EXAMINATION QUESTIONS ON VIBRATION AND WAVE

Question 1

(a) A transverse pulse travelling along a stretched string is inverted when it reflects from a rigidly fixed end, but returns the same way up when it reflects from a free end. Account for this difference.
(b) In a 3D cinema the two lenses of the viewing glasses are polarising filters with their transmission axes set at right angles to one another. Account for how this lets each eye receive a separate image.
(c) A radio station in Dar es Salaam broadcasts at a frequency of 90.0MHz. Taking the speed of electromagnetic waves as 3.00×10^8m/s, calculate the wavelength of the broadcast and the time taken for the signal to reach a receiver 75km away.

Question 2

(a) Light passing through a wide doorway shows no noticeable spreading, while light passing through a slit a fraction of a millimetre wide spreads markedly beyond the straight-through direction. Account for this.
(b) A wave rolling in from deep water towards a gently shelving beach near Bagamoyo slows down and grows steeper as the water becomes shallower. Account for this.
(c) At a workshop one machine alone produces a sound level of 82dB at a worker's position, and a second machine alone produces 78dB at the same position. Calculate the sound level at that position when both machines run together. Take the threshold of hearing as $\mathrm{I_0} = 1.0 \times 10^{-12}\mathrm{W/m^2}$.

Question 3

(a) The loudness of sound is measured on the decibel scale, which is logarithmic rather than linear. Account for the use of a logarithmic scale.
(b) Waves rolling into the harbour at Tanga strike the vertical harbour wall and travel back out, crossing the incoming waves and producing a choppy criss-cross pattern on the water. Account for this.
(c) Monochromatic light of wavelength 600nm falls normally on a diffraction grating ruled with 400 lines per millimetre. Calculate the highest order of diffraction maximum that can be observed.

Question 4

(a) Two transverse pulses travelling in opposite directions along a stretched string meet, overlap, and then continue on their way, each emerging with its original shape, size and speed. Account for this.
(b) A traffic police officer points a radar gun at a car approaching along the road. Account for how the gun is able to determine the car's speed.
(c) Plane-polarised light is incident on an analysing polaroid. Find the angle between the transmission axis of the polaroid and the plane of polarisation of the light for which the transmitted intensity is one quarter of the incident intensity.

Question 5

(a) An astronaut standing on the airless surface of the Moon would not hear a rock fall and shatter a few metres away. Account for this.
(b) The feathers of a peacock and the wing-cases of certain beetles show brilliant, shifting colours, yet contain no coloured pigment. Account for this.
(c) A transverse wave of amplitude 8.0mm and frequency 50Hz travels along a stretched string of linear mass density 0.020kg/m held under a tension of 80N. Calculate the speed of the wave and the average power it carries along the string.

Question 6

(a) A guitarist can change the frequency of the wave he sends along a string, but not the speed at which it travels. Account for the fact that the wave speed is fixed by the string while the frequency is fixed by the source.
(b) Looking at a distant streetlamp at night through the fine wire mesh of a window screen, an observer sees the single lamp spread into a regular pattern of bright spots. Account for this.
(c) An organ pipe closed at one end has a length of 0.55m. Taking the speed of sound in air as 340m/s, calculate the three lowest frequencies at which the pipe resonates.

Question 7

(a) A diffraction grating ruled with many thousands of lines produces much sharper and narrower bright lines than a double slit illuminated with the same light. Explain this.
(b) A bat hunting at dusk emits ultrasonic pulses and listens for the echoes. Explain how the bat can tell whether a moth ahead of it is flying towards it or away from it.
(c) Two waves of the same frequency travel in the same direction along a stretched rope. Their amplitudes are 3.0cm and 5.0cm and there is a constant phase difference of 60° between them. Calculate the amplitude of the resultant wave.

Question 8

(a) In a stationary wave on a stretched string, all the particles in one loop reach their extreme displacements at the same instant, while a particle in the next loop reaches its extreme on the opposite side of the string at that same instant. Explain.
(b) A military jet flying low and very fast over Dar es Salaam is followed by a sudden, loud bang heard at the ground. Account for this bang.
(c) Monochromatic light of wavelength 600nm passes through a single slit of width 0.20mm, and the diffraction pattern is observed on a screen 2.0m away. Calculate the angular position of the first dark fringe and the width of the central bright band on the screen.

Question 9

(a) In a two-slit interference pattern the bright fringes are all of very nearly the same brightness, but in a single-slit diffraction pattern the bright bands grow steadily fainter on either side of the centre. Account for this difference.
(b) Two children make a telephone from two empty tins joined by a length of string. When the string is pulled taut a child speaking into one tin is heard clearly at the other; when the string goes slack, nothing is heard. Account for this.
(c) A car travelling at 30m/s towards a vertical cliff sounds a horn of frequency 480Hz. Taking the speed of sound in air as 340m/s, calculate the frequency of the echo heard by the driver.

Question 10

(a) During a storm the flash of lightning is seen well before the thunder is heard, although both are produced at the same instant by the same lightning stroke. Account for the delay.
(b) A blacksmith heats a steel tool; as it cools in the air its polished surface passes through a sequence of colours, from pale straw through brown and purple to blue. Account for these colours.
(c) A wire fixed at both ends sounds a particular fundamental note, its length and mass per unit length kept constant. Determine the factor by which the tension must be increased to raise the frequency of the fundamental in the ratio 3:2.

Question 11

(a) A mass oscillating on a spring and a swinging pendulum bob appear to be very different systems, yet both execute simple harmonic motion. Account for the feature they share that makes this so.
(b) When a glass screen-protector is pressed onto a phone screen, patches of coloured rings and bands appear in the regions where the two surfaces almost, but not quite, touch. Account for these patterns.
(c) A motorcyclist rides at 20m/s directly towards a stationary siren that emits a steady note of frequency 660Hz. Taking the speed of sound in air as 340m/s, calculate the frequency of the note heard by the motorcyclist.

Question 12

(a) In a Newton's-rings experiment the dark and bright fringes are seen as a series of concentric circles centred on the point of contact. Account for the circular shape of the fringes.

(b) A diving board continues to move up and down for a short while after the diver has sprung off it. Account for this continued motion.

(c) A machine treated as a point source produces a sound level of 88dB at a distance of 2.0m in the open air. Calculate the sound level at a distance of 8.0m from the machine.

Question 13

(a) An organ pipe closed at one end sounds a note an octave lower than an open pipe of the same length. Account for this.
(b) A baby placed in a bouncer, a seat hung from a doorframe by an elastic strap, bounces up and down with a steady, regular rhythm. Account for this rhythmic bouncing.
(c) In a Young's double-slit experiment the bright fringes on the screen are 1.8mm apart when the apparatus is in air. The whole apparatus is then immersed in water of refractive index 1.33. Calculate the new spacing of the bright fringes.

Question 14

(a) If the amplitude of a simple harmonic oscillator is doubled, its maximum speed and its maximum acceleration both double, yet its period is unchanged. Account for this.
(b) In the great domed hall of a certain building, a person whispering quietly at one particular point can be heard clearly by someone far away at another particular point, though listeners between the two hear nothing. Account for this.
(c) A soap film of refractive index 1.33 and thickness 1.10×10^{-7}m is illuminated normally by white light. Calculate the wavelength in the visible range that is most strongly reflected.

Question 15

(a) Sound travels faster through warm air than through cold air. Account for this.
(b) Seen through a windscreen covered with fine scratches left by the wipers, the headlights of oncoming cars at night appear drawn out into bright streaks and spikes of light. Account for this.
(c) A 0.60kg mass hung from a single light spring oscillates vertically with a period of 0.80s. The mass is then hung from two such springs side by side, both supporting it together. Calculate the period of the vertical oscillations of the mass on the pair of springs.

Question 16

(a) Account for the fact that the total energy of a simple harmonic oscillator is proportional to the square of its amplitude.
(b) When a thin straight wire is held a short distance in front of one eye and a distant bright vertical lamp is viewed past its edge, a series of fine light and dark bands is seen. Account for these bands.
(c) An organ pipe open at both ends is to sound a fundamental note of frequency 256Hz. Taking the speed of sound in air as 340m/s, calculate the length of the pipe and the frequency of its second harmonic.

Question 17

(a) In Young's double-slit experiment a single narrow slit is placed between the lamp and the double slit. Account for the need for this single slit.
(b) A person who hears a recording of their own voice for the first time usually finds that it sounds unfamiliar and somewhat thinner than the voice they are used to hearing themselves speak in. Account for this.
(c) A 0.30kg mass hangs from a vertical spring of force constant 30N/m and is set oscillating with an amplitude of 0.080m. Calculate the period of the oscillation and the maximum and minimum tension in the spring during the motion. Take $g = 9.8\text{m/s}^2$.

Question 18

(a) Account for the fact that the kinetic energy of a simple harmonic oscillator varies with time at twice the frequency of its displacement.
(b) A doctor's stethoscope makes the faint sounds of a patient's heartbeat loud enough to hear clearly. Account for how it does this.

(c) Monochromatic light falls normally on a diffraction grating ruled with 500 lines per millimetre. The second-order maximum is observed at an angle of 36.0° to the straight-through direction. Calculate the wavelength of the light.

Question 19

(a) Sunlight reflected from a smooth water surface is found to be partially polarised, and at one particular angle of incidence it is completely polarised. Account for these observations.
(b) A heavy load hangs from the long cable of a crane. When the load is set swinging, the operator notices that paying out more cable makes the swinging slower and lazier. Account for this.

(c) A radar station sends out a pulse of microwaves and receives the echo reflected from an aircraft 1.20×10^{-4}s later. Taking the speed of electromagnetic waves as 3.00×10^{8}m/s, calculate the distance of the aircraft from the station.

Question 20

(a) Radio waves, visible light and X-rays are put to very different uses and have very different effects, yet they are all regarded as the same kind of wave. Account for this.
(b) Looking through polarised sunglasses at the toughened glass rear window of a car in daylight, a passenger sees a pattern of dark spots and blotches spread across the glass. Account for this.
(c) Calculate the length of a simple pendulum that beats seconds, that is, has a period of 2.0s, at a place where $g = 9.8\text{m/s}^2$. If this pendulum is then lengthened by 1.0%, calculate the resulting percentage change in its period.

Question 21

(a) Explain what is meant by the natural frequency of an oscillating system, and state the physical quantities that fix it for a mass on a spring and for a simple pendulum.
(b) At a football match the spectators rise and sit in turn, sending a 'wave' that travels right round the stadium, although no spectator leaves their seat. Account for the fact that the wave travels round the ground while the spectators do not.
(c) A girl standing in front of a high cliff claps her hands once and hears the echo 1.5s later. Taking the speed of sound in air as 340m/s, calculate her distance from the cliff. She then walks 60m directly towards the cliff and claps again; calculate the time after which she now hears the echo.

Question 22

(a) Explain why sound travels faster through a solid such as steel, and through a liquid such as water, than it does through air.
(b) A bungee jumper, after the first plunge, bounces up and down several times, each bounce smaller than the one before, until at last she hangs almost at rest. Explain this behaviour in terms of oscillations.
(c) Water waves travelling across a pond move at 1.8m/s and pass a fixed post 2.5 times every second. Calculate the wavelength of the waves, and determine the phase difference between two points lying 0.30m apart along the direction in which the waves travel.

Question 23

(a) A wave travelling along a light rope reaches a join where the light rope is tied to a much heavier rope. Explain why part of the wave is reflected at the join and part is transmitted into the heavier rope.
(b) A person standing near the corner of a building can hear someone speaking just around the corner, even though the speaker cannot be seen. Explain how the sound reaches the listener.
(c) A particle executing simple harmonic motion has a maximum speed of 0.30m/s and a maximum acceleration of 4.5m/s^2. Calculate the angular frequency, the period and the amplitude of the motion.

Question 24

(a) During simple harmonic motion the displacement, the velocity and the acceleration of the body all vary with time. Explain how the velocity and the acceleration are each related, in phase, to the displacement.

(b) A survey ship finds the depth of water beneath it by sending a pulse of sound down to the sea-bed and detecting the pulse that returns. Account for the way this method reveals the depth.
(c) A long string has a mass per unit length of 2.0×10^{-3}kg/m and is held under a tension of 50N. A vibrator at one end sends transverse waves of frequency 100Hz along it. Calculate the speed of the waves on the string and their wavelength.

Question 25

(a) The note of the siren of an ambulance is heard to fall in pitch at the moment the ambulance passes a stationary listener. Explain why the pitch falls.
(b) When a stone is dropped into a still pond, circular ripples spread outwards and grow steadily weaker the further they travel from the point where the stone entered. Account for the weakening of the ripples as they spread.
(c) A 0.25kg mass hung from a light spring oscillates vertically with a period of 0.50s. Calculate the force constant of the spring, and determine the further mass that must be added to the 0.25kg so that the period becomes 1.0s.

Question 26

(a) Use Huygens' principle to explain how a wavefront advances through a medium.
(b) A musician's metronome marks time with a steady tick, and the musician changes the tempo by sliding the small weight up or down its swinging arm. Explain, in terms of oscillations, why moving the weight changes the rate of ticking.
(c) The speed of sound in air is 340m/s when the air temperature is 15°C, that is 288K. Calculate the speed of sound when the air warms to 35°C. Hence calculate the wavelength in air of the note of a 480Hz tuning fork at each of these two temperatures.

Question 27

(a) Distinguish between the loudness and the pitch of a musical note, and state the physical property of the sound wave on which each of them depends.
(b) A heavy adult and a light child sit in turn on the same playground swing. Explain why each of them completes one full to-and-fro swing in the same time.
(c) Transverse wave pulses travel at 40m/s along a long cable stretched between two fixed posts 100m apart. A pulse is sent from one post; calculate the time after which the pulse, reflected from the far post, returns to its starting point. The tension in the cable is then increased to four times its original value; calculate the new return time.

Question 28

(a) Explain why the wavefronts produced by a small source of waves are very nearly circular close to the source, but appear practically straight to an observer far away from it.
(b) In a hospital, an image of an unborn baby is formed using ultrasound rather than ordinary audible sound. Suggest why ultrasound is used for this purpose.
(c) A body of mass 0.50kg executes simple harmonic motion with a total energy of 0.16J and an amplitude of 0.080m. Calculate the force constant of the system and the maximum speed of the body.

Question 29

(a) Explain why the amplitude of a forced oscillation becomes very large when the frequency of the driving force is close to the natural frequency of the system.
(b) A voice in an empty room with bare walls and floor rings on and sounds hollow, but once the room is furnished with curtains, carpets and soft chairs the hollow ringing disappears. Account for this difference.
(c) A stretched string vibrating as a stationary wave has a distance of 0.12m between a node and the antinode next to it, and the string vibrates 25 times each second. Calculate the wavelength of the waves on the string and the speed at which they travel along it.

Question 30

(a) Explain why an echo can be heard as a sound separate from the original only when the reflecting surface is sufficiently far from the listener.
(b) When two children turn a skipping rope steadily, the rope settles into a single broad loop, with the ends almost still and the middle sweeping out the widest path. Explain why this pattern forms.
(c) A particle moving with simple harmonic motion has a speed of 8.0cm/s when it is 6.0cm from the centre of its path, and a speed of 6.0cm/s when it is 8.0cm from the centre. Calculate the amplitude and the period of the motion.

Question 31

(a) Explain why a body executing simple harmonic motion spends, over each complete cycle, more of its time near the extremes of its path than near the centre.
(b) Boats moored in the sheltered water behind a harbour breakwater are still seen to rock gently, although the breakwater stands directly between them and the open sea. Account for the gentle rocking of the boats.
(c) Monochromatic light of wavelength 590nm is incident normally on a diffraction grating, and the first-order bright image is seen at an angle of 18.0° to the straight-through direction. Calculate the spacing of the grating lines and the number of lines per millimetre of the grating.

Question 32

(a) In a two-slit interference pattern there are dark fringes at which light arrives from both slits and yet no light is seen. Explain how light from the two slits can combine to give darkness, and account for what becomes of the light energy at a dark fringe.
(b) A hacksaw blade clamped firmly to the edge of a bench and twanged vibrates with a steady buzz; sliding more of the blade out beyond the edge of the bench makes the buzz deepen. Explain this behaviour in terms of oscillations.
(c) A wire 0.90m long is fixed at both ends and is made to vibrate in its third harmonic, sounding a note of frequency 200Hz. Calculate the wavelength of the waves on the wire and the speed at which they travel along it.

Question 33

(a) Explain why a stationary wave has points, the nodes, at which the medium never moves at all, even though each of the two waves that combine to form it would, on its own, move every point of the medium.
(b) A tuning fork sounds the very same musical note whether it is struck hard or struck gently. Explain why striking it harder changes the loudness of the note but not the note itself.
(c) In a Young's double-slit experiment the two slits are 0.40mm apart and the fringes are observed on a screen 1.5m away. The bright fringes are found to be 2.1mm apart. Calculate the wavelength of the light used.

Question 34

(a) A diffraction grating and a glass prism both spread white light into a spectrum, but the grating bends red light most and violet least, while the prism does the reverse. Account for the fact that the grating spreads the colours in the opposite order to the prism.
(b) A child pushes and pulls the end of a long springy coil lying on the floor, sending a pulse of compression running along its length to the far end. Account for the way the pulse travels along the coil.
(c) A spring of force constant 18N/m is cut into two equal pieces. Calculate the force constant of each piece, and the period of vertical oscillation of a 0.40kg mass hung from one of the pieces.

Question 35

(a) Explain why a mass suspended from a vertical spring oscillates with exactly the same period as it would if the same spring and mass were laid horizontally, even though gravity pulls on the hanging mass.
(b) On a misty night a bright street lamp is seen surrounded by a faint set of coloured rings. Suggest why these coloured rings appear.
(c) A continuous train of waves of frequency 20Hz travels along a stretched cord at a speed of 8.0m/s. Calculate the wavelength of the waves, and the number of complete waves contained in a 12m length of the cord.

Question 36

(a) Explain why the energy carried past a point each second by a progressive wave depends on the square of the amplitude of the wave.
(b) When a car approaches from far away at night, its two headlamps appear at first as a single point of light, and only when the car is much nearer do they become distinguishable as two separate lamps. Suggest why the two lamps cannot be told apart until the car is close.
(c) A particle executes simple harmonic motion with an amplitude of 0.10m. Calculate the displacement at which its kinetic energy is three times its potential energy, and the fraction of the total energy that is kinetic when the displacement of the particle is 0.080m.

Question 37

(a) A mass oscillating on a spring is replaced by a mass twice as large on the same spring. Explain why the period of oscillation increases by a factor of $\sqrt{2}$ rather than doubling.
(b) When a whip is cracked, the wave sent travelling down it moves faster, and with greater amplitude, as it nears the thin tip. Account for the speeding-up of the wave as it travels towards the tip.
(c) Unpolarised light of intensity $16\mathrm{W/m^2}$ is incident on a pair of polaroids whose transmission axes are set at 60° to each other. Calculate the intensity of the light transmitted through the first polaroid, and the intensity of the light emerging from the second.

Question 38

(a) Account for the equal spacing of the bright fringes across the screen in a Young's double-slit interference pattern.
(b) A child bounces steadily up and down on a pogo stick, a vertical toy with a strong spring for hopping, each bounce taking the same time as the one before. Explain, in terms of oscillations, what makes the bouncing so regular.
(c) A transverse wave of amplitude 1.5cm and frequency 12Hz travels along a stretched string at a speed of 9.0m/s. Calculate the maximum speed of a particle of the string, and state how it compares with the speed of the wave itself.

Question 39

(a) Explain why a train of water waves changes its direction of travel when it crosses, at an angle, into a region where the water is shallower and the waves move more slowly.
(b) A small boat at sea, rocked to one side by a passing wave, continues to roll to and fro with a steady rhythm of its own after the wave has gone. Account for this rolling.
(c) In a Newton's-rings experiment the plano-convex lens has a radius of curvature of 1.2m. The radius of the eighth dark ring, counted from the centre, is measured to be 2.4mm. Calculate the wavelength of the light used.

Question 40

(a) Explain why the sky appears blue to an observer on the ground during the day.
(b) Waves approaching a straight beach are often seen to arrive very nearly parallel to the shoreline, even when, far out, they were travelling towards the shore at a noticeable angle. Explain why the waves arrive nearly parallel to the shore.
(c) A particle moving with simple harmonic motion has a period of 0.40s and travels a total distance of 24cm in each complete oscillation. Calculate the amplitude of the motion, its maximum speed and its maximum acceleration.

ANSWERS

Question 1

(a) At a fixed end the string cannot move. The arriving pulse pulls upward on the support, and by Newton's third law the support pulls downward on the string with an equal force, which launches a reflected pulse on the opposite side, an inversion. At a free end no such reaction force acts: the end is free to overshoot, and the reflected pulse keeps its original orientation. Reflection at a fixed boundary introduces a phase change of π; reflection at a free boundary introduces none.

(b) The projector sends two images to the screen, each carried by light polarised in one of two perpendicular planes. Each lens transmits only the light whose plane of polarisation matches its own transmission axis and blocks the perpendicular component almost completely. So each eye receives only its intended image, and the brain fuses the two slightly different views into one three-dimensional impression.

(c) The wavelength follows from $c = f\lambda$:

$$\lambda = \frac{c}{f} = \frac{3.00 \times 10^8 \text{m/s}}{90.0 \times 10^6 \text{Hz}} = 3.33\text{m}$$

The signal travels at the speed of light, so the time to cover 75km is:

$$t = \frac{d}{c} = \frac{75 \times 10^3 \text{m}}{3.00 \times 10^8 \text{m/s}} = 2.5 \times 10^{-4} \text{s}$$

Question 2

(a) Diffraction, the spreading of a wave as it passes an aperture, becomes appreciable only when the width of the aperture is comparable with the wavelength. The wavelength of light is around 5×10^{-7}m, vastly smaller than a doorway, so the spreading is utterly negligible and the light appears to travel in straight lines. A slit a fraction of a millimetre wide is far closer to the scale of the wavelength, so the spreading becomes noticeable.

(b) The speed of a water wave in shallow water depends on the depth, falling as the water becomes shallower. As the wave advances up the beach its speed therefore decreases. Its frequency is fixed by the distant source, so since $v = f\lambda$ the wavelength must shorten. The same energy is now packed into a shorter, slower wave, so the crest rises and the wave steepens.

(c) The intensity for a level L is $I = I_0 \times \log^{-1}\left(\frac{L}{10}\right)$. For the two machines:

$$I_1 = (1.0 \times 10^{-12} \text{W/m}^2) \times \log^{-1}(8.2) = 1.58 \times 10^{-4} \text{W/m}^2$$
$$I_2 = (1.0 \times 10^{-12} \text{W/m}^2) \times \log^{-1}(7.8) = 0.63 \times 10^{-4} \text{W/m}^2$$

Intensities from independent sources add:

$$I = I_1 + I_2 = 2.21 \times 10^{-4} \text{W/m}^2$$

Converting the combined intensity back to a sound level:

$$L = 10 \log(2.21 \times 10^8) = 83.5\text{dB}$$

Question 3

(a) The range of intensities the ear responds to is enormous: from the faintest audible sound to the threshold of pain spans a factor of about 10^{12}. A linear scale would be hopelessly unwieldy over so vast a range. A logarithmic scale compresses it into a convenient span of roughly 0 to 120. It also matches the ear itself, whose response to intensity is roughly logarithmic, so that equal multiplicative steps in intensity are heard as roughly equal steps in loudness.

(b) The vertical wall reflects the incoming waves with little loss, sending a reflected train back across the path of the incoming train. At every point the water carries both trains at once, and by the principle of superposition the resultant displacement is the sum of the two. Because the trains travel in different directions, their crests and troughs overlap in a continually shifting pattern, giving the choppy criss-cross appearance.

(c) The grating spacing is the reciprocal of the line density:

$$d = \frac{1\text{mm}}{400} = 2.5 \times 10^{-6} \text{m}$$

The grating equation is $d \sin\theta = m\lambda$. The largest order corresponds to $\sin\theta = 1$:

$$m_{max} = \frac{d}{\lambda} = \frac{2.5 \times 10^{-6} \text{m}}{600 \times 10^{-9} \text{m}} = 4.17$$

The order must be a whole number, so the highest maximum that can be observed is the **fourth** order.

Question 4

(a) While the pulses overlap, the displacement of each point of the string is the algebraic sum of the displacements the two pulses would separately produce, which is the principle of superposition. The medium responds to each pulse independently, so neither is altered by the other. Once they have passed through each other, each pulse continues exactly as before, since no energy has been exchanged between them.

(b) The gun emits microwaves of known frequency, which reflect from the moving car and return. Because the car is moving, the reflected waves are Doppler-shifted: the car first acts as a moving receiver and then as a moving re-emitter, so the echo is shifted twice. The gun measures the frequency difference between the emitted and returned waves; this difference is proportional to the car's speed, from which the speed is found.

(c) By Malus's law the transmitted intensity is $I = I_0(\cos\theta)^2$. Setting $I = \frac{1}{4}I_0$:

$$(\cos\theta)^2 = \frac{1}{4}$$

$$\cos\theta = \frac{1}{2}$$

The required angle is therefore:

$$\theta = 60°$$

Question 5

(a) Sound is a mechanical wave: it travels only by the successive disturbance of the particles of a material medium, each layer pushing on the next. The Moon has no atmosphere, so between the shattering rock and the astronaut there is no medium to carry the compressions and rarefactions. With nothing to vibrate, no sound wave can form, and the event reaches the astronaut as silence.

(b) The colours come from interference, not pigment. The surfaces carry extremely fine, regularly spaced structures whose spacing is comparable with the wavelength of light. Light reflected from successive layers interferes; for a given viewing angle one wavelength is reinforced while others are cancelled, so a colour is seen. As the viewing angle changes, the path difference changes, the reinforced wavelength changes, and the colour shifts. This is structural colour.

(c) The wave speed is set by the tension and the linear density:

$$v = \sqrt{\frac{T}{\mu}} = \sqrt{\frac{80\text{N}}{0.020\text{kg/m}}} = 63.2\text{m/s}$$

The average power carried by the wave is $P = \frac{1}{2}\mu\omega^2A^2v$, with $\omega = 2\pi f = 314\text{rad/s}$. Substituting:

$$P = \frac{1}{2}(0.020\text{kg/m})(314\text{rad/s})^2(0.0080\text{m})^2(63.2\text{m/s}) = 3.99\text{W}$$

Question 6

(a) The speed of a transverse wave on a string is $v = \sqrt{T/\mu}$: it is fixed entirely by the tension and the mass per unit length, properties of the string itself, and is independent of how the wave is made. The frequency is the rate at which the source shakes the end of the string, each oscillation launching one cycle, so the source alone fixes it. The wavelength then adjusts to satisfy $\lambda = v/f$.

(b) The regularly spaced openings of the mesh act as a two-dimensional diffraction grating. Light from the lamp passing through the many openings is diffracted, and the diffracted beams interfere. In directions where the path difference between neighbouring openings is a whole number of wavelengths the interference is constructive, giving a bright spot; the regular spacing of the mesh produces a regular pattern of such spots.

(c) A pipe closed at one end has a node at the closed end and an antinode at the open end, so the fundamental wavelength is four times the length:

$$\lambda_1 = 4L = 4(0.55\text{m}) = 2.20\text{m}$$

$$f_1 = \frac{v}{\lambda_1} = \frac{340\text{m/s}}{2.20\text{m}} = 154.5\text{Hz}$$

A closed pipe resonates only in the odd harmonics, so the next two resonances are the third and fifth harmonics:

$$f_3 = 3f_1 = 463.6\text{Hz}$$

$$f_5 = 5f_1 = 772.7\text{Hz}$$

Question 7

(a) In a double slit only two beams interfere, so the intensity varies gradually with angle and the bright fringes are broad. In a grating, light from many thousands of slits interferes. At an exact maximum every beam arrives in phase. A very slight turn away from that direction is enough for the many beams, spread across the whole grating, to fall progressively out of step and cancel completely. The maxima are therefore confined to very narrow ranges of angle, and the more slits there are, the sharper the line.

(b) The echo returning from the moth is Doppler-shifted. If the moth is flying towards the bat it acts as an approaching reflector and the echo returns at a higher frequency than the emitted pulse; if it is flying away, the echo returns at a lower frequency. By comparing the pitch of the echo with that of its own pulse, the bat senses whether the moth is closing in or escaping.

(c) For two waves of the same frequency the resultant amplitude is:

$$A_R = \sqrt{{A_1}^2 + {A_2}^2 + 2A_1A_2\cos\varphi}$$

Substituting $A_1 = 3.0\text{cm}$, $A_2 = 5.0\text{cm}$ and $\varphi = 60°$:

$$A_R = \sqrt{(3.0\text{cm})^2 + (5.0\text{cm})^2 + 2(3.0\text{cm})(5.0\text{cm})(0.50)}$$

$$A_R = 7.0\text{cm}$$

Question 8

(a) In a stationary wave every particle oscillates with the same frequency, but the amplitude varies with position, being zero at the nodes and greatest at the antinodes. Within one loop all the particles pass through equilibrium together and reach their extremes together: they oscillate exactly in phase, differing only in amplitude. Across a node the sign of the displacement reverses, so particles in adjacent loops are always half a cycle out of phase, reaching their extremes on opposite sides at the same instant.

(b) When the jet moves faster than the speed of sound it outruns the waves it produces. These waves cannot move ahead of it; instead they pile up and overlap along a cone-shaped shock front trailing the aircraft, across which there is a sharp, intense pressure change. When this shock front sweeps over an observer on the ground, the abrupt pressure change is heard as the single loud bang.

(c) The first dark fringe of a single slit occurs where $a\sin\theta = \lambda$:

$$\sin\theta = \frac{\lambda}{a} = \frac{600 \times 10^{-9}\text{m}}{0.20 \times 10^{-3}\text{m}} = 3.0 \times 10^{-3}$$

So $\theta = 3.0 \times 10^{-3}\text{rad}$.

The central band runs from this fringe on one side to the matching fringe on the other, so on a screen at distance D its width is:

$$w = 2D\sin\theta = 2(2.0\text{m})(3.0 \times 10^{-3}) = 1.2 \times 10^{-2}\text{m}$$

The central bright band is therefore 12mm wide.

Question 9

(a) In two-slit interference the two beams have effectively equal amplitude wherever they meet, so wherever they arrive in phase they reinforce to the same maximum brightness, and all the bright fringes are alike. In single-slit diffraction the slit acts as very many tiny sources across its width. At the centre all their contributions arrive in phase and add fully; away from the centre they are increasingly out of step, so only a partial reinforcement survives, and the bright bands fade with distance from the centre.

(b) Speech sets the base of the first tin vibrating; the vibration passes into the string and travels along it as a wave to the second tin, whose base then re-radiates the sound. A string can carry such a wave only while it is taut, because the wave needs tension to supply the restoring force, its speed being $v = \sqrt{T/\mu}$. When the string goes slack the tension falls almost to zero, the string can no longer transmit the vibration, and the sound fails to cross.

(c) The sound is Doppler-shifted twice: once as the moving car sends sound to the cliff, and again as the moving driver receives the reflected sound. For a source and observer moving towards a stationary reflector at the same speed u, the two shifts combine to give:

$$f' = f\frac{v + u}{v - u}$$

$$f' = 480\text{Hz} \times \frac{340\text{m/s} + 30\text{m/s}}{340\text{m/s} - 30\text{m/s}} = 573\text{Hz}$$

The driver therefore hears the echo at:

$$f' = 573\text{Hz}$$

Question 10

(a) The flash and the thunder are produced together, but they reach the observer at very different speeds. Light travels at about $3 \times 10^8\text{m/s}$, so the flash arrives almost instantly. Sound travels through air at only about 340m/s, nearly a million times slower, so the thunder takes an appreciable time to cover the same distance. The further away the stroke, the longer the gap between the flash and the thunder.

(b) Heating in air grows a very thin, transparent film of oxide on the steel, and the film thickens the longer and hotter the heating. Light reflected from the top of the oxide film and light reflected from the steel beneath it interfere. The film thickness fixes the path difference, and so fixes which wavelength is reinforced and which cancelled, so the surface shows an interference colour. As the film thickens the reinforced wavelength changes, carrying the surface through the observed sequence of colours.

(c) The fundamental frequency of a wire fixed at both ends is $f = \frac{1}{2L}\sqrt{T/\mu}$. With the length and the linear density unchanged, $f \propto \sqrt{T}$, and therefore $T \propto f^2$.

The frequency is to rise in the ratio $f_2/f_1 = 3/2$, so:

$$\frac{T_2}{T_1} = \left(\frac{f_2}{f_1}\right)^2 = \left(\frac{3}{2}\right)^2 = 2.25$$

The tension must be increased to 2.25 times its original value.

Question 11

(a) Both systems, when displaced from equilibrium, experience a restoring force directed back towards the equilibrium position and proportional to the displacement from it. For the spring it is the elastic force $F = -kx$; for the pendulum at small angles it is the tangential component of gravity, which is likewise proportional to the displacement. Whenever the restoring force is proportional to the displacement and oppositely directed, the acceleration obeys $a = -\omega^2 x$, which is the defining condition of simple harmonic motion. The two systems differ in their physical make-up but share this one essential property.

(b) Where the protector and the screen nearly touch there is a very thin film of trapped air whose thickness varies from point to point. Light reflected from the lower surface of the protector and light reflected from the screen beneath it interfere. Where the air gap makes the path difference a whole number of wavelengths the interference is constructive for that colour, and where it makes an odd number of half-wavelengths it is destructive; since the gap thickness varies across the contact region, different colours are reinforced at different places, producing the coloured rings and bands. This is the same effect as Newton's rings.

(c) The observer moves towards a stationary source, so the observed frequency is raised. With the observer's speed u_o added to the speed of sound:

$$f' = f\frac{v + u_o}{v}$$

$$f' = 660\text{Hz} \times \frac{(340 + 20)\text{m/s}}{340\text{m/s}} = 660\text{Hz} \times \frac{360\text{m/s}}{340\text{m/s}}$$

The motorcyclist therefore hears:

$$f' = 699\text{Hz}$$

Question 12

(a) The fringes form a thin-film interference pattern, and a given fringe is the locus of all points at which the air gap between the curved lens and the flat plate has one particular thickness. The lens surface is part of a sphere resting on a plane, so the air gap has the same thickness at every point lying at a given distance from the point of contact. Points of equal gap thickness therefore lie on circles centred on the contact point, and so each fringe, bright or dark, is itself a circle.

(b) Springing off leaves the board both displaced and moving, so it has been given elastic potential energy and kinetic energy. With the diver gone, no further force drives it; the board simply oscillates freely at its own natural frequency, energy passing back and forth between the elastic strain of the bent board and the kinetic energy of its motion. This is a free oscillation, and it dies away only as the energy is gradually lost to air resistance and internal friction.

(c) For a point source the intensity falls with the inverse square of the distance:

$$\frac{I_2}{I_1} = \left(\frac{r_1}{r_2}\right)^2 = \left(\frac{2.0\text{m}}{8.0\text{m}}\right)^2 = \frac{1}{16}$$

The corresponding change in sound level is:

$$\Delta L = 10\log\left(\frac{I_2}{I_1}\right) = 10\log\left(\frac{1}{16}\right) = -12.0\text{dB}$$

The level at 8.0m is therefore:

$$L_2 = 88\text{dB} - 12.0\text{dB} = 76.0\text{dB}$$

Question 13

(a) An open pipe has a displacement antinode at each end, so its fundamental fits half a wavelength into the pipe: the fundamental wavelength is twice the pipe length. A pipe closed at one end has a node at the closed end and an antinode at the open end, so its fundamental fits only a quarter of a wavelength into the same length: its fundamental wavelength is four times the pipe length, twice that of the open pipe. Since frequency is inversely proportional to wavelength, the closed pipe sounds at half the frequency, which is one octave lower.

(b) The baby and the elastic strap together form a mass-on-a-spring system. The strap stretches under the baby's weight to an equilibrium position; if the baby is displaced below it the extra tension provides a restoring force proportional to the displacement, and above it the reduced tension does the same. The restoring force being proportional to displacement, the baby executes simple harmonic motion, with a regular period $T = 2\pi\sqrt{m/k}$ set by the baby's mass and the stiffness of the strap, hence the steady rhythm.

(c) The fringe spacing is $y = \frac{\lambda D}{d}$, so it is proportional to the wavelength of the light. On immersion in water the wavelength shortens to $\lambda_w = \lambda/n$, the frequency being unchanged, so the fringe spacing shrinks in the same ratio:

$$y_w = \frac{y}{n} = \frac{1.8\text{mm}}{1.33} = 1.35\text{mm}$$

Question 14

(a) In simple harmonic motion the maximum speed is $v_{max} = \omega A$ and the maximum acceleration is $a_{max} = \omega^2 A$; both are directly proportional to the amplitude, so doubling the amplitude doubles each of them. The period $T = 2\pi/\omega$ is fixed by the angular frequency alone, and ω is set by the properties of the system, the stiffness and mass or the length and gravity, not by how far the oscillator is displaced. Doubling the amplitude does not change ω, so the period is unchanged: the oscillator simply covers twice the distance each cycle, moving twice as fast to do so in the same time.

(b) The smooth curved wall of the dome reflects sound well. Sound spreading out from the whisper at one point strikes the curved surface and is reflected, and the curvature is such that the reflected sound is brought together again, or focused, at the second point. The whisper's energy, instead of spreading thinly in every direction, is concentrated by these reflections onto that one listening point, so it arrives there loud enough to hear, while points away from the focus receive only the faint direct sound.

(c) Light reflected from the top surface of the film undergoes a phase change of π, while light reflected from the bottom surface does not. Constructive interference in the reflected light, giving a strongly reflected colour, therefore requires:

$$2nt = \left(m + \frac{1}{2}\right)\lambda$$

The optical path is $2nt = 2(1.33)(1.10 \times 10^{-7}\text{m}) = 2.93 \times 10^{-7}\text{m}$. Taking the lowest order, $m = 0$:

$$\lambda = 2 \times (2.93 \times 10^{-7}\text{m}) = 5.85 \times 10^{-7}\text{m}$$

The most strongly reflected visible wavelength is 585nm.

(The next order, m = 1, would give 195nm, which lies in the ultraviolet.)

Question 15

(a) The speed of sound in a gas depends on how rapidly the molecules pass the disturbance from one to the next. In warmer air the molecules move with greater average speed, so they collide more frequently and transmit the compressions and rarefactions more quickly. The speed of sound in a gas is in fact proportional to the square root of the absolute temperature, so a rise in temperature raises the speed.

(b) Each fine scratch is a narrow transparent track of a width not enormously greater than the wavelength of light, and it diffracts the light passing through it, spreading that light out in the direction across the scratch. The bright point of a distant headlight is therefore smeared into a streak of light running perpendicular to the scratch. Because the wiper scratches lie along many arcs at different orientations, the diffracted light spreads in a range of directions, producing the spikes and streaks seen around each headlight.

(c) For a mass on a spring $T = 2\pi\sqrt{m/k}$, so the period is inversely proportional to the square root of the stiffness. Two identical springs side by side, sharing the load, act as a single spring of twice the stiffness:

$$k_p = 2k$$

The period therefore shortens by a factor of $\sqrt{2}$:

$$T_2 = \frac{T_1}{\sqrt{2}} = \frac{0.80\text{s}}{\sqrt{2}} = 0.57\text{s}$$

Question 16

(a) At the extremes of the motion the oscillator is momentarily at rest, so all its energy is potential. For a system with restoring force $F = -kx$, the potential energy stored at displacement x is $\frac{1}{2}kx^2$, and at the extreme, where $x = A$, this is $\frac{1}{2}kA^2$. Since the total energy is constant throughout the motion, in the absence of damping, and equals this maximum potential energy, the total energy is $\frac{1}{2}kA^2$, proportional to the square of the amplitude. Doubling the amplitude therefore quadruples the energy of the oscillation.

(b) The wire is a narrow obstacle of width comparable with the wavelength of light. As light from the distant lamp passes the wire it is diffracted, bending into the region behind the wire. Light arriving at a point in that region by the two paths passing the opposite sides of the wire has a path difference that depends on the direction; where the path difference is a whole number of wavelengths the waves reinforce, and where it is an odd number of half-wavelengths they cancel. This alternation produces the regular pattern of light and dark bands.

(c) An open pipe has a displacement antinode at each end, so its fundamental wavelength is twice the length of the pipe, $\lambda_1 = 2L$. Hence:

$$L = \frac{v}{2f_1} = \frac{340\text{m/s}}{2 \times 256\text{Hz}} = 0.664\text{m}$$

An open pipe sounds all the harmonics, so the second harmonic has twice the fundamental frequency:

$$f_2 = 2f_1 = 2 \times 256\text{Hz} = 512\text{Hz}$$

Question 17

(a) For a steady interference pattern the light reaching the two slits must be coherent: the two slits must always be illuminated by waves with a constant phase relationship. An ordinary lamp is an extended source whose different parts emit independently, so light reaching the two slits from it directly would have no fixed phase relationship. The single narrow slit acts as one small, effectively point-like source: the light spreading from it reaches both slits as parts of the same wavefront, so the two slits are illuminated coherently and a stable pattern is formed.

(b) When a person speaks, the sound reaches their own ears by two routes, through the air and directly through the bones and tissues of the head. Bone conduction carries the lower frequencies particularly well, so the voice a speaker normally hears is enriched in low frequencies and sounds fuller and deeper to them. A recording captures only the air-conducted sound; played back, it reaches the ears by the air route alone, without the bone-conducted bass, so it sounds thinner and unfamiliar to the speaker, though it is the voice that others always hear.

(c) The period depends only on the mass and the stiffness:

$$T = 2\pi\sqrt{\frac{m}{k}} = 2\pi\sqrt{\frac{0.30\text{kg}}{30\text{N/m}}} = 0.63\text{s}$$

At the equilibrium position the spring tension balances the weight, $mg = 0.30\text{kg} \times 9.8\text{m/s}^2 = 2.94\text{N}$. During the motion the tension varies above and below this by $kA = 30\text{N/m} \times 0.080\text{m} = 2.4\text{N}$. The greatest tension, at the lowest point, is:

$$F_{max} = mg + kA = 2.94\text{N} + 2.4\text{N} = 5.34\text{N}$$

And the least tension, at the highest point, is:

$$F_{min} = mg - kA = 2.94\text{N} - 2.4\text{N} = 0.54\text{N}$$

Question 18

(a) The displacement completes one full cycle in each period. The kinetic energy depends on the square of the speed. In one period of the displacement the oscillator passes through the equilibrium position twice, once moving each way, and at each passage the speed, and hence the kinetic energy, reaches its maximum; the kinetic energy falls to zero twice as well, at the two extremes. The kinetic energy therefore rises and falls twice in every single cycle of the displacement, so it varies at twice the frequency.

(b) The chest-piece picks up the heartbeat sounds, and the hollow tubes of the stethoscope carry the sound waves directly to the doctor's ears. Confined within the narrow tubes, the sound energy cannot spread out and weaken as it would in the open air, where it would diminish with the square of the distance. Almost all of the collected sound energy is therefore delivered to the ears instead of being dispersed, so the heartbeat is heard far more loudly than it would be by simply listening near the chest.

(c) The grating spacing is:

$$d = \frac{1\text{mm}}{500} = 2.0 \times 10^{-6}\text{m}$$

From the grating equation $d\sin\theta = m\lambda$, with $m = 2$ and $\theta = 36.0°$:

$$\lambda = \frac{d \sin\theta}{m} = \frac{(2.0 \times 10^{-6}\text{m})(\sin 36.0°)}{2}$$
$$\lambda = 5.88 \times 10^{-7}\text{m}$$

The wavelength of the light is 588nm.

Question 19

(a) When light strikes a surface, the component of its vibration parallel to the surface is reflected more strongly than the component lying in the plane of incidence. The reflected beam is therefore richer in the parallel vibration, that is, partially polarised. At one particular angle of incidence, Brewster's angle, the reflected and refracted rays are at right angles to each other; the vibration in the plane of incidence is then not reflected at all, and the reflected light consists solely of the vibration parallel to the surface, so it is completely polarised.

(b) The hanging load behaves as a simple pendulum, with the cable serving as the pendulum's length. The period of a simple pendulum is $T = 2\pi\sqrt{l/g}$: it increases as the square root of the length. Paying out more cable lengthens the pendulum, so the period grows and each swing takes longer, the load swinging more slowly. The mass of the load does not enter the formula, so it is the cable length alone that the operator alters when the swinging becomes lazier.

(c) The pulse travels out to the aircraft and back, so in the measured time it covers twice the distance to the aircraft:

$$2d = ct$$
$$d = \frac{ct}{2} = \frac{(3.00 \times 10^{8}\text{m/s})(1.20 \times 10^{-4}\text{s})}{2}$$

The aircraft is therefore:

$$d = 1.80 \times 10^{4}\text{m} = 18.0\text{km}$$

Question 20

(a) All three are electromagnetic waves: each consists of oscillating electric and magnetic fields, at right angles to each other and to the direction of travel, and each is produced by accelerating electric charges. They all travel through a vacuum at the same speed, 3×10^{8}m/s, and all show the wave behaviours of reflection, refraction, interference, diffraction and polarisation. They differ only in frequency, and so in wavelength; this difference gives them their different uses and effects, but their common nature makes them one single family, the electromagnetic spectrum.

(b) Toughened glass is made by cooling its surfaces rapidly, which leaves the glass with permanent built-in stresses that vary from place to place. Stressed glass is birefringent: it affects the two perpendicular components of a light wave differently, altering the polarisation of light passing through it by an amount that depends on the local stress. The daylight reaching the window is already partly polarised, and the sunglasses act as an analysing polariser. Where the stress in the glass has turned the polarisation away from the sunglasses' axis, less light passes and a dark blotch is seen; the varying stress thus shows itself as a pattern of light and dark patches.

(c) From the period formula $T = 2\pi\sqrt{l/g}$, the length is:

$$l = \frac{gT^2}{4\pi^2} = \frac{(9.8\text{m/s}^2)(2.0\text{s})^2}{4\pi^2} = 0.993\text{m}$$

Since $T \propto \sqrt{l}$, a small fractional change in length produces half that fractional change in period:

$$\frac{\Delta T}{T} = \frac{1}{2}\frac{\Delta l}{l} = \frac{1}{2}(1.0\%) = 0.5\%$$

The period increases by 0.5%.

Question 21

(a) The natural frequency of a system is the frequency at which it oscillates on its own once it has been displaced and released, with no external driving force acting. It is fixed entirely by the properties of the system. For a mass on a spring it is set by the force constant of the spring and the mass, through $f = \frac{1}{2\pi}\sqrt{k/m}$. For a simple pendulum it is set by the length of the pendulum and the acceleration of free fall, through $f = \frac{1}{2\pi}\sqrt{g/l}$.

(b) Each spectator only rises and sits down again about their own seat; the disturbance, the act of standing, is passed on from each spectator to the next, so the pattern of standing sweeps around the ground. What travels round the stadium is the disturbance and the energy carried with it, while the spectators, who form the medium, merely oscillate in place. This is the defining feature of a wave: the wave profile and its energy move forward, the medium itself does not.

(c) The clap travels from the girl to the cliff and back, covering twice her distance from the cliff, so:

$$d = \frac{vt}{2} = \frac{(340\text{m/s})(1.5\text{s})}{2} = 255\text{m}$$

After walking 60m closer, her new distance from the cliff is $255\text{m} - 60\text{m} = 195\text{m}$. The echo now covers twice this distance:

$$t' = \frac{2d'}{v} = \frac{2(195\text{m})}{340\text{m/s}} = 1.15\text{s}$$

Question 22

(a) Sound is carried forward by each particle of the medium disturbing its neighbours, and the speed depends on how stiffly the particles are coupled and on the density of the medium. In a solid or a liquid the particles are much more closely bound, and the elastic restoring forces between them far stronger, than in a gas, so a disturbance is handed on from particle to particle far more rapidly. Solids and liquids are also denser, which on its own would slow the sound, but the great increase in stiffness outweighs the increase in density, so the net result is a much higher speed.

(b) On the stretched cord the jumper oscillates up and down: below the equilibrium extension the cord pulls her up, above it her weight pulls her down, giving a restoring force and so an oscillation that is close to simple harmonic. The motion is damped. At each bounce some mechanical energy is lost, to air resistance and to internal friction within the stretching cord, and is carried away as heat. With less energy in each successive cycle the amplitude steadily falls, bounce by bounce, until the surplus energy is spent and she hangs at rest at the equilibrium extension.

(c) The wavelength follows from $v = f\lambda$:

$$\lambda = \frac{v}{f} = \frac{1.8\text{m/s}}{2.5\text{Hz}} = 0.72\text{m}$$

One whole wavelength corresponds to a phase difference of 2π radians, so two points a distance d apart differ in phase by:

$$\Delta\varphi = 2\pi\left(\frac{d}{\lambda}\right) = 2\pi\left(\frac{0.30\text{m}}{0.72\text{m}}\right)$$

which gives:

$$\Delta\varphi = \frac{5\pi}{6}\text{rad} = 2.6\text{rad}$$

Question 23

(a) The join is a boundary between two media along which the wave travels at different speeds, the wave being slower on the heavier rope. A wave meeting such a boundary cannot pass on undisturbed: the two ropes cannot keep their motions matched at the join unless some of the wave is sent back. Part of the energy is therefore carried forward into the heavy rope as a transmitted wave, and the rest is returned along the light rope as a reflected wave. Because it meets a heavier, slower medium, the reflected pulse is also inverted.

(b) Sound spreads out when it passes the edge of an obstacle or an opening, an effect called diffraction, and the spreading is large when the wavelength is comparable with, or larger than, the obstacle. The wavelength of ordinary speech is about a metre, comparable with the size of a wall or a doorway, so the sound diffracts strongly and bends round the corner to reach the listener. Light has a wavelength far smaller than any everyday obstacle, so it diffracts only negligibly and travels on in straight lines; hence the speaker cannot be seen.

(c) For simple harmonic motion the maximum speed is $v_{max} = \omega A$ and the maximum acceleration is $a_{max} = \omega^2 A$. Dividing one by the other eliminates the amplitude:

$$\omega = \frac{a_{max}}{v_{max}} = \frac{4.5\text{m/s}^2}{0.30\text{m/s}} = 15\text{rad/s}$$

The period follows from the angular frequency:

$$T = \frac{2\pi}{\omega} = \frac{2\pi}{15\text{rad/s}} = 0.42\text{s}$$

and the amplitude from $v_{max} = \omega A$:

$$A = \frac{v_{max}}{\omega} = \frac{0.30\text{m/s}}{15\text{rad/s}} = 0.020\text{m}$$

Question 24

(a) Take the displacement as $x = A\sin\omega t$. The velocity is the rate of change of displacement, $v = A\omega\cos\omega t$, and the acceleration is $a = -A\omega^2\sin\omega t$. The velocity is a quarter of a cycle, a phase angle of $\frac{\pi}{2}$, ahead of the displacement: it is greatest as the body crosses the centre, where the displacement is zero, and zero at the extremes, where the displacement is greatest. The acceleration is exactly half a cycle, a phase angle of π, out of step with the displacement: it is always directed opposite to the displacement, greatest at the extremes and zero at the centre.

(b) The ship sends a short pulse of sound vertically downwards. The pulse travels down to the sea-bed, is reflected there, and travels back up to a detector on the ship. The instrument measures the time between sending the pulse and receiving its echo. In that time the pulse has covered the depth twice, and since the speed of sound in seawater is known, the depth is half the product of that speed and the measured time. A longer delay therefore means deeper water.

(c) The speed of a transverse wave on a string is set by the tension and the linear mass density:

$$v = \sqrt{\frac{T}{\mu}} = \sqrt{\frac{50\text{N}}{2.0 \times 10^{-3}\text{kg/m}}} = 158\text{m/s}$$

The wavelength then follows from $v = f\lambda$:

$$\lambda = \frac{v}{f} = \frac{158\text{m/s}}{100\text{Hz}} = 1.58\text{m}$$

Question 25

(a) While the ambulance approaches, each successive compression of the sound wave is sent out from a point closer to the listener than the one before, so the compressions arrive more closely spaced: the effective wavelength is shortened and the frequency raised. While the ambulance recedes, each compression is sent out from a point further away, so they arrive more widely spaced: the wavelength is lengthened and the frequency lowered. At the instant of passing, the source changes from approaching to receding, so the pitch heard drops from the raised value to the lowered one.

(b) The stone gives the water a fixed amount of energy, and this energy is spread along the expanding circular ripple. As the ripple travels outwards its circumference grows, so the same energy must be shared along an ever-greater length of crest, and the energy carried by each portion of the ripple steadily decreases. Since the energy of a wave depends on the square of its amplitude, this falling energy shows itself as a falling amplitude, and the ripples grow weaker as they spread. Some energy is also lost in overcoming the viscosity of the water.

(c) For a mass on a spring $T = 2\pi\sqrt{m/k}$, which rearranges to give the force constant:

$$k = \frac{4\pi^2 m}{T^2} = \frac{4\pi^2(0.25\text{kg})}{(0.50\text{s})^2} = 39.5\text{N/m}$$

Since $T \propto \sqrt{m}$, doubling the period to 1.0s requires the mass to become four times as large, that is $4 \times 0.25\text{kg} = 1.0\text{kg}$.

Checking with the period formula:

$$m' = \frac{kT'^2}{4\pi^2} = \frac{(39.5\text{N/m})(1.0\text{s})^2}{4\pi^2} = 1.0\text{kg}$$

The further mass that must be added is therefore:

$$\Delta m = 1.0\text{kg} - 0.25\text{kg} = 0.75\text{kg}$$

Question 26

(a) Huygens' principle states that every point on a wavefront may be regarded as a source of secondary spherical wavelets, which spread out forwards with the speed of the wave, and that the new wavefront a short time later is the surface that just touches, the envelope of, all these secondary wavelets. To find how a wavefront advances, one draws from each point of the present wavefront a small wavelet, every wavelet of the same radius, equal to the wave speed multiplied by the time elapsed, and then draws the surface tangent to them all. That tangent surface is the wavefront in its new position.

(b) The metronome arm carrying its weight is an oscillating system that swings to and fro, and the rate of ticking is simply its natural frequency. That natural frequency depends on how the mass is distributed about the pivot. Sliding the weight outwards, away from the pivot, increases the rotational inertia of the arm, so each swing takes longer and the ticking slows. Sliding the weight inwards, towards the pivot, reduces the rotational inertia, so each swing is quicker and the ticking speeds up. The musician sets the tempo simply by choosing where the weight sits.

(c) The speed of sound in a gas is proportional to the square root of its absolute temperature, $v \propto \sqrt{T}$. Hence the speed at the higher temperature is:

$$v_2 = v_1\sqrt{\frac{T_2}{T_1}} = (340\text{m/s})\sqrt{\frac{308\text{K}}{288\text{K}}} = 352\text{m/s}$$

The wavelength of the 480Hz note follows from $\lambda = v/f$. At 15°C:

$$\lambda_1 = \frac{v_1}{f} = \frac{340\text{m/s}}{480\text{Hz}} = 0.708\text{m}$$

and at 35°C:

$$\lambda_2 = \frac{v_2}{f} = \frac{352\text{m/s}}{480\text{Hz}} = 0.733\text{m}$$

Question 27

(a) The pitch of a note, how high or low it sounds, is determined by the frequency of the sound wave: a higher frequency is heard as a higher pitch. The loudness of the note, how strong or faint it sounds, is determined by the intensity of the wave, and so by the amplitude of the vibration, since the intensity depends on the square of the amplitude: a larger amplitude is heard as a louder sound. The two properties are independent: two notes may share the same pitch but differ in loudness, or share the same loudness but differ in pitch, because each depends on a different feature of the wave.

(b) A person sitting on a swing is a simple pendulum, and the period of a simple pendulum, $T = 2\pi\sqrt{l/g}$, depends only on the length of the swing and on the acceleration of free fall, not on the mass of the person. A heavier person is pulled towards the lowest point by a larger gravitational restoring force, but that same person also has correspondingly greater inertia and so is harder to set moving; the two effects cancel exactly. The time for one complete to-and-fro swing is therefore the same for the heavy adult and the light child on a swing of the same length.

(c) The pulse travels to the far post and back, a total distance of $2 \times 100\text{m}$, at 40m/s:

$$t = \frac{2L}{v} = \frac{2(100\text{m})}{40\text{m/s}} = 5.0\text{s}$$

The speed of a wave on the cable is $v = \sqrt{T/\mu}$, so $v \propto \sqrt{T}$. Raising the tension to four times its value multiplies the speed by $\sqrt{4} = 2$, giving a new speed of 80m/s. The new return time is therefore:

$$t' = \frac{2L}{v'} = \frac{2(100\text{m})}{80\text{m/s}} = 2.5\text{s}$$

Question 28

(a) A wavefront is the surface reached by all the parts of the wave that are in step with one another. Close to a small source the wave has spread only a short distance, and the wavefront is a small circle, or sphere, centred on the source, with a pronounced curvature. As the wave travels outwards the radius of this circle grows; far from the source the radius is very large, and any small portion of so large a circle, which is the part an observer actually sees, is almost indistinguishable from a straight line. The wavefront therefore appears practically straight, a plane wavefront, at a great distance from the source.

(b) Ultrasound has a far higher frequency than audible sound, and therefore a much shorter wavelength. A wave can resolve, and so form a clear image of, detail only about as small as its own wavelength; the very short wavelength of ultrasound allows the fine structure of the baby to be made out, which long-wavelength audible sound could never show. Ultrasound is also strongly reflected at the boundaries between the different soft tissues, providing the echoes from which the image is built, and it can be sent in a narrow, well-directed beam. It is, in addition, safe for both mother and baby.

(c) The total energy of a simple harmonic oscillator is $E = \frac{1}{2}kA^2$, which gives the force constant:

$$k = \frac{2E}{A^2} = \frac{2(0.16\text{J})}{(0.080\text{m})^2} = 50\text{N/m}$$

At the centre of the motion all the energy is kinetic, $E = \frac{1}{2}m{v_{max}}^2$, which gives the maximum speed:

$$v_{max} = \sqrt{\frac{2E}{m}} = \sqrt{\frac{2(0.16\text{J})}{0.50\text{kg}}} = 0.80\text{m/s}$$

Question 29

(a) A driven system takes in energy from the driving force most effectively when it is pushed in step with its own natural motion. When the driving frequency matches the natural frequency, each push from the driving force acts in the same direction as the system's velocity, and so does positive work on it cycle after cycle, feeding energy in steadily. The amplitude grows until the rate at which energy is fed in is balanced by the rate at which it is lost to damping; when the damping is light this balance is reached only at a very large amplitude. At driving frequencies away from the natural frequency the force is repeatedly out of step, often opposing the motion, so far less energy is delivered and the amplitude stays small.

(b) Sound striking a surface is partly reflected and partly absorbed. Bare hard walls and floors reflect sound strongly and absorb very little, so a voice is reflected back and forth many times and dies away only slowly; the sound rings on and the room seems hollow. Soft furnishings, such as curtains, carpets and padded chairs, are good absorbers: their soft, porous surfaces convert the energy of the sound into heat at each encounter. Once the room is furnished the reflected sound is quickly absorbed, the ringing dies away rapidly, and the hollow quality disappears.

(c) In a stationary wave the distance from a node to the antinode next to it is one quarter of a wavelength, so:

$$\lambda = 4 \times 0.12\text{m} = 0.48\text{m}$$

The string vibrates 25 times each second, so $f = 25\text{Hz}$, and the speed of the waves follows from $v = f\lambda$:

$$v = f\lambda = (25\text{Hz})(0.48\text{m}) = 12\text{m/s}$$

Question 30

(a) The ear and brain cannot separate two sounds that arrive within about a tenth of a second of each other; such sounds merge into one. For a reflected sound to be heard as a distinct echo it must therefore arrive at least about a tenth of a second after the direct sound. The reflected sound has to travel from the listener to the surface and back again, so the surface must be far enough away for this extra journey to take more than about a tenth of a second; at the speed of sound in air this means a reflecting surface more than roughly 17m away. A nearer surface returns the sound too soon, and it is heard only as a prolonging of the original.

(b) The turned skipping rope is held at its two ends and carries a stationary wave. The held ends cannot move, so they must be nodes, points of zero displacement, while the middle is free to swing out fully and is an antinode. The simplest stationary wave that has a node at each end with one antinode between them is a single loop, half a wavelength long; this is the fundamental mode, the one set up at the steady, moderate rate at which the children naturally turn the rope. The rope therefore settles into one broad loop, its ends almost still and its middle sweeping out the widest path.

(c) For simple harmonic motion the speed at a displacement x is given by $v^2 = \omega^2(A^2 - x^2)$. Applying this at the two stated points:

$$(8.0\text{cm/s})^2 = \omega^2(A^2 - (6.0\text{cm})^2)$$
$$(6.0\text{cm/s})^2 = \omega^2(A^2 - (8.0\text{cm})^2)$$

Dividing the first equation by the second eliminates ω:

$$\frac{64}{36} = \frac{A^2 - 36}{A^2 - 64}$$

Cross-multiplying and solving for the amplitude:

$$64(A^2 - 64) = 36(A^2 - 36)$$
$$A = 10\text{cm}$$

Substituting back into the first equation gives the angular frequency:

$$\omega^2 = \frac{64}{A^2 - 36} = \frac{64}{64} = 1.0$$
$$\omega = 1.0\text{rad/s}$$

And hence the period:

$$T = \frac{2\pi}{\omega} = \frac{2\pi}{1.0\text{rad/s}} = 6.3\text{s}$$

Question 31

(a) In simple harmonic motion the speed is greatest at the centre and falls to zero at the extremes, since $v = \omega\sqrt{A^2 - x^2}$. Near the extremes the body is moving slowly, so it takes a relatively long time to cover any given small stretch of its path there; near the centre it is moving quickly and passes through in a short time. Over one complete cycle, therefore, more of the total time is spent in the slow-moving regions near the two extremes than in the fast-moving region near the centre.

(b) The breakwater has an end, and the open water beyond that end acts as a gap past which the sea waves travel. Waves passing the end of the wall are diffracted: they spread sideways into the region of calm water behind the wall. Because the wavelength of the sea waves is not small compared with the wall, this spreading is appreciable, so some of the wave energy curls round into the sheltered water and reaches the moored boats, setting them gently rocking. The waves there are weaker than in the open sea, since only the diffracted part of the wave arrives.

(c) The grating equation is $d\sin\theta = m\lambda$. For the first-order image, $m = 1$, the line spacing is:

$$d = \frac{m\lambda}{\sin\theta} = \frac{(1)(590 \times 10^{-9}\mathrm{m})}{\sin 18.0°} = 1.91 \times 10^{-6}\mathrm{m}$$

This spacing is 1.91×10^{-3}mm, so the number of lines per millimetre of the grating is:

$$N = \frac{1\mathrm{mm}}{1.91 \times 10^{-3}\mathrm{mm}} = 524$$

Question 32

(a) At a dark fringe the light from the two slits arrives exactly out of step, the crest of one wave coinciding with the trough of the other. By the principle of superposition the two displacements cancel, so the resultant disturbance, and with it the intensity, is zero. The light energy is not destroyed but redistributed. Where the two waves arrive in step, at the bright fringes, they reinforce, and the intensity there is greater than the sum of the two separate intensities. Averaged over the whole pattern the total energy is exactly that supplied by the two slits; interference simply moves energy from the dark fringes to the bright ones.

(b) The clamped blade, with its free end, is an oscillating system; once twanged it vibrates freely at its own natural frequency, which is heard as the pitch of the buzz. That natural frequency depends on the stiffness of the vibrating part and on its mass. Sliding more of the blade out beyond the bench lengthens the vibrating portion: a longer projecting blade bends more easily, so it is less stiff, and it also carries more mass. Both changes lower the natural frequency, and so the buzz is heard to deepen.

(c) A string fixed at both ends and vibrating in its third harmonic carries three half-wavelengths along its length, so:

$$\lambda = \frac{2L}{3} = \frac{2(0.90\mathrm{m})}{3} = 0.60\mathrm{m}$$

The speed of the waves on the wire then follows from $v = f\lambda$:

$$v = f\lambda = (200\mathrm{Hz})(0.60\mathrm{m}) = 120\mathrm{m/s}$$

Question 33

(a) A stationary wave is formed by two identical waves travelling through the medium in opposite directions. At a node the two waves always arrive exactly out of step: whatever displacement one wave would give to that point, the other gives an equal and opposite displacement at the same instant, and this holds at every instant. By the principle of superposition the two displacements cancel completely and permanently, so the medium at a node never moves. Each wave on its own would indeed move that point, but their contributions there are equal and opposite at all times, and their sum is always zero.

(b) Striking the fork harder gives its prongs more energy and so a larger amplitude of vibration, which is heard as a louder sound. But the frequency at which the prongs vibrate, and it is the frequency that fixes the musical note, is the natural frequency of the fork, set by the stiffness and the mass of the prongs. This natural frequency does not depend on the amplitude; the oscillation is isochronous. So however hard or gently the fork is struck, it vibrates at the same frequency and sounds the same note, differing only in loudness.

(c) The spacing of the bright fringes in a Young's pattern is $y = \frac{\lambda D}{d}$, which rearranges to give the wavelength:

$$\lambda = \frac{yd}{D} = \frac{(2.1 \times 10^{-3}\mathrm{m})(0.40 \times 10^{-3}\mathrm{m})}{1.5\mathrm{m}}$$

which gives:

$$\lambda = 5.6 \times 10^{-7}\mathrm{m}$$

Question 34

(a) A prism spreads light by refraction. Glass has a slightly greater refractive index for violet light than for red, so violet light is slowed and bent the more; violet is deviated most and red least. A grating spreads light by diffraction and interference: the direction of a bright order is fixed by $d\sin\theta = m\lambda$, so the angle of deviation increases with wavelength. Red light, having the longer wavelength, satisfies the grating equation at a larger angle than violet, so with a grating red is deviated most and violet least. The two instruments order the colours oppositely because they spread the light by entirely different mechanisms.

(b) Pushing and pulling the end of the coil squeezes the nearby turns together, forming a compression, and then draws them apart, forming a rarefaction. Each disturbed turn pushes and pulls on the next, so the compression is handed on from turn to turn and a longitudinal wave travels to the far end. Each individual turn of the coil only moves a short way back and forth about its own position; it is the pattern of compression, together with the energy carried with it, that travels the length of the coil.

(c) The force constant measures the force needed for each unit of extension. A given pull stretches the whole spring twice as far as it stretches either half alone, so each half is twice as stiff as the whole spring:

$$k' = 2k = 2(18\mathrm{N/m}) = 36\mathrm{N/m}$$

The period of the 0.40kg mass on one half-spring is then:

$$T = 2\pi\sqrt{\frac{m}{k'}} = 2\pi\sqrt{\frac{0.40\mathrm{kg}}{36\mathrm{N/m}}} = 0.66\mathrm{s}$$

Question 35

(a) When a mass hangs on a vertical spring, gravity stretches the spring to a new equilibrium position at which the spring's pull exactly balances the weight. If the mass is then displaced from this position, the extra force that restores it is supplied entirely by the change in the spring's tension, just as it is for a horizontal spring displaced from its own equilibrium. The constant weight is already balanced and plays no further part in the motion. The restoring force per unit displacement is therefore the force constant of the spring in both cases, so the period $T = 2\pi\sqrt{m/k}$ is the same; gravity only shifts the equilibrium position, it does not change the period.

(b) The mist consists of a great many tiny water droplets. Light from the street lamp passing the droplets is diffracted, spreading slightly as it passes each small droplet, and the diffracted light from the many droplets interferes. The angle through which the light is bent depends on its wavelength, so each colour is reinforced at a slightly different angle away from the lamp. The eye therefore sees the lamp ringed by faint bands in which the colours are separated, a corona produced by diffraction.

(c) The wavelength follows from $v = f\lambda$:

$$\lambda = \frac{v}{f} = \frac{8.0\mathrm{m/s}}{20\mathrm{Hz}} = 0.40\mathrm{m}$$

The number of complete waves contained in a 12m length is that length divided by one wavelength:

$$n = \frac{12\mathrm{m}}{0.40\mathrm{m}} = 30$$

Question 36

(a) As a wave passes, each particle of the medium is set oscillating, and the energy handed to a particle is the energy of its oscillation. The energy of an oscillating particle is proportional to the square of its amplitude: its greatest kinetic energy is $\frac{1}{2}m{v_{max}}^2$, and with $v_{max} = \omega A$ this is proportional to A^2. The wave passes this energy on from particle to particle, so the energy carried past any point each second, which is the power of the wave, is likewise proportional to the square of the amplitude. Doubling the amplitude therefore carries four times the energy, although the speed and frequency of the wave are unchanged.

(b) Light from each headlamp, on entering the eye, passes through the pupil, which is a small circular opening, and is diffracted there. Instead of forming a perfect point on the retina, each lamp forms a small spread-out patch of light. When the car is far away the two lamps are very close together in angle, so their two patches overlap and merge into what looks like a single light. As the car comes nearer the angle between the lamps grows; once it is large enough for the two patches to be seen apart, the lamps are made out as two separate lights.

(c) The total energy is $E = \frac{1}{2}kA^2$ and the potential energy at a displacement x is $\frac{1}{2}kx^2$, so $\frac{PE}{E} = \left(\frac{x}{A}\right)^2$. When the kinetic energy is three times the potential energy, the potential energy is one quarter of the total:

$$\left(\frac{x}{A}\right)^2 = \frac{1}{4}$$

$$x = \frac{A}{2} = \frac{0.10\mathrm{m}}{2} = 0.050\mathrm{m}$$

The kinetic energy is whatever part of the total is not potential, so the fraction that is kinetic is $1 - \left(\frac{x}{A}\right)^2$. At a displacement of 0.080m:

$$\frac{KE}{E} = 1 - \left(\frac{0.080\mathrm{m}}{0.10\mathrm{m}}\right)^2 = 0.36$$

Question 37

(a) The period of a mass on a spring is $T = 2\pi\sqrt{m/k}$. The period depends not on the mass itself but on its square root. Doubling the mass therefore multiplies the period not by 2 but by $\sqrt{2}$. The heavier mass does have more inertia and so is accelerated less by the same restoring force, which lengthens the period; but the period responds only to the square root of the mass, because it is $\sqrt{m}$, and not m, that appears in the formula. The period therefore rises by a factor of $\sqrt{2}$, about 1.4, and not by a factor of 2.

(b) A whip is not uniform: it tapers, growing thinner and lighter towards the tip. The speed of a transverse wave along it is $v = \sqrt{T/\mu}$, which is greater where the mass per unit length is smaller. As the wave runs towards the thin tip the mass per unit length steadily decreases, so the wave travels faster and faster. At the same time the energy carried by the wave is being delivered to less and less mass, so the amplitude of the motion grows, until the tip is flicked so violently that it produces the sharp crack.

(c) An ideal polaroid transmits one half of the intensity of unpolarised light, whatever its orientation, so after the first polaroid:

$$I_1 = \frac{I_0}{2} = \frac{16\mathrm{W/m^2}}{2} = 8.0\mathrm{W/m^2}$$

The light is now plane-polarised. By Malus's law the second polaroid, with its axis at 60° to the first, transmits:

$$I_2 = I_1(\cos 60°)^2 = (8.0\mathrm{W/m^2})(0.50)^2 = 2.0\mathrm{W/m^2}$$

Question 38

(a) A bright fringe is formed wherever the path difference between the light from the two slits is a whole number of wavelengths: 0, λ, 2λ, 3λ and so on. In passing from one bright fringe to the next, the path difference increases by exactly one wavelength. For a screen far from the slits the path difference changes in direct proportion to the distance moved along the screen, so equal steps of one wavelength in path difference correspond to equal steps of distance on the screen. The bright fringes are therefore evenly spaced, the spacing being $\frac{\lambda D}{d}$.

(b) The child and the spring of the pogo stick together form a mass-on-a-spring system. When the spring is compressed below the equilibrium position it pushes back harder than the child's weight, and the restoring force is proportional to how far the spring is compressed. A restoring force proportional to the displacement is the condition for simple harmonic motion, whose period $T = 2\pi\sqrt{m/k}$ is fixed by the mass of the child and the stiffness of the spring, and does not depend on how hard the child bounces. Each bounce therefore takes the same time, giving the steady rhythm.

(c) A particle of the string performs simple harmonic motion, so its maximum speed is $v_{max} = \omega A$, with $\omega = 2\pi f$:

$$v_{max} = 2\pi f A = 2\pi(12\mathrm{Hz})(0.015\mathrm{m}) = 1.1\mathrm{m/s}$$

The wave itself travels along the string at 9.0m/s. The maximum speed of a particle, 1.1m/s, is therefore much smaller than the speed of the wave; the two are quite distinct quantities.

Question 39

(a) When a train of waves crosses obliquely into shallower water it travels more slowly there. The wavefronts meet the boundary at an angle, so one end of a wavefront enters the shallow water and is slowed while the other end, still in deeper water, continues at the higher speed. Each wavefront therefore swings round as it crosses, and the whole train changes its direction of travel. The frequency is unchanged, so since $v = f\lambda$ the slower waves also have a shorter wavelength; the change of direction is the result of the two ends of each wavefront advancing unequally.

(b) When a wave tilts the boat to one side, the upward push of the water, the buoyancy force, and the weight of the boat no longer act along the same line, and together they produce a turning effect that acts to right the boat. This restoring turning effect grows with the angle of tilt, so once the wave has passed the boat rolls to and fro about its upright position in an oscillation that is close to simple harmonic. The rhythm of the roll is the boat's own natural period, fixed by its shape and by the way its mass is distributed, not by the wave that started the rolling.

(c) For Newton's rings the radius of the n-th dark ring is given by $r_n^2 = nR\lambda$, which rearranges to give the wavelength:

$$\lambda = \frac{r_n^2}{nR} = \frac{(2.4 \times 10^{-3}\mathrm{m})^2}{(8)(1.2\mathrm{m})}$$

which gives:

$$\lambda = 6.0 \times 10^{-7}\mathrm{m}$$

Question 40

(a) Sunlight passing through the atmosphere is scattered by the air molecules, which are very much smaller than the wavelength of light. For scattering by such small particles the amount of scattering is far greater for short wavelengths than for long ones, being inversely proportional to the fourth power of the wavelength (Rayleigh scattering). Blue light, having a shorter wavelength than red, is therefore scattered much more strongly. Light reaching the observer's eye from all directions of the sky is this scattered light, in which the blue is dominant, so the sky appears blue.

(b) As waves approach a shelving beach the water grows shallower towards the shore, and waves travel more slowly in shallower water. When a wavefront comes in at an angle, the end of it nearer the shore is in shallower water and moves more slowly than the end still in deeper water. The slow end is held back while the fast end advances, so the wavefront steadily swings round until it lies almost parallel to the shoreline. By the time the waves reach the beach they are travelling nearly straight in, whatever angle they had far out; this turning is the refraction of the waves.

(c) In one complete oscillation the particle travels from one extreme to the other and back again, covering the amplitude four times, so the total path length is 4A:

$$A = \frac{24\text{cm}}{4} = 6.0\text{cm}$$

The angular frequency is $\omega = 2\pi/T$, and the maximum speed is $v_{max} = \omega A$:

$$\omega = \frac{2\pi}{T} = \frac{2\pi}{0.40\text{s}} = 15.7\text{rad/s}$$

$$v_{max} = \omega A = (15.7\text{rad/s})(0.060\text{m}) = 0.94\text{m/s}$$

and the maximum acceleration is $a_{max} = \omega^2 A$:

$$a_{max} = \omega^2 A = (15.7\text{rad/s})^2(0.060\text{m}) = 14.8\text{m/s}^2$$