraying the paint droplets are charged as they leave the nozzle. Explain how this makes the paint form an even coat on an earthed metal object, even reaching the surfaces that face away from the spray.
(c) A parallel-plate capacitor with plates of area $250cm^2$ separated by 5.0mm of air is charged to 1.0kV. Calculate (i) the capacitance, (ii) the energy stored, and (iii) the energy density in the field, and verify that the energy density multiplied by the volume between the plates equals the stored energy.

Question 14

(a) Account for the fact that capacitors connected in series all carry the same charge, whereas capacitors connected in parallel all have the same voltage across them.
(b) Why is it that a plastic comb which has been run through dry hair can bend a thin stream of water falling from a tap, although the water carries no net charge?
(c) A 3.0μF capacitor is connected in series with a parallel combination of a 2.0μF and a 4.0μF capacitor, across a 60V supply. Find (i) the equivalent capacitance, (ii) the charge drawn from the supply, and (iii) the charge on, and the voltage across, the 2.0μF and 4.0μF capacitors.

Question 15

(a) Explain why a good capacitor dielectric should possess both a high dielectric constant and a high dielectric strength.
(b) Account for the fitting of sharp pointed conductors, called static wicks, on the trailing edges of an aircraft's wings and tail.
(c) Point charges of +2.0μC, +2.0μC and −3.0μC are placed at the corners of an equilateral triangle of side 10cm. Calculate the total electric potential energy of the configuration.

Question 16

(a) Give reasons why the whole of the charge given to a solid metal conductor resides on its outer surface.
(b) Explain why a supercapacitor can store far more charge than an ordinary capacitor of the same size, and why it is increasingly used to deliver short bursts of power.
(c) A 500μF capacitor is charged to 80V and then discharged through a 2.0kΩ resistor. Determine (i) the time constant, (ii) the initial discharge current, (iii) the energy dissipated in the resistor, and (iv) the time for the charge to fall to half its initial value.

Question 17

(a) What does the inverse-square law say about the force between two point charges? Give one experimental observation that supports it.
(b) Explain how the pressing of a key on a capacitive computer keyboard is detected electrically, without any mechanical contact being made.
(c) A small sphere of mass 0.20g carrying a charge of +5.0nC hangs by a light thread between two vertical parallel plates 5.0cm apart with 2.0kV across them. Calculate (i) the field between the plates, (ii) the horizontal force on the sphere, and (iii) the angle the thread makes with the vertical. ($g = 9.8m/s^2$.)

Question 18

(a) Using Gauss's law, show that the electric field just outside a charged conducting surface is σ/ε_0.
(b) Account for the practice of immersing high-voltage capacitors in oil.
(c) The plates of a parallel-plate capacitor each carry a surface charge density of $2.0\mu C/m^2$, have area $100cm^2$ and are 2.0mm apart in air. Determine (i) the field between the plates, (ii) the potential difference, (iii) the capacitance, and (iv) the charge on each plate.

Question 19

(a) Explain why, when a small charged sphere is connected by a wire to a large uncharged one, most of the charge passes to the large sphere.
(b) Give reasons why the terminals of high-voltage equipment are made large and well rounded rather than small and sharp.
(c) A sphere of radius 5.0cm is charged to a potential of 9.0×10^4V and then connected by a long thin wire to a distant uncharged sphere of radius 10cm. Calculate (i) the initial charge on the first sphere, (ii) their common potential after connection, and (iii) the final charge on each sphere.

Question 20

(a) Account for the fact that the energy stored in a capacitor increases fourfold when the voltage across it is doubled.
(b) Explain why clothes made of synthetic fibres cling to the body and crackle when they are taken off in dry weather.
(c) A 10μF capacitor charged to 100V is discharged through a small electric motor that raises a 50g mass. If 60% of the energy stored in the capacitor becomes gravitational potential energy of the mass, calculate the height through which the mass is raised. ($g = 9.8m/s^2$.)

Question 21

(a) Explain why the power dissipated in a resistor can be found from any of the expressions $P = VI$, $P = I^2R$ or $P = V^2/R$, and state which is the most convenient when (i) the current and resistance are known, and (ii) the voltage and resistance are known.
(b) Give reasons why the heating element of an electric kettle delivers the same heat whether it is run on a 12V steady (d.c.) supply or on a 12V (rms) alternating supply.
(c) A lamp marked "60W, 240V" is connected to the 240V, 50Hz mains. Calculate (i) its operating resistance, (ii) the rms current, (iii) the peak current, and (iv) the peak instantaneous power, and state the frequency at which the power delivered to the lamp fluctuates.

Question 22

(a) Account for the fact that the free electrons in a wire carrying alternating current merely oscillate back and forth about fixed positions and never travel from the source to the appliance.
(b) Explain why the connecting wires of a circuit carrying a large current stay cool, while the thin lamp filament carrying the same current becomes white-hot.
(c) A copper wire of cross-sectional area $2.0mm^2$ carries an alternating current of peak value 4.0A at 50Hz. Taking the free-electron density as $8.5\times10^{28}/m^3$, determine (i) the peak drift velocity of the electrons and (ii) the amplitude of their back-and-forth oscillation. ($e = 1.6\times10^{-19}C$.)

Question 23

(a) Explain why the source used to charge a battery must have an electromotive force greater than that of the battery being charged.
(b) Account for the fact that a laptop charger contains not only a transformer but also a rectifier and a smoothing capacitor.
(c) The full-wave rectified output of a transformer provides a steady 14V, which charges a 12V battery of internal resistance 0.30Ω through a 1.2Ω protective resistor. Calculate (i) the charging current, (ii) the potential difference across the battery terminals, and (iii) the rate at which energy is stored chemically in the battery.

Question 24

(a) Account for the fact that a coil of wire offers a much greater opposition to alternating current than to direct current at the same voltage.
(b) Give reasons why an ammeter is made with a very low resistance and connected in series, whereas a voltmeter is made with a very high resistance and connected in parallel.
(c) An iron-cored coil of resistance 30Ω draws a current of 1.0A when connected to a 60V, 50Hz a.c. supply. Determine (i) its impedance, (ii) its inductive reactance, (iii) its inductance, and (iv) the power it dissipates.

Question 25

(a) Give reasons why an inductive load draws a larger current than a purely resistive load that delivers the same useful power at the same voltage.
(b) Explain why thick cables are used for the wiring that carries large currents, such as the supply to an electric cooker, while thin wire suffices for a lamp.
(c) A single-phase motor takes 6.0A at a power factor of 0.75 from the 240V, 50Hz mains, the supply cable having a resistance of 0.40Ω. Calculate (i) the true power, (ii) the apparent power, (iii) the current that a purely resistive load of the same true power would draw, and (iv) the extra power wasted in the cable owing to the motor's lower power factor.

Question 26

(a) Explain why a capacitor passes alternating current but blocks direct current, whereas an inductor passes direct current freely but opposes alternating current.
(b) Account for the use of a tightly coiled tungsten filament, sealed in an inert gas or a vacuum, in an electric lamp.
(c) A 10μF capacitor and a 2.0kΩ resistor are connected in series across a 12V, 50Hz a.c. supply. Find (i) the capacitive reactance, (ii) the impedance, (iii) the rms current, and (iv) the current that would flow if the same 12V were applied as direct current.

Question 27

(a) State, with a reason, why the value marked on a fuse and quoted for the mains supply is the rms value of the current or voltage, and not the peak value.
(b) Explain why running an electric heater for an hour costs far more than running a television for the same time on the same mains.
(c) A 240V (rms), 50Hz socket protected by a 13A fuse supplies an electric heater of resistance 24Ω. Calculate (i) the rms current drawn, (ii) the peak current, (iii) whether the fuse blows, and (iv) the energy used in 2.0 hours, in kilowatt-hours, and its cost at 290 TZS per kWh.

Question 28

(a) Account for the fact that, when resistors are joined in series, the same current passes through each, whereas when they are joined in parallel the same voltage acts across each.
(b) Explain why a transformer hums with a sound whose pitch is twice the frequency of the mains supplying it.
(c) A 100Ω resistor and a 200Ω resistor connected in parallel are placed in series with a coil of inductance 0.30H across a 240V, 50Hz supply. Determine (i) the resistance of the parallel combination, (ii) the inductive reactance of the coil, (iii) the impedance of the circuit, and (iv) the current drawn from the supply.

Question 29

(a) Explain why a wire carrying a current remains electrically neutral, even though it contains a stream of moving electrons.
(b) Account for the fact that a filament lamp run on the mains shows no visible flicker, although the current through it falls to zero a hundred times every second.
(c) An a.c. generator of rms EMF 250V and internal resistance 1.0Ω supplies a heater of resistance 49Ω. Find (i) the rms current, (ii) the terminal voltage, (iii) the power delivered to the heater, and (iv) the power wasted inside the generator.

Question 30

(a) Give two reasons why alternating current is preferred to direct current for the generation and distribution of electrical energy.
(b) Explain what is meant by the kilowatt-hour, and why electrical energy is sold in this unit rather than in joules.
(c) A 240V, 50Hz supply feeds the primary of an ideal step-down transformer of turns ratio 20:1, whose secondary drives a heating coil of resistance 0.60Ω. Calculate (i) the secondary voltage, (ii) the secondary current, (iii) the power delivered to the coil, (iv) the current drawn from the mains, and (v) the energy used in 5.0 hours, in kilowatt-hours.

Question 31

(a) Explain why the current is the same at all points in a simple series circuit, even when the circuit contains components of very different resistance.
(b) Account for the fact that the reading of a hot-wire a.c. ammeter does not change when the frequency of the supply is altered, provided the rms current stays the same.
(c) A 60Ω resistor and a 30Ω resistor are connected in parallel across a 240V, 50Hz supply. Calculate (i) the combined resistance, (ii) the rms current from the supply, (iii) the peak current, and (iv) the total power dissipated.

Question 32

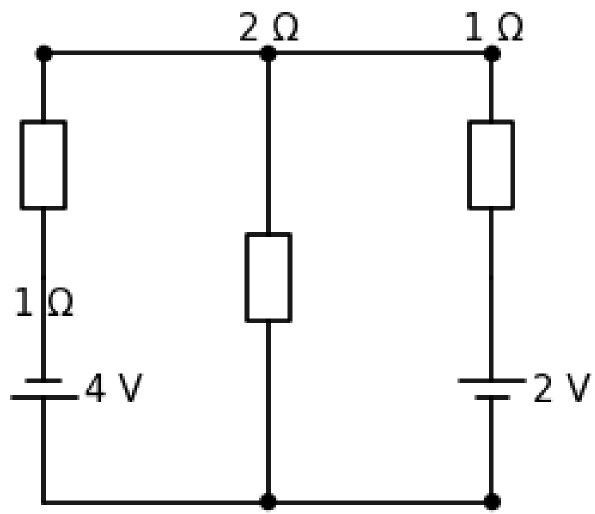
(a) Account for the fact that a pure inductor or a pure capacitor consumes no power on average, whereas a resistor does.
(b) Give reasons why a car battery can deliver a very large current to the starter motor, while a torch battery cannot.
(c) A pure inductor of reactance 60Ω is connected to a 240V, 50Hz supply, first by itself and then in series with an 80Ω resistor. Calculate, for each case, the current drawn and the average power dissipated.

Question 33

(a) Account for the fact that a potentiometer can measure the EMF of a cell accurately, whereas a voltmeter connected across the cell cannot.
(b) Explain why a choke (an iron-cored coil), rather than a resistor, is used to limit the current in a fluorescent-lamp fitting, and the advantage this gives in running cost.
(c) A 40Ω heating element is run from the 240V, 50Hz mains in series with a choke of inductance 0.15H and negligible resistance. Calculate (i) the reactance of the choke, (ii) the impedance, (iii) the current, and (iv) the power now dissipated in the heater, and compare it with the power dissipated without the choke.

Question 34

(a) Explain why, in a series a.c. circuit, the instantaneous voltages across the components always add up to the instantaneous supply voltage, while their rms (or peak) values do not.
(b) Account for the fact that, when two batteries of unequal EMF are connected in parallel, the stronger one may drive a current backwards through the weaker one.
(c) The figure shows two batteries and three resistors connected in two loops: a battery of EMF 4.0V in series with a 1.0Ω resistor forms the left branch, a battery of EMF 2.0V in series with a 1.0Ω resistor forms the right branch, and a 2.0Ω resistor forms the central branch common to both loops. Using Kirchhoff's laws, determine the current in each of the three branches.



Question 35

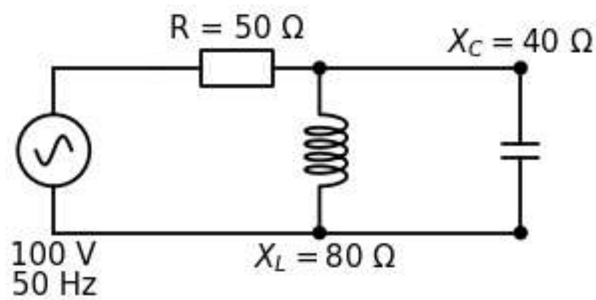
(a) Account for the fact that the primary of a transformer draws only a very small current when nothing is connected to its secondary.
(b) Explain why the cables carrying current to a distant load must be made thick, or the load will receive a reduced voltage.
(c) A transformer supplies 5.0kW at 250V to a feeder cable of resistance 0.20Ω that carries the power to a load. Calculate (i) the current in the feeder, (ii) the voltage drop along it, (iii) the voltage at the load, and (iv) the power wasted in the feeder.

Question 36

(a) Give reasons why a fuse is rated by the current it can carry rather than by the voltage across it.
(b) Explain why an inductor and a capacitor of equal reactance, carrying the same current, have voltages across them that are exactly out of step.
(c) A circuit draws a current of 5.0A at a power factor of 0.80 lagging from the 240V, 50Hz mains. Determine (i) the true power, (ii) the reactive power, (iii) the impedance, and (iv) the resistance and the reactance of the circuit.

Question 37

(a) Explain why the impedance of a circuit that contains both inductance and capacitance depends on the frequency of the supply.
(b) Give reasons why the lights in a house may dim for a moment when a high-power appliance is switched on, if the supply wiring is thin.
(c) In the circuit shown, a 50Ω resistor is connected in series with a parallel combination of a pure inductor of reactance 80Ω and a pure capacitor of reactance 40Ω, across a 100V, 50Hz supply. Determine (i) the combined reactance of the parallel section, (ii) the impedance of the whole circuit, (iii) the current drawn from the supply, and (iv) the voltage across the parallel section.



Question 38

(a) Give reasons why connecting several cells in series increases the EMF available to a circuit.
(b) Explain why the reactance of an inductor is zero for direct current.
(c) A transformer rated 2.0kVA, 240V/24V has a full-load copper loss of 40W and an iron loss of 30W. At full load and a power factor of 0.90, calculate (i) the secondary current, (ii) the output power, (iii) the input power, and (iv) the efficiency.

Question 39

(a) Account for the fact that a step-down transformer can provide a larger current than is drawn from the supply.
(b) Account for the fact that the electrical energy used by a household is measured in kilowatt-hours rather than in joules.
(c) An 80Ω resistor and a coil of inductance 0.30H (of negligible resistance) are connected in series across a 240V, 50Hz supply. Find (i) the inductive reactance, (ii) the impedance, (iii) the current, (iv) the voltage across the resistor and across the coil, and show that these two voltages do not add arithmetically to 240V.

Question 40

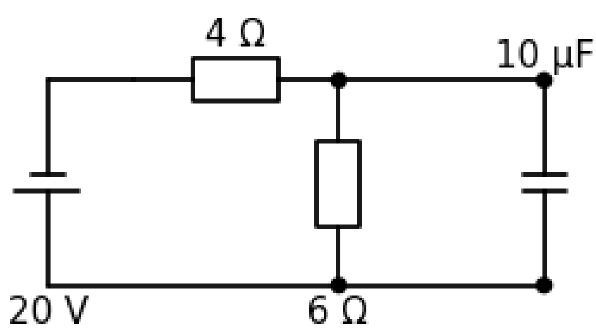
(a) Explain why the value of the unknown resistance found with a balanced Wheatstone bridge does not depend on the EMF of the supply used.
(b) Explain why an a.c. source delivers less average power to an inductive load than to a purely resistive load that draws the same current at the same voltage.
(c) The figure shows a Wheatstone bridge with arms P = 10Ω, Q = 20Ω, R = 15Ω and S = 25Ω, a galvanometer of resistance 30Ω connecting the junctions B and D, and a 6.0V battery of negligible internal resistance across A and C. Determine (i) whether the bridge is balanced, and (ii) the current through the galvanometer.

Question 41

(a) Explain why a capacitor passes a larger alternating current as the frequency of the supply is increased.
(b) Account for the fact that a capacitor intended for use on the 240V mains is marked with a working voltage of 400V or more.
(c) A 2.0μF capacitor is connected across a 240V, 50Hz supply. Calculate (i) its reactance, (ii) the rms current, (iii) the maximum charge on its plates, and (iv) the maximum energy stored in it.

Question 42

(a) Explain why, in the steady state, no current flows through a capacitor in a d.c. circuit, and why the voltage across it equals that across the component it is connected in parallel with.
(b) Account for the fact that a large capacitor connected across a supply helps to keep the voltage steady when the load current changes suddenly.
(c) In the circuit shown, a 4.0Ω resistor and a 6.0Ω resistor are connected in series across a 20V battery of negligible internal resistance, and a 10μF capacitor is connected across the 6.0Ω resistor. In the steady state, determine (i) the current in the resistors, (ii) the voltage across the 6.0Ω resistor, (iii) the charge on the capacitor, and (iv) the energy stored in it.



Question 43

(a) Explain why the electric field along a current-carrying wire is weak and almost uniform, unlike the strong field close to an isolated point charge.
(b) Give reasons why a thick wire carries a larger current than a thin wire of the same length and material when the same voltage is applied across it.
(c) A potential difference of 1.5V is applied across a copper wire 2.0m long and of cross-sectional area 1.0mm^2. (Resistivity of copper = $1.7\times10^{-8}\Omega$ m; free-electron density = $8.5\times10^{28}m^{-3}$.) Calculate (i) the resistance of the wire, (ii) the current, (iii) the electric field along the wire, and (iv) the drift velocity of the electrons.

Question 44

(a) Explain why a charged particle gains kinetic energy as it moves through a potential difference, and state where this energy comes from.
(b) Account for the fact that increasing the accelerating voltage in a cathode-ray tube produces a faster electron beam.
(c) An electron, starting from rest, is accelerated through a potential difference of 4.0kV across a gap 5.0cm wide. Calculate (i) the kinetic energy it gains (in joules and in electron-volts), (ii) its final speed, (iii) the uniform electric field in the gap, and (iv) the force on the electron. ($e = 1.6\times10^{-19}$C, $m_e = 9.11\times10^{-31}$kg.)

Question 45

(a) Explain why the output of a full-wave rectifier, although always in one direction, is not steady and must be smoothed.
(b) Account for the fact that a capacitor of larger capacitance smooths the rectified output more effectively.
(c) A transformer steps the mains down to a peak of 12V, which is full-wave rectified and then smoothed by a 2200μF reservoir capacitor across a 60Ω load. Determine (i) the mean (d.c.) output voltage before smoothing, (ii) the charge on the reservoir capacitor when charged to the peak, (iii) the energy then stored in it, and (iv) the mean load current before smoothing.

Question 46

(a) Account for the fact that a potential difference must be maintained across a conductor for a steady current to flow through it.
(b) Give reasons why the connecting wires of a circuit are usually taken to have no resistance.
(c) A 12V battery of internal resistance 0.50Ω drives a current through an external resistor of 5.5Ω, which is a uniform wire 10cm long. Find (i) the current, (ii) the terminal voltage of the battery, (iii) the electric field along the resistor wire, and (iv) the power dissipated in the resistor.

Question 47

(a) Explain why a charged capacitor can deliver a large current for a brief instant when it is discharged through a low resistance.
(b) Account for the fact that discharging a given amount of stored energy in a shorter time produces a larger average power.
(c) A 50μF capacitor charged to 100V is discharged completely through a small motor in 0.50s. Calculate (i) the charge that flows, (ii) the energy released, (iii) the average current, and (iv) the average power delivered.

Question 48

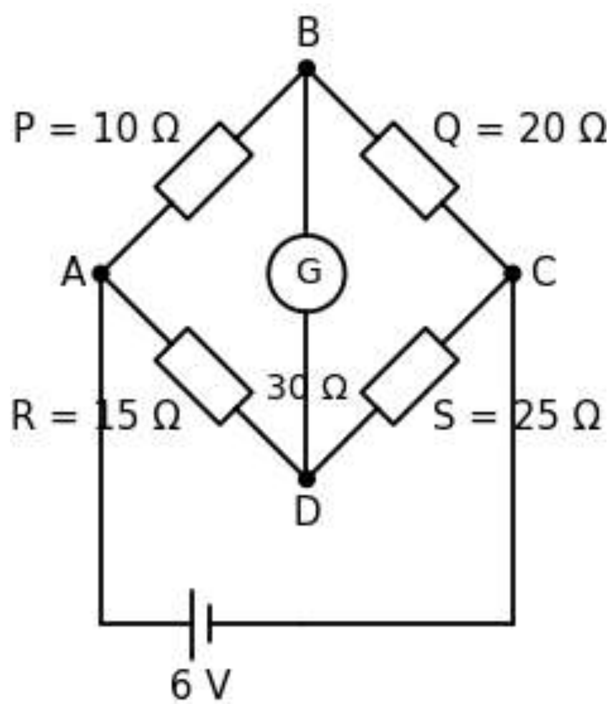
(a) Explain why a series combination of an inductor and a capacitor responds most strongly at one particular frequency.
(b) Account for the fact that, in an oscillating LC circuit, the energy is stored entirely in the capacitor at one instant and entirely in the inductor a quarter of a cycle later.
(c) A coil of inductance 0.40H is connected in series with a 10μF capacitor across a supply of variable frequency. Determine (i) the resonant frequency, and (ii) the reactance of the inductor and of the capacitor at that frequency.

Question 49

(a) Give reasons why connecting two capacitors in series gives a combined capacitance smaller than either of them.
(b) Explain why, once the capacitors in a d.c. circuit are fully charged, the whole of the battery's EMF appears across them.
(c) A 4.0μF capacitor and a 2.0μF capacitor are connected in series across a 30V battery. Find (i) the combined capacitance, (ii) the charge on each capacitor, and (iii) the voltage across each.

Question 50

(a) Account for the fact that a capacitor blocks a steady direct current but allows an alternating current to pass.
(b) Explain why a capacitor draws a current from an a.c. supply yet consumes no power on average.
(c) A 20μF capacitor is connected first to a 100V battery and then to a 100V, 50Hz a.c. supply. Calculate (i) the charge it holds and the steady current it passes on the battery, and (ii) its reactance, the rms current, and the average power on the a.c. supply.



SOLUTIONS

Question 1

(a) The plates carry equal and opposite charges spread evenly over their facing surfaces, so each small patch of one plate has charge directly opposite it on the other; every such pair contributes the same field, and these add to give a field of equal strength and direction at all points between the plates (except near the edges), which is what is meant by a uniform field. An isolated point charge, by contrast, sends its field outward in all directions, and as the distance grows the same field lines spread over an ever larger spherical surface, so the field weakens as the inverse square of the distance.

(b) The diaphragm and the fixed plate together form a capacitor holding a fixed charge. A sound wave moves the diaphragm in and out, changing the plate separation and hence the capacitance. Since the charge Q is kept constant while C varies, the voltage $V = \frac{Q}{C}$ across the plates rises and falls in step with the sound, producing an electrical signal that copies the sound wave.

(c) (i) The capacitance is:

$$C = \frac{\varepsilon_0 A}{d} = \frac{8.85 \times 10^{-12}\,\text{F m}^{-1} \times 0.015\text{m}^2}{2.0 \times 10^{-3}\text{m}} = 66.4\text{pF}$$

(ii) the charge stored is:

$$Q = CV = 66.4 \times 10^{-12}\text{F} \times 200\text{V} = 1.33 \times 10^{-8}\text{C} = 13.3\text{nC}$$

(iii) the field between the plates is:

$$E = \frac{V}{d} = \frac{200\text{V}}{2.0 \times 10^{-3}\text{m}} = 1.0 \times 10^{5}\text{V m}^{-1}$$

(iv) and the energy stored is:

$$U = \frac{1}{2}CV^2 = \frac{1}{2} \times 66.4 \times 10^{-12}\text{F} \times (200\text{V})^2 = 1.33 \times 10^{-6}\text{J}$$

The capacitance is 66.4pF, the charge 13.3nC, the field 1.0×10^{5}V m^{-1}, and the energy stored 1.33μJ.

Question 2

(a) As charge is added to a capacitor the voltage across it rises in proportion, starting from zero when it is empty and reaching V only as the last of the charge arrives. The early charge is therefore moved across a small potential difference and the later charge across a larger one, so the average potential difference during charging is only $\frac{1}{2}$V. The work done, and hence the energy stored, is the charge times this average voltage, $\frac{1}{2}$QV, and not QV.

(b) The negative rod repels the free electrons in the metal cap and stem down into the leaves, so both leaves acquire a negative charge; being like charges they repel and the leaves diverge. When the rod is removed the electrons spread back evenly over the conductor, the leaves lose their excess charge and fall together again. No charge is transferred, because the rod never touches the electroscope.

(c) (i) The charge is conserved when the capacitors are joined, so the common voltage is the total charge divided by the total capacitance:

$$V = \frac{C_1V_1}{C_1 + C_2} = \frac{4.0\ \mu\text{F} \times 300\text{V}}{4.0\ \mu\text{F} + 2.0\ \mu\text{F}} = 200\text{V}$$

(ii) the final charges are then:

$$Q_1 = C_1V = 4.0\ \mu\text{F} \times 200\text{V} = 800\ \mu\text{C}, \qquad Q_2 = C_2V = 2.0\ \mu\text{F} \times 200\text{V} = 400\ \mu\text{C}$$

(iii) and the energy lost is the difference between the initial and final stored energies:

$$\Delta U = 1/2\, C_1V_1^2 - 1/2\,(C_1 + C_2)V^2 = 0.18\text{J} - 0.12\text{J} = 0.06\text{J}$$

The common voltage is 200V, the final charges are 800μC and 400μC, and 0.06J is lost (as heat and radiation in the connecting wires).

Question 3

(a) The potential at a point is the work done in bringing unit positive charge to it from a chosen reference point. For a point charge the field, and so the force on a test charge, dies away to nothing only at an infinite distance, so infinity is the natural place where a charge is free of the influence and is taken to have zero potential. The charge itself cannot be used as the zero, because there the field is infinite and the potential would be unbounded.

(b) When the switch opens, the current it was carrying cannot stop at once, and the voltage across the contacts tends to rise sharply and strike a spark. A capacitor connected across the contacts gives this charge somewhere to go: it absorbs the sudden surge, so the voltage across the gap rises only slowly and stays below the value needed to spark. The contacts are thereby protected from burning away.

(c) (i) The time constant is:

$$\tau = RC = 10 \times 10^{3}\Omega \times 50 \times 10^{-6}\text{F} = 0.50\text{s}$$

(ii) the initial discharge current is:

$$I_0 = \frac{V}{R} = \frac{100\text{V}}{10 \times 10^{3}\Omega} = 10\text{mA}$$

(iii) after one time constant the charge falls to a fraction e^{-1} of the initial charge $Q_0 = CV = 5.0\text{mC}$:

$$Q = Q_0e^{-1} = 5.0\text{mC} \times 0.368 = 1.84\text{mC}$$

(iv) and all the initial stored energy ends up as heat in the resistor:

$$W = \frac{1}{2}CV^2 = \frac{1}{2} \times 50 \times 10^{-6}\text{F} \times (100\text{V})^2 = 0.25\text{J}$$

The time constant is 0.50s, the initial current 10mA, the charge after one time constant 1.84mC, and the heat dissipated 0.25J.

Question 4

(a) Capacitance is defined as the ratio of the charge stored to the voltage produced, $C = \frac{Q}{V}$. If more charge is placed on the plates the voltage rises in exact proportion, so the ratio is unchanged; the capacitance measures only how much charge the capacitor holds per volt, which is fixed by the area of the plates, their separation and the dielectric between them. It is therefore a property of the capacitor's construction, not of how much it happens to be charged.

(b) On a humid day a thin film of water, slightly conducting because of dissolved ions, settles on the sphere and on nearby insulators. This damp film provides a path along which the charge slowly leaks away to earth and to the air. On a dry day the film is absent, the surfaces stay good insulators, and the charge is held for much longer.

(c) (i) The capacitance of an isolated sphere is:

$$C = 4\pi\varepsilon_0 R = 4\pi \times 8.85 \times 10^{-12}\text{F m}^{-1} \times 0.15\text{m} = 16.7\text{pF}$$

(ii) the surface field reaches the breakdown value when $E = \frac{V}{R}$, so the greatest potential is:

$$V_{max} = ER = 3.0 \times 10^6\text{V m}^{-1} \times 0.15\text{m} = 4.5 \times 10^5\text{V}$$

(iii) the corresponding charge and (iv) energy are:

$$Q = CV_{max} = 16.7 \times 10^{-12}\text{F} \times 4.5 \times 10^5\text{V} = 7.5\,\mu\text{C}$$

$$U = \frac{1}{2}CV_{max}^2 = \frac{1}{2} \times 16.7 \times 10^{-12}\text{F} \times (4.5 \times 10^5\text{V})^2 = 1.7\text{J}$$

The sphere has a capacitance of 16.7pF, can be raised to 450kV, holding 7.5μC and storing 1.7J before the air breaks down.

Question 5

(a) Electric field is a vector while electric potential is a scalar. At the midpoint the two equal positive charges push outward in exactly opposite directions with equal strength, so their field vectors cancel and the resultant field is zero. Their potentials, however, are scalars that simply add: each charge raises the potential at the midpoint by the same positive amount, so the total is twice that of one charge and is certainly not zero.

(b) Glass is a dielectric whose molecules become polarised in the field of the plates, lining up so that their induced surface charges partly cancel that field. For a fixed charge this lowers the voltage across the plates, and since $C = \frac{Q}{V}$ a smaller voltage for the same charge means a larger capacitance. The glass therefore raises the capacitance above its air value by the factor of its dielectric constant.

(c) (i) The capacitance in air, and then with the mica, are:

$$C_0 = \frac{\varepsilon_0 A}{d} = \frac{8.85 \times 10^{-12}\text{F m}^{-1} \times 0.020\text{m}^2}{1.0 \times 10^{-3}\text{m}} = 177\text{pF}, \qquad C = \varepsilon_r C_0 = 5.0 \times 177\text{pF} = 885\text{pF}$$

(ii) the charge is fixed at $Q = C_0 V_0 = 17.7\text{nC}$ (the supply is disconnected), so the new voltage is:

$$V = \frac{Q}{C} = \frac{17.7 \times 10^{-9}\text{C}}{885 \times 10^{-12}\text{F}} = 20\text{V}$$

(iii) the field falls from $E_0 = \frac{V_0}{d} = 1.0 \times 10^5\text{V m}^{-1}$ to $E = \frac{V}{d} = 2.0 \times 10^4\text{V m}^{-1}$, and

(iv) the stored energy changes from:

$$U_0 = 1/2\, C_0 V_0^2 = 0.885\,\mu\text{J} \;\text{ to }\; U = 1/2\, CV^2 = 0.177\,\mu\text{J}$$

so the capacitance rises from 177pF to 885pF, the voltage falls to 20V, the field falls fivefold, and the stored energy drops by 0.71μJ (the mica being pulled in does the work).

Question 6

(a) The free electrons in a conductor move until no sideways push remains on them. If the field met the surface at a slant it would have a component along the surface, which would drive the surface electrons sideways and they would keep moving. Equilibrium is reached only when this tangential component has vanished, that is when the field meets the surface exactly at right angles.

(b) A defibrillator must pour a large amount of energy into the heart in a few milliseconds, far faster than the charging supply could deliver it. A capacitor is charged slowly and then discharged all at once, releasing its stored energy $\frac{1}{2}CV^2$ in a single brief, powerful pulse of controlled size. Connecting the patient straight to the supply could neither give so sharp a pulse nor limit the energy so safely.

(c) (i) For capacitors in series the reciprocals add:

$$\frac{1}{C} = \frac{1}{2.0\mu F} + \frac{1}{3.0\mu F} + \frac{1}{6.0\mu F} = 1.0\,\mu F^{-1} \Rightarrow C = 1.0\,\mu F$$

(ii) the same charge sits on each capacitor in series:

$$Q = CV = 1.0\,\mu F \times 90V = 90\,\mu C$$

(iii) and the voltage across each follows from $V = \frac{Q}{C}$:

$$V_2 = \frac{90\,\mu C}{2.0\,\mu F} = 45V, \quad V_3 = \frac{90\,\mu C}{3.0\,\mu F} = 30V, \quad V_6 = \frac{90\,\mu C}{6.0\,\mu F} = 15V$$

The combined capacitance is 1.0μF, each carries 90μC, and the voltages are 45V, 30V and 15V, which add to the 90V supply.

Question 7

(a) The force between two charges is governed by Coulomb's law, $F = \frac{Q_1 Q_2}{4\pi\varepsilon r^2}$, in which the medium enters through its permittivity ε. Oil has a higher permittivity than air, because its molecules polarise and partly screen the two charges from each other. This screening weakens the force, so the same charges the same distance apart push or pull less strongly in oil than in air, smaller by the factor of the oil's relative permittivity.

(b) An electrolytic capacitor uses an extremely thin layer of oxide, formed electrically on a metal foil, as its dielectric. Because the capacitance grows as the plate separation shrinks, this very small spacing yields an enormous capacitance from a modest area of foil, far more than a practical air gap could give. The thinness of the oxide layer is what packs so much capacitance into so small a case.

(c) (i) The initial force follows from Coulomb's law:

$$F_1 = \frac{kQ_1Q_2}{r^2} = \frac{8.99 \times 10^9 \times 6.0 \times 10^{-9} \times 2.0 \times 10^{-9}}{(0.04m)^2} = 6.7 \times 10^{-5}N$$

(ii) joining the two identical spheres lets the total charge share equally between them:

$$q = \frac{Q_1 + Q_2}{2} = \frac{6.0nC + 2.0nC}{2} = 4.0nC$$

(iii) so the new force is:

$$F_2 = \frac{kq^2}{r^2} = \frac{8.99 \times 10^9 \times (4.0 \times 10^{-9})^2}{(0.04m)^2} = 9.0 \times 10^{-5}N$$

The force rises from 67μN to 90μN, because sharing makes the charges equal and a given total charge gives the greatest force when split equally.

Question 8

(a) The plates of a charged capacitor carry opposite charges and so attract each other, and to pull them apart this attraction must be worked against. With the capacitor disconnected the charge Q stays fixed, but a larger separation lowers the capacitance, and since the stored energy is $\frac{Q^2}{2C}$ a smaller C means more stored energy. The work done in separating the plates is exactly this increase in the electrostatic energy held in the field.

(b) The casing is a conductor, and in equilibrium any external field makes its free electrons rearrange until the field they set up cancels the external field within the metal and the space it encloses. The interior is therefore left with essentially no field, whatever fields act outside, so the sensitive circuit within is shielded.

(c) (i) The original capacitance and charge are:

$$C_0 = \frac{\varepsilon_0 A}{d} = \frac{8.85 \times 10^{-12} \times 0.010}{1.0 \times 10^{-3}} = 88.5pF, \qquad Q = C_0V_0 = 4.43nC$$

(ii) tripling the gap thirds the capacitance:

$$C' = \frac{\varepsilon_0 A}{d'} = \frac{8.85 \times 10^{-12} \times 0.010}{3.0 \times 10^{-3}} = 29.5pF$$

(iii) the charge is unchanged, so the voltage rises:

$$V' = \frac{Q}{C'} = \frac{4.43 \times 10^{-9}\text{C}}{29.5 \times 10^{-12}\text{F}} = 150\text{V}$$

(iv) and the work done equals the gain in stored energy:

$$W = 1/2\, C'V'^2 - 1/2\, C_0 V_0^2 = 0.332\ \mu\text{J} - 0.111\ \mu\text{J} = 0.22\ \mu\text{J}$$

The capacitance falls from 88.5pF to 29.5pF, the voltage rises to 150V, and 0.22μJ of work is done in separating the plates.

Question 9

(a) By Gauss's law the field at a distance r from the centre depends only on the charge enclosed within a sphere of that radius and on the symmetry. For a uniformly charged sphere all the charge lies inside any external spherical surface, and the spherical symmetry makes the field the same in every direction, so it is identical to the field of a single point charge of the same total amount placed at the centre. Outside the sphere, then, its size makes no difference to the field.

(b) The battery can supply only a limited current, so it gives up its energy slowly; the capacitor, once charged, can release its stored energy in a tiny fraction of a second. Since power is energy divided by time, the same energy delivered in a far shorter time means a far greater power, so the brief discharge through the bulb is intensely bright. The battery alone, spread over a long time, could never reach that peak brightness.

(c) (i) The energy stored is:

$$U = \frac{1}{2}CV^2 = \frac{1}{2} \times 2.0 \times 10^{-6}\text{F} \times (200\text{V})^2 = 0.040\text{J}$$

(ii) if this is delivered in 2.0ms the average power is:

$$P = \frac{U}{t} = \frac{0.040\text{J}}{2.0 \times 10^{-3}\text{s}} = 20\text{W}$$

(iii) which is four times the 5.0W the battery can supply, which is why the flash is so bright for its brief instant.

Question 10

(a) When first connected the uncharged capacitor offers no opposing voltage, so a large charging current flows and charge piles onto the plates. As the charge grows the capacitor's voltage rises and opposes the supply, so the current falls; once the capacitor's voltage equals the supply voltage there is no net push, the current falls to zero, and no steady current flows thereafter. A capacitor thus passes a brief charging current but blocks a steady direct current.

(b) The dome of a Van de Graaff generator has a very small capacitance, so even at several hundred thousand volts it holds only a tiny charge and stores very little energy; a touch discharges it in one harmless tingle. The mains sits at a much lower voltage but can drive a large current continuously for as long as contact lasts, delivering far more energy to the body. It is the sustained current and energy, not the high voltage alone, that make the mains lethal.

(c) (i) The time constant is:

$$\tau = RC = 1.5 \times 10^6\Omega \times 2.0 \times 10^{-6}\text{F} = 3.0\text{s}$$

(ii) the maximum (final) charge is:

$$Q_{max} = CV = 2.0 \times 10^{-6}\text{F} \times 9.0\text{V} = 18\ \mu\text{C}$$

(iii) after $t = 3.0\text{s} = \tau$ the charge has risen to:

$$Q = Q_{max}(1 - e^{-t/\tau}) = 18\ \mu\text{C} \times (1 - e^{-1}) = 11.4\ \mu\text{C}$$

(iv) and the charging current there is:

$$I = \frac{V}{R}e^{-t/\tau} = \frac{9.0\text{V}}{1.5 \times 10^6\Omega} \times e^{-1} = 2.2\ \mu\text{A}$$

The time constant is 3.0s, the maximum charge 18μC, the charge after 3.0s is 11.4μC, and the current there is 2.2μA.

Question 11

(a) In a uniform field the force on the charged particle is constant in both size and direction, so it gives the particle a constant acceleration at right angles to its original motion while leaving the motion along the original direction unchanged. This is exactly the situation of a projectile, which moves steadily forward while gravity accelerates it steadily downward. The combination of uniform velocity in one direction and uniform acceleration at right angles to it traces out a parabola in both cases.

(b) The dielectric of a capacitor can withstand only a certain field before it breaks down and starts to conduct. If the applied voltage is raised too high, the field across the dielectric exceeds its dielectric strength, a spark punches through it, and the capacitor is permanently ruined. The marked working voltage is set safely below this breakdown value.

(c) (i) The field between the plates is:

$$E = \frac{V}{d} = \frac{200V}{1.0 \times 10^{-2}m} = 2.0 \times 10^{4}V\ m^{-1}$$

(ii) the vertical acceleration is:

$$a = \frac{eE}{m} = \frac{1.6 \times 10^{-19}C \times 2.0 \times 10^{4}V\ m^{-1}}{9.11 \times 10^{-31}kg} = 3.5 \times 10^{15}m\ s^{-2}$$

(iii) the time spent between the plates is $t = \frac{L}{v} = \frac{4.0\times10^{-2}m}{4.0\times10^{7}m\ s^{-1}} = 1.0 \times 10^{-9}s$, so the deflection is:

$$y = 1/2\, at^2 = 1/2 \times 3.5 \times 10^{15}m\ s^{-2} \times (1.0 \times 10^{-9}s)^2 = 1.8 \times 10^{-3}m$$

(iv) and the vertical velocity gained is:

$$v_y = at = 3.5 \times 10^{15}m\ s^{-2} \times 1.0 \times 10^{-9}s = 3.5 \times 10^{6}m\ s^{-1}$$

The field is 2.0×10^4V m^{-1}, the acceleration 3.5×10^{15}m s^{-2}, the deflection 1.8mm, and the vertical velocity 3.5×10^6m s^{-1}.

Question 12

(a) By Gauss's law the field around a long straight charged wire is found from a cylindrical surface whose area, 2πrL, grows in proportion to the radius r. The same enclosed charge is therefore spread over a surface that increases as r, so the field falls as 1/r. For a point charge the surface is a sphere whose area grows as r^2, so its field falls off faster, as $1/r^2$.

(b) The inner and outer conductors of a coaxial cable are separated by an insulator, so they behave as the two plates of a capacitor and store charge whenever a voltage exists between them. This capacitance must be charged and discharged every time the signal changes, which draws current; at high frequencies the changes are so rapid that this current noticeably weakens and distorts the signal, so such cables are designed to keep their capacitance low.

(c) (i) The capacitance of the cable is:

$$C = \frac{2\pi\varepsilon_0 L}{\ln(b/a)} = \frac{2\pi \times 8.85 \times 10^{-12}F\ m^{-1} \times 50m}{\ln(4.0/1.0)} = 2.0nF$$

(ii) and the charge stored at 200V is:

$$Q = CV = 2.0 \times 10^{-9}F \times 200V = 4.0 \times 10^{-7}C = 0.40\ \mu C$$

The cable has a capacitance of 2.0nF and stores 0.40μC at 200V.

Question 13

(a) Although the charge sits on the plates, the stored energy can be located in the space between them where the field exists, because the energy is exactly accounted for by that field. The stronger the field, the more energy it holds per unit volume, the amount being $1/2\,\varepsilon_0 E^2$. It is therefore natural and self-consistent to regard the electric field itself as the seat of the stored energy.

(b) The droplets all carry the same sign of charge, so they repel one another and spread out into a fine, even mist instead of clumping together. The metal object is earthed, so the charged droplets are attracted to it and follow the field lines, which curve round to its far side, carrying paint even onto the hidden surfaces. The result is an even coat with very little waste.

(c) (i) The capacitance is:

$$C = \frac{\varepsilon_0 A}{d} = \frac{8.85 \times 10^{-12} \times 0.025}{5.0 \times 10^{-3}} = 44.3pF$$

(ii) the energy stored is:

$$U = 1/2\, CV^2 = 1/2 \times 44.3 \times 10^{-12}F \times (1.0 \times 10^{3}V)^2 = 22.1\ \mu J$$

(iii) the field is $E = \frac{V}{d} = 2.0 \times 10^{5}V\ m^{-1}$, so the energy density is:

$$u = 1/2\, \varepsilon_0 E^2 = 1/2 \times 8.85 \times 10^{-12} \times (2.0 \times 10^{5}V\ m^{-1})^2 = 0.177J\ m^{-3}$$

and the volume between the plates is $Ad = 0.025m^2 \times 5.0 \times 10^{-3}m = 1.25 \times 10^{-4}m^3$, so:

$$u \times (Ad) = 0.177J\ m^{-3} \times 1.25 \times 10^{-4}m^3 = 22.1\ \mu J$$

which is equal to the stored energy, confirming that the energy resides in the field.

Question 14

(a) In a series arrangement the plates between neighbouring capacitors are joined by a conductor that began neutral, so charge pushed onto one plate draws an equal and opposite charge onto the next; every capacitor in the chain therefore carries the same charge. In a parallel arrangement all the upper plates are joined to one terminal and all the lower plates to the other, so each capacitor has the full supply voltage across it, and they share the same voltage while generally carrying different charges.

(b) A water molecule is polar, having its positive and negative ends slightly apart. The charged comb attracts the opposite end of each molecule a little more strongly than it repels the near end, so the stream as a whole feels a net pull toward the comb and bends visibly toward it, even though the water carries no net charge.

(c) (i) The parallel pair combine, and then add in series with the 3.0μF:

$$C_p = 2.0\,\mu F + 4.0\,\mu F = 6.0\,\mu F, \qquad \frac{1}{C} = \frac{1}{3.0} + \frac{1}{6.0} = \frac{1}{2.0}\,\mu F^{-1} \Rightarrow C = 2.0\,\mu F$$

(ii) the charge drawn from the supply is:

$$Q = CV = 2.0\,\mu F \times 60V = 120\,\mu C$$

(iii) this charge sits on the series 3.0μF and on the parallel block, across which the voltage is $V_p = \frac{120\,\mu C}{6.0\,\mu F} = 20V$, so:

$$Q_2 = C_2 V_p = 2.0\,\mu F \times 20V = 40\,\mu C, \qquad Q_4 = C_4 V_p = 4.0\,\mu F \times 20V = 80\,\mu C$$

The equivalent capacitance is 2.0μF, the supply delivers 120μC, and the 2.0μF and 4.0μF capacitors carry 40μC and 80μC at 20V.

Question 15

(a) A high dielectric constant is wanted because it multiplies the capacitance, so the capacitor holds more charge and stores more energy at a given voltage. A high dielectric strength is wanted because it sets the greatest field, and hence the greatest voltage, the capacitor can take before the dielectric breaks down. A good dielectric needs both: a high constant so that it stores much, and a high strength so that it can be worked at a high voltage without failing.

(b) In flight an aircraft picks up static charge from friction with the air, which would build up to a high voltage and discharge noisily through the radio aerials, interfering with communications. At the sharp tips of the static wicks the field becomes very intense, so the charge leaks quietly away into the air at the points before it can accumulate, keeping the aircraft near the potential of its surroundings.

(c) The potential energy is the sum over the three pairs, $U = \frac{k}{r}(q_1q_2 + q_1q_3 + q_2q_3)$, with $r = 0.10$m and the products, in units of $10^{-12}C^2$, equal to $(2)(2) + (2)(-3) + (2)(-3) = -8$:

$$U = \frac{8.99 \times 10^9 N\,m^2C^{-2} \times (-8.0 \times 10^{-12}C^2)}{0.10m} = -0.72J$$

The total electric potential energy is −0.72J, the negative sign showing that the configuration is bound (net attractive).

Question 16

(a) The excess charges on a conductor are free to move and they all repel one another, so they spread as far apart as they can, which means onto the outer surface. Any charge remaining within the body or on an inner surface would feel a net outward push from the others and would move outward too, so in equilibrium none is left inside; the whole of the charge resides on the outer surface.

(b) A supercapacitor uses electrodes of enormous effective surface area together with an extremely thin charge-separation layer, and since the capacitance grows both with area and with thinness of the gap, this packs a very large capacitance into a small volume. It can therefore store a large charge and release it rapidly, which suits it to delivering short bursts of power and to bridging brief interruptions where a battery would respond too slowly.

(c) (i) The time constant is:

$$\tau = RC = 2.0 \times 10^3\Omega \times 500 \times 10^{-6}F = 1.0s$$

(ii) the initial discharge current is:

$$I_0 = \frac{V}{R} = \frac{80V}{2.0 \times 10^3\Omega} = 40mA$$

(iii) all the stored energy ends up as heat in the resistor:

$$W = 1/2\,CV^2 = 1/2 \times 500 \times 10^{-6}F \times (80V)^2 = 1.6J$$

(iv) and the time for the charge to halve is:

$$t_{1/2} = 0.693\,\tau = 0.693 \times 1.0\text{s} = 0.69\text{s}$$

The time constant is 1.0s, the initial current 40mA, the energy dissipated 1.6J, and the half-time 0.69s.

Question 17

(a) The law says: The electrostatic force between two point charges falls off as the inverse square of their separation, so that doubling the distance reduces the force to a quarter. Strong evidence for this is that the field, and hence the force, is exactly zero everywhere inside a hollow charged conductor; such complete cancellation occurs only if the law of force is precisely inverse-square, so this null result is a sensitive test that confirms it.

(b) A key on a capacitive keyboard sits above a small capacitor; pressing the key brings one conducting layer closer to another, or brings a conducting finger near, which changes the plate separation and so the capacitance. The electronics continually sense each key's capacitance and register a keystroke the moment it changes, so no mechanical contact is needed.

(c) (i) The field between the plates is:

$$E = \frac{V}{d} = \frac{2.0 \times 10^{3}\text{V}}{5.0 \times 10^{-2}\text{m}} = 4.0 \times 10^{4}\text{V m}^{-1}$$

(ii) the horizontal force on the sphere is:

$$F = qE = 5.0 \times 10^{-9}\text{C} \times 4.0 \times 10^{4}\text{V m}^{-1} = 2.0 \times 10^{-4}\text{N}$$

(iii) and the thread settles at the angle for which this force balances against the weight:

$$\tan\theta = \frac{F}{mg} = \frac{2.0 \times 10^{-4}\text{N}}{2.0 \times 10^{-4}\text{kg} \times 9.8\text{m s}^{-2}} = 0.102 \;\Rightarrow\; \theta = 5.8°$$

The field is 4.0×10^{4}V m^{-1}, the force 2.0×10^{-4}N, and the thread hangs at 5.8° to the vertical.

Question 18

(a) Take a small Gaussian pillbox with one flat face just inside the conductor, where the field is zero, and the other just outside the surface. All the flux therefore passes out through the outer face alone, while the charge enclosed is the surface density σ times the area of that face. Gauss's law, flux equals enclosed charge divided by ε_0, then gives $E \times \text{area} = \frac{\sigma \times \text{area}}{\varepsilon_0}$, so the field just outside the surface is $E = \frac{\sigma}{\varepsilon_0}$.

(b) A high-voltage capacitor is immersed in oil because oil is a dielectric of much greater dielectric strength than air and can withstand the intense field between the plates without breaking down. The oil also seeps into any tiny air gaps, where breakdown would otherwise begin, and carries heat away. The capacitor can therefore be worked safely at a far higher voltage than it could in air.

(c) (i) The field between the plates is:

$$E = \frac{\sigma}{\varepsilon_0} = \frac{2.0 \times 10^{-6}\text{C m}^{-2}}{8.85 \times 10^{-12}\text{F m}^{-1}} = 2.26 \times 10^{5}\text{V m}^{-1}$$

(ii) the potential difference is:

$$V = Ed = 2.26 \times 10^{5}\text{V m}^{-1} \times 2.0 \times 10^{-3}\text{m} = 452\text{V}$$

(iii) the capacitance is:

$$C = \frac{\varepsilon_0 A}{d} = \frac{8.85 \times 10^{-12} \times 0.010}{2.0 \times 10^{-3}} = 44.3\text{pF}$$

(iv) and the charge on each plate is:

$$Q = \sigma A = 2.0 \times 10^{-6}\text{C m}^{-2} \times 0.010\text{m}^{2} = 2.0 \times 10^{-8}\text{C} = 20\text{nC}$$

The field is 2.26×10^{5}V m^{-1}, the voltage 452V, the capacitance 44.3pF, and the charge 20nC.

Question 19

(a) The charge a sphere holds at a given potential is proportional to its capacitance, which is itself proportional to its radius. When the two spheres are joined by a wire they must come to the same potential, and since the larger sphere has the greater capacitance it takes the greater share of the charge at that common potential. Most of the charge therefore flows onto the large sphere.

(b) At a sharp point or small radius the surface charge crowds together and the field becomes very intense, easily reaching the value at which the surrounding air breaks down and the charge leaks away. By making high-voltage terminals large and well rounded, the surface charge is spread out and the field is kept low, so the air does not break down and the charge, and hence the high voltage, is held without leakage or corona.

(c) (i) The capacitance of the first sphere and its initial charge are:

$$C_1 = 4\pi\varepsilon_0 R_1 = 4\pi \times 8.85 \times 10^{-12} \times 0.05 = 5.56\text{pF}, \qquad Q = C_1 V_1 = 0.50\,\mu\text{C}$$

(ii) on connection the charge is shared at a common potential set by the combined capacitance $4\pi\varepsilon_0(R_1 + R_2) = 16.7\text{pF}$:

$$V = \frac{Q}{4\pi\varepsilon_0(R_1 + R_2)} = \frac{0.50 \times 10^{-6}\text{C}}{16.7 \times 10^{-12}\text{F}} = 3.0 \times 10^4\text{V}$$

(iii) and the final charges are in proportion to the radii:

$$Q_1 = C_1 V = 0.167\,\mu\text{C}, \qquad Q_2 = 4\pi\varepsilon_0 R_2 V = 0.334\,\mu\text{C}$$

The first sphere starts with 0.50μC; after connection the common potential is 3.0×10^4V and the charges are 0.167μC and 0.334μC, twice as much on the sphere of twice the radius.

Question 20

(a) The energy stored in a capacitor is $1/2\,CV^2$, which depends on the square of the voltage. Doubling the voltage therefore multiplies the stored energy by $2^2 = 4$, and not merely by two, because raising the voltage both doubles the charge and doubles the potential difference through which it is held. The stored energy thus increases fourfold.

(b) In dry weather, pulling synthetic clothing over the body rubs unlike materials together and transfers charge, leaving the garment and the skin oppositely charged. The opposite charges attract, so the cloth clings to the body, and where the charge jumps the small gaps it makes tiny sparks that are heard as crackles. In humid weather the charge leaks away through the moist air, so the clinging and crackling disappear.

(c) (i) The energy stored in the capacitor is:

$$U = 1/2\,CV^2 = 1/2 \times 10 \times 10^{-6}\text{F} \times (100\text{V})^2 = 0.050\text{J}$$

(ii) of which 60% is converted to gravitational potential energy:

$$E_{\text{useful}} = 0.60 \times 0.050\text{J} = 0.030\text{J}$$

(iii) so the mass is raised through:

$$h = \frac{E_{\text{useful}}}{mg} = \frac{0.030\text{J}}{0.050\text{kg} \times 9.8\text{m s}^{-2}} = 0.061\text{m} = 6.1\text{cm}$$

The capacitor stores 0.050J, delivers 0.030J usefully, and raises the mass through about 6.1cm.

Question 21

(a) The power dissipated in a resistor is the product of the voltage across it and the current through it, $P = VI$. Because the resistor obeys Ohm's law, $V = IR$, this can be rewritten in two further forms: putting $V = IR$ gives $P = I^2R$, while putting $I = V/R$ gives $P = V^2/R$. All three are equivalent; $P = I^2R$ is the most convenient when the current and resistance are known, and $P = V^2/R$ when the voltage and resistance are known.

(b) The heat produced in a resistor depends on the square of the current, and the rms value of an alternating current is defined as exactly the steady direct current that gives the same average heating. A 12V (rms) supply therefore drives the same average heating power through the element as a 12V steady supply, so the kettle boils in the same time on either. The element responds only to the mean-square current, so it cannot tell the two apart.

(c) (i) The operating resistance is:

$$R = \frac{V^2}{P} = \frac{(240\text{V})^2}{60\text{W}} = 960\Omega$$

(ii) the rms current and (iii) the peak current are:

$$I_{\text{rms}} = \frac{P}{V} = \frac{60\text{W}}{240\text{V}} = 0.25\text{A}, \qquad I_0 = I_{\text{rms}}\sqrt{2} = 0.25\text{A} \times \sqrt{2} = 0.35\text{A}$$

(iv) the peak instantaneous power is twice the average power:

$$P_{\text{peak}} = V_0 I_0 = 2P_{\text{av}} = 2 \times 60\text{W} = 120\text{W}$$

The resistance is 960Ω, the rms current 0.25A, the peak current 0.35A, and the peak power 120W; the power delivered fluctuates at twice the mains frequency, that is at 100Hz.

Question 22

(a) In an alternating current the driving voltage reverses many times a second, so the force on the free electrons reverses with it. The electrons are therefore pushed first one way and then the other, oscillating back and forth about fixed average positions instead of advancing steadily along the wire. Over a complete cycle their net displacement is zero, so no electron ever travels from the source to the appliance.

(b) The connecting wires are thick and of low-resistance copper, so although they carry the full current their resistance is small and they dissipate little heat, since the heating is I^2R. The lamp filament is a thin wire of high resistance carrying the same current, so far more power is dissipated in it; this concentrated heating drives the filament to white heat while the leads stay cool.

(c) (i) The peak drift velocity is:

$$v_0 = \frac{I_0}{neA} = \frac{4.0\text{A}}{8.5\times10^{28}\text{m}^{-3}\times1.6\times10^{-19}\text{C}\times2.0\times10^{-6}\text{m}^2} = 1.5\times10^{-4}\text{m s}^{-1}$$

(ii) since the velocity varies as $v_0\sin\omega t$, the displacement oscillates with amplitude:

$$x_0 = \frac{v_0}{\omega} = \frac{1.5\times10^{-4}\text{m s}^{-1}}{2\pi\times50\text{Hz}} = 4.7\times10^{-7}\text{m}$$

The peak drift velocity is 1.5×10^{-4}m s^{-1}, and the electrons oscillate to and fro with an amplitude of only about 4.7×10^{-7}m.

Question 23

(a) To charge a battery, current must be driven backwards through it, into its positive terminal, against its own EMF. This is possible only if the charging source has the greater EMF, so that the net driving voltage, the difference of the two EMFs, is positive and can push the charging current through the combined internal resistances. If the source EMF were equal to or less than the battery's, no charging current would flow.

(b) The mains supplies alternating current at a high voltage, but a laptop needs a low, steady direct voltage. The transformer first steps the voltage down to the value required; the rectifier then turns the alternating current into a one-way (direct) current; and the smoothing capacitor finally levels out the pulses of the rectified output into a nearly steady direct voltage. All three stages are needed to convert the mains into usable d.c.

(c) (i) The charging current is set by the excess of the supply EMF over the battery EMF, divided by the total resistance:

$$I = \frac{V_{dc} - E}{R + r} = \frac{14\text{V} - 12\text{V}}{1.2\Omega + 0.30\Omega} = 1.33\text{A}$$

(ii) the terminal voltage of the battery while charging is:

$$V = E + Ir = 12\text{V} + 1.33\text{A}\times0.30\Omega = 12.4\text{V}$$

(iii) and the rate at which energy is stored chemically is:

$$P_{chem} = EI = 12\text{V}\times1.33\text{A} = 16.0\text{W}$$

The charging current is 1.33A, the battery terminal voltage 12.4V, and chemical energy is stored at 16.0W.

Question 24

(a) To a direct current a coil presents only the resistance of its wire, because once the current is steady the coil's magnetic field is unchanging and induces no back-EMF. To an alternating current, however, the ever-changing current sets up a continually changing flux that induces a back-EMF opposing the change; this appears as inductive reactance. The total opposition, the impedance $Z = \sqrt{R^2 + X_L^2}$, is therefore much greater than the resistance alone.

(b) An ammeter is connected in series, so the whole current passes through it; it must therefore have a very low resistance, so that inserting it scarcely alters the current it is meant to measure. A voltmeter is connected in parallel across a component; it must have a very high resistance, so that it draws almost no current and does not disturb the voltage it is meant to read.

(c) (i) The impedance is:

$$Z = \frac{V}{I} = \frac{60\text{V}}{1.0\text{A}} = 60\Omega$$

(ii) the inductive reactance and (iii) the inductance follow:

$$X_L = \sqrt{Z^2 - R^2} = \sqrt{(60\Omega)^2 - (30\Omega)^2} = 52.0\Omega, \qquad L = \frac{X_L}{2\pi f} = \frac{52.0\Omega}{2\pi \times 50\text{Hz}} = 0.165\text{H}$$

(iv) and the power dissipated (only in the resistance) is:

$$P = I^2R = (1.0\text{A})^2 \times 30\Omega = 30\text{W}$$

The impedance is 60Ω, the reactance 52.0Ω, the inductance 0.165H, and the power dissipated 30W.

Question 25

(a) The useful (true) power of an a.c. load is VI cos φ, so the current it needs is I = P/(V cos φ). An inductive load has a power factor cos φ less than one, so for the same true power and voltage it must draw a larger current than a purely resistive load, whose power factor is one. The extra current carries the reactive power that surges back and forth without doing useful work.

(b) The heat generated in a cable is I^2R, so a cable carrying a large current would overheat unless its resistance were kept low. Since resistance falls as the cross-sectional area rises (R = ρL/A), thick cables are used for large currents to keep the resistance, and so the heating and the voltage drop, small; a thin wire is adequate for a lamp, which draws only a small current.

(c) (i) The true power and (ii) the apparent power are:

$$P = VI\cos\phi = 240\text{V} \times 6.0\text{A} \times 0.75 = 1080\text{W}, \qquad S = VI = 240\text{V} \times 6.0\text{A} = 1440\text{VA}$$

(iii) a purely resistive load of the same true power would draw:

$$I_R = \frac{P}{V} = \frac{1080\text{W}}{240\text{V}} = 4.5\text{A}$$

(iv) so the extra power wasted in the cable because of the larger current is:

$$\Delta P = (I^2 - I_R^2)R = (6.0^2 - 4.5^2)\text{A}^2 \times 0.40\Omega = 6.3\text{W}$$

The true power is 1080W, the apparent power 1440VA, the resistive load would draw only 4.5A, and the low power factor wastes an extra 6.3W in the cable.

Question 26

(a) A capacitor passes alternating current because its plates are charged and discharged repeatedly as the voltage alternates, so a current flows in the wires at every instant; but it blocks direct current, because once charged to a steady voltage no further charge moves. An inductor passes direct current freely, since a steady current meets only the small resistance of its wire; but it opposes alternating current, because the changing current induces a back-EMF (inductive reactance) that grows with frequency.

(b) Tungsten is used because it has a very high melting point and can be raised to white heat without melting. The filament is tightly coiled so that a long length, and hence a high resistance, is fitted into a small space, concentrating the heat. It is sealed in an inert gas or a vacuum so that the hot metal does not burn away by oxidising in the air.

(c) (i) The capacitive reactance is:

$$X_C = \frac{1}{2\pi fC} = \frac{1}{2\pi \times 50\text{Hz} \times 10 \times 10^{-6}\text{F}} = 318\Omega$$

(ii) the impedance and (iii) the rms current are:

$$Z = \sqrt{R^2 + X_C^2} = \sqrt{(2000\Omega)^2 + (318\Omega)^2} = 2.03 \times 10^3\Omega, \qquad I_{rms} = \frac{V}{Z} = \frac{12\text{V}}{2.03 \times 10^3\Omega} = 5.9\text{mA}$$

(iv) on direct current the capacitor blocks the steady current once charged, so:

$$I_{dc} = 0$$

The reactance is 318Ω, the impedance 2.03kΩ, the rms current 5.9mA, and the direct current is zero.

Question 27

(a) The rms value is used because it fixes the average heating effect, on which the safe carrying of current and the rating of equipment depend; the peak value, reached only for an instant, would overstate the steady effect by a factor √2. Marking the mains as 240V and a fuse as 13A in rms terms therefore states the values that govern the real power and heating, which is what matters for safety and design.

(b) The cost of running an appliance is its power multiplied by the time for which it runs, charged at a fixed rate per kilowatt-hour. A heater converts electrical energy into heat at a high rate, often two or three kilowatts, while a television uses only a small fraction of this. For the same running time the heater therefore uses far more energy and costs far more, even on the same mains.

(c) (i) The rms current and (ii) the peak current are:

$$I_{rms} = \frac{V}{R} = \frac{240V}{24\Omega} = 10A, \qquad I_0 = I_{rms}\sqrt{2} = 10A \times \sqrt{2} = 14.1A$$

(iii) the fuse responds to the rms (heating) current of 10A, which is below its 13A rating, so it does not blow, even though the current momentarily peaks at 14.1A. (iv) The power and the energy used are:

$$P = VI_{rms} = 240V \times 10A = 2400W = 2.4kW$$

$$\text{Energy} = Pt = 2.4kW \times 2.0h = 4.8kWh, \qquad \text{Cost} = 4.8kWh \times 290\ \text{TZS kWh}^{-1} = 1392\ \text{TZS}$$

The rms current is 10A and the peak 14.1A; the fuse holds, and the heater uses 4.8kWh, costing 1392 TZS.

Question 28

(a) When resistors are joined in series there is only one path, so the same current must pass through each in turn, while the supply voltage divides among them in proportion to their resistances. When they are joined in parallel each is connected directly across the same two supply terminals, so each has the full supply voltage across it, while the total current divides among the branches. Hence series elements share the current and parallel elements share the voltage.

(b) The iron core is magnetised first one way and then the other as the alternating current reverses, and a magnetised iron bar changes its length very slightly (magnetostriction). The core therefore expands and contracts twice in every cycle of the mains, setting the air around it vibrating; because two such movements occur per cycle, the hum is at twice the mains frequency, that is 100Hz on a 50Hz supply.

(c) (i) The parallel resistance is:

$$R_p = \frac{R_1R_2}{R_1 + R_2} = \frac{100\Omega \times 200\Omega}{100\Omega + 200\Omega} = 66.7\Omega$$

(ii) the inductive reactance and (iii) the impedance are:

$$X_L = 2\pi fL = 2\pi \times 50Hz \times 0.30H = 94.2\Omega, \qquad Z = \sqrt{R_p^2 + X_L^2} = \sqrt{(66.7\Omega)^2 + (94.2\Omega)^2} = 115\Omega$$

(iv) and the current drawn from the supply is:

$$I = \frac{V}{Z} = \frac{240V}{115\Omega} = 2.1A$$

The parallel resistance is 66.7Ω, the reactance 94.2Ω, the impedance 115Ω, and the supply current 2.1A.

Question 29

(a) A current-carrying wire contains equal amounts of positive charge (the fixed lattice ions) and negative charge (the free electrons), so the two cancel and the wire is electrically neutral. A current is merely the drift of the electrons through this stationary positive background; as one electron moves on, another enters to take its place, so the total charge in any region stays zero. The movement of charge does not mean any accumulation of charge.

(b) A filament lamp receives its power in pulses, the current falling to zero a hundred times a second, but the filament has thermal inertia and cannot cool appreciably in the hundredth of a second between pulses, so it glows at a nearly steady temperature. The eye, too, cannot follow such rapid changes (persistence of vision), so the lamp appears to shine steadily and no flicker is seen.

(c) (i) The rms current is:

$$I = \frac{E}{R + r} = \frac{250V}{49\Omega + 1.0\Omega} = 5.0A$$

(ii) the terminal voltage and (iii) the power delivered to the heater are:

$$V = IR = 5.0A \times 49\Omega = 245V, \qquad P_R = I^2R = (5.0A)^2 \times 49\Omega = 1225W$$

(iv) and the power wasted inside the generator is:

$$P_r = I^2r = (5.0A)^2 \times 1.0\Omega = 25W$$

The current is 5.0A, the terminal voltage 245V, the heater receives 1225W, and 25W is wasted inside the generator.

Question 30

(a) First, an alternating voltage can be stepped up and down efficiently by a transformer, so it can be sent at very high voltage (and therefore low current and small I^2R loss) and then reduced to a safe value for the consumer; a direct voltage cannot be changed so simply. Second, alternating current is generated directly and cheaply by rotating-coil generators. These advantages make a.c. far more convenient for large-scale generation and distribution.

(b) The kilowatt-hour is the unit in which electrical energy is sold; one kilowatt-hour is the energy used by an appliance of power one kilowatt running for one hour. It is preferred to the joule because the joule is inconveniently small for household amounts of energy, whereas one kilowatt-hour, equal to 3.6×10^6J, is a practical size that gives manageable numbers on a bill.

(c) (i) The secondary voltage and (ii) the secondary current are:

$$V_s = \frac{V_p}{20} = \frac{240V}{20} = 12V, \qquad I_s = \frac{V_s}{R} = \frac{12V}{0.60\Omega} = 20A$$

(iii) the power delivered to the coil and (iv) the primary current are:

$$P = V_s I_s = 12V \times 20A = 240W, \qquad I_p = \frac{P}{V_p} = \frac{240W}{240V} = 1.0A$$

(v) and the energy used in 5.0 hours is:

$$\text{Energy} = Pt = 0.240kW \times 5.0h = 1.2kWh$$

The secondary gives 12V at 20A, delivering 240W to the coil; the mains supplies 1.0A, and 1.2kWh is used in 5.0 hours.

Question 31

(a) In a single series loop there is only one path for charge, and since charge cannot pile up at any point, the same number of coulombs passes every cross-section each second. The current is therefore the same at all points, however the resistances differ; it is the voltage that divides among the components, not the current.

(b) A hot-wire ammeter responds to the heating produced by the current, which depends only on the mean square of the current and not on how rapidly it alternates. Provided the rms value is unchanged, the heating, and hence the reading, stays the same whatever the frequency, so the meter reads alike at 50Hz and at any other frequency.

(c) (i) The combined resistance is:

$$R_p = \frac{R_1 R_2}{R_1 + R_2} = \frac{60\Omega \times 30\Omega}{60\Omega + 30\Omega} = 20\Omega$$

(ii) the rms current and (iii) the peak current are:

$$I_{rms} = \frac{V}{R_p} = \frac{240V}{20\Omega} = 12A, \qquad I_0 = I_{rms}\sqrt{2} = 17.0A$$

(iv) and the total power dissipated is:

$$P = \frac{V^2}{R_p} = \frac{(240V)^2}{20\Omega} = 2880W$$

The combined resistance is 20Ω, the rms current 12A, the peak current 17.0A, and the power 2880W.

Question 32

(a) In a pure inductor or capacitor the current is a quarter-cycle out of step with the voltage, so during each cycle the component takes energy from the supply for one quarter and returns all of it during the next; the average power is therefore zero. A resistor has its current in step with its voltage, so it takes energy at every instant and turns it irreversibly into heat, giving a positive average power.

(b) A car battery is built with very low internal resistance, using large plates of big area set close together, so that even a modest terminal voltage can drive the hundreds of amperes the starter needs, since I = E/(R + r) is large when r is tiny. A torch battery has a high internal resistance, so it can supply only a small current; demanding a large current would collapse its terminal voltage and exhaust it quickly.

(c) By itself the pure inductor draws:

$$I = \frac{V}{X_L} = \frac{240V}{60\Omega} = 4.0A, \qquad P = 0 \ \ \text{(wattless)}$$

In series with the 80Ω resistor the impedance, current and power become:

$$Z = \sqrt{R^2 + X_L^2} = \sqrt{(80\Omega)^2 + (60\Omega)^2} = 100\Omega, \qquad I = \frac{240V}{100\Omega} = 2.4A$$

$$P = I^2R = (2.4A)^2 \times 80\Omega = 461W$$

Alone the inductor passes 4.0A but dissipates no power; with the resistor the current falls to 2.4A and 461W is dissipated, all of it in the resistor.

Question 33

(a) A potentiometer balances the cell's EMF against a known potential drop along a wire, the balance being found when the galvanometer shows no deflection. At balance no current is drawn from the cell, so there is no internal 'lost volts' (Ir), and the reading gives the true EMF. A voltmeter must draw some current to deflect, so it always reads the terminal voltage, which is a little less than the EMF.

(b) A choke limits the current by its inductive reactance, which opposes the alternating current without dissipating energy, since a pure inductor takes no average power. A resistor used instead would waste much energy as heat (I^2R). The choke therefore controls the current while keeping the running cost low, which is why fluorescent fittings use one.

(c) (i) The reactance of the choke is:

$$X_L = 2\pi fL = 2\pi \times 50Hz \times 0.15H = 47.1\Omega$$

(ii) the impedance and (iii) the current are:

$$Z = \sqrt{R^2 + X_L^2} = \sqrt{(40\Omega)^2 + (47.1\Omega)^2} = 61.8\Omega, \qquad I = \frac{240V}{61.8\Omega} = 3.88A$$

(iv) the power now dissipated in the heater is:

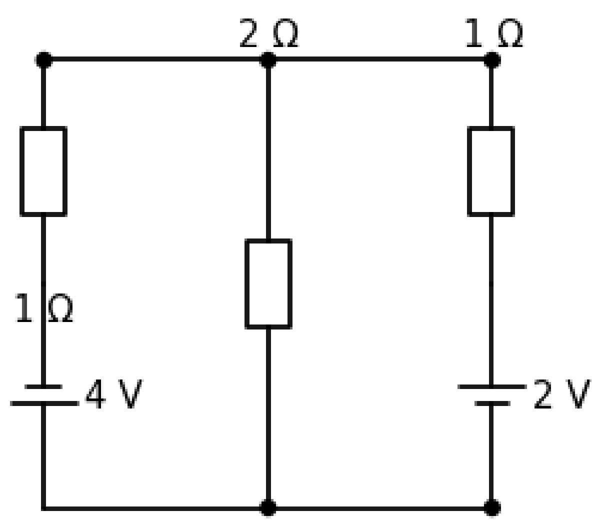
$$P = I^2R = (3.88A)^2 \times 40\Omega = 602W$$

Without the choke the current would be 240V/40Ω = 6.0A and the power 1440W, so the choke cuts the power from 1440W to about 602W while itself wasting almost none.

Question 34

(a) In a series a.c. circuit the instantaneous voltages are the actual voltages present at the same moment, and at every instant they must add up to the instantaneous supply voltage, since Kirchhoff's voltage law holds at each instant. Their rms or peak values, however, are reached at different instants because the components are out of phase, so these maxima never occur together and cannot simply be added; they must be combined as phasors instead.

(b) When two batteries of unequal EMF are joined in parallel, the one with the higher EMF tends to drive current round the loop, and if its EMF exceeds the other's sufficiently it forces current backwards through the weaker battery, charging it. Solving the network by Kirchhoff's laws then gives a negative value for the current in the weaker branch, showing that it flows opposite to the assumed direction.



(c) Let I_1 and I_2 be the currents from the 4.0V and 2.0V branches and I_3 the current in the central 2.0Ω resistor. Kirchhoff's current law at the top node gives $I_3 = I_1 + I_2$. Applying the voltage law to the left and right loops:

$$4.0V = (1.0\Omega)I_1 + (2.0\Omega)I_3, \qquad 2.0V = (1.0\Omega)I_2 + (2.0\Omega)I_3$$

Solving these three equations simultaneously:

$$I_1 = 1.6A, \qquad I_2 = -0.4A, \qquad I_3 = 1.2A$$

The 4.0V battery supplies 1.6A and the central resistor carries 1.2A; the negative sign for I_2 shows that 0.4A actually flows backwards through the 2.0V branch, which is being charged by the stronger battery.

Question 35

(a) On open circuit the secondary delivers no current, so the primary need draw only the small magnetising current that sets up the alternating flux in the core. This current is small because the primary winding has a large inductive reactance, which limits it; only enough flows to magnetise the core and supply the small iron losses. The transformer therefore takes very little power when nothing is connected to its secondary.

(b) The current to a distant load flows through the resistance of the long cable, which causes a voltage drop (IR) along it and wastes power as heat (I^2R). A thin cable has a large resistance, so the drop is big and the load receives a reduced voltage. Thick cables, of low resistance, are used so that the voltage drop and the power loss are kept small.

(c) (i) The current in the feeder is:

$$I = \frac{P}{V} = \frac{5000W}{250V} = 20A$$

(ii) the voltage drop and (iii) the voltage at the load are:

$$V_{drop} = IR = 20A \times 0.20\Omega = 4.0V, \qquad V_{load} = 250V - 4.0V = 246V$$

(iv) and the power wasted in the feeder is:

$$P_{loss} = I^2R = (20A)^2 \times 0.20\Omega = 80W$$

The feeder carries 20A, drops 4.0V, leaves 246V at the load, and wastes 80W.

Question 36

(a) A fuse protects a circuit by melting when the current becomes too large, and it melts because of the heating I^2R it suffers, which depends on the current it carries. Its safe rating is therefore set by that current, not by the voltage across it, which is almost zero while the fuse is intact. A fuse is accordingly marked with a current rating.

(b) The two carry the same current, but the voltage across the inductor leads the current by a quarter-cycle while the voltage across the capacitor lags it by a quarter-cycle. The two voltages are therefore half a cycle (180°) apart, exactly out of step: when one is at its positive peak the other is at its negative peak, so they oppose and tend to cancel.

(c) (i) The true power and (ii) the reactive power are:

$$P = VI\cos\phi = 240V \times 5.0A \times 0.80 = 960W, \qquad Q = VI\sin\phi = 240V \times 5.0A \times 0.60 = 720VAR$$

(iii) the impedance is $Z = \frac{V}{I} = \frac{240V}{5.0A} = 48\Omega$, so (iv) the resistance and reactance are:

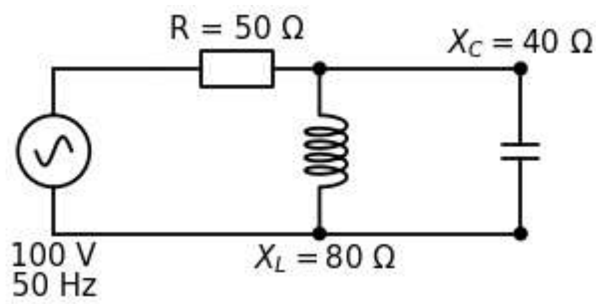
$$R = Z\cos\phi = 48\Omega \times 0.80 = 38.4\Omega, \qquad X = Z\sin\phi = 48\Omega \times 0.60 = 28.8\Omega$$

The true power is 960W, the reactive power 720VAR, the impedance 48Ω, the resistance 38.4Ω and the reactance 28.8Ω.

Question 37

(a) Impedance combines resistance with reactance, and the reactances depend on frequency: the inductive reactance $X_L = 2\pi fL$ rises with frequency while the capacitive reactance $X_C = \frac{1}{2\pi fC}$ falls. As the frequency changes, these reactances change, so the net reactance, and hence the impedance $Z = \sqrt{R^2 + (X_L - X_C)^2}$, also changes. The impedance of such a circuit is therefore a function of frequency.

(b) Switching on a high-power appliance draws a large current, which flows through the resistance of the house wiring and supply cable and causes a larger voltage drop (IR) there, so less voltage is left for the lamps and they dim. If the wiring is thin, and so of higher resistance, the effect is marked; thick, low-resistance wiring reduces it.



(c) (i) The inductor and capacitor in parallel present a net reactance:

$$X_{LC} = \frac{X_L X_C}{X_L - X_C} = \frac{80\Omega \times 40\Omega}{80\Omega - 40\Omega} = 80\Omega \text{ (capacitive)}$$

(ii) the impedance of the whole circuit and (iii) the supply current are:

$$Z = \sqrt{R^2 + X_{LC}^2} = \sqrt{(50\Omega)^2 + (80\Omega)^2} = 94.3\Omega, \qquad I = \frac{V}{Z} = \frac{100V}{94.3\Omega} = 1.06A$$

(iv) and the voltage across the parallel section is:

$$V_{LC} = IX_{LC} = 1.06A \times 80\Omega = 84.8V$$

The parallel section behaves as an 80Ω capacitive reactance; the supply delivers 1.06A, with 84.8V across the parallel combination.

Question 38

(a) Cells in series are joined so that the positive terminal of one connects to the negative of the next, so their EMFs all act in the same sense round the circuit and simply add. A chain of n equal cells therefore has a total EMF n times that of a single cell, able to drive current through a larger total resistance (their internal resistances add as well).

(b) The reactance of an inductor is $X_L = 2\pi fL$, proportional to the frequency. For direct current the frequency is zero, so the reactance is zero; once the current is steady the flux is unchanging and induces no back-EMF, and the inductor opposes the current only by the small resistance of its wire.

(c) (i) The secondary current is fixed by the rated apparent power:

$$I_s = \frac{S}{V_s} = \frac{2000VA}{24V} = 83.3A$$

(ii) the output power and (iii) the input power are:

$$P_{out} = S\cos\phi = 2000VA \times 0.90 = 1800W, \qquad P_{in} = P_{out} + 40W + 30W = 1870W$$

(iv) so the efficiency is:

$$\eta = \frac{P_{out}}{P_{in}} = \frac{1800W}{1870W} = 96.3\%$$

The secondary current is 83.3A, the output 1800W, the input 1870W, and the efficiency 96.3%.

Question 39

(a) An ideal transformer conserves power, so $V_p I_p = V_s I_s$. A step-down transformer has a lower secondary voltage, and since the power is unchanged the secondary current must be correspondingly larger. It therefore supplies a larger current than is drawn from the mains, at a lower voltage, which is why it is used where a large current at low voltage is needed.

(b) The energy a household uses is the power of its appliances multiplied by the time they run, and over a month this is an enormous number of joules. The kilowatt-hour, the energy used by a one-kilowatt appliance in one hour and equal to 3.6×10^6J, is a far more convenient size for such amounts, giving manageable figures, so the meter and the bill are reckoned in kilowatt-hours.

(c) (i) The inductive reactance is:

$$X_L = 2\pi fL = 2\pi \times 50Hz \times 0.30H = 94.2\Omega$$

(ii) the impedance and (iii) the current are:

$$Z = \sqrt{R^2 + X_L^2} = \sqrt{(80\Omega)^2 + (94.2\Omega)^2} = 123.6\Omega, \qquad I = \frac{240V}{123.6\Omega} = 1.94A$$

(iv) the voltages across the resistor and the coil are:

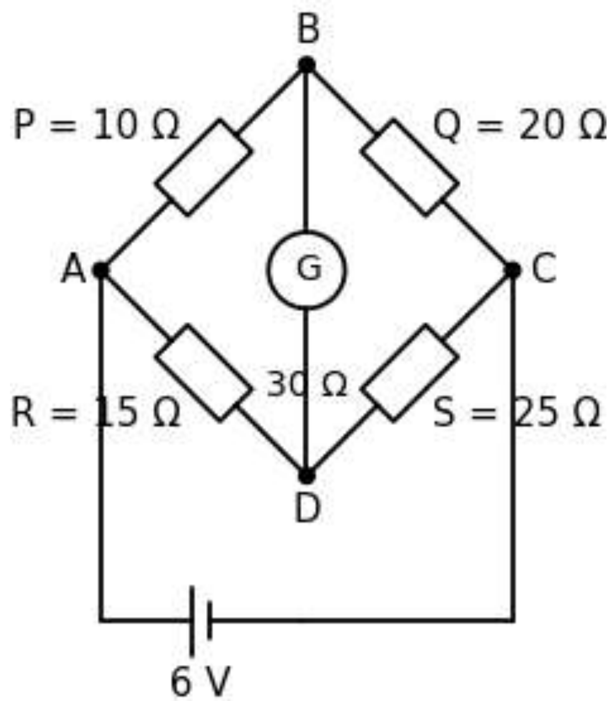
$$V_R = IR = 1.94A \times 80\Omega = 155.3V, \qquad V_L = IX_L = 1.94A \times 94.2\Omega = 183.0V$$

These add arithmetically to 338.3V, far more than the 240V supply; but because they are 90° out of phase they combine as a phasor sum, $\sqrt{V_R^2 + V_L^2} = \sqrt{(155.3V)^2 + (183.0V)^2} = 240V$, which equals the supply voltage.

Question 40

(a) A Wheatstone bridge is balanced when the ratio of the resistances in one pair of arms equals that in the other, $\frac{P}{Q} = \frac{R}{S}$, for then the two junctions are at the same potential and the galvanometer shows no deflection. At balance no current flows through the galvanometer, so the condition involves only the four arm resistances; the supply EMF affects the currents in the arms but not the ratio at which balance occurs, so the unknown resistance found is independent of the EMF.

(b) For a purely resistive load the current is in phase with the voltage, so the average power is $V_{rms}I_{rms}$, the same as a d.c. source of that voltage would deliver. For an inductive load the current lags the voltage, so only the part of it in phase with the voltage does work; the average power is $V_{rms}I_{rms}\cos\phi$, which is less, the reactive part of the current merely surging back and forth without delivering net power.



(c) The bridge is not balanced, since $\frac{P}{Q} = \frac{10}{20} = 0.5$ while $\frac{R}{S} = \frac{15}{25} = 0.6$, so a current flows through the galvanometer. With the galvanometer removed and A at 6.0V, C at 0, the potentials of B and D are:

$$V_B = E\frac{Q}{P+Q} = 6.0V \times \frac{20}{30} = 4.00V, \qquad V_D = E\frac{S}{R+S} = 6.0V \times \frac{25}{40} = 3.75V$$

so the open-circuit (Thévenin) voltage across BD is $V_{th} = V_B - V_D = 0.25V$.

The Thévenin resistance, with the battery short-circuited, is the two pairs of arms in parallel:

$$R_{th} = \frac{PQ}{P+Q} + \frac{RS}{R+S} = 6.67\Omega + 9.375\Omega = 16.0\Omega$$

and the galvanometer current is:

$$I_g = \frac{V_{th}}{R_{th} + R_g} = \frac{0.25V}{16.0\Omega + 30\Omega} = 5.4 \times 10^{-3}A = 5.4mA$$

The bridge is unbalanced and the galvanometer carries about 5.4mA.

Question 41

(a) The reactance of a capacitor is $X_C = \frac{1}{2\pi fC}$, which falls as the frequency rises. For a fixed voltage the current it passes is $\frac{V}{X_C}$, so a higher frequency, giving a smaller reactance, lets a larger current flow. In effect the plates charge and discharge more times each second, so more charge moves to and fro per second, which is a larger current.

(b) The figure 240V is the rms value; the voltage actually rises to a peak of $V_0 = V_{rms}\sqrt{2} \approx 339V$ at the crest of every cycle. The dielectric of the capacitor must withstand this peak without breaking down, so a working voltage safely above it, such as 400V, is chosen; a capacitor rated at only 240V would be over-stressed and could fail.

(c) (i) The reactance is:

$$X_C = \frac{1}{2\pi fC} = \frac{1}{2\pi \times 50Hz \times 2.0 \times 10^{-6}F} = 1.59 \times 10^3\Omega$$

(ii) the rms current is:

$$I_{rms} = \frac{V}{X_C} = \frac{240V}{1.59 \times 10^3\Omega} = 0.151A$$

(iii) the maximum charge, at the peak voltage $V_0 = 240V \times \sqrt{2} = 339V$, is:

$$Q_{max} = CV_0 = 2.0 \times 10^{-6}F \times 339V = 6.79 \times 10^{-4}C = 679\,\mu C$$

(iv) and the maximum energy stored is:

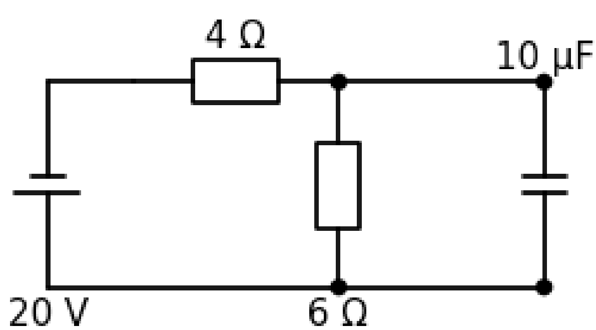
$$E_{max} = 1/2\,CV_0^2 = 1/2 \times 2.0 \times 10^{-6}F \times (339V)^2 = 0.115J$$

The reactance is 1.59kΩ, the rms current 0.151A, the maximum charge 679μC, and the maximum energy 0.115J.

Question 42

(a) Once a capacitor in a d.c. circuit is fully charged, no more charge moves onto its plates, so no current passes through its branch. Being connected in parallel with a component, it must share the same potential difference as that component; in the steady state it therefore simply settles at the voltage across whatever it is connected across.

(b) A large capacitor across the supply holds a reservoir of charge at the supply voltage. If the load suddenly draws more current the capacitor releases charge to help meet the surge, and if the demand falls it takes charge back up, so the terminal voltage is held steadier than the supply alone could manage.



(c) (i) In the steady state no current flows through the capacitor, so the current is that of the two resistors in series:

$$I = \frac{V}{R_1 + R_2} = \frac{20V}{4.0\Omega + 6.0\Omega} = 2.0A$$

(ii) the voltage across the 6.0Ω resistor is:

$$V_6 = IR_2 = 2.0A \times 6.0\Omega = 12V$$

(iii) the capacitor, in parallel with it, charges to this voltage, so:

$$Q = CV_6 = 10 \times 10^{-6}F \times 12V = 120\,\mu C$$

(iv) and the energy stored is:

$$E = 1/2\,CV_6^2 = 1/2 \times 10 \times 10^{-6}F \times (12V)^2 = 7.2 \times 10^{-4}J$$

The current is 2.0A, the voltage across the 6.0Ω resistor 12V, the charge on the capacitor 120μC, and the energy stored 0.72mJ.

Question 43

(a) Along a current-carrying wire the potential falls only gradually from one end to the other, so the field $E = \frac{V}{L}$ is small, and because the wire is uniform it is the same at every point. An isolated point charge, by contrast, concentrates its field close to itself, where it is very strong, and the field falls off as the inverse square of the distance; the two situations are quite different.

(b) The resistance of a wire is $R = \frac{\rho L}{A}$, so for the same length and material a thicker wire (larger A) has a smaller resistance. With the same voltage applied, the smaller resistance allows a larger current, since $I = \frac{V}{R}$, so the thick wire carries more current than the thin one.

(c) (i) The resistance is:

$$R = \frac{\rho L}{A} = \frac{1.7 \times 10^{-8}\Omega\text{m} \times 2.0\text{m}}{1.0 \times 10^{-6}\text{m}^2} = 0.034\Omega$$

(ii) the current and (iii) the field along the wire are:

$$I = \frac{V}{R} = \frac{1.5\text{V}}{0.034\Omega} = 44\text{A}, \qquad E = \frac{V}{L} = \frac{1.5\text{V}}{2.0\text{m}} = 0.75\text{V m}^{-1}$$

(iv) and the drift velocity of the electrons is:

$$v_d = \frac{I}{neA} = \frac{44\text{A}}{8.5 \times 10^{28}\text{m}^{-3} \times 1.6 \times 10^{-19}\text{C} \times 1.0 \times 10^{-6}\text{m}^2} = 3.2 \times 10^{-3}\text{m s}^{-1}$$

The resistance is 0.034Ω, the current 44A, the field 0.75V m^{-1}, and the drift velocity only 3.2×10^{-3}m s^{-1}.

Question 44

(a) An electric field exerts a force on a charge, and as the charge moves through a potential difference V the field does work qV on it. By the work-energy principle this work becomes kinetic energy, so $qV = 1/2\,mv^2$. The energy comes from the field, and ultimately from the source that maintains the potential difference, which gives up energy as the charge gains it.

(b) In a cathode-ray tube the electrons are accelerated through a potential difference and gain kinetic energy eV. A larger voltage does more work on each electron, so it reaches a higher speed, $v = \sqrt{\frac{2eV}{m}}$, before striking the screen; raising the accelerating voltage therefore produces a faster beam.

(c) (i) The kinetic energy gained is:

$$KE = eV = 1.6 \times 10^{-19}\text{C} \times 4.0 \times 10^3\text{V} = 6.4 \times 10^{-16}\text{J} = 4.0 \times 10^3\text{eV}$$

(ii) the final speed follows from $KE = 1/2\,mv^2$:

$$v = \sqrt{\frac{2\,KE}{m}} = \sqrt{\frac{2 \times 6.4 \times 10^{-16}\text{J}}{9.11 \times 10^{-31}\text{kg}}} = 3.7 \times 10^7\text{m s}^{-1}$$

(iii) the field in the gap and (iv) the force on the electron are:

$$E = \frac{V}{d} = \frac{4.0 \times 10^3\text{V}}{5.0 \times 10^{-2}\text{m}} = 8.0 \times 10^4\text{V m}^{-1}, \qquad F = eE = 1.6 \times 10^{-19}\text{C} \times 8.0 \times 10^4\text{V m}^{-1} = 1.3 \times 10^{-14}\text{N}$$

The electron gains 6.4×10^{-16}J (4.0keV), reaching 3.7×10^7m s^{-1}; the field is 8.0×10^4V m^{-1} and the force 1.3×10^{-14}N.

Question 45

(a) A full-wave rectifier turns both halves of each cycle the same way up, so its output is always positive; but it still rises and falls between zero and the peak a hundred times a second, so it is far from steady. To run electronic circuits, which need a nearly constant voltage, this pulsating output must be smoothed.

(b) A smoothing capacitor charges up to the peak and then feeds the load while the rectified voltage dips, so the output falls only a little before the next peak tops it up. A larger capacitor stores more charge for a given voltage ($Q = CV$), so it loses a smaller fraction of its charge, and hence of its voltage, during each dip, leaving a smaller ripple.

(c) (i) The mean output of a full-wave rectifier before smoothing is:

$$V_{dc} = \frac{2V_0}{\pi} = \frac{2 \times 12V}{\pi} = 7.64V$$

(ii) the charge on the reservoir capacitor at the peak and (iii) the energy stored are:

$$Q = CV_0 = 2200 \times 10^{-6}F \times 12V = 2.64 \times 10^{-2}C = 26.4mC, \qquad E = 1/2\,CV_0^2 = 1/2 \times 2200 \times 10^{-6}F \times (12V)^2 = 0.158J$$

(iv) and the mean load current before smoothing is:

$$I_{mean} = \frac{V_{dc}}{R} = \frac{7.64V}{60\Omega} = 0.127A$$

The mean output is 7.64V, the capacitor stores 26.4mC and 0.158J at the peak, and the mean load current is 127mA.

Question 46

(a) A steady current is a steady drift of the free electrons, and this drift is driven by the electric field set up inside the conductor by the potential difference across its ends. If the potential difference is removed, the field vanishes, the driving force disappears, and the electrons soon lose their net drift; a potential difference must therefore be maintained to keep a steady current flowing.

(b) Connecting wires are made of a good conductor, usually copper, and of generous thickness, so their resistance is extremely small compared with the components in the circuit. The voltage dropped across them is therefore negligible, and they are taken to have no resistance in order to simplify the analysis.

(c) (i) The current is:

$$I = \frac{E}{R + r} = \frac{12V}{5.5\Omega + 0.50\Omega} = 2.0A$$

(ii) the terminal voltage and (iii) the field along the 10cm resistor wire are:

$$V = IR = 2.0A \times 5.5\Omega = 11V, \qquad E = \frac{V}{L} = \frac{11V}{0.10m} = 110V\,m^{-1}$$

(iv) and the power dissipated in the resistor is:

$$P = I^2R = (2.0A)^2 \times 5.5\Omega = 22W$$

The current is 2.0A, the terminal voltage 11V, the field along the resistor 110V m^{-1}, and the power 22W.

Question 47

(a) A charged capacitor stores energy in the electric field between its plates, equal to $1/2\,CV^2$. When it is discharged this energy is released, and if it is released through a low resistance the charge flows out very quickly, giving a large current for a brief instant, enough to drive a device such as a small motor or a camera flash.

(b) The average current is the charge that flows divided by the time taken, and the average power is the energy delivered divided by the time. Releasing a given charge and a given energy in a shorter time therefore gives both a larger average current and a larger average power, which is why a rapid discharge is so vigorous.

(c) (i) The charge that flows and (ii) the energy released are:

$$Q = CV = 50 \times 10^{-6}F \times 100V = 5.0 \times 10^{-3}C = 5.0mC, \qquad E = 1/2\,CV^2 = 1/2 \times 50 \times 10^{-6}F \times (100V)^2 = 0.25J$$

(iii) the average current and (iv) the average power, over the 0.50s discharge, are:

$$I_{avg} = \frac{Q}{t} = \frac{5.0 \times 10^{-3}C}{0.50s} = 10mA, \qquad P_{avg} = \frac{E}{t} = \frac{0.25J}{0.50s} = 0.50W$$

The capacitor passes 5.0mC, releases 0.25J, and delivers an average of 10mA and 0.50W during the discharge.

Question 48

(a) An inductor and a capacitor in series resonate when their reactances are equal and opposite, $X_L = X_C$, for then they cancel and the circuit offers least opposition and responds most strongly. This happens at one particular natural frequency, $f_0 = \frac{1}{2\pi\sqrt{LC}}$, at which energy passes most freely back and forth between the two components.

(b) The capacitor stores energy in its electric field, $1/2\,CV^2$, and the inductor in its magnetic field, $1/2\,LI^2$. As the oscillation proceeds the current and charge are a quarter-cycle out of step, so when the charge is greatest (current zero) all the energy is in the capacitor, and a quarter-cycle later, when the current is greatest (charge zero), all of it is in the inductor.

(c) (i) The resonant frequency is:

$$f_0 = \frac{1}{2\pi\sqrt{LC}} = \frac{1}{2\pi\sqrt{0.40H \times 10 \times 10^{-6}F}} = 79.6Hz$$

(ii) at this frequency the two reactances are:

$$X_L = 2\pi f_0 L = 2\pi \times 79.6Hz \times 0.40H = 200\Omega, \qquad X_C = \frac{1}{2\pi f_0 C} = \frac{1}{2\pi \times 79.6Hz \times 10 \times 10^{-6}F} = 200\Omega$$

The resonant frequency is 79.6Hz, where the inductive and capacitive reactances are equal at 200Ω, confirming resonance.

Question 49

(a) When two capacitors are joined in series the same charge is pushed onto each, while the supply voltage divides between them; the combined capacitance is found from $\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2}$. Because the reciprocals add, the combined capacitance is always smaller than either capacitor on its own, much as adding resistors in parallel gives a smaller resistance.

(b) While the capacitors are charging a current flows from the battery, but once they are fully charged no steady current flows. With no current there is no voltage dropped across the battery's internal resistance or any series resistor, so the whole of the battery's EMF appears across the capacitor combination.

(c) (i) The combined capacitance is:

$$C = \frac{C_1 C_2}{C_1 + C_2} = \frac{4.0\ \mu F \times 2.0\ \mu F}{4.0\ \mu F + 2.0\ \mu F} = 1.33\ \mu F$$

(ii) the same charge sits on each capacitor in series:

$$Q = CV = 1.33 \times 10^{-6}F \times 30V = 40\ \mu C$$

(iii) and the voltage across each follows from $V = \frac{Q}{C}$:

$$V_4 = \frac{40\ \mu C}{4.0\ \mu F} = 10V, \qquad V_2 = \frac{40\ \mu C}{2.0\ \mu F} = 20V$$

The combined capacitance is 1.33μF, each capacitor carries 40μC, and the voltages are 10V and 20V, which add to the 30V supply.

Question 50

(a) A capacitor blocks a steady direct current because, once it has charged to the supply voltage, no further charge can cross the gap between its plates and the current stops. It passes an alternating current because the continually reversing voltage keeps charging and discharging the plates, so charge flows back and forth in the connecting wires the whole time.

(b) On a.c. the capacitor opposes the current by its reactance $X_C = \frac{1}{2\pi fC}$, and a current flows; but the current leads the voltage by a quarter-cycle, so during each cycle the capacitor takes energy from the supply for one quarter and returns all of it during the next. The average power is therefore zero: the capacitor stores and returns energy but consumes none.

(c) (i) On the 100V battery the capacitor charges to:

$$Q = CV = 20 \times 10^{-6}F \times 100V = 2.0 \times 10^{-3}C = 2.0mC$$

after which the steady current and the power are both zero. (ii) On the 100V (rms), 50Hz supply the reactance, rms current and average power are:

$$X_C = \frac{1}{2\pi fC} = \frac{1}{2\pi \times 50Hz \times 20 \times 10^{-6}F} = 159\Omega, \qquad I_{rms} = \frac{V}{X_C} = \frac{100V}{159\Omega} = 0.628A$$

with an average power of zero, since the current and voltage are a quarter-cycle out of phase. On d.c. the capacitor holds 2.0mC and passes no current; on a.c. it draws 0.628A but consumes no power.