

EXAMINATION QUESTIONS ON EXTENDED MECHANICS

Question 1

- (a) A solid cylinder and a hollow cylinder of the same mass and radius are released simultaneously from rest at the top of an inclined plane. Both roll without slipping. The solid cylinder reaches the bottom first. Account for this observation.
- (b) A wet umbrella is spun rapidly and water droplets fly off the edges. Explain why the droplets leave the umbrella and why they travel in straight lines after leaving.
- (c) A uniform solid disc of mass 4.0kg and radius 0.30m is mounted on a frictionless axle. A light cord is wrapped around the rim and pulled with a constant force of 12N. Calculate the angular acceleration of the disc and the angular velocity after 5.0s from rest.

Question 2

- (a) An ice skater spins slowly with arms outstretched, then pulls her arms close to her body and spins much faster. No external torque acts on her. Account for the increase in angular velocity.
- (b) Water rises higher in a narrow glass tube than in a wide one when both are dipped into the same beaker. Explain why.
- (c) A soap bubble of radius 3.0cm ($\gamma = 0.025\text{Nm}^{-1}$) is blown at the end of a tube in a room where atmospheric pressure is $1.01 \times 10^5\text{Pa}$. Calculate the total pressure inside the bubble.

Question 3

- (a) A flywheel in a machine continues to rotate for a long time after the motor is switched off. A bicycle wheel with the same mass but smaller radius stops much sooner under similar frictional conditions. Account for this difference.
- (b) During a storm in Bagamoyo, a corrugated iron roof is lifted off a house even though the wind blows horizontally. Explain how a horizontal wind produces a vertical force on the roof.
- (c) A steel ball of radius 2.0mm ($\rho = 7800\text{kgm}^{-3}$) is dropped into glycerine ($\rho = 1260\text{kgm}^{-3}$, $\eta = 1.5\text{Nsm}^{-2}$). Calculate the terminal velocity of the ball. Take $g = 9.8\text{ms}^{-2}$.

Question 4

- (a) A rotating platform slows down when a person walks from the centre to the edge, but speeds up when the person walks back to the centre. Explain why.
- (b) A pond skater walks on water, but a small coin placed gently on the same surface sinks. Both are denser than water. Explain why the insect is supported but the coin is not.
- (c) A uniform rod of mass 3.0kg and length 1.2m is pivoted at one end and released from a horizontal position. Calculate the angular velocity of the rod as it passes through the vertical position. Take $g = 9.8\text{ms}^{-2}$.

Question 5

- (a) A ball rolling on a flat surface reaches the edge of a table and falls off. Even while falling, the ball continues to spin. Explain why the spin persists during the fall.
- (b) A doctor listens with a stethoscope and hears a faint murmur in a patient's artery. Explain what physical change in the artery causes this sound.
- (c) Water ($\eta = 1.0 \times 10^{-3}\text{Nsm}^{-2}$) flows through a horizontal pipe of internal radius 4mm and length 1.5m under a pressure difference of 600Pa. Calculate the volume flow rate and the maximum velocity at the centre of the pipe.

Question 6

- (a) Two gears of different radii are meshed together. When the smaller gear is turned, the larger gear rotates more slowly but with greater torque. Account for this observation.
- (b) Oil spilled from a tanker spreads rapidly on the ocean surface rather than remaining as a thick blob. Explain why, using the concept of surface energy.
- (c) A solid sphere of mass 2.0kg and radius 0.10m rolls without slipping down an incline of height 3.0m. Calculate the speed of the sphere at the bottom. Take $g = 9.8\text{ms}^{-2}$.

Question 7

- (a) A spinning top remains upright while spinning fast but falls over when it slows down. Account for this behaviour using the concept of angular momentum.
- (b) A capillary tube dipped in mercury shows a depression rather than a rise. Explain why mercury behaves differently from water in a glass tube.
- (c) A flat circular disc of radius 12cm rotates on a thin film of oil ($\eta = 0.80\text{Nsm}^{-2}$, thickness 1.0mm) at 120 revolutions per minute. Calculate the torque required to maintain the rotation.

Question 8

- (a) A solid sphere and a thin hollow spherical shell of equal mass and radius are given the same angular velocity. More work is needed to stop the shell than to stop the solid sphere. Account for this.
- (b) When Kipute blows air between two sheets of paper held parallel to each other, the sheets move toward each other instead of apart. Explain this observation.
- (c) A cylindrical tank of cross-sectional area 0.50m^2 is filled with water to a height of 4.0m. A small orifice of area 2.0cm^2 is opened at the base. Calculate the initial speed of efflux and the initial volume flow rate. Take $g = 9.8\text{ms}^{-2}$.

Question 9

- (a) A door is much easier to open by pushing near the handle than near the hinge, even though the same force is applied. Account for this observation.
- (b) A newly waxed car repels rainwater, which forms small beads on the surface. An unwaxed car allows water to spread into flat patches. Explain the difference.
- (c) Two spheres P and Q are made of the same material. Sphere P has radius r and sphere Q has radius $3r$. Both are dropped into the same viscous liquid. Find the ratio of their terminal velocities.

Question 10

- (a) A helicopter's tail rotor prevents the body of the helicopter from spinning. Explain why the body would spin without the tail rotor.
- (b) Fine clay particles stirred into a glass of water remain suspended for hours, while coarse sand grains settle within seconds. Explain this difference.
- (c) A uniform solid cylinder of mass 5.0kg and radius 0.20m rotates about its axis at 10rad s^{-1} . A braking torque of 0.50Nm is applied. Calculate the time taken for the cylinder to stop and the number of revolutions it makes before stopping.

Question 11

- (a) A uniform disc can be made to rotate about an axis through its centre or about an axis through its edge. The disc requires more torque to achieve the same angular acceleration when rotated about the edge. Account for this difference.
- (b) When a fast-flowing river passes under a bridge with narrow arches, the water level drops slightly as it passes through the arches. Explain why the water level decreases at the narrow section.

- (c) A venturi meter in a water pipeline has a pipe diameter of 15cm and a throat diameter of 5cm. A mercury manometer ($\rho = 13600\text{kgm}^{-3}$) shows a height difference of 3.0cm. Calculate the volume flow rate of water ($\rho = 1000\text{kgm}^{-3}$). Take $g = 9.8\text{ms}^{-2}$.

Question 12

- (a) A solid ball and a hollow ball of the same mass and radius roll along a flat surface at the same speed. The hollow ball is harder to stop. Account for this.
- (b) A student holds a funnel connected to a tap and lets water flow out of the wide end gently. She then flips the funnel so water exits the narrow end. The water shoots out much faster. Explain why.
- (c) A spherical drop of mercury ($\gamma = 0.5\text{Nm}^{-1}$, $\rho = 13,600\text{kgm}^{-3}$, $c = 140\text{Jkg}^{-1}\text{K}^{-1}$) of radius 2mm breaks into 8 identical smaller drops. Calculate the work done and the temperature rise if all the work is converted to heat.

Question 13

- (a) A ball rolling without slipping on a rough surface has both translational and rotational kinetic energy. If the same ball were sliding without rolling at the same speed on a frictionless surface, it would have less total kinetic energy. Account for this difference.
- (b) Honey pours slowly from a jar on a cold morning but flows freely when warmed. Explain why heating reduces the resistance to flow.
- (c) An aircraft of mass 8000kg has wings of total area 30m^2 . The air speed over the upper surface is 90ms^{-1} . Calculate the minimum speed of air under the lower surface for the aircraft to become airborne. Take $\rho_{\text{air}} = 1.2\text{kgm}^{-3}$ and $g = 9.8\text{ms}^{-2}$.

Question 14

- (a) When a car engine is started, the crankshaft must overcome the rotational inertia of the flywheel before the engine runs smoothly. Explain why a heavier flywheel requires more energy to start but provides smoother operation once running.
- (b) A person standing near the edge of a railway platform feels pulled toward a passing express train. Explain this effect.
- (c) A capillary tube of internal radius 0.5mm is dipped vertically into water ($\gamma = 0.073\text{Nm}^{-1}$, $\rho = 1000\text{kgm}^{-3}$, $\theta = 0^\circ$). Calculate the height of rise. If the tube is only 2cm long, explain what happens. Take $g = 9.8\text{ms}^{-2}$.

Question 15

- (a) A merry-go-round spins freely. A child standing at the edge jumps off tangentially. The merry-go-round speeds up slightly after the child leaves. Account for this.
- (b) A soap bubble is always spherical regardless of how it was blown. Explain why.
- (c) A uniform disc of mass 8.0kg and radius 0.40m rotates at 6.0rad s^{-1} about its axis. A 2.0kg lump of clay is dropped vertically onto the disc at the rim and sticks. Calculate the new angular velocity.

Question 16

- (a) A tightrope walker carries a long horizontal pole. Explain how the pole helps maintain balance.
- (b) Water flows through a horizontal pipe that widens at one section. At the wider section, the water moves more slowly. Yet the pressure at the wider section is higher than at the narrow section. Explain why slower-moving fluid has higher pressure.
- (c) A flat plate of area 0.04m^2 is pulled at constant velocity of 0.6ms^{-1} over a layer of oil ($\eta = 0.30\text{Nsm}^{-2}$) of thickness 2.0mm on a flat surface. Calculate the force required.

Question 17

- (a) A hammer thrown through the air tumbles, but one specific point inside the hammer follows a smooth parabolic path. Explain what this point is and why only this point follows the parabolic trajectory.
- (b) A water company needs to triple the flow rate through a pipe but can only increase the pressure by 50%. Explain why a small increase in pipe radius would be more effective than a large increase in pressure.
- (c) A solid sphere of mass 0.50kg and radius 0.05m rolls without slipping along a horizontal surface at 4.0ms^{-1} and then rolls up an incline. Calculate the maximum height reached on the incline. Take $g = 9.8\text{ms}^{-2}$.

Question 18

- (a) Two identical discs rotate about the same axis. One spins at ω and the other is stationary. They are pressed together and reach a common angular velocity. Explain why kinetic energy is lost in this process even though angular momentum is conserved.
- (b) An atomiser produces a fine spray when air is blown across the top of a vertical tube dipping into a liquid. Explain how the spray is produced.
- (c) A glass ball of radius 1.0mm and density 2500kgm^{-3} is timed falling through oil ($\rho_f = 800\text{kgm}^{-3}$). It takes 6.5s to fall 30cm at terminal velocity. Calculate the viscosity of the oil. Take $g = 9.8\text{ms}^{-2}$.

Question 19

- (a) A ball is thrown with backspin (spinning backward while moving forward). As it bounces, it reverses direction. Account for this observation.
- (b) A uniform rod pivoted at one end swings faster than the same rod pivoted at its centre when released from the same angular displacement. Account for this.
- (c) A turntable of moment of inertia 0.80kgm^2 rotates at 3.0rad s^{-1} . A ring of mass 2.0kg and radius 0.25m is gently placed on the turntable coaxially. Calculate the new angular velocity and the kinetic energy lost.

Question 20

- (a) A solid cylinder rolls down a rough incline without slipping, but the same cylinder slides down a smooth incline without rolling. In both cases, the cylinder starts from rest at the same height. The sliding cylinder reaches the bottom faster. Explain why.
- (b) Blood flows slowly in capillaries despite moving quickly in the aorta. Explain how the total cross-sectional area of capillaries accounts for this.
- (c) Two soap bubbles of radii 2.0cm and 5.0cm ($\gamma = 0.04\text{Nm}^{-1}$) are connected by a tube. Explain what happens and calculate the radius of curvature of the common interface.

Question 21

- (a) When a spinning gyroscope is placed on a pedestal with one end of its axle supported, it does not fall but instead moves sideways in a slow circle. Account for this behaviour.
- (b) A glass capillary tube is dipped into water and then slowly lifted out while the bottom end remains submerged. The water inside the tube remains higher than the water outside. Explain why the water does not drain out.
- (c) A string is wound around a solid cylinder of mass 4.0kg and radius 0.15m. The cylinder is free to rotate about a horizontal axis. When the string is released, a 2.0kg mass hangs from it and falls. Calculate the acceleration of the falling mass and the tension in the string. Take $g = 9.8\text{ms}^{-2}$.

Question 22

- (a) Two rods of the same mass and length are welded into a T-shape and an L-shape respectively. Both are rotated about an axis through one end. One has a larger moment of inertia than the other. Explain why the distribution of mass relative to the axis determines which shape is harder to rotate.
- (b) Water rises in a paper towel when one end is dipped in water, but does not rise in a waxed paper strip. Explain the difference.
- (c) Water flows upward through a vertical pipe that widens from radius 3cm at the bottom to 6cm at the top. The sections are 4m apart. The flow speed at the bottom is 6ms^{-1} and the pressure there is $2.5 \times 10^5\text{Pa}$. Find the pressure at the top. Take $\rho = 1000\text{kgm}^{-3}$ and $g = 9.8\text{ms}^{-2}$.

Question 23

- (a) A uniform metre ruler balanced at its centre with a heavy mass on one end tips over. The same ruler with the mass at the centre remains balanced. Explain why the position of the mass affects the stability of the ruler.
- (b) Explain why a spinning cricket ball curves sideways during flight.
- (c) A hollow sphere of mass M and radius R rolls without slipping along a horizontal surface with speed v . Show that the total kinetic energy is $\frac{5}{6}Mv^2$.

Question 24

- (a) An Atwood machine with a massive pulley takes longer to accelerate than one with a light pulley, even though the hanging masses are the same. Account for this.
- (b) A large raindrop falls faster than a small one during a downpour. Explain why, using the concept of terminal velocity.
- (c) A wire frame with a sliding wire of length 6cm has a soap film ($\gamma = 0.03\text{Nm}^{-1}$). The sliding wire is pulled outward by 4cm. Calculate the work done and the force on the sliding wire.

Question 25

- (a) A solid sphere rolling at speed v on a horizontal surface has more total kinetic energy than a block sliding at the same speed v on a frictionless surface. Both have the same mass. Account for the extra energy in the rolling sphere.
- (b) Explain why the excess pressure inside a small soap bubble is greater than inside a large soap bubble, even though both are made from the same solution.
- (c) A solid cylinder of mass 10kg and radius 0.25m is mounted horizontally. A rope is wound around it and a 5.0kg mass is attached to the free end. The system is released from rest. Calculate the speed of the hanging mass after it has fallen 2.0m. Take $g = 9.8\text{ms}^{-2}$.

Question 26

- (a) A car's engine is connected to the wheels through a gearbox. In low gear, the wheels turn slowly but with great torque. In high gear, the wheels turn fast but with less torque. Explain this trade-off.
- (b) A pitot tube mounted on an aircraft measures airspeed using a pressure difference. Explain how the speed of the aircraft is related to this pressure difference.
- (c) Glycerine ($\eta = 1.5\text{Nsm}^{-2}$) flows through a horizontal tube of radius 3mm and length 0.8m. The pressure difference is $5 \times 10^4\text{Pa}$. Calculate the volume flow rate and the Reynolds number. Is the flow laminar? Take $\rho = 1260\text{kgm}^{-3}$.

Question 27

- (a) A rigid body rotating about an axis has all its particles moving at different linear speeds but the same angular velocity. Account for this.
- (b) A water tank has two identical small holes at different depths. The water from the lower hole hits the ground further from the tank than water from the upper hole when both are open. Explain why.
- (c) A uniform solid disc of mass 6.0kg and radius 0.30m rotates at 8.0rad s^{-1} . Calculate the rotational kinetic energy and the torque required to bring it to rest in 4.0s.

Question 28

- (a) A mechanic uses two hands to turn a steering wheel. Each hand pushes in opposite directions on opposite sides of the wheel. Explain why this is more effective than pushing with one hand on one side.
- (b) A soap film formed on a wire frame with a sliding wire pulls the wire inward when released. Explain why the film exerts a force on the wire.
- (c) A sphere of density 2ρ and radius r is released in a liquid of density ρ and viscosity η . Show that the terminal velocity is $v_t = \frac{2r^2\rho g}{9\eta}$.

Question 29

- (a) A solid ball rolls up an incline and comes momentarily to rest before rolling back down. At the instant it is at rest, it has zero translational velocity but still has potential energy. Account for where the kinetic energy went.
- (b) Two flywheels of the same mass rotate at the same angular velocity. One is a solid disc and the other is a thin ring. The ring stores more kinetic energy. Account for this difference.
- (c) A solid cylinder and a solid sphere of the same mass and radius start from rest and roll without slipping down an incline of height h . Find the ratio of their speeds at the bottom.

Question 30

- (a) A rolling ball on a rough surface gradually slows down and eventually stops. Explain the role of friction in both maintaining the rolling motion and eventually stopping it.
- (b) A cook notices that fresh cooking oil flows easily from the bottle, but the same oil after repeated use in a deep fryer becomes thick and pours slowly. Explain what has changed in the oil.
- (c) A pulley of mass 3.0kg and radius 0.20m is treated as a uniform disc. Masses of 5.0kg and 3.0kg hang on opposite sides. Calculate the acceleration of the system and the tensions in the two strings. Take $g = 9.8\text{ms}^{-2}$.

Question 31

- (a) A constant torque applied to a wheel for a short time produces a small change in angular velocity. The same torque applied for twice as long produces twice the change. Account for this using the rotational analogue of Newton's second law.
- (b) When a garden hose with a nozzle is used, the water exits at high speed and the hose recoils backward. Explain how Bernoulli's principle and Newton's third law each contribute to this observation.
- (c) An air bubble of radius 0.5mm is formed at the bottom of a lake at a depth of 20m. Calculate the total pressure inside the bubble. Take $\gamma = 0.073\text{Nm}^{-1}$, $\rho = 1000\text{kgm}^{-3}$, $P_{\text{atm}} = 1.01 \times 10^5\text{Pa}$, and $g = 9.8\text{ms}^{-2}$.

Question 32

- (a) A uniform rod is easier to balance vertically on a finger if it is long than if it is short. Account for this.
- (b) A thick milkshake is much harder to sip through a narrow straw than through a wide one. Explain why.

- (c) A uniform solid sphere of mass 3.0kg and radius 0.10m is rotating at 20rad s^{-1} about an axis through its centre. Calculate its angular momentum and the torque needed to stop it in 5.0s.

Question 33

- (a) A ball placed on a smooth incline slides without rotating, but the same ball on a rough incline both rotates and translates simultaneously. Explain the role of friction in producing rotation during rolling without slipping.
- (b) Explain why a liquid drop in free fall (such as inside the International Space Station) is perfectly spherical, while a drop resting on a table is flattened.
- (c) A large open tank has water filled to 6m. A hole is opened at a depth of 2m below the surface. The jet emerges horizontally. Calculate the horizontal distance from the tank where the jet hits the ground. Take $g = 9.8\text{ms}^{-2}$.

Question 34

- (a) A flat circular disc has a moment of inertia about an axis perpendicular to its face that equals the sum of the moments about two perpendicular diameters. Explain physically why the resistance to rotation about the face axis is greater than about a diameter.
- (b) A farmer in Singida breaks up the top layer of dry soil to prevent water from evaporating. Explain how breaking the top layer reduces water loss.
- (c) A disc of radius 0.20m is placed on a thin oil film ($\eta = 0.40\text{Nsm}^{-2}$, thickness 0.5mm) and pushed at constant velocity 0.50ms^{-1} . The disc has area 0.04m^2 . Calculate the force needed.

Question 35

- (a) A cannon mounted on a wheeled cart fires a ball. The cart rolls backward. Explain why the angular momentum of the system is not conserved about a horizontal axis through the cannon's barrel, but the linear momentum is conserved.
- (b) A water tank with a tap at the bottom delivers water more slowly as the tank empties. Explain why the flow rate decreases as the water level drops.
- (c) A uniform solid cylinder of mass M and radius R rolls without slipping down an incline angled at θ to the horizontal. Show that the acceleration is $a = \frac{2g\sin\theta}{3}$.

Question 36

- (a) A long-handled spanner makes it easier to loosen a tight bolt than a short-handled one. Explain why, using the relationship between torque, force, and the distance from the pivot.
- (b) Raindrops are approximately spherical, not teardrop-shaped as commonly depicted. Explain why surface tension produces this shape.
- (c) A 2.0kg sphere ($\rho = 7800\text{kgm}^{-3}$) falls through oil ($\rho = 900\text{kgm}^{-3}$, $\eta = 0.50\text{Nsm}^{-2}$) and reaches terminal velocity. Calculate the terminal velocity. Take $g = 9.8\text{ms}^{-2}$.

Question 37

- (a) A playground roundabout is pushed by a child running alongside it. Even after the child stops pushing, the roundabout continues to accelerate briefly before beginning to decelerate. Explain why the deceleration does not begin immediately.
- (b) Two ships sailing parallel to each other in close proximity are drawn toward each other. Explain this effect.
- (c) A body of moment of inertia 0.60kgm^2 rotates at 10rad s^{-1} . A constant frictional torque reduces it to 4rad s^{-1} in 8s. Calculate the frictional torque, the number of revolutions in those 8s, and the energy dissipated.

Question 38

- (a) A coin spinning on a table eventually wobbles and falls flat. Explain why the coin cannot maintain its spinning motion indefinitely.
- (b) A child blows a soap bubble and notices that the bubble slowly shrinks when the blowing tube is left open. Explain why the bubble contracts rather than remaining the same size.
- (c) Two horizontal capillary tubes of equal length are connected in series. The first has radius R and the second has radius $2R$. Find the ratio of the pressure drop across the first tube to that across the second tube.

Question 39

- (a) A solid sphere and a solid cylinder of equal mass and radius roll at the same speed on a horizontal surface. Both experience the same rolling friction. The cylinder takes longer to stop. Account for this.
- (b) A factory uses a long, narrow pipe to transport oil. In winter, the flow rate drops dramatically even though the pump pressure has not changed. Explain why.
- (c) A motor drives a grinding wheel of moment of inertia 0.40kgm^2 from rest to 50rad s^{-1} in 10s at constant angular acceleration. Calculate the torque exerted by the motor and the power delivered at the end of the 10s.

Question 40

- (a) Two identical flywheels rotate at the same angular velocity. One is suddenly coupled to a stationary identical flywheel. Explain what happens to the angular velocity and the kinetic energy of the system.
- (b) During a flood, water rushes through a narrow gap in a wall much faster than in the wide river behind it. Explain why.
- (c) A pitot tube connected to a water manometer ($\rho = 1000\text{kgm}^{-3}$) measures the speed of air ($\rho = 1.2\text{kgm}^{-3}$) flowing through a duct. The manometer shows a height difference of 8.0cm. Calculate the air speed. Take $g = 9.8\text{ms}^{-2}$.

Question 41

- (a) A ball thrown with topspin bounces at a lower angle than a ball thrown without spin. Account for this.
- (b) A swimming pool overflow system uses a wide channel to carry water. Replacing it with a narrower channel reduces the flow significantly, even though the height difference driving the flow remains the same. Explain why.
- (c) A solid sphere of mass 2.5kg and radius 0.08m is suspended from a string and acts as a pendulum. Using the parallel axis theorem, calculate the moment of inertia about the pivot point (at the top of the string, 0.60m from the centre of the sphere).

Question 42

- (a) Two soap bubbles of different sizes are connected by a tube. The smaller bubble gets smaller and the larger bubble gets larger until only one remains. Explain why.
- (b) A diver at depth experiences pressure from all directions, not just from above. Explain why fluid pressure at a point is the same in all directions.
- (c) A solid sphere of mass 1.5kg and radius 0.06m rolls without slipping at 5.0ms^{-1} on a horizontal surface and then rolls up a 30° incline. Calculate the distance along the incline the sphere travels before stopping. Take $g = 9.8\text{ms}^{-2}$.

Question 43

- (a) A rotating platform is used in a physics experiment. When sand is gently sprinkled onto the rotating platform from above, the platform slows down. Explain why.

- (b) A pipe carrying water develops a small leak at the top. Water squirts upward from the leak. Explain why the water emerges from the leak instead of air being drawn in.
- (c) A uniform rod of mass 2.0kg and length 0.80m is pivoted at its centre and initially at rest. A 0.05kg bullet travelling at 200ms^{-1} horizontally strikes one end of the rod and embeds itself. Calculate the angular velocity of the rod immediately after impact.

Question 44

- (a) A gyroscope spinning inside an aircraft helps maintain the aircraft's orientation. Explain how the gyroscope resists changes in direction.
- (b) During a severe storm, windows sometimes shatter outward rather than inward, even though the wind is blowing against the outside of the building. Explain why.
- (c) Three identical spheres A, B, and C have the same radius but are made of different materials with densities 8000kgm^{-3} , 4000kgm^{-3} , and 2000kgm^{-3} respectively. All three are dropped simultaneously into the same viscous liquid of density 1000kgm^{-3} . Rank them in order of terminal velocity from highest to lowest, and find the ratio $v_A : v_B : v_C$.

Question 45

- (a) A rotating disc on a viscous oil film gradually slows down and eventually stops. Explain the role of viscosity in removing the disc's rotational kinetic energy.
- (b) A dam is built much thicker at the base than at the top. Explain the physics behind this design.
- (c) A solid ball of mass 0.30kg and radius 0.02m rolls without slipping along a table at 3.0ms^{-1} . It rolls off the edge and falls 1.5m to the ground. Calculate the speed of the ball just before it hits the ground. Take $g = 9.8\text{ms}^{-2}$.

Question 46

- (a) A spinning hard-boiled egg is briefly touched to stop it and then released. It stays still. A raw egg treated the same way starts spinning again after the finger is removed. Account for this difference.
- (b) A laboratory technician drops identical steel balls into two tall jars of different liquids. The ball in one jar takes much longer to reach the bottom. Explain how this observation reveals which liquid has higher viscosity.
- (c) In a blood transfusion, the bottle is positioned so that the blood level is 1.3m above a needle of internal diameter 0.36mm and length 30mm. If 4.5cm^3 of blood passes through the needle in one minute, calculate the viscosity of blood. Take $\rho_{\text{blood}} = 1060\text{kgm}^{-3}$ and $g = 9.8\text{ms}^{-2}$.

Question 47

- (a) A solid sphere and a solid cylinder of equal mass roll without slipping from rest down the same incline. Without calculation, explain which reaches the bottom with greater translational speed and why.
- (b) A Bunsen burner draws air into its barrel through a small side hole when gas flows through the narrow jet at the base. Explain how the flowing gas draws air into the burner.
- (c) A water tower maintains a constant water level at a height of 10m above a circular outlet at the base. Calculate the diameter of the outlet needed to deliver water at a rate of 26.4m^3 per minute. Take $g = 9.8\text{ms}^{-2}$.

Question 48

- (a) A ball rolling along a surface strikes a low step. If the ball is large enough that its centre is above the top of the step, it rolls over. If the ball's centre is below the top, it bounces back. Account for this difference using the concept of torque about the contact point.

- (b) When Kipute pours water gently from a bottle, it flows smoothly down the side. When she tilts the bottle steeply and pours fast, the flow becomes noisy and irregular. Explain what causes this transition.
- (c) An air bubble of radius 1.0cm rises at a steady speed of 0.20cms^{-1} through a liquid of density 1470kgm^{-3} . Calculate the viscosity of the liquid. Take $\rho_{\text{air}} = 1.2\text{kgm}^{-3}$ and $g = 9.8\text{ms}^{-2}$.

Question 49

- (a) A bicycle wheel with light spokes and a heavy rim has a larger moment of inertia than a wheel with heavy spokes and a light rim, even though both have the same total mass. Account for this.
- (b) A patient's blood pressure is always measured at the upper arm, level with the heart. Explain why measuring at the ankle would give a higher reading that does not reflect the heart's actual pumping pressure.
- (c) A pipe carrying liquid develops deposits on its inner walls, reducing the effective radius by 8%. Calculate the factor by which the pressure difference must increase to maintain the same flow rate.

Question 50

- (a) A disc rolling without slipping on a rough surface reaches the base of a smooth (frictionless) incline and begins to climb. Explain what happens to the translational and rotational motion of the disc as it moves up the smooth incline.
- (b) Water from a tap flows as a smooth, clear stream near the tap but breaks into separate droplets further down. Explain why the stream breaks up as it falls.
- (c) A spherical raindrop of radius 2.0mm and density 1000kgm^{-3} falls from a height of 500m. It accelerates with decreasing acceleration until reaching terminal velocity at a height of 250m, then falls at constant speed for the remaining distance. Calculate the work done by gravity on the drop in each half of the journey. Take $g = 9.8\text{ms}^{-2}$.

Question 51

- (a) A water tank has a small hole in its wall and is continuously refilled to maintain a constant water level. A plumber suggests enlarging the hole to get more water. Explain why the speed of the emerging water does not increase when the hole is enlarged, but the volume of water collected per second does increase.
- (b) Campers are advised never to touch the inside of a wet canvas tent during rain. Explain why touching the inside causes water to drip through.
- (c) Water flows steadily through a horizontal tube of cross-sectional area 25cm^2 . The static pressure is $1.3 \times 10^5\text{Pa}$ and the total pressure is $1.4 \times 10^5\text{Pa}$. Calculate the velocity of the water and the mass of water flowing past a point per second. Take $\rho = 1000\text{kgm}^{-3}$.

Question 52

- (a) A thin uniform rod is held horizontally at one end and released. The free end accelerates downward faster than a freely falling object. Explain how a point on the rod can accelerate faster than g even though the only forces acting are gravity and the pivot reaction.
- (b) A skydiver falls much faster before opening the parachute than after. Explain how the parachute reduces the terminal velocity.
- (c) In a Millikan-type experiment, an uncharged oil drop of radius $2.0 \times 10^{-5}\text{m}$ and density $1.2 \times 10^3\text{kgm}^{-3}$ falls through air of viscosity $1.8 \times 10^{-5}\text{Nsm}^{-2}$. Neglecting the buoyancy of air, calculate the terminal speed of the drop and the viscous force at that speed. Take $g = 9.8\text{ms}^{-2}$.

Question 53

- (a) A mechanic applies a large force to a wrench trying to loosen a seized bolt, but the bolt does not move. Despite the large torque, no work is done on the bolt. Explain why.

- (b) A large bottle is emptied through a capillary siphon. The same bottle filled with petrol empties 2.5 times faster than when filled with water through the same siphon. Explain what this tells us about the relative viscosities of water and petrol.
- (c) Water flows through a horizontal pipe that narrows from a diameter of 0.60m at section A to 0.20m at section B. The pressure difference between A and B is equivalent to 1.0m column of water. Calculate the volume of water flowing per second. Take $g = 9.8\text{ms}^{-2}$ and $\rho = 1000\text{kgm}^{-3}$.

Question 54

- (a) A ball rolling without slipping on a rough incline slows down and eventually comes to rest with both $v = 0$ and $\omega = 0$ at the same instant. Explain why the translational and rotational motions stop simultaneously rather than one stopping before the other.
- (b) A glass rod dipped in water and withdrawn carries a thin film of water clinging to it. The same rod dipped in mercury and withdrawn comes out perfectly clean and dry. Explain this difference.
- (c) Water ($\eta = 1.0 \times 10^{-3}\text{Nsm}^{-2}$, $\rho = 1000\text{kgm}^{-3}$) flows through a pipe of diameter 2.0cm. Calculate the maximum volume flow rate before the flow becomes turbulent. Take the critical Reynolds number as 2000.

Question 55

- (a) A child pushes a playground roundabout through two full revolutions. Another child pushes with the same force but through only half a revolution. Explain why the first child transfers more energy to the roundabout.
- (b) A glass capillary tube dipped in paraffin shows a smaller rise than the same tube dipped in water. Explain why different liquids rise to different heights in the same capillary tube.
- (c) A glass capillary tube is dipped vertically into paraffin ($\gamma = 0.026\text{Nm}^{-1}$, $\rho = 800\text{kgm}^{-3}$, angle of contact = 26°). The paraffin rises 4.5cm. Calculate the internal diameter of the tube. Take $g = 9.8\text{ms}^{-2}$.

Question 56

- (a) A horizontal pipe of uniform cross-section carries water from one end to the other. According to Bernoulli's theorem, the pressure should remain constant along the pipe since the speed and height do not change. However, pressure measurements reveal a steady decrease from inlet to outlet. Explain this observation.
- (b) A flag mounted on a pole flutters vigorously when strong winds blow but hangs limply on a calm day. Explain why moving air causes the flag to flutter rather than remain flat.
- (c) A pressure gauge attached to a closed water pipe reads $2.5 \times 10^5\text{Pa}$. When the valve is opened and water begins to flow, the reading drops to $2.0 \times 10^5\text{Pa}$. Calculate the speed of water flowing through the pipe. Take $\rho = 1000\text{kgm}^{-3}$.

Question 57

- (a) A parachutist jumps from a height of 2000m and descends gently, while a stone released from the same height crashes into the ground at high speed. Explain why the two objects behave so differently despite falling from the same height.
- (b) Water emerges horizontally from a small hole in the side of a tank, but the stream curves downward and hits the ground some distance from the base. Explain why the water follows a curved path rather than continuing in a straight horizontal line.
- (c) Two identical spherical drops of water fall through air, each at terminal velocity V . The drops collide and merge into a single spherical drop. Determine the terminal velocity of the combined drop in terms of V .

Question 58

- (a) A student claims that replacing a water tank with a wider one, while keeping the water level the same, will reduce the efflux velocity from a hole at the base. Explain why this claim is incorrect.
- (b) Water flows steadily through a horizontal pipe that narrows from a wide section to a narrow section. Measurements show that water moves faster in the narrow section. Explain why the speed increases even though nothing is pushing the water harder.
- (c) Water is maintained at a constant height of 10m in a large tank. A small hole in the side of the tank is 3.0m below the surface. The jet emerges horizontally. Calculate the speed of efflux and the horizontal distance from the tank at which the jet strikes the ground. Take $g = 9.8\text{ms}^{-2}$.

Question 59

- (a) Soap solution is a more effective cleansing agent than plain water. Explain why, using the concept of surface tension.
- (b) Explain why the excess pressure inside a soap bubble of a given radius is exactly twice that inside an air bubble of the same radius in water.
- (c) The excess pressure inside a soap bubble of radius 1.0cm is balanced by the pressure due to a column of oil of specific gravity 0.8 and height 2.0mm. Calculate the surface tension of the soap solution. Take $g = 9.8\text{ms}^{-2}$.

Question 60

- (a) A large open tank has a small circular hole at its base. When the tank is full, the water jet from the hole shoots out forcefully. As the tank drains and the water level falls, the jet becomes progressively weaker. Explain this observation.
- (b) A container resting on a smooth surface has a small hole in one of its walls. When the container is filled with liquid and the hole is unplugged, the liquid flows out and the container begins to move in the opposite direction. Explain why the container experiences this backward thrust.
- (c) A large cylindrical tank contains water to a depth of 0.30m. A hole of cross-sectional area 6.5cm^2 at the bottom allows water to drain. Calculate the initial drainage rate. At a point 30cm below the hole, determine the cross-sectional area of the falling water stream. Take $g = 9.8\text{ms}^{-2}$.

Question 61

- (a) A daladala mechanic in Dar es Salaam squirts oil horizontally from a can held at waist height. The oil lands further away than when the can is held near the ground. Explain why the height at which the can is held affects the horizontal distance the oil travels before hitting the ground.
- (b) A horizontal pipe carrying water has a narrow constriction in the middle. Small vertical tubes open to the atmosphere are attached at the wide section and at the constriction. The water level in the vertical tube at the constriction is lower than at the wide section. Explain this observation.
- (c) A syringe containing water is held horizontally with its nozzle at a height $h = 1.25\text{m}$ above the ground. The diameter of the piston is 5 times that of the nozzle. The piston is pushed at a constant speed of 0.20ms^{-1} . Calculate the speed at which the water hits the ground. Take $g = 9.8\text{ms}^{-2}$.

Question 62

- (a) A nurse at Muhimbili National Hospital notices that replacing an IV needle with one of slightly smaller radius causes the drip rate to fall dramatically. Explain why a small decrease in needle radius produces such a large drop in flow rate.
- (b) Two capillary tubes of different radii but the same length are connected in series, and liquid flows through them under streamline conditions. The pressure drop is not shared equally between the two tubes. Explain why the narrower tube carries the larger share of the total pressure drop.

- (c) Three capillaries of the same length but of internal radii $3r$, $4r$, and $5r$ are connected in series. Liquid flows through them under streamline conditions. If the pressure difference across the third capillary is 8.1mm of the liquid, find the pressure difference across the first capillary.

Question 63

- (a) On a still morning in Dar es Salaam, tiny fog droplets seem to hang almost motionless in the air, while larger raindrops during the afternoon downpour fall rapidly. Explain why very small drops fall so much more slowly than large ones.
- (b) A glass sphere falls through glycerine and reaches terminal velocity. When the experiment is repeated with the glycerine warmed to a higher temperature, the same sphere reaches a higher terminal velocity. Explain this observation.
- (c) An oil drop of density 900kgm^{-3} falls through air ($\eta = 1.8 \times 10^{-5}\text{Nsm}^{-2}$, $\rho_{\text{air}} = 1.2\text{kgm}^{-3}$) at a terminal velocity of $3.2 \times 10^{-2}\text{ms}^{-1}$. Calculate the radius of the drop. If the drop splits into a smaller drop of half the original radius, calculate the new terminal velocity. Take $g = 9.8\text{ms}^{-2}$.

Question 64

- (a) The accumulation of snow or ice on an aeroplane wing during flight may reduce the lift. Explain how this surface change affects the wing's ability to generate lift.
- (b) When a liquid flows through a narrow constriction in a pipe, its speed increases. Explain where the liquid gets the extra kinetic energy for this increased speed.
- (c) Water flows steadily through a horizontal pipe made of two sections joined end to end. The first section is 21cm long with a diameter of 0.225cm , and the second is 7cm long with a diameter of 0.075cm . If the pressure difference across the entire combination is 14cm of water, find the pressure difference across each section. Take $g = 9.8\text{ms}^{-2}$ and $\rho = 1000\text{kgm}^{-3}$.

Question 65

- (a) Automobile manufacturers recommend using oils of different viscosities in different seasons. Explain why the same oil is not suitable for both hot and cold weather.
- (b) A car engine runs more freely and smoothly as it heats up after a cold start. Explain why warming up reduces the resistance to motion inside the engine.
- (c) In an experiment to determine the coefficient of viscosity of motor oil, a glass sphere of mass $1.2 \times 10^{-4}\text{kg}$ and diameter $4.0 \times 10^{-3}\text{m}$ reaches a terminal velocity of $5.4 \times 10^{-2}\text{ms}^{-1}$. The density of the motor oil is 900kgm^{-3} . Calculate the coefficient of viscosity. Take $g = 9.8\text{ms}^{-2}$.

Question 66

- (a) A water tank at a school in Mwanza is continuously fed by a supply pipe at the top. Students open a tap at the base and notice that the water level drops at first but then remains steady, even though the tap is still running. Explain why the water level stabilises at a particular height.
- (b) Under a constant pressure head, a liquid flows through a pipe at rate Q . The pipe is replaced by one that is twice as long and half the diameter. Explain qualitatively why the new flow rate is dramatically less than Q .
- (c) Under a given pressure head, the rate of flow of liquid through a pipe is Q . If the length of the pipe is doubled and its diameter is halved, calculate the new rate of flow in terms of Q .

Question 67

- (a) Most people assume an aeroplane can only fly right-side up, yet stunt pilots routinely fly upside down. Explain how an aeroplane generates lift even when it is inverted.

- (b) During takeoff and landing, pilots extend wing flaps that increase both the wing area and the angle of attack. Explain how extending flaps helps the aircraft fly safely at lower speeds.
- (c) Water is supplied to a house at ground level through a pipe of inner diameter 1.5cm at an absolute pressure of $6.5 \times 10^5 \text{Pa}$ and a velocity of 5ms^{-1} . The pipe leading to the second floor bathroom 8m above has an inner diameter of 0.75cm. Calculate the flow velocity and the pressure at the second floor. Take $\rho = 1000 \text{kgm}^{-3}$ and $g = 9.8 \text{ms}^{-2}$.

Question 68

- (a) A horizontal pipeline increases uniformly from 0.080m diameter to 0.160m diameter in the direction of flow. A student predicts that the pressure gauge reading at the wider section should be lower because “more space means less pressure.” Explain why this prediction is incorrect.
- (b) A large cylindrical tank of cross-sectional area A contains water to a height H_1 above a small hole of area a at the base, where $a \ll A$. The hole is opened and the tank drains. Show that the time for the water level to fall from H_1 to H_2 is:

$$t = \frac{A}{a} \sqrt{\frac{2}{g}} (\sqrt{H_1} - \sqrt{H_2})$$

- (c) Water flows through the pipeline described in (a) at 96 litres per second. A pressure gauge at the 0.080m section reads $3.5 \times 10^5 \text{Pa}$. Calculate the reading at the 0.160m section. Take $\rho = 1000 \text{kgm}^{-3}$.

Question 69

- (a) Water flows into a tank of large cross-sectional area at a constant rate while draining out through a hole at the base. The water level initially rises but eventually stabilises at a fixed height. Explain why the level stops rising.
- (b) Two small spheres of the same material fall through a viscous liquid. The diameter of one sphere is three times that of the other. After both have reached terminal velocity, explain which sphere takes longer to fall the same vertical distance and why.
- (c) Water flows into a tank at a rate of $2 \times 10^{-4} \text{m}^3 \text{s}^{-1}$ and drains out through a hole of area 2cm^2 at the base. Calculate the equilibrium height of water in the tank. Take $g = 9.8 \text{ms}^{-2}$.

Question 70

- (a) It is much easier to throw a curve with a tennis ball than with a cricket ball. Explain why.
- (b) A horizontal cylinder filled with water of volume V is fitted with a piston at one end and a small orifice of cross-sectional area a at the other, where a is much less than the piston area. A constant force pushes the piston and squeezes all the water out through the orifice in time t . Neglecting friction and viscosity, show that the work done by the force is $W = \frac{\rho V^3}{2a^2 t^2}$.
- (c) A steel ball of radius 2.0mm and density 7800kgm^{-3} is dropped into glycerine of density 1260kgm^{-3} and viscosity 0.83Nsm^{-2} . Calculate the terminal velocity. Take $g = 9.8 \text{ms}^{-2}$.

Question 71

- (a) Water leaks horizontally from a hole in the side of a large tank. A plumber plugs the hole and drills a new one at a lower position on the same wall. The jet from the lower hole emerges faster but hits the ground closer to the tank. Explain why a faster jet can land closer to the base.
- (b) An oil drop falls through air with a terminal velocity of $5 \times 10^{-4} \text{ms}^{-1}$. Neglecting the density of air compared to that of oil, explain why a drop of half the radius would have a terminal velocity exactly one quarter of this value.

- (c) Water leaks horizontally from a small hole in the side of a large open tank. The hole is 80cm below the water surface and 120cm above the ground. Calculate the speed at which water emerges from the hole and the horizontal distance from the tank at which the stream hits the ground. Take $g = 9.8\text{ms}^{-2}$.

Question 72

- (a) A high diver can perform more somersaults before entering the water if she tucks her body tightly rather than keeping it outstretched. Explain why tucking allows more somersaults in the same fall time.
- (b) There is no single, unique value of the moment of inertia for a given rigid body. Explain why and identify what determines the value.
- (c) A disc of moment of inertia $2.5 \times 10^{-4}\text{kgm}^2$ rotates freely about an axis through its centre at 20 revolutions per minute. A piece of wax of mass 0.04kg is dropped gently onto the disc at a distance of 0.05m from the axis. Calculate the new angular velocity in revolutions per minute.

Question 73

- (a) A flywheel is designed so that most of its mass is concentrated at the rim rather than near the hub. Explain why this design is preferred for a flywheel whose purpose is to store rotational energy and smooth out speed fluctuations.
- (b) A uniform solid sphere has the same moment of inertia about every axis passing through its centre. Explain why a sphere has this property while a cylinder does not.
- (c) A grinding wheel of radius 0.40m and mass 3.0kg rotates at 3600 revolutions per minute. Calculate the torque required to bring it to rest in 10s. Treat the wheel as a uniform disc.

Question 74

- (a) A rotating object must always have a non-zero moment of inertia about its axis of rotation. Explain why it is physically impossible for a real object to rotate with zero moment of inertia.
- (b) A solid cylinder rolls without slipping down an inclined plane. Explain why the acceleration of the cylinder down the incline is less than $g\sin\theta$.
- (c) A solid cylinder of mass M and radius of gyration k rolls without slipping from rest down an inclined plane of angle α and length ℓ . Find the speed at the bottom in terms of α , ℓ , k , g , and the radius R .

Question 75

- (a) A hollow cylinder and a solid cylinder have the same mass and the same outer radius. Both are set spinning about their central axes to the same angular velocity. Explain which one requires more work to reach that angular velocity and why.
- (b) A uniform semicircular hoop of mass M and radius R lies in a plane. By integration, show that the centre of mass of the hoop is located at a distance $\frac{2R}{\pi}$ from the centre of the full circle along the axis of symmetry.
- (c) A thin circular ring of mass M and radius r rotates about its axis with angular velocity ω_1 . Two small objects, each of mass m , are gently attached to the ring at diametrically opposite points. Calculate the new angular velocity in terms of M , m , and ω_1 .

Question 76

- (a) A rotating object does not necessarily have a net torque acting on it. Explain why an object can rotate at constant angular velocity with zero net torque, and give a physical example.
- (b) Mechanics sometimes extend the handle of a wrench by slipping a section of pipe over it to loosen a seized bolt. Explain why this procedure makes the bolt easier to turn but risks damaging it.
- (c) Masses $m_1 = 0.20\text{kg}$ and $m_2 = 0.25\text{kg}$ are suspended from a light cord that passes over a wheel of radius 0.15m and moment of inertia 0.12kgm^2 . The masses start at the same height. Calculate the speed of each

mass and the angular velocity of the wheel when the vertical distance between them is 0.30m. Take $g = 9.8\text{ms}^{-2}$.

Question 77

- (a) A ball sliding without rotating on a frictionless surface has translational kinetic energy but no rotational kinetic energy. If it then moves onto a rough surface and begins to roll without slipping, its translational speed decreases. Explain where the “lost” translational energy has gone.
- (b) The Earth moves faster when closer to the Sun and slower when farther away. Explain why, using conservation of angular momentum.
- (c) If the Earth were to suddenly contract to half its present radius without any external torque acting on it, calculate the new length of the day. Assume the Earth is a uniform solid sphere.

Question 78

- (a) Opening a door requires pushing at a single point far from the hinge, while twisting open a bottle cap requires gripping with thumb and fingers on opposite sides. Explain why different techniques are needed for these two tasks.
- (b) A stationary object can have a non-zero angular acceleration at the instant a torque is first applied to it. Explain why an object at rest can simultaneously have $\omega = 0$ and $\alpha \neq 0$.
- (c) The Earth has a radius of $6.37 \times 10^6\text{m}$ and a mean density of 5450kgm^{-3} . Calculate the moment of inertia of the Earth about its axis of rotation. Treat the Earth as a uniform solid sphere.

Question 79

- (a) When building a house, masons plaster brick walls with cement. Explain how the plastering improves the strength and water resistance of the wall, referring to the role of surface tension and adhesion.
- (b) Molecules at the surface of a liquid have more potential energy than molecules deep within the liquid. Explain why a molecule at the surface is in a higher energy state than one surrounded by neighbours on all sides.
- (c) During a rainstorm, 64 identical small raindrops, each of radius r , combine to form a single large drop. Calculate the ratio of the total surface energy of the 64 small drops to the surface energy of the single large drop.

Question 80

- (a) A barometer uses a narrow tube, which introduces an error in the reading because capillary rise adds to the true liquid column height. Explain how using two tubes of different radii and comparing their readings allows the true pressure to be determined.
- (b) A thin glass plate resting on the surface of water requires a measurable downward force, beyond its own weight, to detach it from the surface. Explain where this extra resistance comes from and why it depends on the perimeter of the plate rather than its area.
- (c) A barometer contains two uniform capillary tubes of radii $6.5 \times 10^{-4}\text{m}$ and $1.24 \times 10^{-3}\text{m}$. The height of water in the narrow tube is 0.20m more than in the wide tube. Calculate the true pressure difference. Take $\gamma = 0.073\text{Nm}^{-1}$, $\theta = 0^\circ$, $\rho = 1000\text{kgm}^{-3}$, and $g = 9.8\text{ms}^{-2}$.

Question 81

- (a) Two soap bubbles of unequal radii are blown at the two ends of a tube and then connected by opening a valve between them. Explain what happens and why.
- (b) Surface tension may be defined as a force per unit length or as energy per unit area. By considering the work done in increasing the surface area of a liquid film, show that these two definitions are dimensionally and physically equivalent.

- (c) A spherical drop of mercury of radius 5mm falls on the ground and breaks into 1000 identical droplets. Calculate the work done in breaking the drop. Take $\gamma = 0.50\text{Nm}^{-1}$.

Question 82

- (a) Hot soup releases flavour molecules more readily than cold soup of the same recipe. Explain how the temperature of the soup affects the transport of flavour molecules to your sense of smell and taste, referring to the role of surface tension.
- (b) When writing with a fountain pen, ink flows from the reservoir down through the nib and onto the paper, even when the pen is held at an angle against gravity. Explain how capillary action enables the ink to reach the paper.
- (c) Calculate the work done against surface tension in blowing a soap bubble of diameter 1.0cm. Take $\gamma = 0.030\text{Nm}^{-1}$.

Question 83

- (a) Two pieces of paper joined by glue are virtually impossible to pull apart cleanly. Explain why the bond formed by the glue is so strong, referring to adhesive forces and surface contact.
- (b) The coefficient of surface tension of a liquid does not change when the surface area is doubled. Explain why surface tension is an intrinsic property of the liquid and not a function of surface area.
- (c) Two spherical mercury droplets of radii 0.1cm and 0.2cm coalesce into a single spherical drop. Calculate the energy released. Take $\gamma = 0.50\text{Nm}^{-1}$.

Question 84

- (a) The angle of contact of mercury with glass is obtuse, while that of water with glass is acute. Explain why the same glass surface produces opposite wetting behaviour in these two liquids.
- (b) When blowing soap bubbles, it is noticeably harder to start a very small bubble than to inflate it further once it has grown. Explain why more effort is needed to form a small bubble than to enlarge an existing one.
- (c) Water rises in a capillary tube to a height of 2.0cm. Calculate the height to which water rises in another capillary tube whose radius is one-third that of the first tube.

Question 85

- (a) During the rainy season in Moshi, rainwater soaks into a cotton towel left outside but rolls off a plastic tarpaulin in nearly spherical droplets. Explain this difference in behaviour.
- (b) The pressure inside a very small air bubble submerged in water is significantly greater than the pressure in the surrounding water at the same depth, while for a large bubble the difference is negligible. Explain why the excess pressure depends on the size of the bubble.
- (c) 1000 spherical droplets of water, each of diameter 10^{-8}m , coalesce to form a single large spherical drop. Calculate the energy liberated. Take $\gamma = 0.073\text{Nm}^{-1}$.

Question 86

- (a) A farmer breaks up the top layer of dry soil to prevent water from evaporating from the moist soil below. Explain how breaking the soil reduces water loss, referring to capillary action.
- (b) A number of small spherical water droplets, each of radius r , coalesce to form a single spherical drop of radius R . The surface energy released heats the combined drop. Show that the rise in temperature is $\Delta\theta = \frac{3\gamma}{\rho c} \left(\frac{1}{r} - \frac{1}{R} \right)$.

- (c) The xylem tubes in the actively growing outer layer of a tree are uniform cylinders with a contact angle of 45° and surface tension $5 \times 10^{-2} \text{Nm}^{-1}$. Calculate the maximum radius of the xylem tubes for sap to rise by capillarity alone to a height of 20m. Take $\rho = 1000 \text{kgm}^{-3}$ and $g = 9.8 \text{ms}^{-2}$.

Question 87

- (a) A soap bubble of radius r is enclosed in a sealed piston chamber at initial air pressure P_0 . The piston is slowly pulled out until the bubble doubles in radius to $2r$. Assuming isothermal conditions, show that the new air pressure in the chamber is $P = \frac{P_0}{8} - \frac{3\gamma}{2r}$.
- (b) A capillary tube is cut to a length shorter than the expected rise height and dipped vertically in water. The water rises to the top but does not overflow. Explain what happens to the meniscus and the angle of contact when the tube is too short for the full equilibrium rise.
- (c) Calculate the excess pressure inside a soap bubble of radius 1.0cm, given $\gamma = 2.5 \times 10^{-2} \text{Nm}^{-1}$. If an air bubble of the same radius is formed at a depth of 40cm inside a container of the same soap solution (relative density 1.2), calculate the total pressure inside the air bubble. Take $P_{\text{atm}} = 1.013 \times 10^5 \text{Pa}$ and $g = 9.8 \text{ms}^{-2}$.

Question 88

- (a) When a small loop of thread is placed on an intact soap film and the film inside the loop is punctured with a pin, the thread immediately springs into a perfect circle. Explain why the thread takes this particular shape.
- (b) Water rises in a glass capillary tube to a height of 9.0cm, while mercury in the same tube is depressed by 3.4cm. Explain why the same tube produces opposite behaviour for these two liquids.
- (c) Using the data in (b), with angles of contact 0° for water and 135° for mercury, determine the ratio of the surface tension of mercury to that of water. Take $\rho_w = 1000 \text{kgm}^{-3}$ and $\rho_{\text{Hg}} = 13,600 \text{kgm}^{-3}$.

Question 89

- (a) A stationary bicycle is much harder to balance on than one that is moving. Explain how the spinning wheels help the rider maintain balance.
- (b) A uniform metre rule is balanced on a pivot at its centre. Two equal masses hung at equal distances from the pivot keep the rule balanced. If both masses are moved further from the pivot by the same amount, the rule remains balanced but is noticeably harder to tilt. Explain why increasing the distance makes the system more resistant to angular acceleration.
- (c) A thin rectangular sheet of aluminium of mass 0.032kg has a length of 0.25m and a width of 0.10m. Calculate its moment of inertia about an axis in the plane of the sheet, parallel to the length and passing through its centre of mass. Then calculate the moment of inertia about an axis parallel to the width and passing through its centre of mass.

SOLUTIONS

Question 1

- (a) Both cylinders have the same mass and radius, but their moments of inertia differ. The solid cylinder has $I = \frac{1}{2}MR^2$ while the hollow cylinder has $I = MR^2$. When rolling without slipping, gravitational potential energy is converted into both translational and rotational kinetic energy. The hollow cylinder, with its larger moment of inertia, diverts a greater fraction of its energy into rotation, leaving less for translation. Consequently, it moves more slowly down the incline. The solid cylinder, with a smaller moment of inertia, retains more energy as translational kinetic energy and reaches the bottom first.
- (b) While the umbrella spins, each water droplet undergoes circular motion. The surface of the umbrella provides the centripetal force required. When the droplet reaches the edge, the surface can no longer provide sufficient centripetal force, and the droplet leaves the umbrella. After leaving, no centripetal force acts on the droplet. By Newton's first law, it continues in a straight line tangent to the circular path at the point of release.

(c) Moment of inertia of the disc:

$$I = \frac{1}{2}MR^2 = \frac{1}{2} \times 4.0\text{kg} \times (0.30\text{m})^2 = 0.18\text{kgm}^2$$

Torque: $\tau = FR = 12\text{N} \times 0.30\text{m} = 3.6\text{Nm}$

Angular acceleration:

$$\alpha = \frac{\tau}{I} = \frac{3.6\text{Nm}}{0.18\text{kgm}^2} = 20\text{rad s}^{-2}$$

Angular velocity after 5.0s from rest:

$$\omega = \omega_0 + \alpha t = 0 + 20\text{rad s}^{-2} \times 5.0\text{s} = 100\text{rad s}^{-1}$$

Question 2

(a) With no external torque, angular momentum is conserved: $I\omega = \text{constant}$. When the skater pulls her arms inward, her mass moves closer to the axis of rotation. This decreases her moment of inertia I . For the product $I\omega$ to remain constant, the angular velocity ω must increase. Hence, she spins faster.

(b) By the capillary rise formula, $h = 2\gamma\cos\theta/(\rho gr)$, the height of rise is inversely proportional to the tube radius. A narrower tube has a smaller r , producing a larger h . Physically, surface tension acts along the circumference of the meniscus ($2\pi r$) while supporting a column of weight proportional to $\pi r^2 h$. In a narrower tube, the weight per unit contact length is smaller, so surface tension can support a taller column.

(c) A soap bubble has two surfaces. Excess pressure:

$$\Delta P = \frac{4\gamma}{r} = \frac{4 \times 0.025\text{Nm}^{-1}}{0.03\text{m}} = 3.33\text{Pa}$$

Total pressure inside the bubble:

$$P = P_{\text{atm}} + \Delta P = 1.01 \times 10^5\text{Pa} + 3.33\text{Pa} = 1.01003 \times 10^5\text{Pa}$$

Question 3

(a) The flywheel has a larger radius than the bicycle wheel. Since both have the same mass, the flywheel has a larger moment of inertia (more mass is distributed farther from the axis). Rotational kinetic energy is $KE = \frac{1}{2}I\omega^2$. At the same angular velocity, the flywheel stores more kinetic energy. Under similar frictional torque, the flywheel takes longer to dissipate this larger energy store. The bicycle wheel, with smaller moment of inertia, stores less energy and stops sooner.

(b) When wind blows horizontally across the top of the roof, the air above the roof moves at high speed. The air inside the house below the roof is stationary. By Bernoulli's equation, the fast-moving air above has lower pressure than the still air below. This pressure difference produces a net upward force on the roof equal to $F = \frac{1}{2}\rho v^2 \times A_{\text{roof}}$. If this upward force exceeds the weight of the roof and the strength of its fastenings, the roof is lifted off.

(c) Terminal velocity:

$$v_t = \frac{2r^2(\rho_s - \rho_f)g}{9\eta} = \frac{2 \times (2.0 \times 10^{-3}\text{m})^2 \times (7800 - 1260)\text{kgm}^{-3} \times 9.8\text{ms}^{-2}}{9 \times 1.5\text{Nsm}^{-2}}$$
$$v_t = 0.038\text{ms}^{-1}$$

Question 4

(a) When no external torque acts, angular momentum is conserved: $L = I\omega = \text{constant}$. When the person walks from the centre to the edge, they move mass farther from the axis, increasing the total moment of inertia I of the system. For $I\omega$ to remain constant, ω must decrease. When the person walks back to the centre, I decreases and ω increases.

(b) The pond skater's legs are hydrophobic and distribute its tiny weight over a large contact perimeter. The surface tension force acting upward along the contact line around each leg is sufficient to support the insect's weight without breaking the surface. A coin has much greater weight concentrated over a short contact perimeter. The surface tension force along this short perimeter is far too small to support the coin's weight, so the surface breaks and the coin sinks.

(c) Using conservation of energy. The rod's centre of mass falls through $h = L/2 = 0.60\text{m}$ from horizontal to vertical:

$$Mgh = \frac{1}{2}I\omega^2$$

$$Mg \times \frac{L}{2} = \frac{1}{2} \times \frac{1}{3} ML^2 \times \omega^2$$

$$g \times \frac{L}{2} = \frac{L^2 \omega^2}{6}$$

$$\omega = \sqrt{\frac{3g}{L}} = \sqrt{\frac{3 \times 9.8 \text{ms}^{-2}}{1.2\text{m}}} = 4.95 \text{rad s}^{-1}$$

Question 5

(a) While on the table, both translational and rotational motion are maintained by the rolling condition. When the ball leaves the edge, gravity acts on its centre of mass (producing translational acceleration downward) but exerts zero torque about the centre of mass (because the gravitational force passes through the centre). With no torque to change the angular velocity, the ball continues to spin at the same rate throughout the fall, by the rotational analogue of Newton's first law.

(b) In a healthy artery, blood flows in a smooth, laminar pattern. When an artery is partially blocked by plaque (atherosclerosis), the internal radius decreases. By the continuity equation, blood speed increases at the narrowed section. This raises the Reynolds number. If it exceeds approximately 2000–3000, the flow transitions from laminar to turbulent. Turbulent flow produces chaotic pressure fluctuations that vibrate the artery wall, creating the audible murmur.

(c) By Poiseuille's formula:

$$Q = \frac{\pi \Delta P R^4}{8 \eta L} = \frac{\pi \times 600 \text{Pa} \times (4 \times 10^{-3} \text{m})^4}{8 \times 1.0 \times 10^{-3} \text{Nsm}^{-2} \times 1.5 \text{m}}$$

$$Q = \frac{\pi \times 600 \text{Pa} \times 2.56 \times 10^{-10} \text{m}^4}{1.2 \times 10^{-2} \text{Nsm}^{-1}} = 4.02 \times 10^{-5} \text{m}^3 \text{s}^{-1}$$

Maximum velocity at the centre:

$$v_{\text{max}} = \frac{\Delta P R^2}{4 \eta L} = \frac{600 \text{Pa} \times (4 \times 10^{-3} \text{m})^2}{4 \times 1.0 \times 10^{-3} \text{Nsm}^{-2} \times 1.5 \text{m}} = 1.6 \text{ms}^{-1}$$

Question 6

(a) When two gears are meshed, the tangential velocity at the contact point is the same for both. For the smaller gear (radius r_1), $v = \omega_1 r_1$. For the larger gear (radius r_2), $v = \omega_2 r_2$. Since $r_2 > r_1$, $\omega_2 < \omega_1$: the larger gear rotates more slowly. However, assuming negligible friction, power is conserved: $\tau_1 \omega_1 = \tau_2 \omega_2$. Since $\omega_2 < \omega_1$, $\tau_2 > \tau_1$: the larger gear delivers greater torque. The gear system trades angular velocity for torque.

(b) The ocean surface is a water-air interface with high surface energy ($\gamma \approx 0.073 \text{Nm}^{-1}$). When oil is placed on this surface, it creates oil-air and oil-water interfaces whose combined surface energy is less than the original water-air surface energy. Since physical systems tend toward minimum energy, the oil spreads spontaneously to replace the high-energy water-air surface with the lower-energy combination, continuing until it forms a thin film.

(c) For a solid sphere rolling without slipping, conservation of energy gives:

$$Mgh = \frac{1}{2} Mv^2 + \frac{1}{2} I\omega^2 = \frac{1}{2} Mv^2 + \frac{1}{2} \times \frac{2}{5} MR^2 \times \frac{v^2}{R^2} = \frac{1}{2} Mv^2 + \frac{1}{5} Mv^2 = \frac{7}{10} Mv^2$$

$$v = \sqrt{\frac{10gh}{7}} = \sqrt{\frac{10 \times 9.8 \text{ms}^{-2} \times 3.0 \text{m}}{7}} = 6.48 \text{ms}^{-1}$$

Question 7

(a) A spinning top has angular momentum $L = I\omega$ directed along its spin axis. While spinning fast, L is large. Any small disturbance (such as a slight tilt) produces a gravitational torque that attempts to change the direction of L . However, a large angular momentum resists rapid changes in direction, so the top remains stable. As friction slows the spin, ω and hence L decrease. The same gravitational torque now produces a larger rate of change of the direction of L , the top wobbles increasingly, and eventually falls over.

(b) In a glass tube, the cohesive forces between mercury atoms are much stronger than the adhesive forces between mercury and glass. Consequently, mercury does not wet the glass surface. The angle of contact exceeds 90° , producing a convex meniscus. The surface tension force acts downward along the contact line (because the meniscus curves downward), pulling the mercury surface below the level of mercury outside the tube. Water, by contrast, has stronger adhesion to glass than cohesion, producing an angle of contact less than 90° and a capillary rise.

(c) Angular velocity:

$$\omega = 2\pi \times \frac{120}{60} = 4\pi \text{ rad s}^{-1}$$

Using the rotating disc torque formula:

$$\tau = \frac{\pi\eta\omega R^4}{2d} = \frac{\pi \times 0.80 \text{ Nsm}^{-2} \times 4\pi \text{ rad s}^{-1} \times (0.12 \text{ m})^4}{2 \times 1.0 \times 10^{-3} \text{ m}}$$
$$\tau = 3.27 \text{ Nm}$$

Question 8

(a) The rotational kinetic energy of a body is $KE = \frac{1}{2}I\omega^2$. The solid sphere has $I = \frac{2}{5}MR^2$ and the hollow shell has $I = \frac{2}{3}MR^2$. At the same angular velocity, the shell has a larger moment of inertia and therefore greater rotational kinetic energy. The work needed to stop a rotating body equals its kinetic energy (by the work-energy theorem). Since the shell has more kinetic energy, more work is needed to stop it.

(b) When air is blown between the two sheets, the air between them moves at high speed. The air on the outer sides of the sheets is stationary. By Bernoulli's equation, the fast-moving air between the sheets has lower pressure than the still air outside. The pressure difference across each sheet produces a net inward force, pushing the sheets toward each other.

(c) By Torricelli's theorem:

$$v = \sqrt{2gh} = \sqrt{2 \times 9.8 \text{ ms}^{-2} \times 4.0 \text{ m}} = 8.85 \text{ ms}^{-1}$$

Initial volume flow rate:

$$Q = av = 2.0 \times 10^{-4} \text{ m}^2 \times 8.85 \text{ ms}^{-1} = 1.77 \times 10^{-3} \text{ m}^3 \text{ s}^{-1}$$

Question 9

(a) Torque is the product of force and the perpendicular distance from the line of action of the force to the pivot: $\tau = F \times d$. When the same force F is applied near the handle (large d), the torque is large. Near the hinge (small d), the torque is small. Since the angular acceleration of the door is $\alpha = \tau/I$, a larger torque produces a greater angular acceleration. Therefore, the door opens more easily when pushed near the handle.

(b) On a waxed surface, the wax is hydrophobic. The cohesive forces within the water are stronger than the adhesive forces between water and wax. The angle of contact exceeds 90° , so water beads up into near-spherical drops. On an unwaxed surface, the adhesive forces between water and the paint are stronger than the cohesive forces. The angle of contact is small, so the water spreads and wets the surface.

(c) Terminal velocity: $v_t \propto r^2$ (when density and fluid are the same).

$$\frac{v_Q}{v_P} = \frac{(3r)^2}{r^2} = 9$$

The terminal velocity of Q is 9 times that of P .

Question 10

(a) When the main rotor spins, it exerts a torque on the air (pushing air downward and around). By Newton's third law, the air exerts an equal and opposite torque on the rotor. This reaction torque is transmitted through the rotor shaft to the helicopter body, tending to rotate the body in the opposite direction to the rotor. The tail rotor produces a sideways thrust that creates a counter-torque about the vertical axis, preventing the body from spinning.

(b) By Stokes' law, the terminal velocity of a particle in a fluid is $v_t \propto r^2$. Fine clay particles have very small radii, so their terminal velocities are extremely small (fractions of a millimetre per second). Even slight air currents keep them suspended. Coarse sand grains have much larger radii, so their terminal velocities are much higher (proportional to r^2), and they settle rapidly under gravity.

(c) Moment of inertia:

$$I = \frac{1}{2}MR^2 = \frac{1}{2} \times 5.0 \text{ kg} \times (0.20 \text{ m})^2 = 0.10 \text{ kgm}^2$$

Angular deceleration:

$$\alpha = \frac{\tau}{I} = \frac{0.50 \text{ Nm}}{0.10 \text{ kgm}^2} = 5.0 \text{ rad s}^{-2}$$

Time to stop:

$$t = \frac{\omega_0}{\alpha} = \frac{10 \text{ rad s}^{-1}}{5.0 \text{ rad s}^{-2}} = 2.0 \text{ s}$$

Angular displacement:

$$\theta = \omega_0 t - \frac{1}{2} \alpha t^2 = 10 \times 2.0 - \frac{1}{2} \times 5.0 \times (2.0)^2 = 20 - 10 = 10 \text{ rad}$$

Number of revolutions:

$$n = \frac{\theta}{2\pi} = \frac{10 \text{ rad}}{2\pi} = 1.59 \approx 1.6 \text{ revolutions}$$

Question 11

(a) The moment of inertia of a disc about its centre is $I_c = \frac{1}{2}MR^2$. By the parallel axis theorem, the moment about an axis through the edge (distance R from the centre) is $I_e = I_c + MR^2 = \frac{1}{2}MR^2 + MR^2 = \frac{3}{2}MR^2$. Since $I_e = 3I_c$, the disc has three times the moment of inertia about the edge. By $\tau = I\alpha$, achieving the same angular acceleration requires three times the torque.

(b) When the river passes through the narrow arches, the continuity equation ($A_1v_1 = A_2v_2$) requires the water speed to increase. By Bernoulli's equation, the increased speed corresponds to a decrease in pressure. Since the water surface is exposed to the atmosphere, a lower pressure within the flowing water means the water surface drops to maintain equilibrium. Hence, the water level decreases at the narrow section.

(c) Given: $A = \pi(0.075\text{m})^2 = 1.767 \times 10^{-2}\text{m}^2$, $a = \pi(0.025\text{m})^2 = 1.963 \times 10^{-3}\text{m}^2$

Pressure difference from manometer:

$$\Delta P = (\rho' - \rho)gh = (13,600 - 1000)\text{kgm}^{-3} \times 9.8\text{ms}^{-2} \times 0.03\text{m} = 3704\text{Pa}$$

$$A^2 - a^2 = (1.767 \times 10^{-2})^2 - (1.963 \times 10^{-3})^2 = 3.084 \times 10^{-4}\text{m}^4$$

$$v_1 = \frac{a}{\sqrt{A^2 - a^2}} \times \sqrt{\frac{2\Delta P}{\rho}} = \frac{1.963 \times 10^{-3}\text{m}^2}{1.756 \times 10^{-2}\text{m}^2} \times \sqrt{\frac{2 \times 3704\text{Pa}}{1000\text{kgm}^{-3}}} = 0.304\text{ms}^{-1}$$

$$Q = Av_1 = 1.767 \times 10^{-2}\text{m}^2 \times 0.304\text{ms}^{-1} = 5.37 \times 10^{-3}\text{m}^3\text{s}^{-1}$$

Question 12

(a) Both balls have the same mass and speed, so their translational kinetic energies are equal. However, a rolling ball also has rotational kinetic energy $\frac{1}{2}I\omega^2$. The hollow ball has $I = \frac{2}{3}MR^2$ while the solid ball has $I = \frac{2}{5}MR^2$. At the same speed (and therefore same $\omega = v/R$), the hollow ball has more rotational kinetic energy. The work needed to stop a body equals its total kinetic energy. Since the hollow ball has more total energy, it is harder to stop.

(b) By the equation of continuity ($A_1v_1 = A_2v_2$), the volume of water passing through any cross-section per unit time must be the same. The narrow end has a much smaller cross-sectional area than the wide end. For the same volume to pass through per unit time, the water must move much faster through the narrow end. No additional force is needed; the speed increase is a direct consequence of mass conservation.

(c) By conservation of volume: $r = R/n^{1/3} = 2 \times 10^{-3}\text{m}/8^{1/3} = 1 \times 10^{-3}\text{m}$

Work done = increase in surface energy:

$$W = \gamma(8 \times 4\pi r^2 - 4\pi R^2) = 4\pi \times 0.5\text{Nm}^{-1} \times (8 \times (10^{-3}\text{m})^2 - (2 \times 10^{-3}\text{m})^2)$$

$$W = 2\pi \times (8 \times 10^{-6}\text{m}^2 - 4 \times 10^{-6}\text{m}^2) = 2.51 \times 10^{-5}\text{J}$$

Mass of original drop:

$$m = \frac{4}{3}\pi R^3 \rho = \frac{4}{3}\pi \times (2 \times 10^{-3}\text{m})^3 \times 13,600\text{kgm}^{-3} = 4.571 \times 10^{-4}\text{kg}$$

Temperature rise:

$$\Delta T = \frac{W}{mc} = \frac{2.51 \times 10^{-5}\text{J}}{4.571 \times 10^{-4}\text{kg} \times 140\text{Jkg}^{-1}\text{K}^{-1}} = 3.92 \times 10^{-4}\text{K}$$

Question 13

(a) A rolling ball has both translational KE ($\frac{1}{2}Mv^2$) and rotational KE ($\frac{1}{2}I\omega^2$). The total kinetic energy is the sum of both. A sliding ball at the same speed has only translational KE ($\frac{1}{2}Mv^2$) and no rotational KE (since it does not rotate). The extra energy in the rolling ball comes from the work done by friction at the contact point during the initial phase when rolling was established. Once rolling without slipping is achieved, friction does no further work, but the rotational energy it created remains.

(b) Viscosity in liquids arises from intermolecular attractive forces that resist the sliding of adjacent layers. At low temperature, molecules have low kinetic energy and intermolecular forces are strong, producing high resistance to flow. When heated, molecules gain kinetic energy and overcome these forces more easily. The resistance between layers decreases, the viscosity drops, and the honey flows more freely.

(c) For level flight, lift = weight:

$$\frac{1}{2}\rho A(v_1^2 - v_2^2) = Mg$$

$$v_1^2 - v_2^2 = \frac{2Mg}{\rho A} = \frac{2 \times 8000\text{kg} \times 9.8\text{ms}^{-2}}{1.2\text{kgm}^{-3} \times 30\text{m}^2} = 4356\text{m}^2\text{s}^{-2}$$

$$v_2^2 = (90\text{ms}^{-1})^2 - 4356\text{m}^2\text{s}^{-2} = 3744\text{m}^2\text{s}^{-2}$$

$$v_2 = \sqrt{3744\text{m}^2\text{s}^{-2}} = 61.2\text{ms}^{-1}$$

Question 14

(a) A heavier flywheel has a larger moment of inertia I . Starting it from rest requires kinetic energy $\frac{1}{2}I\omega^2$, which is larger for larger I . Hence more energy (and time) is needed to bring it to operating speed. However, once running, the large I stores more kinetic energy. When the engine encounters a sudden load (such as a compression stroke), the flywheel releases stored energy to maintain the angular velocity nearly constant. A lighter flywheel stores less energy and allows greater speed fluctuations. Thus, a heavier flywheel smooths out variations in angular velocity.

(b) The fast-moving train drags nearby air along with it. By Bernoulli's equation, this fast-moving air has lower pressure than the still air on the far side of the person. The pressure difference across the person creates a net force directed toward the train.

(c) Height of rise:

$$h = \frac{2\gamma\cos\theta}{\rho g r} = \frac{2 \times 0.073\text{Nm}^{-1} \times 1}{1000\text{kgm}^{-3} \times 9.8\text{ms}^{-2} \times 5 \times 10^{-4}\text{m}} = 0.0298\text{m} = 2.98\text{cm}$$

Since the tube is only 2cm long (shorter than 2.98cm), the water rises to the top but does not overflow. The meniscus at the top adjusts its curvature (becomes flatter), increasing the effective radius of curvature until the reduced excess pressure is just sufficient to support a 2cm column.

Question 15

(a) Before the child jumps, the total angular momentum is $L = I_{\text{total}}\omega$. When the child jumps off tangentially, she carries angular momentum with her ($L_{\text{child}} = mvr$, where r is the radius). By conservation of angular momentum, the remaining angular momentum of the merry-go-round must increase to compensate: $L_{\text{merry}} = L - L_{\text{child}}$. Since the merry-go-round now has a smaller moment of inertia (the child's mass is gone) but must retain its share of angular momentum, its angular velocity increases.

(b) A soap bubble has two surfaces (inner and outer), each with surface energy proportional to the surface area. Among all closed shapes enclosing a given volume, a sphere has the smallest surface area. Surface tension drives the film toward the configuration of minimum total surface energy. Therefore, regardless of the initial shape, the bubble relaxes into a sphere.

(c) Initial angular momentum:

$$L_i = I_{\text{disc}}\omega_i = \frac{1}{2} \times 8.0\text{kg} \times (0.40\text{m})^2 \times 6.0\text{rad s}^{-1} = 3.84\text{kgm}^2\text{s}^{-1}$$

After clay sticks at the rim, the new moment of inertia:

$$I_f = I_{\text{disc}} + m_{\text{clay}}R^2 = 0.64\text{kgm}^2 + 2.0\text{kg} \times (0.40\text{m})^2 = 0.96\text{kgm}^2$$

By conservation of angular momentum:

$$\omega_f = \frac{L_i}{I_f} = \frac{3.84\text{kgm}^2\text{s}^{-1}}{0.96\text{kgm}^2} = 4.0\text{rad s}^{-1}$$

Question 16

(a) The long pole has a large moment of inertia about the tightrope (because its mass is distributed far from the axis). By $\tau = I\alpha$, a large I means that any gravitational torque caused by a slight tilt produces only a small angular acceleration. This gives the walker more time to shift body weight and correct the tilt before it becomes dangerous. Without the pole, the smaller moment of inertia allows faster tilting, making balance harder.

(b) By Bernoulli's equation for horizontal flow: $P + \frac{1}{2}\rho v^2 = \text{constant}$. The total of pressure energy and kinetic energy per unit volume is constant along a streamline. At the wider section, the speed is lower (by continuity), so the kinetic energy term $\frac{1}{2}\rho v^2$ is smaller. For the sum to remain constant, the pressure P must be larger. At the narrow section, the speed is higher, so the pressure must be lower.

(c) Velocity gradient:

$$\frac{dv}{dy} = \frac{v}{d} = \frac{0.6\text{ms}^{-1}}{2.0 \times 10^{-3}\text{m}} = 300\text{s}^{-1}$$

By Newton's law of viscosity:

$$F = \eta A \frac{dv}{dy} = 0.30\text{Nsm}^{-2} \times 0.04\text{m}^2 \times 300\text{s}^{-1} = 3.6\text{N}$$

Question 17

(a) The point is the centre of mass of the hammer. External forces (gravity and air resistance) act on the body as a whole. The motion of the centre of mass depends only on these external forces, regardless of how the body rotates. Gravity produces a uniform downward acceleration, so the centre of mass follows a parabolic trajectory (just like a point particle). All other points rotate around the centre of mass while it follows this parabola, (which is why the hammer appears to tumble).

(b) By Poiseuille's formula, $Q \propto \Delta P \times R^4$. The flow rate depends on the fourth power of the radius but only the first power of the pressure. To triple Q by pressure alone (keeping R constant), ΔP must triple, which requires a 200% increase. To triple Q by increasing R alone (keeping ΔP constant), R^4 must triple, so R must increase by a factor of $3^{1/4} = 1.32$, only a 32% increase. The R^4 dependence makes radius changes far more effective than pressure changes.

(c) For a solid sphere rolling without slipping:

$$\begin{aligned} \frac{1}{2}Mv^2 + \frac{1}{2} \times \frac{2}{5}MR^2 \times \frac{v^2}{R^2} &= Mgh \\ \frac{7}{10}Mv^2 &= Mgh \\ h = \frac{7v^2}{10g} &= \frac{7 \times (4.0\text{ms}^{-1})^2}{10 \times 9.8\text{ms}^{-2}} = 1.14\text{m} \end{aligned}$$

Question 18

(a) Angular momentum is conserved: $I\omega = (I + I)\omega_f$, giving $\omega_f = \omega/2$. Initial KE: $\frac{1}{2}I\omega^2$. Final KE: $\frac{1}{2}(2I)(\omega/2)^2 = \frac{1}{4}I\omega^2$. Half the kinetic energy is lost. The lost energy is converted to heat by friction between the disc surfaces as they slide against each other before reaching the common angular velocity. Angular momentum is conserved because no external torque acts, but kinetic energy is not conserved because internal friction is an inelastic process.

(b) When the rubber bulb is squeezed, air rushes through the horizontal tube at high speed. At the junction with the vertical tube, the fast-moving air creates low pressure (by Bernoulli's principle). The atmospheric pressure on the liquid surface in the reservoir is greater than the pressure at the junction. This pressure difference pushes the liquid up the vertical tube. At the junction, the fast airstream breaks the liquid into fine droplets, producing a spray.

(c) Terminal velocity:

$$v_t = \frac{d}{t} = \frac{0.30\text{m}}{6.5\text{s}} = 0.0462\text{ms}^{-1}$$

Rearranging the terminal velocity formula:

$$\eta = \frac{2r^2(\rho_s - \rho_f)g}{9v_t} = \frac{2 \times (1.0 \times 10^{-3}\text{m})^2 \times (2500 - 800)\text{kgm}^{-3} \times 9.8\text{ms}^{-2}}{9 \times 0.0462\text{ms}^{-1}} = 0.0801\text{Nsm}^{-2}$$

Question 19

(a) When the ball with backspin hits the ground, the point of contact has two velocity components: a forward translational velocity and a backward velocity due to the spin. If the backward spin velocity exceeds the forward translational velocity at the contact point, the friction force from the ground acts forward on the ball. This forward friction decelerates the forward motion, momentarily stops it, and then accelerates the ball backward. The ball reverses direction. The angular momentum about the contact point is partially transferred to linear momentum in the reverse direction.

(b) When pivoted at one end, the moment of inertia is $I_{\text{end}} = \frac{1}{3}ML^2$. When pivoted at the centre, $I_{\text{centre}} = \frac{1}{12}ML^2$. The restoring torque due to gravity depends on the displacement of the centre of mass from below the pivot. For the rod pivoted at one end, the centre of mass is $\frac{L}{2}$ from the pivot, giving torque $\tau = \frac{MgL\sin\theta}{2}$. The angular acceleration is $\alpha = \frac{\tau}{I} = \frac{\frac{MgL\sin\theta}{2}}{\frac{1}{3}ML^2} = \frac{3g\sin\theta}{2L}$. For the rod pivoted at the centre, there is no restoring torque (the centre of mass is at the pivot). The rod pivoted at the end swings; the one at the centre does not swing at all under gravity. Hence, the end-pivoted rod swings faster.

(c) Moment of inertia of the ring: $I_{\text{ring}} = mr^2 = 2.0\text{kg} \times (0.25\text{m})^2 = 0.125\text{kgm}^2$

Total initial angular momentum:

$$L = I_{\text{turntable}}\omega_i = 0.80\text{kgm}^2 \times 3.0\text{rad s}^{-1} = 2.4\text{kgm}^2\text{s}^{-1}$$

After the ring is placed on the turntable:

$$I_f = 0.80\text{kgm}^2 + 0.125\text{kgm}^2 = 0.925\text{kgm}^2$$

$$\omega_f = \frac{L}{I_f} = \frac{2.4\text{kgm}^2\text{s}^{-1}}{0.925\text{kgm}^2} = 2.59\text{rad s}^{-1}$$

Kinetic energy lost:

$$KE_i = \frac{1}{2} \times 0.80\text{kgm}^2 \times (3.0\text{rad s}^{-1})^2 = 3.6\text{J}$$

$$KE_f = \frac{1}{2} \times 0.925\text{kgm}^2 \times (2.59\text{rad s}^{-1})^2 = 3.10\text{J}$$

$$\Delta KE = 3.6\text{J} - 3.10\text{J} = 0.50\text{J}$$

Question 20

(a) When the cylinder rolls without slipping, gravitational PE is converted into both translational and rotational KE: $Mgh = \frac{1}{2}Mv^2 + \frac{1}{2}I\omega^2$. Some energy goes into spinning the cylinder. When the cylinder slides without rolling, all the PE converts to translational KE only: $Mgh = \frac{1}{2}Mv^2$. The sliding cylinder has more translational speed at the bottom because no energy is “lost” to rotation. Therefore, it reaches the bottom faster.

(b) By the equation of continuity ($Av = \text{constant}$), the product of cross-sectional area and flow speed is constant throughout the circulatory system. The aorta has a small total cross-sectional area, so blood flows quickly. Capillaries are individually very narrow, but there are billions of them. Their combined total cross-sectional area is vastly larger than the aorta’s. For the same volume of blood to pass through per unit time, the speed in the capillaries must be correspondingly lower.

(c) The smaller bubble has greater excess pressure ($\Delta P = 4\gamma/r$). When connected, air flows from the smaller (higher pressure) bubble into the larger (lower pressure) bubble. The smaller bubble shrinks and the larger one grows.

Radius of curvature of the common interface:

$$\frac{1}{R} = \frac{1}{r_1} - \frac{1}{r_2} = \frac{1}{0.02\text{m}} - \frac{1}{0.05\text{m}} = 50\text{m}^{-1} - 20\text{m}^{-1} = 30\text{m}^{-1}$$

$$R = \frac{1}{30}\text{m} = 0.0333\text{m} = 3.33\text{cm}$$

Question 21

(a) The spinning gyroscope has angular momentum $L = I\omega$ directed along its spin axis. Gravity exerts a torque about the support point, trying to pull the axle downward. This torque is perpendicular to L , so it changes the direction of L without changing its magnitude. The result is that the angular momentum vector rotates horizontally in a slow circle. This is precession. The gyroscope does not fall because the gravitational torque is entirely used up in changing the direction of the large angular momentum, not in creating a new downward rotation.

(b) When the capillary tube is lifted, the water column remains supported by surface tension at the meniscus inside the tube. The excess pressure across the curved meniscus ($\Delta P = \frac{2\gamma\cos\theta}{r}$) supports the weight of the water column. As long as the bottom of the tube remains submerged and the meniscus is intact, the water does not drain out because the surface tension force at the top meniscus balances the hydrostatic pressure of the column.

(b) Let the acceleration of the falling mass be a and the tension in the string be T .

For the hanging mass: $mg - T = ma \rightarrow 2 \times 9.8 - T = 2a \rightarrow 19.6 - T = 2a \dots (1)$

For the cylinder: $\tau = I\alpha \rightarrow TR = \frac{1}{2}MR^2 \times \frac{a}{R} \rightarrow T = \frac{1}{2}Ma = \frac{1}{2} \times 4\text{kg} \times a = 2a \dots (2)$

Substituting (2) into (1): $19.6\text{N} - 2a = 2a \rightarrow 4a = 19.6\text{N} \rightarrow a = 4.9\text{ms}^{-2}$

$$T = 2 \times 4.9 = 9.8\text{N}$$

Question 22

(a) The moment of inertia of a body about a given axis depends on how far its mass is distributed from that axis ($I = \sum m_i r_i^2$). For the T-shape and L-shape, the same total mass is distributed differently relative to the pivot at one end. The shape that has more mass farther from the pivot has a larger moment of inertia. By $\tau = I\alpha$, a larger I requires more torque for the same angular acceleration, making that shape harder to rotate.

(b) Paper towel fibres are hydrophilic (water-attracting). The adhesive forces between water and the fibres are stronger than the cohesive forces within the water. The angle of contact is small (less than 90°), so capillary rise occurs in the narrow spaces between fibres. Waxed paper has hydrophobic fibres. The cohesive forces in water are stronger than the adhesive forces with wax. The angle of contact exceeds 90° , so capillary rise does not occur.

(c) By continuity: $v_2 = v_1 \times r_1^2 / r_2^2 = 6\text{ms}^{-1} \times (3\text{cm})^2 / (6\text{cm})^2 = 1.5\text{ms}^{-1}$

By Bernoulli's equation:

$$P_2 = P_1 + \frac{1}{2}\rho(v_1^2 - v_2^2) - \rho g(h_2 - h_1)$$

$$P_2 = 2.5 \times 10^5 \text{Pa} + \frac{1}{2} \times 1000 \text{kgm}^{-3} \times ((6\text{ms}^{-1})^2 - (1.5\text{ms}^{-1})^2) - 1000 \text{kgm}^{-3} \times 9.8\text{ms}^{-2} \times 4\text{m}$$

$$P_2 = 2.28 \times 10^5 \text{Pa}$$

Question 23

(a) When the heavy mass is at one end, it creates a large gravitational torque about the pivot (the centre). Any slight displacement is amplified because the torque increases as the ruler tilts. The ruler is in unstable equilibrium and tips over. When the mass is at the centre (at the pivot), it contributes zero torque regardless of the tilt angle, because its distance from the pivot is zero. The ruler remains balanced because only the ruler's own symmetrically distributed weight acts, and this produces a restoring effect for small displacements.

(b) When a cricket ball spins while moving forward, it drags the surrounding air in the direction of spin. On one side, the spin adds to the oncoming airflow (faster air, lower pressure). On the opposite side, the spin opposes the airflow (slower air, higher pressure). The pressure difference produces a net sideways force (Magnus effect) that curves the ball's path.

(c) Total KE = translational KE + rotational KE:

$$\text{KE} = \frac{1}{2}Mv^2 + \frac{1}{2}I\omega^2 = \frac{1}{2}Mv^2 + \frac{1}{2} \times \frac{2}{3}MR^2 \times \frac{v^2}{R^2}$$

$$\text{KE} = \frac{1}{2}Mv^2 + \frac{1}{3}Mv^2 = \frac{3}{6}Mv^2 + \frac{2}{6}Mv^2 = \frac{5}{6}Mv^2$$

Question 24

(a) In an Atwood machine, the net torque on the pulley is $\tau = (m_1 - m_2)gR$. The angular acceleration is $\alpha = \tau / (I_{\text{pulley}} + (m_1 + m_2)R^2)$. A massive pulley has a large I_{pulley} , increasing the denominator and reducing α . The system accelerates more slowly because the net force must accelerate not only the hanging masses but also spin up the pulley. A light pulley contributes negligible I and the system accelerates as if the pulley were not there.

(b) The terminal velocity of a raindrop is $v_t \propto r^2$ (by Stokes' law). A large raindrop has a larger radius and therefore a much higher terminal velocity. The weight of a drop grows as r^3 while the viscous drag grows only as r . A larger drop must reach a higher speed before drag balances its weight. Hence, large drops fall faster.

(c) The soap film has two surfaces. Force on the wire:

$$F = 2\gamma l = 2 \times 0.03 \text{Nm}^{-1} \times 0.06 \text{m} = 3.6 \times 10^{-3} \text{N}$$

Work done:

$$W = F \times d = 3.6 \times 10^{-3} \text{N} \times 0.04 \text{m} = 1.44 \times 10^{-4} \text{J}$$

Question 25

(a) The sliding block has only translational kinetic energy: $KE = \frac{1}{2}Mv^2$. The rolling sphere has both translational and rotational kinetic energy: $KE = \frac{1}{2}Mv^2 + \frac{1}{2}I\omega^2 = \frac{1}{2}Mv^2 + \frac{1}{5}Mv^2 = \frac{7}{10}Mv^2$. The extra $\frac{1}{5}Mv^2$ is rotational energy, which exists because the sphere is spinning. This extra energy was originally supplied by friction at the contact point when the rolling motion was first established.

(b) The excess pressure inside a soap bubble is $\Delta P = 4\gamma/r$, which is inversely proportional to the radius. A smaller bubble has a smaller r , giving a larger ΔP . Physically, the smaller bubble has a more tightly curved surface. The surface tension forces along this tightly curved surface have a larger inward component per unit area, requiring a greater internal pressure to maintain equilibrium.

(c) Using conservation of energy. Let the speed of the hanging mass be v after falling $h = 2.0\text{m}$:

$$\begin{aligned} mgh &= \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2 = \frac{1}{2}mv^2 + \frac{1}{2} \times \frac{1}{2}MR^2 \times \frac{v^2}{R^2} = \frac{1}{2}mv^2 + \frac{1}{4}Mv^2 \\ 5.0\text{kg} \times 9.8\text{ms}^{-2} \times 2.0\text{m} &= \frac{1}{2} \times 5.0\text{kg} \times v^2 + \frac{1}{4} \times 10\text{kg} \times v^2 \\ 98\text{J} &= 2.5v^2 + 2.5v^2 = 5.0v^2 \\ v &= \sqrt{\frac{98}{5.0}} \text{ms}^{-1} = 4.43\text{ms}^{-1} \end{aligned}$$

Question 26

(a) A gearbox conserves power: $P = \tau\omega$. In low gear, a small gear drives a large gear. The output angular velocity ω is low, but since power is conserved, the output torque $\tau = P/\omega$ is high. In high gear, a large gear drives a small gear. The output ω is high but τ is low. The gearbox trades angular velocity for torque (or vice versa) while conserving power (minus friction losses).

(b) The pitot tube has an opening facing directly into the airflow, bringing the air to rest at the stagnation point. By Bernoulli's equation, the stagnation pressure exceeds the static pressure by $\frac{1}{2}\rho v^2$. A manometer measures this pressure difference ΔP . The airspeed is then $v = \sqrt{\frac{2\Delta P}{\rho}}$.

(c) By Poiseuille's formula:

$$Q = \frac{\pi\Delta PR^4}{8\eta L} = \frac{\pi \times 5 \times 10^4 \text{Pa} \times (3 \times 10^{-3} \text{m})^4}{8 \times 1.5 \text{Nsm}^{-2} \times 0.8 \text{m}} = 1.325 \times 10^{-6} \text{m}^3 \text{s}^{-1}$$

$$\text{Average velocity: } v_{\text{avg}} = \frac{Q}{\pi R^2} = 1.325 \times \frac{10^{-6} \text{m}^3 \text{s}^{-1}}{\pi \times 9 \times 10^{-6} \text{m}^2} = 0.0469 \text{ms}^{-1}$$

Reynolds number:

$$R_e = \frac{\rho v_{\text{avg}} \times 2R}{\eta} = \frac{1260 \text{kgm}^{-3} \times 0.0469 \text{ms}^{-1} \times 6 \times 10^{-3} \text{m}}{1.5 \text{Nsm}^{-2}} = 0.237$$

Since $R_e = 0.237 \ll 2000$, the flow is **laminar**.

Question 27

(a) In a rigid body, all particles are locked in position relative to each other. When the body rotates, every particle moves in a circle about the axis. All particles complete one full revolution in the same time, so they all have the same angular velocity ω . However, each particle's linear speed is $v = \omega r$, where r is its distance from the axis. Particles farther from the axis travel larger circles in the same time, so they move at higher linear speeds.

(b) By Torricelli's theorem, the efflux speed from a hole at depth h below the surface is $v = \sqrt{2gh}$. The lower hole is at greater depth, so its efflux speed is higher. After exiting horizontally, the water follows a projectile path. Although the lower hole has less

fall height (less time in the air), its much greater horizontal speed more than compensates, and the jet strikes the ground further from the tank.

(c) Moment of inertia: $I = \frac{1}{2}MR^2 = \frac{1}{2} \times 6.0\text{kg} \times (0.30\text{m})^2 = 0.27\text{kgm}^2$

Rotational KE: $KE = \frac{1}{2}I\omega^2 = \frac{1}{2} \times 0.27\text{kgm}^2 \times (8.0\text{rad s}^{-1})^2 = 8.64\text{J}$

Angular deceleration: $\alpha = \frac{\omega}{t} = \frac{8.0\text{rads}^{-1}}{4.0\text{s}} = 2.0\text{rad s}^{-2}$

Torque: $\tau = I\alpha = 0.27\text{kgm}^2 \times 2.0\text{rad s}^{-2} = 0.54\text{Nm}$

Question 28

(a) When two hands push in opposite directions on opposite sides of the wheel, they form a couple: two equal, opposite, parallel forces. A couple produces pure rotation with no net translational force. The total torque of the couple is $\tau = F \times d$, where d is the diameter of the wheel (the distance between the forces). With one hand, the torque is $\tau = F \times r$ (only the radius). Since the diameter is twice the radius, the couple provides twice the torque for the same force, making it more effective.

(b) The soap film has two surfaces, each with surface tension γ . Surface tension acts to minimise the total surface area. The sliding wire is free to move, so the surface tension forces pull it inward (toward the frame), reducing the film area and hence the total surface energy. The force on the wire is $F = 2\gamma l$, where l is the wire length.

(c) Sphere density = 2ρ , fluid density = ρ :

$$v_t = \frac{2r^2(\rho_s - \rho_f)g}{9\eta} = \frac{2r^2(2\rho - \rho)g}{9\eta} = \frac{2r^2\rho g}{9\eta}$$

Question 29

(a) At the instant the ball is momentarily at rest on the incline, both its translational and rotational kinetic energies are zero ($v = 0$ and $\omega = 0$, since $v = R\omega$ for rolling without slipping). All the initial kinetic energy (translational + rotational) has been converted into gravitational potential energy Mgh , where h is the height gained on the incline.

(b) Both have the same mass M and the same angular velocity ω . The rotational kinetic energy is $KE = \frac{1}{2}I\omega^2$. The solid disc has $I = \frac{1}{2}MR^2$ while the thin ring has $I = MR^2$. Since $I_{\text{ring}} > I_{\text{disc}}$, the ring has greater kinetic energy at the same ω . The ring stores more energy because all its mass is concentrated at the maximum distance from the axis, maximising the mr^2 contribution.

(c) For rolling without slipping, $Mgh = \frac{1}{2}Mv^2 + \frac{1}{2}I\omega^2$ with $\omega = \frac{v}{R}$:

Cylinder ($I = \frac{1}{2}MR^2$): $Mgh = \frac{1}{2}Mv_c^2 + \frac{1}{4}Mv_c^2 = \frac{3}{4}Mv_c^2 \rightarrow v_c = \sqrt{\frac{4gh}{3}}$

Sphere ($I = \frac{2}{5}MR^2$): $Mgh = \frac{1}{2}Mv_s^2 + \frac{1}{5}Mv_s^2 = \frac{7}{10}Mv_s^2 \rightarrow v_s = \sqrt{\frac{10gh}{7}}$

$$\frac{v_s}{v_c} = \frac{\sqrt{\frac{10gh}{7}}}{\sqrt{\frac{4gh}{3}}} = \sqrt{\frac{10}{7} \cdot \frac{3}{4}} = \sqrt{\frac{10}{4} \cdot \frac{3}{7}} = 1.035$$

The sphere is about 3.5% faster than the cylinder.

Question 30

(a) Static friction at the contact point is essential for rolling without slipping. It provides the torque that maintains the rotational motion in step with the translational motion ($v = R\omega$). Without friction (on a smooth surface), the ball would slide without rotating. However, rolling friction (a separate, smaller effect caused by deformation at the contact point) dissipates energy and gradually slows both the translation and rotation. Eventually, all kinetic energy is converted to heat and the ball stops.

(b) Repeated heating at high temperatures causes the oil molecules to undergo chemical changes: polymerisation (molecules joining into longer chains) and oxidation. Longer-chain molecules have stronger intermolecular forces and resist sliding past each other more strongly. This increases the viscosity of the oil, making it thicker and harder to pour.

(c) Let a be the acceleration, T_1 and T_2 the tensions on the 5.0kg and 3.0kg sides.

Pulley: $I = \frac{1}{2} \times 3.0\text{kg} \times (0.20\text{m})^2 = 0.06\text{kgm}^2$

For 5.0kg mass: $5 \times 9.8 - T_1 = 5a \rightarrow 49 - T_1 = 5a \dots (1)$

For 3.0kg mass: $T_2 - 3.0 \times 9.8 = 3a \rightarrow T_2 - 29.4 = 3a \dots (2)$

For pulley: $(T_1 - T_2)R = I\alpha = \frac{Ia}{R} \rightarrow T_1 - T_2 = \frac{Ia}{R^2} = 0.06 \times \frac{a}{0.04} = 1.5a \dots (3)$

Adding (1) + (2): $49 - 29.4 - (T_1 - T_2) = 8.0a$

Substituting (3): $19.6 - 1.5a = 8.0a \rightarrow 9.5a = 19.6 \rightarrow a = 2.06\text{ms}^{-2}$

$$T_1 = 49 - 5.0 \times 2.06 = 49 - 10.3 = 38.7\text{N}$$

$$T_2 = 29.4 + 3.0 \times 2.06 = 29.4 + 6.18 = 35.6\text{N}$$

Question 31

(a) The rotational analogue of Newton's second law is $\tau = I\alpha$, where $\alpha = \frac{\Delta\omega}{\Delta t}$. Rearranging: $\tau\Delta t = I\Delta\omega$. The product $\tau\Delta t$ is the angular impulse. For a constant torque, doubling the time doubles the angular impulse and therefore doubles the change in angular velocity. This is the rotational equivalent of the linear impulse-momentum theorem ($F\Delta t = m\Delta v$).

(b) Bernoulli's principle: The nozzle narrows the exit area. By continuity, the water speed increases at the nozzle. By Bernoulli's equation, the pressure drops, and the water emerges as a fast jet. Newton's third law: The hose pushes water forward at high momentum. By Newton's third law, the water pushes back on the hose with an equal and opposite force. This reaction force causes the hose to recoil backward.

(c) Excess pressure (one surface): $\Delta P = \frac{2\gamma}{r} = 2 \times \frac{0.073\text{Nm}^{-1}}{5 \times 10^{-4}\text{m}} = 292\text{Pa}$

Hydrostatic pressure at depth 20m: $P_h = \rho gh = 1000\text{kgm}^{-3} \times 9.8\text{ms}^{-2} \times 20\text{m} = 196,000\text{Pa}$

Total pressure:

$$P = P_{\text{atm}} + P_h + \Delta P = 1.01 \times 10^5\text{Pa} + 1.96 \times 10^5\text{Pa} + 292\text{Pa} = 2.97 \times 10^5\text{Pa}$$

Question 32

(a) A long rod has a larger moment of inertia about its base than a short rod of the same mass. By $\tau = I\alpha$, the gravitational torque (which causes tilting) produces a smaller angular acceleration in the long rod. The long rod tilts more slowly, giving the person more time to make corrective adjustments with the finger. A short rod tilts quickly and is harder to correct in time.

(b) By Poiseuille's formula, $Q = \frac{\pi\Delta PR^4}{8\eta L}$. The flow rate depends on the fourth power of the straw radius. Doubling the radius increases the flow by a factor of $2^4 = 16$. A thick milkshake has high viscosity η , which already reduces Q . Through a narrow straw (small R), the R^4 factor makes the flow rate extremely small. A wider straw dramatically increases R^4 and restores a usable flow rate.

(c) Moment of inertia: $I = \frac{2}{5}MR^2 = \frac{2}{5} \times 3.0\text{kg} \times (0.10\text{m})^2 = 0.012\text{kgm}^2$

Angular momentum: $L = I\omega = 0.012\text{kgm}^2 \times 20\text{rad s}^{-1} = 0.24\text{kgm}^2\text{s}^{-1}$

Torque to stop in 5.0s: $\tau = \frac{L}{t} = \frac{0.24\text{kgm}^2\text{s}^{-1}}{5.0\text{s}} = 0.048\text{Nm}$

Question 33

(a) On a smooth incline, there is no friction at the contact point. The only force along the incline is the component of gravity, which accelerates the ball's centre of mass downward. With no friction, there is no torque about the centre of mass, so the ball does not rotate. On a rough incline, static friction acts at the contact point, directed up the incline. This friction provides a torque about the centre of mass that initiates and maintains rotation. The friction does not dissipate energy (it is static, with no sliding), but it converts some translational energy into rotational energy, producing rolling without slipping.

(b) In free fall, no external contact forces act on the drop. The only force shaping it is surface tension, which minimises the surface area. For a given volume, a sphere has the smallest surface area. Hence, the drop is perfectly spherical. On a table, the drop's weight presses its base flat against the surface, and the normal reaction from the table distorts the shape. The drop is flattened because gravity competes with surface tension.

(c) The hole is at depth 2m below the surface, so the efflux speed is:

$$v = \sqrt{2gh} = \sqrt{2 \times 9.8\text{ms}^{-2} \times 2\text{m}} = \sqrt{39.2\text{m}^2\text{s}^{-2}} = 6.26\text{ms}^{-1}$$

The hole is at height $6\text{m} - 2\text{m} = 4\text{m}$ above the ground. Time to fall:

$$t = \sqrt{\frac{2H}{g}} = \sqrt{2 \times \frac{4\text{m}}{9.8\text{ms}^{-2}}} = 0.903\text{s}$$

Horizontal distance: $x = vt = 6.26\text{ms}^{-1} \times 0.903\text{s} = 5.65\text{m}$

Question 34

(a) When the disc rotates about an axis perpendicular to its face (through the centre), every particle of the disc is at its maximum possible distance from the axis (the full radial distance r). When the disc rotates about a diameter, particles near the diameter are very close to the axis and contribute little to I , while only particles near the rim contribute significantly. On average, less mass is far from the axis when rotating about a diameter. Since $I = \sum m_i r_i^2$, the perpendicular-face axis gives a larger I and greater resistance to rotation.

(b) In untilled soil, narrow spaces between soil particles act as capillary tubes, drawing water upward from moist subsoil to the dry surface, where it evaporates. Tilling breaks the top layer into larger, irregular clumps with much wider gaps. Capillary rise is inversely proportional to gap width ($h \propto 1/r$). The wide gaps cannot sustain capillary rise, so the connection between the moist subsoil and the surface is broken. The moisture stays underground.

(c) Velocity gradient: $\frac{dv}{dy} = \frac{v}{d} = \frac{0.50\text{ms}^{-1}}{5 \times 10^{-4}\text{m}} = 1000\text{s}^{-1}$

Force: $F = \eta A \left(\frac{dv}{dy}\right) = 0.40\text{Nsm}^{-2} \times 0.04\text{m}^2 \times 1000\text{s}^{-1} = 16\text{N}$

Question 35

(a) Linear momentum is conserved because no external horizontal force acts on the system (the cart is on a smooth surface). The forward momentum of the ball equals the backward momentum of the cart-cannon system. Angular momentum about the barrel axis is not conserved because gravity exerts an external torque on the system about that axis (the ball follows a projectile path after leaving the barrel, and gravity acts on it throughout the flight).

(b) The flow rate from the tap is governed by Torricelli's theorem: $v = \sqrt{2gh}$, where h is the height of water above the tap. As water drains out, the water level drops, h decreases, and the efflux speed v decreases. Since the volume flow rate is $Q = av$ (where a is the tap area), a lower v means a lower Q . The flow rate decreases continuously as the tank empties.

(c) For rolling without slipping: $a = R\alpha$ and $v = R\omega$.

Forces along the incline: $Mg\sin\theta - f = Ma \dots (1)$

Torque about centre: $fR = I\alpha = \frac{1}{2}MR^2 \times a/R \rightarrow f = \frac{1}{2}Ma \dots (2)$

Substituting (2) into (1): $Mg\sin\theta - \frac{1}{2}Ma = Ma \rightarrow Mg\sin\theta = \frac{3}{2}Ma$

$$a = \frac{2g\sin\theta}{3}$$

Question 36

(a) Torque equals force times perpendicular distance from the pivot: $\tau = Fd$. A long-handled spanner has a larger distance d from the hand to the bolt. For the same applied force F , a larger d produces a larger torque. Since the bolt resists turning with a fixed frictional torque, a larger applied torque is more likely to overcome this resistance.

(b) Surface tension pulls every part of the liquid surface inward, minimising the total surface area. Among all closed shapes enclosing a given volume, a sphere has the smallest surface area. Consequently, surface tension moulds the drop into a sphere. The "teardrop" shape is incorrect for free-falling drops; it occurs only when a drop is attached to a surface and being elongated by gravity just before detaching.

(c) From mass and density: $m = \frac{4}{3}\pi r^3 \rho_s$

$$r = \left(\frac{3m}{4\pi\rho_s}\right)^{1/3} = \left(\frac{3 \times 2.0\text{kg}}{4\pi \times 7800\text{kgm}^{-3}}\right)^{1/3} = 0.0396\text{m}$$

Terminal velocity:

$$v_t = \frac{2r^2(\rho_s - \rho_f)g}{9\eta} = \frac{2 \times (0.0396\text{m})^2 \times (7800 - 900)\text{kgm}^{-3} \times 9.8\text{ms}^{-2}}{9 \times 0.50\text{Nsm}^{-2}}$$

$$v_t = 47.1\text{ms}^{-1}$$

Note: This very high terminal velocity confirms that Stokes' law would not be valid for this large, fast sphere (the Reynolds number would be far above 1). In practice, the actual terminal velocity would be much lower due to turbulent drag.

Question 37

(a) The child's push is applied as a force at the rim for a certain time. By the angular impulse-momentum theorem ($\tau\Delta t = \Delta L$), the push delivers an angular impulse that increases the angular momentum of the roundabout. Even after the child stops pushing, the roundabout has gained angular momentum (and hence angular velocity) from the impulse already delivered. It does not decelerate immediately because the only remaining torque is friction, which is typically much smaller than the child's push. The roundabout continues to accelerate briefly if the angular velocity has not yet caught up with the impulse already received, then friction gradually decelerates it.

(b) Water between the two ships is confined to a narrow channel. By the equation of continuity, the water in this narrow space flows faster than the water on the outer sides of the ships. By Bernoulli's equation, faster-moving water has lower pressure. The pressure between the ships is therefore lower than the pressure on the outer sides. This pressure difference pushes both ships inward, toward each other.

(c) Angular deceleration:

$$\alpha = \frac{\omega_f - \omega_i}{t} = \frac{4.0 - 10}{8.0} \text{ rad s}^{-2} = -0.75 \text{ rad s}^{-2}$$

Frictional torque: $\tau = I|\alpha| = 0.60 \text{ kgm}^2 \times 0.75 \text{ rad s}^{-2} = 0.45 \text{ Nm}$

Angular displacement:

$$\theta = \omega_i t + \frac{1}{2} \alpha t^2 = 10 \times 8.0 + \frac{1}{2} (-0.75)(8.0)^2 = 56 \text{ rad}$$

Revolutions: $n = \frac{56}{2\pi} = 8.91 \approx 8.9$ revolutions

Energy dissipated:

$$\Delta KE = \frac{1}{2} I \omega_i^2 - \frac{1}{2} I \omega_f^2 = \frac{1}{2} \times 0.60 \text{ kgm}^2 \times ((10)^2 - (4)^2) \text{ rad}^2 \text{ s}^{-2} = 0.30 \times 84 = 25.2 \text{ J}$$

Question 38

(a) Friction between the coin's edge and the table surface dissipates kinetic energy. Air resistance also removes energy, though more slowly. As the rotational kinetic energy decreases, the angular momentum decreases, and the gyroscopic stability that kept the coin upright weakens. Eventually, the angular momentum is too small to resist the gravitational torque, the coin begins to wobble (precession with increasing amplitude), and it falls flat.

(b) The excess pressure inside the soap bubble is $\Delta P = 4\gamma/r$. This pressure is higher than atmospheric pressure outside. When the blowing tube is left open, the air inside the bubble has a direct path to the atmosphere. The higher internal pressure pushes air out through the tube. As air escapes, the bubble's volume decreases and it contracts. The bubble shrinks until the internal pressure equalises with the atmosphere (which, for a soap film, means the bubble disappears entirely).

(c) In series, the flow rate Q is the same through both tubes. By Poiseuille's formula:

$$\Delta P = \frac{8\eta L Q}{\pi R^4}$$

For tube 1 (radius R): $\Delta P_1 = \frac{8\eta L Q}{\pi R^4}$

For tube 2 (radius $2R$): $\Delta P_2 = \frac{8\eta L Q}{\pi (2R)^4} = \frac{8\eta L Q}{16\pi R^4}$

$$\frac{\Delta P_1}{\Delta P_2} = \frac{16\pi R^4}{\pi R^4} = 16$$

The pressure drop across the narrow tube is **16 times** that across the wide tube.

Question 39

(a) At the same translational speed v , both bodies also rotate at $\omega = v/R$ (rolling condition). The total kinetic energy of a rolling body is $KE = \frac{1}{2} Mv^2 (1 + I/(MR^2))$. For the sphere ($I = \frac{2}{5} MR^2$): $KE = \frac{7}{10} Mv^2$. For the cylinder ($I = \frac{1}{2} MR^2$): $KE = \frac{3}{4} Mv^2$. The cylinder has more total kinetic energy because its moment of inertia is larger. Under the same rolling friction (which dissipates energy at the same rate for both at any given speed), the cylinder has more energy to lose before stopping. Therefore, the cylinder takes longer to stop.

(b) The viscosity of oil increases as temperature drops (viscosity of liquids is inversely related to temperature). In winter, the oil is colder and more viscous. By Poiseuille's formula ($Q = \pi \Delta P R^4 / (8\eta L)$), the flow rate is inversely proportional to viscosity. Higher η in winter means lower Q , even though ΔP , R , and L are unchanged.

(c) Angular acceleration: $\alpha = \frac{(\omega - \omega_0)}{t} = \frac{(50 - 0) \text{rad s}^{-1}}{10 \text{s}} = 5.0 \text{rad s}^{-2}$

Torque: $\tau = I\alpha = 0.40 \text{kgm}^2 \times 5.0 \text{rad s}^{-2} = 2.0 \text{Nm}$

Power at end: $P = \tau\omega = 2.0 \text{Nm} \times 50 \text{rad s}^{-1} = 100 \text{W}$

Question 40

(a) By conservation of angular momentum (no external torque): $I\omega = (I + I)\omega_f$, giving $\omega_f = \omega/2$. The angular velocity halves. Initial KE: $\frac{1}{2}I\omega^2$. Final KE: $\frac{1}{2}(2I)(\omega/2)^2 = \frac{1}{4}I\omega^2$. Half the kinetic energy is lost, converted to heat by friction between the disc surfaces as they slide against each other before reaching the common velocity.

(b) By the equation of continuity ($A_1v_1 = A_2v_2$), the same volume of water must pass through the narrow gap per unit time as through the wide river. Since the gap has a much smaller cross-sectional area, the water must move much faster to maintain the same volume flow rate.

(c) $\frac{1}{2}\rho_{\text{air}}v^2 = \rho_{\text{water}}gh$

$$v = \sqrt{\frac{2\rho_{\text{water}}gh}{\rho_{\text{air}}}} = \sqrt{\frac{2 \times 1000 \text{kgm}^{-3} \times 9.8 \text{ms}^{-2} \times 0.08 \text{m}}{1.2 \text{kgm}^{-3}}} = 36.1 \text{ms}^{-1}$$

Question 41

(a) When a ball with topspin hits the ground, the spinning surface at the contact point moves forward (in the same direction as the ball's translational motion). Friction at the contact point acts backward on the ball (opposing the relative motion of the contact surface). This backward friction has two effects: it slightly reduces the ball's forward speed, and it increases the forward rotation. The net result is that after the bounce, the ball has more forward spin and a flatter (lower) trajectory angle than a non-spinning ball, because friction redirected some of the vertical bounce energy into forward motion.

(b) By Poiseuille's formula, the flow rate through a channel is proportional to R^4 (the fourth power of the effective radius). Replacing the wide channel with a narrower one drastically reduces R . Since the flow depends on R^4 , even a modest reduction in width causes a dramatic reduction in flow rate.

(c) Moment of inertia of sphere about its centre: $I_{\text{cm}} = \frac{2}{5}MR^2 = \frac{2}{5} \times 2.5 \text{kg} \times (0.08 \text{m})^2 = 6.4 \times 10^{-3} \text{kgm}^2$

Distance from centre of sphere to pivot: $d = 0.60 \text{m}$

By the parallel axis theorem:

$$I_{\text{pivot}} = I_{\text{cm}} + Md^2 = 6.4 \times 10^{-3} \text{kgm}^2 + 2.5 \text{kg} \times (0.60 \text{m})^2 = 0.906 \text{kgm}^2$$

Question 42

(a) The excess pressure inside a soap bubble is $\Delta P = 4\gamma/r$. The smaller bubble has a larger excess pressure (smaller r , larger ΔP). When connected, air flows from the region of higher pressure (small bubble) to the region of lower pressure (large bubble). The small bubble shrinks (increasing its pressure further) and the large bubble grows (decreasing its pressure). This is a positive feedback process that continues until the small bubble disappears entirely.

(b) Consider a small cube of fluid at rest at depth h . If the pressure on one face were greater than on the opposite face, there would be a net force on the cube, and it would accelerate. Since the fluid is at rest (no acceleration), the pressure on all faces must be equal. This argument holds for any orientation of the cube, proving that pressure at a point in a static fluid is the same in all directions (Pascal's result).

(c) Total KE on the horizontal surface:

$$\text{KE} = \frac{7}{10}Mv^2 = \frac{7}{10} \times 1.5 \text{kg} \times (5.0 \text{ms}^{-1})^2 = 26.25 \text{J}$$

On the incline, all KE converts to PE: $\text{KE} = Mgh = Mg \times d \sin 30^\circ$

$$d = \frac{\text{KE}}{Mg \sin 30^\circ} = \frac{26.25 \text{J}}{1.5 \text{kg} \times 9.8 \text{ms}^{-2} \times 0.5} = 3.57 \text{m}$$

Question 43

(a) The sand falls vertically and has zero angular momentum about the platform's axis before landing. When it lands on the rotating platform, friction accelerates it to the platform's angular velocity, giving it angular momentum. By conservation of angular momentum for the system, the angular momentum gained by the sand must come from the platform. The platform's angular momentum decreases. Since $L = I\omega$ and the total I has increased (added mass), ω decreases.

(b) Inside the pipe, the water is under pressure (greater than atmospheric). At the leak, the pressurised water pushes outward against atmospheric pressure. Since the internal pressure exceeds atmospheric pressure, water is forced out through the leak. Air would only be drawn in if the internal pressure were lower than atmospheric, which does not occur in a pressurised water pipe.

(c) Before impact, the bullet has linear momentum $p = mv = 0.05\text{kg} \times 200\text{ms}^{-1} = 10\text{kgms}^{-1}$.

Its angular momentum about the pivot (centre of rod) at distance $L/2 = 0.40\text{m}$:

$$L_i = mv \times \frac{L}{2} = 10\text{kgms}^{-1} \times 0.40\text{m} = 4.0\text{kgm}^2\text{s}^{-1}$$

After impact, the bullet is embedded at the end. Total moment of inertia:

$$I = I_{\text{rod}} + m_{\text{bullet}} \left(\frac{L}{2}\right)^2 = \frac{1}{12}ML^2 + m\left(\frac{L}{2}\right)^2$$

$$I = \frac{1}{12} \times 2.0\text{kg} \times (0.80\text{m})^2 + 0.05\text{kg} \times (0.40\text{m})^2 = 0.1147\text{kgm}^2$$

By conservation of angular momentum:

$$\omega = \frac{L_i}{I} = \frac{4.0\text{kgm}^2\text{s}^{-1}}{0.1147\text{kgm}^2} = 34.9\text{rad s}^{-1}$$

Question 44

(a) The spinning gyroscope has a large angular momentum $L = I\omega$ directed along its spin axis. By the law of conservation of angular momentum, L remains constant in direction unless an external torque acts. When the aircraft turns or tilts, no torque is applied to the gyroscope (it is mounted on gimbals). The gyroscope's spin axis continues to point in the original direction, providing a fixed reference against which the aircraft's orientation can be measured.

(b) During high winds, the fast-moving air outside the building has lower pressure than the still air inside (by Bernoulli's equation). The internal air pressure exceeds the external air pressure on the windward side. This pressure difference pushes windows outward from inside. If the pressure difference exceeds the strength of the glass, the window shatters outward.

(c) Terminal velocity: $v_t = \frac{2r^2(\rho_s - \rho_f)g}{9\eta}$. Since r , g , and η are the same for all three:

$$v_t \propto (\rho_s - \rho_f)$$

$$v_A \propto (8000 - 1000) = 7000, v_B \propto (4000 - 1000) = 3000, v_C \propto (2000 - 1000) = 1000$$

Ranking: $v_A > v_B > v_C$

$$v_A : v_B : v_C = 7000 : 3000 : 1000 = 7 : 3 : 1$$

Question 45

(a) The oil film lies between the rotating disc and the fixed surface below. The oil layer in contact with the disc moves with the disc, while the oil layer in contact with the fixed surface remains stationary. By Newton's law of viscosity, the velocity difference between these layers produces a viscous drag force that opposes the disc's rotation. This drag force creates a retarding torque on the disc. The torque does negative work on the disc, steadily converting its rotational kinetic energy into heat energy in the oil. As the kinetic energy decreases, the angular velocity decreases. The disc eventually stops when all its rotational kinetic energy has been transferred to the oil as heat.

(b) The pressure of water at a depth h below the surface is $P = \rho gh$. The pressure increases linearly with depth. At the base of the dam, the water pressure is maximum. The dam wall must withstand this pressure. A thicker base provides greater structural strength to resist the larger force per unit area at greater depth. Near the top, the water pressure is small and a thinner wall suffices.

(c) On the table, the ball has translational and rotational KE:

$$KE_{\text{table}} = \frac{7}{10}Mv^2 = \frac{7}{10} \times 0.30\text{kg} \times (3.0\text{ms}^{-1})^2 = 1.89\text{J}$$

During the fall, the ball gains translational KE from gravity (rotational KE is unchanged since gravity exerts no torque about the centre of mass):

$$KE_{\text{fall}} = Mgh = 0.30\text{kg} \times 9.8\text{ms}^{-2} \times 1.5\text{m} = 4.41\text{J}$$

Total KE just before hitting the ground:

$$KE_{\text{total}} = KE_{\text{table}} + KE_{\text{fall}} = 1.89\text{J} + 4.41\text{J} = 6.30\text{J}$$

This total KE is the sum of translational and rotational:

$$KE_{\text{total}} = \frac{1}{2}Mv_{\text{final}}^2 + \frac{1}{2}I\omega^2$$

The rotational KE is unchanged from the table: $\frac{1}{2}I\omega^2 = \frac{1}{5}Mv_{\text{table}}^2 = \frac{1}{5} \times 0.30 \times 9 = 0.54\text{J}$

$$\frac{1}{2}Mv_{\text{final}}^2 = KE_{\text{total}} - \frac{1}{2}I\omega^2 = 6.30\text{J} - 0.54\text{J} = 5.76\text{J}$$

$$v_{\text{final}} = \sqrt{\frac{2 \times 5.76\text{J}}{0.30\text{kg}}} = 6.2\text{ms}^{-1}$$

Note: This is the translational speed. The total speed (including the tangential speed due to rotation at the surface) involves the spin, but in examination context, the “speed of the ball” refers to the speed of the centre of mass, which is 6.2ms^{-1} .

Question 46

(a) The hard-boiled egg is a rigid body. When stopped, every particle inside it stops simultaneously. When released, there is no internal motion to restart the spin.

The raw egg has a liquid interior. When the shell is stopped, the liquid inside continues to rotate (it was not in direct contact with the finger). When the finger is released, the still-rotating liquid drags the shell back into motion through viscous friction between the liquid and the inner wall of the shell.

(b) At terminal velocity, the net force on the ball is zero: the weight equals the sum of the upthrust and viscous drag. By Stokes' law, the viscous drag is $F = 6\pi\eta r v_t$. A higher viscosity produces a larger drag force at any given speed, so the ball reaches terminal velocity at a lower speed and takes longer to fall the same distance. The liquid in which the ball takes longer has the higher viscosity.

(c) Given: $h = 1.3\text{m}$, $r = 0.18\text{mm} = 1.8 \times 10^{-4}\text{m}$, $L = 0.03\text{m}$, $Q = 4.5\text{cm}^3/\text{min} = 7.5 \times 10^{-8}\text{m}^3\text{s}^{-1}$

Pressure difference driving the flow:

$$\Delta P = \rho gh = 1060\text{kgm}^{-3} \times 9.8\text{ms}^{-2} \times 1.3\text{m} = 13504\text{Pa}$$

By Poiseuille's formula:

$$\eta = \frac{\pi\Delta Pr^4}{8QL} = \frac{\pi \times 13,504\text{Pa} \times (1.8 \times 10^{-4}\text{m})^4}{8 \times 7.5 \times 10^{-8}\text{m}^3\text{s}^{-1} \times 0.03\text{m}} = 2.33 \times 10^{-3}\text{Nsm}^{-2}$$

Question 47

(a) When rolling without slipping, gravitational PE converts into both translational and rotational KE. The fraction going to rotation depends on the moment of inertia. The solid sphere has $I = \frac{2}{5}MR^2$ and the solid cylinder has $I = \frac{1}{2}MR^2$. The cylinder has a larger moment of inertia, so it diverts a greater fraction of its energy into rotation, leaving less for translation. The sphere retains more translational KE and reaches the bottom with the greater translational speed.

(b) Gas flows at high speed through the narrow jet at the base of the burner. By Bernoulli's equation, the fast-moving gas creates a region of low pressure at the jet. The atmospheric pressure outside the side hole is greater than the pressure at the jet. This pressure difference pushes air through the side hole into the barrel, where it mixes with the gas before combustion.

(c) By Torricelli's theorem: $v = \sqrt{2gh} = \sqrt{2 \times 9.8\text{ms}^{-2} \times 10\text{m}} = 14\text{ms}^{-1}$

Required flow rate: $Q = 26.4\text{m}^3/\text{min} = 26.4/60 = 0.44\text{m}^3\text{s}^{-1}$

$$Q = Av = \frac{\pi d^2}{4} \times v$$

$$d = \sqrt{\frac{4Q}{\pi v}} = \sqrt{\frac{4 \times 0.44\text{m}^3\text{s}^{-1}}{\pi \times 14\text{ms}^{-1}}} = 0.20\text{m}$$

The outlet diameter is 0.20m (20 cm).

Question 48

(a) When the ball strikes the step, the step exerts a reaction force at the contact point (the edge of the step). The torque of this reaction force about the ball's centre determines whether the ball rotates over the step or is pushed back.

If the ball's centre is above the step edge, the reaction force at the edge passes below the centre. This produces a torque that rotates the ball forward over the step. If the ball's centre is below the step edge, the reaction force passes above the centre. This produces a torque that rotates the ball backward, and the ball bounces back.

(b) At low pouring speed, the Reynolds number of the flow is small and the flow is laminar: smooth, orderly, and quiet. When the bottle is tilted steeply, the flow speed increases, raising the Reynolds number. When it exceeds the critical value (approximately 2000), the flow transitions from laminar to turbulent. Turbulent flow is chaotic, with eddies and irregular pressure fluctuations that produce the gurgling noise.

(c) At terminal velocity, the net force on the bubble is zero. The buoyant force (upward) equals the weight of the bubble plus the viscous drag (downward on a rising bubble):

$$\text{Upthrust} = \frac{4}{3}\pi r^3 \rho_f g, \text{ Weight} = \frac{4}{3}\pi r^3 \rho_{\text{air}} g, \text{ Drag} = 6\pi\eta r v_t$$

Since $\rho_{\text{air}} \ll \rho_f$, the weight of the bubble is negligible:

$$6\pi\eta r v_t = \frac{4}{3}\pi r^3 (\rho_f - \rho_{\text{air}})g$$
$$\eta = \frac{2r^2(\rho_f - \rho_{\text{air}})g}{9v_t} = \frac{2 \times (0.01\text{m})^2 \times (1470 - 1.2)\text{kgm}^{-3} \times 9.8\text{ms}^{-2}}{9 \times 0.002\text{ms}^{-1}}$$
$$\eta = 16.0\text{Nsm}^{-2}$$

Question 49

(a) The moment of inertia of a body is $I = \sum m_i r_i^2$, where r_i is each particle's distance from the axis. In the wheel with a heavy rim, most of the mass sits at the maximum distance (the rim radius R), contributing $m_{\text{rim}} R^2$ to I . In the wheel with heavy spokes, most of the mass is distributed between $r = 0$ (the hub) and $r = R$ (the rim), at an average distance much less than R . Since the r^2 weighting penalises mass near the centre, the heavy-spoke wheel has a smaller I despite the same total mass.

(b) At the ankle, the column of blood between the heart and the ankle adds hydrostatic pressure $\Delta P = \rho gh$, where h is the vertical distance from the heart to the ankle. This additional pressure raises the reading above the value produced by the heart's pumping alone. At the upper arm (level with the heart), $h \approx 0$ and no hydrostatic correction is needed. The arm reading therefore reflects the heart's pumping pressure directly.

(c) By Poiseuille's formula, $Q = \pi\Delta P R^4 / (8\eta L)$. For constant Q : $\Delta P \propto 1/R^4$.

New radius: $R' = R - 0.08R = 0.92R$

$$\frac{\Delta P'}{\Delta P} = \frac{R^4}{R'^4} = \frac{R^4}{(0.92R)^4} = \frac{1}{(0.92)^4} = \frac{1}{0.7164} = 1.396$$

The pressure difference must increase by a factor of approximately 1.40 (a 40% increase) to maintain the same flow rate through a pipe whose radius has decreased by only 8%.

Question 50

(a) On the smooth incline, no friction acts. There is no tangential force along the surface, so no torque acts about the disc's centre of mass. By the rotational analogue of Newton's first law, the angular velocity ω remains constant throughout the climb. However, the component of gravity along the incline decelerates the translational motion, so the translational speed v decreases. The rolling condition $v = R\omega$ breaks immediately because v changes while ω does not. The disc slides and spins simultaneously as it climbs. Eventually $v = 0$ and the disc stops translating, but it is still spinning at the original ω .

(b) As the stream falls, it accelerates under gravity. By the equation of continuity ($Av = \text{constant}$), the stream must narrow as it speeds up. The narrowing increases the surface area to volume ratio ($A - V$ ratio $\propto \frac{1}{r}$). Surface tension, which always acts to minimise surface area, eventually breaks the thin stream into separate droplets, because a series of spherical drops has less total surface energy than a long, thin cylinder of the same volume.

(c) Mass of the raindrop:

$$m = \frac{4}{3}\pi r^3 \rho = \frac{4}{3}\pi \times (2.0 \times 10^{-3}\text{m})^3 \times 1000\text{kgm}^{-3} = 3.351 \times 10^{-5}\text{kg}$$

The work done by gravity depends only on the force (mg) and the vertical displacement, regardless of the speed at which the drop travels.

First half (500m to 250m, distance = 250m):

$$W_1 = mgh_1 = 3.351 \times 10^{-5}\text{kg} \times 9.8\text{ms}^{-2} \times 250\text{m} = 0.082\text{J}$$

Second half (250m to ground, distance = 250m):

$$W_2 = mgh_2 = 3.351 \times 10^{-5}\text{kg} \times 9.8\text{ms}^{-2} \times 250\text{m} = 0.082\text{J}$$

The work done by gravity is the **same** in both halves (0.082J).

In the first half, this work goes partly into increasing the drop's kinetic energy and partly into overcoming air resistance. In the second half (constant speed, zero net acceleration), all of gravity's work goes entirely into overcoming air resistance.

Question 51

(a) By Torricelli's theorem, the efflux speed is $v = \sqrt{2gh}$, where h is the depth of the hole below the water surface. This depends only on h and g , not on the size of the hole. Enlarging the hole does not change v . However, the volume flow rate is $Q = Av$. A larger hole has a larger cross-sectional area A , so Q increases even though v stays the same.

(b) The wet canvas has water filling the tiny gaps between the fibres. Surface tension at each gap supports the water and prevents it from passing through, because the water surface curves into a meniscus at each gap and the excess pressure ($\Delta P = 2\gamma/r$) holds the water in place. When the inside is touched, the finger pushes the water surface through the gaps, breaking the menisci. Once broken, the surface tension barrier is destroyed and water drips through freely.

(c) Total pressure = static pressure + dynamic pressure:

$$\frac{1}{2}\rho v^2 = P_{\text{total}} - P_{\text{static}} = 1.4 \times 10^5\text{Pa} - 1.3 \times 10^5\text{Pa} = 1.0 \times 10^4\text{Pa}$$

$$v = \sqrt{\frac{2 \times 1.0 \times 10^4\text{Pa}}{1000\text{kgm}^{-3}}} = 4.47\text{ms}^{-1}$$

Mass flow rate:

$$\dot{m} = \rho Av = 1000\text{kgm}^{-3} \times 25 \times 10^{-4}\text{m}^2 \times 4.47\text{ms}^{-1} = 11.2\text{kgs}^{-1}$$

Question 52

(a) When the rod is released, gravity acts at its centre of mass (distance $L/2$ from the pivot). The gravitational torque about the pivot is $\tau = Mg(L/2)$. The angular acceleration is $\alpha = \tau/I = Mg(L/2)/(\frac{1}{3}ML^2) = 3g/(2L)$. The linear acceleration of the free end (at distance L from the pivot) is $a_{\text{tip}} = \alpha L = 3g/2$. This is 1.5 times g , which exceeds the acceleration of a freely falling object. The explanation is that the pivot reaction force prevents the upper end from falling freely, so the gravitational torque concentrates the rotational effect at the far end, where the lever arm is greatest.

(b) Before the parachute opens, the skydiver's cross-sectional area is small. The drag force ($F = 6\pi\eta rv$ for Stokes drag, or more generally proportional to Av^2 at high speeds) is small, so a high speed is needed before drag equals the weight. Opening the parachute dramatically increases the effective cross-sectional area. The drag force at any given speed is now much larger. Equilibrium between weight and drag is reached at a much lower speed, giving a much lower terminal velocity.

(c) At terminal velocity, weight = drag (buoyancy neglected):

$$mg = 6\pi\eta rv_t$$

$$\frac{4}{3}\pi r^3 \rho g = 6\pi\eta rv_t$$

$$v_t = \frac{2r^2\rho g}{9\eta} = \frac{2 \times (2.0 \times 10^{-5}\text{m})^2 \times 1200\text{kgm}^{-3} \times 9.8\text{ms}^{-2}}{9 \times 1.8 \times 10^{-5}\text{Nsm}^{-2}} = 0.058\text{ms}^{-1}$$

Viscous force at terminal velocity:

$$F = 6\pi\eta rv_t = 6\pi \times 1.8 \times 10^{-5}\text{Nsm}^{-2} \times 2.0 \times 10^{-5}\text{m} \times 0.058\text{ms}^{-1} = 3.94 \times 10^{-10}\text{N}$$

Question 53

(a) Rotational work is $W = \tau\theta$, where θ is the angular displacement through which the torque acts. If the bolt does not move, $\theta = 0$, and therefore $W = 0$ regardless of how large the torque is. A force that produces no displacement does no work. The mechanic's effort is absorbed by elastic deformation in the wrench and the bolt, but no rotation occurs and no energy is transferred to the bolt.

(b) Through the same siphon (same radius R , same length L , same pressure difference ΔP), Poiseuille's formula gives $Q \propto 1/\eta$. A liquid that flows faster (empties the bottle in less time) has a lower viscosity. Since petrol empties 2.5 times faster, its flow rate is 2.5 times larger. Therefore, the viscosity of petrol is 2.5 times smaller than that of water: $\eta_{\text{petrol}} = \eta_{\text{water}}/2.5$.

(c) Pressure difference: $\Delta P = \rho gh = 1000\text{kgm}^{-3} \times 9.8\text{ms}^{-2} \times 1.0\text{m} = 9800\text{Pa}$

Areas: $A_A = \pi(0.30\text{m})^2 = 0.2827\text{m}^2$, $A_B = \pi(0.10\text{m})^2 = 0.03142\text{m}^2$

By continuity: $v_A = v_B \times \frac{A_B}{A_A} = \frac{v_B}{9}$

By Bernoulli (horizontal):

$$P_A - P_B = \frac{1}{2}\rho(v_B^2 - v_A^2) = \frac{1}{2}\rho v_B^2 \left(1 - \frac{1}{81}\right) = \frac{1}{2}\rho v_B^2 \times \frac{80}{81}$$

$$9800\text{Pa} = \frac{1}{2} \times 1000\text{kgm}^{-3} \times v_B^2 \times 0.9877$$

$$v_B^2 = 19.84\text{m}^2\text{s}^{-2}$$

$$v_B = 4.45\text{ms}^{-1}$$

$$Q = A_B v_B = 0.03142\text{m}^2 \times 4.45\text{ms}^{-1} = 0.140\text{m}^3\text{s}^{-1}$$

Question 54

(a) The rolling condition requires $v = R\omega$ at all times during rolling without slipping. As the ball decelerates on the rough incline, friction simultaneously reduces both v (through the translational equation of motion) and ω (through the rotational equation). The constraint $v = R\omega$ ensures that both quantities decrease in proportion. When v reaches zero, $\omega = v/R = 0$ simultaneously. One cannot reach zero without the other, because that would violate the rolling condition, which is maintained as long as the surface is rough enough to prevent slipping.

(b) Water wets glass because the adhesive forces between water molecules and glass are stronger than the cohesive forces within the water. The angle of contact is small (less than 90°), so water clings to the glass surface and forms a thin film when withdrawn. Mercury does not wet glass because the cohesive forces between mercury atoms are much stronger than the adhesive forces between mercury and glass. The angle of contact exceeds 90° , so mercury pulls away from the glass surface and the rod emerges clean.

(c) At the critical point: $R_e = \frac{\rho v_c r}{\eta}$

$$v_c = \frac{R_e \times \eta}{\rho r} = \frac{2000 \times 1.0 \times 10^{-3}\text{Nsm}^{-2}}{1000\text{kgm}^{-3} \times 0.01\text{m}} = \frac{2.0\text{Nsm}^{-1}}{10\text{kgm}^{-2}} = 0.20\text{ms}^{-1}$$

Maximum volume flow rate:

$$Q = A v_c = \pi r^2 v_c = \pi \times (0.01\text{m})^2 \times 0.20\text{ms}^{-1} = 6.28 \times 10^{-5}\text{m}^3\text{s}^{-1}$$

Question 55

(a) The work done by a torque is $W = \tau\theta$, where θ is the angular displacement. Both children push with the same force at the same distance from the axis, producing the same torque τ . But the first child pushes through $\theta = 2 \times 2\pi = 4\pi$ radians while the second pushes through $\theta = \pi$ radians. Since $W = \tau\theta$, the first child does four times as much work. More energy is transferred to the roundabout because the torque acts through a greater angular displacement.

(b) The height of capillary rise is $h = 2\gamma\cos\theta/(\rho gr)$. For the same tube (same r), the rise depends on three liquid-specific properties: the surface tension γ , the angle of contact θ , and the density ρ . Paraffin has a lower surface tension, a non-zero contact angle, and lower density than water. The combined effect of the lower $\gamma\cos\theta$ relative to ρ produces a smaller rise. Different liquids rise to different heights because these three properties differ from liquid to liquid.

(c) By the capillary rise formula: $h = \frac{2\gamma\cos\theta}{\rho gr}$

$$r = \frac{2\gamma\cos\theta}{\rho gh} = \frac{2 \times 0.026\text{Nm}^{-1} \times \cos 26^\circ}{800\text{kgm}^{-3} \times 9.8\text{ms}^{-2} \times 0.045\text{m}} = 1.325 \times 10^{-4}\text{m}$$

Internal diameter: $d = 2r = 2.65 \times 10^{-4}\text{m} = 0.265\text{mm}$

Question 56

(a) Bernoulli's theorem applies to an ideal (non-viscous) fluid. Real water has viscosity, which causes internal friction between adjacent layers. This friction dissipates energy along the pipe, converting kinetic energy into heat. To maintain the same flow speed (as required by continuity in a uniform pipe), the pressure must drop along the length to compensate for the energy lost to viscous friction. The steady pressure decrease from inlet to outlet reflects this continuous energy loss.

(b) When wind blows past the flag, the air on both sides moves at varying speeds due to the flag's flexible shape. Any slight ripple creates regions where air moves faster on one side (lower pressure) and slower on the other (higher pressure). The resulting pressure difference pushes the flag sideways. Once displaced, the flag creates new ripples with new pressure differences, sustaining a continuous fluttering motion. On a calm day, with no airflow, no pressure differences develop and the flag hangs limp.

(c) When the valve is closed, the water is stationary and the gauge reads the static pressure. When water flows, Bernoulli's equation gives:

$$P_{\text{static}} + \frac{1}{2}\rho v^2 = P_{\text{closed}}$$

$$\frac{1}{2}\rho v^2 = P_{\text{closed}} - P_{\text{flowing}} = 2.5 \times 10^5 \text{Pa} - 2.0 \times 10^5 \text{Pa} = 5.0 \times 10^4 \text{Pa}$$

$$v = \sqrt{\frac{2 \times 5.0 \times 10^4 \text{Pa}}{1000 \text{kgm}^{-3}}} = 10 \text{ms}^{-1}$$

Question 57

(a) Both objects experience the same gravitational acceleration initially. As they fall, air resistance builds up. The stone is compact and dense, so its terminal velocity is very high and it accelerates nearly freely for most of the fall. The parachutist deploys a large canopy that presents an enormous surface area to the air. The drag force on the parachute equals the person's weight at a much lower speed. The parachutist reaches a low terminal velocity and descends gently, while the stone reaches a high terminal velocity and strikes the ground with great force.

(b) After emerging horizontally from the hole, the water has a horizontal velocity but no vertical velocity. Gravity immediately begins to accelerate it downward. The horizontal velocity remains constant (no horizontal force acts) while the vertical velocity increases steadily. The combination of constant horizontal motion and uniformly accelerated vertical motion produces a parabolic curve, exactly as in projectile motion.

(c) By conservation of volume: $\frac{4}{3}\pi R^3 = 2 \times \frac{4}{3}\pi r^3$, giving $R = r \times 2^{1/3}$.

Terminal velocity: $v_t \propto r^2$.

$$\frac{V_{\text{combined}}}{V} = \frac{R^2}{r^2} = \frac{(r \times 2^{1/3})^2}{r^2} = 2^{2/3} = \sqrt[3]{4}$$

$$V_{\text{combined}} = 2^{2/3}V = \sqrt[3]{4}V \approx 1.587V$$

Question 58

(a) The student is incorrect. By Torricelli's theorem, $v = \sqrt{2gh}$, the efflux velocity depends only on the depth h of the hole below the water surface and not on the width or shape of the tank. A wider tank holds more water (so it takes longer to drain), but the speed at which water exits is determined solely by the height of water above the hole.

(b) By the equation of continuity ($A_1 v_1 = A_2 v_2$), the same volume of water must pass through every cross-section per unit time. At the narrow section, the cross-sectional area is smaller. For the same volume to pass through per unit time, the water must move faster. The extra kinetic energy comes from a drop in pressure: by Bernoulli's equation, the pressure in the narrow section is lower than in the wide section.

(c) The hole is 3.0m below the surface. Efflux speed:

$$v = \sqrt{2gh} = \sqrt{2 \times 9.8 \text{ms}^{-2} \times 3.0 \text{m}} = 7.67 \text{ms}^{-1}$$

The hole is $10\text{m} - 3.0\text{m} = 7.0\text{m}$ above the ground. Time to fall:

$$t = \sqrt{\frac{2H}{g}} = \sqrt{\frac{2 \times 7.0 \text{m}}{9.8 \text{ms}^{-2}}} = 1.20 \text{s}$$

Horizontal distance:

$$x = vt = 7.67\text{ms}^{-1} \times 1.20\text{s} = 9.2\text{m}$$

Question 59

(a) Plain water has a high surface tension ($\gamma \approx 0.073\text{Nm}^{-1}$), which prevents it from spreading into narrow gaps and pores in dirty fabric or skin. Soap molecules reduce the surface tension of water dramatically. The low-surface-tension soap solution penetrates small gaps, loosens dirt particles, and surrounds them. The soap molecules also have hydrophilic (water-attracting) and hydrophobic (water-repelling) ends, which attach to grease and dirt and suspend them in the water for removal.

(b) A soap bubble has an inner and an outer liquid surface. Each surface contributes an excess pressure of $2\gamma/R$. The total excess pressure is the sum: $\Delta P_{\text{soap}} = 2 \times 2\gamma/R = 4\gamma/R$. An air bubble in water has only one liquid surface (the water-air interface on the outside). It contributes a single excess pressure: $\Delta P_{\text{air}} = 2\gamma/R$. Therefore, $\Delta P_{\text{soap}} = 2 \times \Delta P_{\text{air}}$.

(c) The excess pressure inside the soap bubble:

$$\Delta P = \frac{4\gamma}{R}$$

This is balanced by the oil column pressure:

$$\frac{4\gamma}{R} = \rho_{\text{oil}}gh = 0.8 \times 1000\text{kgm}^{-3} \times 9.8\text{ms}^{-2} \times 2.0 \times 10^{-3}\text{m} = 15.68\text{Pa} = \Delta P$$

$$\gamma = \frac{\Delta P \times R}{4} = \frac{15.68\text{Pa} \times 0.01\text{m}}{4} = 0.0392\text{Nm}^{-1}$$

Question 60

(a) By Torricelli's theorem, the efflux speed is $v = \sqrt{2gh}$, where h is the height of water above the hole. When the tank is full, h is large and the jet emerges at high speed. As the tank drains, h decreases, and the efflux speed decreases proportionally to \sqrt{h} . The volume flow rate $Q = av$ also decreases. Consequently, the jet becomes progressively weaker.

(b) Before the hole is unplugged, the liquid pushes equally on all internal walls. When the hole is opened, the liquid at the hole escapes, so there is no longer any outward force on the wall at the hole. The opposite wall, however, still has liquid pressing against it. This unbalanced force pushes the container away from the hole. By Newton's third law, the escaping liquid carries momentum in one direction and the container gains equal momentum in the opposite direction.

(c) Initial efflux speed:

$$v = \sqrt{2gh} = \sqrt{2 \times 9.8\text{ms}^{-2} \times 0.30\text{m}} = 2.42\text{ms}^{-1}$$

Initial drainage rate:

$$Q = av = 6.5 \times 10^{-4}\text{m}^2 \times 2.42\text{ms}^{-1} = 1.57 \times 10^{-3}\text{m}^3\text{s}^{-1}$$

At 30cm below the hole, the water has fallen freely through 0.30m. Its speed there:

$$v_2 = \sqrt{v^2 + 2g \times 0.30\text{m}} = \sqrt{5.88 + 5.88} = 3.43\text{ms}^{-1}$$

By continuity ($a_1v_1 = a_2v_2$):

$$a_2 = \frac{a_1v_1}{v_2} = \frac{6.5 \times 10^{-4}\text{m}^2 \times 2.42\text{ms}^{-1}}{3.43\text{ms}^{-1}} = 4.59 \times 10^{-4}\text{m}^2 = 4.6\text{cm}^2$$

Question 61

(a) The oil exits horizontally and then follows a projectile path. The horizontal distance depends on two factors: the horizontal speed (which is the same in both cases, since the can produces the same exit speed) and the time the oil is in the air. At waist height, the oil falls a greater vertical distance, taking more time to reach the ground. More time in the air means more horizontal distance before landing. At ground level, the fall height is small, the air time is short, and the oil lands close to the can.

(b) At the wide section, the water moves slowly (by continuity). At the constriction, the water moves fast. By Bernoulli's equation, faster-moving fluid has lower pressure. The vertical tubes act as pressure indicators: the water rises to a height proportional to the local pressure. Since the pressure at the constriction is lower, the water level in its vertical tube is lower than at the wide section.

(c) By continuity, the nozzle speed:

$$v_{\text{nozzle}} = v_{\text{piston}} \times \frac{A_{\text{piston}}}{A_{\text{nozzle}}} = 0.20\text{ms}^{-1} \times \frac{(5d)^2}{d^2} = 0.20\text{ms}^{-1} \times 25 = 5.0\text{ms}^{-1}$$

The water exits horizontally at 5.0ms^{-1} and falls 1.25m. Vertical velocity on hitting the ground:

$$v_y = \sqrt{2gh} = \sqrt{2 \times 9.8 \text{ms}^{-2} \times 1.25 \text{m}} = 4.95 \text{ms}^{-1}$$

Speed on hitting the ground:

$$v = \sqrt{v_x^2 + v_y^2} = \sqrt{(5.0 \text{ms}^{-1})^2 + (4.95 \text{ms}^{-1})^2} = 7.04 \text{ms}^{-1}$$

Question 62

(a) By Poiseuille's formula, the flow rate through the needle is $Q = \pi \Delta P R^4 / (8\eta L)$. The flow depends on the fourth power of the radius. Even a small decrease in R causes R^4 to drop dramatically. For example, reducing the radius by 20% reduces R^4 to $(0.8)^4 = 0.41$, cutting the flow by nearly 60%. This extreme sensitivity to radius explains why the drip rate falls so sharply with a slightly smaller needle.

(b) In series, the same volume flow rate Q passes through both tubes. By Poiseuille's formula, the pressure drop across each tube is $\Delta P = 8\eta L Q / (\pi R^4)$. For the same Q and L , $\Delta P \propto 1/R^4$. The narrower tube has a smaller R and therefore a much larger ΔP , carrying the larger share of the total pressure drop.

(c) In series, Q is constant. $\Delta P \propto 1/R^4$ for each tube.

$$\frac{\Delta P_1}{\Delta P_3} = \frac{R_3^4}{R_1^4} = \frac{(5r)^4}{(3r)^4} = \frac{625}{81} = 7.716$$

$$\Delta P_1 = 7.716 \times 8.1 \text{mm} = 62.5 \text{mm of liquid}$$

Question 63

(a) By Stokes' law, the terminal velocity of a spherical drop is $v_t \propto r^2$. Tiny fog droplets have very small radii, so their terminal velocities are extremely low (fractions of a millimetre per second). Even the slightest air currents keep them suspended. Large raindrops have much greater radii, so their terminal velocities are proportionally much higher (r^2 dependence), and they fall rapidly.

(b) At terminal velocity, the viscous drag equals the net downward force (weight minus upthrust). The drag force is $F = 6\pi\eta r v_t$, so $v_t \propto 1/\eta$. When glycerine is warmed, its viscosity η decreases (intermolecular forces weaken at higher temperature). A lower η means a smaller drag force at any given speed, so the sphere must reach a higher speed before drag balances its weight. Hence, the terminal velocity increases.

(c) Finding the radius:

$$v_t = \frac{2r^2(\rho_s - \rho_{\text{air}})g}{9\eta}$$

$$r^2 = \frac{9\eta v_t}{2(\rho_s - \rho_{\text{air}})g} = \frac{9 \times 1.8 \times 10^{-5} \text{Nsm}^{-2} \times 3.2 \times 10^{-2} \text{ms}^{-1}}{2 \times (900 - 1.2) \text{kgm}^{-3} \times 9.8 \text{ms}^{-2}} = 2.944 \times 10^{-10} \text{m}^2$$

$$r = 1.72 \times 10^{-5} \text{m}$$

For a drop of half the radius: $v_t \propto r^2$

$$v_t' = v_t \times \left(\frac{r}{2}\right)^2 = 3.2 \times 10^{-2} \text{ms}^{-1} \times \frac{1}{4} = 8.0 \times 10^{-3} \text{ms}^{-1}$$

Question 64

(a) Snow or ice accumulation roughens the normally smooth wing surface and alters its aerodynamic shape. The rough surface disrupts the smooth flow of air over the wing, causing the boundary layer to separate earlier. This reduces the speed difference between air flowing over the top and bottom of the wing. By Bernoulli's equation, a smaller speed difference means a smaller pressure difference, which reduces the lift. In severe cases, the disrupted airflow causes a stall.

(b) By Bernoulli's equation, the total energy per unit volume along a streamline is constant: $P + \frac{1}{2}\rho v^2 = \text{constant}$ (for horizontal flow). When the fluid enters the constriction and speeds up, its kinetic energy per unit volume increases. This increase comes at the expense of pressure energy. The pressure in the constriction drops, and it is this pressure drop that provides the energy for the increased speed.

(c) By Poiseuille's formula, for tubes in series (same Q): $\Delta P \propto \frac{L}{R^4}$.

$$\frac{\Delta P_1}{\Delta P_2} = \frac{\frac{L_1}{R_1^4}}{\frac{L_2}{R_2^4}} = \frac{L_1 R_2^4}{L_2 R_1^4} = \frac{21\text{cm} \times (0.0375\text{cm})^4}{7\text{cm} \times (0.1125\text{cm})^4} = 0.03704$$

$$\Delta P_1 + \Delta P_2 = 14\text{cm}$$

$$0.03704\Delta P_2 + \Delta P_2 = 14\text{cm}$$

$$\Delta P_2 = \frac{14}{1.03704} = 13.5\text{cm of water}$$

$$\Delta P_1 = 14 - 13.5 = 0.5\text{cm of water}$$

Question 65

(a) Engine oil must lubricate moving parts by maintaining a film between metal surfaces. In hot weather, oil becomes thinner (lower viscosity) and may fail to maintain an adequate film, leading to increased wear. In cold weather, oil becomes thicker (higher viscosity) and resists flow, making it harder to start the engine and increasing energy loss to internal friction. Different viscosity grades are designed so that the oil remains within the optimal viscosity range for the prevailing temperature.

(b) The engine contains oil that lubricates moving parts. Cold oil has high viscosity, producing large frictional forces between the oil layers and the moving surfaces. As the engine runs, it generates heat that warms the oil. The viscosity of the oil decreases with increasing temperature. Lower viscosity means less internal friction between the oil layers, so the engine encounters less resistance and runs more smoothly.

(c) First, find the density of the sphere:

$$\rho_s = \frac{m}{V} = \frac{m}{\frac{4}{3}\pi r^3} = \frac{1.2 \times 10^{-4}\text{kg}}{\frac{4}{3}\pi \times (2.0 \times 10^{-3}\text{m})^3} = 3580\text{kgm}^{-3}$$

Terminal velocity formula rearranged for viscosity:

$$\eta = \frac{2r^2(\rho_s - \rho_f)g}{9v_t} = \frac{2 \times (2.0 \times 10^{-3}\text{m})^2 \times (3580 - 900)\text{kgm}^{-3} \times 9.8\text{ms}^{-2}}{9 \times 5.4 \times 10^{-2}\text{ms}^{-1}}$$

$$\eta = 0.432\text{Nsm}^{-2}$$

Question 66

(a) The tank is continuously fed at a constant rate Q_{in} . The outflow rate depends on the water height: $Q_{out} = a\sqrt{2gh}$. Initially, the water level is high and $Q_{out} > Q_{in}$, so the level drops. As h decreases, Q_{out} decreases. At a particular height, $Q_{out} = Q_{in}$: water leaves at the same rate it enters. Below this height, $Q_{out} < Q_{in}$ and the level would rise back up. Hence, the system self-corrects to the equilibrium height where inflow equals outflow.

(b) By Poiseuille's formula, $Q = \pi\Delta PR^4/(8\eta L)$. Doubling L halves the flow. Halving the diameter halves R , so R^4 drops by a factor of $2^4 = 16$. The combined effect reduces the flow by a factor of $2 \times 16 = 32$. The R^4 dependence is devastating: a small reduction in radius produces a dramatic reduction in flow.

(c) By Poiseuille's formula: $Q \propto \frac{R^4}{L}$. New length = $2L$, new radius = $\frac{R}{2}$:

$$Q' = Q \times \left(\frac{R}{2}\right)^4 \times \frac{L}{2L} = Q \times \frac{1}{16} \times \frac{1}{2} = \frac{Q}{32}$$

Question 67

(a) In normal flight, the curved upper surface and angled wing cause air to move faster over the top than the bottom, creating lower pressure above and hence lift. When inverted, the wing's curved surface faces downward and would normally push the aircraft down. However, the pilot tilts the nose upward, increasing the angle of attack. This forces air to deflect downward off the underside of the wing (now the top surface), producing an upward reaction force. The lift from this high angle of attack overcomes the unfavourable wing curvature.

(b) Lift is proportional to $\frac{1}{2}\rho Av^2 \times C_L$, where A is the wing area and C_L is the lift coefficient (which increases with angle of attack). Extending flaps increases both A and C_L simultaneously. A larger product $A \times C_L$ means the same lift can be generated at a lower speed v . This allows the aircraft to take off and land at safely reduced speeds without stalling.

(c) By continuity: $v_2 = v_1 \times \left(\frac{d_1}{d_2}\right)^2 = 5\text{ms}^{-1} \times \left(\frac{1.5\text{cm}}{0.75\text{cm}}\right)^2 = 20\text{ms}^{-1}$

By Bernoulli (with height difference $h = 8\text{m}$):

$$P_2 = P_1 + \frac{1}{2}\rho(v_1^2 - v_2^2) - \rho gh$$

$$P_2 = 6.5 \times 10^5\text{Pa} + \frac{1}{2} \times 1000\text{kgm}^{-3} \times ((5)^2 - (20)^2)\text{m}^2\text{s}^{-2} - 1000\text{kgm}^{-3} \times 9.8\text{ms}^{-2} \times 8\text{m}$$

$$P_2 = 3.84 \times 10^5\text{Pa}$$

Question 68

(a) The student's prediction reverses the causality. By the equation of continuity, water slows down in the wider section (larger area, lower speed). By Bernoulli's equation, slower-moving fluid has higher pressure. The gauge at the wider section reads higher, not lower. The "more space means less pressure" reasoning confuses gas behaviour (where expansion reduces pressure) with incompressible fluid flow (where pressure and speed trade off along a streamline).

(b) By Torricelli: $v = \sqrt{2gh}$. Volume outflow: $-A \frac{dh}{dt} = a\sqrt{2gh}$

Separating variables: $\frac{dh}{\sqrt{h}} = -\frac{a}{A}\sqrt{2g} dt$

Integrating from H_1 to H_2 and 0 to t :

$$\int_{H_1}^{H_2} \frac{dh}{\sqrt{h}} = \int_0^t -\frac{a}{A}\sqrt{2g} dt$$

$$[2\sqrt{h}]_{H_1}^{H_2} = -\frac{a}{A}\sqrt{2g} \times t$$

$$2(\sqrt{H_2} - \sqrt{H_1}) = -\frac{a}{A}\sqrt{2g} \times t$$

$$t = \frac{A}{a} \sqrt{\frac{2}{g}} (\sqrt{H_1} - \sqrt{H_2})$$

(c) Areas: $A_1 = \pi(0.04\text{m})^2 = 5.027 \times 10^{-3}\text{m}^2$, $A_2 = \pi(0.08\text{m})^2 = 2.011 \times 10^{-2}\text{m}^2$

By continuity: $v_1 = v_2 \times \frac{A_2}{A_1}$. Since $A_2 = 4A_1$: $v_1 = 4v_2$

Flow rate: $Q = 96 \times 10^{-3}\text{m}^3\text{s}^{-1} = 0.096\text{m}^3\text{s}^{-1}$

$$v_1 = \frac{Q}{A_1} = \frac{0.096}{5.027} \times 10^{-3} = 19.1\text{ms}^{-1}$$

$$v_2 = \frac{Q}{A_2} = \frac{0.096}{2.011} \times 10^{-2} = 4.77\text{ms}^{-1}$$

By Bernoulli:

$$P_2 = P_1 + \frac{1}{2}\rho(v_1^2 - v_2^2) = 3.5 \times 10^5\text{Pa} + 500\text{kgm}^{-3} \times ((19.1)^2 - (4.77)^2)\text{m}^2\text{s}^{-2}$$

$$P_2 = 5.21 \times 10^5\text{Pa}$$

Question 69

(a) Water flows in at a constant rate Q_{in} . The outflow rate is $Q_{\text{out}} = a\sqrt{2gh}$, which increases with height. Initially h is small and $Q_{\text{out}} < Q_{\text{in}}$, so water accumulates and h rises. As h increases, Q_{out} increases. The level stops rising when $Q_{\text{out}} = Q_{\text{in}}$: the outflow through the hole exactly matches the inflow. Any further rise would make $Q_{\text{out}} > Q_{\text{in}}$, causing the level to drop back. The system is self-regulating.

(b) Terminal velocity is $v_t \propto r^2$. The larger sphere (diameter $3d$, radius $3r/2$) has a terminal velocity 9 times that of the smaller sphere (radius $r/2$). Since both fall the same distance and the larger sphere moves 9 times faster, the smaller sphere takes 9 times as long.

(c) At equilibrium: $Q_{\text{in}} = Q_{\text{out}} = a\sqrt{2gh}$

$$h = \frac{Q_{in}^2}{2ga^2} = \frac{(2 \times 10^{-4} \text{m}^3 \text{s}^{-1})^2}{2 \times 9.8 \text{ms}^{-2} \times (2 \times 10^{-4} \text{m}^2)^2} = 0.051 \text{m} = 5.1 \text{cm}$$

Question 70

(a) A tennis ball is lighter and has a rough, fuzzy surface. When spinning, the rough surface drags more air with it, creating a larger asymmetry in air speed between the two sides. By the Magnus effect, this larger speed difference produces a larger pressure difference and hence a greater sideways force. A cricket ball is heavier (requiring more force to deflect) and smoother (less surface drag), so the same spin produces less curve.

(b) Let the piston area be A_p and the orifice area be a . The piston moves at speed $\frac{V}{A_p t}$ (to push volume V in time t , the piston distance is $\frac{V}{A_p}$). By continuity, the efflux speed is $v = \frac{V}{at}$.

By Bernoulli (piston to orifice, horizontal, $a \ll A_p$ so piston speed is negligible):

$$P_{\text{piston}} = \frac{1}{2} \rho v^2 = \frac{1}{2} \rho \frac{V^2}{a^2 t^2}$$

Work done by force $F = P_{\text{piston}} \times A_p$ through distance $d = \frac{V}{A_p}$:

$$W = F \times d = P_{\text{piston}} \times A_p \times \frac{V}{A_p} = P_{\text{piston}} \times V = \frac{1}{2} \rho \frac{V^2}{a^2 t^2} \times V$$

$$W = \frac{\rho V^3}{2a^2 t^2}$$

(c) Terminal velocity:

$$v_t = \frac{2r^2(\rho_s - \rho_f)g}{9\eta} = \frac{2 \times (2.0 \times 10^{-3} \text{m})^2 \times (7800 - 1260) \text{kgm}^{-3} \times 9.8 \text{ms}^{-2}}{9 \times 0.83 \text{Nsm}^{-2}}$$

$$v_t = 0.069 \text{ms}^{-1}$$

Question 71

(a) By Torricelli's theorem, a lower hole has a greater depth h below the surface, giving a higher efflux speed $v = \sqrt{2gh}$. However, the lower hole is also closer to the ground, so the jet has less fall height H and less air time $t = \sqrt{\frac{2H}{g}}$. The horizontal distance is

$x = vt = \sqrt{2gh} \times \sqrt{\frac{2H}{g}} = 2\sqrt{hH}$. For a tank of total height L : $h + H = L$, and $x = 2\sqrt{h(L-h)}$, which is maximised when $h = \frac{L}{2}$.

A hole below the midpoint has high speed but very little fall height, so x decreases. Hence, a faster jet can land closer.

(b) Terminal velocity is $v_t = 2r^2(\rho_s - \rho_f)g/(9\eta)$, so $v_t \propto r^2$. A drop of half the radius has $v_t' = v_t \times (r/2)^2/r^2 = v_t/4$. The terminal velocity is exactly one quarter because the weight (which drives the fall) decreases as r^3 while the drag force (which opposes it) decreases only as r . The net effect is that smaller drops require much less drag to reach equilibrium, which occurs at a much lower speed.

(c) Efflux speed: $v = \sqrt{2gh} = \sqrt{2 \times 9.8 \text{ms}^{-2} \times 0.80 \text{m}} = 3.96 \text{ms}^{-1}$

Time to fall $H = 1.20 \text{m}$: $t = \sqrt{\frac{2H}{g}} = \sqrt{2 \times 1.20 \text{m}/9.8 \text{ms}^{-2}} = 0.495 \text{s}$

Horizontal distance: $x = vt = 3.96 \text{ms}^{-1} \times 0.495 \text{s} = 1.96 \text{m}$

Question 72

(a) When the diver leaves the board, her angular momentum is fixed (no external torque acts during the fall). Angular momentum is $L = I\omega$. Tucking reduces the moment of inertia I by bringing her mass closer to the axis of rotation. For L to remain constant, ω must increase. A higher angular velocity means more revolutions completed in the same fall time.

(b) The moment of inertia depends on the axis of rotation chosen. It is defined as $I = \sum m_i r_i^2$, where r_i is each particle's perpendicular distance from the axis. A different axis gives different distances r_i for every particle, producing a different value of I . Therefore, a rigid body has infinitely many possible moments of inertia, one for each possible axis.

(c) Initial angular velocity: $\omega_1 = 20 \times 2\pi/60 = 2.094 \text{rad s}^{-1}$

After the wax is added:

$$I_2 = I_1 + mr^2 = 2.5 \times 10^{-4} \text{kgm}^2 + 0.04 \text{kg} \times (0.05 \text{m})^2 = 3.5 \times 10^{-4} \text{kgm}^2$$

By conservation of angular momentum:

$$\omega_2 = \frac{I_1 \omega_1}{I_2} = \frac{2.5 \times 10^{-4} \times 2.094}{3.5 \times 10^{-4}} = 1.496 \text{rad s}^{-1}$$

$$n_2 = \frac{1.496 \times 60}{2\pi} = 14.3 \text{ rev/min}$$

Question 73

(a) The rotational kinetic energy stored is $KE = \frac{1}{2}I\omega^2$. For a given mass and angular velocity, concentrating the mass at the rim maximises I (since $I = \sum m_i r_i^2$ and rim mass sits at the maximum r). A larger I stores more energy at the same speed. When the engine encounters a sudden load, the flywheel releases this stored energy to maintain ω nearly constant. Conversely, when the engine surges, the large I resists the speed increase. Both effects smooth out speed fluctuations.

(b) A uniform solid sphere has perfect symmetry: every axis through its centre is identical to every other (by rotational symmetry of the sphere). Since $I = \frac{2}{5}MR^2$ for any such axis, the value is the same for all axes through the centre. A cylinder does not have this property because it has different symmetry about its central axis ($I = \frac{1}{2}MR^2$) compared to a diameter through the centre ($I = \frac{1}{4}MR^2 + \frac{1}{12}ML^2$). The cylinder's geometry distinguishes between these axes.

(c) Treating the wheel as a disc: $I = \frac{1}{2}MR^2 = \frac{1}{2} \times 3.0 \text{kg} \times (0.40 \text{m})^2 = 0.24 \text{kgm}^2$

$$\omega_0 = 3600 \times 2\pi/60 = 120\pi \text{ rad s}^{-1}$$

$$\alpha = \frac{\omega_0}{t} = \frac{120\pi}{10} = 12\pi \text{ rad s}^{-2}$$

$$\tau = I\alpha = 0.24 \text{kgm}^2 \times 12\pi \text{ rad s}^{-2} = 9.05 \text{Nm}$$

Question 74

(a) Moment of inertia is $I = \sum m_i r_i^2$. Every real object consists of particles with non-zero mass. For $I = 0$, every particle would need $r_i = 0$, meaning all the mass lies exactly on the axis. A real three-dimensional object always has mass distributed at some distance from any chosen axis. Therefore, $I > 0$ for any real rotating object.

(b) On a frictionless incline, the acceleration would be $g \sin \theta$ (all PE converts to translational KE). When rolling without slipping, part of the PE converts to rotational KE. Less energy is available for translation, so the translational acceleration is reduced. The fraction diverted to rotation depends on $I/(MR^2)$: for a solid cylinder ($I = \frac{1}{2}MR^2$), one-third of the gravitational component goes to rotation, giving $a = \frac{2}{3}g \sin \theta$.

(c) By conservation of energy: $Mg\ell \sin \alpha = \frac{1}{2}Mv^2 + \frac{1}{2}I\omega^2$

With $I = Mk^2$ and $\omega = v/R$:

$$Mg\ell \sin \alpha = \frac{1}{2}Mv^2 + \frac{1}{2}Mk^2 \times \frac{v^2}{R^2} = \frac{1}{2}Mv^2 \left(1 + \frac{k^2}{R^2} \right)$$

$$v = \sqrt{\frac{2g\ell \sin \alpha}{1 + \frac{k^2}{R^2}}}$$

Question 75

(a) The hollow cylinder has all its mass at the maximum distance from the axis ($I = MR^2$), while the solid cylinder has mass distributed from the axis to the rim ($I = \frac{1}{2}MR^2$). At the same angular velocity, $KE = \frac{1}{2}I\omega^2$, so the hollow cylinder has twice the kinetic energy. By the work-energy theorem, the work required to bring it to that angular velocity is twice as large.

(b) Place the diameter along the x -axis. A small element at angle θ from the axis of symmetry subtends $d\theta$ and has mass $dm = \frac{M}{\pi R} R d\theta = \frac{M}{\pi} d\theta$. Its y -coordinate (along the axis of symmetry) is $y = R \sin \theta$.

$$y_{\text{cm}} = \frac{1}{M} \int_0^\pi y \, dm = \frac{1}{M} \int_0^\pi R \sin\theta \times \frac{M}{\pi} d\theta = \frac{R}{\pi} \int_0^\pi \sin\theta \, d\theta$$

$$y_{\text{cm}} = \frac{R}{\pi} [-\cos\theta]_0^\pi = \frac{R}{\pi} (-\cos\pi + \cos 0) = \frac{R}{\pi} (1 + 1)$$

$$y_{\text{cm}} = \frac{2R}{\pi}$$

(c) Initial angular momentum: $L = I_1 \omega_1 = MR^2 \omega_1$ (ring about its axis: $I = MR^2$ since $r = R$... using the variable in the question, $I = Mr^2$).

After adding two masses at the ring: $I_2 = Mr^2 + 2mr^2 = (M + 2m)r^2$

By conservation of angular momentum:

$$\omega_2 = \frac{Mr^2 \omega_1}{(M + 2m)r^2} = \frac{M \omega_1}{M + 2m}$$

Question 76

(a) An object rotating at constant angular velocity has $\alpha = 0$, so by $\tau_{\text{net}} = I\alpha$, the net torque is zero. A torque is needed to change the angular velocity, not to maintain it. Example: a frictionless flywheel spinning at constant ω requires no torque to keep spinning. This is the rotational analogue of Newton's first law.

(b) The pipe extension increases the effective handle length d . By $\tau = Fd$, the same hand force F produces a much larger torque. This larger torque is more likely to exceed the frictional resistance of the seized bolt. However, the bolt head and threads are designed to withstand only a certain maximum torque. The extended handle can easily produce a torque that exceeds the bolt's structural strength, stripping the threads or shearing the bolt head.

(c) Using conservation of energy. The heavier mass falls $\frac{h}{2} = 0.15\text{m}$ while the lighter rises 0.15m :

$$(m_2 - m_1)g \times 0.15\text{m} = \frac{1}{2}(m_1 + m_2)v^2 + \frac{1}{2}I\omega^2$$

With $v = \omega R$: $\omega = \frac{v}{R} = \frac{v}{0.15\text{m}}$

$$(0.25 - 0.20)\text{kg} \times 9.8\text{ms}^{-2} \times 0.15\text{m} = \frac{1}{2}(0.45\text{kg})v^2 + \frac{1}{2}(0.12\text{kgm}^2) \frac{v^2}{(0.15\text{m})^2}$$

$$v = 0.159\text{ms}^{-1}$$

$$\omega = \frac{v}{R} = \frac{0.159\text{ms}^{-1}}{0.15\text{m}} = 1.06\text{rad s}^{-1}$$

Question 77

(a) When the sliding ball moves onto the rough surface, friction at the contact point acts backward (opposing the translational motion) and simultaneously creates a torque that initiates rotation. The translational speed decreases while the rotational speed increases until the rolling condition $v = R\omega$ is satisfied. The "lost" translational kinetic energy has been converted into rotational kinetic energy. The total kinetic energy has decreased slightly because friction dissipated some energy as heat during the transition.

(b) No external torque acts on the Earth about the Sun (gravity passes through the Sun, producing zero torque about it). Angular momentum is conserved: $L = mr^2\omega = \text{constant}$. When the Earth is closer to the Sun, r is smaller. For $r^2\omega$ to remain constant, ω must increase. When farther away, r is larger and ω decreases.

(c) For a uniform solid sphere: $I = \frac{2}{5}MR^2$. If $R' = \frac{R}{2}$:

$$I' = \frac{2}{5}M \left(\frac{R}{2}\right)^2 = \frac{1}{4} \times \frac{2}{5}MR^2 = \frac{I}{4}$$

By conservation of angular momentum: $I\omega = I'\omega'$

$$\omega' = \frac{I\omega}{I'} = \frac{I\omega}{\frac{I}{4}} = 4\omega$$

Since $\omega \propto \frac{1}{r}$;

Period: $T' = \frac{T}{4} = \frac{24 \text{ hours}}{4} = 6 \text{ hours}$

The day would shrink from 24 hours to **6 hours**.

Question 78

(a) Opening a door requires a single torque about the hinge axis. A single force applied far from the hinge (large lever arm) produces this torque efficiently. Twisting a bottle cap requires a couple: two equal, opposite, parallel forces that produce pure rotation without translation. A single finger cannot create a couple. Thumb and finger on opposite sides produce equal forces in opposite directions, forming a couple whose torque equals $F \times d$ (where d is the cap diameter).

(b) At the instant a torque is first applied to a stationary object, $\omega = 0$ but $\alpha = \tau/I \neq 0$. Angular acceleration is the rate of change of angular velocity. The object has not yet acquired any angular velocity, but it is about to. (This is analogous to a ball at the instant of release from rest: its velocity is zero, but its acceleration is g .)

(c) Mass of Earth: $M = \rho \times \frac{4}{3}\pi R^3 = 5450 \text{kgm}^{-3} \times \frac{4}{3}\pi \times (6.37 \times 10^6 \text{m})^3 = 5.90 \times 10^{24} \text{kg}$

$$I = \frac{2}{5}MR^2 = \frac{2}{5} \times 5.90 \times 10^{24} \text{kg} \times (6.37 \times 10^6 \text{m})^2 = 9.58 \times 10^{37} \text{kgm}^2$$

Question 79

(a) Brick walls have porous surfaces with tiny gaps and cracks. Without plastering, water can enter these gaps by capillary action and penetrate the wall. Cement plaster fills the pores and creates a smooth, continuous surface. The adhesive forces between cement and brick bond them together, strengthening the wall structurally. The smooth plaster surface eliminates the narrow capillary channels, preventing water from being drawn into the wall by surface tension.

(b) A molecule inside the liquid is surrounded by neighbours on all sides. The attractive forces from all directions cancel out, and the molecule is in its lowest energy state. A molecule at the surface has neighbours only on the sides and below, but none above (the air above exerts negligible attraction). There is a net inward pull on the surface molecule. To bring a molecule from the interior to the surface, work must be done against this inward pull. This work increases the molecule's potential energy. Hence, surface molecules are in a higher energy state.

(c) By volume conservation: $64 \times \frac{4}{3}\pi r^3 = \frac{4}{3}\pi R^3$, giving $R = 4r$.

Total surface energy of 64 small drops: $E_1 = 64 \times 4\pi r^2 \gamma = 256\pi r^2 \gamma$

Surface energy of one large drop: $E_2 = 4\pi(4r)^2 \gamma = 64\pi r^2 \gamma$

$$\frac{E_1}{E_2} = \frac{256\pi r^2 \gamma}{64\pi r^2 \gamma} = 4$$

Question 80

(a) In a narrow capillary tube, the liquid rises by an amount $h_c = \frac{2\gamma \cos\theta}{\rho g r}$, which adds to the true height. If two tubes of radii r_1 and r_2 are used, the observed heights are $H_1 = H_{\text{true}} + h_{c1}$ and $H_2 = H_{\text{true}} + h_{c2}$. The difference $H_1 - H_2 = h_{c1} - h_{c2} = \frac{2\gamma \cos\theta}{\rho g \left(\frac{1}{r_1} - \frac{1}{r_2}\right)}$, which is known. This allows h_{c1} and h_{c2} to be calculated and subtracted from each reading, yielding the true pressure.

(b) Surface tension acts along the contact line (the perimeter where the liquid, glass, and air meet). The force is $F = \gamma \times \text{perimeter} = \gamma \times 2\pi r$ for a circular plate (factor of 2 for wetting on both sides). This force pulls the plate downward into the liquid. The extra force needed to detach the plate equals this surface tension force. It depends on the perimeter because surface tension is a force per unit length, not a force per unit area.

(c) The difference in readings is due to the difference in capillary rises:

$$\begin{aligned} H_1 - H_2 &= \frac{2\gamma \cos\theta}{\rho g} \left(\frac{1}{r_1} - \frac{1}{r_2} \right) \\ &= \frac{2 \times 0.073 \text{Nm}^{-1} \times 1}{1000 \text{kgm}^{-3} \times 9.8 \text{ms}^{-2}} \times \left(\frac{1}{6.5 \times 10^{-4} \text{m}} - \frac{1}{1.24 \times 10^{-3} \text{m}} \right) = 0.01091 \text{m} \end{aligned}$$

Observed difference = 0.20m. True pressure difference (in metres of water):

$$H_{\text{true}} = H_{\text{narrow}} - h_{c,\text{narrow}}$$

The capillary rise in the narrow tube:

$$h_{c1} = \frac{2\gamma\cos\theta}{\rho g r_1} = \frac{2 \times 0.073 \times 1}{1000 \times 9.8 \times 6.5 \times 10^{-4}} = 0.0229\text{m}$$

The capillary rise in the wide tube:

$$h_{c2} = \frac{2\gamma\cos\theta}{\rho g r_2} = 0.01202\text{m}$$

The true height difference (correcting the narrow tube): the 0.20m excess reading is partly due to the capillary difference $h_{c1} - h_{c2} = 0.0229 - 0.0120 = 0.0109\text{m}$.

True pressure difference:

$$\Delta P = \rho g(0.20\text{m} - 0.0109\text{m}) = 1000\text{kgm}^{-3} \times 9.8\text{ms}^{-2} \times 0.189\text{m} = 1852\text{Pa}$$

Question 81

(a) The excess pressure inside a soap bubble is $\Delta P = 4\gamma/r$. The smaller bubble has a smaller radius and therefore a higher internal pressure. When connected, air flows from the higher-pressure (smaller) bubble to the lower-pressure (larger) bubble. The smaller bubble shrinks while the larger one grows. This continues until the smaller bubble collapses and only the larger bubble remains.

(b) Consider a rectangular film on a wire frame with a sliding wire of length l . The film has two surfaces, each exerting force γl , for a total force $F = 2\gamma l$. If the wire is pulled outward by distance dx , the increase in area is $dA = 2l dx$ (two surfaces). Work done: $dW = F dx = 2\gamma l dx = \gamma \times 2l dx = \gamma dA$.

Therefore: $\gamma = dW/dA$ (energy per unit area) and $\gamma = F/(2l)$ (force per unit length of one-sided contact). Both give the same γ with dimensions $\text{Nm}^{-1} = \text{Jm}^{-2}$.

(c) By volume conservation: $1000 \times \frac{4}{3}\pi r^3 = \frac{4}{3}\pi R^3$, so $R = 10r = 10 \times 5 \times 10^{-3}\text{m} = 0.05\text{m}$.

Work done = increase in surface area $\times \gamma$:

$$W = \gamma(1000 \times 4\pi r^2 - 4\pi R^2) = 4\pi \times 0.50\text{Nm}^{-1} \times (1000 \times (5 \times 10^{-3}\text{m})^2 - (0.05\text{m})^2)$$

$$W = 0.1414\text{J}$$

Question 82

(a) At higher temperature, the surface tension of the soup decreases. A lower surface tension allows the soup surface to break more easily, releasing vapour bubbles that carry volatile flavour molecules into the air above the bowl. These molecules reach the nose (smell) and tongue (taste) more readily. Cold soup has higher surface tension, which keeps the surface intact and traps flavour molecules, reducing the aroma and perceived flavour.

(b) The nib of a fountain pen contains narrow channels (slits) between the tines. These channels act as capillary tubes. The adhesive forces between ink and the nib material (usually metal or plastic) are stronger than the cohesive forces within the ink. This produces a small angle of contact, enabling capillary rise. The ink is drawn through the narrow channels by capillary action from the reservoir toward the paper, even against gravity, because the surface tension force along the contact line exceeds the weight of the ink column in the channel.

(c) A soap bubble has two surfaces. Surface area: $A = 2 \times 4\pi r^2$ with $r = 0.5\text{cm} = 5 \times 10^{-3}\text{m}$:

$$W = \gamma \times 2 \times 4\pi r^2 = 0.030\text{Nm}^{-1} \times 8\pi \times (5 \times 10^{-3}\text{m})^2 = 1.88 \times 10^{-5}\text{J}$$

Question 83

(a) Glue is a liquid when applied and flows into the microscopic pores, cracks, and irregularities on both surfaces. As it sets, strong adhesive bonds form between the glue molecules and the molecules of each surface. The large contact area (including all the pores filled by the glue) means that an enormous number of adhesive bonds form across the joint. To pull the pieces apart, all these bonds must be broken simultaneously. The total force required is proportional to the true contact area, which is much larger than the apparent surface area due to the penetration into pores.

(b) Surface tension is a property of the liquid-air (or liquid-liquid) interface arising from intermolecular forces. Each molecule at the surface experiences the same imbalance of forces regardless of how large the surface is. Doubling the surface area doubles the number of surface molecules but does not change the force per unit length experienced by each molecule. The coefficient γ characterises the strength of the intermolecular interaction at the interface, not the total force on the entire surface.

(c) By volume conservation: $\frac{4}{3}\pi r_1^3 + \frac{4}{3}\pi r_2^3 = \frac{4}{3}\pi R^3$

$$R^3 = r_1^3 + r_2^3 = (0.001\text{m})^3 + (0.002\text{m})^3 = 10^{-9} + 8 \times 10^{-9} = 9 \times 10^{-9}\text{m}^3$$

$$R = (9 \times 10^{-9})^{1/3} = 2.080 \times 10^{-3} \text{m}$$

Energy released = decrease in surface energy:

$$\Delta E = \gamma(4\pi r_1^2 + 4\pi r_2^2 - 4\pi R^2) = 4\pi \times 0.50 \text{Nm}^{-1} \times ((0.001)^2 + (0.002)^2 - (2.080 \times 10^{-3})^2) \text{m}^2$$

$$\Delta E = 4.23 \times 10^{-6} \text{J}$$

Question 84

(a) The angle of contact depends on the relative strengths of cohesive forces (within the liquid) and adhesive forces (between the liquid and the solid). For water on glass, adhesive forces are stronger than cohesive forces. Water molecules are attracted to the glass more than to each other, so the liquid spreads and the meniscus is concave (acute angle). For mercury on glass, cohesive forces within mercury are much stronger than adhesive forces to glass. Mercury molecules pull away from the glass, forming a convex meniscus (obtuse angle).

(b) The excess pressure needed to inflate a bubble is $\Delta P = 4\gamma/r$. When r is very small (just starting), ΔP is very large. The person must blow hard to generate this high pressure. As the bubble grows, r increases and ΔP decreases. Less effort is required to continue inflating. This is why the hardest part of blowing a bubble is the initial formation.

(c) By the capillary rise formula, $h \propto \frac{1}{r}$. If $r_2 = \frac{r_1}{3}$:

$$h_2 = h_1 \times \frac{r_1}{r_2} = 2.0 \text{cm} \times 3 = 6.0 \text{cm}$$

Question 85

(a) Cotton is made of hydrophilic (water-attracting) cellulose fibres. The adhesive forces between water and cotton exceed the cohesive forces within water. The angle of contact is small, and water is drawn into the narrow spaces between fibres by capillary action, soaking the towel. A plastic tarpaulin is hydrophobic. The cohesive forces within water are stronger than the adhesive forces to the plastic surface. The angle of contact exceeds 90° , so water beads up into nearly spherical droplets and rolls off.

(b) The excess pressure inside a bubble is $\Delta P = 2\gamma/r$. For a very small bubble (say $r \sim 0.1 \text{mm}$), this gives $\Delta P \sim 1460 \text{Pa}$, which is about 1.4% of atmospheric pressure and measurable. For a large bubble (say $r \sim 1 \text{cm}$), $\Delta P \sim 14.6 \text{Pa}$, which is 0.014% of atmospheric pressure and negligible. The inverse dependence on r means excess pressure becomes significant only when the radius is very small.

(c) Volume conservation: $1000 \times \frac{4}{3} \pi r^3 = \frac{4}{3} \pi R^3$, so $R = 10r$.

With $r = 0.5 \times 10^{-8} \text{m} = 5 \times 10^{-9} \text{m}$, $R = 5 \times 10^{-8} \text{m}$.

Energy liberated = decrease in surface area $\times \gamma$:

$$\Delta E = \gamma(1000 \times 4\pi r^2 - 4\pi R^2) = 4\pi \times 0.073 \text{Nm}^{-1} \times (1000 \times (5 \times 10^{-9} \text{m})^2 - (5 \times 10^{-8} \text{m})^2)$$

$$\Delta E = 2.06 \times 10^{-14} \text{J}$$

Question 86

(a) In untilled soil, narrow spaces between soil particles act as capillary tubes. Water is drawn upward from the moist subsoil to the dry surface by capillary action ($h = 2\gamma \cos\theta / (\rho g r)$). At the surface, it evaporates. Tilling breaks the top layer into larger clumps with much wider gaps. Capillary rise is inversely proportional to the gap width. The wide gaps cannot sustain capillary rise, so the connection between the moist subsoil and the surface is broken. The water stays underground.

(b) Let n small drops of radius r form one large drop of radius R .

Volume conservation: $n \times \frac{4}{3} \pi r^3 = \frac{4}{3} \pi R^3$, so $n = \frac{R^3}{r^3}$

Surface energy released:

$$\Delta E = \gamma(n \times 4\pi r^2 - 4\pi R^2) = 4\pi\gamma(nr^2 - R^2) = 4\pi\gamma\left(\frac{R^3}{r^3} \times r^2 - R^2\right) = 4\pi\gamma R^3 \left(\frac{1}{r} - \frac{1}{R}\right)$$

This energy heats the combined drop of mass $m = \frac{4}{3} \pi R^3 \rho$:

$$\Delta E = mc\Delta\theta = \frac{4}{3} \pi R^3 \rho c \Delta\theta$$

Equating:

$$4\pi\gamma R^3 \left(\frac{1}{r} - \frac{1}{R} \right) = \frac{4}{3} \pi R^3 \rho c \Delta\theta$$

$$\Delta\theta = \frac{3\gamma}{\rho c} \left(\frac{1}{r} - \frac{1}{R} \right)$$

(c) By the capillary rise formula:

$$r = \frac{2\gamma \cos\theta}{\rho g h} = \frac{2 \times 5 \times 10^{-2} \text{Nm}^{-1} \times \cos 45^\circ}{1000 \text{kgm}^{-3} \times 9.8 \text{ms}^{-2} \times 20 \text{m}} = 3.61 \times 10^{-7} \text{m} \approx 0.36 \mu\text{m}$$

Question 87

(a) Initial state: bubble of radius r , chamber pressure P_0 , pressure inside bubble $P_0 + \frac{4\gamma}{r}$.

Final state: bubble of radius $2r$, chamber pressure P , pressure inside bubble $P + \frac{4\gamma}{2r} = P + \frac{2\gamma}{r}$.

By Boyle's law (isothermal): $P_i V_i = P_f V_f$

$$\left(P_0 + \frac{4\gamma}{r} \right) \times \frac{4}{3} \pi r^3 = \left(P + \frac{2\gamma}{r} \right) \times \frac{4}{3} \pi (2r)^3$$

$$\left(P_0 + \frac{4\gamma}{r} \right) r^3 = \left(P + \frac{2\gamma}{r} \right) \times 8r^3$$

$$P_0 + \frac{4\gamma}{r} = 8P + \frac{16\gamma}{r}$$

$$8P = P_0 + \frac{4\gamma}{r} - \frac{16\gamma}{r} = P_0 - \frac{12\gamma}{r}$$

$$P = \frac{P_0}{8} - \frac{3\gamma}{2r}$$

(b) The expected rise height is $h_e = 2\gamma \cos\theta / (\rho g r)$. If the tube length $L < h_e$, the water rises to the top but cannot rise further. The meniscus adjusts: it flattens, increasing its radius of curvature from $r/\cos\theta$ to a new larger value. This means the angle of contact increases from θ to a new value θ' such that $L = 2\gamma \cos\theta' / (\rho g r)$. The water does not overflow because the meniscus adjusts its curvature to support exactly the column L .

(c) Excess pressure in the soap bubble (two surfaces):

$$\Delta P = \frac{4\gamma}{r} = \frac{4 \times 2.5 \times 10^{-2} \text{Nm}^{-1}}{0.01 \text{m}} = 10 \text{Pa}$$

For the air bubble at depth 40cm in soap solution (one surface):

$$\Delta P_{\text{bubble}} = \frac{2\gamma}{r} = \frac{2 \times 2.5 \times 10^{-2} \text{Nm}^{-1}}{0.01 \text{m}} = 5 \text{Pa}$$

Hydrostatic pressure at 40cm depth:

$$P_h = \rho g h = 1200 \text{kgm}^{-3} \times 9.8 \text{ms}^{-2} \times 0.40 \text{m} = 4704 \text{Pa}$$

Total pressure inside the air bubble:

$$P = P_{\text{atm}} + P_h + \Delta P_{\text{bubble}} = 1.013 \times 10^5 \text{Pa} + 4704 \text{Pa} + 5 \text{Pa} = 1.060 \times 10^5 \text{Pa}$$

Question 88

(a) The remaining soap film outside the loop has surface tension pulling equally in all directions. Before puncturing, the tension on both sides of the thread cancels out. After the film inside the loop is destroyed, there is no longer any inward pull from inside the loop. The surface tension from the intact film outside pulls the thread outward from every point along its length. The shape that maximises the enclosed area for a given perimeter is a circle. The thread springs into a circle because surface tension minimises the remaining film area, which is equivalent to maximising the area enclosed by the thread.

(b) Water wets glass (adhesive forces stronger than cohesive forces, acute contact angle), so it is pulled upward in the tube by surface tension, forming a concave meniscus. Mercury does not wet glass (cohesive forces stronger than adhesive forces, obtuse

contact angle), so the surface tension pulls the mercury surface downward inside the tube, forming a convex meniscus. The same tube produces opposite behaviour because the two liquids have opposite relationships between cohesive and adhesive forces with glass.

(c) For water: $h_w = \frac{2\gamma_w \cos\theta_w}{\rho_w g r} \dots (1)$

For mercury: $h_{Hg} = \frac{2\gamma_{Hg} \cos\theta_{Hg}}{\rho_{Hg} g r} \dots (2)$

Dividing (2) by (1):

$$\frac{h_{Hg}}{h_w} = \frac{\gamma_{Hg} \cos\theta_{Hg} \times \rho_w}{\gamma_w \cos\theta_w \times \rho_{Hg}}$$

$$\frac{\gamma_{Hg}}{\gamma_w} = \frac{h_{Hg}}{h_w} \times \frac{\cos\theta_w}{\cos\theta_{Hg}} \times \frac{\rho_{Hg}}{\rho_w}$$

Using $h_{Hg} = -3.4\text{cm}$, $h_w = 9.0\text{cm}$:

$$\frac{\gamma_{Hg}}{\gamma_w} = \frac{-3.4}{9.0} \times \frac{\cos 0^\circ}{\cos 135^\circ} \times \frac{13600}{1000}$$

$$\frac{\gamma_{Hg}}{\gamma_w} = 7.27$$

Question 89

(a) The spinning wheels have angular momentum directed along their axes. A large angular momentum resists changes in direction (gyroscopic stability). Any small tilt of the bicycle produces a gravitational torque, but the angular momentum vector resists this change, keeping the bike upright. On a stationary bicycle, the wheels have no angular momentum. Any small tilt is immediately amplified by gravity with no gyroscopic resistance, making balance very difficult.

(b) The moment of inertia of the system about the pivot is $I = I_{\text{ruler}} + 2md^2$, where d is the distance of each mass from the pivot. Moving the masses further from the pivot increases d and therefore increases I . The angular acceleration for a given torque is $\alpha = \tau/I$. A larger I means a smaller α for the same tilt (which produces the same gravitational torque). The system resists angular acceleration more strongly.

(c) For a thin rectangular sheet of mass M , length a , and width b :

Axis parallel to the length (through centre): $I_l = \frac{1}{12} Mb^2$

$$I_l = \frac{1}{12} \times 0.032\text{kg} \times (0.10\text{m})^2 = \frac{1}{12} \times 0.032 \times 0.01 = 2.67 \times 10^{-5} \text{kgm}^2$$

Axis parallel to the width (through centre): $I_w = \frac{1}{12} Ma^2$

$$I_w = \frac{1}{12} \times 0.032\text{kg} \times (0.25\text{m})^2 = \frac{1}{12} \times 0.032 \times 0.0625 = 1.67 \times 10^{-4} \text{kgm}^2$$