# EXAMINATION QUESTIONS ON PARTICLE MECHANICS

**Question 1**

(a) A heavy crate is dragged across a rough warehouse floor by two ropes pulled by two workers. One pulls northward, the other pulls eastward. The crate moves in a direction between north and east. Account for the direction of the crate's motion.

(b) A goalkeeper catches a fast-moving football and is pushed backward slightly. A spectator says the ball *"transferred its energy to the goalkeeper."* Discuss whether this description is physically complete, and identify what quantity is actually conserved during the catch.

(c) A crane supports a $200\text{kg}$ load using two cables. One cable makes $40°$ with the horizontal and the other makes $60°$ with the horizontal, both on the same side of the vertical. Take $g = 9.8\text{m/s}^2$. Calculate the tension in each cable.

**Question 2**

(a) A motorcycle stunt rider loops inside a vertical circular track. At the topmost point, the rider is upside down yet does not fall. Give a physical account of why the rider remains on the track.

(b) When a heavy lorry and a small car collide head-on, the occupants of the small car are usually more seriously injured. Both vehicles experience the same magnitude of force during the collision. Account for the greater injury in the small car.

(c) A $0.06\text{kg}$ tennis ball travelling horizontally at $40\text{m/s}$ is struck by a racket and returned in the opposite direction at $55\text{m/s}$. The contact time is $0.008\text{s}$. Calculate the average force exerted by the racket on the ball.

**Question 3**

(a) A javelin thrower releases the javelin at an angle to the horizontal. After release, the javelin continues to rise for a while before curving downward. Explain why the javelin rises initially and what causes it to eventually descend.

(b) A precision scale calibrated in a laboratory at Iringa (altitude $1600\text{m}$) is transported to a laboratory in Bagamoyo (near sea level). A scientist notices that the same standard mass gives a slightly different reading. Account for this discrepancy.

(c) A particle is in equilibrium under the action of three forces: $F_1 = 10\text{N}$ acting along the positive x-axis, $F_2 = 15\text{N}$ acting at $120°$ to $F_1$, and a third force $F_3$. Calculate the magnitude and direction of $F_3$.

**Question 4**

(a) A satellite in low Earth orbit completes one revolution approximately every $90$ minutes. An observer claims the satellite is "not affected by gravity" because it does not fall. Discuss why this claim is physically incorrect.

(b) During a road accident investigation, investigators estimate the speed of a vehicle before impact by measuring the length of skid marks on the road. Explain the physical principle behind this method.

(c) A ball is kicked from the edge of a flat rooftop $15\text{m}$ above the ground with a velocity of $20\text{m/s}$ at $30°$ above the horizontal. Take $g = 9.8\text{m/s}^2$. Calculate the speed of the ball just before it hits the ground.

**Question 5**

(a) A coin placed on a spinning turntable stays in place at low speed but flies off when the turntable spins too fast. Explain this observation.

(b) An athlete throws a discus at an angle to the horizontal. Explain why the range of the discus depends on the angle of projection, and identify the angle that gives maximum range on level ground.

(c) A $0.50\text{kg}$ object moves at $4.0\text{m/s}$ on a smooth horizontal surface and collides head-on with a $0.30\text{kg}$ object moving at $3.0\text{m/s}$ in the opposite direction. After the collision, the $0.50\text{kg}$ object moves at $1.0\text{m/s}$ in its original direction. Calculate the velocity of the $0.30\text{kg}$ object after the collision and determine whether kinetic energy is conserved.

**Question 6**

(a) A stone tied to a string is whirled in a horizontal circle. When the string breaks, the stone flies off in a straight line tangent to the circle, not outward. Account for this observation.

(b) Explain why the gravitational force between two ordinary objects (such as two books on a table) exists but cannot be felt or measured without extremely sensitive instruments.

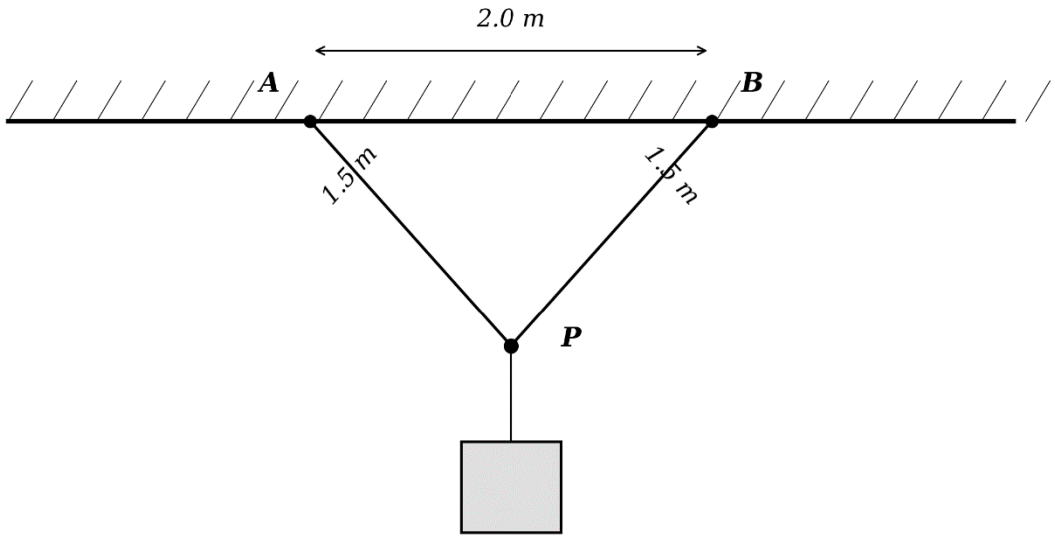
(c) A mortar fires a shell at $100\text{m/s}$ at $53°$ to the horizontal from level ground. Take $g = 9.8\text{m/s}^2$. Calculate the horizontal distance at which the shell is at a height of $100\text{m}$ for the first time.

**Question 7**

(a) A car travels over a hump-backed bridge and the driver feels momentarily lighter at the top. Give a physical explanation for this sensation.

(b) An ice hockey puck slides a great distance across the ice after being struck, yet a football kicked with similar effort on a grass field stops much sooner. Both objects were given a similar initial push. Account for the difference in the distances covered.

(c) Two ropes are attached to a ceiling at points $2.0\text{m}$ apart. A $15\text{kg}$ mass hangs from the junction of two ropes, each $1.5\text{m}$ long. Take $g = 9.8\text{m/s}^2$. Calculate the tension in each rope.




**Question 8**

(a) A planet moves faster at perihelion and slower at aphelion. Explain why this change in speed occurs, using the concept of angular momentum conservation.

(b) Explain why a satellite does not need to fire its engines to remain in orbit, yet an aeroplane must continuously run its engines to stay aloft.

(c) A $3.0\text{kg}$ body moving at $8.0\text{m/s}$ to the right collides elastically and head-on with a $1.0\text{kg}$ body at rest. Calculate the velocity of each body after the collision.

**Question 9**

(a) Explain why the escape velocity from a planet depends on the planet's mass and radius but not on the mass of the escaping object.

(b) A child on a playground roundabout (merry-go-round) moves to the outer edge and finds it harder to hold on. Account for this observation.

(c) A projectile is launched from ground level at $35\text{m/s}$ at $50°$ above the horizontal. Take $g = 9.8\text{m/s}^2$. Calculate the total time the projectile spends above a height of $20\text{m}$.

**Question 10**

(a) Two ropes attached to a ring pull in different directions. A third rope is attached such that the ring remains stationary. Explain what condition the three forces must satisfy and how the third force is determined.

(b) Water from a hosepipe follows a curved path after leaving the nozzle, while a laser beam travels in a straight line through the same space. Account for this difference.

(c) A $800\text{kg}$ car rounds a flat circular bend of radius $50\text{m}$. The coefficient of static friction between the tyres and the road is $0.45$. Take $g = 9.8\text{m/s}^2$. Calculate the maximum safe speed for the car on this bend.

**Question 11**

(a) A satellite in a higher orbit moves more slowly than one in a lower orbit, yet more energy was needed to place it there. Explain why this is not contradictory.

(b) In a cricket match, a batsman swings through the ball rather than just blocking it. Explain how the follow-through affects the ball's speed after being struck.

(c) A $5.0\text{kg}$ object hangs from two strings. One string makes $30°$ with the ceiling and the other makes $45°$ with the ceiling. Take $g = 9.8\text{m/s}^2$. Calculate the tension in each string.

**Question 12**

(a) Explain why a bullet fired horizontally from a gun and a bullet dropped from the same height at the same instant hit the ground at the same time (neglecting air resistance).

(b) A car entering a roundabout at moderate speed does not skid, but the same car going too fast may skid outward. Explain the role of friction in each case.

(c) The Moon orbits the Earth at a mean distance of $3.84 \times 10^8\text{m}$. Take $G = 6.67 \times 10^{-11}\text{Nm}^2\text{kg}^{-2}$ and $M_E = 6.0 \times 10^{24}\text{kg}$. Calculate the speed at which the Moon must travel to maintain its orbit.

**Question 13**

(a) A truck moving at constant velocity on a highway has zero acceleration. A student concludes that no forces act on it. Explain why this conclusion is incorrect.

(b) A long jumper takes off at an angle to the horizontal. Explain why the jumper's centre of mass follows a parabolic path regardless of the body movements performed during flight.

(c) A $0.025\text{kg}$ bullet is fired horizontally into a $4.975\text{kg}$ wooden pendulum bob hanging at rest. The bob swings upward to a height of $0.10\text{m}$. Take $g = 9.8\text{m/s}^2$. Calculate the initial speed of the bullet.

**Question 14**

(a) A ball thrown vertically upward with speed less than the escape velocity always returns to the ground. Explain why, using energy considerations.

(b) During a tug-of-war, neither team moves for several seconds despite both teams pulling very hard. Account for the force situation during this period.

(c) A $0.80$kg ball is attached to a $1.2$m string and swung in a vertical circle. Calculate the minimum speed at the top of the circle for the ball to complete the circular motion. Take $g = 9.8\text{m/s}^2$.

**Question 15**

(a) Explain why two satellites at the same altitude but with different masses orbit the Earth at exactly the same speed.

(b) Explain why there is a maximum height from which a person can jump safely onto the ground, and why landing on a soft surface increases this safe height.

(c) A satellite orbits a planet at a height where the gravitational field strength is $3.2$N/kg. The planet has a radius of $5.0 \times 10^6$m and a mass of $4.0 \times 10^{24}$kg. Take $G = 6.67 \times 10^{-11}\text{Nm}^2\text{kg}^{-2}$. Calculate the altitude of the satellite above the planet's surface.

**Question 16**

(a) Two balls of different masses are dropped simultaneously from the same height. In the absence of air resistance, they hit the ground at the same time. Explain why mass does not affect the time of fall.

(b) A roundabout (merry-go-round) at a playground spins at constant angular velocity. A child sitting near the centre feels a gentle outward pull, while another child at the edge feels a strong outward pull. Account for this difference.

(c) A shell explodes at the highest point of its parabolic trajectory into two equal fragments. One fragment falls vertically from rest. Determine the direction and speed of the other fragment immediately after the explosion, given that the shell had a horizontal velocity of $200$m/s at the point of explosion.

**Question 17**

(a) Explain why a banked road allows a car to round a curve at high speed even when the road surface is icy and friction is negligible.

(b) A large asteroid approaching the Earth has a negative gravitational potential energy with respect to the Earth. Explain what this negative value physically means.

(c) A stone is projected at $25$m/s at $37°$ above the horizontal from the top of a $45$m cliff. Take $g = 9.8\text{m/s}^2$. Calculate the time taken for the stone to reach the ground below.

**Question 18**

(a) A person travelling in a car notices that when the car suddenly brakes, their body lurches forward but the seatbelt holds them back. Using the concept of impulse, explain how the seatbelt protects the person.

(b) A tennis player follows through with the racket after hitting the ball. Explain how this follow-through increases the speed of the ball.

(c) A $2.0$kg mass moves in a horizontal circle on the inner surface of a smooth conical bowl. The half-angle of the cone is $50°$ (measured from the vertical axis). Take $g = 9.8\text{m/s}^2$. Calculate the speed of the mass if the radius of the circular path is $0.30$m.

**Question 19**

(a) A ball that just barely completes a vertical circle on a string is released at the top. Explain why it appears to "float" momentarily before falling.

(b) Explain why a very high coefficient of friction between the tyres and the road can be both an advantage and a disadvantage for a car negotiating a level curved road.

(c) A truck has its wheels 1.8m apart laterally and its centre of gravity 1.0m above the ground. It rounds a level curve of radius 50m. Special high-grip tyres give a coefficient of static friction of 0.95. Take $g = 9.8\text{m/s}^2$.

(i) Calculate the maximum speed before skidding.
(ii) Calculate the maximum speed before overturning.
(iii) Determine which happens first, and explain why the high-grip tyres make this vehicle more dangerous on this curve, not less.

**Question 20**

(a) Explain why a geostationary satellite appears stationary to an observer on Earth, even though it is moving at over 3km/s.

(b) A fire hose directs a powerful jet of water at a burning building. Explain why the firefighter holding the hose is pushed backward.

(c) A ball of mass 0.4kg is attached to a string and swung in a vertical circle of radius 0.8m. The speed at the lowest point is 6.0m/s. Take $g = 9.8\text{m/s}^2$. Calculate the tension in the string at the lowest point.

**Question 21**

(a) Explain why the acceleration due to gravity on the surface of a planet depends on both the mass and the radius of the planet.

(b) When a gun is fired, the bullet moves forward at high speed while the gun recoils backward at a much lower speed. Explain why the speeds are different even though the forces on the bullet and the gun are equal.

(c) A stone is projected horizontally at 12m/s from a bridge and hits the water 2.4s later. Take $g = 9.8\text{m/s}^2$. Calculate the speed at which the stone enters the water.

**Question 22**

(a) Explain why all geostationary satellites must orbit at the same altitude above the Earth.

(b) A student drops a heavy ball and a light ball simultaneously from the same height. In the absence of air resistance, both hit the ground at the same time. Explain why.

(c) A 0.05kg bullet travelling at 300m/s strikes a 2.95kg stationary block on a smooth surface and embeds itself. The block then slides up a rough incline of $30°$ with coefficient of kinetic friction 0.25. Calculate how far up the incline the block travels before stopping.

**Question 23**

(a) Explain why the gravitational potential energy of a satellite in orbit is negative while its kinetic energy is positive.

(b) A sprinter pushes backward on the starting blocks at the beginning of a race. Explain how this causes the sprinter to accelerate forward.

(c) Two ropes support a 40kg traffic light at a junction. One rope makes $35°$ and the other makes $55°$ with the horizontal. Take $g = 9.8\text{m/s}^2$. Calculate the tension in each rope.

**Question 24**

(a) Explain why a body moving in a circular path at constant speed requires a net force, even though its speed is not changing.

(b) A lorry loaded with goods is more difficult to stop than the same lorry when empty, even if both are travelling at the same speed. Explain why, using the concept of momentum.

(c) A mortar fires a shell at $80\text{m/s}$ at $50°$ to the horizontal from the edge of a cliff $60\text{m}$ above flat ground. Take $g = 9.8\text{m/s}^2$. Calculate the horizontal distance from the base of the cliff to where the shell lands.

**Question 25**

(a) Explain why Kepler's second law (equal areas in equal times) is a consequence of the conservation of angular momentum.

(b) Passengers in a car lean sideways when the car goes around a sharp bend. Explain why this happens.

(c) A $3.0\text{kg}$ object moving east at $5.0\text{m/s}$ collides with a $2.0\text{kg}$ object moving north at $7.5\text{m/s}$. The objects stick together. Calculate the speed and direction of the combined object after the collision.

**Question 26**

(a) Explain why the gravitational field strength is zero at the centre of the Earth but the gravitational potential is not zero there.

(b) A person standing on a weighing scale inside a freely falling lift would observe a reading of zero. Explain why.

(c) A rocket of total mass $5000\text{kg}$ (including fuel) is launched vertically. The exhaust gases are expelled at $800\text{m/s}$ relative to the rocket at a rate of $20\text{kg/s}$. Take $g = 9.8\text{m/s}^2$. Calculate the initial acceleration of the rocket.

**Question 27**

(a) Explain why a projectile launched at $45°$ to the horizontal achieves the maximum range on level ground.

(b) Explain why the period of a satellite in orbit does not depend on the mass of the satellite.

(c) A block of mass $4.0\text{kg}$ on a smooth horizontal surface is connected by a light string over a frictionless pulley to a hanging mass of $1.0\text{kg}$. Take $g = 9.8\text{m/s}^2$. Calculate the acceleration of the system and the tension in the string.

**Question 28**

(a) Explain why the total mechanical energy of a satellite in circular orbit is negative.

(b) A cricket ball hit into the air reaches the same height whether the batsman hits it gently or hard, as long as the vertical component of the initial velocity is the same. Explain why.

(c) A force of F at angle $\theta$ above the horizontal is just sufficient to move a block of mass m at constant velocity on a rough horizontal surface with coefficient of kinetic friction $\mu$. Show that $F = \frac{\mu mg}{\cos\theta + \mu\sin\theta}$.

**Question 29**

(a) Explain the physical difference between mass and weight, and why an astronaut in orbit has mass but appears to have no weight.

(b) Explain why a car with a higher centre of mass is more likely to overturn on a curved road than a car with a lower centre of mass.

(c) A planet has a surface gravity of $4.5\text{m/s}^2$ and a radius of $4.8 \times 10^6\text{m}$. Take $G = 6.67 \times 10^{-11}\text{Nm}^2\text{kg}^{-2}$. Calculate the minimum speed at which a spacecraft must be launched from this planet to escape its gravitational field.

**Question 30**

(a) Explain why a ball kicked off a cliff follows a parabolic path, while the same ball rolling along a flat field follows a straight path.

(b) In the sport of ice curling, a heavy stone slides across the ice with very little friction. If two identical stones collide elastically and head-on, the first stone stops and the second moves off at the first stone's original speed. Explain why this outcome conserves both momentum and kinetic energy.

(c) The Moon orbits the Earth at a mean distance of $3.84 \times 10^8$m. Take $M_E = 6.0 \times 10^{24}$kg and $G = 6.67 \times 10^{-11}\text{Nm}^2\text{kg}^{-2}$. Calculate the orbital period of the Moon in days.

**Question 31**

(a) Explain why it is impossible for a body to escape from a planet if its total mechanical energy is negative.

(b) During a rainstorm, a person walking forward notices that the rain appears to come at an angle from the front, even though the rain is actually falling vertically. Explain this observation.

(c) A car of mass $1200$kg rounds a banked curve of radius $80$m. The banking angle is $20°$. Take $g = 9.8\text{m/s}^2$. Calculate the design speed of the curve.

**Question 32**

(a) Explain why the value of g at the equator is slightly less than at the poles.

(b) Explain why a parachutist reaches terminal velocity and stops accelerating, even though gravity continues to act.

(c) A $0.2$kg ball travelling at $8$m/s to the right bounces off a wall and returns at $6$m/s to the left. The contact time with the wall is $0.05$s. Calculate the average force exerted by the wall on the ball.

**Question 33**

(a) Explain why the relationship $v_{escape} = \sqrt{2} \times v_{orbital}$ holds at any distance from a planet, not just at the surface.

(b) A bus driver brakes gradually to stop rather than braking suddenly. Explain the advantage of gradual braking for the passengers, using the impulse-momentum theorem.

(c) A $60$kg skier starts from rest at the top of a $25°$ slope and skis down for $200$m. The coefficient of kinetic friction between the skis and the snow is $0.10$. Take $g = 9.8\text{m/s}^2$. Calculate the speed of the skier at the bottom.

**Question 34**

(a) Explain why a stone thrown at an angle to the horizontal has its minimum speed at the highest point of its trajectory.

(b) Explain why a moving satellite does not need any fuel to maintain its orbit (in the absence of atmospheric drag).

(c) Two bodies of masses $m_1 = 3.0$kg and $m_2 = 5.0$kg are connected by a compressed spring and held together on a smooth horizontal surface. When released, $m_1$ moves to the left at $4.0$m/s. Calculate the energy stored in the spring before release.

**Question 35**

(a) Explain why the approximate formula $g' \approx g\left(1 - \frac{2h}{r_E}\right)$ gives a percentage error that increases rapidly with height.

(b) Explain why a head-on collision between two cars moving at $60$km/h each is not the same as one car hitting a rigid wall at $120$km/h.

(c) A stone is thrown from ground level at $30$m/s at $37°$ above the horizontal. At what time after launch is the speed of the stone equal to $25$m/s? Take $g = 9.8\text{m/s}^2$.

**Question 36**

(a) Explain why the escape velocity from the surface of the Moon is much lower than from the surface of the Earth.

(b) A cricket fielder catches a fast-moving ball by drawing their hands backward. Explain how this technique reduces the force on the fielder's hands.

(c) A $1500\text{kg}$ car enters a vertical loop of radius $12\text{m}$ at a speed of $20\text{m/s}$ at the bottom. Take $g = 9.8\text{m/s}^2$. Using energy conservation, calculate the speed at the top and hence determine whether the car completes the loop.

**Question 37**

(a) A car travels at constant speed over a hump-backed bridge. Explain why the driver feels lighter at the top of the bridge than on flat road.

(b) A bullet embeds itself in a wooden block, and a large amount of kinetic energy is lost. Explain where this "lost" kinetic energy goes.

(c) A stone is projected at $40\text{m/s}$ at an angle of $35°$ above the horizontal from the top of a cliff $70\text{m}$ above flat ground. Take $g = 9.8\text{m/s}^2$. Calculate the speed of the stone just before it strikes the ground.

**Question 38**

(a) A spacecraft uses a "gravity assist" to gain speed by passing close to a planet, without using any fuel. Explain how the spacecraft gains kinetic energy in this manoeuvre.

(b) Two equal forces act on an object, yet the object does not accelerate. Explain how this is possible.

(c) A $0.40\text{kg}$ ball attached to a string of length $0.60\text{m}$ is whirled in a vertical circle. At the top of the circle, the tension in the string is $5.0\text{N}$. Take $g = 9.8\text{m/s}^2$. Calculate the speed of the ball at the top.

**Question 39**

(a) Explain why a satellite in geostationary orbit must orbit at a specific altitude and cannot be placed at any other height.

(b) A goalkeeper dives to catch a fast ball and rolls on the ground after catching it. Explain how rolling helps protect the goalkeeper from injury.

(c) Three concurrent forces act on a bolt at the origin: $F_1 = 25\text{N}$ along the positive x-axis, $F_2 = 18\text{N}$ at $130°$ from the positive x-axis, and $F_3 = 12\text{N}$ at $240°$ from the positive x-axis. Calculate the magnitude and direction of the equilibrant.

**Question 40**

(a) An object is projected horizontally from the top of a tall building. Explain why the horizontal distance covered increases with the height of the building, even though the initial horizontal speed is unchanged.

(b) Explain why a person standing on a bathroom scale inside a freely falling lift would observe a reading of zero.

(c) A $2.0\text{kg}$ object moving at $6.0\text{m/s}$ to the east collides with a $3.0\text{kg}$ object moving at $4.0\text{m/s}$ to the north. The objects stick together after the collision. Calculate the velocity and direction of the combined object.

**Question 41**

(a) Explain why the acceleration due to gravity on the surface of the Moon ($1.6\text{m/s}^2$) is much less than on the surface of the Earth ($9.8\text{m/s}^2$), even though the Moon's radius is only about $\frac{1}{3.7}$ of the Earth's.

(b) A football is kicked into the air. At the highest point, the velocity of the ball is not zero. Explain why.

(c) A $1500$kg car travels at $25$m/s around a flat circular bend of radius $60$m. Take $g = 9.8\text{m/s}^2$. Calculate the minimum coefficient of static friction between the tyres and the road required to prevent the car from skidding.

**Question 42**

(a) Explain why the total mechanical energy of a satellite in circular orbit is negative, and what this negative value physically signifies.

(b) A car moving at constant speed on a straight road uses more fuel than the same car parked with its engine idling. Explain why, even though neither car is accelerating.

(c) A ball is thrown horizontally at $15$m/s from the top of a tower. It hits the ground at an angle of $60°$ below the horizontal. Take $g = 9.8\text{m/s}^2$. Calculate the height of the tower.

**Question 43**

(a) Why does launching a golf ball upward increase its horizontal distance?

(b) A coin placed on a smooth rotating turntable slides off when the turntable spins too fast. Explain why the coin slides off and why it moves outward rather than in any other direction.

(c) A satellite orbits the Earth at a height where the gravitational field strength is $4.9$N/kg. Take $M_E = 6.0 \times 10^{24}$kg, $r_E = 6.4 \times 10^{6}$m, and $G = 6.67 \times 10^{-11}\text{Nm}^2\text{kg}^{-2}$. Calculate the altitude of the satellite above the Earth's surface.

**Question 44**

(a) Why is a rope thrown outward from a cliff rather than dropped straight down when trying to reach someone below?

(b) A large ship and a small rowing boat collide at low speed. The rowing boat is severely damaged while the ship is barely affected. Explain this difference.

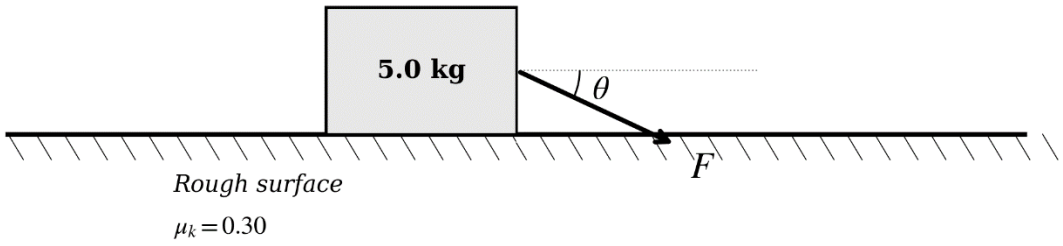
(c) A body of mass $0.5$kg is attached to a string of length $1.0$m and swung in a vertical circle. Calculate the minimum speed at the bottom of the circle for the body to just complete the full circle. Take $g = 9.8\text{m/s}^2$.

**Question 45**

(a) A planet at perihelion moves faster than at aphelion. Explain why the planet's speed changes during its orbit, even though no tangential force appears to act on it.

(b) A truck carrying sand has a small hole in its floor. As the truck moves, sand leaks out at a steady rate. If the engine continues to provide the same driving force, explain what happens to the truck's acceleration over time.

(c) A force F acts at angle $\theta$ below the horizontal on a $5.0$kg block resting on a rough horizontal surface. The coefficient of kinetic friction is $\mu_k = 0.30$. If $F = 40$N and $\theta = 25°$, calculate the acceleration of the block. Take $g = 9.8\text{m/s}^2$.




**Question 46**

(a) Explain why two satellites at the same altitude orbit the Earth at exactly the same speed, regardless of their different masses.

(b) A volleyball player "cushions" the ball by moving their arms backward as the ball arrives. Explain how this technique reduces the force on the arms.

(c) A ball is projected from ground level at $28\text{m/s}$ at $40°$ above the horizontal. Take $g = 9.8\text{m/s}^2$. Calculate the velocity (magnitude and direction) of the ball $2.0\text{s}$ after launch.

**Question 47**

(a) A person weighs slightly less at the equator than at the North Pole. Explain the two physical reasons for this difference.

(b) A tennis ball is served at very high speed. The racket is in contact with the ball for only a few milliseconds. Explain why a short contact time results in a large force on the ball.

(c) A $0.020\text{kg}$ bullet travelling at $350\text{m/s}$ embeds itself in a $2.48\text{kg}$ stationary wooden block on a smooth surface. The block then slides onto a rough patch of floor with coefficient of kinetic friction $0.40$. Take $g = 9.8\text{m/s}^2$. Calculate the distance the block slides on the rough patch before stopping.

**Question 48**

(a) Explain why the gravitational field strength decreases as one moves deeper below the Earth's surface, and why it is zero at the centre.

(b) A car enters a roundabout too fast and the rear of the car swings outward. Explain what causes this to happen.

(c) A particle is held in equilibrium by three coplanar forces. One force of $50\text{N}$ acts vertically downward and another of $30\text{N}$ acts horizontally to the right. Calculate the magnitude and direction of the third force.

**Question 49**

(a) Explain why a satellite that gradually loses energy due to atmospheric drag moves to a lower orbit and speeds up, rather than slowing down and drifting outward.

(b) A person catches a cricket ball bare-handed without pulling the hands back. Explain why this is painful.

(c) A $1200\text{kg}$ car travels at $20\text{m/s}$ on a banked curve of radius $100\text{m}$. The banking angle is $15°$. Take $g = 9.8\text{m/s}^2$. Calculate the friction force on the car and state whether it acts up or down the slope.

**Question 50**

(a) Explain why it is dangerous for a car to travel at high speed on a wet curved road, even if the same speed is safe on a dry road.

(b) A comet moves in a highly elliptical orbit around the Sun. It moves very fast near the Sun and very slowly when far away. Explain why the comet's speed varies so dramatically.

(c) A $2.0\text{kg}$ mass hangs from a spring balance inside a lift. The balance reads $24.0\text{N}$ when the lift is accelerating. Take $g = 9.8\text{m/s}^2$. Determine whether the lift is accelerating upward or downward, and calculate the magnitude of the acceleration.

**Question 51**

(a) Explain why a pilot at the bottom of a vertical loop feels much heavier than normal, while at the top of the loop the pilot may feel almost weightless.

(b) A loaded minibus takes much longer to stop than an empty one travelling at the same speed, even though both have similar brakes. Explain why.

(c) A planet has a radius twice that of Earth and a density half that of Earth. Calculate the gravitational field strength on the surface of this planet, in terms of g.

**Question 52**

(a) Two forces of $10\text{N}$ act on an object. Explain why the resultant can have any value from $0$ to $20\text{N}$ depending on the angle between the forces.

(b) An astronaut on the Moon can throw a ball much farther than on Earth with the same effort. However, the astronaut cannot run much faster on the Moon than on Earth. Explain the difference.

(c) A $0.10\text{kg}$ ball moving at $10\text{m/s}$ strikes a stationary $0.10\text{kg}$ ball in an elastic head-on collision. Calculate the velocities of both balls after the collision.

**Question 53**

(a) Explain why a satellite in a lower orbit completes more orbits per day than a satellite in a higher orbit.

(b) A helicopter hovers at a constant height above the ground. Explain why the helicopter's engines must work continuously even though it is not accelerating.

(c) A $3.0\text{kg}$ block on a smooth $30°$ incline is connected by a light string over a smooth pulley at the top of the incline to a $2.0\text{kg}$ block hanging vertically. Take $g = 9.8\text{m/s}^2$. Calculate the acceleration of the system.

**Question 54**

(a) Explain why a person pushing a heavy box on a rough floor finds it easier to push horizontally than to push downward at an angle.

(b) During a collision between two vehicles, crumple zones at the front of the car deform. Explain how this deformation protects the occupants.

(c) A $1400\text{kg}$ car travels at $18\text{m/s}$ over the crest of a hill with radius of curvature $40\text{m}$. Take $g = 9.8\text{m/s}^2$. Calculate the normal reaction on the car at the crest, and state whether the car maintains contact with the road.

**Question 55**

(a) Explain why the gravitational potential energy of a satellite becomes less negative (increases) as the satellite moves to a higher orbit.

(b) A student observes that a freely falling ball near the top of a tall building and a satellite orbiting the Earth are both "falling under gravity." Explain the key difference between these two situations.

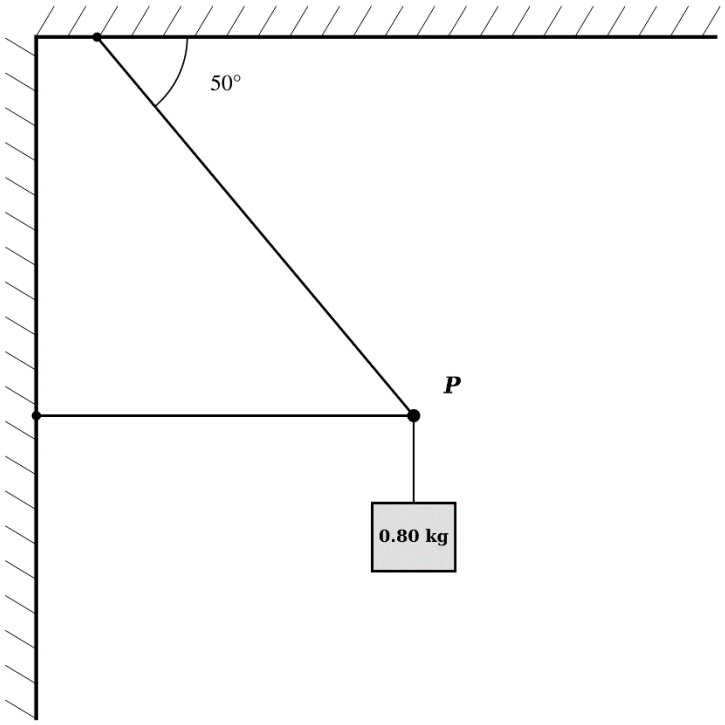
(c) A $5.0\text{kg}$ mass and a $3.0\text{kg}$ mass are connected by a light string passing over a smooth pulley. The system is released from rest. Take $g = 9.8\text{m/s}^2$. Calculate the distance each mass travels in the first $2.0\text{s}$.

**Question 56**

(a) A rifle bullet travels in an almost flat trajectory over short distances, while a thrown ball follows a clearly curved arc. Both are projectiles. Explain why their paths appear so different.

(b) A road is banked at an angle designed for a particular speed. Explain what happens to a car travelling at this exact design speed on the banked curve.

(c) A body of mass $0.80\text{kg}$ is suspended from two strings. One string is horizontal and attached to a wall. The other is attached to the ceiling and makes $50°$ with the horizontal. Take $g = 9.8\text{m/s}^2$. Calculate the tension in each string.




**Question 57**

(a) Two identical trucks, one loaded and one empty, travel at the same speed toward a junction. Explain why the loaded truck needs to start braking earlier than the empty truck to stop at the same point.

(b) Explain why the gravitational field of the Earth is approximately uniform near the surface but clearly non-uniform at the scale of satellite orbits.

(c) A ball is projected from ground level and lands $60\text{m}$ away on the same level. The maximum height reached is $20\text{m}$. Take $g = 9.8\text{m/s}^2$. Calculate the initial speed and angle of projection.

**Question 58**

(a) Explain why a pendulum swings more slowly on the Moon than on Earth, even though the length of the string is the same.

(b) A spacecraft in orbit around the Earth fires its engines to speed up briefly. Explain what happens to the orbit immediately after the burn.

(c) A $0.030\text{kg}$ bullet travelling at $400\text{m/s}$ passes through a $1.5\text{kg}$ stationary wooden plank and emerges at $100\text{m/s}$. Calculate the speed of the plank after the bullet passes through it and the kinetic energy lost.

**Question 59**

(a) Explain why a long jumper achieves a greater distance by running fast before jumping rather than jumping from a standing position.

(b) The value of g inside the Earth decreases linearly with depth (assuming uniform density), while above the surface it decreases as $\frac{1}{r^2}$. Explain why the two patterns are different.

(c) A satellite has a period of $12$ hours orbiting a planet of mass $5.0 \times 10^{24}\text{kg}$. Take $G = 6.67 \times 10^{-11}\text{Nm}^2\text{kg}^{-2}$. Calculate the orbital radius.

**Question 60**

(a) Explain why a planet's escape velocity can be calculated from its mass and radius alone, without knowing anything about the object that is escaping.

(b) A car travelling on a flat road enters a dip (valley) at speed. Explain why the driver feels heavier at the bottom of the dip.

(c) A 4.0kg body slides from rest down a smooth curved ramp of height 3.0m onto a rough horizontal floor with $\mu_k = 0.50$. Take $g = 9.8\text{m/s}^2$. Calculate how far the body slides on the floor before stopping.

**Question 61**

(a) A bucket of water is swung in a vertical circle. At the top, the water does not spill. Explain why, and state what happens if the speed at the top is too low.

(b) A hammer thrower spins several times before releasing the hammer. Explain why more spins lead to a longer throw.

(c) A ball is dropped from a height of 5m onto a hard floor. It bounces repeatedly. The coefficient of restitution between the ball and floor is 0.80. Calculate the height reached after the third bounce and the total distance travelled by the ball before the fourth bounce.

**Question 62**

(a) Explain why a heavy lorry moving at low speed can have the same momentum as a light car moving at high speed, and discuss which one is more dangerous in a collision.

(b) A person weighs 700N at the surface of the Earth. Explain qualitatively how this weight would change if the Earth suddenly shrank to half its radius without losing any mass.

(c) A ball is thrown at 16m/s at 30° above the horizontal from the top of a building 25m tall. At the same instant, a person at ground level 40m from the base of the building starts running at constant speed to catch the ball. Take $g = 9.8\text{m/s}^2$. Calculate the speed at which the person must run.

**Question 63**

(a) Explain why it is impossible for a single force acting alone to keep a body in equilibrium.

(b) Explain why the path of a projectile in reality deviates from a perfect parabola.

(c) A 3kg block travelling at 4m/s on a smooth surface collides with a 2kg block at rest. After the collision, the 3kg block moves at 1.6m/s in the same direction. Calculate the coefficient of restitution.

**Question 64**

(a) A package is dropped from a moving helicopter. Explain why the package does not land directly below the helicopter (assume the helicopter continues at constant velocity).

(b) A train moving at constant speed on a curved track tilts slightly inward. Explain why the track is designed this way.

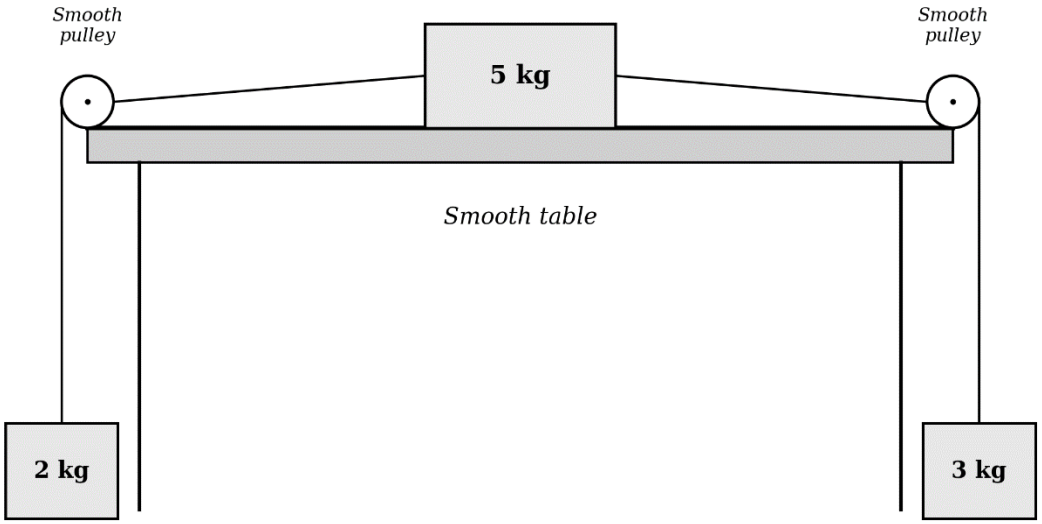
(c) A projectile is launched from level ground and must pass over a 15m wall located 30m away. The launch speed is 25m/s. Take $g = 9.8\text{m/s}^2$. Calculate the minimum angle of projection required to clear the wall.

**Question 65**

(a) Explain why a satellite in a polar orbit can eventually photograph the entire surface of the Earth, while a satellite in an equatorial orbit cannot.

(b) A space station rotates to simulate gravity for the astronauts inside. Explain how this artificial gravity works and why the astronauts must stand on the outer wall rather than the inner wall.

(c) A 5kg mass on a smooth table is connected by a string passing over a smooth pulley at the edge of the table to a 3kg mass hanging freely. The 5kg mass is also connected by another string over another smooth pulley on the opposite edge to a 2kg mass hanging freely. All strings are light and the pulleys are smooth. Take $g = 9.8\text{m/s}^2$. Calculate the acceleration of the system and the tension in each string.




**Question 66**

(a) As an aircraft prepares to land, the pilot tilts (banks) the wings. Explain how banking helps the aircraft change direction and align with the runway.

(b) A cricket fielder always draws his hands backward while catching a fast-moving ball. Explain why this technique reduces the chance of injury to the hands.

(c) A bullet of mass 12g travelling horizontally at $150\text{ms}^{-1}$ embeds itself in a block of wood of mass 1.188kg suspended freely by two strings. Calculate the velocity of the block immediately after impact and the kinetic energy lost in the collision.

**Question 67**

(a) For a projectile launched at a given angle above the horizontal, explain what happens to the horizontal range if the initial velocity is doubled.

(b) A satellite orbiting the Earth in an elliptical orbit moves faster when closer to the Earth and slower when farther away. Explain why the kinetic energy of the satellite changes as it moves between these two positions.

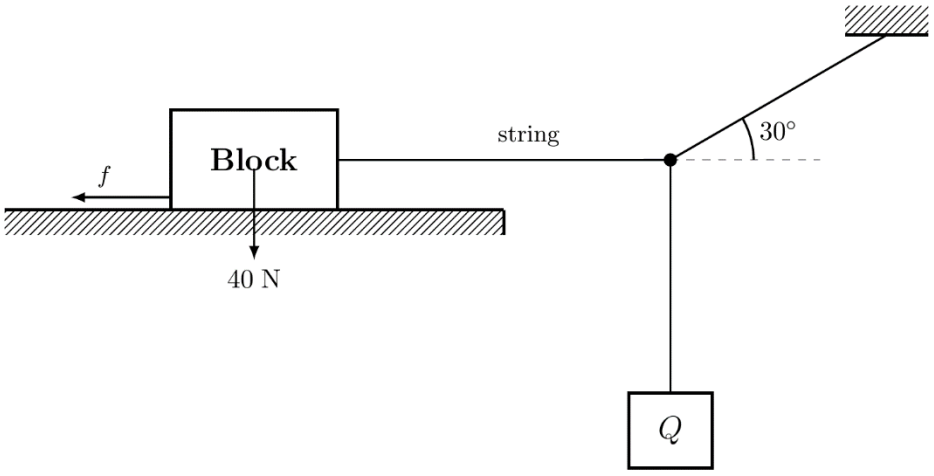
(c) A ball is kicked with an initial velocity of $8.0\text{ms}^{-1}$ such that it just passes over a barrier which is 2.2m high. Neglecting air resistance, calculate the horizontal velocity of the ball and the total time of flight. Take $g = 9.8\text{ms}^{-2}$.

**Question 68**

(a) The gravitational potential energy of an object at a point in a gravitational field is always negative. Explain what the negative sign signifies and why it is never zero at any finite distance from the attracting body.

(b) A person standing perfectly still on level ground takes a step forward. Before the step, the person's momentum was zero, but afterwards the person clearly has some momentum in the forward direction. Explain whether the conservation of momentum is violated in this situation.

(c) A block of weight $40\text{N}$ rests on a horizontal table. A light horizontal string is attached to the block and runs to a knot at the edge of the table. At the knot, two other strings are attached: one goes upward to a fixed support on the ceiling at $30°$ to the horizontal, and the other hangs vertically supporting a weight Q, as shown in the figure. The maximum friction force between the block and the table is $12\text{N}$. Determine the maximum value of Q.




**Question 69**

(a) A rocket in outer space, far from any planet, fires its engines and accelerates forward even though there is no air to push against. Explain how the rocket is able to accelerate in a vacuum.

(b) The Earth is not a perfect sphere but is slightly flattened at the poles and bulges at the equator. Explain what causes this equatorial bulge.

(c) A ball is thrown upward with an initial velocity of $33\text{ms}^{-1}$ from a point on a hillside which slopes upward uniformly at $28°$ to the horizontal. The ball is launched at $65°$ to the horizontal. Calculate the distance up the slope at which the ball strikes the hill and the time of flight. Take $\text{g} = 9.8\text{ms}^{-2}$.

**Question 70**

(a) In a perfectly inelastic collision, the colliding bodies stick together and move as one unit after impact. Explain why kinetic energy is not conserved in such a collision even though momentum is conserved, and describe how the "lost" kinetic energy is accounted for.

(b) When two vehicles collide at an intersection, crash investigators use the direction of skid marks left by the combined wreckage together with momentum conservation to reconstruct the collision. Explain why the direction of the post-collision skid marks depends on the ratio of the momenta of the two vehicles before impact.

(c) A cyclist moving at $72\text{kmh}^{-1}$ is decelerating at a rate of $5\text{ms}^{-2}$ while travelling along a circular path of radius $100\text{m}$. Calculate the magnitude and direction of the net acceleration of the cyclist.

**Question 71**

(a) A rocket in deep space ejects exhaust gases to accelerate. Explain, using Newton's third law and conservation of momentum, why the rocket accelerates even though there is nothing external for the exhaust to push against.

(b) A spacecraft orbiting the Earth fires its engines to increase speed and move to a higher orbit. Explain why the gravitational potential energy of the spacecraft increases by more than the kinetic energy it gains, and discuss where the extra energy comes from.

(c) A cannon of mass $1300\text{kg}$ fires a $72\text{kg}$ ball in a horizontal direction with a muzzle speed of $55\text{ms}^{-1}$. The cannon recoils freely. Calculate the recoil velocity of the cannon and the velocity of the ball relative to the Earth.

**Question 72**

(a) An insect is released from rest at the top of a smooth spherical bowl turned upside down. It slides over the surface. Explain why the insect must eventually lose contact with the surface, and identify the condition that determines where this happens.

(b) Between any two massive bodies, such as the Earth and the Moon, there exists a point where the net gravitational pull on an object is zero. Explain what determines the location of this neutral point and why it lies closer to the less massive body.

(c) A spacecraft is launched from the Earth to the Moon. The mass of the Earth is 81 times that of the Moon, and the distance from the centre of the Earth to the centre of the Moon is about $4.0 \times 10^5\text{km}$. Calculate the distance from the centre of the Earth at which the resultant gravitational force on the spacecraft becomes zero.

**Question 73**

(a) A fireman fighting a fire in a tall building must aim the nozzle of his hose well above the window he wants the water to reach. Explain why the water must be launched at an angle above the horizontal rather than aimed directly at the target.

(b) A rifle bullet travels 200m to its target in a fraction of a second. To the naked eye, the bullet appears to travel in a perfectly straight line. Explain why the bullet's path is actually curved even though it appears straight.

(c) A fireman stands at a horizontal distance of $38\text{m}$ from the edge of a burning building. He aims a stream of water at $60°$ above the horizontal into a first-floor window which is $20\text{m}$ above the ground. The water strikes this floor $2\text{m}$ from the outer edge. Calculate the speed at which the water leaves the nozzle of the fire hose. Take $\text{g} = 9.8\text{ms}^{-2}$.

**Question 74**

(a) A body is projected at an angle of $60°$ to the horizontal and is required to pass just above two vertical walls each of height $7\text{m}$. Explain why, for a given launch speed, there are exactly two points along the trajectory at which the projectile is at this height, and why the two walls must be placed at these two positions.

(b) In forensic ballistics, investigators fire test bullets through stacks of identical wooden boards to estimate the bullet's initial speed. Each board reduces the bullet's speed by the same fraction. Explain why a bullet that loses one-twentieth of its speed per board is stopped by far fewer than twenty boards.

(c) A body is projected from the ground at $60°$ to the horizontal with a velocity of $100\text{ms}^{-1}$. It passes just above two vertical walls each of height $7\text{m}$. Calculate the distance between the two walls. Take $\text{g} = 9.8\text{ms}^{-2}$.

**Question 75**

(a) When a system of connected bodies is in static equilibrium, the net force on the entire system is zero. Explain why the total momentum of such a system remains constant, and discuss what happens to the momentum of individual parts if one of the connections suddenly breaks.

(b) The escape velocity from the Earth's surface has a fixed value regardless of the direction in which the object is launched. Explain why the direction of launch does not affect the escape velocity.

(c) A $0.060\text{kg}$ tennis ball travelling horizontally at $35\text{ms}^{-1}$ is struck by a racket and returned in the opposite direction at $45\text{ms}^{-1}$. The contact time between the racket and the ball is $0.006\text{s}$.

(i) Calculate the impulse exerted on the ball by the racket.

(ii) Calculate the average force exerted on the ball during contact.

**Question 76**

(a) A rocket accelerates by continuously ejecting exhaust gases at high speed. Explain how the thrust force on the rocket depends on both the rate of mass ejection and the exhaust velocity, and why the acceleration of the rocket increases as fuel is consumed even if the thrust remains constant.

(b) A rocket is launched from the Earth toward the Sun. At some point along the journey, the net gravitational force on the rocket due to the Earth and the Sun is zero. Explain why this equilibrium point lies much closer to the Earth than to the Sun.

(c) A rocket of mass $20\text{kg}$ (excluding fuel) carries $180\text{kg}$ of fuel. The exhaust velocity is $1.6\text{kms}^{-1}$.

(i) Calculate the minimum rate of fuel consumption that enables the rocket to lift off from the ground.

(ii) If the actual rate of fuel consumption is $2.0\text{kgs}^{-1}$, determine the ultimate vertical speed gained by the rocket. Take $\text{g} = 9.8\text{ms}^{-2}$ and assume g remains constant during the burn.

**Question 77**

(a) A block rests on a rough inclined plane at a small angle. As the angle is gradually increased, the block remains stationary up to a certain critical angle, then begins to slide. Explain why the block can remain stationary over a range of angles rather than sliding as soon as the plane is tilted.

(b) A ball projected at $30°$ and another projected at $60°$ with the same speed from level ground land at the same horizontal distance. Explain why two different angles of projection can produce the same range.

(c) A helicopter is rising vertically at a uniform velocity of $14.7\text{ms}^{-1}$. When it is $196\text{m}$ above the ground, a ball is projected from it with a horizontal velocity of $8.5\text{ms}^{-1}$ relative to the helicopter.

(i) Calculate the time taken for the ball to reach the ground.

(ii) Determine the horizontal distance from the point directly below the helicopter (at the instant of projection) to where the ball hits the ground.

(iii) Find the speed of the ball when it hits the ground.

**Question 78**

(a) Two automobiles of equal mass approach an intersection at right angles. After colliding, they stick together and leave parallel skid marks. Explain why the direction of these skid marks reveals the ratio of the speeds of the two vehicles before impact.

(b) An elevator carrying passengers decelerates as it approaches a lower floor. Explain how the apparent weight of each passenger changes during this deceleration phase compared to when the elevator moves at constant speed.

(c) Two automobiles of equal mass approach an intersection at right angles. One is travelling east at $13\text{ms}^{-1}$ and the other is travelling north at speed v. They collide and stick together, leaving skid marks at $55°$ north of east. The speed limit for both roads is $15.6\text{ms}^{-1}$.

(i) Determine the speed v of the northward-moving vehicle.

(ii) Was the driver of the northward-moving vehicle within the speed limit? Justify your answer.

**Question 79**

(a) The gravitational field strength g near the Earth's surface decreases with altitude according to the inverse square law. Explain why, for most projectile motion problems near the surface, we treat g as constant.

(b) At sea level, the gravitational field strength is $9.81\text{Nkg}^{-1}$. At the summit of a certain mountain, the measured value is $9.70\text{Nkg}^{-1}$. Explain how the difference between these two values can be used to estimate the height of the mountain. Identify one physical reason why the actual measured value at the summit might differ from the prediction.

(c) A ball is kicked from the top of a building of height $1000\text{m}$ at a velocity of $80\text{ms}^{-1}$ at $60°$ above the horizontal. It enters a window of another building of height $400\text{m}$ across the street.

(i) Calculate the time taken for the ball to reach the window.

(ii) Determine the horizontal distance between the two buildings. Take $\text{g} = 9.8\text{ms}^{-2}$.

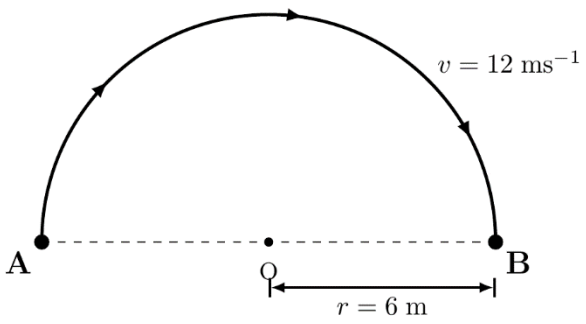
**Question 80**

(a) A satellite moves in a circular orbit around the Earth under the influence of gravity alone. Explain why the satellite's speed remains constant even though a net force acts on it continuously. Relate your explanation to the work done by gravity.

(b) A particle moves at constant speed along a semicircular path. Explain why the average velocity over the semicircular path is not equal to the speed, and discuss how this distinction is important in analysing circular motion.

(c)

(i) A particle moves along a semicircular path AB of radius 6m with a constant speed of $12\text{ms}^{-1}$. Calculate its average velocity over the semicircular path.




(ii) Show that the angular momentum of a satellite of mass $\text{M}_\text{s}$ revolving around the Earth of mass $\text{M}_\text{E}$ in a circular orbit of radius r is equal to $\text{M}_\text{s}\sqrt{\text{GM}_\text{E}\text{r}}$.

**Question 81**

(a) When a satellite is launched from the Earth's surface, it must acquire both a vertical velocity to reach the desired altitude and a horizontal velocity to maintain a stable orbit. Explain why simply launching a satellite vertically with a very high speed does not produce a circular orbit.

(b) A spaceship in a circular orbit close to the Earth's surface needs additional velocity to escape the Earth's gravitational field entirely. Explain why the required additional speed is less than the orbital speed itself.

(c) A satellite of mass $50\text{kg}$ is to be placed into a circular orbit $500\text{km}$ above the Earth's surface. After launch, the satellite reaches the orbital altitude with a vertical velocity of $2000\text{ms}^{-1}$ and zero horizontal velocity.

(i) Calculate the orbital speed required for a stable circular orbit at this height.

(ii) Determine the magnitude of the impulse needed to redirect the satellite from its vertical motion into the correct circular orbit. Take $\text{M}_\text{E} = 5.97 \times 10^{24}\text{kg}$, $\text{R}_\text{E} = 6.4 \times 10^{6}\text{m}$, $\text{G} = 6.67 \times 10^{-11}\text{Nm}^2\text{kg}^{-2}$.

**Question 82**

(a) Two projectiles are launched simultaneously from different positions in the same vertical plane. Explain why the condition for the two projectiles to collide in mid-air is independent of the value of g.

(b) A diver jumping into a swimming pool from a $10\text{m}$ platform experiences a much larger average force from the water than a diver jumping from the $3\text{m}$ platform. Explain this difference using the concepts of impulse and momentum.

(c) Two objects A and B are projected simultaneously in the same vertical plane. Object A is projected from a height of $2\text{m}$ above the ground at $30°$ above the horizontal. Object B is projected from the ground directly below A with a velocity of $30\text{ms}^{-1}$ at $60°$ above the horizontal. If the two objects collide in mid-air, determine the time taken until they collide.

**Question 83**

(a) A marksman aims a rifle directly at a small target on the same horizontal level some distance away. Explain why the bullet will always strike below the target, and discuss how the marksman should compensate for this effect.

(b) A hunter aims a rifle directly at a monkey hanging from a tree branch. At the exact instant the gun is fired, the monkey releases the branch and begins to fall freely. Explain whether the bullet will hit the monkey.

(c) A marksman fires a rifle at a small target $200\text{m}$ away on the same horizontal level. The muzzle velocity is $1000\text{ms}^{-1}$.

(i) Calculate how far below the target the bullet strikes if the rifle is aimed directly at the target.

(ii) How high above the target the marksman must aim to hit it. Take $\text{g} = 9.8\text{ms}^{-2}$.

**Question 84**

(a) A person standing at the equator experiences a slightly lower apparent weight than a person of the same mass standing at the North Pole. Explain this difference using the concept of centripetal acceleration.

(b) A spaceship in a low circular orbit just above the Earth's atmosphere has a well-defined orbital speed. Explain why a speed increase of about $41\%$ above the orbital speed allows the spaceship to escape the Earth entirely.

(c)

(i) Calculate the length of the day if the Earth were to rotate fast enough that a person standing at the equator would experience zero apparent weight. Take $\text{R}_\text{E} = 6.4 \times 10^6\text{m}$ and $\text{g} = 9.8\text{ms}^{-2}$.

(ii) A spaceship is in a circular orbit close to the Earth's surface with orbital speed $7.9\text{kms}^{-1}$. Calculate the additional speed the spaceship must acquire to escape the Earth's gravitational field.

**Question 85**

(a) The gravitational potential energy of a body at the surface of a planet is negative when measured relative to infinity. Explain what this negative value means physically, and why a body must be given a minimum kinetic energy to escape the planet's gravitational pull entirely.

(b) A rocket launched vertically from a planet's surface loses a fraction of its kinetic energy to atmospheric drag during ascent. Explain how energy conservation determines the maximum height the rocket reaches.

(c) Take $\text{M}_\text{E} = 6 \times 10^{24}\text{kg}$, $\text{R}_\text{E} = 6.4 \times 10^6\text{m}$, and $\text{G} = 6.67 \times 10^{-11}\text{Nm}^2\text{kg}^{-2}$.

(i) Calculate the work done to move a $10\text{kg}$ body from the Earth's surface to infinity.

(ii) If the same body falls from rest at infinity toward the Earth, calculate its speed when it strikes the Earth's surface.

(iii) A rocket is fired vertically from the surface of Mars with a speed of $2\text{kms}^{-1}$. If $20\%$ of its initial kinetic energy is lost to atmospheric resistance, calculate how far the rocket rises above the surface. Take $g_{\text{Mars}} = 3.7\text{ms}^{-2}$ and assume it remains constant.

**Question 86**

(a) When an object moving in a circular path is suddenly released, it travels in a straight line tangent to the circle at the point of release. Explain why the object does not fly radially outward.

(b) The driver of a truck travelling at speed v suddenly notices a wall at a distance r directly ahead. The driver has two options: brake to stop, or maintain speed and swerve in a circular arc of radius r to avoid the wall. In both cases the maximum friction force is the same. Explain which option gives the driver a better chance of avoiding a collision.

(c) A stone is whirled in a horizontal circle of radius $1.5\text{m}$ by means of a string, at a height of $2\text{m}$ above the ground. The string breaks and the stone flies off horizontally, striking the ground at a point $10\text{m}$ from the point directly below the break. Calculate the centripetal acceleration of the stone just before the string broke. Take $g = 9.8\text{ms}^{-2}$.

**Question 87**

(a) In a binary star system, two stars orbit their common centre of mass under their mutual gravitational attraction. Explain why each star experiences a centripetal acceleration directed toward the centre of mass, and discuss how the orbital radii of the two stars depend on their masses.

(b) If one star in a binary system is much more massive than the other, explain how the position of the centre of mass shifts and how this changes the orbital motion of each star. Relate your answer to the Earth and Sun system.

(c)

(i) Calculate the height above the Earth's surface at which the acceleration due to gravity becomes half its value at the surface. Take $R_E = 6400\text{km}$.

(ii) A satellite is placed in a circular orbit around Jupiter so that it always remains above the same point on Jupiter's equator. Jupiter rotates once every $9.84$ hours, its mass is $1.9 \times 10^{27}\text{kg}$, and its mean radius is $6.99 \times 10^{7}\text{m}$. Calculate the altitude of the satellite above Jupiter's surface. Take $G = 6.67 \times 10^{-11}\text{Nm}^2\text{kg}^{-2}$.

**Question 88**

(a) When a stationary object explodes into several fragments, the total momentum of all fragments must be zero. Explain why individual fragments can nevertheless have large momenta, and discuss how momentum conservation constrains the directions and speeds of the fragments.

(b) A person standing inside a lift holds a briefcase at waist height and then releases it. The briefcase does not fall to the floor but instead remains floating at the same height relative to the person. Explain what this observation reveals about the motion of the lift.

(c) A bomb of mass M is suspended at rest from a cable. The cable is cut and the bomb simultaneously explodes into three pieces. The first piece (mass $M/2$) moves horizontally in the positive x-direction at speed $V_0$. The second piece (mass $M/3$) moves at speed $2V_0$ at an angle of $37°$ above the positive x-axis. Determine the speed and direction of the third piece (mass $M/6$).

**Question 89**

(a) A ball thrown at a certain angle reaches a maximum height during its flight. Explain how the launch angle and initial velocity together determine the maximum height, and discuss why there is a maximum horizontal range for a given speed regardless of the angle chosen.

(b) A helicopter hovers by using its rotor blades to push air downward. Explain, using Newton's third law and the concept of momentum change per unit time, how the downward thrust of air supports the weight of the helicopter.

(c)

(ii) A ball is thrown at $40\text{ms}^{-1}$ inside a hall whose ceiling is $25\text{m}$ high. Determine the maximum horizontal distance the ball can travel without hitting the ceiling. Take $g = 9.8\text{ms}^{-2}$.

(iii) A helicopter hovers in still air. Its rotor blades sweep a circle of radius $2\text{m}$ and push air vertically downward at $8\text{ms}^{-1}$. Taking the density of air as $1.3\text{kgm}^{-3}$, estimate the mass of the helicopter. Take $g = 9.8\text{ms}^{-2}$.

**Question 90**

(a) When a person pulls an object along a rough horizontal surface by a rope inclined above the horizontal, the normal reaction force is less than the weight of the object. Explain why this occurs and discuss how the reduced normal reaction affects the friction force.

(b) Roads and railway lines are banked on curves. Explain how banking allows a vehicle to negotiate a curve safely even without friction, and describe the role of the normal reaction force in providing the centripetal force.

(c)

(i) Ambitious is pulling an object of mass $20\text{kg}$ across a horizontal plane at constant velocity. The force she exerts makes an angle of $30°$ with the horizontal. If the coefficient of sliding friction is $0.57$, calculate the magnitude of the force Ambitious exerts. Take $g = 9.8\text{ms}^{-2}$.

(ii) A highway designed for an average speed of $72\text{kmh}^{-1}$ has a curve of radius $50\text{m}$. Calculate the minimum banking angle so that cars at this speed do not depend on friction. Take $g = 9.8\text{ms}^{-2}$.

**Question 91**

(a) Two projectiles are launched from the same point with the same speed but at different angles above the horizontal. Explain why, in general, their horizontal ranges differ, and discuss the special relationship between the two angles that produce the same range.

(b) Gravel is dropped vertically onto a horizontal conveyor belt that moves at constant speed. Explain, using the concept of impulse, why a horizontal force must act on the gravel to accelerate it to the belt's speed.

(c)

(i) Particle P is projected at angle $\alpha$ with velocity $u_P$ and particle Q is projected at angle $\beta$ with velocity $u_Q$ from the same point. The vertical components of their initial velocities are equal. Find the ratio of their horizontal ranges in terms of $\alpha$ and $\beta$.

(ii) Gravel is dropped vertically onto a horizontal conveyor belt at a rate of $75\text{kgs}^{-1}$. The belt moves at a constant speed of $2\text{ms}^{-1}$. Calculate the force needed to keep the belt moving at constant speed, and the rate at which kinetic energy is given to the gravel.

**Question 92**

(a) A satellite moves in an elliptical orbit around the Earth. Explain why the kinetic energy of the satellite changes as it moves around its orbit, relating your answer to the work done by gravity.

(b) The Moon has no significant atmosphere while the Earth retains a thick one. Explain the reason for this difference, using the concepts of escape velocity and surface gravity.

(c) Two stars, each of one solar mass ($2 \times 10^{30}\text{kg}$), are approaching each other for a head-on collision. When they are $10^9\text{km}$ apart, their speeds are negligible. Each star has a radius of $10^4\text{km}$. Calculate the speed with which they collide. Take $G = 6.67 \times 10^{-11}\text{Nm}^2\text{kg}^{-2}$.

**Question 93**

(a) A car moves on a circular track while its speed is increasing. Explain why the resultant acceleration is not directed toward the centre of the circle, and discuss what determines the angle between the resultant acceleration and the velocity direction.

(b)

(i) A projectile is thrown from the base of an inclined plane that makes $30°$ with the horizontal. Calculate the angle of projection, measured from the horizontal, that gives the maximum range along the surface of the incline.

(ii) A car moves at $30\text{ms}^{-1}$ on a circular track of radius $500\text{m}$ while its speed increases at $2\text{ms}^{-2}$. Calculate the magnitude of the resultant acceleration and the angle it makes with the velocity direction.

**Question 94**

(a) A projectile is fired at $45°$ to achieve maximum horizontal range. At the highest point of its trajectory, the velocity is entirely horizontal. Explain why the kinetic energy at the highest point is not zero, and show that it is exactly half the kinetic energy at launch.

(b) A conveyor belt at a shopping mall carries passengers at constant speed regardless of how many people are standing on it. Explain what physical condition the driving mechanism must satisfy for the belt to maintain constant speed under varying loads.

(c) A projectile is launched with a kinetic energy of $500\text{J}$ at the angle that gives maximum horizontal range. Calculate the kinetic energy of the projectile at the highest point of its trajectory.

**Question 95**

(a) An object released from rest slides down a smooth inclined plane. Explain why the speed at the bottom depends only on the vertical height and not on the angle or length of the slope.

(b) A boy throws a stone vertically upward and catches it when it returns. A student claims this is an example of projectile motion. Evaluate this claim, clearly stating the conditions that define projectile motion.

(c) A ball is thrown from a point $2\text{m}$ above the ground, at a horizontal distance of $3\text{m}$ from a smooth vertical wall. The ball is launched at $20\text{ms}^{-1}$ at $30°$ above the horizontal toward the wall. After an elastic collision with the wall, the ball bounces back. Determine the horizontal distance from the wall to the point where the ball hits the ground. Take $\text{g} = 9.8\text{ms}^{-2}$.

**Question 96**

(a) A magician pulls a tablecloth out from under a set of plates without disturbing them. The trick works best when the cloth is pulled very quickly. Explain why a fast pull succeeds while a slow pull drags the plates along.

(b) A rocket accelerates by ejecting exhaust gases at high speed. Explain why the acceleration of the rocket increases over time even if the thrust force remains constant.

(c) A rocket of initial total mass $100\text{kg}$ launches vertically and ejects mass at a constant rate of $20\text{kgs}^{-1}$. The thrust force is constant at $1500\text{N}$. Calculate the acceleration at launch and write an expression for the acceleration at time $\text{t}$. Take $\text{g} = 9.8\text{ms}^{-2}$.

**Question 97**

(a) When a spacecraft uses a planet's gravity to change its speed and direction (a gravitational slingshot), the spacecraft can gain a large amount of kinetic energy even though the planet's speed barely changes. Explain why, using conservation of momentum.

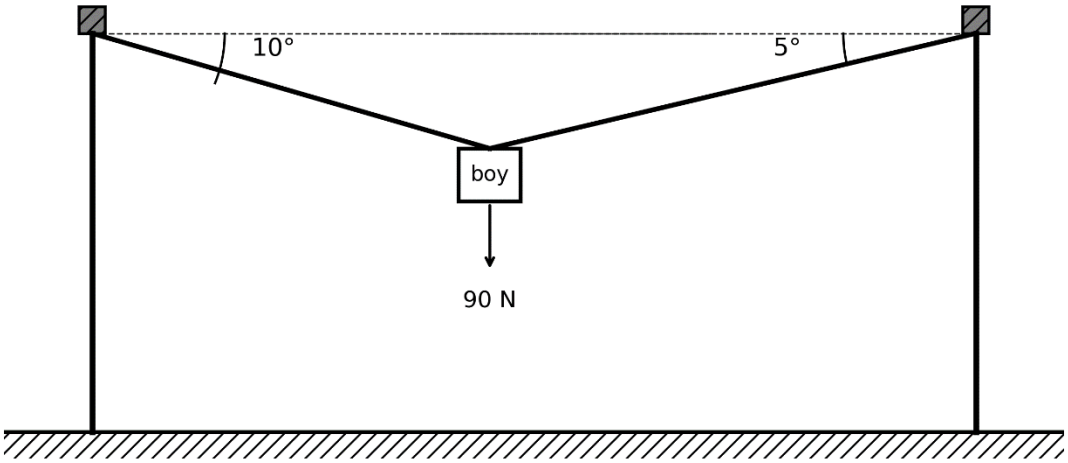
(b) A satellite in a circular orbit experiences no change in speed, yet a continuous force acts on it. Explain how this is consistent with the work-energy theorem. Contrast this with the case of a satellite in an elliptical orbit where the speed does change.

(c) Two identical spacecraft, each of mass $500\text{kg}$, are in the same circular orbit at height $400\text{km}$ above the Earth. They collide head-on and stick together. Calculate the orbital speed before the collision. Determine the speed of the combined wreckage immediately after the collision and discuss what happens to the wreckage. Take $M_E = 5.97 \times 10^{24}\text{kg}$, $R_E = 6.4 \times 10^{6}\text{m}$, $G = 6.67 \times 10^{-11}\text{Nm}^2\text{kg}^{-2}$.

**Question 98**

(a) Two important factors must be considered when placing a satellite into geostationary orbit. Identify these two factors and explain why each is necessary for the satellite to remain permanently above the same point on the Earth's equator.

(b) A rope is stretched between two vertical poles. A boy weighing $90\text{N}$ hangs from the rope, causing it to sag. The left segment makes an angle of $10°$ below the horizontal and the right segment makes $5°$ below the horizontal. Calculate the tension in each segment of the rope.




**Question 99**

(a) A stone thrown horizontally from the top of a cliff and a stone dropped vertically from the same height reach the ground at the same time. Explain what determines the horizontal distance the thrown stone travels before hitting the ground.

(b) In a football match, a player some distance from the kicker starts running the instant the ball is kicked, hoping to reach the ball just as it lands. Explain what physical quantities the player must estimate to decide at what speed to run.

(c) In a football game, a ball is kicked from the ground at $19.5\text{ms}^{-1}$ at $45°$ above the horizontal. A player standing $55\text{m}$ away in the direction of the kick starts running toward the ball at the instant it is kicked. Calculate the minimum average speed the player must maintain to reach the ball just before it hits the ground. Take $g = 9.8\text{ms}^{-2}$.

**Question 100**

(a) A person standing on a moving platform throws a ball. Explain how the velocity of the platform affects the ball's trajectory as seen by a stationary observer on the ground.

(b) A gun is kept on a straight horizontal road and used to fire a shell at a car travelling along the same road at a uniform speed. At the instant of firing, the car is some distance from the gun. Explain what conditions must be satisfied for the shell to hit the moving car.

(c) A gun fires a shell at $45°$ above the horizontal at a car travelling along the same straight road at $72\text{kmh}^{-1}$. At the instant of firing, the car is $500\text{m}$ from the gun. Calculate the minimum muzzle speed of the shell required to hit the car. Take $g = 9.8\text{ms}^{-2}$.

**Question 101**

(a) Two objects connected by a string over a frictionless pulley rest on the two slopes of a rough double wedge. Explain why the coefficient of friction affects the range of conditions for which equilibrium is possible.

(b) A ball attached to a string moves in a horizontal circle (conical pendulum). Explain why the string cannot be perfectly horizontal during this motion.

(c) Two equal masses, each of mass $m$, are connected by a light string passing over a frictionless pulley at the peak of a rough double wedge. One slope makes $60°$ with the horizontal and the other makes $30°$. The coefficient of friction $\mu$ is the same on both surfaces. If the masses move at constant velocity, find $\mu$.

**Question 102**

(a) A block rests on an inclined plane of angle $\theta$ and is held in place by a horizontal force. Explain why the normal reaction force in this case is not simply $mg\cos\theta$.

(b) A satellite in a low circular orbit has a well-defined orbital speed. If the satellite fires a short burst of its engines to increase its speed, it moves into a higher orbit. Explain why the satellite ends up moving slower in the higher orbit despite having been given extra energy.

(c) A block of mass $m$ is held at rest on a rough inclined plane of angle $\theta$ by a horizontal force $F$. The coefficient of static friction is $\mu_s$. Show that the maximum horizontal force that can be applied before the block begins to slide up the plane is:

$$F_{max} = \frac{mg(\sin\theta + \mu_s\cos\theta)}{\cos\theta - \mu_s\sin\theta}$$

**Question 103**

(a) In a conical pendulum, the bob moves in a horizontal circle while the string traces out a cone. Explain why the vertical component of the string tension must equal the weight of the bob.

(b) A loaded truck negotiating a banked curve at too high a speed tends to slide outward, while the same truck moving too slowly tends to slide inward. Explain both observations, stating direction of friction in each case.

(c) A conical pendulum consists of a $0.2\text{kg}$ bob attached to a $1.5\text{m}$ string. The string makes an angle of $25°$ with the vertical. Calculate the tension in the string, the speed of the bob, and the period of revolution. Take $g = 9.8\text{ms}^{-2}$.

**Question 104**

(a) A firework rocket explodes at the highest point of its trajectory, splitting into several fragments. Explain why the centre of mass of all the fragments continues to follow the original parabolic path even after the explosion.

(b) A cannon mounted on a wheeled carriage fires a shell horizontally. Explain, using conservation of momentum, why the carriage recoils backward, and how the recoil speed depends on the ratio of the shell's mass to the carriage's mass.

(c) A projectile of mass $3\text{kg}$ is launched from the ground at $50\text{ms}^{-1}$ at $60°$ above the horizontal. At the highest point of its trajectory, the projectile splits into two fragments. One fragment, of mass $1\text{kg}$, falls vertically downward with zero initial speed relative to the ground. Calculate the velocity of the remaining $2\text{kg}$

fragment immediately after the split, and the horizontal distance from the launch point to where it lands. Take $g = 9.8 ms^{-2}$.

**Question 105**

(a) When a stone is thrown at a window, the glass shatters into many pieces. When a high-speed bullet hits the same glass, it passes through leaving a clean round hole without shattering the rest of the pane. Explain this difference using the concepts of impulse and contact time.

(b) Imagine that gravity were to suddenly vanish everywhere. Describe what would immediately happen to objects resting on the Earth's surface, the Moon in its orbit, and the Earth's atmosphere. Explain each prediction using the relevant law of motion.

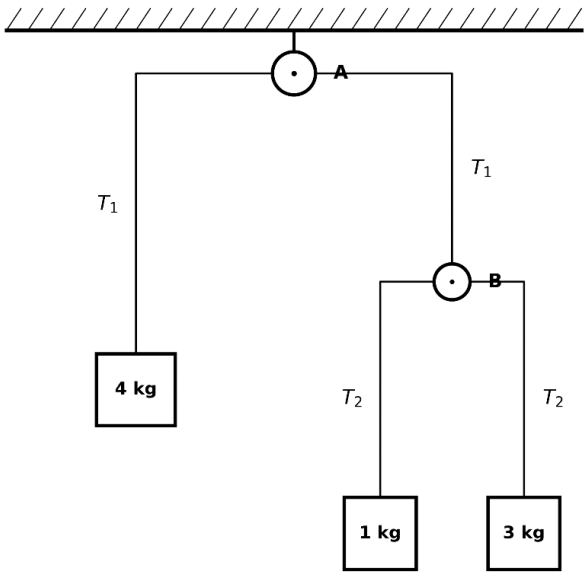
(c) A fireman stands $20m$ from the base of a burning building. The water leaves the hose at a height of $1m$ above the ground at $55°$ to the horizontal. The water just enters an open window $18m$ above the ground. Calculate the speed of the water as it leaves the hose. Take $g = 9.8 ms^{-2}$.

**Question 106**

(a) Explain how the motion of a body whirled in a horizontal circle differs from the motion of a body whirled in a vertical circle. In your answer, discuss the variation of speed and string tension in each case.

(b) When two objects collide and stick together, the final speed of the combined mass is less than the initial speed of the faster object. Explain why this is always the case.

(c) The figure shows a frictionless pulley A fixed to the ceiling. On one side hangs a 4kg mass. On the other side hangs a small frictionless pulley B from which a 1kg mass and a 3kg mass are suspended. All strings are light and inextensible. Find the acceleration of each mass and the tensions in the strings. Take $g = 9.8 ms^{-2}$.

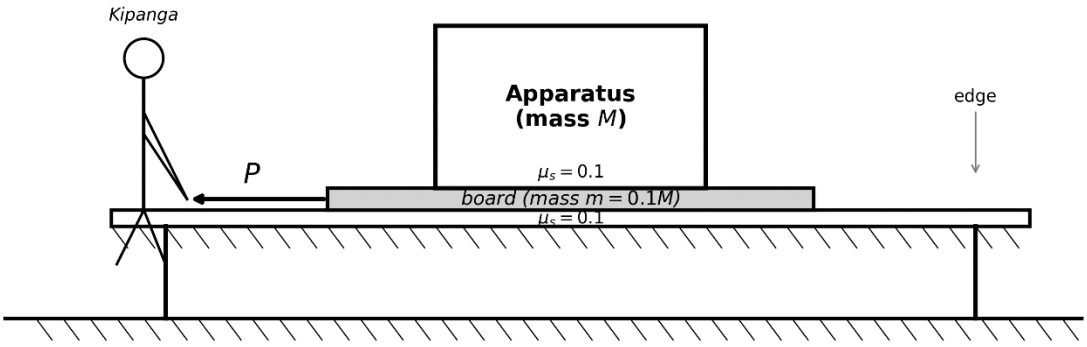



**Question 107**

(a) A satellite that gradually loses energy due to atmospheric drag speeds up rather than slowing down. Explain why this seemingly paradoxical result follows from the relationship between orbital speed and radius.

(b) In Mr. Akilikubwa's laboratory, a heavy measuring apparatus of mass M sits on a thin wooden board of mass $m = 0.1M$ that rests on a long laboratory bench. The coefficient of static friction between all contact

surfaces is $\mu_s = 0.1$. Kipanga wants to remove the board without lifting the apparatus. Mr. Akilikubwa tells him he must pull the board quickly, not slowly. Explain why the friction force between the board and the bench is greater than the friction force between the apparatus and the board, even though the coefficient of friction is the same at both surfaces.




(c) Using the arrangement described in (b):

(i) Draw a free body diagram for the board, clearly showing all forces acting on it, and a separate free body diagram for the apparatus.

(ii) Calculate the minimum value of P required to cause any motion in the system. Express your answer in terms of M and g.

(iii) Calculate the force P required to slide the board out from under the apparatus while the apparatus remains stationary. Express your answer in terms of M and g.

(iv) Kipanga pulls the board out quickly and the apparatus is left at rest on the bench, 5cm from the edge. The coefficient of kinetic friction between the apparatus and the bench is also 0.1. Determine whether the apparatus slides off the edge of the bench.

# SOLUTIONS

**Question 1**

(a) Each rope exerts a force in its own direction. Since the two forces act at $90°$ to each other, their vector sum (the resultant) lies between north and east at an angle determined by the relative magnitudes of the two forces. The crate accelerates in the direction of this resultant, not in the direction of either individual force.

(b) The description is incomplete. While kinetic energy is indeed partially transferred to the goalkeeper, kinetic energy is not conserved in this collision (it is inelastic, since the ball and goalkeeper move together after the catch). The quantity that is conserved is **momentum**, not kinetic energy. Some kinetic energy is converted to thermal energy, sound, and deformation of the ball. Conservation of momentum is the correct principle for analysing such interactions.

(c) Weight: $W = 200\text{kg} \times 9.8\text{m/s}^2 = 1960\text{N}$

Let $T_1$ be the tension at $40°$ and $T_2$ at $60°$.

Vertically: $T_1 \sin 40° + T_2 \sin 60° = 1960\text{N}$

Horizontally: $T_1 \cos 40° = T_2 \cos 60°$

From horizontal: $T_1 = T_2 \times \frac{\cos 60°}{\cos 40°} = 0.6527 T_2$

Substituting: $0.6527 T_2 \times 0.6428 + T_2 \times 0.8660 = 1960\text{N}$

$0.4196 T_2 + 0.8660 T_2 = 1960\text{N}$

$1.2856 T_2 = 1960\text{N}$

$\mathbf{T_2 = 1524.6N}$

$\mathbf{T_1} = 0.6527 \times 1524.6\text{N} = \mathbf{995.2N}$

**Question 2**

(a) At the top of the loop, both the rider's weight and the normal reaction from the track point downward (toward the centre of the loop). Together they provide the centripetal force needed for circular motion. As long as the rider's speed is high enough that the required centripetal force ($\frac{mv^2}{r}$) exceeds or equals the weight (mg), the track pushes on the rider and the rider stays in contact.

(b) By Newton's third law, the force on the car equals the force on the lorry during the collision, and both forces act for the same duration. Therefore both vehicles experience the same impulse ($F\Delta t$), and hence the same magnitude of change in momentum. However, because the car has a much smaller mass than the lorry, the same change in momentum produces a much larger change in velocity for the car $\left(\Delta v = \frac{\Delta p}{m}\right)$. This sudden, large change in velocity is what causes severe deceleration of the occupants inside the car, leading to greater injury. The lorry, having a much larger mass, undergoes a much smaller change in velocity for the same change in momentum, so its occupants are less affected.

(c) Taking the initial direction as positive:

$u = +40\text{m/s}$, $v = -55\text{m/s}$

$$\Delta p = m(v - u) = 0.06\text{kg} \times (-55 - 40)\text{m/s} = 0.06\text{kg} \times (-95\text{m/s}) = -5.7\text{kgm/s}$$

$$F = \frac{\Delta p}{\Delta t} = \frac{-5.7\text{kgm/s}}{0.008\text{s}} = -712.5\text{N}$$

The average force is 712.5N in the direction of the return.

**Question 3**

(a) At the moment of release, the javelin has an upward component of velocity. This upward velocity causes it to rise. However, gravitational acceleration acts downward at all times, progressively reducing the upward velocity. Eventually the upward velocity reaches zero (at the peak), after which gravity accelerates the javelin downward and it descends.

(b) Gravitational acceleration g varies with altitude: g is slightly smaller at higher altitudes because the object is farther from the Earth's centre. At Iringa (1600m), g is slightly less than at Bagamoyo (sea level). Since the scale measures gravitational force and converts it to mass using its stored value of g, the same mass produces a slightly greater force at sea level, giving a slightly higher reading in Bagamoyo.

(c) For equilibrium, $F_1 + F_2 + F_3 = 0$, so $F_3 = -(F_1 + F_2)$.

Resultant of $F_1$ and $F_2$:

$$R_x = 10 + 15\cos 120° = 10\text{N} + 15 \times (-0.5)\text{N} = 10\text{N} - 7.5\text{N} = 2.5\text{N}$$

$$R_y = 0 + 15\sin 120° = 12.99\text{N}$$

$$R = \sqrt{(2.5\text{N})^2 + (12.99\text{N})^2} = 13.2\text{N}$$

$F_3 = 13.2\text{N}$, directed opposite to R.

Direction of R: $\alpha = \tan^{-1}\left(\frac{12.99}{2.5}\right) = 79.1°$ above the positive x-axis.

Therefore $F_3$ acts at $79.1° + 180° = 259.1°$ from the positive x-axis (or 79.1° below the negative x-axis).

**Question 4**

(a) The satellite is very much affected by gravity. In fact, gravity is the only force acting on it, and it provides the centripetal force that keeps the satellite in orbit. The satellite is in continuous free fall toward the Earth, but because it has a large horizontal velocity, it falls "around" the Earth rather than into it. The curved surface of the Earth drops away as fast as the satellite falls.

(b) The kinetic energy of the vehicle before impact is converted to thermal energy through friction during skidding: $\frac{1}{2}mv^2 = \mu mgs$, where s is the skid distance. Since mass cancels, $v = \sqrt{2\mu gs}$. By measuring the skid length and knowing the friction coefficient of the road, the initial speed can be calculated. Longer skid marks indicate higher speed before braking.

(c) Using conservation of mechanical energy:

$$\frac{1}{2}mu^2 + mgh = \frac{1}{2}mv^2$$

$$v^2 = u^2 + 2gh = (20\text{m/s})^2 + 2 \times 9.8\text{m/s}^2 \times 15\text{m} = 694\text{m}^2/\text{s}^2$$

$$v = \sqrt{694\text{m}^2/\text{s}^2} = \mathbf{26.3m/s}$$

**Question 5**

(a) The coin requires centripetal force to move in a circle, which is provided by static friction between the coin and the turntable surface. The required centripetal force is $m\omega^2 r$, which increases with angular velocity $\omega$. The maximum available friction is $\mu_s mg$, which is fixed. When the turntable spins fast enough that the centripetal force demanded exceeds the maximum friction available, friction can no longer hold the coin and it slides off.

(b) The range is $R = \frac{u^2 \sin 2\theta}{g}$, which depends on the angle $\theta$ through the factor $\sin 2\theta$. A steep angle gives great height but short horizontal travel; a shallow angle gives fast horizontal motion but little time of flight. The maximum range occurs when $\sin 2\theta = 1$, which gives $2\theta = 90°$, hence $\theta = 45°$. In this angle both vertical motion and horizontal motion are well optimized.

(c) Taking the direction of the 0.50kg object as positive:

$$0.50\text{kg} \times 4.0\text{m/s} + 0.30\text{kg} \times (-3.0\text{m/s}) = 0.50\text{kg} \times 1.0\text{m/s} + 0.30\text{kg} \times v$$

$$2.0\text{kgm/s} - 0.9\text{kgm/s} = 0.5\text{kgm/s} + 0.30\text{kg} \times v$$

$$1.1\text{kgm/s} = 0.5\text{kgm/s} + 0.30v$$

$v = \frac{0.6\text{kgm/s}}{0.30\text{kg}} = 2.0\text{m/s}$ (in the original direction of the 0.50kg object)

KE before $= \frac{1}{2} \times 0.50\text{kg} \times (4.0\text{m/s})^2 + \frac{1}{2} \times 0.30\text{kg} \times (3.0\text{m/s})^2 = 4.0\text{J} + 1.35\text{J} = 5.35\text{J}$

KE after $= \frac{1}{2} \times 0.50\text{kg} \times (1.0\text{m/s})^2 + \frac{1}{2} \times 0.30\text{kg} \times (2.0\text{m/s})^2 = 0.25\text{J} + 0.60\text{J} = 0.85\text{J}$

Since $5.35\text{J} \neq 0.85\text{J}$, kinetic energy is not conserved. The collision is inelastic.

**Question 6**

(a) While the string is intact, the tension provides a centripetal force directed toward the centre, continuously pulling the stone inward to maintain circular motion. The stone's velocity at any instant is tangent to the circle. When the string breaks, the centripetal force vanishes. By Newton's first law, the stone continues moving in the direction of its velocity at the instant of breaking, which is tangential. There is no outward force; the apparent "outward tendency" during circular motion is simply the stone's inertia resisting the inward change of direction.

(b) The gravitational constant G is extremely small ($6.67 \times 10^{-11}\text{Nm}^2\text{kg}^{-2}$). For two books, the product of masses is too small to produce significant amount of the gravitational force. Gravity becomes significant only when at least one mass is enormous (like a planet), because the large mass compensates for the tiny value of G.

(c) $u_y = 100\text{m/s} \times \sin 53° = 79.86\text{m/s}$

At height 100m: $100 = 79.86t - \frac{1}{2} \times 9.8 \times t^2$

$4.9t^2 - 79.86t + 100 = 0$

Solve the quadratic equation gives:

$$t=1.367\text{s or } t=14.93$$

The smaller time corresponds to when the projectile reaches 100 m on its way up (first time), while the larger time corresponds to when it passes 100 m again on its way down (second time).

First time: $t = 1.367\text{s}$

Horizontal distance: $x = u_x t = 100\text{m/s} \times \cos 53° \times 1.367\text{s} = 82.3\text{m}$

**Alternative solution:** You may use trajectory equation with $s_y = 100\text{m}$, where the smaller value of $s_x$ corresponds to the required horizontal distance.

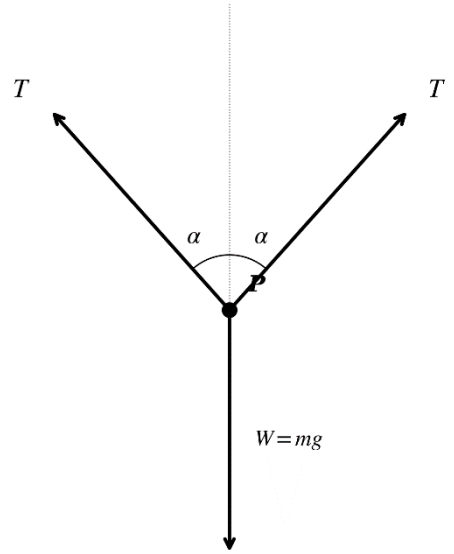
**Question 7**

(a) At the top of the bridge, the car follows a curved (circular) path. The centripetal acceleration is directed downward. Part of the driver's weight provides this centripetal acceleration, so the normal reaction from the seat (which the driver feels as

"weight") is reduced: $R = mg - \frac{mv^2}{r}$. Since the sensation of weight comes from the contact force, not from gravity itself, the reduced contact force makes the driver feel lighter.

(b) The puck and the football both obey Newton's first law: they continue moving until a net force stops them. On ice, friction is extremely small, so the decelerating force on the puck is very small and it takes a long time (and distance) to stop. On grass, friction is much larger, producing a larger decelerating force. By $v^2 = u^2 - 2as$, a larger deceleration a results in a shorter stopping distance s for the same initial speed.

(c) Consider the following Free Body Diagram:




From the geometry of the provided diagram in the question: the two attachment points are 2.0m apart, each rope is 1.5m. The junction hangs below the midpoint. Half-width = 1.0m. Each rope: $\cos\alpha = \frac{1.0}{1.5} = 0.6667$, so $\alpha = 48.2°$ from vertical.

By symmetry, both tensions are equal: $T_1 = T_2 = T$.

Vertically: $2T\cos\alpha = mg$

$$T = \frac{mg}{\cos\alpha} = \frac{15kg \times 9.8m/s^2}{0.6667} = \mathbf{110.3N}$$

**Question 8**

(a) The gravitational force on the planet is directed radially toward the Sun. Because this force has no component perpendicular to the radius, it produces zero torque about the Sun. With zero torque, angular momentum $L = mvr$ is conserved. At perihelion, the radius r is smallest, so the speed v must be largest to keep mvr constant. At aphelion, r is largest, so v is smallest.

(b) A satellite in orbit is in free fall. Gravity provides the centripetal force needed for circular motion, and no work is done (the force is perpendicular to the velocity). The satellite maintains its speed and orbit indefinitely without fuel. An aeroplane, by contrast, is not in free fall. It relies on lift from its wings to support its weight, and this lift is produced only when the aircraft moves through air. Air resistance continuously decelerates the aircraft, so the engines must constantly provide thrust to maintain speed and therefore lift.

(c) For elastic head-on collision:

$$v_1 = \frac{(m_1 - m_2)u_1}{m_1 + m_2} = \frac{(3.0-1.0)kg \times 8.0m/s}{(3.0+1.0)kg} = \frac{2.0kg \times 8.0m/s}{4.0kg} = 4.0m/s \text{ (to the right)}$$

$$v_2 = \frac{2m_1u_1}{m_1 + m_2} = \frac{2 \times 3.0kg \times 8.0m/s}{4.0kg} = \frac{48.0kgm/s}{4.0kg} = 12.0m/s \text{ (to the right)}$$

**Question 9**

(a) Escape velocity is derived from energy conservation: $\frac{1}{2}mv_E^2 = \frac{GMm}{r}$. The escaping mass m appears on both sides and cancels, giving $v_E = \sqrt{\frac{2GM}{r}}$. This depends only on M (mass of the planet) and r (radius of the planet). A heavier escaping object needs more energy, but it also has proportionally more kinetic energy at the same speed. These two effects balance exactly, leaving the required speed independent of mass.

(b) At the outer edge, the radius r of the circular path is larger. The centripetal force needed to maintain circular motion is $F = m\omega^2 r$, which increases with r (since $\omega$ is the same everywhere on the roundabout). Therefore, the child must hold on with a greater force at the outer edge. If the child's grip cannot provide this increased force, the child will slide off.

(c) $u_y = 35\text{m/s} \times \sin 50° = 35 \times 0.766 = 26.81\text{m/s}$

At height 20m: $20 = 26.81t - \frac{1}{2} \times 9.8t^2$

$4.9t^2 - 26.81t + 20 = 0$

$t_1 = 0.891\text{s}, t_2 = 4.58\text{s}$

Time above 20m: $\Delta t = 4.58\text{s} - 0.891\text{s} = 3.69\text{s}$

**Question 10**

(a) For the ring to remain stationary (in equilibrium), the vector sum of all three forces must be zero. This means the third force must be equal in magnitude and opposite in direction to the resultant of the first two forces. This third force is called the **equilibrant**. It can be found by resolving the first two forces into components, finding the resultant, and then reversing its direction.

(b) The water has mass and is therefore subject to gravitational acceleration. After leaving the nozzle, the water follows a parabolic (projectile) path because gravity pulls it downward while it continues to move forward. The laser beam consists of massless photons that travel at the speed of light. Over the short distances involved, the gravitational deflection of light is completely negligible, so the beam appears perfectly straight.

(c) The maximum centripetal force equals the maximum static friction:

$$\frac{mv_{max}^2}{r} = \mu_s mg$$

Mass cancels:

$$v_{max} = \sqrt{\mu_s gr} = \sqrt{0.45 \times 9.8\text{m/s}^2 \times 50\text{m}} = 14.8\text{m/s}$$

**Question 11**

(a) Orbital speed is $v = \sqrt{\frac{GM}{R}}$. A larger R gives a smaller v because gravity is weaker at greater distances and less speed is needed to maintain the orbit. However, total energy is $E = -\frac{GMm}{2R}$, and a larger R makes E less negative (higher total energy). The apparent contradiction dissolves when one realises that the higher satellite has less kinetic energy but much more (less negative) potential energy, and the total energy is indeed greater.

(b) The follow-through increases the time of contact between bat and ball. By the impulse-momentum theorem ($F\Delta t = \Delta p$), a longer contact time means the bat transfers a greater impulse to the ball for the same average force. A greater impulse produces a greater change in the ball's momentum and hence a higher final speed.

(c) Weight: $W = 5.0\text{kg} \times 9.8\text{m/s}^2 = 49.0\text{N}$

Let $T_1$ be at 30° from ceiling (i.e., 60° from horizontal) and $T_2$ at 45° from ceiling (i.e., 45° from horizontal).

Horizontally: $T_1 \sin 30° = T_2 \sin 45°$

$$T_1 \times 0.5 = T_2 \times 0.7071$$

$$T_1 = 1.414T_2$$

Vertically: $T_1 \cos 30° + T_2 \cos 45° = 49.0\text{N}$

$$1.414T_2 \times 0.866 + T_2 \times 0.7071 = 49.0\text{N}$$

$$1.224T_2 + 0.7071T_2 = 49.0\text{N}$$

$$T_2 = 25.4\text{N}$$

$$T_1 = 1.414 \times 25.4\text{N} = 35.9\text{N}$$

**Question 12**

(a) Both bullets experience the same gravitational acceleration ($g = 9.8\text{m/s}^2$) in the vertical direction. The fired bullet has a horizontal velocity, but this does not affect the vertical motion because horizontal and vertical motions are independent. Both bullets start with zero vertical velocity, fall the same height, and accelerate downward at the same rate. Therefore, they reach the ground at the same time. The fired bullet simply lands farther from the firing point.

(b) At moderate speed, the centripetal force required ($\frac{mv^2}{r}$) is less than the maximum static friction available ($\mu_s mg$). Friction provides the necessary centripetal force and the car follows the curve safely. At excessive speed, the centripetal force required exceeds the maximum friction available. Friction can no longer supply the needed centripetal force, so the car cannot maintain the curved path and skids outward (in accordance with Newton's first law).

(c) $v = \sqrt{\frac{GM_E}{R}} = \sqrt{\frac{6.67\times10^{-11}\text{Nm}^2\text{kg}^{-2}\times6.0\times10^{24}\text{kg}}{3.84\times10^{8}\text{m}}} = 1021\text{m/s} = 1.02\text{km/s}$

**Question 13**

(a) The conclusion is incorrect. Zero acceleration means zero net force, not zero individual forces. The truck experiences several forces simultaneously: the engine's driving force forward, air resistance backward, friction at the tyres, weight downward, and normal reaction upward. These forces add up to zero (they are in equilibrium), producing zero net force and hence zero acceleration. This is consistent with Newton's first law.

(b) Once the jumper is airborne, the only external force acting is gravity (air resistance is negligible). By Newton's second law, the centre of mass accelerates at g downward regardless of how the jumper moves their limbs. Internal forces (muscle movements) can change the position of body parts relative to the centre of mass but cannot change the trajectory of the centre of mass itself. Therefore, the centre of mass follows a parabolic path dictated solely by the initial velocity and gravity.

(c) After impact, the bullet-bob system has velocity v. This velocity sends the system to height 0.10m:

$$\frac{1}{2}(m_b + M)v^2 = (m_b + M)gh$$

$$v = \sqrt{2gh} = \sqrt{2 \times 9.8\text{m/s}^2 \times 0.10\text{m}} = 1.4\text{m/s}$$

By conservation of momentum during the collision:

$$m_b u = (m_b + M)v$$

$$0.025\text{kg} \times u = (0.025 + 4.975)\text{kg} \times 1.4\text{m/s} = 7.0\text{kgm/s}$$

$$u = \frac{7.0\text{kgm/s}}{0.025\text{kg}} = 280\text{m/s}$$

**Question 14**

(a) The ball's total mechanical energy is $E = \frac{1}{2}mv^2 - \frac{GMm}{r}$. If the initial speed is less than the escape velocity, E is negative. To reach infinity (where PE = 0), the ball would need $E \geq 0$. Since the total energy is negative and conserved, the ball can never reach infinity. It rises until all its kinetic energy is converted to potential energy, then falls back under gravity.

(b) Both teams exert large forces, but the forces are equal in magnitude and opposite in direction. The net horizontal force on the rope is zero. By Newton's second law, zero net force means zero acceleration, so neither team moves. This is a state of equilibrium. The large individual forces exist (the rope is under high tension), but they cancel each other.

(c) At the top, the minimum speed occurs when the tension is zero and gravity alone provides the centripetal force:

$$mg = \frac{mv_{min}^2}{r}$$

$$v_{min} = \sqrt{gr} = \sqrt{9.8\text{m/s}^2 \times 1.2\text{m}} = 3.43\text{m/s}$$

**Question 15**

(a) The orbital speed is derived from $\frac{mv^2}{R} = \frac{GMm}{R^2}$. The satellite mass m cancels from both sides, giving $v = \sqrt{\frac{GM}{R}}$, which depends only on M (Earth's mass) and R (orbital radius). A heavier satellite needs more centripetal force, but gravity provides exactly that extra force because gravitational force is proportional to m. The two effects cancel, leaving the speed independent of satellite mass.

(b) When a person lands, their momentum must be reduced to zero. The force experienced is $F = \frac{mv}{\Delta t}$, where v is the landing speed and $\Delta t$ is the stopping time. From a greater height, the landing speed is higher (more KE), so the force is greater. If this force exceeds what bones and joints can withstand, injury results. A soft surface increases the stopping time $\Delta t$, reducing the force for the same momentum change.

(c) $g = \frac{GM}{R^2}$, so $R = \sqrt{\frac{GM}{g}}$

$$R = \sqrt{\frac{6.67 \times 10^{-11}\text{Nm}^2\text{kg}^{-2} \times 4.0 \times 10^{24}\text{kg}}{3.2\text{N/kg}}} = 9.131 \times 10^6\text{m}$$

Altitude: $h = R - r = 9.131 \times 10^6\text{m} - 5.0 \times 10^6\text{m} = 4.13 \times 10^6\text{m} = 4130\text{km}$

**Question 16**

(a) The gravitational force on a body is $F = mg$. By Newton's second law, the acceleration is $a = \frac{F}{m} = \frac{mg}{m} = g$. The mass cancels because gravity provides a force proportional to mass and inertia resists acceleration proportionally to mass. These two effects balance exactly, giving all objects the same acceleration regardless of mass. Since both balls start from rest and have the same acceleration, they reach the ground at the same time.

(b) Both children have the same angular velocity $\omega$, but the child at the edge has a larger radius r. The centripetal acceleration is $a = \omega^2 r$, which increases with r. A larger centripetal acceleration requires a larger centripetal force. What the child feels as an "outward pull" is actually the body's tendency to continue in a straight line (inertia). The child at the edge must exert a greater inward force to maintain the circular path, and hence feels a stronger pull.

(c) At the highest point, the shell moves horizontally at $200\text{m/s}$ with total momentum $= m \times 200\text{m/s}$.

After explosion, one fragment (mass $\frac{m}{2}$) falls vertically, so its horizontal momentum $= 0$.

By conservation of horizontal momentum:

$$m \times 200\text{m/s} = \frac{m}{2} \times 0 + \frac{m}{2} \times v$$

$v = \frac{m \times 200\text{m/s}}{\frac{m}{2}} = 400\text{m/s}$ (horizontally, in the same direction as the original motion)

**Question 17**

(a) On a banked road, the road surface is tilted inward. The normal reaction from the surface therefore has a horizontal component directed toward the centre of the curve. This horizontal component provides the centripetal force needed for circular motion, without relying on friction. At the design speed, the horizontal component of the normal reaction alone equals the required centripetal force: $R\sin\theta = \frac{mv^2}{r}$.

(b) Negative gravitational potential energy means the asteroid-Earth system is gravitationally bound. Energy must be supplied to the asteroid to separate it to infinity (where PE = 0). The magnitude of the negative PE indicates how much energy would be needed to completely escape the Earth's gravitational influence. As the asteroid approaches Earth, its PE becomes more negative (it falls deeper into the gravitational well), and this lost PE is converted to kinetic energy, causing the asteroid to speed up.

(c) $u_y = 25\text{m/s} \times \sin 37° = 25 \times 0.6018 = 15.05\text{m/s}$ (upward)

Vertical displacement: $-45\text{m}$ (downward from launch point)

$$-45 = 15.05t - \frac{1}{2} \times 9.8 \times t^2$$

$$4.9t^2 - 15.05t - 45 = 0$$

$$t = 4.93\text{s}$$

**Question 18**

(a) Without a seatbelt, the person would hit the dashboard or windscreen, stopping in a very short time $\Delta t$. Since the impulse $F\Delta t = \Delta p$ is fixed (same change in momentum), a very short stopping time means a very large force, causing serious injury. The seatbelt stretches slightly and decelerates the person over a longer time, reducing the peak force. By increasing $\Delta t$, the seatbelt decreases F for the same $\Delta p$, greatly reducing the risk of injury.

(b) The follow-through increases the time of contact between the racket and the ball. Since impulse equals force multiplied by time ($F\Delta t = \Delta p$), a longer contact time means a greater impulse is delivered to the ball. A greater impulse produces a greater change in momentum and hence a higher speed.

(c) On the conical surface, the normal reaction R acts perpendicular to the surface.

Vertically: $R\cos 50° = mg$

$$R = \frac{mg}{\cos 50°} = \frac{2.0\text{kg} \times 9.8\text{m/s}^2}{\cos 50°} = \frac{19.6\text{N}}{0.6428} = 30.49\text{N}$$

Horizontally: $R\sin 50° = \frac{mv^2}{r}$

$$v^2 = \frac{R\sin 50° \times r}{m} = \frac{30.49\text{N} \times \sin 50° \times 0.30\text{m}}{2.0\text{kg}} = 3.503\text{m}^2/\text{s}^2$$

$$v = \sqrt{3.503\text{m}^2/\text{s}^2} = 1.87\text{m/s}$$

**Alternatively,** you can solve directly by dividing Rsin50 by Rcos50° to get $v = \sqrt{rg\tan 50°} = \sqrt{0.3 \times 9.8\tan 50°} = 1.87\text{m/s}$

**Question 19**

(a) When a ball just barely completes the vertical circle, the tension at the top is zero and the speed at the top is $v = \sqrt{gr}$. At this instant, the centripetal acceleration equals g, meaning gravity alone provides exactly the centripetal force needed. The ball is effectively in free fall along a circular arc as its acceleration is purely gravitational. There is no contact force, so the ball momentarily behaves as a weightless projectile, giving the appearance of "floating."

(b) **Advantage:** A high coefficient of friction increases the maximum speed before the car skids ($v_{skid} = \sqrt{\mu gr}$). The car can negotiate sharper curves at higher speeds without losing grip.

**Disadvantage:** If the coefficient of friction exceeds $\frac{a}{2h}$, the car reaches the overturning speed before it reaches the skidding speed. The tyres grip so well that instead of sliding safely, the car tips over. In this case, high friction removes the "safety valve" of skidding and exposes the vehicle to the more severe risk of overturning.

(b) The answer of each part is as follows:

(i) $v_{skid} = \sqrt{\mu_s gr} = \sqrt{0.95 \times 9.8\text{m/s}^2 \times 50\text{m}} = 21.6\text{m/s}$

(ii) $v_{overturn} = \sqrt{\frac{arg}{2h}} = \sqrt{\frac{1.8\text{m} \times 50\text{m} \times 9.8\text{m/s}^2}{2 \times 1.0\text{m}}} = 21\text{m/s}$

(iii) Since $v_{overturn}(21.0\text{m/s}) < v_{skid}(21.6\text{m/s})$, the truck **overturns first**.

**Explanation:**

The high-grip tyres prevent skidding so effectively that the truck reaches its overturning limit first. With ordinary tyres, the truck would skid which is unpleasant but recoverable. With high-grip tyres, it overturns, that is catastrophic.

**Question 20**

(a) The satellite orbits in the equatorial plane with a period of exactly 24 hours, the same as the Earth's rotation period. Because the satellite completes one orbit in the same time the Earth completes one rotation, and both move in the same direction (they both move with the same angular velocity), the satellite remains directly above the point on the equator directly below it. From the observer's perspective, the satellite neither falls behind nor moves ahead, so it appears stationary.

(b) The hose accelerates a large mass of water from rest to a high forward velocity in a short time. As a result, the water gains a large forward momentum. By conservation of momentum, the total momentum of the system (water + hose + firefighter) must remain constant. Since the water gains forward momentum, the hose and firefighter must gain an equal magnitude of backward momentum to keep the total unchanged. Therefore, the firefighter is pushed backward. The faster the water flows and the greater the mass of water ejected per second, the greater the rate of change of momentum, and hence the stronger the backward push on the firefighter.

(c) At the lowest point, both the tension and the weight act along the radius, with tension directed upward (toward the centre) and weight directed downward (away from the centre):

$$T - mg = \frac{mv^2}{r}$$

$$T = mg + \frac{mv^2}{r} = 0.4\text{kg} \times 9.8\text{m/s}^2 + \frac{0.4\text{kg} \times (6.0\text{m/s})^2}{0.8\text{m}} = 21.9\text{N}$$

**Question 21**

(a) Surface gravity is given by $g = \frac{GM}{r^2}$. A larger mass M increases g because there is more matter to exert gravitational pull. A larger radius r decreases g because the surface is farther from the centre. Therefore, g depends on both quantities, and the net effect is determined by which factor dominates.

(b) By Newton's third law, the force on the bullet equals the force on the gun, and both act for the same duration. Therefore the impulse ($F\Delta t$) is the same on both. Since impulse equals change in momentum, both gain equal magnitudes of momentum. However, since momentum = mass × velocity, and the gun has a much larger mass than the bullet, the gun's velocity must be much smaller for its momentum to equal the bullet's momentum.

(c) Horizontal velocity remains constant: $v_x = 12\text{m/s}$

Vertical velocity at impact: $v_y = gt = 9.8\text{m/s}^2 \times 2.4\text{s} = 23.52\text{m/s}$

Speed at impact: $v = \sqrt{v_x^2 + v_y^2} = \sqrt{(12\text{m/s})^2 + (23.52\text{m/s})^2} = 26.4\text{m/s}$

**Question 22**

(a) The geostationary orbit requires a period of exactly 24 hours. From Kepler's third law, $T^2 = \frac{4\pi^2}{GM_E}R^3$, the orbital radius R is uniquely determined by the period T. Since T is fixed at 24 hours and $GM_E$ is a constant, there is only one possible value of R, and therefore only one altitude. No other height gives a 24-hour period.

(b) The gravitational force on each ball is $F = mg$. By Newton's second law, the acceleration is $a = \frac{F}{m} = \frac{mg}{m} = g$. The mass cancels, giving both balls the same acceleration regardless of their mass. Since they start from rest at the same height and have the same acceleration, they reach the ground at the same time.

(c) First, find the velocity of the block+bullet after impact using conservation of momentum:

$$0.05\text{kg} \times 300\text{m/s} = (0.05 + 2.95)\text{kg} \times v$$

$$v = 5.0\text{m/s}$$

On the incline, the **deceleration** comes from the component of gravity along the slope plus friction:

$$a = -(g\sin 30° + \mu g\cos 30°) = -\left(9.8\text{m/s}^2 \times 0.5 + 0.25 \times 9.8\text{m/s}^2 \times 0.866\right) = -7.022\text{m/s}^2$$

Distance up the incline: $v^2 = u^2 + 2as$.

$$0 = (5.0\text{m/s})^2 - 2 \times 7.022\text{m/s}^2 \times s$$

$$s = \frac{25\text{m}^2/\text{s}^2}{14.044\text{m/s}^2} = 1.78\text{m}$$

**Question 23**

(a) The kinetic energy is positive because it depends on the square of velocity, which is always positive ($\frac{1}{2}mv^2 > 0$). The gravitational potential energy is negative because the satellite is bound to the Earth. Energy must be supplied to move it from its orbit to infinity, where PE is defined as zero. Since it is currently below infinity in the potential well, its PE is less than zero and therefore negative.

(b) When the sprinter pushes backward on the blocks, by Newton's third law, the blocks push forward on the sprinter with an equal and opposite force. This forward reaction force from the blocks accelerates the sprinter forward. The sprinter does not accelerate because of the force they exert, but because of the reaction force exerted on them.

(c) Let $T_1$ be the tension in the rope at 35° and $T_2$ in the rope at 55°.

Weight: $W = 40\text{kg} \times 9.8\text{m/s}^2 = 392\text{N}$

Vertically: $T_1\sin 35° + T_2\sin 55° = 392\text{N}$

Horizontally: $T_1\cos 35° = T_2\cos 55°$

From the horizontal equation: $T_1 = T_2 \times \frac{\cos 55°}{\cos 35°} = T_2 \times \frac{0.5736}{0.8192} = 0.7002T_2$

Substituting into the vertical equation:

$$0.7002T_2 \times 0.5736 + T_2 \times 0.8192 = 392\text{N}$$

$$0.4016T_2 + 0.8192T_2 = 392\text{N}$$

$$T_2 = 321.1\text{N}$$

$$T_1 = 0.7002 \times 321.1\text{N} = 224.8\text{N}$$

**Question 24**

(a) Speed is constant, but the direction of motion changes continuously. Since velocity is a vector (having both magnitude and direction), a change in direction means the velocity is changing. By Newton's second law, a changing velocity requires a net force. This force is directed toward the centre of the circle and is called the centripetal force.

(b) Momentum equals mass times velocity ($p = mv$). At the same speed, the loaded lorry has a greater mass and therefore greater momentum. To stop the lorry, its momentum must be reduced to zero. Since the change in momentum is greater for the loaded lorry, a greater impulse ($F\Delta t$) is needed, making it harder to stop.

(c) $u_x = 80\text{m/s} \times \cos 50° = 51.42\text{m/s}$

$$u_y = 80\text{m/s} \times \sin 50° = 61.28\text{m/s}$$

Vertical displacement when landing: $s_y = -60\text{m}$ (below launch point)

$$-60 = 61.28t - \frac{1}{2} \times 9.8 \times t^2$$

$$4.9t^2 - 61.28t - 60 = 0$$

$$t = 13.42\text{s}$$

Horizontal distance: $x = u_x t = 51.42\text{m/s} \times 13.42\text{s} = 690\text{m}$

**Question 25**

(a) The gravitational force on a planet is directed radially toward the Sun. Because this force has no component perpendicular to the radius vector, it exerts zero torque about the Sun. With zero torque, angular momentum is conserved ($L = mvr =$ constant). The rate of area swept is $\frac{dA}{dt} = \frac{1}{2}rv = \frac{L}{2m}$, which is constant. Therefore, equal areas are swept in equal times.

(b) Before the bend, both the car and the passengers have momentum in the straight-ahead direction. When the car turns, friction between the tyres and road provides the centripetal force to change the car's direction. However, the passengers' bodies, not being rigidly attached, tend to continue in the original straight-line direction (Newton's first law). Relative to the turning car, the passengers appear to lean outward.

(c) Momentum is conserved in each direction:

East: $p_x = 3.0\text{kg} \times 5.0\text{m/s} = 15.0\text{kgm/s}$

North: $p_y = 2.0\text{kg} \times 7.5\text{m/s} = 15.0\text{kgm/s}$

Total mass: $3.0 + 2.0 = 5.0\text{kg}$

$$v_x = \frac{15.0\text{kgm/s}}{5.0\text{kg}} = 3.0\text{m/s}$$

$$v_y = \frac{15.0\text{kgm/s}}{5.0\text{kg}} = 3.0\text{m/s}$$

Speed: $v = \sqrt{(3.0\text{m/s})^2 + (3.0\text{m/s})^2} = 4.24\text{m/s}$

Direction: $\theta = \tan^{-1}\left(\frac{3.0}{3.0}\right) = 45°$ north of east.

**Question 26**

(a) At the centre, mass is distributed symmetrically in all directions around the point. The gravitational pulls from all directions cancel, giving zero net force and hence zero field strength. However, gravitational potential is a scalar that measures the energy needed per unit mass to escape to infinity. At the centre, an object is at the deepest point of the gravitational potential well, so the potential has its most negative value ($-\frac{3GM}{2r_E}$). Zero force does not mean zero energy.

(b) In a freely falling lift, both the person and the scale accelerate downward at g. There is no relative acceleration between the person and the scale. As a result, the person exerts no force on the scale, so the scale reads zero. The person's weight (gravitational pull) still exists, but it is entirely used to accelerate the person downward, leaving no contact force to register on the scale.

(c) Thrust: $F_{thrust} = v_{exhaust} \times \frac{dm}{dt} = 800\text{m/s} \times 20\text{kg/s} = 16000\text{N}$

Weight: $W = 5000\text{kg} \times 9.8\text{m/s}^2 = 49000\text{N}$

Net upward force: $F_{net} = 16000\text{N} - 49000\text{N} = -33000\text{N}$

Since the thrust is less than the weight, the rocket does not lift off. The initial acceleration is zero (the rocket remains on the ground).

**Question 27**

(a) The range formula is $R = \frac{u^2 \sin 2\theta}{g}$. Since $\sin 2\theta$ has a maximum value of 1 when $2\theta = 90°$, the maximum range occurs at $\theta = 45°$. At this angle, the vertical and horizontal components of the initial velocity are equal, giving the optimum balance between height (time in the air) and horizontal velocity.

(b) The period is derived from $\frac{mv^2}{R} = \frac{GMm}{R^2}$. The satellite mass m appears on both sides and cancels. The resulting expression $T = 2\pi\sqrt{\frac{R^3}{GM}}$ depends only on the orbital radius R and the mass M of the central body, not on the satellite mass. A heavier satellite needs more centripetal force, but gravity provides proportionally more force, so the motion is the same.

(c) For the system (both masses accelerate together):

Net force = weight of hanging mass = $1.0\text{kg} \times 9.8\text{m/s}^2 = 9.8\text{N}$

Total mass = $4.0\text{kg} + 1.0\text{kg} = 5.0\text{kg}$

$$a = \frac{9.8\text{N}}{5.0\text{kg}} = 1.96\text{m/s}^2$$

For the hanging mass: $mg - T = ma$

$$T = m(g - a) = 1.0\text{kg} \times (9.8 - 1.96)\text{m/s}^2 = 1.0\text{kg} \times 7.84\text{m/s}^2 = 7.84\text{N}$$

**Question 28**

(a) The total energy is $E = K + U = \frac{GMm}{2R} - \frac{GMm}{R} = -\frac{GMm}{2R}$. The potential energy (negative) has twice the magnitude of the kinetic energy (positive). When added, the result is negative. This negative total energy indicates that the satellite is gravitationally bound and cannot escape to infinity without additional energy.

(b) The maximum height depends only on the vertical component of the initial velocity: $H = \frac{u_y^2}{2g}$. The horizontal component does not affect the vertical motion because horizontal and vertical motions are independent (gravity acts only vertically). As long as $u_y$ is the same, the height is the same regardless of the horizontal component.

(c) Vertically: $R + F\sin\theta = mg$, so $R = mg - F\sin\theta$

Horizontally (constant velocity, so net force = 0): $F\cos\theta = \mu R$

Substituting R: $F\cos\theta = \mu(mg - F\sin\theta)$

$$F\cos\theta = \mu mg - \mu F\sin\theta$$

$$F\cos\theta + \mu F\sin\theta = \mu mg$$

$$F(\cos\theta + \mu\sin\theta) = \mu mg$$

$$F = \frac{\mu mg}{\cos\theta + \mu\sin\theta}$$

**Question 29**

(a) Mass is the quantity of matter in a body, measured in kg. It is a scalar and does not change with location. Weight is the gravitational force on the body ($W = mg$), measured in N. It depends on the local value of g. An astronaut in orbit has the same mass as on Earth, but appears weightless because both the astronaut and the spacecraft are in free fall together. There is no contact force between them, so the astronaut experiences no sensation of weight.

(b) When a car rounds a curve, the centripetal force (friction at the tyres) acts at road level. This force and the weight of the car (acting at the centre of mass) create a turning effect (torque) about the outer wheels. The torque tending to overturn the car is proportional to the height of the centre of mass. A higher centre of mass produces a larger overturning torque for the same centripetal force, making the car more likely to tip over before it skids.

(c) Escape velocity: $v_E = \sqrt{2g_{surface}r} = \sqrt{2 \times 4.5\text{m/s}^2 \times 4.8 \times 10^6\text{m}} = 6573\text{m/s} = 6.57\text{km/s}$

**Question 30**

(a) Off the cliff, the ball has a horizontal velocity (from the kick) and simultaneously experiences a vertical gravitational acceleration. These two independent motions combine to produce a curved (parabolic) trajectory. On the flat field, the ball has only horizontal motion. Gravity acts vertically downward and is balanced by the normal reaction from the ground, so there is no vertical acceleration. With motion in only one direction, the path is a straight line.

(b) Before: total momentum = mv (first stone) + 0 (second stone) = mv. After: total momentum = 0 (first stone) + mv (second stone) = mv. Momentum is conserved. Before: total KE = ½mv². After: total KE = ½mv². KE is conserved. Both conservation laws are satisfied because the entire momentum and kinetic energy are transferred from the first stone to the second.

(c) $T = 2\pi\sqrt{\frac{R^3}{GM_E}} = 2 \times 3.14 \times \sqrt{\frac{(3.84\times10^8\text{m})^3}{6.67\times10^{-11}\text{Nm}^2\text{kg}^{-2}\times6.0\times10^{24}\text{kg}}} = 2.363 \times 10^6\text{s}$

Converting to days: $\frac{2.363\times10^6\text{s}}{86400\text{s/day}} = 27.3 \text{ days}$

**Question 31**

(a) At infinity, both kinetic energy and potential energy are zero, giving total energy of zero. If a body's total energy is negative, it would need to gain energy to reach zero total energy at infinity. Since no external energy source is available (for an unpowered projectile), the body cannot reach infinity. It will rise, slow down, and eventually fall back. Only a body with total energy greater than or equal to zero can escape.

(b) The rain falls vertically, but the person moves forward horizontally. The velocity of the rain relative to the person is the vector sum of the rain's downward velocity and the person's forward velocity (reversed). This resultant velocity is directed downward and backward relative to the person. Therefore, the rain appears to come from the front at an angle.

(c) At the design speed, the horizontal component of the normal reaction provides the centripetal force, and friction is zero.

Resolving: $R\sin\theta = \frac{mv^2}{r}$ and $R\cos\theta = mg$

Dividing: $\tan\theta = \frac{v^2}{rg}$

$$v = \sqrt{rg\tan\theta} = \sqrt{80\text{m} \times 9.8\text{m/s}^2 \times \tan20^\circ} = 16.9\text{m/s}$$

**Question 32**

(a) Two effects contribute. First, the Earth is slightly flattened at the poles (oblate), so the equatorial radius is larger than the polar radius. Since $g \propto \frac{1}{r^2}$, the larger equatorial radius gives a smaller g. Second, at the equator, part of the gravitational pull is used to provide the centripetal acceleration for the Earth's rotation. This reduces the effective g. At the poles, no centripetal acceleration is needed (the rotation radius is zero), so the full gravitational pull is felt. Both effects make g smaller at the equator.

(b) As the parachutist falls, air resistance increases with speed. Initially, gravity exceeds air resistance, producing a net downward force and hence acceleration. As speed increases, air resistance grows until it equals the gravitational force. At this point, the net force is zero, so by Newton's second law, the acceleration is zero. The parachutist continues at constant speed (terminal velocity). Gravity still acts, but it is exactly balanced by air resistance.

(c) Taking rightward as positive:

$u = +8\text{m/s}$, $v = -6\text{m/s}$

Change in momentum: $\Delta p = mv - mu = 0.2\text{kg} \times (-6\text{m/s}) - 0.2\text{kg} \times 8\text{m/s}$

$$= -1.2\text{kgm/s} - 1.6\text{kgm/s} = -2.8\text{kgm/s}$$

Average force: $F = \frac{\Delta p}{\Delta t} = \frac{-2.8\text{kgm/s}}{0.05\text{s}} = -56\text{N}$

The magnitude of the average force is 56N (directed to the left, toward the ball).

**Question 33**

(a) At any distance R from a planet, the orbital velocity is $v_{orb} = \sqrt{\frac{GM}{R}}$ and the escape velocity is $v_{esc} = \sqrt{\frac{2GM}{R}}$. Dividing: $\frac{v_{esc}}{v_{orb}} = \sqrt{2}$, regardless of the value of R. The factor $\sqrt{2}$ arises from the ratio of the coefficients in the two formulas, and R cancels completely. This relationship is therefore universal.

(b) The change in momentum of each passenger is the same whether braking is sudden or gradual (the passenger goes from the bus's speed to rest). Since impulse $= F\Delta t = \Delta p$, and $\Delta p$ is fixed, a longer braking time $\Delta t$ means a smaller force F on the passengers. Gradual braking increases the stopping time and therefore reduces the force experienced, making it safer and more comfortable.

(c) Along the slope, net acceleration:

$$a = g\sin 25° - \mu g\cos 25° = 9.8\text{m/s}^2(\sin 25° - 0.10\cos 25°) = 3.254\text{m/s}^2$$

$$v^2 = u^2 + 2as = 0 + 2 \times 3.254\text{m/s}^2 \times 200\text{m} = 1301.6\text{m}^2/\text{s}^2$$

$$v = \sqrt{1301.6\text{m}^2/\text{s}^2} = 36.1\text{m/s}$$

**Question 34**

(a) Throughout the flight, the horizontal component of velocity remains constant (no horizontal force acts). The vertical component decreases as the ball rises (due to gravitational deceleration) and increases as it falls. At the highest point, the vertical component is momentarily zero. Since the speed is $\sqrt{v_x^2 + v_y^2}$, and $v_y = 0$ at the highest point, the speed equals the horizontal component alone, which is its smallest possible value.

(b) A satellite in circular orbit is in free fall. The gravitational force acts perpendicular to the velocity (directed toward the centre), so it does no work. Since no work is done, the satellite neither gains nor loses energy. Its speed remains constant and its direction changes continuously in exactly the right way to maintain the circular path. No fuel is needed because gravity provides the centripetal force for free.

(c) Using conservation of momentum (initial momentum = 0):

$$0 = 3.0\text{kg} \times (-4.0\text{m/s}) + 5.0\text{kg} \times v_2$$

$v_2 = \frac{3.0 \times 4.0}{5.0} = 2.4\text{m/s}$ (to the right)

Energy stored in the spring = total KE after release:

$$E = \frac{1}{2} \times 3.0\text{kg} \times (4.0\text{m/s})^2 + \frac{1}{2} \times 5.0\text{kg} \times (2.4\text{m/s})^2$$

$$= 24.0\text{J} + 14.4\text{J} = 38.4\text{J}$$

**Question 35**

(a) The approximate formula is derived from the exact formula using the binomial expansion, which drops terms of order $\left(\frac{h}{r_E}\right)^2$ and higher. These dropped terms are negligible only when $\frac{h}{r_E} \ll 1$. As h increases, $\frac{h}{r_E}$ increases, and the dropped terms become increasingly significant. Therefore, the difference between the approximate and exact values grows, producing a rapidly increasing percentage error.

(b) In the head-on collision of two identical cars at 60km/h, each car decelerates from 60km/h to 0km/h on impact (by symmetry, the wreckage is stationary). The change in velocity for each car is 60km/h. Hitting a rigid wall at 120km/h would require the car to decelerate from 120km/h to 0km/h, a change of 120km/h, which is twice as severe. The head-on collision is therefore equivalent to hitting a wall at 60km/h, not 120km/h.

(c) $u_x = 30\cos 37° = 30 \times 0.7986 = 23.96\text{m/s}$ (constant)

$u_y = 30\sin 37° = 30 \times 0.6018 = 18.05\text{m/s}$ (changes: $v_y = u_y - gt$)

Using: $v^2 = v_x^2 + v_y^2$

At time t, speed is: $v^2 = u_x^2 + (u_y - gt)^2$

$$25^2 = 23.96^2 + (18.05 - 9.8t)^2$$

$$(18.05 - 9.8t)^2 = 50.9$$

$$18.05 - 9.8t = \pm 7.13$$

$t = 1.11\text{s}$ or $t = 2.57\text{s}$

The speed is $25\text{m/s}$ at $t = 1.11\text{s}$ (on the way up) and $t = 2.57\text{s}$ (on the way down).

**Question 36**

(a) Escape velocity is $v_E = \sqrt{\frac{2GM}{r}}$. The Moon has a much smaller mass than the Earth (about $\frac{1}{81}$ of Earth's mass) and a smaller radius (about $\frac{1}{3.7}$ of Earth's radius). The smaller mass dominates, giving a weaker gravitational field. As a result, less kinetic energy is needed to escape from the Moon, and hence a lower speed is sufficient.

(b) Drawing the hands backward increases the time over which the ball decelerates. The change in momentum is the same regardless of how the ball is caught. Since impulse $= F\Delta t = \Delta p$, increasing $\Delta t$ decreases the force F for the same $\Delta p$. Therefore, the force on the fielder's hands is reduced.

(c) At the bottom, KE $= \frac{1}{2} \times 1500\text{kg} \times (20\text{m/s})^2 = 300000\text{J}$

At the top (height $= 2r = 24\text{m}$), by energy conservation:

$$\frac{1}{2}mv_{top}^2 = \frac{1}{2}mv_{bottom}^2 - mg(2r)$$

$$v_{top}^2 = (20\text{m/s})^2 - 2 \times 9.8\text{m/s}^2 \times 24\text{m} = -70.4\text{m}^2/\text{s}^2$$

Since $v_{top}^2$ is negative, the car does not have enough speed to reach the top. The car fails to complete the loop.

**Question 37**

(a) At the top of the bridge, the car follows a circular arc whose centre of curvature is below the road. The centripetal acceleration is therefore directed downward. Because part of the driver's weight is required to provide this centripetal acceleration, the normal reaction R from the seat is less than the driver's weight: $R = mg - \frac{mv^2}{r}$. Since what the driver feels as "weight" is the contact force R rather than gravity itself, the reduced R creates the sensation of feeling lighter than on the flat road where the normal reaction is equal to the driver's weight

(b) The collision between the bullet and block is perfectly inelastic. The kinetic energy that disappears from the motion of the objects is not destroyed but is converted into other forms of energy. Most of it becomes thermal energy (the bullet and the surrounding wood heat up due to friction and deformation). A small portion is converted into sound energy (the impact is audible) and into permanent deformation of the wood and bullet.

(c) The stone falls through a net vertical height of $70\text{m}$. Using conservation of energy:

$$\frac{1}{2}mu^2 + mgh = \frac{1}{2}mv^2$$

$$v^2 = u^2 + 2gh$$

$$v^2 = (40\text{m/s})^2 + 2 \times 9.8\text{m/s}^2 \times 70\text{m} = 2972\text{m}^2/\text{s}^2$$

$$v = \sqrt{2972\text{m}^2/\text{s}^2} = 54.5\text{m/s}$$

**Question 38**

(a) In the planet's reference frame, the spacecraft approaches and leaves with the same speed (the gravitational interaction is elastic). However, in the Sun's reference frame, the planet is itself moving along its orbit. The spacecraft effectively "borrows" a small amount of kinetic energy from the planet's orbital motion. Because the planet's mass is enormously larger than the spacecraft's, the planet's speed decreases by an insignificant amount, while the spacecraft gains a significant increase in speed.

(b) This is possible when the two equal forces act in exactly opposite directions. In that case, their vector sum (the resultant) is zero. Since there is no net force, there is no acceleration, and the object remains in equilibrium. This is the principle behind the equilibrant: a force equal in magnitude but opposite in direction to a given force produces equilibrium.

(c) At the top, both the tension and the weight act toward the centre (downward), so together they provide the centripetal force:

$$T + mg = \frac{mv^2}{r}$$

$$v^2 = \frac{(T + mg)r}{m}$$

$$v^2 = \frac{(5.0\text{N} + 0.40\text{kg} \times 9.8\text{m/s}^2) \times 0.60\text{m}}{0.40\text{kg}} = 13.38\text{m}^2/\text{s}^2$$

$$v = \sqrt{13.38\text{m}^2/\text{s}^2} = 3.66\text{m/s}$$

**Question 39**

(a) A geostationary satellite must have a period of exactly 24 hours to match the Earth's rotation. From Kepler's third law, $T^2 = \frac{4\pi^2}{GM_E}R^3$, the orbital radius R is uniquely determined by the period T. Since both T and $GM_E$ are fixed constants, only one value of R satisfies the equation. Therefore, only one altitude gives a 24-hour period, and all geostationary satellites must orbit at this height (approximately 36000km).

(b) When the goalkeeper dives, the body gains significant momentum toward the ground. Upon landing, this momentum must be reduced to zero. By rolling, the goalkeeper extends the time and distance over which the body decelerates after hitting the ground. By the impulse-momentum relationship ($F\Delta t = \Delta p$), since the change in momentum is fixed, a longer deceleration time $\Delta t$ produces a smaller average impact force F on the body. Rolling also spreads the contact over a larger area of the body rather than concentrating it on a single point such as the elbow or shoulder. As a result, both the force and the pressure on any one part of the body are reduced, greatly lowering the risk of injury from the dive.

(c) Resolving into components:

$$R_x = 25\text{N} + 18\cos 130°\text{N} + 12\cos 240°\text{N} = 7.43\text{N}$$

$$R_y = 0 + 18\sin 130°\text{N} + 12\sin 240°\text{N} = 3.40\text{N}$$

Resultant: $R = \sqrt{(7.43\text{N})^2 + (3.40\text{N})^2} = 8.17\text{N}$

Direction of resultant: $\theta = \tan^{-1}\left(\frac{3.40}{7.43}\right) = 24.6°$ above the positive x-axis.

The equilibrant has the same magnitude (8.17N) but acts in the opposite direction: $24.6° + 180° = 204.6°$ from the positive x-axis.

**Question 40**

(a) When an object is projected horizontally, it moves forward at constant horizontal velocity while simultaneously falling under gravity. A taller building means the object has a longer time to fall before reaching the ground. Because the horizontal distance equals the horizontal velocity multiplied by the time of flight ($x = u_x \times t$), a longer fall time produces a greater horizontal distance, even though $u_x$ is the same.

(b) In a freely falling lift, both the person and the scale accelerate downward at g. Because there is no relative acceleration between the person and the scale, the person exerts no force on the scale. As a result, the scale reads zero. The person's weight (mg) still exists as a gravitational force, but it is entirely used to accelerate the person downward, leaving no contact force to register on the scale.

(c) Momentum is conserved in each direction separately.

Eastward: $p_x = 2.0\text{kg} \times 6.0\text{m/s} + 0 = 12.0\text{kgm/s}$

Northward: $p_y = 0 + 3.0\text{kg} \times 4.0\text{m/s} = 12.0\text{kgm/s}$

Total mass: $m = 2.0\text{kg} + 3.0\text{kg} = 5.0\text{kg}$

Velocity of combined object:

$$v = \frac{\sqrt{p_x^2 + p_y^2}}{m} = \frac{\sqrt{(12.0\text{kgm/s})^2 + (12.0\text{kgm/s})^2}}{5.0\text{kg}} = 3.39\text{m/s}$$

Direction: $\theta = \tan^{-1}\left(\frac{12.0}{12.0}\right) = 45°$ north of east.

**Question 41**

(a) Surface gravity is $g = \frac{GM}{r^2}$. The Moon's smaller radius would, by itself, increase g (because the surface is closer to the centre). However, the Moon's mass is only about $\frac{1}{81}$ of the Earth's mass, which greatly reduces g. Since mass appears in the numerator

and radius squared in the denominator, the much smaller mass of the Moon dominates over the slightly smaller radius. As a result, the net effect is a much lower surface gravity.

(b) The ball was kicked at an angle, so it has both a horizontal and a vertical component of velocity at launch. At the highest point, the vertical component is zero (the ball has momentarily stopped rising). However, the horizontal component remains unchanged throughout the flight because no horizontal force acts on the ball (neglecting air resistance). Therefore, at the highest point, the ball still has a non-zero horizontal velocity, and hence its velocity is not zero.

(c) For the car to follow the circular path, static friction must provide the centripetal force:

$$\mu_s mg = \frac{mv^2}{r}$$

$$\mu_s = \frac{v^2}{gr}$$

$$\mu_s = \frac{(25\text{m/s})^2}{9.8\text{m/s}^2 \times 60\text{m}} = 1.06$$

Since $\mu_s > 1$ is unrealistic for ordinary tyres on road, the car cannot safely negotiate this bend at $25\text{m/s}$ on a flat road. It would skid.

**Question 42**

(a) The total energy of a satellite is $E = K + U = \frac{GMm}{2R} - \frac{GMm}{R} = -\frac{GMm}{2R}$. The potential energy (negative) has twice the magnitude of the kinetic energy (positive). When they are added, the result is negative. This negative total energy signifies that the satellite is gravitationally bound to the Earth. To free the satellite completely (send it to infinity with zero speed), energy equal to $|E|$ must be supplied. A satellite with negative total energy cannot escape without an external energy input.

(b) The parked car needs only enough fuel to keep the engine running against internal friction (very low energy demand). The moving car, however, must overcome continuous air resistance and road friction. These resistive forces constantly remove kinetic energy from the car, which must be replaced by the engine. Because the driving force must continuously do work against these resistive forces to maintain constant speed, the fuel consumption is much higher. Even though the acceleration is zero in both cases, the moving car dissipates energy at a much higher rate.

(c) At impact, the horizontal velocity is $v_x = 15\text{m/s}$ (unchanged). The angle below horizontal is $60°$, so:

$$\tan 60° = \frac{v_y}{v_x}$$

$$v_y = v_x \tan 60° = 15\text{m/s} \times 1.732 = 25.98\text{m/s}$$

Since the ball starts with zero vertical velocity, using $v_y^2 = 2gh$:

$$h = \frac{v_y^2}{2g} = \frac{(25.98\text{m/s})^2}{2 \times 9.8\text{m/s}^2} = 34.4\text{m}$$

**Question 43**

(a) Launching the ball upward allows it to remain in the air longer, increasing the time available for horizontal motion. The longer time of flight allows the ball to travel a greater horizontal distance before landing.

(b) The coin needs a centripetal force directed toward the centre to move in a circle. On a smooth turntable, there is very little friction to provide this force. As the angular speed increases, the required centripetal force ($m\omega^2 r$) increases. When the required force exceeds the available friction, the friction can no longer maintain the circular path. The coin then continues moving in the direction of its instantaneous velocity (tangentially), which carries it outward and away from the centre. It appears to slide "outward" because the tangential direction at any point on a circle leads away from the centre.

(c) Using $g = \frac{GM_E}{R^2}$, making R the subject:

$$R = \sqrt{\frac{GM_E}{g}}$$

$$R = \sqrt{\frac{6.67 \times 10^{-11}\text{Nm}^2\text{kg}^{-2} \times 6.0 \times 10^{24}\text{kg}}{4.9\text{N/kg}}} = 9.037 \times 10^6\text{m}$$

Altitude:

$$h = R - r_E = 9.037 \times 10^6\text{m} - 6.4 \times 10^6\text{m} = 2.637 \times 10^6\text{m} = 2637\text{km}$$

**Question 44**

(a) Throwing the rope outward gives it a horizontal component of velocity. This allows it to travel away from the cliff while falling. If thrown straight downward, it would simply fall vertically close to the cliff face.

(b) By Newton's third law, the force on the boat equals the force on the ship during the collision, and both act for the same duration. Therefore, both experience the same impulse and hence the same magnitude of change in momentum. However, because the boat has a much smaller mass, this momentum change produces a much larger change in velocity ($\Delta v = \frac{\Delta p}{m}$). As a result, the boat decelerates violently, causing severe structural damage, while the ship's much larger mass means its velocity changes negligibly.

(c) At the top, for the body to just complete the circle, the minimum condition is $R = 0$ (the string goes slack). Then gravity alone provides the centripetal force:

$$mg = \frac{mv_{top}^2}{r}$$

$$v_{top} = \sqrt{gr} = \sqrt{9.8\text{m/s}^2 \times 1.0\text{m}} = 3.13\text{m/s}$$

Using energy conservation from bottom to top (height change = 2r):

$$\frac{1}{2}mv_{bottom}^2 = \frac{1}{2}mv_{top}^2 + mg(2r)$$

$$v_{bottom} = \sqrt{v_{top}^2 + 4gr} = \sqrt{gr + 4gr} = \sqrt{5gr}$$

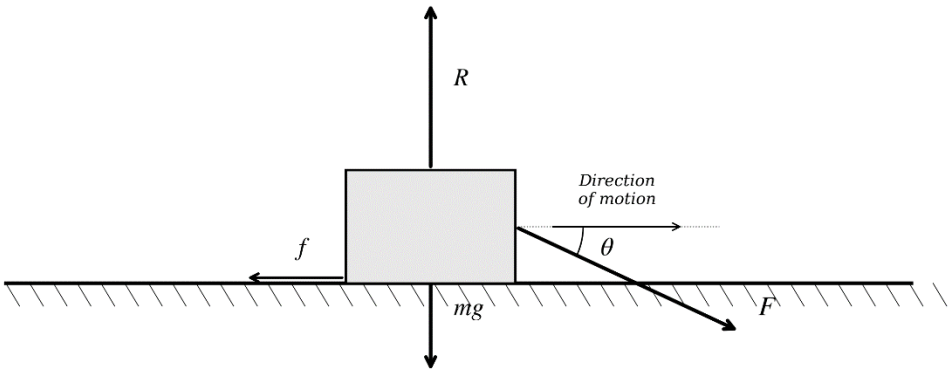
$$v_{bottom} = \sqrt{5 \times 9.8\text{m/s}^2 \times 1.0\text{m}} = 7.0\text{m/s}$$

**Question 45**

(a) The gravitational force on the planet is directed radially toward the Sun and is therefore always perpendicular to the planet's displacement along the orbit only for circular orbits. For an elliptical orbit, the gravitational force has a component along the direction of motion whenever the planet is not at perihelion or aphelion. Between aphelion and perihelion (as the planet moves inward), the radial force has a component in the direction of motion, doing positive work and thus increasing the planet's speed. Between perihelion and aphelion (as the planet moves outward), the component opposes the motion, doing negative work and slowing the planet down. This is consistent with conservation of energy: as the planet falls closer to the Sun, it loses potential energy and gains kinetic energy, and vice versa.

(b) As sand leaks out, the total mass of the truck decreases over time. If the engine provides the same driving force F and the resistive forces remain approximately constant, the net force stays roughly the same. By Newton's second law, $a = \frac{F_{net}}{m}$. Because m decreases while $F_{net}$ remains constant, the acceleration increases over time. The truck gradually speeds up faster and faster.

(c)




Since F acts below the horizontal, it pushes the block into the surface, increasing the normal reaction.

Vertically: $R = mg + F\sin\theta = 5.0\text{kg} \times 9.8\text{m/s}^2 + 40\text{N} \times \sin 25° = 65.9\text{N}$

Friction: $f = \mu_k R = 0.30 \times 65.9\text{N} = 19.77\text{N}$

Horizontally: $F\cos\theta - f = ma$

$$a = \frac{F\cos\theta - f}{m} = \frac{40\text{N} \times \cos 25° - 19.77\text{N}}{5.0\text{kg}} = 3.30\text{m/s}^2$$

**Question 46**

(a) Orbital speed is derived from the condition that gravity provides the centripetal force: $\frac{mv^2}{R} = \frac{GM_E m}{R^2}$. The satellite's mass m appears on both sides and cancels, giving $v = \sqrt{\frac{GM_E}{R}}$. This expression depends only on the Earth's mass and the orbital radius, not on the satellite's mass. A heavier satellite requires more centripetal force, but gravity provides exactly that extra force because the gravitational force is proportional to m. Hence the speed is the same for all satellites at the same altitude.

(b) The change in the ball's momentum when caught is fixed. By the impulse-momentum relationship, $F\Delta t = \Delta p$. By moving the arms backward, the player increases the time $\Delta t$ over which the ball decelerates. Because $\Delta p$ is constant, a larger $\Delta t$ results in a smaller average force F on the arms. Thus the cushioning technique reduces the impact force.

(c) Components at launch: $v_x = 28\text{m/s} \times \cos 40° = 21.45\text{m/s}$, $v_{y,0} = 28\text{m/s} \times \sin 40° = 17.99\text{m/s}$

At $t = 2.0\text{s}$:

$$v_x = 21.45\text{m/s (unchanged)}$$

$$v_y = v_{y,0} - gt = 17.99\text{m/s} - 9.8\text{m/s}^2 \times 2.0\text{s} = -1.61\text{m/s}$$

The negative sign means the ball is now moving downward.

Magnitude of velocity (speed):

$$v = \sqrt{(21.45\text{m/s})^2 + (-1.61\text{m/s})^2} = 21.5\text{m/s}$$

Direction: $\alpha = \tan^{-1}\left(\frac{1.61}{21.45}\right) = 4.3°$ below the horizontal.

**Question 47**

(a) **First reason:** The Earth is not a perfect sphere but is slightly flattened at the poles (oblate). The equatorial radius is larger than the polar radius. Since $g \propto \frac{1}{r^2}$, the larger equatorial radius gives a smaller g at the equator.

**Second reason:** At the equator, the person moves in a circle due to the Earth's rotation. Part of the gravitational pull must provide the centripetal acceleration for this circular motion. As a result, the effective g is reduced by $\omega^2 r_E$. At the poles, the person is on the rotation axis, so no centripetal acceleration is needed and the full gravitational pull is felt.

Both effects make the effective g smaller at the equator, and hence the person weighs less there.

(b) The ball gains a large momentum in a very short time. By the impulse-momentum theorem, $F = \frac{\Delta p}{\Delta t}$. Because $\Delta t$ is extremely small (a few milliseconds), even a modest change in momentum produces a very large force.

(c) First, find the velocity after impact using conservation of momentum:

$$m_b u = (m_b + M)v$$

$$v = \frac{m_b u}{m_b + M} = \frac{0.020\text{kg} \times 350\text{m/s}}{0.020\text{kg} + 2.48\text{kg}} = 2.8\text{m/s}$$

On the rough patch, friction decelerates the block.

$$F = ma = \mu mg$$

$$a = \mu g \text{ (opposing the motion)}$$

Using $v^2 = u^2 - 2\mu gs$, with the block coming to rest ($v = 0$), and $a = -\mu g$:

$$s = \frac{u^2}{2\mu g} = \frac{(2.8\text{m/s})^2}{2 \times 0.40 \times 9.8\text{m/s}^2} = 1.0\text{m}$$

**Alternative solution**

$$\text{Work done by friction} = \text{loss of kinetic energy}$$

$$f \times s = \frac{1}{2}mu^2$$

$$\mu mg \times s = \frac{1}{2}mu^2$$

$$s = \frac{u^2}{2\mu g} = \frac{(2.8\text{m/s})^2}{2 \times 0.40 \times 9.8\text{m/s}^2} = 1.0\text{m}$$

**Question 48**

(a) Below the Earth's surface, by the Shell Theorem, the spherical shell of mass above the object exerts zero net gravitational force on it. Only the mass within the sphere of radius equal to the object's distance from the centre contributes to gravity. As the object moves deeper, this contributing mass decreases (since less mass lies beneath the object). Therefore, g decreases with depth. At the centre, all the mass is distributed symmetrically around the point, so the gravitational pulls from all directions cancel completely, giving $g = 0$.

(b) When the car enters the roundabout, all parts of the car must follow a curved path, requiring centripetal force directed toward the centre. The front tyres, being steered, change direction first and friction at the front provides centripetal force. However, the rear of the car, with its greater distance from the turning axis and its inertia, tends to continue in a straight line (in accordance with Newton's first law). If the speed is too high, the rear tyres cannot provide enough friction to keep the back end on the curved path, and the rear swings outward. This is called oversteer.

(c) For equilibrium, the third force must balance the other two. It must have an upward component of $50\text{N}$ and a leftward component of $30\text{N}$.

Magnitude:

$$F_3 = \sqrt{(50\text{N})^2 + (30\text{N})^2} = 58.3\text{N}$$

Direction:

$\theta = \tan^{-1}\left(\frac{30}{50}\right) = 31.0°$ from the vertical, toward the left.

Equivalently, $59.0°$ above the negative x-axis (or $121°$ from the positive x-axis).

**Question 49**

(a) When drag removes energy, the total energy $E = -\frac{GMm}{2R}$ becomes more negative. A more negative E corresponds to a smaller R, so the satellite spirals inward to a lower orbit. At this lower orbit, the required orbital speed $v = \sqrt{\frac{GM}{R}}$ is higher because R is smaller. Therefore, the satellite speeds up. This occurs because the gravitational potential energy lost in falling to a lower orbit exceeds the energy lost to drag plus the kinetic energy gained. The satellite trades altitude for speed.

(b) The ball has large momentum that must be reduced to zero when caught. The change in momentum is fixed. Without pulling the hands back, the stopping time $\Delta t$ is very short (the ball stops almost instantly). By the impulse-momentum relationship ($F = \frac{\Delta p}{\Delta t}$), a very short $\Delta t$ produces a very large force F on the hands, causing pain. If the catcher draws the hands back, $\Delta t$ increases and the force is reduced.

(c) The design speed for this bank: $v_d = \sqrt{rg\tan 15°} = \sqrt{100 \times 9.8 \times 0.2679} = 16.2\text{m/s}$

Since the car's speed ($20\text{m/s}$) exceeds the design speed ($16.2\text{m/s}$), the banking alone cannot provide enough centripetal force. Friction must act **down the slope** (toward the centre) to supplement the centripetal force.

Required centripetal force: $\frac{mv^2}{r} = \frac{1200\text{kg} \times (20\text{m/s})^2}{100\text{m}} = 4800\text{N}$

Force balance toward centre (horizontally): $R\sin 15° + f\cos 15° = 4800\text{N}$

Force balance vertically: $R\cos 15° - f\sin 15° = mg = 1200\text{kg} \times 9.8\text{m/s}^2 = 11760\text{N}$

From vertical: $R = \frac{11760\text{N} + f\sin 15°}{\cos 15°}$

Substituting into horizontal:

$$\frac{(11760 + f\sin15°)\sin15°}{\cos15°} + f\cos15° = 4800$$

$$11760\tan15° + f\sin15°\tan15° + f\cos15° = 4800$$

$$3151 + f(0.06699 + 0.9659) = 4800$$

$$f = 1596\text{N (acting down the slope, toward the centre)}$$

**Question 50**

(a) On a curved road, friction between the tyres and the road surface provides the centripetal force needed to follow the curve. Water on the road acts as a lubricant, significantly reducing the coefficient of friction between the tyres and the surface. As a result, the maximum friction available ($\mu$mg) is much smaller on a wet road. Because the centripetal force required ($\frac{mv^2}{r}$) depends on the square of the speed, a speed that was safe on dry road may demand more friction than the wet surface can provide, causing the car to skid.

(b) The gravitational force on the comet is always directed toward the Sun. In a highly elliptical orbit, the distance from the Sun varies enormously. Near the Sun, the comet is deep in the gravitational potential well. As it falls inward from a great distance, gravitational potential energy is converted to kinetic energy, so it speeds up greatly. When moving away from the Sun, kinetic energy is converted back to potential energy, and the comet slows dramatically. This is consistent with conservation of energy: the total energy remains constant, but the balance between kinetic and potential energy shifts as the distance changes.

(c) True weight: $W = 2.0\text{kg} \times 9.8\text{m/s}^2 = 19.6\text{N}$

The balance reads 24.0N, which is greater than 19.6N. Since the apparent weight exceeds the true weight, the lift must be accelerating **upward** (the balance must push the mass upward more than gravity alone requires).

$$R - mg = ma$$

$a = \frac{R-mg}{m} = \frac{24.0\text{N}-19.6\text{N}}{2.0\text{kg}} = 2.2\text{m/s}^2$ upward

**Question 51**

(a) At the bottom of the loop, the centripetal acceleration is directed upward (toward the centre of the loop). The seat must push the pilot upward with a force that both supports the weight and provides the centripetal force: $R = mg + \frac{mv^2}{r}$. Because R exceeds mg, the pilot feels heavier than normal. At the top of the loop, the centripetal acceleration is directed downward. Gravity assists in providing the centripetal force, so the seat needs to push less: $R = \frac{mv^2}{r} - mg$. If the speed is close to the minimum value for the loop, R is very small, and the pilot feels almost weightless.

(b) The loaded minibus has a much greater momentum ($p = mv$) than the empty one at the same speed. Since the braking force is similar, the impulse delivered per second is similar. Because the loaded minibus has more momentum to remove, it takes a longer time to stop.

(c) For the planet: $r_p = 2r_E$ and $\rho_p = \frac{1}{2}\rho_E$

Using $g = \frac{4}{3}\pi G\rho r$:

$$\frac{g_p}{g_E} = \frac{\rho_p \times r_p}{\rho_E \times r_E} = \frac{\frac{1}{2}\rho_E \times 2r_E}{\rho_E \times r_E} = 1$$

$$g_p = g$$

The surface gravity is equal to g (the same as Earth's).

**Question 52**

(a) When two forces act on an object, the resultant depends on the angle between them. When both forces act in the same direction (0°), their magnitudes add directly: $R = 10 + 10 = 20\text{N}$. When they act in opposite directions (180°), their magnitudes subtract: $R = 10 - 10 = 0\text{N}$. At any intermediate angle, the resultant lies between these extremes, as given by the parallelogram law. Therefore the resultant ranges from 0 to 20N.

(b) Throwing distance depends on the range formula $R = \frac{u^2\sin2\theta}{g}$. On the Moon, g is smaller, so the range is greater than on the Earth for the same launch speed. However, running speed depends on how fast the legs can push the body forward, which

is not determined by gravity. The Moon's lower gravity makes each stride longer (the astronaut stays airborne longer between steps), but the maximum leg-push speed is essentially the same. Therefore, throwing range improves dramatically but running speed does not.

(c) For an elastic head-on collision between equal masses, the velocities are simply exchanged. The moving ball stops completely, and the stationary ball moves off at the original velocity.

**$v_1 = 0$m/s, $v_2 = 10$m/s (in the original direction)**

**Question 53**

(a) A lower orbit has a smaller radius R. From $T = 2\pi\sqrt{\frac{R^3}{GM}}$, a smaller R gives a shorter period T. With a shorter period, the satellite completes each orbit faster, and therefore fits more orbits into a 24-hour day. Additionally, the orbital speed $v = \sqrt{\frac{GM}{R}}$ is higher at smaller R, so the satellite moves faster along a shorter path, both effects reducing the period.

(b) The helicopter must continuously push air downward to generate an upward thrust (lift) equal to its weight. Each parcel of air pushed downward quickly moves away and must be replaced by new air. The engines power the rotor blades, which continuously accelerate air downward, giving it momentum. By Newton's third law, this produces an equal upward force on the helicopter. If the engines stopped, no air would be pushed downward, the upward force would vanish, and the helicopter would fall.

(c) For the 3.0kg block on the incline, the driving force down the incline is the component of its weight along the slope: $3.0\text{kg} \times 9.8\text{m/s}^2 \times \sin 30° = 14.7\text{N}$

For the 2.0kg hanging block, the driving force is its weight: $2.0\text{kg} \times 9.8\text{m/s}^2 = 19.6\text{N} > 14.7\text{N}$

The hanging block accelerates downward, pulling the block on the incline upward along the slope. The net driving force is:

$$F_{net} = 19.6\text{N} - 14.7\text{N} = 4.9\text{N}$$

$$a = \frac{4.9\text{N}}{(3.0 + 2.0)\text{kg}} = 0.98\text{m/s}^2$$

**Question 54**

(a) When pushing at a downward angle, the applied force has a downward component that presses the box harder into the floor. This increases the normal reaction R, which in turn increases the friction force ($f = \mu R$). As a result, more of the applied force is wasted overcoming the increased friction. When pushing horizontally, there is no downward component, so the normal reaction equals just the weight, and friction is at its minimum for the given conditions. Therefore, horizontal pushing requires less total effort to move the box.

(b) Crumple zones are designed to deform (crush) during a collision. This deformation increases the distance over which the car decelerates. By the work-energy theorem, $F \times d = \Delta KE$. Since the change in kinetic energy is fixed (determined by the initial speed), a larger deformation distance d results in a smaller average force F. Equivalently, the crumpling increases the stopping time, which by the impulse-momentum theorem ($F\Delta t = \Delta p$) reduces the force. The smaller force is transmitted to the occupants, reducing the severity of injuries.

(c) At the crest: $mg - R = \frac{mv^2}{r}$

$$R = mg - \frac{mv^2}{r} = 1400\text{kg} \times 9.8\text{m/s}^2 - \frac{1400\text{kg} \times (18\text{m/s})^2}{40\text{m}} = 2380\text{N}$$

Since $R > 0$, the car maintains contact with the road.

**Question 55**

(a) Gravitational potential energy is $U = -\frac{GMm}{R}$. As R increases (higher orbit), the denominator becomes larger, making $\frac{GMm}{R}$ smaller. Since U is the negative of this quantity, U becomes less negative (closer to zero). Physically, the satellite moves farther from the Earth and is less tightly bound, so its potential energy is higher (less negative). Energy must be supplied to move the satellite to a higher orbit, and this energy appears as an increase in potential energy.

(b) Both the ball and the satellite are acted upon by gravity alone. The key difference is the horizontal velocity. The ball has negligible horizontal velocity, so it falls in a nearly straight vertical path toward the ground. The satellite has a very large horizontal velocity (about 7.7km/s), so although it falls toward the Earth at the same rate, its forward motion carries it far enough that the curved surface of the Earth drops away beneath it at the same rate. As a result, the satellite continually falls but never gets closer to the surface. It "falls around" the Earth.

(c) Net driving force: $(5.0-3.0)\text{kg}\times 9.8\text{m/s}^2 = 19.6\text{N}$

Total mass: $(5.0+3.0)\text{kg} = 8.0\text{kg}$

$$a = \frac{19.6\text{N}}{8.0\text{kg}} = 2.45\text{m/s}^2$$

$$s = \frac{1}{2}at^2 = \frac{1}{2}\times 2.45\text{m/s}^2 \times (2.0\text{s})^2 = \frac{1}{2}\times 2.45\times 4.0 = 4.9\text{m}$$

Each mass travels 4.9m in 2.0s.

**Question 56**

(a) Both the bullet and the ball follow parabolic paths under gravity. The bullet's horizontal velocity (hundreds of metres per second) is vastly greater than the ball's. Over short distances, the bullet's time of flight is extremely small, so it falls only a tiny distance vertically before reaching the target. This tiny vertical drop is unnoticeable, making the path appear flat. The ball's much lower velocity means a much longer flight time, during which gravity pulls it through a clearly visible vertical distance, producing an obviously curved arc.

(b) At the design speed, the horizontal component of the normal reaction from the banked surface alone provides exactly the centripetal force needed for the circular path: $R\sin\theta = \frac{mv^2}{r}$. No friction is needed at all. The car follows the curve smoothly without any tendency to slide either inward or outward. The vertical component of R supports the weight: $R\cos\theta = mg$. This is the ideal condition for which the road was designed.

(c) Let $T_1$ be the tension in the ceiling string (at $50°$ to horizontal) and $T_2$ the tension in the horizontal string.

Vertically: $T_1\sin 50° = mg = 0.80\text{kg}\times 9.8\text{m/s}^2 = 7.84\text{N}$

$$T_1 = \frac{7.84\text{N}}{\sin 50°} = 10.23\text{N}$$

Horizontally: $T_2 = T_1\cos 50° = 10.23\text{N}\times 0.6428 = 6.58\text{N}$

**Question 57**

(a) Both trucks have similar braking forces (similar brakes). The loaded truck has greater mass and therefore greater momentum at the same speed ($p = mv$). Since the braking force is similar, the rate of momentum removal ($F = \frac{\Delta p}{\Delta t}$) is similar for both. Because the loaded truck has more momentum to remove, it requires a longer time and a greater distance to stop. Therefore, it must begin braking earlier.

(b) Near the surface, the variation of distance from the Earth's centre is very small compared to the Earth's radius. Because the change in r is negligible, the change in $g = \frac{GM}{r^2}$ is also negligible, and the field appears uniform. At the scale of satellite orbits, the distances involved are comparable to or larger than the Earth's radius. The variation in r is significant, so g changes noticeably with position, and the radial (non-uniform) nature of the field becomes apparent.

(c) Maximum height: $H = \frac{u_y^2}{2g}$, so $u_y = \sqrt{2gH} = \sqrt{2\times 9.8\text{m/s}^2\times 20\text{m}} = 19.80\text{m/s}$

$$\mathbf{R} = \frac{u^2\sin 2\theta}{g} = \frac{2u^2\sin\theta\cos\theta}{g} = \frac{2\times(u\cos\theta)(u\sin\theta)}{g} = \frac{\mathbf{2u_x u_y}}{\mathbf{g}}$$

So $u_x = \frac{Rg}{2u_y} = \frac{60\text{m}\times 9.8\text{m/s}^2}{2\times 19.80\text{m/s}} = 14.85\text{m/s}$

$$u = \sqrt{u_x^2+u_y^2} = \sqrt{(14.85\text{m/s})^2+(19.80\text{m/s})^2} = 24.7\text{m/s}$$

$$\theta = \tan^{-1}\left(\frac{19.80}{14.85}\right) = 53.1°$$

**Question 58**

(a) The period of a pendulum is $T = 2\pi\sqrt{\frac{L}{g}}$. On the Moon, g is smaller (about 6 times smaller) than on Earth. Because T is proportional to $\frac{1}{\sqrt{g}}$, a smaller g gives a longer period. With a longer period, each swing takes more time, so the pendulum appears to swing more slowly. The length L is the same, but the weaker gravitational restoring force means the pendulum accelerates more slowly toward the equilibrium position.

(b) After the burn, the satellite has more kinetic energy than required for its current circular orbit. The orbit becomes elliptical, with the burn point as the perigee (lowest point) of the new ellipse. The satellite climbs to a higher point (apogee) on the opposite side of the orbit. The period increases because the semi-major axis of the elliptical orbit is larger than the original circular radius.

(c) By conservation of momentum:

$$0.030\text{kg} \times 400\text{m/s} = 0.030\text{kg} \times 100\text{m/s} + 1.5\text{kg} \times \text{v}$$

$$\text{v} = \frac{9.0\text{kgm/s}}{1.5\text{kg}} = 6.0\text{m/s}$$

KE before: $\frac{1}{2} \times 0.030\text{kg} \times (400\text{m/s})^2 = 2400\text{J}$

KE after: $\frac{1}{2} \times 0.030\text{kg} \times (100\text{m/s})^2 + \frac{1}{2} \times 1.5\text{kg} \times (6.0\text{m/s})^2 = 177\text{J}$

KE lost: $2400\text{J} - 177\text{J} = 2223\text{J}$

**Question 59**

(a) The range of a jump is $\text{R} = \frac{\text{u}^2 \sin 2\theta}{\text{g}}$, where u is the takeoff speed. Running builds up a large horizontal velocity before the jump. At takeoff, this horizontal velocity combines with the vertical push from the legs to produce a higher overall launch velocity u. Since range is proportional to $\text{u}^2$, even a modest increase in launch velocity produces a significant increase in horizontal distance. From a standing position, the launch velocity comes only from the legs, giving a much smaller u and hence a much shorter range.

(b) Above the surface, the entire mass of the Earth attracts the object. As r increases, the force decreases as $\frac{1}{\text{r}^2}$ (inverse square law) because the same mass pulls from a greater distance. Below the surface, the Shell Theorem means that only the mass within radius r contributes. This contributing mass is proportional to $\text{r}^3$ (for uniform density). The field strength is $\text{g} = \frac{\text{GM}\prime}{\text{r}^2} = \frac{\text{G} \times \frac{4}{3}\pi\rho\text{r}^3}{\text{r}^2} = \frac{4}{3}\pi\text{G}\rho\text{r}$, which is proportional to r. Therefore g decreases linearly with decreasing r inside, rather than as $\frac{1}{\text{r}^2}$.

(c) $\text{T} = 12\text{h} = 43200\text{s}$

$$\text{R}^3 = \frac{\text{GMT}^2}{4\pi^2} = \frac{6.67 \times 10^{-11}\text{Nm}^2\text{kg}^{-2} \times 5.0 \times 10^{24}\text{kg} \times (43200\text{s})^2}{4 \times (3.14)^2} = 1.577 \times 10^{22}\text{m}^3$$

$$\text{R} = (1.577 \times 10^{22})^{1/3} = 2.51 \times 10^7\text{m} = 25100\text{km}$$

**Question 60**

(a) The escape velocity is derived from energy conservation: $\frac{1}{2}\text{mv}_\text{E}^2 = \frac{\text{GMm}}{\text{r}}$. The mass m of the escaping object appears on both sides and cancels, giving $\text{v}_\text{E} = \sqrt{\frac{2\text{GM}}{\text{r}}}$. Because m cancels completely, the escape velocity depends only on M (the planet's mass) and r (the planet's radius). A heavier object needs more energy to escape, but it also has proportionally more kinetic energy at the same speed. These effects balance exactly.

(b) At the bottom of the dip, the car follows a circular arc whose centre of curvature is above the road. The centripetal acceleration is directed upward. The road must push the car upward with a force greater than the weight to provide both the support against gravity and the centripetal acceleration: $\text{R} = \text{mg} + \frac{\text{mv}^2}{\text{r}}$. Because R exceeds mg, the driver's body is pushed into the seat with greater force than normal, creating the sensation of feeling heavier.

(c) Speed at the bottom of the ramp (energy conservation, smooth ramp):

$$\text{v} = \sqrt{2\text{gh}} = \sqrt{2 \times 9.8\text{m/s}^2 \times 3.0\text{m}} = 7.67\text{m/s} = \text{initial velocity on horizontal floor}$$

On the rough floor, friction decelerates the body:

$$\text{u}^2 = 2\mu_\text{k}\text{gs} \ (\text{from } \text{v}^2 = \text{u}^2 - 2\mu_\text{k}\text{gs}; \text{with } \text{v} = 0)$$

$$\text{s} = \frac{\text{u}^2}{2\mu_\text{k}\text{g}} = \frac{58.8\text{m}^2/\text{s}^2}{2 \times 0.50 \times 9.8\text{m/s}^2} = 6.0\text{m}$$

**Question 61**

(a) At the top of the circle, the water requires centripetal force directed downward (toward the centre of the circle, which is below the bucket). Gravity provides part or all of this centripetal force. If the speed is high enough that $\frac{mv^2}{r} \geq mg$, gravity alone (or gravity plus the bucket pushing down) is sufficient. The water presses against the bottom of the bucket and stays inside. If the speed is too low ($\frac{mv^2}{r} < mg$), gravity exceeds the centripetal force needed, and the water falls out of the bucket.

(b) Each spin increases the tangential speed of the hammer. The hammer moves in a circle, and with each revolution the thrower's muscles add kinetic energy to the system. After several spins, the hammer has a much higher tangential speed than after just one spin. Since the range of the hammer (once released as a projectile) is proportional to the square of the release speed ($R = \frac{u^2 \sin 2\theta}{g}$), even a modest increase in speed produces a significant increase in distance.

(c) Height after each bounce: $\mathbf{h_n = e^{2n} h_0}$

After 3rd bounce: $h_3 = (0.80)^6 \times 5m = 0.2621 \times 5m = 1.31m$

Total distance before 4th bounce:

Drop: $5m$

After 1st bounce: up $e^2 \times 5 = 0.64 \times 5 = 3.2m$, down $3.2m$

After 2nd bounce: up $e^4 \times 5 = 0.4096 \times 5 = 2.048m$, down $2.048m$

After 3rd bounce: up $1.31m$, down $1.31m$

Total $= 5 + 2(3.2 + 2.048 + 1.31) = 18.1m$

**Question 62**

(a) Momentum is $p = mv$. A lorry with large m and small v can have the same product mv as a car with small m and large v. In a collision, however, the damage depends on the kinetic energy ($\frac{1}{2}mv^2$), not just momentum. For the same momentum, the faster-moving car has more kinetic energy (because KE increases with $v^2$ or $K.E \propto \frac{1}{mass}$ for constant momentum). Therefore, the car is more dangerous in terms of the energy released during impact, even though both have the same momentum.

(b) Surface gravity is $g = \frac{GM}{r^2}$. If the radius halves while mass stays the same, the new g becomes $\frac{GM}{(r/2)^2} = \frac{4GM}{r^2} = 4g$. The person's weight would become $4 \times 700N = 2800N$, four times heavier. The person's mass remains unchanged, but the stronger surface gravity quadruples the weight.

(c) Ball's horizontal velocity: $u_x = 16m/s \times \cos 30° = 13.86m/s$

Ball's vertical velocity: $u_y = 16m/s \times \sin 30° = 8m/s$ (upward)

Time for ball to reach ground (falling $25m$ below launch):

$$-25 = 8t - \frac{1}{2} \times 9.8t^2$$

$$4.9t^2 - 8t - 25 = 0$$

$$t = 3.22s$$

Horizontal distance from building base: $x_{ball} = 13.86 \times 3.22 = 44.6m$

Person is $40m$ from the base, so must cover $44.6 - 40 = 4.6m$ (away from building) in $3.22s$:

$$v_{person} = \frac{4.6m}{3.22s} = 1.43m/s$$

**Question 63**

(a) Equilibrium requires zero net force. A single force has a definite magnitude and direction and therefore produces a non-zero net force. There is no way to make a single force sum to zero by itself. At least two forces are needed (equal and opposite) to achieve equilibrium.

(b) The parabolic path assumes that the only force acting is constant gravity directed vertically downward, and that the Earth's surface is flat. In reality, air resistance acts on the projectile, opposing its motion. This reduces the horizontal velocity (which

the parabolic model assumes is constant) and alters the vertical motion, making the trajectory asymmetric: the descending branch is steeper than the ascending branch.

(c) By conservation of momentum:

$$3\text{kg} \times 4\text{m/s} = 3\text{kg} \times 1.6\text{m/s} + 2 \times v_2$$

$$12\text{kgm/s} = 4.8\text{kgm/s} + 2v_2$$

$$v_2 = 3.6\text{m/s}$$

Coefficient of restitution:

$$e = \frac{\text{speed of separation}}{\text{speed of approach}} = \frac{v_2 - v_1}{u_1 - u_2} = \frac{3.6\text{m/s} - 1.6\text{m/s}}{4\text{m/s} - 0} = 0.50$$

**Question 64**

(a) At the moment of release, the package has the same horizontal velocity as the helicopter. After release, no horizontal force acts on it (ignoring air resistance), so it continues forward at the helicopter's speed. Since both the helicopter and the package move forward at the same horizontal velocity, the package remains directly below the helicopter throughout the fall and lands at a point directly below the helicopter's position at impact time, which is ahead of the release point as seen from the ground.

(b) On a curved track, the train requires a centripetal force directed toward the centre of the curve. By tilting (banking) the track inward, the normal reaction from the track has a horizontal component directed toward the centre. This component provides the centripetal force without relying on sideways friction between the wheels and rails. Without banking, the rails would need to push the wheels sideways, causing rapid wear on both the wheels and the track. Banking also improves passenger comfort, because the resultant of the normal reaction and gravity points more nearly through the passengers' seats rather than sideways.

(c) The ball must satisfy $y \geq 15\text{m}$ at $x = 30\text{m}$:

By using trajectory equation:

$$15 = 30\tan\theta - \frac{9.8 \times 900}{2 \times 625\cos^2\theta}$$

$$15 = 30\tan\theta - \frac{7.056}{\cos^2\theta}$$

Let $p = \tan\theta$, so $\frac{1}{\cos^2\theta} = \mathbf{1 + p^2}$**:**

$$15 = 30p - 7.056(1 + p^2)$$

$$7.056p^2 - 30p + 22.056 = 0$$

$$p = 3.307 \text{ or } p = 0.946$$

$$\theta = 73.2° \text{ or } \theta = 43.4°$$

The minimum angle to clear the wall is 43.4° (the lower of the two solutions).

**Question 65**

(a) A polar orbit passes over the North and South Poles. As the Earth rotates beneath the satellite, different strips of longitude pass under the orbital plane on each revolution. After enough orbits (12 to 14 per day for a low-orbit satellite), the entire surface has passed beneath the satellite.

An equatorial orbit stays above the equator at all times. No matter how many orbits the satellite completes, it never passes over regions at high latitudes. Therefore, it can only photograph a narrow band around the equator.

(b) The rotating station creates a centripetal acceleration directed inward (toward the rotation axis). For an astronaut standing on the outer wall, the wall pushes inward on the astronaut's feet, providing the centripetal force. The astronaut's body resists this inward push (due to inertia), pressing outward against the wall. This outward press on the feet mimics the sensation of standing on a floor under gravity. The outer wall must be used because that is where the reaction force pushes inward. On the inner wall, the geometry is reversed and no outward-pressing sensation exists.

(c) The 3kg mass pulls one way, the 2kg mass pulls the opposite way. The 5kg mass sits between them on the table.

Net driving force: $(3 - 2)\text{kg} \times 9.8\text{m/s}^2 = 9.8\text{N}$

Total mass: $(5 + 3 + 2)\text{kg} = 10\text{kg}$

$$a = \frac{9.8\text{N}}{10\text{kg}} = 0.98\text{m/s}^2$$

For the 3kg mass (accelerating downward): $3g - T_1 = 3a$

$$T_1 = 3(9.8 - 0.98) = 26.5\text{N}$$

For the 2kg mass (accelerating upward): $T_2 - 2g = 2a$

$$T_2 = 2(9.8 + 0.98) = 21.6\text{N}$$

**Question 66**

(a) In level flight, the lift force on the wings acts vertically upward, balancing the weight. When the pilot banks the wings, the lift tilts inward. The vertical component of the tilted lift still supports the weight, while the horizontal component provides a centripetal force directed toward the centre of the turn. This centripetal force changes the aircraft's direction, allowing it to curve toward the runway.

(b) The ball arrives with large momentum. To stop it, the fielder must reduce its momentum to zero, which requires an impulse ($J = \Delta p$). By the impulse-momentum theorem, $F\Delta t = \Delta p$. Drawing the hands backward increases the time $\Delta t$ over which the ball decelerates. For the same momentum change, a longer time means a smaller average force on the hands, reducing the chance of injury.

(c) By conservation of momentum:

$$m_1 v_1 = (m_1 + m_2)V$$

$$V = \frac{0.012\text{kg} \times 150\text{ms}^{-1}}{0.012\text{kg} + 1.188\text{kg}} = \frac{1.8\text{kgms}^{-1}}{1.2\text{kg}} = 1.5\text{ms}^{-1}$$

KE before: $KE_i = \frac{1}{2} \times 0.012\text{kg} \times (150\text{ms}^{-1})^2 = 135\text{J}$

KE after: $KE_f = \frac{1}{2} \times 1.2\text{kg} \times (1.5\text{ms}^{-1})^2 = 1.35\text{J}$

KE lost: $\Delta KE = 135\text{J} - 1.35\text{J} = 133.65\text{J}$

**Question 67**

(a) The horizontal range of a projectile is $R = u^2 \sin 2\theta / g$. The range depends on the square of the initial velocity. If the initial velocity is doubled ($u \to 2u$), the range becomes $R' = (2u)^2 \sin 2\theta / g = 4u^2 \sin 2\theta / g = 4R$. The range quadruples.

(b) In an elliptical orbit, the satellite moves under gravity alone. As the satellite moves closer to the Earth, it falls deeper into the gravitational potential well, losing gravitational PE. By conservation of energy, this lost PE is converted to KE, so the satellite speeds up. As it moves away, KE converts back to PE and the satellite slows down.

(c) At the maximum height (top of barrier), vertical velocity $v_y = 0$:

$$v_y^2 = u_y^2 - 2gh$$

$$u_y = \sqrt{2gh} = \sqrt{2 \times 9.8\text{ms}^{-2} \times 2.2\text{m}} = 6.57\text{ms}^{-1}$$

Horizontal velocity: $u_x = \sqrt{u^2 - u_y^2} = \sqrt{(8.0\text{ms}^{-1})^2 - (6.57\text{ms}^{-1})^2} = 4.57\text{ms}^{-1}$

Time to reach max height: $t_{up} = \frac{u_y}{g} = \frac{6.57\text{ms}^{-1}}{9.8\text{ms}^{-2}} = 0.670\text{s}$

Total time of flight: $T = 2t_{up} = 2 \times 0.670\text{s} = 1.34\text{s}$

**Question 68**

(a) Gravitational PE is defined as the work done against gravity to bring a mass from infinity (where PE is zero by convention) to the point in question. Since gravity is always attractive, a mass moving from infinity toward the attracting body moves in the direction of the gravitational force, meaning gravity does positive work. The mass therefore loses PE. Starting from zero at infinity, the PE becomes negative as the mass approaches. It can never be zero at any finite distance because gravity, though weakening with distance, never vanishes entirely.

(b) Conservation of momentum is not violated. The person pushes backward on the Earth with their foot, and by Newton's third law, the Earth pushes the person forward. The person gains forward momentum $+p$ while the Earth gains an equal backward

momentum $-p$. The total momentum of the person-Earth system remains zero. The Earth's velocity change is imperceptibly small because its mass is extremely large.

(c) At the knot, three forces act: horizontal tension $T_H$ (from the block string), tension $T_1$ at $30°$ above horizontal (to ceiling), and weight Q vertically downward.

Horizontal equilibrium at the knot:

$$T_H = T_1 \cos 30°$$

Vertical equilibrium at the knot:

$$T_1 \sin 30° = Q$$

For the block on the table, the horizontal string pulls it with force $T_H$, resisted by friction:

$$T_H \leq f_{max} = 12N$$

At maximum: $T_H = 12N$

$$T_1 = \frac{T_H}{\cos 30°} = \frac{12N}{\cos 30°} = 13.86N$$

$$Q_{max} = T_1 \sin 30° = 13.86N \times 0.5 = 6.93N$$

**Question 69**

(a) The rocket engines expel exhaust gases backward at high speed. By Newton's third law, the gases exert an equal and opposite force (thrust) on the rocket, pushing it forward. No external medium is required. The thrust depends only on the rate at which mass is ejected and the exhaust velocity: $F = v_e \times dm/dt$. In a vacuum, there is no air resistance, so the thrust is entirely available for acceleration.

(b) The Earth rotates about its axis once every 24 hours. Every part of the Earth undergoes circular motion and requires a centripetal acceleration directed toward the axis. At the equator, this centripetal acceleration is greatest (largest radius of rotation). The material at the equator tends to move outward (away from the axis), bulging the equator. At the poles, the radius of rotation is zero and there is no outward tendency. Over geological time, the Earth's material has deformed to accommodate this, producing the oblate spheroid shape.

(c) Here $\theta = 65°$ (from horizontal) and $\beta = 28°$ (uphill, positive).

Range along the incline:

$$L = \frac{u^2}{g\cos^2\beta}(\sin(2\theta - \beta) - \sin\beta)$$

$$L = \frac{(33ms^{-1})^2}{9.8ms^{-2} \times \cos^2 28°}(\sin(2 \times 65° - 28°) - \sin 28°)$$

$$L = 72.5m$$

Time of flight:

$$T = \frac{2u\sin(\theta - \beta)}{g\cos\beta} = \frac{2 \times 33ms^{-1} \times \sin(65° - 28°)}{9.8ms^{-2} \times \cos 28°}$$

$$T = 4.59s$$

**Question 70**

(a) In a perfectly inelastic collision, momentum is conserved: $m_1v_1 + m_2v_2 = (m_1 + m_2)V$. However, kinetic energy is not conserved because the collision involves permanent deformation, heat generation, and sound. The "lost" kinetic energy is converted into internal energy (deformation of the bodies, heating of materials, sound waves). The amount lost can be calculated from the difference between the initial and final kinetic energies.

(b) The combined wreckage moves in the direction of the total momentum vector of the system before impact. If vehicle 1 has momentum $p_1$ (eastward, say) and vehicle 2 has momentum $p_2$ (northward), the wreckage moves in the direction of $\vec{p}_1 + \vec{p}_2$. The angle of the skid marks from the east direction is $\tan^{-1}\left(\frac{p_2}{p_1}\right)$. Thus the angle depends on the ratio of the momenta (mv for each).

(c) Speed: $v = 72kmh^{-1} = 20ms^{-1}$

Centripetal acceleration: $a_c = \frac{v^2}{r} = \frac{(20ms^{-1})^2}{100m} = 4ms^{-2}$ (directed toward centre)

Tangential deceleration: $a_t = 5\text{ms}^{-2}$ (directed backward along path)

Net acceleration:

$$a = \sqrt{a_c^2 + a_t^2} = \sqrt{(4\text{ms}^{-2})^2 + (5\text{ms}^{-2})^2} = 6.4\text{ms}^{-2}$$

Angle from the velocity direction (toward centre):

$$\theta = \tan^{-1}\left(\frac{a_c}{a_t}\right) = \tan^{-1}\left(\frac{4}{5}\right) = 38.7°$$

The net acceleration makes $38.7°$ with the backward tangential direction, toward the centre of the circular path.

**Question 71**

(a) The rocket exerts a backward force on the exhaust gases, accelerating them rearward. By Newton's third law, the gases exert an equal forward force on the rocket. By conservation of momentum, the backward momentum of the exhaust equals the forward momentum gained by the rocket. No external medium is needed because the interaction is between the rocket and its own exhaust, not between the rocket and anything external.

(b) The total energy of a satellite in orbit is $E = KE + PE$, where PE is negative. When the engines fire, they add energy to the satellite. As the satellite moves to a higher orbit, it moves further from the Earth and its gravitational PE increases (becomes less negative). For a circular orbit, $KE = -E$ and $PE = 2E$, so the PE increase is twice the total energy increase while the KE actually decreases. The extra energy comes from the fuel burned by the engines.

(c) By conservation of momentum (cannon + ball initially at rest):

$$0 = m_{ball}v_{ball} + m_{cannon}v_{cannon}$$

The muzzle speed $55\text{ms}^{-1}$ is relative to the cannon. Let the cannon's recoil velocity be $v_c$ (backward) and the ball's velocity relative to Earth be $v_b$ (forward):

$$v_b - (-v_c) = 55\text{ms}^{-1}$$

so $v_b + v_c = 55\text{ms}^{-1}$ … (1)

$72v_b = 1300v_c$ … (2)

From (2): $v_b = 18.06v_c$. Substituting into (1):

$$18.06v_c + v_c = 55$$

$\rightarrow v_c = \frac{55}{19.06} = 2.89\text{ms}^{-1}$

$v_b = 52.1\text{ms}^{-1}$

**Question 72**

(a) As the insect slides down the curved surface, it gains speed. At any point, Newton's second law in the radial direction gives: $mg\cos\theta - R = \frac{mv^2}{r}$, where $\theta$ is measured from the vertical and R is the normal reaction. As v increases and $\cos\theta$ decreases, R must decrease. The insect loses contact when $R = 0$, which occurs when $mg\cos\theta = \frac{mv^2}{r}$. Using energy conservation to relate v to $\theta$, this gives $\cos\theta = \frac{2}{3}$, or $\theta \approx 48°$.

(b) At the neutral point, the gravitational pulls from the two bodies are equal and opposite. If d is the distance between centres and x is the distance from the larger body to the neutral point: $\frac{GM}{x^2} = \frac{Gm}{(d-x)^2}$, giving $\frac{x}{d-x} = \sqrt{\frac{M}{m}}$. Since $M > m$, $x > d - x$, meaning the neutral point is closer to the less massive body. The lighter body needs less distance for its weaker field to match the stronger field of the heavier body.

(c) Let x be the distance from the Earth's centre:

$$\frac{M_E}{x^2} = \frac{M_M}{(d-x)^2}$$

$$\frac{x}{d-x} = \sqrt{\frac{M_E}{M_M}} = \sqrt{81} = 9$$

$$x = 9(d-x) = 9d - 9x$$

$\rightarrow 10x = 9d \rightarrow x = 0.9d = 0.9 \times 4 \times 10^5 km = 3.6 \times 10^5 km$

**Question 73**

(a) Water leaving the nozzle is a projectile: gravity immediately pulls it downward. If aimed directly at the window, the water would follow a parabolic path that curves below the target and miss. By aiming above the window, the fireman gives the water an upward component of velocity that compensates for the downward pull of gravity during the flight. The water rises initially, then curves downward through the window at the correct height.

(b) The bullet is a projectile. From the moment it leaves the barrel, gravity pulls it downward. Over 200m, the bullet drops by $h = \frac{1}{2}gt^2$. Since the bullet is moving with very high velocity ( say $1000 ms^{-1}$), the flight time is over 200m is too small ($t = \frac{200}{1000} = 0.2s$), giving very small height drop ($h = \frac{1}{2} \times 9.8 \times 0.04 = 0.196m \approx 20cm$). This height drop (20cm) over 200m is too small to notice with the naked eye, which is why the path appears straight. But it is genuinely curved.

(c) The water must reach a point $40m$ horizontally (38m + 2m) and $20m$ vertically. Using the trajectory equation:

$$y = x\tan\theta - \frac{gx^2}{2u^2\cos^2\theta}$$

$$20 = 40\tan 60° - \frac{9.8 \times (40)^2}{2u^2\cos^2 60°}$$

$$u^2 = 636.4 m^2 s^{-2}$$

$$u = 25.2 ms^{-1}$$

**Question 74**

(a) A projectile's vertical position is $y = u_y t - \frac{1}{2}gt^2$. Setting $y = 7m$ gives a quadratic equation in $t$ with two solutions: one on the way up and one on the way down. These two times correspond to two horizontal positions ($x_1 = u_x t_1$ and $x_2 = u_x t_2$). The walls must be placed at precisely these two positions. For any height below the maximum, there are always exactly two such points (the trajectory is symmetric about its peak for level-ground launch).

(b) Each board removes the same fraction (1/20) of the current speed, not the same absolute amount. After the first board, the speed is $19v/20$. After the second, $(19/20)^2 v$. After n boards, the speed is $(19/20)^n v$. The kinetic energy is proportional to $v^2$, so it decreases as $(19/20)^{2n}$. Each successive board encounters a slower bullet with less KE. However, each board absorbs the same fraction of the remaining speed, meaning the absolute KE removed per board decreases. The bullet is stopped when its KE is fully absorbed. Because the percentage loss compounds geometrically, far fewer than 20 boards are needed.

(c) $u_x = 100\cos 60° = 50 ms^{-1}$, $u_y = 100\sin 60° = 86.6 ms^{-1}$

At height $y = 7m$: $7 = 86.6t - 4.9t^2$

$$4.9t^2 - 86.6t + 7 = 0$$

$t_1 = 0.0816s$ and $t_2 = 17.59s$

Distance between walls:

$$d = u_x(t_2 - t_1) = 50 ms^{-1} \times (17.59 - 0.0816)s = 875m$$

Alternatively, you can solve it by using trajectory equation to get $x_1$ and $x_2$ and then calculate $x_2 - x_1$ to get the final answer.

**Question 75**

(a) In static equilibrium, every part of the system is at rest. Since $p = mv$ and $v = 0$ for every component, the total momentum is zero and remains zero. If a connection suddenly breaks, the internal forces that maintained equilibrium vanish. Each freed part now has a net unbalanced external force (typically its weight) acting on it. By Newton's second law, each part accelerates and gains momentum. However, the total momentum of the entire system remains zero because no new external force acts on the system as a whole.

(b) Escape velocity is derived from energy conservation: $\frac{1}{2}mv_{esc}^2 = GMm/R$. The mass m of the escaping object cancels on both sides, giving $v_{esc} = \sqrt{2GM/R}$. This depends only on the planet's mass and radius, not on the direction. The direction of launch does not matter because gravity is a conservative force: the work done against gravity depends only on the initial and final positions (surface to infinity), not on the path taken.

(c)(i) Taking the initial direction of the ball as positive:

$$J = \Delta p = m(v_f - v_i) = 0.060kg \times (-45 ms^{-1} - 35 ms^{-1}) = -4.8 kgms^{-1}$$

The impulse has magnitude $4.8\text{kgms}^{-1}$, directed opposite to the initial motion (toward the racket side).

(ii) Average force:

$$F_{avg} = \frac{J}{\Delta t} = \frac{4.8\text{kgms}^{-1}}{0.006\text{s}} = 800\text{N}$$

**Question 76**

(a) From Newton's second law in its general form, thrust $= v_e \times \frac{dm}{dt}$, where $v_e$ is the exhaust velocity and $\frac{dm}{dt}$ is the rate of mass ejection. Higher exhaust speed or faster mass ejection produces greater thrust. As fuel is consumed, the total mass M of the rocket decreases. Since $a = \frac{F_{net}}{M}$ and the thrust is constant, a decreasing M produces an increasing acceleration.

(b) At the equilibrium point, $\frac{GM_E}{x^2} = \frac{GM_S}{(d-x)^2}$, giving $\frac{x}{(d-x)} = \sqrt{\frac{M_E}{M_S}}$. Since $M_S \gg M_E$ (the Sun is about 333,000 times more massive), $\sqrt{\frac{M_E}{M_S}} \ll 1$, so $x \ll d - x$. The equilibrium point lies very close to the Earth because the Earth's much weaker field can only match the Sun's field at a point where the Earth is nearby and the Sun is very far away.

(c)(i) For liftoff, thrust ≥ total weight:

$$v_e \times \frac{dm}{dt} \geq (M + m_{fuel})g$$

$$\frac{dm}{dt} \geq \frac{(20 + 180)\text{kg} \times 9.8\text{ms}^{-2}}{1600\text{ms}^{-1}} = 1.225\text{kgs}^{-1}$$

The minimum rate of fuel consumption is $1.225\text{kgs}^{-1}$.

(c)(ii) Burn time: $t = \frac{180\text{kg}}{2.0\text{kgs}^{-1}} = 90\text{s}$

Using the rocket equation:

$$v = v_e \ln\frac{M_0}{M_f} - gt = 1600\text{ms}^{-1} \times \ln\frac{200\text{kg}}{20\text{kg}} - 9.8\text{ms}^{-2} \times 90\text{s} = 2803\text{ms}^{-1} \approx 2.8\text{kms}^{-1}$$

**Question 77**

(a) Static friction is a self-adjusting force: it takes whatever value is needed to prevent motion, up to its maximum $f_{max} = \mu_s R$. At a small angle, the component of gravity along the plane ($mg\sin\theta$) is small, and static friction matches it exactly, keeping the block stationary. As the angle increases, $mg\sin\theta$ grows and friction increases to match. The block remains stationary as long as $mg\sin\theta \leq \mu_s mg\cos\theta$. At the critical angle ($\tan\theta_c = \mu_s$), friction reaches its maximum and can no longer match the gravitational component. Beyond this angle, the block slides.

(b) The horizontal range is $R = u^2\sin 2\theta/g$. The range depends on $\sin 2\theta$, not on $\theta$ alone. The sine function has the property that $\sin 2\theta = \sin(180° - 2\theta)$, which means $\sin 2\theta_1 = \sin 2\theta_2$ whenever $\theta_1 + \theta_2 = 90°$. For example, $\sin 60° = \sin 120°$, so 30° and 60° give the same range. Physically, the steep angle (60°) gives a high trajectory with long flight time but small horizontal speed, while the shallow angle (30°) gives a low trajectory with short flight time but large horizontal speed. The two effects compensate exactly for complementary angles.

(c)(i) At projection, the ball has: $v_x = 8.5\text{ms}^{-1}$ (horizontal) and $v_y = +14.7\text{ms}^{-1}$ (upward, inherited from helicopter).

Vertical displacement to ground: $-196\text{m}$ (downward).

$$-196 = 14.7t - \frac{1}{2}(9.8)t^2$$

$$4.9t^2 - 14.7t - 196 = 0$$

$$t = 8\text{s}$$

(c)(ii) Horizontal distance: $x = v_x \times t = 8.5\text{ms}^{-1} \times 8\text{s} = 68\text{m}$

(c)(iii) At ground: $v_x = 8.5\text{ms}^{-1}$, $v_y = 14.7\text{ms}^{-1} - 9.8\text{ms}^{-2} \times 8\text{s} = -63.7\text{ms}^{-1}$

$$v = \sqrt{v_x^2 + v_y^2} = 64.3\text{ms}^{-1}$$

**Question 78**

(a) After a perfectly inelastic collision, the wreckage moves in the direction of the total momentum. If two vehicles of equal mass approach at right angles, one with speed $v_1$ and the other with speed $v_2$, the total momentum has components $mv_1$ and $mv_2$ along the two original directions. The skid marks point along the resultant momentum, at an angle $\tan\theta = \frac{mv_2}{mv_1} = \frac{v_2}{v_1}$ from the first vehicle's direction. Since the masses are equal, they cancel, and hence the angle directly reveals the speed ratio.

(b) The elevator is moving downward and decelerating (slowing down), which means its acceleration is directed upward. By Newton's second law applied to a passenger: $R - mg = ma$ (upward acceleration), so $R = m(g + a)$. The scale reading R exceeds the true weight mg. The passengers feel heavier during this phase than when the elevator moves at constant speed (where $R = mg$).

(c)(i) The skid marks are at $55°$ north of east, meaning the angle is measured from the eastward direction. The eastward momentum ($m \times 13$) is the adjacent component and the northward momentum ($m \times v$) is the opposite component:

$$\tan 55° = \frac{mv}{m \times 13} = \frac{v}{13}$$

$$v = 13\text{ms}^{-1} \times \tan 55° = 18.6\text{ms}^{-1}$$

(c)(ii) The speed limit is $15.6\text{ms}^{-1}$. Since $v = 18.6\text{ms}^{-1} > 15.6\text{ms}^{-1}$, the northward-moving driver was **exceeding the speed limit** by $3\text{ms}^{-1}$.

**Question 79**

(a) The variation of g with altitude is $g' = g\left(R_E/(R_E + h)\right)^2$. For typical projectile heights h are very small compared to Earth's radius, making the change in g negligible. So the constant-g approximation introduces an error far smaller than other neglected effects (air resistance, wind), and hence it is entirely justified for near-surface problems.

(b) Using $g' = g\left(\frac{R_E}{R_E+h}\right)^2 : \frac{R_E+h}{R_E} = \sqrt{\frac{g}{g'}}$, giving $h = R_E\left(\sqrt{\frac{g}{g'}} - 1\right)$ from which the height of the mountain can be estimated. For example, with $g = 9.81$ and $g' = 9.70$: $h = 6400\text{km} \times \left(\sqrt{\frac{9.81}{9.70}} - 1\right) = 36.3\text{km}$. One reason the actual value might differ: the density of rock beneath the mountain differs from the average Earth density, so the local gravitational field is affected by the mass distribution of nearby geological structures.

(c)(i) The ball starts at $1000\text{m}$ and must reach $400\text{m}$: vertical drop $= 600\text{m}$.

$$-600 = u_y t - \frac{1}{2}gt^2$$

$$u_y = 80\sin 60° = 69.3\text{ms}^{-1}$$

$$4.9t^2 - 69.3t - 600 = 0$$

$$t = 20.2\text{s}$$

(c)(ii) $u_x = 80\cos 60° = 40\text{ms}^{-1}$

$$x = u_x \times t = 40\text{ms}^{-1} \times 20.2\text{s} = 808\text{m}$$

**Question 80**

(a) Gravity acts on the satellite directed toward the Earth's centre, which is perpendicular to the satellite's velocity (tangent to the orbit). Work done is $W = F \cdot d\cos\theta$. Since $\theta = 90°$, $\cos 90° = 0$, and gravity does zero work on the satellite. By the work-energy theorem, zero work means zero change in kinetic energy. Therefore, the speed remains constant even though a force acts continuously.

(b) Speed is the magnitude of velocity (a scalar), while average velocity is displacement divided by time (a vector). Over a semicircle of radius r, the distance travelled is $\pi r$ but the displacement is $2r$ (a straight line from start to end). Average speed $= \frac{\pi r}{t}$ while average velocity $= \frac{2r}{t}$. Since $\pi > 2$, average speed exceeds the magnitude of average velocity. In circular motion, the speed may be constant while the velocity changes direction continuously.

(c)(i) Time for semicircle: $t = \frac{\pi r}{v} = \pi \times \frac{6\text{m}}{12\text{ms}^{-1}} = \frac{\pi}{2} = 1.571\text{s}$

Displacement: $2r = 12\text{m}$ (diameter, from A to B)

Average velocity: $v_{avg} = \frac{12\text{m}}{1.571\text{s}} = 7.64\text{ms}^{-1}$ (directed from A to B)

(c)(ii) For circular orbit: $\frac{GM_E M_s}{r^2} = \frac{M_s v^2}{r}$, so $v = \sqrt{\frac{GM_E}{r}}$.

Angular momentum: $L = M_s vr = M_s r\sqrt{\frac{GM_E}{r}} = M_s\sqrt{GM_E r}$

**Question 81**

(a) A vertical launch gives the satellite only radial velocity (directed away from Earth). For a circular orbit, the satellite needs a velocity entirely tangential to the orbit (perpendicular to the radius). A vertically launched object will rise, slow down, stop, and fall back along the same radial line. To enter a circular orbit, the satellite must be turned so that its velocity is horizontal at the orbital altitude, which requires a separate horizontal impulse.

(b) Orbital speed is $v_{orb} = \sqrt{\frac{GM}{r}}$ and escape speed is $v_{esc} = \sqrt{\frac{2GM}{r}} = \sqrt{2} \times v_{orb}$. The additional speed needed is $v_{esc} - v_{orb} = (\sqrt{2} - 1)v_{orb} = 0.414v_{orb}$. This is only 41.4% of the orbital speed, which is less than the orbital speed itself. The satellite already has most of the energy needed to escape.

(c)(i) Orbital radius: $r = R_E + h = 6.4 \times 10^6 m + 5 \times 10^5 m = 6.9 \times 10^6 m$

$$v_{orb} = \sqrt{\frac{GM_E}{r}} = \sqrt{\frac{6.67 \times 10^{-11} \times 5.97 \times 10^{24}}{6.9 \times 10^6}} = 7597 ms^{-1} \approx 7.6 kms^{-1}$$

(c)(ii) The satellite has vertical momentum $p_v = 50kg \times 2000 ms^{-1} = 1 \times 10^5 kgms^{-1}$ and needs horizontal momentum $p_h = 50kg \times 7597 ms^{-1} = 3.799 \times 10^5 kgms^{-1}$ with zero vertical momentum.

The impulse must cancel the vertical momentum and provide the horizontal momentum:

$$J = \sqrt{p_v^2 + p_h^2} = \sqrt{(10^5 kgms^{-1})^2 + (3.799 \times 10^5 kgms^{-1})^2} = 3.93 \times 10^5 kgms^{-1}$$

**Question 82**

(a) In the reference frame of one projectile, the other approaches in a straight line (because both experience the same gravitational acceleration g, which cancels in the relative motion). The relative velocity is constant (a straight line), so the collision condition depends only on whether the initial relative velocity points from one launch point to the other. This condition involves only the initial velocities and positions, not g. Hence, the collision condition is independent of g.

(b) The 10m diver hits the water with greater speed ($v = \sqrt{2gh}$) than the 3m diver. Greater speed means greater momentum at entry. The water must reduce this momentum to zero. By the impulse-momentum theorem ($F_{avg} \times \Delta t = \Delta p$), a larger momentum change requires a larger impulse. If the stopping time $\Delta t$ is similar in both cases (comparable depth of submersion), the average force on the 10m diver must be larger.

(c) Let the velocity of A be $u_A$ at 30° from height 2m. At collision time t:

Position of A: $x_A = u_A cos30° \times t$, $y_A = 2 + u_A sin30° \times t - \frac{1}{2}gt^2$

Position of B (from ground, directly below A): $x_B = 30cos60° \times t = 15t$, $y_B = 30sin60° \times t - \frac{1}{2}gt^2 = 25.98t - 4.9t^2$

For collision: $y_A = y_B$ and $x_A = x_B$ (but A moves horizontally, B moves at angle — they must meet at same point).

Since both experience the same g, the $\frac{1}{2}gt^2$ terms cancel in the vertical equation:

$$2 + u_A sin30° \times t = 30sin60° \times t$$

$$2 = (30sin60° - u_A sin30°)t = (25.98 - 0.5u_A)t$$

From horizontal: $u_A cos30° \times t = 30cos60° \times t \rightarrow u_A = \frac{30cos60°}{cos30°} = 17.32 ms^{-1}$

Substituting back: $2 = (25.98 - 0.5 \times 17.32)t = (25.98 - 8.66)t = 17.32t$

$$t = \frac{2}{17.32} = 0.115s$$

**Question 83**

(a) The bullet is a projectile: gravity pulls it downward throughout its flight. If aimed directly at the target, the bullet follows a parabolic path that curves below the line of aim. Over the flight time $t = \frac{x}{v}$, the bullet drops $h = \frac{1}{2}gt^2$. To compensate, the marksman aims slightly above the target so the bullet's parabolic path intersects the target at the correct height.

(b) Yes, the bullet hits the monkey. At the instant of firing, the bullet is aimed directly at the monkey. Both the bullet and the monkey begin falling under gravity at the same instant. In time t, both fall the same vertical distance $\frac{1}{2}gt^2$ below their initial positions. The bullet's path would have pointed directly at the monkey's original position if gravity were absent. However, since both the bullet and the monkey fall by the same amount under gravity, their relative positions remain aligned. Therefore, the bullet and the monkey meet at the same point below the original line of aim.

(c)(i) Time of flight: $t = \frac{x}{v} = \frac{200m}{1000ms^{-1}} = 0.2s$

Drop: $h = \frac{1}{2}gt^2 = \frac{1}{2} \times 9.8ms^{-2} \times (0.2s)^2 = 0.196m = 19.6cm$

(c)(ii) The marksman must aim $19.6cm$ above the target. At this small angle, the upward component compensates for the gravitational drop during the flight time. The bullet rises $19.6cm$ above the direct line while simultaneously dropping $19.6cm$ due to gravity, arriving at target height.

**Question 84**

(a) At the equator, a person undergoes circular motion with the Earth's rotation. The centripetal acceleration required is $a_c = \omega^2 R_E$, directed toward the Earth's axis. The normal reaction R (apparent weight) and the true weight mg must provide this centripetal acceleration: $mg - R = m\omega^2 R_E$, so $R = m(g - \omega^2 R_E) < mg$. At the North Pole, the person is on the rotation axis ($r = 0$), requires no centripetal acceleration, and $R = mg$.

(b) Escape speed is $v_{esc} = \sqrt{2}v_{orb}$. An orbiting spaceship at speed $v_{orb}$ needs to reach $\sqrt{2}v_{orb} = 1.414v_{orb}$. The additional speed is $0.414v_{orb}$, which is $41.4\%$ of the orbital speed. This is because the spaceship already possesses significant kinetic energy in orbit. Escape requires only the additional energy to bring the total mechanical energy from negative (bound orbit) to zero (just escaping).

(c)(i) For zero apparent weight: $mg = m\omega^2 R_E$, so $\omega = \sqrt{\frac{g}{R_E}}$

$$\omega = \sqrt{\frac{9.8ms^{-2}}{6.4 \times 10^6 m}} = 1.238 \times 10^{-3} rads^{-1}$$

$$T = \frac{2\pi}{\omega} = \frac{2\pi}{1.238 \times 10^{-3} rads^{-1}} = 5075s = 1.41 \text{ hours}$$

(c)(ii) $v_{esc} = \sqrt{2} \times v_{orb} = 1.414 \times 7.9kms^{-1} = 11.17kms^{-1}$

Additional speed: $\Delta v = (11.17 - 7.9)kms^{-1} = 3.27kms^{-1}$

**Question 85**

(a) Gravitational PE is zero at infinity (by convention) and negative at all finite distances because work must be done against gravity to move from any point to infinity. The negative value represents a bound state: the body is trapped in the gravitational well. To escape, the body must be given enough kinetic energy to climb out of the well entirely, reaching zero total energy at infinity. The minimum KE needed equals the magnitude of the PE at the surface: $KE_{min} = \frac{GMm}{R}$.

(b) At launch, the rocket has kinetic energy $KE_0 = \frac{1}{2}mv^2$. As the rocket rises, this energy is converted into two forms: gravitational potential energy (mgh) gained by climbing to height h, and thermal energy lost to atmospheric drag ($E_{drag}$). By conservation of energy:

$$\frac{1}{2}mv^2 = mgh_{max} + E_{drag}$$

The rocket reaches its maximum height when all the initial KE has been used up. Rearranging:

$$h_{max} = \frac{v^2}{2g} - \frac{E_{drag}}{mg}$$

The first term is the height the rocket would reach without drag. The second term is the height reduction caused by drag. The greater the fraction of KE lost to drag, the lower the maximum height.

(c)(i) Work to move to infinity:

$$W = \frac{GM_E m}{R_E} = \frac{6.67 \times 10^{-11} Nm^2 kg^{-2} \times 6 \times 10^{24} kg \times 10kg}{6.4 \times 10^6 m} = 6.25 \times 10^8 J$$

(c)(ii) By energy conservation: $\frac{1}{2}mv^2 = W$

$$v = \sqrt{\frac{2W}{m}} = \sqrt{\frac{2 \times 6.25 \times 10^8 J}{10kg}} = 11.2 kms^{-1}$$

(c)(iii) Initial KE: $\frac{1}{2}mv^2 = \frac{1}{2}m(2000ms^{-1})^2 = 2 \times 10^6 m$ J

After drag loss (20%): $KE_{remaining} = \frac{80}{100} \times 2 \times 10^6 m = 1.6 \times 10^6 m$ J

$$mgh = 1.6 \times 10^6 m$$

$$h = \frac{1.6 \times 10^6 m}{g_{Mars}} = \frac{1.6 \times 10^6 m}{3.7 ms^{-2}} = 4.32 \times 10^5 m = 432 km$$

**Question 86**

(a) By Newton's first law, an object in motion continues in a straight line unless acted upon by a force. During circular motion, the centripetal force continuously deflects the object inward toward the centre. At the instant the string breaks, the centripetal force vanishes. No inward force acts, so the object continues with whatever velocity it had at that instant. That velocity is tangent to the circle (perpendicular to the radius).

Note: There is no outward force and never was; the sensation of being "thrown outward" is simply the object continuing in a straight line while the circular constraint is removed.

(b) Braking: the friction force $f = \mu mg$ decelerates the truck. Using $v^2 = 2as$, the stopping distance is $s = \frac{v^2}{2\mu g}$.

Swerving: the same friction provides centripetal force $\mu mg = \frac{mv^2}{r}$, giving the minimum radius $r_{min} = \frac{v^2}{\mu g}$.

Comparing: $s = \frac{v^2}{2\mu g} = \frac{r_{min}}{2}$. The stopping distance is half the minimum turning radius. Since the wall is at distance r, braking requires $r > \frac{v^2}{2\mu g}$ while swerving requires $r > \frac{v^2}{\mu g}$.

Hence, **braking is the better option** as it needs only half the distance.

(c) After the string breaks, the stone is a horizontal projectile from height $h = 2m$.

Time to hit ground:

$$t = \sqrt{\frac{2h}{g}} = \sqrt{\frac{2 \times 2m}{9.8 ms^{-2}}} = \sqrt{0.4082 s^2} = 0.639 s$$

Horizontal speed at break:

$$v = \frac{x}{t} = \frac{10m}{0.639s} = 15.65 ms^{-1}$$

Centripetal acceleration:

$$a_c = \frac{v^2}{r} = \frac{(15.65 ms^{-1})^2}{1.5m} = \frac{244.9 m^2 s^{-2}}{1.5m} = 163 ms^{-2}$$

**Question 87**

(a) Each star moves in a circle about the common centre of mass. Circular motion requires a centripetal force directed toward the centre of the circle, which is the centre of mass. This force is provided by the gravitational attraction of the other star. By the definition of centre of mass, $m_1 r_1 = m_2 r_2$, where $r_1$ and $r_2$ are the orbital radii. The less massive star orbits at a larger radius, and the more massive star orbits at a smaller radius.

(b) The centre of mass lies at $r_1 = \frac{m_2 d}{m_1 + m_2}$ from the heavier star. If $m_1 \gg m_2$, the centre of mass lies very close to the heavier star. The heavier star barely moves (tiny orbit) while the lighter star traces a large orbit. In the Earth-Sun system, the Sun is about 333,000 times the Earth's mass. The centre of mass lies inside the Sun, so the Sun wobbles imperceptibly while the Earth traces a large orbit. This is why we say the Earth orbits the Sun.

(c)(i) At height h: $g' = g\left(\frac{R_E}{R_E + h}\right)^2$

For $g' = \frac{g}{2}$:

$$\frac{1}{2} = \frac{R_E^2}{(R_E + h)^2}$$

$$(R_E + h)^2 = 2R_E^2$$

$$R_E + h = R_E\sqrt{2}$$

$$h = R_E(\sqrt{2} - 1) = 6400\text{km} \times 0.4142 = 2651\text{km}$$

(c)(ii) For geostationary orbit: $T = 9.84 \times 3600\text{s} = 35424\text{s}$

$$R^3 = \frac{GMT^2}{4\pi^2} = \frac{6.67 \times 10^{-11}\text{Nm}^2\text{kg}^{-2} \times 1.9 \times 10^{27}\text{kg} \times (35{,}424\text{s})^2}{4\pi^2}$$

$$R^3 = 4.028 \times 10^{24}\text{m}^3$$

$$R = (4.028 \times 10^{24}\text{m}^3)^{1/3} = 1.591 \times 10^8\text{m}$$

Altitude: $h = R - R_J = 1.591 \times 10^8\text{m} - 6.99 \times 10^7\text{m} = 8.92 \times 10^7\text{m} = 89200\text{km}$

**Question 88**

(a) Momentum is a vector quantity. The total momentum is the vector sum of all individual momenta. For the total to be zero, the momentum vectors must cancel when added. This allows individual fragments to have large momenta provided they point in directions that sum to zero. For example, in a two-fragment explosion, the fragments must fly in exactly opposite directions with equal magnitudes; whereas In a three-fragment explosion, the three momentum vectors must form a closed triangle.

(b) Both the person and the briefcase are inside the lift. If the briefcase does not fall relative to the person, it means the briefcase has the same acceleration as the person and the lift. The only way a released object can remain stationary relative to its surroundings is if everything is in free fall together: the lift, the person, and the briefcase all accelerate downward at g. The lift cable must have snapped or the lift is in free fall with $a = g$.

(c) At the instant of explosion, the bomb is at rest (just cut from cable, before gravity acts appreciably). Total initial momentum is zero.

x-direction: $0 = \frac{M}{2}V_0 + \frac{M}{3}(2V_0)\cos 37° + \frac{M}{6}v_x$

$$0 = \frac{V_0}{2} + \frac{2V_0 \times 0.7986}{3} + \frac{v_x}{6} = 0.5V_0 + 0.5324V_0 + \frac{v_x}{6}$$

$$v_x = -6 \times 1.0324V_0 = -6.195V_0$$

y-direction: $0 = 0 + \frac{M}{3}(2V_0)\sin 37° + \frac{M}{6}v_y$

$$0 = \frac{2V_0 \times 0.6018}{3} + \frac{v_y}{6} = 0.4012V_0 + \frac{v_y}{6}$$

$$v_y = -6 \times 0.4012V_0 = -2.407V_0$$

Speed: $\mathbf{v_3} = \sqrt{v_x^2 + v_y^2} = \sqrt{(6.195V_0)^2 + (2.407V_0)^2} = \mathbf{6.65V_0}$

Direction: $\theta = \tan^{-1}\left(\frac{-2.407}{-6.195}\right) = \tan^{-1}(0.3885) = 21.2°$ below the negative x-axis.

The third piece moves at $6.65V_0$ at $21.2°$ below the negative x-direction (into the third quadrant).

**Question 89**

(a) Maximum height depends on the vertical component of initial velocity: $H = \frac{u^2\sin^2\theta}{2g}$. An initial velocity at steeper angle gives a larger vertical component $u\sin\theta$, and hence greater height. The horizontal range is $R = \frac{u^2\sin 2\theta}{g}$. For a given speed u, the range depends on $\sin 2\theta$, which has a maximum value of 1 when $2\theta = 90°$ (i.e. $\theta = 45°$). No angle can make $\sin 2\theta > 1$, so the range cannot exceed $\frac{u^2}{g}$ regardless of the angle chosen.

(b) The rotor blades push a mass of air downward each second, giving it momentum. By Newton's third law, the air pushes upward on the rotor with an equal force. This upward force is the thrust. The thrust equals the rate of momentum given to the air: $F =$

$\left(\frac{dm}{dt}\right) \times v$, where $\frac{dm}{dt}$ is the mass of air pushed down per second and v is its downward speed. For hovering, this thrust must equal the helicopter's weight.

(c)(i) The ball must not exceed height $25m$. Maximum height: $H = \frac{u^2 \sin^2\theta}{2g}$.

Setting $H = 25m$: $\sin^2\theta = \frac{2gH}{u^2} = 2 \times 9.8ms^{-2} \times \frac{25m}{(40ms^{-1})^2} = 0.30625$

$$\sin\theta = 0.5534$$

$$\theta = 33.6°$$

Range at this angle:

$$R = \frac{u^2 \sin 2\theta}{g} = \frac{(40ms^{-1})^2 \times \sin 67.2°}{9.8ms^{-2}} = 150.5m$$

(c)(ii) Volume of air pushed down per second:

$$V = A \times v = \pi(2m)^2 \times 8ms^{-1} = 32\pi\, m^3 s^{-1}$$

Mass per second: $\frac{dm}{dt} = \rho V = 1.3kgm^{-3} \times 32\pi\, m^3 s^{-1} = 130.7kgs^{-1}$

Thrust: $F = \left(\frac{dm}{dt}\right) \times v = 130.7kgs^{-1} \times 8ms^{-1} = 1045.6N$

For hovering: $mg = F$

$$m = \frac{F}{g} = \frac{1045.6N}{9.8ms^{-2}} = 106.7kg$$

**Question 90**

(a) The rope has a vertical component pulling upward on the object: $F\sin\theta$ (where $\theta$ is the angle above horizontal). This partially supports the weight. The surface needs to provide less normal reaction: $R = mg - F\sin\theta$. Since kinetic friction is $f = \mu R$, a smaller R means a smaller friction force. Pulling at an angle therefore reduces friction, making the object easier to move than if pulled horizontally.

(b) On a banked curve, the road surface tilts inward. The normal reaction R acts perpendicular to the road surface, which is no longer vertical. Its horizontal component $R\sin\theta$ (where $\theta$ is the banking angle) points toward the centre of the curve, providing the centripetal force. Its vertical component $R\cos\theta$ supports the weight. No friction is needed when $\tan\theta = \frac{v^2}{rg}$: the geometry of the bank naturally provides exactly the right centripetal force.

(c)(i) At constant velocity, net force is zero. Horizontal: $F\cos 30° = \mu R$. Vertical: $R = mg - F\sin 30°$.

Substituting:

$$F\cos 30° = \mu(mg - F\sin 30°)$$

$$F\cos 30° + \mu F\sin 30° = \mu mg$$

$$F(\cos 30° + \mu \sin 30°) = \mu mg$$

$$F = \frac{\mu mg}{(\cos 30° + \mu \sin 30°)} = \frac{0.57 \times 20kg \times 9.8ms^{-2}}{0.866 + 0.57 \times 0.5} = 97.1N$$

(c)(ii) Speed: $v = 72kmh^{-1} = 20ms^{-1}$

For banking without friction: $\tan\theta = \frac{v^2}{rg}$

$$\tan\theta = \frac{(20ms^{-1})^2}{50m \times 9.8ms^{-2}} = 0.8163$$

$$\theta = \tan^{-1}(0.8163) = 39.2°$$

**Question 91**

(a) The range is $R = u^2 \sin 2\theta / g$. For a given u, the range depends on $\sin 2\theta$, which varies with $\theta$. Different angles give different values of $\sin 2\theta$ and hence different ranges. However, $\sin 2\theta = \sin(180° - 2\theta)$, so two angles $\theta_1$ and $\theta_2$ give the same range whenever $\theta_1 + \theta_2 = 90°$. These are called complementary angles (e.g. 30° and 60°, or 20° and 70°).

(b) The gravel lands on the belt with zero horizontal velocity. The belt moves at constant speed v. To bring each piece of gravel from rest to speed v, its horizontal momentum must increase by mv. By the impulse-momentum theorem, a horizontal force must act on the gravel over some time interval. This force comes from kinetic friction between the belt surface and the gravel. The belt's motor must supply a driving force to compensate for this friction and maintain constant speed.

(c)(i) Equal vertical components: $u_P \sin\alpha = u_Q \sin\beta$.

$$R_P = \frac{2u_P^2 \sin\alpha\cos\alpha}{g}, \quad R_Q = \frac{2u_Q^2 \sin\beta\cos\beta}{g}$$

$$\frac{R_P}{R_Q} = \frac{u_P^2 \sin\alpha\cos\alpha}{u_Q^2 \sin\beta\cos\beta} = \frac{u_P(u_P \sin\alpha)\cos\alpha}{u_Q(u_Q \sin\beta)\cos\beta} = \frac{u_P \cos\alpha}{u_Q \cos\beta}$$

From: $u_P \sin\alpha = u_Q \sin\beta$; $\frac{u_P}{u_Q} = \frac{\sin\beta}{\sin\alpha}$:

$$\frac{\mathbf{R_P}}{\mathbf{R_Q}} = \frac{\sin\beta\cos\alpha}{\sin\alpha\cos\beta} = \frac{\sin\beta}{\cos\beta} \times \frac{1}{\frac{\sin\alpha}{\cos\alpha}} = \mathbf{\frac{\tan\beta}{\tan\alpha}}$$

(c)(ii) Force to maintain belt speed:

$$F = v \times \frac{dm}{dt} = 2\text{ms}^{-1} \times 75\text{kgs}^{-1} = 150\text{N}$$

Rate of kinetic energy given to gravel:

$$P_{KE} = \frac{1}{2} \times \frac{dm}{dt} \times v^2 = \frac{1}{2} \times 75\text{kgs}^{-1} \times (2\text{ms}^{-1})^2 = 150\text{Js}^{-1} = 150\text{W}$$

**Question 92**

(a) In an elliptical orbit, the distance between the satellite and Earth changes continuously. When the satellite moves closer to Earth, gravity does positive work (the force has a component along the direction of motion). Positive work increases kinetic energy, so the satellite speeds up. When moving away, gravity does negative work, and the satellite slows down. The total mechanical energy remains constant throughout.

(b) Escape velocity from a body's surface is $v_{esc} = \sqrt{2GM/R}$. The Moon has both a smaller mass and a smaller radius than Earth, but the mass effect dominates, giving the Moon a much lower escape velocity ($\approx 2.4\text{kms}^{-1}$) compared to Earth ($\approx 11.2\text{kms}^{-1}$). Gas molecules at a given temperature have a distribution of speeds. On the Moon, many molecules (especially lighter ones) exceed the low escape velocity and leave permanently. Over billions of years, the Moon has lost virtually all its atmosphere. On Earth, only the lightest molecules occasionally exceed the higher escape velocity, so the atmosphere is retained.

(c) By conservation of energy. Taking the initial separation as $d = 10^9\text{km} = 10^{12}\text{m}$ and the final separation (surfaces touching) as $2R = 2 \times 10^4\text{km} = 2 \times 10^7\text{m}$:

$$-\frac{Gm^2}{d} + 0 = -\frac{Gm^2}{2R} + 2 \times \frac{1}{2}mv^2$$

$$mv^2 = Gm^2\left(\frac{1}{2R} - \frac{1}{d}\right)$$

Since $d \gg 2R$, $\frac{1}{d}$ is negligible:

$$v^2 \approx \frac{Gm}{2R} = \frac{6.67 \times 10^{-11}\text{Nm}^2\text{kg}^{-2} \times 2 \times 10^{30}\text{kg}}{2 \times 10^7\text{m}} = 6.67 \times 10^{12}\text{m}^2\text{s}^{-2}$$

$$v = \sqrt{6.67 \times 10^{12}\text{m}^2\text{s}^{-2}} = 2.58 \times 10^6\text{ms}^{-1} \approx 2580\text{kms}^{-1}$$

**Question 93**

(a) The car has two accelerations: centripetal acceleration $a_c = \frac{v^2}{r}$ directed toward the centre (perpendicular to velocity) and tangential acceleration $a_t$ along the velocity direction (from the speed increase). Since these are perpendicular, the resultant is $a = \sqrt{a_c^2 + a_t^2}$. The angle between the resultant and the velocity is $\theta = \tan^{-1}\left(\frac{a_c}{a_t}\right)$, directed toward the centre. As the speed increases, $a_c$ grows and $\theta$ increases.

(c)(i) Optimal angle: $\theta_{opt} = 45° + \frac{\beta}{2} = 45° + \frac{30°}{2} = 45° + 15° = 60°$

(c)(ii) Centripetal acceleration: $a_c = \frac{v^2}{r} = \frac{(30ms^{-1})^2}{500m} = 1.8ms^{-2}$

Tangential acceleration: $a_t = 2ms^{-2}$

Resultant: $a = \sqrt{(1.8ms^{-2})^2 + (2ms^{-2})^2} = 2.69ms^{-2}$

Angle from velocity: $\theta = \tan^{-1}\left(\frac{a_c}{a_t}\right) = \tan^{-1}\left(\frac{1.8}{2}\right) = 42°$ toward the centre.

**Question 94**

(a) At the highest point, the vertical velocity is zero but the horizontal velocity $v_x = u\cos 45° = \frac{u}{\sqrt{2}}$ is unchanged (no air resistance, no horizontal force) leading to non-zero KE.

$$KE_{top} = \frac{1}{2}m\left(\frac{u}{\sqrt{2}}\right)^2 = \frac{\frac{1}{2}mu^2}{2} = \frac{1}{2} \times KE_{launch}$$

Hence kinetic energy at the top is exactly half the launch kinetic energy.

(b) As more passengers step onto the belt, the total friction force on the belt increases (more mass to accelerate from rest to belt speed). The driving mechanism must supply a force equal to the total friction acting backward on the belt. For constant speed, the drive force must continuously match the friction force, which increases with the number of passengers. The motor must therefore increase its output automatically in response to the increased load.

(c) At 45°, $KE_{top} = \frac{1}{2} \times KE_{launch} = \frac{1}{2} \times 500J = 250J$

**Question 95**

(a) By conservation of energy, the PE lost equals the KE gained: $mgh = \frac{1}{2}mv^2$. The mass cancels, giving $v = \sqrt{2gh}$. This depends only on h, not on the angle or length of the slope. A steeper slope is shorter but has a larger component of gravity along it; a gentler slope is longer but has a smaller component. Both effects compensate exactly, producing the same final speed.

(b) Projectile motion is defined as motion under the influence of gravity alone (no other forces), with a constant horizontal velocity and a constant vertical acceleration g. A stone thrown vertically upward satisfies these conditions: the only force acting is gravity, and the acceleration is constant at g downward throughout the flight. It is a special case of projectile motion where the horizontal velocity is zero. The student's claim is correct.

(c) $u_x = 20\cos 30° = 17.32ms^{-1}$, $u_y = 20\sin 30° = 10ms^{-1}$

Time to reach wall (3m away): $t_1 = \frac{3m}{17.32ms^{-1}} = 0.1732s$

Height at wall: $y_1 = 2m + 10ms^{-1} \times 0.1732s - \frac{1}{2} \times 9.8ms^{-2} \times (0.1732s)^2 = 3.585m$

Vertical velocity at wall: $v_y = 10ms^{-1} - 9.8ms^{-2} \times 0.1732s = 8.303ms^{-1}$ (upward)

After elastic bounce: $v_x = -17.32ms^{-1}$ (reversed), $v_y = 8.303ms^{-1}$ (unchanged)

The ball now moves away from the wall at height $3.585m$ with $v_x = 17.32ms^{-1}$ (away) and $v_y = 8.303ms^{-1}$ (upward).

Time to hit ground from this point ($y = 0$):

$$0 = 3.585 + 8.303t - 4.9t^2$$

$$4.9t^2 - 8.303t - 3.585 = 0$$

$$t = 2.051s$$

Horizontal distance from wall: $x = 17.32ms^{-1} \times 2.051s = 35.5m$

**Question 96**

(a) The plates have inertia: they resist changes in motion. When the cloth is pulled, friction between the cloth and plates acts on the plates. A fast pull means the cloth is in contact with the plates for a very short time. By the impulse-momentum theorem, $J = F\Delta t$: even though friction is large, the extremely short contact time produces a tiny impulse, and the plates barely move. A slow pull keeps friction acting for a long time, producing a large impulse that drags the plates along with the cloth.

(b) Thrust is $F = v_e \times \frac{dm}{dt}$. If both the exhaust velocity and the burn rate are constant, the thrust is constant. However, acceleration is $a = \frac{(F-Mg)}{M}$ (for vertical flight) or $a = \frac{F}{M}$ (in space). As fuel is consumed, the total mass M decreases. With constant F and decreasing M, the acceleration increases.

(c) At launch ($t = 0$): mass $= 100\text{kg}$

$$a_0 = \frac{F - mg}{m} = \frac{1500\text{N} - 100\text{kg} \times 9.8\text{ms}^{-2}}{100\text{kg}} = 5.2\text{ms}^{-2}$$

At time t: mass $= (100 - 20t)\text{kg}$

$$\mathbf{a(t)} = \frac{F - mg}{m} = \frac{F}{m} - g = \frac{\mathbf{1500N}}{\mathbf{(100 - 20t)kg}} - \mathbf{9.8ms^{-2}}$$

**Question 97**

(a) The planet has an enormous mass compared to the spacecraft. In the planet's reference frame, the spacecraft swings around and leaves at the same speed it arrived (elastic gravitational interaction). But the planet is moving relative to the Sun. In the Sun's frame, the spacecraft gains speed from the planet's motion. By conservation of momentum for the planet-spacecraft system, the planet loses a tiny amount of momentum. Because the planet's mass is so large, its speed change is negligible, while the spacecraft gains a significant velocity boost.

(b) In a circular orbit, gravity acts perpendicular to the velocity (radially inward while velocity is tangential). The work done by a force perpendicular to displacement is zero: $W = Fd\cos 90° = 0$. By the work-energy theorem, zero work means no change in KE, so speed is constant. In an elliptical orbit, gravity is not always perpendicular to the velocity. When the satellite moves toward Earth, the component of gravity along the motion does positive work, increasing KE and speed. When moving away, gravity does negative work, decreasing speed.

(c) Orbital radius: $R = R_E + h = 6.4 \times 10^6\text{m} + 4 \times 10^5\text{m} = 6.8 \times 10^6\text{m}$

Orbital speed:

$$v = \sqrt{\frac{GM_E}{R}} = \sqrt{\frac{6.67 \times 10^{-11}\text{Nm}^2\text{kg}^{-2} \times 5.97 \times 10^{24}\text{kg}}{6.8 \times 10^6\text{m}}} = 7653\text{ms}^{-1}$$

Head-on collision (equal masses, equal speeds, opposite directions):

$$p_{total} = mv - mv = 0$$

After sticking together: $2m \times V = 0$, so $V = 0$.

The wreckage has zero velocity. With no orbital speed, it cannot maintain orbit and falls directly toward the Earth under gravity. It crashes into the Earth (or burns up in the atmosphere).

**Question 98**

(a) Two conditions: (1) The orbital period must equal 24 hours (one sidereal day) so that the satellite completes one orbit in the same time the Earth completes one rotation. If the periods differ, the satellite drifts east or west relative to the ground. (2) The orbit must lie in the equatorial plane and the satellite must orbit in the same direction as the Earth's rotation (west to east). If the orbit is inclined, the satellite oscillates north and south of the equator, appearing to move in a figure-eight pattern as seen from the ground.

(b) At the hanging point, three forces act: tension $T_L$ in the left segment at 10° below horizontal, tension $T_R$ in the right segment at 5° below horizontal, and the boy's weight 90N downward.

Horizontal: $T_L\cos 10° = T_R\cos 5°$

$$T_L = T_R \times \frac{\cos 5°}{\cos 10°} = T_R \times \frac{0.9962}{0.9848} = 1.0116\,T_R$$

Vertical: $T_L\sin 10° + T_R\sin 5° = 90\text{N}$

$$1.0116\,T_R \times 0.1736 + T_R \times 0.08716 = 90\text{N}$$

$$T_R(0.1756 + 0.08716) = 90\text{N}$$

$$T_R \times 0.2628 = 90\text{N}$$

$$T_R = 342.5\text{N}$$

$$T_L = 1.0116 \times 342.5\text{N} = 346.5\text{N}$$

**Question 99**

(a) Both stones fall the same vertical distance h and take the same time $t = \sqrt{2h/g}$ to reach the ground (the horizontal velocity does not affect the fall time). The horizontal distance depends on two factors: the initial horizontal velocity u and the fall time t. Since the fall time is fixed by the height, the horizontal distance is $x = ut$. A greater horizontal velocity or a greater height (which gives more fall time) increases the distance.

(b) The player must estimate: (1) the range of the ball; that is how far from the kicker it will land, which depends on the kick speed and angle; (2) the time of flight; that is how long the ball stays in the air, which determines how much time the player has to run; and (3) the direction of the kick. From these, the player calculates the distance between their starting position and the landing point, then divides by the time of flight to find the minimum running speed.

(c) Range of the ball:

$$R = \frac{u^2 \sin 2\theta}{g} = \frac{(19.5\text{ms}^{-1})^2 \times \sin 90°}{9.8\text{ms}^{-2}} = 38.8\text{m}$$

Time of flight:

$$T = \frac{2u\sin\theta}{g} = \frac{2 \times 19.5\text{ms}^{-1} \times \sin 45°}{9.8\text{ms}^{-2}} = 2.81\text{s}$$

The player is $55\text{m}$ away from the kicker. The ball lands $38.8\text{m}$ from the kicker. The player must cover $55 - 38.8 = 16.2\text{m}$ in $2.81\text{s}$:

$$v_{min} = \frac{16.2\text{m}}{2.81\text{s}} = 5.77\text{ms}^{-1}$$

**Question 100**

(a) At the instant the ball is thrown, it already has the platform's horizontal velocity (by inertia). The throw adds an additional velocity. To a stationary ground observer, the ball's velocity is the vector sum of the platform velocity and the throw velocity. For example, if the platform moves east and the ball is thrown north relative to the platform, the ground observer sees the ball move northeast. The trajectory shape (parabolic) is the same, but it is tilted relative to what the thrower sees.

(b) The shell follows a parabolic trajectory with a definite range and flight time. The car moves at constant speed along the same road. For the shell to hit the car, the shell must land at the exact position where the car is at that instant. This requires: (1) the shell's range must equal the car's initial distance plus the distance the car travels during the shell's flight time, and (2) the shell and car must arrive at the same point at the same time.

(c) Speed of car: $v_c = 72\text{kmh}^{-1} = 20\text{ms}^{-1}$

At 45°, the shell's range is $R = \frac{u^2}{g}$ and flight time is $T = \frac{2u\sin 45°}{g} = \frac{u\sqrt{2}}{g}$.

During time T, the car moves an additional $v_c \times T$ from the gun.

The shell must reach: $500\text{m} + v_c T$

$$\frac{u^2}{g} = 500\text{m} + 20\text{ms}^{-1} \times \frac{u\sqrt{2}}{g}$$

$$u^2 = 500 \times 9.8 + 20\sqrt{2} \times u = 4900 + 28.28u$$

$$u^2 - 28.28u - 4900 = 0$$

$$u = 85.6\text{ms}^{-1}$$

**Question 101**

(a) On each slope, the forces along the plane are: the gravity component down the slope ($mg\sin\theta$), friction (which can act either up or down the slope), and the string tension. Friction can take any value up to $\mu R$, and can reverse direction. This creates a range of tensions for which neither mass slides. A higher $\mu$ widens this range; more friction means a larger band of weight ratios and angles that maintain equilibrium. Without friction ($\mu = 0$), equilibrium requires a single precise condition.

(b) The bob has weight mg acting vertically downward. The only other force is the string tension, which acts along the string toward the pivot. If the string were horizontal, the tension would be entirely horizontal and there would be no upward force to balance the weight. The bob would accelerate downward. For equilibrium in the vertical direction, the string must have an upward component: $T\cos\theta = mg$. This requires $\theta < 90°$, so the string can never be horizontal.

(c) For constant velocity (zero acceleration), the net force along each slope is zero.

On the 60° slope (mass slides down, friction acts up):

$$mg\sin60° - T - \mu mg\cos60° = 0 \quad \ldots(1)$$

On the 30° slope (mass is pulled up, friction acts down):

$$T - mg\sin30° - \mu mg\cos30° = 0 \quad \ldots(2)$$

Adding (1) and (2) to eliminate T:

$$mg\sin60° - mg\sin30° = \mu mg(\cos60° + \cos30°)$$

$$\mu = \frac{\sin60° - \sin30°}{\cos60° + \cos30°} = 0.27$$

**Question 102**

(a) When only gravity acts, the normal reaction balances the perpendicular component of weight: $R = mg\cos\theta$. A horizontal force F has a component perpendicular to the slope: $F\sin\theta$ (pushing into the surface). This adds to the normal reaction: $R = mg\cos\theta + F\sin\theta$.

(b) The total energy of a satellite in circular orbit is $E = -\frac{GMm}{2r}$. When engines fire, they add energy, making E less negative. The satellite moves to a larger radius r. But orbital speed is $v = \sqrt{\frac{GM}{r}}$, which decreases with increasing r. The added energy goes primarily into gravitational PE (which increases as $-\frac{GMm}{r}$, becoming less negative). The KE ($= \frac{GMm}{2r}$) actually decreases. The satellite trades kinetic energy for potential energy as it climbs to the higher orbit.

(c) Forces on the block perpendicular to slope:

$$R = mg\cos\theta + F\sin\theta$$

Forces along the slope (about to slide up, friction acts downward):

$$F\cos\theta = mg\sin\theta + \mu_s R = mg\sin\theta + \mu_s(mg\cos\theta + F\sin\theta)$$

$$F\cos\theta - \mu_s F\sin\theta = mg\sin\theta + \mu_s mg\cos\theta$$

$$F(\cos\theta - \mu_s\sin\theta) = mg(\sin\theta + \mu_s\cos\theta)$$

$$F_{max} = \frac{mg(\sin\theta + \mu_s\cos\theta)}{\cos\theta - \mu_s\sin\theta}$$

**Question 103**

(a) The bob moves in a horizontal circle. In the vertical direction there is no acceleration. By Newton's second law in the vertical direction: the net vertical force must be zero. The only upward force is $T\cos\theta$ and the only downward force is mg. Therefore $T\cos\theta = mg$.

(b) At the design speed, the horizontal component of the normal reaction exactly provides the centripetal force. At a higher speed, the required centripetal force ($\frac{mv^2}{r}$) exceeds the horizontal component of R. The deficit must be supplied by friction acting down the bank (inward), and the truck tends to slide outward. At a lower speed, the centripetal requirement is less than the horizontal component of R. The excess pushes the truck inward, and friction must act up the bank (outward) to prevent sliding.

(c) Vertical: $T\cos25° = mg$

$$T = \frac{mg}{\cos25°} = \frac{0.2kg \times 9.8ms^{-2}}{0.9063} = 2.16N$$

Horizontal (centripetal): $T\sin25° = \frac{mv^2}{r}$

Radius of circle: $r = L\sin25° = 1.5m \times 0.4226 = 0.634m$

$$v^2 = \frac{T\sin25° \times r}{m} = \frac{2.16N \times 0.4226 \times 0.634m}{0.2kg} = 2.894m^2s^{-2}$$

$$v = 1.70ms^{-1}$$

Period: $T_{period} = \frac{2\pi r}{v} = 2\pi \times \frac{0.634\text{m}}{1.70\text{ms}^{-1}} = 2.34\text{s}$

**Question 104**

(a) At the highest point, only gravity acts on the projectile (the same force that acted before the explosion). The explosion is an internal event: internal forces cannot change the motion of the centre of mass. By Newton's second law applied to the centre of mass, $F_{ext} = Ma_{cm}$, and the only external force is gravity. Therefore, $a_{cm} = g$ downward, the same as before the explosion. The centre of mass continues along the original parabola.

(b) Before firing, the total momentum of the cannon-shell system is zero (both at rest). After firing, momentum is conserved: $0 = m_s v_s - m_c v_c$, so $v_c = \left(\frac{m_s}{m_c}\right) v_s$. The recoil speed is proportional to the shell-to-cannon mass ratio. A heavier cannon recoils more slowly; a heavier shell increases the recoil.

(c) At the highest point: $v_x = u\cos 60° = 50\text{ms}^{-1} \times 0.5 = 25\text{ms}^{-1}$ (horizontal), $v_y = 0$

Momentum before split: $p = 3\text{kg} \times 25\text{ms}^{-1} = 75\text{kgms}^{-1}$ (horizontal)

After split: the 1kg fragment has zero horizontal velocity.

$$75\text{kgms}^{-1} = 1\text{kg} \times 0 + 2\text{kg} \times v_2$$

$v_2 = 37.5\text{ms}^{-1}$(horizontal)

Height at highest point: $H = \frac{u^2 \sin^2 60°}{2g} = \frac{(50\text{ms}^{-1})^2 \times 0.75}{2 \times 9.8\text{ms}^{-2}} = \frac{1875\text{m}^2\text{s}^{-2}}{19.6\text{ms}^{-2}} = 95.66\text{m}$

Time from peak to ground: $H = \frac{1}{2} g t_2^2$, so $t_2 = \sqrt{\frac{2H}{g}} = \sqrt{2 \times \frac{95.66\text{m}}{9.8\text{ms}^{-2}}} = 4.42\text{s}$

Time from launch to peak: $t_1 = \frac{u\sin 60°}{g} = 50 \times \frac{0.866}{9.8} = 4.42\text{s}$

Horizontal distance of launch to peak: $x_1 = 25\text{ms}^{-1} \times 4.42\text{s} = 110.5\text{m}$

Horizontal distance of peak to landing (2kg fragment at $37.5\text{ms}^{-1}$):

$$x_2 = 37.5\text{ms}^{-1} \times 4.42\text{s} = 165.8\text{m}$$

Total distance from launch: $x_1 + x_2 = 110.5\text{m} + 165.8\text{m} = 276.3\text{m}$

**Question 105**

(a) The stone moves slowly and stays in contact with the glass for a long time. The impulse spreads across the entire pane, stressing it everywhere and causing it to shatter. The bullet passes through in a tiny fraction of a second. The contact time is so short that only the small area in the bullet's path receives the impulse. The rest of the glass has no time to respond, leaving a clean hole.

(b) Objects on the surface: with gravity gone, no downward force acts. Objects continue with the rotational velocity they had and fly off tangentially.

The Moon: gravity provides the centripetal force. Without it, by Newton's first law, the Moon continues in a straight line tangent to its orbit.

The atmosphere: gas molecules are held near the surface by gravity. Without it, they disperse into space following their thermal velocities.

(c) The water must reach $x = 20\text{m}$, $y = 18\text{m} - 1\text{m} = 17\text{m}$ above the nozzle:

By using trajectory equation:

$$17\text{m} = 20\text{m} \times \tan 55° - \frac{9.8\text{ms}^{-2} \times (20\text{m})^2}{2u^2 \cos^2 55°}$$

$$u^2 = 515\text{m}^2\text{s}^{-2}$$

$$u = 22.7\text{ms}^{-1}$$

**Question 106**

(a) In a horizontal circle, speed is constant because gravity has no component along the motion. The tension is constant: $T = \frac{mv^2}{r}$.

In a vertical circle, gravity has a component along the motion, so speed varies; fastest at the bottom, slowest at the top. The tension is $T_{bottom} = \frac{mv_{bottom}^2}{r} + mg$ at the bottom and $T_{top} = \frac{mv_{top}^2}{r} - mg$ at the top. Tension is maximum at the bottom and minimum at the top.

(b) By conservation of momentum: $m_1v_1 = (m_1 + m_2)V$, so $V = \frac{m_1v_1}{m_1+m_2} < v_1$. The momentum is shared among a larger mass, reducing the speed.

(c) Let $a_1$ = downward acceleration of the 4kg mass (= upward acceleration of pulley B). Let $a_2$ = acceleration of the 3kg mass downward relative to pulley B. Let $T_1$ = tension over pulley A, $T_2$ = tension over pulley B.

Constraint (massless pulley B): $T_1 = 2T_2$

Absolute accelerations (downward positive): 4kg: $a_1$; 3kg: $(-a_1 + a_2)$; 1kg: $(-a_1 - a_2)$.

Equations of motion:

$$4g - T_1 = 4a_1 \quad \dots(1)$$

$$3g - T_2 = 3(-a_1 + a_2) \quad \dots(2)$$

$$g - T_2 = (-a_1 - a_2) \quad \dots(3)$$

From (2): $T_2 = 3g + 3a_1 - 3a_2$. From (3): $T_2 = g + a_1 + a_2$.

Equating: $3g + 3a_1 - 3a_2 = g + a_1 + a_2 \rightarrow g + a_1 = 2a_2$ … (4)

From (1) and (2): $T_1 = 4g - 4a_1$, so $T_2 = 2g - 2a_1$.

Setting equal to (3) expression: $2g - 2a_1 = g + a_1 + a_2 \rightarrow g = 3a_1 + a_2$ … (5)

From (4): $a_2 = (g + a_1)/2$. Substituting into (5):

$$g = 3a_1 + \frac{g + a_1}{2} = 3a_1 + \frac{g}{2} + \frac{a_1}{2} = \frac{7a_1}{2} + \frac{g}{2}$$

$$\frac{g}{2} = \frac{7a_1}{2}$$

$\rightarrow a_1 = \frac{g}{7} = 1.4 ms^{-2}$

$a_2 = \frac{\left(g + \frac{g}{7}\right)}{2} = \frac{4g}{7} = 5.6 ms^{-2}$

**Accelerations:**

4kg: $\frac{g}{7} = 1.4 ms^{-2}$ downward

3kg: $-\frac{g}{7} + \frac{4g}{7} = \frac{3g}{7} = 4.2 ms^{-2}$ downward

1kg: $-\frac{g}{7} - \frac{4g}{7} = -\frac{5g}{7} = 7 ms^{-2}$ upward

**Tensions:**

$$T_2 = g + a_1 + a_2 = 9.8 + 1.4 + 5.6 = 16.8N$$

$$T_1 = 2T_2 = 33.6N$$

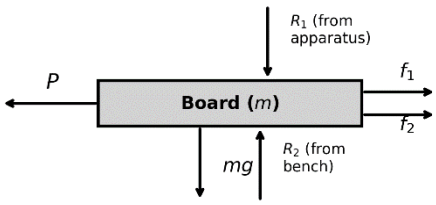
**Question 107**

(a) For a circular orbit, total energy is $E = -GMm/(2r)$ and orbital speed is $v = \sqrt{GM/r}$. As the satellite loses energy to drag, E becomes more negative, so r decreases. A smaller r means a larger $v = \sqrt{GM/r}$. The satellite spirals inward and speeds up. The energy lost to drag exceeds the KE gained because the PE decreases by twice the KE increase.

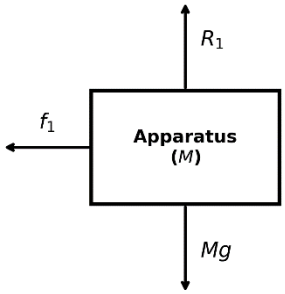
(b) The board is sandwiched between the apparatus above and the bench below. The friction force between the apparatus and the board is $f_1 = \mu_s Mg$ (due to the weight of the apparatus pressing on the board). The friction force between the board and the bench is $f_2 = \mu_s(M + m)g = \mu_s(M + 0.1M)g = 1.1\mu_s Mg$ because the bench supports both the apparatus and the board. Since $1.1\mu_s Mg > \mu_s Mg$, the friction at the lower surface is greater even though $\mu_s$ is the same. This is because the bench carries more weight.

(c)(i) Free body diagram:

**FBD of Board**




**FBD of Apparatus**




(c)(ii)

For the system to start moving as a whole, P must overcome the friction between the board and the bench (the only external friction on the system):

$$\mathbf{P_{min}} = f_2 = \mu_s(M + m)g = 0.1 \times (M + 0.1M)g = 0.1 \times 1.1Mg = \mathbf{0.11Mg}$$

(c)(iii) For the board to slide out while the apparatus remains stationary, P must overcome both friction surfaces acting on the board:

Friction from apparatus on board (top): $f_1 = \mu_s Mg = 0.1Mg$

Friction from bench on board (bottom): $f_2 = \mu_s(M + m)g = 0.11Mg$

$$\mathbf{P} = f_1 + f_2 = 0.1Mg + 0.11Mg = \mathbf{0.21Mg}$$

(c)(iv) While Kipanga pulls the board, kinetic friction from the sliding board acts on the apparatus in the direction of the pull: $f_{board \to app} = \mu_k Mg = 0.1Mg$ (kinetic, because the board slides under the apparatus).

The bench resists this with static friction on the apparatus: $f_{bench \to app} \leq \mu_s Mg = 0.1Mg$.

Since the driving force (0.1Mg) exactly equals the maximum resisting force (0.1Mg), the apparatus is on the verge of motion but never actually accelerates. It acquires no velocity during the pull.

After the board is removed, the apparatus sits stationary on the bench, 5cm from the edge, with zero velocity. No horizontal force acts to start it moving.

**The apparatus does not slide off the edge.**