Chapter 4

EQUILIBRANT FORCES

INTRODUCTION

In the previous chapters, motion was the star of the show: buses accelerating, balls flying, bicycles braking, and students occasionally running when the emergency assembly bell suddenly rings and everyone remembers that walking is no longer fast enough. But Physics has another equally interesting side: situations where *nothing seems to happen*. A book sits quietly on a table, a signboard hangs without drama, and you may be sitting perfectly still in class while appearing deeply attentive. Yet behind this peaceful appearance, forces are actually having a silent tug-of-war.

When forces balance each other, motion does not change. Not because forces disappeared, but because they cancel perfectly. At O-Level you met balanced forces in simple situations. Now at A-Level, we sharpen that idea with a powerful concept; the **equilibrant force**: *the single force that would exactly oppose all others and bring a system into equilibrium.* Engineers use it when designing bridges, pilots rely on it unknowingly during steady flight, and even your body depends on it just to stand upright without falling like an unplugged robot.

Do not worry if this sounds serious already; equilibrium may sound calm, but the Physics behind it is lively, elegant, and sometimes surprisingly funny. By the end of this chapter, you will not only understand why objects stay still or move steadily, but you will also begin to see balance everywhere; from classroom desks to skyscrapers… and perhaps even in your study habits (which, hopefully, will now include balanced revision!).

Welcome to the first chapter of **Particle Mechanics** where stillness finally gets the attention it deserves.

From Balanced Forces to the “Balance-Maker”: Meet the Equilibrant

In O-level Physics, you learnt the comforting idea of **balanced forces**: *when forces cancel, the object does not accelerate*. A book on a table stays put because the downward weight is balanced by the upward normal reaction. A picture frame hangs peacefully because the tension in the string balances its weight. So far, so good; O-level teaches you how to *recognise* equilibrium.

Now A-level takes that familiar idea and upgrades it into a **tool**. Instead of only saying “the forces are balanced”, we begin asking: *What single force would I need to add to make the body perfectly balanced?* That single “balance-maker” is called the **equilibrant force**. It is not a new mysterious type of force; it is simply the force that would exactly cancel the **resultant** (net force) of all the other forces.

Concisely: **The equilibrant force** *is the single force equal in magnitude and opposite in direction to the resultant of all other forces acting on a body.*

If the forces already balance, the resultant is zero and the equilibrant is also zero, meaning there is no extra force needed. But if the forces do not balance, then the equilibrant tells you exactly what to add (magnitude and direction) to restore equilibrium. This is why equilibrant forces matter in A-level Particle Mechanics: they help us analyse real situations like signboards, ladders, cables, cranes, bridges, and any object that must stay steady while forces pull in different directions including you, sitting on a chair, pretending you are not tired, while gravity quietly keeps pulling you downward.

RESULTANT FORCE VERSUS EQUILIBRANT FORCE

Before we talk about *equilibrium*, we must be fluent in the two “opposite twins” of force analysis: the **resultant** and the **equilibrant**. They are not enemies; they are simply two ways of describing the same situation, depending on whether you are *adding forces* or *cancelling forces*.

Resultant force (net force)

The **resultant force** *is the single force that has the same effect as all the forces acting together on a body.*

$$\text{Resultant force}, \mathbf{F_R} = F_1 + F_2 + F_3 + \ldots$$

Meaning: *if you replace all the separate forces by one force $\boldsymbol{F_R}$, the body would accelerate in the same way.*

Key idea:

- If $F_R \neq 0$, the body accelerates in the direction of F_R.

- If $F_R = 0$, the body has zero acceleration (it stays at **rest** or moves with **constant velocity**).

So equilibrium is simply the case when $\mathbf{F_R = 0.}$

Equilibrant force

The **equilibrant force** *is the single force that would bring the body into equilibrium by cancelling the resultant of all the other forces.*

$$\text{Equilibrant force}, F_E = -F_R$$

This means:

- F_E has the same magnitude as F_R.
- F_E acts in the opposite direction to F_R.

So if you apply the equilibrant, the new resultant becomes zero:

$$F_R + F_E = 0$$

Remember this:

- Resultant tells you: *"What is the overall push?"*
- Equilibrant tells you: *"What single push would cancel that overall push?"*

To have better understanding on this; suppose two forces act on a body:

- 12 N to the right
- 7 N to the left

Then, resultant force: $F_R = 12\text{N} - 7\text{N} = 5\text{N}$ to the right

And equilibrant force: $F_E = 5\text{N}$ to the left

So the equilibrant is the force that would make the body balanced.

For now, give theory a short holiday and let us sharpen our understanding with two worked examples.

BINDER example 1

A lamp is suspended from the ceiling by a vertical cord. The tension in the cord is 25 N upward while the weight of the lamp is 25 N downward.

(a) Determine the resultant force acting on the lamp.
(b) State the equilibrant force.

Solution

Upward force = 25N

Downward force = 25N

Resultant, $F_R = 25\text{N} - 25\text{N};\ F_R = 0\text{N}$

(a) So the resultant force is 0N (the lamp is in equilibrium).
(b) Since the resultant force is zero, no additional force is required to balance the system.

$$\text{Equilibrant force} = 0\text{N}.$$

Making Sense of the Answer: *If forces already balance, the system is already in equilibrium. The equilibrant simply tells us that nothing extra is needed; balance has already been achieved.*

Thinking Like a Physicist: *Do not assume an equilibrant must always exist physically. Sometimes it is only a conceptual force used to check whether equilibrium already exists.*

REAL example 2

Kipute is sitting quietly on a chair revising Physics. She says: *"I am at rest because the upward force from the chair balances my weight."*

Kipanga disagrees: *"If gravity is pulling you downward, there must be a stronger downward force. Otherwise you would float!"*

Explain who is correct and clarify the roles of resultant and equilibrant forces in this situation.

Solution

Kipute is correct.

Explanation:

When she sits at rest: Weight (W) acts downward. Normal reaction (R_n) from the chair acts upward. These two forces are equal in magnitude and opposite in direction, so the resultant force is zero. This means there is no acceleration and the system is in equilibrium. Kipanga's idea is incorrect because equilibrium does not require a stronger force but only equal and opposite forces.

Clarification:

The upward reaction from the chair acts as the equilibrant force to her weight. It balances the downward gravitational force that would otherwise cause downward acceleration.

Making Sense of the Answer: *Objects at rest are not force-free. Usually, forces are present but balanced. Sitting comfortably is actually a perfect demonstration of equilibrium though it may not feel like Physics at the time.*

Thinking Like a Physicist: *Whenever you see something stationary, ask:*

- *What forces are acting?*
- *Which ones balance each other?*

Having given the worked examples their full say, we can now welcome the next subtopic and see what fresh ideas it brings along.

THE TWO EQUILIBRIUM EQUATIONS

Up to now, we have talked about equilibrium in a simple way: if the resultant force on a body is zero, the body either remains at rest or continues moving with constant velocity. That idea works perfectly when all forces act along one straight line. But real life is rarely that cooperative. Forces often act in different directions: a hanging sign pulls downward while its cables pull sideways and upward, a ladder leans against a wall, a crane cable pulls at an angle, or even your school bag straps pull in directions you did not plan when you overloaded the bag.

To deal with such situations, Physics does not panic. Instead, it separates forces into two perpendicular components (directions): horizontal component (x-direction) and vertical component (y-direction). This process is called **resolution of forces**. To have better understanding on this consider the following two cases.

Case 1: Angle measured from the horizontal

In most cases, angles are measured from the horizontal, as shown in the figure below.

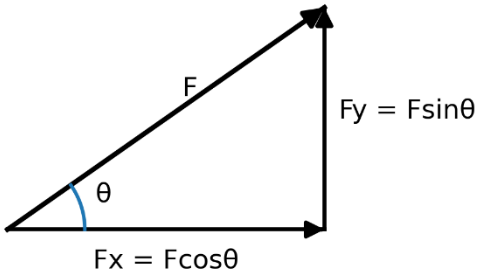


Figure: *A force resolved into horizontal and vertical components when the angle is measured from the horizontal.*

From the diagram, a force **F** act at angle **θ** above the horizontal (**measured from positive x-axis**).

From right-triangle geometry:

$$\cos\theta = \frac{\text{adjacent}}{\text{hypotenuse}} = \frac{F_x}{F}$$

From which:

$$\mathbf{F_x = F\cos\theta \ = Horizontal\ component}$$

Analogously:

$$\sin\theta \ = \frac{\text{opposite}}{\text{hypotenuse}} = \frac{F_y}{F}$$

From which:

$$\mathbf{F_y = F\sin\theta \ = Vertical\ component}$$

Case 2: Angle Measured from the Vertical

In some situations, angles are measured from the vertical, as shown in the figure below.

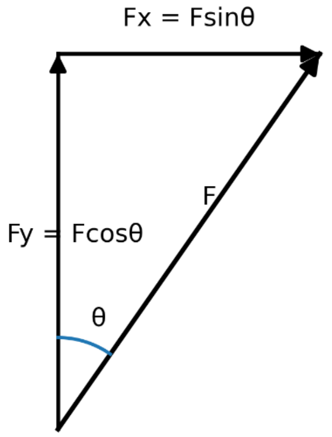


Figure: *Resolution of a force when the angle is measured from the vertical; note that the component expressions reverse.*

In this case:

$$\sin\theta \ = \frac{\text{opposite}}{\text{hypotenuse}} = \frac{F_x}{F}$$

From which:

$$\mathbf{F_x = F\sin\theta \ = Horizontal\ component}$$

Similarly:

$$\cos\theta \ = \frac{\text{adjacent}}{\text{hypotenuse}} = \frac{F_y}{F}$$

From which:

$$\mathbf{F_y = F\cos\theta \ = Vertical\ component}$$

Important Alert!

In the second case, *the components reverse.*

This happens because the horizontal component is now opposite to the angle, while the vertical component is adjacent to it. So, always ***remember to check which axis the angle is measured from.***

It is also worth noting that, from geometry (Pythagoras' theorem), the magnitude of a force is related to its components in both cases by:

$$F^2 = (F_x)^2 + (F_y)^2 = (F\cos\theta)^2 + (F\sin\theta)^2$$

Hence:

$$\mathbf{F} = \sqrt{(\mathbf{F_x})^2 + (\mathbf{F_y})^2} \text{or } \mathbf{F} = \sqrt{(\mathbf{F\cos\theta})^2 + (\mathbf{F\sin\theta})^2}$$

This confirms that the resultant magnitude is independent of whether the angle is measured from the horizontal or the vertical.

Before the ideas start colliding in our heads, let us calm them down with a few simple worked examples.

BINDER Example 3

A force of 50N acts on a body at an angle of 30° above the horizontal. Calculate the horizontal and vertical components of this force.

Solution

Since the angle was measured from horizontal:

Horizontal component, $F_x = F\cos\theta = 50\cos30° = \mathbf{43.3N}$

Vertical component, $F_y = F\sin\theta = 50\sin30° = \mathbf{25N}$

Making Sense of the Answer: *The horizontal component is larger because the force leans more toward the horizontal. If the angle were 60°, the vertical component would dominate.*

Thinking Like a Physicist: *Component analysis allows us to treat two-dimensional force problems as two separate one-dimensional problems. This simplification is one of Physics' most elegant problem-solving techniques.*

BINDER Example 4

A hanging sign is supported by a cable. The cable pulls on the sign with a force of 120N and makes an angle of 40° with the vertical. Without worrying yet about the detailed theory of tension, resolve this force into:

(a) the horizontal component, and

(b) the vertical component.

Solution

Since the angle was measured from vertical:

Horizontal component, $F_x = F\sin\theta = 120\sin40° = \mathbf{77.1N}$

Vertical component, $F_y = F\cos\theta = 120\cos40° = \mathbf{91.9N}$

Making Sense of the Answer: *Notice how the component equations reversed compared to Example 3. This happens because now the vertical component is adjacent to the angle (cosine) while the horizontal is opposite (sine).*

Thinking Like a Physicist: *The physics doesn't change with how we measure angles, but the mathematics adjusts accordingly. Staying alert to reference axes prevents sign errors and incorrect component calculations.*

The worked examples have done their job nicely; now let us invite the next concept to step forward and show us its charm.

Fundamental Condition for Equilibrium in Two Dimensions

From Newton’s second law:

$$\text{Resultant force} = \Sigma F = ma;\ \text{where } \Sigma \text{ stands for the } \textit{summation of}.$$

When a resultant force (ΣF) acts diagonally, it has components in both horizontal and vertical directions. If these components are unbalanced, the body will experience acceleration in both directions, leading to **motion in two dimensions**.

For horizontal motion:

$$\Sigma F_x = ma_x$$

For vertical motion:

$$\Sigma F_y = ma_y$$

But for equilibrium, acceleration is zero. Thus:

$$a_x = 0 \text{ and } a_y = 0$$

Hence:

$$\Sigma F_x = 0$$

$$\Sigma F_y = 0$$

The final result is the set of **two equilibrium equations for forces acting in two dimensions**.

The equations mean that for equilibrium to be maintained:

- The algebraic sum of horizontal components of all forces must be zero.
- The algebraic sum of vertical components of all forces must be zero.

When either of these conditions fails, equilibrium is broken. If:

- $\Sigma F_x \neq 0$; horizontal acceleration occurs resulting to sideway motion.
- $\Sigma F_y \neq 0$; vertical acceleration occurs resulting to vertical (upward or downward) motion.

Equilibrium is therefore a multidirectional condition.

You have to remember that:

A body could move with constant velocity and still satisfy: $\Sigma F_x = 0$ and $\Sigma F_y = 0$.

To truly understand and enjoy these ideas, let us serve them in the form of worked example.

BINDER Example 5

Three coplanar forces act on a particle in equilibrium: 10N eastward, 6N northward, and a third force F at an angle. If the particle is in equilibrium, find the magnitude and direction of the third force F.

Solution

For equilibrium: $\Sigma F_x = 0$ and $\Sigma F_y = 0$

Taking east as positive x-direction and north as positive y-direction.

Applying horizontal equilibrium equation: $\Sigma F_x = 0$

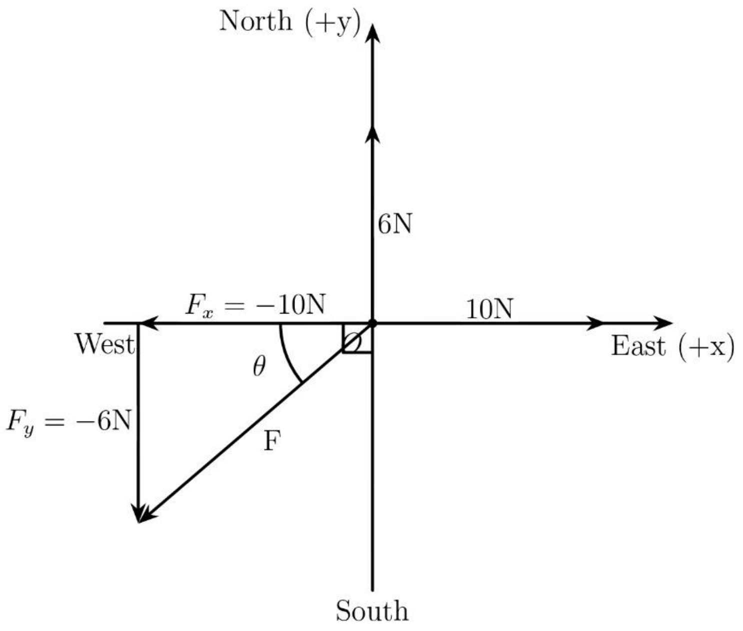
$$F_x + 10 = 0;\ F_x = -10\text{N} = \text{Horizontal component of the third force}$$

The negative sign means the horizontal component acts ***westward****.*

Applying vertical component equilibrium equation: $\Sigma F_y = 0$

$$F_y + 6 = 0;\ F_y = -6\text{N} = \text{Vertical component of the third force}$$

Here, *the negative sign means the vertical component acts* ***southward****.*



From the diagram:

$$\tan\theta = \frac{F_y}{F_x} = \frac{-6}{-10} = 0.6;\ \theta = \tan^{-1} 0.6 = 31°$$

Also, using $F = \sqrt{(F_x)^2 + \left(F_y\right)^2} = \sqrt{(-10)^2 + (-6)^2} = 11.7\text{N}$

The magnitude is 11.7N.

The direction is 31° south of west.

Making Sense of the Answer: *The third force must "cancel" the combined effect of the other two. Since we have eastward and northward forces, the equilibrant must pull westward (to balance eastward force) and southward (to balance northward force).*

Thinking Like a Physicist: *Equilibrium means the vector sum is zero. Treating x and y directions separately transforms a vector problem into two simple algebraic equations.*

With this worked example now neatly packed away, let us move on and meet the next subtopic; it has been waiting patiently to join the conversation.

FREE-BODY DIAGRAMS (FBDs)

Before applying equilibrium equations, we need a clear picture of what forces are actually acting on a body. In real situations, forces often come from different sources: gravity pulls downward, surfaces push upward, strings pull at angles, and friction resists motion. To analyse such situations properly, Physics isolates the body from its surroundings and represents all the forces acting on it in a simple sketch called a **free-body diagram (FBD)**. This diagram acts as the bridge between the physical situation and the mathematical equations used to analyse equilibrium. It is a bit like cooking **pilau** without the spices; you may still call it pilau, but everyone knows something important is missing. In the same way, solving equilibrium problems without a free-body diagram often produces an answer, but it often lacks balance, clarity, and sometimes correctness.

Concisely, a free-body diagram *is a diagram of a body isolated from its surroundings, showing* ***only the forces acting on it***. It is the bridge between the real situation and the equilibrium equations: $\Sigma F_x = 0$ and $\Sigma F_y = 0$.

Is a Free-Body Diagram Always Necessary?

It is not always necessary to draw a free-body diagram before solving equilibrium problems, especially in very simple situations where the forces are obvious and act along a single straight line. In such cases, the equilibrium equations can sometimes be written directly without first sketching a diagram.

However, as soon as forces act in different directions, particularly at angles, drawing a free-body diagram becomes extremely important. The diagram helps to:

- Identify all forces acting on the body.
- Show their correct directions.
- Avoid missing or inventing forces.
- Guide the correct resolution of forces into components.

Without a proper free-body diagram, mistakes often occur not in algebra but in identifying forces and their directions.

For this reason, although not always compulsory, drawing a free-body diagram is considered best practice in A-Level Physics and is strongly recommended before applying the equilibrium equations.

How to Draw a Free-Body Diagram (Step-by-Step)

Step 1: Choose the body

Decide exactly what you are analysing (a block, a ring, a hanging load, a trolley, etc.). Treat it as a **particle** (a point).

Step 2: Isolate it

Imagine the body is cut free from contact surfaces, strings, and supports. The surroundings are removed, but the **forces they exert remain**.

Step 3: Replace each interaction by a force arrow

Draw arrows starting from the particle. Each arrow represents a force on the body.

Step 4: Label every force

Use standard symbols (**W** for weight, **R** for normal reaction, **T** for tension, **f** for friction force). If a force is at an angle, show the angle clearly.

Step 5: Choose axes

Usually: **x** for horizontal, **y** for vertical. Choose positive directions. If an incline is involved, you may choose axes parallel and perpendicular to the plane (later in the chapter).

What to Include (and What NOT to Include) in FBDs

A correct free-body diagram includes only the relevant forces acting on the body and nothing more. The following guidelines help ensure clarity and accuracy.

Include:

- Forces **acting on** the body (pushes, pulls, weight, contact forces).
- Directions of forces.
- Angles where needed.

Do NOT include:

- Motion arrows (velocity) as if they were forces.
- Forces exerted **by** the body on other objects (unless that other object is your chosen body).
- Extra forces invented to “make it balance.”

Always obey the following **rule:** *Every force must have a clearly identifiable source (agent). If you cannot say what is causing the force, you should question whether it really exists.*

Common FBD Mistakes

Errors in equilibrium analysis frequently originate from incorrect free-body diagrams rather than algebraic mistakes. The following are common pitfalls students should avoid.

- **Mixing action and reaction on the same diagram**

  Example: If you draw forces on a block resting on a table, include the weight of the block and the upward reaction from the table. Do not include the force the block exerts on the table, because that force acts on the table, not on the block.

  So, *always draw only forces acting on the object you are analysing, not the forces it exerts on other bodies.*

- **Friction drawn in the wrong direction**

  Friction opposes motion: it acts opposite to actual movement or to the direction an object is trying to move.

- **Assuming the Normal Reaction Is Always Vertical**

  The normal reaction force acts perpendicular (at right angle) to the contact surface and is therefore **vertical only when the surface is horizontal** (the normal reaction is not necessarily vertical).

- **Leaving out one force**

  In equilibrium problems, missing a force is the fastest way to destroy the conditions $\Sigma F_x = 0$ and $\Sigma F_y = 0$.

Before these ideas start colliding in our heads, let us calm them down with a few useful worked examples.

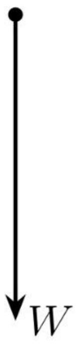
BINDER Example 6

A small metal ball of mass 0.50kg is released and allowed to fall freely through the air (air resistance neglected).

(a) Draw the free-body diagram of the ball.
(b) State the resultant force on the ball.
(c) Show that the acceleration of the ball is equal to the acceleration due to gravity, g.

Solution

(a)



(b) Resultant force is downward and equals the weight, $W = 0.5N \times 9.8N/kg = 4.9N$ downward.

(c) Using Newton's second law in the vertical direction:

$$\Sigma F_y = ma_y$$
$$W = ma_y$$

But $W = mg$

$$mg = ma_y$$

So: $\mathbf{a_y = g}$

Making Sense of the Answer: The ball accelerates because there is no other force to cancel its weight. **Thinking Like a Physicist:** If you cannot name the agent producing a force (string, surface, air), do not draw it.

BINDER Example 7

A particle is acted on by two horizontal forces: 20N to the right and 12N to the left.

(a) Draw the free-body diagram.
(b) Find the resultant force.
(c) A student adds an extra 8N force to the left on the diagram and says, *"The particle must be in equilibrium."* Explain what is wrong with this thinking.
(d) State the equilibrant force for the given forces.

Solution

(a) FBD

(b) Taking right as positive:

$$F_R = 20N - 12N = 8N$$

The resultant force is 8N to the right.

(c) The extra 8N force has no physical source (no agent). Forces cannot be added simply because we want equilibrium; they must come from a real interaction (a string, a push, contact, etc.). The particle is not automatically in equilibrium; it accelerates if the resultant is not zero.

(d) The equilibrant is the single force that would cancel the resultant: $F_E = 8N$ to the left

Making Sense of the Answer: *Equilibrium is a condition produced by real forces, not a wish added to the diagram.*

Thinking Like a Physicist: *Never "repair" a diagram by inventing forces. First ask: what object could apply that force?*

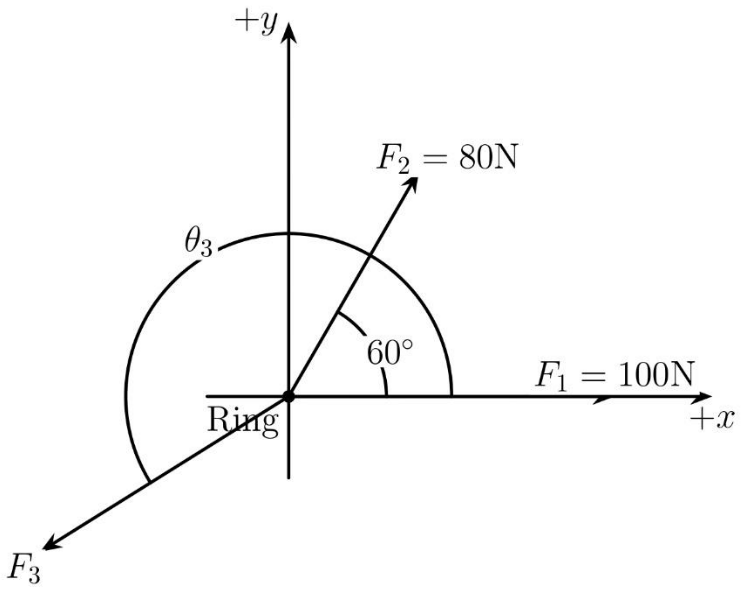
BINDER Example 8

Three coplanar forces act on a small ring at rest at a junction of light cords. Force $\mathbf{F_1 = 100N}$ acts horizontally to the right. Force $\mathbf{F_2 = 80N}$ acts at **60° above the horizontal** (measured counterclockwise from the positive x-axis). Force $\mathbf{F_3}$ has unknown magnitude and unknown direction.

(a) Determine the **magnitude** of $\mathbf{F_3}$ required for equilibrium.
(b) Determine the **direction** of $\mathbf{F_3}$ (measured counterclockwise from the positive x-axis).

Solution

Since F_1 and F_2 together act generally northeast, the equilibrant force F_3 must act generally southwest. So, the free body diagram for the example 8 will be as follows:



Resolving F_1 and F_2 into components

$\mathbf{F_1}$ **= 100N to the right (**0° from the positive x − axis**)**

- $(F_1)_x = 100N$ (By using formula it is $100N\cos 0° = 100N$)
- $(F_1)_y = 0$ (By using formula it is $100N\sin 0° = 0N$)

$\mathbf{F_2}$ **= 80N at 60°**

- $(F_2)_x = 80\cos 60 = 40N$
- $(F_2)_y = 80\sin 60 = 69.3N$

The equilibrium condition: $\Sigma F_x = 0$ and $\Sigma F_y = 0$
Then;

$$\Sigma F_x = 100N + 40N + (F_3)_x = 0;\ \mathbf{(F_3)_x = -140N}$$

And;

$$\Sigma F_y = 0N + 69.3N + (F_3)_y = 0;\ \mathbf{(F_3)_y = -69.3N}$$

The magnitude of F_3 is given by the following relationship:

$$F_3 = \sqrt{((F_3)_x)^2 + \left((F_3)_y\right)^2} = \sqrt{(-140)^2 + (-69.3)^2} = 156.21N$$

(a) The magnitude is $156.21\mathrm{N}$.

From geometry of free body diagram;

$$\tan(\theta_3 - 180) = \frac{(F_3)_y}{(F_3)_x} = \frac{-69.3}{-140} = 0.495$$

$$\theta_3 - 180 = \tan^{-1} 0.495 = 26.33°;\ \theta_3 = 26.33° + 180° = 206.33°$$

(b) The direction of F_3 is $206.33°$ from the positive x-axis (counterclockwise).

Making Sense of the Answer: *Since F_1 and F_2 together act generally northeast, the equilibrant F_3 must act generally southwest to maintain equilibrium. Its magnitude simply matches the resultant of F_1 and F_2, but its direction is opposite.*

Thinking Like a Physicist: *Before calculating, always predict the direction. If the combined forces point northeast, the equilibrant must point southwest. This quick check helps prevent sign and angle mistakes.*

As the worked examples quietly leave the table, the next subtopic arrives; not to overwhelm us, but to be understood and enjoyed!

STANDARD FORCE MODELS

In equilibrium problems, correctly identifying the forces acting on a body is often more important than the mathematics that follows. Fortunately, many physical situations involve a small number of common force types that appear repeatedly. Recognising these standard force models allows you to draw accurate free-body diagrams quickly and apply the equilibrium equations with confidence. Among the most frequently encountered forces are **weight (W)**, **normal reaction (R), tension (T)**, and **friction**. Each has characteristic origins, directions, and physical interpretations, and mastering them forms an essential foundation for solving problems in Particle Mechanics.

Understanding these force models not only improves diagram accuracy but also prevents common mistakes such as inventing forces, misdirecting forces, or overlooking important interactions. Once these forces are correctly identified, the equilibrium equations: $\Sigma F_x = 0$ and $\Sigma F_y = 0$ become straightforward to apply.

And just as a well-prepared free-body diagram was compared to the essential spices in **pilau**, these standard force models are the actual ingredients themselves. Without recognising them properly, you may still attempt the analysis, but the final result can feel as disappointing as pilau cooked without the right spices; it may fill the stomach, but it certainly does not excite the taste buds, much like solving Physics problems without properly identifying the forces.

Weight, W

Weight is the gravitational force exerted on a body due to the Earth's gravitational field. It is one of the most fundamental forces encountered in Particle Mechanics because it acts on virtually every object near the Earth's surface.

The magnitude of the weight is given by: $W = mg$

Where:

m = mass of the body

g = acceleration due to gravity (approximately 9.8 m/s² near the Earth's surface).

Recognising weight correctly in free-body diagrams is essential. It is often the first force to identify, and errors in its direction or magnitude can lead to incorrect equilibrium analysis.

It is very important to understand that:

Weight always acts *vertically downward*, towards the centre of the Earth, regardless of the body's orientation or motion. *Even when an object rests on an inclined plane, hangs from a string, or moves through the air, its weight* ***still acts vertically downward****.*

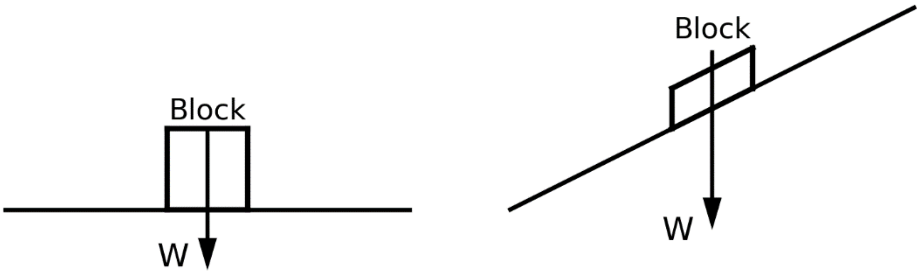


Figure: *The weight of a body always acts vertically downward towards the centre of the Earth, regardless of whether the body rests on a horizontal surface, an inclined plane, or is suspended in space. Surface orientation does not affect the direction of weight.*

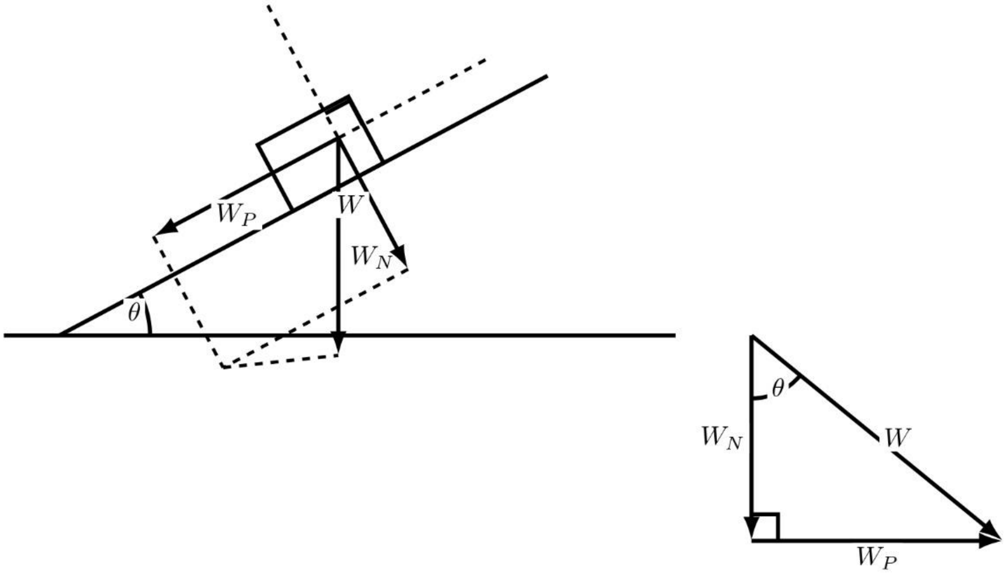
Components of Weight Parallel and Normal to an Inclined Plane

When a body rests on an inclined plane, its weight still acts vertically downward toward the centre of the Earth. However, because the surface is tilted, it is often more convenient to analyse this single gravitational force by resolving it into two components: one **parallel to the plane** and one **perpendicular (normal) to the plane**. This approach simplifies equilibrium analysis because each component directly relates to the forces (friction and normal reaction) that usually oppose them: friction acts parallel to the surface (opposite to the weight's parallel component), while the normal reaction acts perpendicular to it (opposite to the weight's perpendicular component).

To have better understanding of this, consider a body of weight, **W = mg** placed on a plane inclined at an angle **θ to the horizontal**. Resolving the weight into components relative to the plane gives:

- **Component parallel to the plane** $= \mathrm{W\sin\theta} = \mathrm{mg\sin\theta}$
- **Component perpendicular to the plane** $= \mathrm{W\cos\theta} = \mathrm{mg\cos\theta}$

These expressions arise from simple right-triangle geometry formed when the vertical weight vector is resolved relative to the inclined surface as shown in the figure.



(a) (b)

Figure: ***(a)*** *Weight (W) of a body on an inclined plane resolved into components parallel to the plane (W_P) and normal (perpendicular) to the plane (W_N).* ***(b)*** *Component triangle showing the geometric relationship between the weight and its parallel and normal components.*

From the figure it is clearly understood that when a body rests on an inclined plane, its weight $W = mg$ acts vertically downward. For analysis relative to the plane, we resolved this single force into two mutually perpendicular components:

1) W_P: component parallel to the plane (down the slope).
2) W_N: component normal (perpendicular) to the plane (into the plane).

From diagram (a), the three vectors form a right-angled triangle: the hypotenuse is W, and other sides are W_N and W_P.

Before we proceed, let us first familiarise ourselves with the key geometric features of the diagram.

1. The plane makes an angle θ with the horizontal.
2. The normal line is perpendicular to the plane, so the normal is tilted by θ from the vertical.
3. Since W is vertical, the angle between W and the normal direction is θ.
   So, in the component triangle (diagram b), θ is the angle between W and W_N.

Now, in the tringle:

Hypotenuse $=$ W, Adjacent to $\theta = W_N$, Opposite to $\theta = W_P$

Therefore:

$$\cos\theta = \frac{\text{adjacent}}{\text{hypotenuse}} = \frac{W_N}{W}$$

$$\sin\theta = \frac{\text{opposite}}{\text{hypotenuse}} = \frac{W_P}{W}$$

Hence:

$$\mathbf{W_N = W\cos\theta = mg\cos\theta = Normal\ (perpendicular) component\ of\ the\ weight}$$

$$\mathbf{W_P = W\sin\theta = mg\sin\theta = Parallel\ component\ \ of\ the\ weight}$$

An important conceptual point is that resolving weight does **not create new forces**. The weight remains a single gravitational force acting vertically downward. The components are simply a convenient way of describing how that force influences motion relative to the plane.

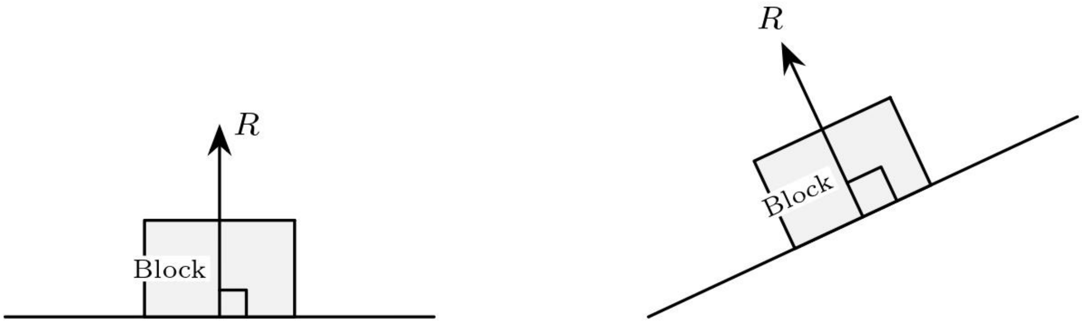
Understanding these components is essential in Particle Mechanics because many real systems: vehicles on slopes, ladders against walls, cables supporting loads, or objects resting on ramps; depend on correctly analysing forces relative to inclined surfaces. Once the weight is resolved appropriately, applying the equilibrium equations: $\mathbf{\Sigma F_{parallel} = 0}$ and $\mathbf{\Sigma F_{perpendicular} = 0}$ becomes straightforward and systematic.

In practice, *whenever an inclined plane appears in a problem, resolving the weight parallel and perpendicular to the plane is usually the first and most important step toward a clear and accurate solution.*

Normal Reaction, R (or R_n)

The **normal reaction** *is the force exerted by a surface on a body in contact with it*. It arises due to the interaction between the surfaces and acts to prevent the two bodies from passing through each other and thus they do not occupy the same space. In equilibrium problems, the normal reaction is one of the most frequently encountered contact forces.

The direction of the normal reaction is always perpendicular (**normal**) to the surface of contact. This is an important point: *the normal reaction is not necessarily vertical.* It becomes vertical **only** when the surface itself is horizontal. When a body rests on an inclined plane, for example, the normal reaction acts perpendicular to that plane rather than vertically upward.



Horizontal surface

Inclined surface

Figure: *The normal reaction always acts perpendicular to the surface of contact. Its direction depends on the orientation of the surface and is not always vertical.*

Correct identification of the normal reaction in free-body diagrams is essential for accurate equilibrium analysis. Confusing its direction with that of weight is a common mistake, especially in problems involving inclined surfaces.

It should also be remembered that the normal reaction is a responsive force. It adjusts its magnitude according to the situation. In some cases, it may equal the weight of the body, but this is not a general rule; additional forces acting on the body can increase or decrease its value. **For example,** *if a downward push is applied in addition to weight, the normal reaction increases; if a lifting force acts upward, the normal reaction decreases.*

The following points summarise the key features of the normal reaction:

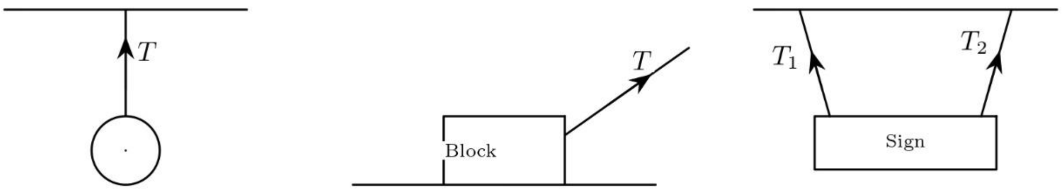
1. It always acts perpendicular to the surface of contact.
2. It exists only when there is physical contact between the body and the surface.
3. Its magnitude is not necessarily equal to the weight of the body. Equality occurs only in special cases, such as when a body rests on a horizontal surface with no other vertical forces acting.
4. For a body of mass, m resting on an inclined plane at an angle θ to the horizontal, **the normal reaction is** $\mathbf{R = mg\cos\theta}$ (normal component of weight) provided that no other forces act in that perpendicular direction.

Tension, T

In Particle Mechanics, **tension** is one of the most common forces in equilibrium problems. Tension appears whenever a body is pulled, supported, or suspended by a string, rope, cable, chain, or wire. From hanging signboards and ceiling lamps to suspension bridges, cranes, elevators, and even the strap of your school bag, tension forces are quietly working to maintain balance.

Concisely, tension *is the **pulling force** transmitted through a **stretched** string, rope, cable, or wire.*

A key feature of tension is its direction. Unlike the normal reaction, which pushes perpendicular to a surface, tension always acts **along the length** of the string or cable and pulls **away from the body** to which it is attached. Therefore, when representing tension in a free-body diagram, always draw the arrow pointing **away from (NOT toward) the body along the string or cable** (see the figure). This reflects the physical fact that a string can only pull, not push. *Drawing the arrow toward the body may suggest compression rather than tension and can lead to **incorrect** interpretation of the forces acting on the system.*



(a) Hanging lamp (b) Block pulled by rope (c) Suspended signboard

Figure: *In all cases, tension acts along the string or cable and always pulls away from the body.*

Modelling assumptions used in A-Level mechanics

To make problems solvable, we often use idealised models:

- **Light (massless) string:**

  The string's weight is negligible, so we ignore its weight in the analysis.

- **Inextensible string:**

  The string does not stretch, so its length remains constant.

- **Smooth pulley (if present):**

  No friction between string and pulley, so the tension remains the same on both sides of the pulley.

Under these ideal conditions, the tension has the same magnitude throughout a continuous string.

Important clarifications about tension

- **Tension is not automatically equal to weight.**

  For a hanging object at rest, tension equals weight only if the object is supported by a single vertical string and no other vertical forces act.

- **More than one string means tensions can share the load.**

  For example, if a body is supported by two strings at angles, each string provides a tension that contributes a **vertical component** supporting the weight.

- **Tension acts at the point of attachment.**

  In particle mechanics we often draw it as acting on the particle, but the physical meaning is **"the string pulls the body at the attachment point."**

- **Tension exists only when the string is taut**

  If the string becomes slack, it can no longer exert a pulling force; therefore, the tension effectively becomes zero.

Friction Force, f

In Particle Mechanics, **friction** is another very common force encountered in many physical situations, including both equilibrium and motion. From walking without slipping and vehicles braking safely to ladders resting against walls and objects remaining stationary on inclined surfaces, friction plays an important role in maintaining stability, control, and balance in everyday life.

Friction is a contact force that arises when two surfaces touch and there is relative motion, or a tendency for relative motion, between them. It acts along the surface of contact and always opposes the direction in which motion occurs or would occur.

A crucial feature of friction is its direction. **Friction always acts parallel to the surface and opposite to the direction of relative motion or the tendency to move.** Therefore, when representing friction in a free-body diagram, always draw the friction force parallel to the surface and opposite to the direction of motion or impending motion (see the figure).

(a) Actual motion (b) Impending motion

Friction on a horizontal surface: *Friction acts along the surface of contact and always opposes either the actual motion of the body or its tendency to move.*

(a) Tendency to slide down the plane (b) Block pulled up the plane

Friction on an inclined plane: *Friction acts parallel to the plane and opposes the tendency of motion, acting up the plane if the body tends to slide down and down the plane if the body is pulled upward.*

Two main types of friction are commonly considered in Particle Mechanics:

1. Static friction

Static friction acts when two surfaces are in contact but there is **no relative motion** between them. Its role is to prevent motion from starting.

An important property of static friction that its magnitude adjusts itself as needed (up to a limiting value) to prevent motion. This is why a gently pushed object may remain at rest as friction increases just enough to maintain equilibrium.

The adjustment obeys the following condition:

$$\mathbf{f \le \mu R}$$

Where:

f is frictional force.

μ is coefficient of static friction.

R is normal reaction between the surfaces.

It is worth to understand that when static friction reaches its maximum possible value just before motion begins, it is called **limiting friction** and the body is said to be at **limiting equilibrium**.

At this point:

$$\mathbf{f = \text{limiting friction} = \mu R}$$

Beyond this point, motion starts and static friction is no longer existing.

2. Kinetic (sliding) friction

Once motion begins, static friction no longer operates because the surfaces are no longer at rest relative to each other. However, a resisting force still exists that opposes the motion. This force is known as **kinetic** (or **sliding) friction**. Its magnitude is usually slightly smaller than the maximum static friction (limiting friction), which explains why an object often moves more easily once it has started sliding.

Important clarifications about friction

1. **Friction always acts parallel to the contact surface and never perpendicular to it**

   Although friction is related to the normal reaction (the perpendicular contact force), friction itself does not act perpendicular to the surface. Instead, it always acts parallel to the surface of contact.

2. **Friction is not always present**

   In many mechanics models, surfaces may be treated as smooth (frictionless). In such cases, friction is assumed to be zero to simplify analysis.

3. **Friction does not always equal μR**

   The relation $f = \mu R$ applies only at limiting equilibrium. In many situations, friction is less than μR.

4. **Friction opposes motion or attempted motion**

   It does not necessarily oppose an applied force directly; it opposes the resulting motion tendency.

5. **Friction exists only when surfaces are in contact**

   If contact is lost, friction immediately becomes zero.

Do not invent friction: A conceptual warning

Friction should never be invented simply to “make forces balance.” It must have a physical cause: contact between surfaces with a tendency to slide. If no such tendency exists, friction is zero. For example: *A book resting on a horizontal table experiences only its weight downward and the normal reaction upward. Since there is no horizontal force tending to cause motion, no friction is required, and the frictional force is therefore zero.*

Combining All Four Forces: W, R, T, and f

Up to this point, we have examined the standard forces of Particle Mechanics one at a time:

- **Weight (W)** acting vertically downward due to gravity,
- **Normal reaction (R)** arising from surface contact and acting perpendicular to that surface,
- **Tension (T)** transmitted through strings, ropes, or cables and always pulling away from the body, and
- **Friction (f)** acting parallel to the surface and opposing relative motion or the tendency of motion.

Individually, each force is straightforward. But real physical situations rarely present forces one by one. A block may rest on a rough inclined plane while attached to a string, a ladder may lean against a wall with friction preventing slipping, or a suspended load may simultaneously experience tension, weight, and contact forces. Understanding equilibrium therefore requires seeing **how these forces work together**.

This integration marks an important step: you are no longer just recognising forces; you are analysing complete physical systems.

A unified view of equilibrium

When all four forces appear together, three fundamental direction rules must always be remembered:

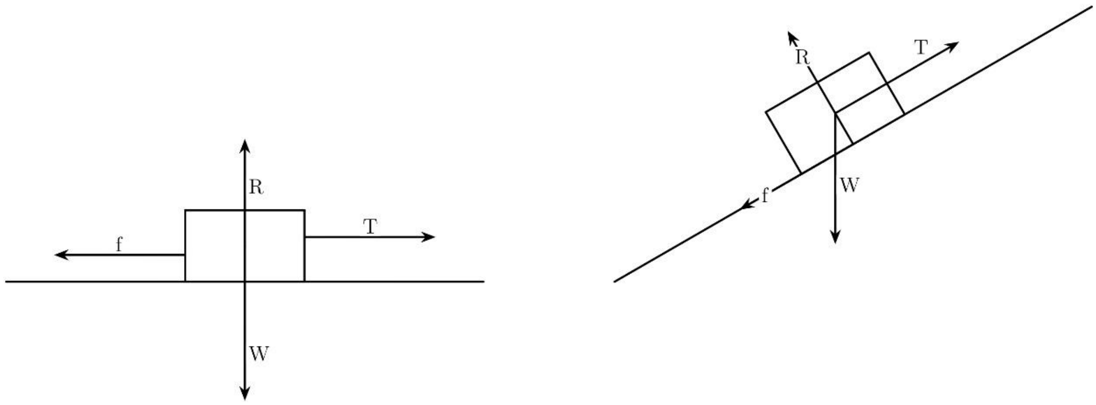
Rule 1: ***Weight (W)*** *always acts vertically downward, regardless of the surface orientation.*

Rule 2: ***Normal reaction (R)*** *always acts perpendicular to the contact surface.*

Rule 3: ***Tension (T)*** *always acts along the string or cable, pulling away from the body.*

Rule 4: ***Friction (f)*** *always acts parallel to the surface and opposes actual or impending motion.*

These directional rules are not optional; they come directly from the physical origin of each force. Correct free-body diagrams depend on respecting these directions.



(a) Horizontal surface (b) Inclined plane

Forces on a block on horizontal and inclined surfaces: *Weight (W), normal reaction (R), tension (T), and friction (f) acting on a body.*

Have you noticed this interesting fact?

Weight comes from gravity,

Reaction comes from contact,

Tension comes from connection,

Friction comes from surface interaction.

Different physical origins; yet all cooperate to maintain equilibrium.

Physics often looks complicated only until the forces are identified correctly. Once the forces are clear, equilibrium analysis becomes systematic rather than mysterious.

Why this integration matters?

Combining W, R, T, and f allows us to analyse many practical situations, including:

1. Bodies on rough inclined planes supported by strings,
2. Ladders resting against walls,
3. Suspended structures in contact with surfaces,
4. Engineering systems where stability depends on multiple interacting forces.

These are not artificial textbook problems; they reflect how structures remain safe and stable in everyday life.

Congratulation!

You have now completed one of the most important subtopics in Particle Mechanics. Mastery of free-body diagrams, the two equilibrium equations ($\Sigma F_x = 0$ and $\Sigma F_y = 0$), and the standard force models (weight, normal reaction, tension, and friction) gives you powerful tools for analysing forces with confidence. These ideas will not only make Particle Mechanics clearer and more enjoyable, but will also support your understanding across many other areas of Physics where balance, motion, and interaction are involved.

From this stage onward, equilibrium problems become less about memorising formulas and more about thinking clearly. And that, ultimately, is the goal of Advanced Physics: not just solving problems, but understanding why systems remain balanced in the first place.

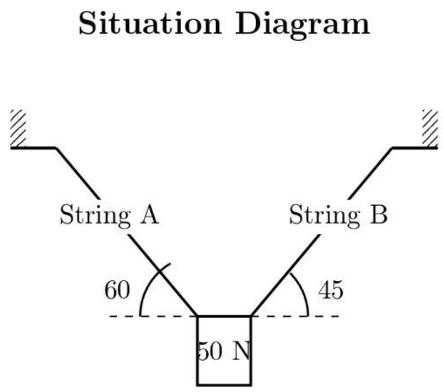
With the ideas now simmering nicely, let us serve them properly through a few worked examples and enjoy the flavour of physics in action.

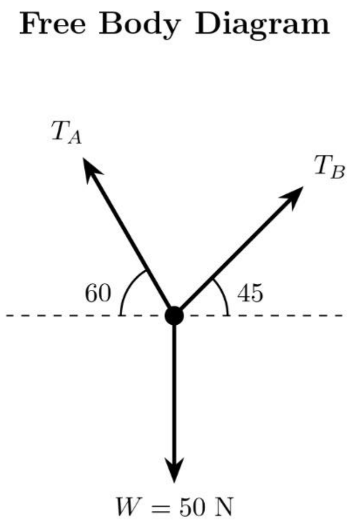
BINDER Example 9

A 50N weight is suspended at rest from a junction by two light strings. **String A** is inclined at 60° to the horizontal and **string B** is inclined at 45° to the horizontal (both above the horizontal). Calculate the tension in each string.

Solution

To be at rest (equilibrium), the two strings must pull upward and outward from opposite sides of the load as shown in the **situation diagram**.





Resolving each force to horizontal and vertical component (refer to the free body diagram):

For string A:

$$\theta = \text{angle measured from positive x} - \text{axis} = 180° - 60° = 120°$$

Horizontal component $= (T_A)_x = T_A\cos120° = -0.5T_A$

Vertical component $= (T_A)_y = T_A\sin120°$

For string B:

$$\theta = \text{angle measured from positive x} - \text{axis} = 45°$$

Horizontal component $= (T_B)_x = T_B\cos45°$

Vertical component $= (T_B)_y = T_B\sin45°$

For weight:

Weight always acts vertically downward; therefore, its horizontal component is zero, while its vertical component is equal in magnitude to the weight but directed downward ($-\mathbf{50}\boldsymbol{N}$).

This can also be shown using the **standard resolution method** as follows:

$$\theta = \text{angle measured from positive x} - \text{axis} = 270°$$

Horizontal component $= W_x = W\cos270° = 0$

Vertical component $= W_y = W\sin270° = -W = -50\text{N}$

From equilibrium equations: : $\Sigma F_x = 0$ and $\Sigma F_y = 0$

Then;

$$\Sigma F_x = -0.5T_A + T_B\cos45° + 0 = 0 \text{ or}$$

$-0.5T_A + T_B\cos45° = 0 \ldots\ldots$ (i)

And;

$$\Sigma F_y = T_A \sin 120° + T_B \sin 45° - 50 = 0 \text{ or}$$

$T_A \sin 120° + T_B \sin 45 = 50 \ldots\ldots$ (ii)

Solving (i) and (ii) simultaneously gives: $T_A = 36.6\text{N}$; $T_B = 25.9\text{N}$

Tension in string A is 36.6N.

Tension in string B is 25.9N.

Making Sense of the Answer: *The tensions are unequal because the strings are inclined at different angles. The string at 60° (string A) provides a larger vertical component, so it carries more of the weight. Their vertical components balance the 50N load, while the horizontal components cancel maintaining equilibrium.*

Thinking Like a Physicist: *Instead of guessing signs,* ***measure angles consistently from the positive x-axis****. For example, using 120° rather than 60° automatically gives the correct horizontal and vertical components. This systematic approach reduces sign errors and keeps equilibrium analysis clear.*

Method Insight: *In this first example, the weight was resolved explicitly using the general vector method to illustrate the procedure clearly. In most later problems, since weight acts vertically downward, we will usually write its components directly without repeating the full resolution each time.*

BINDER Example 10

A 5kg block rests on a horizontal table. A horizontal force of 12N is applied to the block. The coefficient of static friction between block and table is $\mu = 0.4$.

(a) Determine whether the block remains in equilibrium.
(b) Find the magnitude of the frictional force acting on the block.

$$\text{Take } g = 9.8\text{m/s}^2$$

Solution

Normal reaction: $R = mg = 5\text{kg} \times 9.8\text{N/kg} = 49\text{N}$

Limiting friction: $f = \mu R = 0.4 \times 49\text{N} = 19.6\text{N}$

But the applied force, $F = 12\text{N}$ which is smaller than the limiting friction.

(a) Since the applied force (12N) is smaller than the limiting friction (19.6N), the static friction is sufficient to prevent motion and hence the block remains in equilibrium.
(b) When the block is at equilibrium, frictional force equals the applied force (12N) and hence the frictional force acting on the block is 12N.

Making Sense of the Answer: *Static friction adjusts itself to oppose the applied force up to its maximum value. Since the applied force (12N) is less than the maximum possible static friction (19.6N), the block does not move, and friction simply matches the applied force to maintain equilibrium.*

Thinking Like a Physicist: *Do not assume friction is always μR. That expression gives only the maximum static friction. Always compare the applied force with this maximum first; if it is smaller, the actual friction equals the applied force, not μR.*

REAL Example 11

Kipanga is pulling his overloaded school bag along the ground using a strap. He can pull horizontally or at an angle upward. Explain why pulling at an angle upward might make it easier to move the bag, even though some of the force is "wasted" going upward instead of forward.

Solution

Pulling the bag at an upward angle reduces the **normal reaction (R)** between the bag and the ground. When Kipanga pulls partly upward, that upward component of the pulling force slightly lifts the bag, so the ground presses on it less strongly.

Since friction depends on the normal reaction ($f = \mu R$), a smaller normal reaction means **less frictional force resisting motion**. Although part of the pulling force is directed upward rather than forward, the reduction in friction usually outweighs this "loss," making the bag easier to move.

Making Sense of the Answer: *Physics often involves trade-offs. Sacrificing some forward force to reduce friction can make pulling easier overall. Efficiency is not always about pushing hardest in one direction.*

Thinking Like a Physicist: *Real-world situations often involve balancing competing effects. The best pulling angle can be analysed mathematically at advanced levels, but the key idea is simple: reducing resistance can be more effective than simply increasing effort.*

REAL Example 12

During a practical lesson, **Kipute** places her Physics textbook on a desk that is tilted at a small angle. The book stays in place.

Kipanga: *"The book must be glued to the desk! Otherwise gravity would pull it down."*

Kipute: *"Don't be silly! There's no glue. But Kipanga has a point, Mr. Akilikubwa; why doesn't the book slide?"*

Mr. Akilikubwa: (smiling) *"Ah! You have discovered friction, the invisible guardian. No glue needed. The desk surface and the book touch, and friction acts like tiny invisible hands holding the book in place, opposing the tendency to slide."*

Kipanga: *"So friction is always there?"*

Mr. Akilikubwa: *"Not quite! Friction only appears when there is a tendency to move. Place the same book on a perfectly horizontal desk......, no tendency to slide, so friction takes a break and remains zero. Tilt the desk slightly, and friction wakes up just enough to prevent motion. Tilt it more, and friction increases up to its maximum strength. Tilt it too much, and friction loses the battle; the book slides. Friction is a responsive force, not a permanent one."*

Kipute: *"So it's like a lazy security guard who only works when needed?"*

Mr. Akilikubwa: (laughing) *"Exactly! And just like a lazy guard, once the thief (motion) escapes and the book starts sliding, the guard (static friction) stops trying and switches to a different mode called kinetic friction, which is usually weaker."*

Question: Based on this dialogue, explain:

(a) Why friction does not act on a book resting on a horizontal table.
(b) Why friction increases as the desk tilt increases (up to a limit)
(c) What happens when the book finally starts sliding.

In (b) and (c), justify your explanation with relevant mathematical equations.

Solution

(a) When the book rests on a horizontal table, weight acts vertically downward and is balanced by the normal reaction acting vertically upward. There is no force component parallel to the surface that would tend to cause motion. Since friction opposes attempted motion, and there is no such attempt, friction is zero.

(b) As the desk tilts, weight develops a component parallel to the surface (down the slope). This component tries to pull the book downward. Static friction responds by acting up the slope with exactly the right magnitude to prevent motion. The steeper the tilt, the larger the parallel component of weight, so friction must increase to maintain equilibrium up to its maximum possible value (limiting friction, $f = \mu R$).

Mathematical justification:

As the desk tilts by angle θ:

- Weight develops a component parallel to surface: $W\sin\theta$ (down the slope)
- This parallel component tries to pull the book downward.
- Static friction responds by acting up the slope: $f = W\sin\theta$ (upward)
- The steeper the tilt (larger value of $\sin\theta$), the larger $W\sin\theta$ becomes, so friction must increase to maintain equilibrium.

However, friction has a maximum value: $f_{max} = \mu R = \mu W\cos\theta$

Friction can increase up to this limit. Beyond this, equilibrium breaks.

(c) When tilt becomes too steep, the parallel component of weight exceeds the maximum static friction. At this point, static friction can no longer prevent motion, and the book begins to slide. Once motion starts, kinetic (sliding) friction takes over, which is typically smaller than the maximum static friction and thus the book continues to slide easily.

Mathematical justification:

When tilt becomes too steep: $\mathbf{W\sin\theta > \mu W\cos\theta}$ or $\mathbf{\tan\theta > \mu}$ (by dividing $W\cos\theta$ both sides)

At this point:

- Static friction reaches its maximum but can no longer prevent motion. So the book begins to slide.
- Static friction no longer operates (surfaces are moving relative to each other).
- Kinetic (sliding) friction takes over: $f_k = \mu_k R$

As $\mu_k < \mu$ (kinetic friction is weaker than maximum static friction), once the book starts sliding, it continues more easily.

Making Sense of the Answer: *Friction is not a fixed force; it adjusts itself as needed (up to a limit). Understanding this adaptive behavior prevents the common mistake of assuming friction always equals μR. The equation f = μR applies only at limiting equilibrium, just before motion begins.*

Thinking Like a Physicist: *Mr. Akilikubwa's "lazy security guard" analogy captures an important truth: friction is a passive, responsive force. It doesn't initiate anything; it merely reacts to prevent motion. This is fundamentally different from active forces like applied pushes or gravity, which act regardless of the situation.*

BINDER Example 13

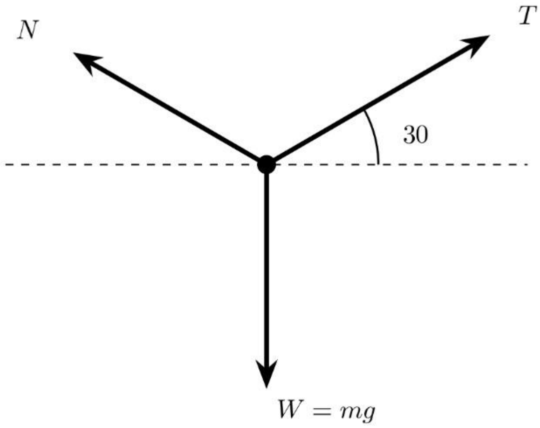
A 10kg block rests on a smooth (frictionless) inclined plane that makes an angle of 30° with the horizontal. The block is held in place by a string parallel to the plane. Take $g = 9.8\text{m/s}^2$.

Calculate:

(a) the normal reaction from the plane and (b) the tension in the string

Solution

Forces acting on the block are shown in the following free body diagram.



Resolving weight into components:

Component parallel to plane (down):

$$W_{\text{parallel}} = mg\sin\theta = 10 \times 9.8\sin30° = 49\text{N}$$

Component perpendicular to plane (into surface):

$$W_{\text{perp}} = mg\cos\theta = 10 \times 9.8\cos30° = 98\text{N}\cos30°$$

Forces acting perpendicular to plane:

- Normal reaction (**away** from surface), R.
- Perpendicular component of weight (**into** surface), 98Ncos30°.

Since the two forces act in opposite directions and the block is at equilibrium (held in place):

$$R = 98N\cos 30° = 84.9N$$

Forces acting parallel to plane:

- Tension (upward), T.
- Parallel component of weight (downward), $W_{\text{parallel}} = 49N$

Again, the two forces act in opposite directions and the block is still at equilibrium. So:

$$T = 49N$$

(a) The normal reaction is 84.9N.
(b) The tension in string is 49N.

Making Sense of the Answer: *The normal reaction* (84.9*N*) *is less than the weight* (98*N*) *because the plane is tilted. Only the component of weight perpendicular to the plane is balanced by R. The weight component parallel to the plane* (49*N*) *tries to pull the block down, so the string must provide equal tension upward along the plane to maintain equilibrium.*

Thinking Like a Physicist: *On an incline, always resolve weight into components parallel and perpendicular to the surface. This transforms a two-dimensional problem into two separate one-dimensional problem. The steeper the incline, the larger the parallel component (harder to hold) and the smaller the perpendicular component (smaller normal reaction).*

BINDER Example 14

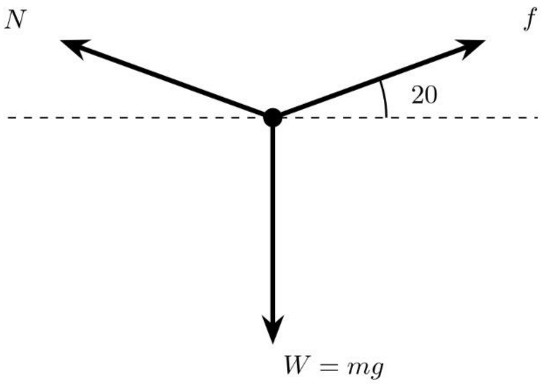
A block of mass 20kg rests on a rough inclined plane inclined at 20° to the horizontal. The block is on the verge of sliding down the plane. The coefficient of static friction between the block and the plane is $\mu = 0.364$. Take $g = 9.8m/s^2$.

Calculate:

(a) The normal reaction between the block and the plane.
(b) The frictional force acting on the block.
(c) Verify that the block is in limiting equilibrium.

Solution

Forces acting on the block are shown in the following free body diagram.



Resolving weight into components:

Component parallel to plane (downward):

$$W_{\text{parallel}} = mg\sin\theta = 20 \times 9.8\sin 20° = 196N\sin 20°$$

Component perpendicular to plane (into surface):

$$W_{perp} = mg\cos\theta = 20 \times 9.8\cos 20° = 196\text{N}\cos 20°$$

Forces acting perpendicular to plane:

- Normal reaction (**away** from surface), R.
- Perpendicular component of weight (**into** surface), 196Ncos20°.

Since the two forces act in opposite directions and the block is at equilibrium (it is just on verge of sliding, not yet moving):

$$R = 196\text{N}\cos 20° = 184.15\text{N}$$

(a) The normal reaction is 184.15N.

Forces acting parallel to plane:

- Friction force (upward), f.
- Parallel component of weight (downward), $W_{parallel} = 196\text{N}\sin 20$.

Again, the two forces act in opposite directions and the block is still at equilibrium. So:

$$f = 196\text{N}\sin 20 = 67.04\text{N}$$

(b) The frictional force is 67.04N.

Limiting equilibrium is found when: $f_{max} = \mu R$

Substituting $f_{max} = 0.364 \times 184.15\text{N} = 67.03\text{N} \approx 67.04\text{N} = f$ (found in (b))

(c) Since the actual frictional force is equal to the limiting friction, the block is in limiting equilibrium.

Alternative verification

Using the condition $\boldsymbol{\mu = \tan\theta}$ at limiting equilibrium.

$\tan 20° = 0.364$

Given $\mu = 0.364$

Since $\mu \approx \tan\theta$, limiting equilibrium is confirmed.

Making Sense of the Answer: *At limiting equilibrium, friction reaches its maximum value, which exactly balances the down-slope component of weight. Any slight increase in angle or decrease in μ would cause sliding. This is why $\mu = \tan\theta$ is a useful relationship.*

Thinking Like a Physicist: *Limiting equilibrium represents the boundary between static equilibrium and motion. Understanding this threshold is crucial in engineering: roads must have sufficient friction (high μ) or low slope (small θ) to prevent vehicles from sliding, especially in rain when μ decreases.*

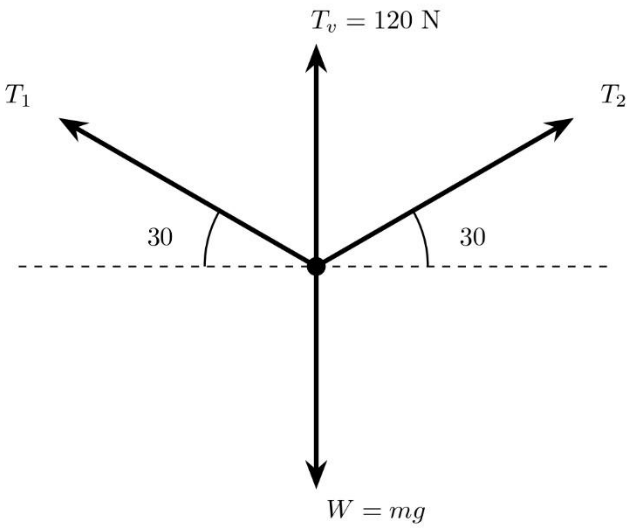
HOT Example 15

A 20kg signboard hangs from a vertical cable attached to its center and two side cables. Each side cable is attached to the edge of the signboard and makes 30° with the horizontal, pulling outward to wall supports. If the vertical cable has tension 120N, calculate the tension in each side cable.

$$\text{Take } g = 9.8\text{m/s}^2.$$

Solution

All forces acting on the signboard are shown in the following free body diagram.



Resolving each force to horizontal and vertical component:

For T_1:

$$\theta = \text{angle measured from positive x} - \text{axis} = 180° - 30° = 150°$$

Horizontal component $= T_1\cos150° = -T_1\cos30°$

Vertical component $= T_1\sin150° = T_1\sin30°$

For T_v:

Horizontal component $= 0$

Vertical component $= 120\text{N}$

For T_2:

$$\theta = \text{angle measured from positive x} - \text{axis} = 30°$$

Horizontal component $= T_2\cos30°$

Vertical component $= T_2\sin30°$

For weight:

Horizontal component $= 0$

Vertical component $= -mg = -20 \times 9.8\text{N} = -196\text{N}$

From equilibrium equations: : $\Sigma F_x = 0$ and $\Sigma F_y = 0$

Then;

$\Sigma F_x = -T_1\cos30° + T_2\cos30° = 0$ or

$T_2\cos30° = T_1\cos30°;\ \mathbf{T_2 = T_1}$

And;

$\Sigma F_y = T_1\sin30° + 120 + T_2\sin30° - 196 = 0$ or

$T_1\sin30° + T_2\sin30° = 76$

But $T_2 = T_1$

$T_1\sin30° + T_1\sin30° = 76;\ 2T_1\sin30° = 76$ or $T_1 = 76\text{N}$

Tension in each side cable is 76N.

Making Sense of the Answer: *The signboard weighs 196N downward, with 120N already supported by the vertical cable. The remaining 76N is shared by the two side cables through their vertical components. The side cables also provide horizontal stabilization, preventing the signboard from swinging.*

Thinking Like a Physicist: *Real engineering often involves over-constraining systems for safety. A signboard could hang from a single cable, but using three distributes the load and provides backup support. If one side cable fails, the board may tilt, but it does not fall immediately.*

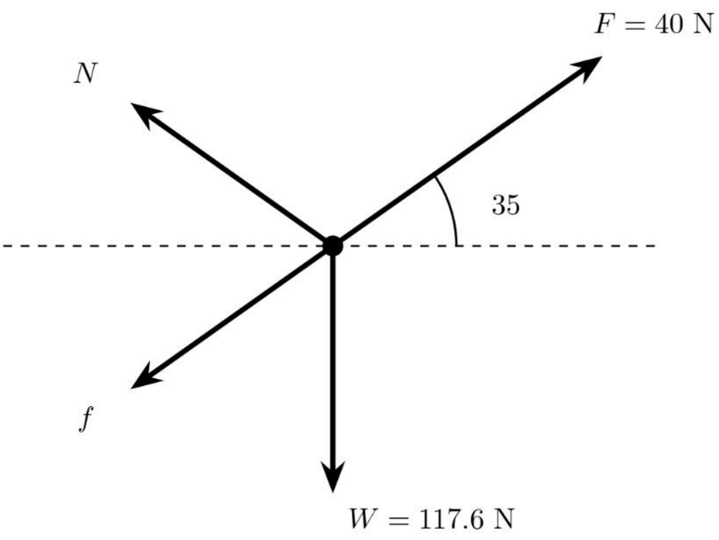
HOT Example 16

A 12kg block rests on a rough plane inclined at $35°$ to the horizontal. A force of $40N$ acts on the block parallel to and up the plane. The block is in equilibrium. The coefficient of static friction between block and plane is $\mu = 0.6$. Take $g = 9.8m/s^2$.

(a) Calculate the normal reaction.
(b) Determine the magnitude and direction of the frictional force.
(c) Verify that the block is not at limiting equilibrium.

Solution

All forces acting on the block are shown in the following free body diagram.



(a) Forces acting perpendicular to plane are only normal reaction, **R** and the perpendicular component of weight.

Since the two forces act in opposite directions;

$$R = mg\cos\theta = 12 \times 9.8 \times \cos 35° = 96.3N$$

The normal reaction is $96.3N$.

(b) Forces acting parallel to the plane.
- Parallel weight component (downward): $W_P = -mg\sin\theta = -117.6\sin 35 = -67.4N$
- Applied force (upward): $F = 40N$
- Frictional force (unknown direction): f

Applying equilibrium equation parallel to the plane:

$$-67.4N + 40N + f = 0; f = 27.4N \text{ (upward as it is positive)}$$

Direction determination: since weight component down the plane ($67.4N$) is larger than the applied force ($40N$) up the plane, the tendency is to slide **down** the plane, so friction must act **up the plane** to prevent the downward motion.

The frictional force is $27.4N$ up the plane.

(c) Limiting friction: $f_{max} = \mu R = 0.6 \times 96.3 = 57.8N$

But actual friction required for equilibrium is $27.4N$, which is less than $57.8N$.

Since the actual frictional force is less than the limiting friction, the block is not at limiting equilibrium.

Making Sense of the Answer: *The applied force ($40N$) helps support the block but isn't strong enough to overcome the down-slope pull ($67.4N$). Friction makes up the difference ($27.4N$). Together, the applied force and friction provide $67.4N$ up the slope, exactly balancing the down-slope weight component. The large safety margin ($57.8N$ maximum vs $27.4N$ actual) means the block is very stable.*

Thinking Like a Physicist: *This problem illustrates an important principle:* ***friction direction depends on motion tendency, not on which forces are present****. Always first determine which way the block would move if friction were absent, then friction acts opposite to that tendency. In this case, removing friction would cause downward motion ($67.4N > 40N$), so friction acts upward.*

With these worked examples now neatly packed away, let us move on and meet the next subtopic; it has been waiting patiently to join the conversation.

MOTION OF CONNECTED BODIES

Introduction

Imagine two students connected by a rope during a tug-of-war revision session. One pulls confidently, the other resists bravely, and suddenly both start moving together, not because either planned it, but because the forces refused to stay balanced. Physics describes such situations as **motion of connected bodies**.

In the previous subtopics, you mastered equilibrium for single objects: books on tables, blocks on inclines, and signboards hanging from cables. But the real world rarely presents isolated objects. Instead, systems work together: elevators carrying people, cranes lifting loads, cars towing trailers, and even your school bag hanging from a desk hook while pulling on the strap.

When two or more objects are physically connected by strings, cables, ropes, chains, or rigid bars, they form what we call a **connected bodies system**. The fascinating aspect of such systems is this: although the objects may have different masses and experience different forces, they often share the same motion. A hanging mass pulls on a string that accelerates a block on a table. The string transmits force between them, and both objects accelerate together (though in different directions).

What makes this different from what we have studied?

Unlike equilibrium situations where forces cancel perfectly, connected bodies often reveal what happens when that balance is slightly disturbed.

Previously, we analysed equilibrium where acceleration was zero. Now we extend our understanding to connected systems where acceleration may occur, but the key principle remains: we apply Newton's laws systematically to each body, paying special attention to how the connection (string or cable) transmits force through tension.

Instead of the equilibrium condition: $\Sigma F = 0$; we now apply Newton’s second law: $\Sigma F = ma$.

Thus, this topic does not replace equilibrium principles; it builds directly on them.

Three common configurations:

Connected bodies can move in three main ways:

1. **Vertical motion:** Bodies connected by a string over a pulley, where one hangs and pulls the other vertically, as in an elevator and its counterweight system.
2. **Horizontal motion:** Bodies connected by a string or tow-bar on a horizontal surface, where one pulls the other along the same plane, similar to a car towing a trailer.
3. **Inclined plane motion:** Bodies connected by a string over a pulley at the top of an inclined plane, where one moves up or down the slope while the other hangs vertically, as seen in some mountain rescue lifting systems.

Let us explore each configuration systematically, building your understanding step by step.

Connected Bodies in Vertical Motion

One of the simplest connected-body systems consists of two masses joined by a light string passing over a smooth pulley. Such a system is often called an **Atwood-type machine**.

Consider two masses m_1 and m_2 connected by a light, inextensible string that passes over a smooth (frictionless) pulley fixed to the ceiling. The masses hang freely on either side of the pulley.

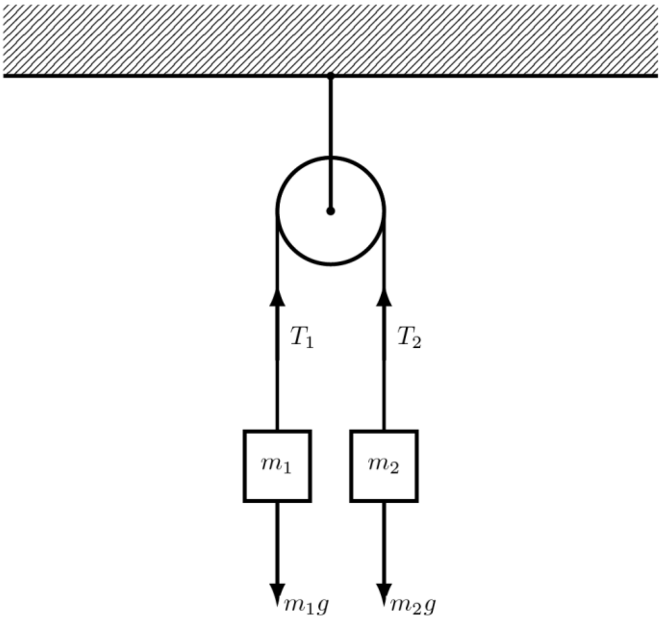


Figure: *Two masses m_1 and m_2 connected by a light inextensible string passing over a smooth pulley. Tensions T_1 and T_2 act along the string, while the weights m_1g and m_2g act vertically downward.*

Three possible scenarios:

1) **If $m_1 = m_2$**: The system is in equilibrium. There is no acceleration; the masses remain stationary (or move with constant velocity if already in motion).
2) **If $m_1 < m_2$**: The heavier mass m_2 pulls the system downward. Mass m_2 accelerates downward while m_1 accelerates upward. Both have the **same** magnitude of acceleration **a**.
3) **If $m_1 > m_2$**: The heavier mass m_1 pulls the system downward. Mass m_1 accelerates downward while m_2 accelerates upward, again with the same acceleration magnitude a.

Deriving the acceleration and tension formulas:

Let us analyse the case where m_2 is heavier than m_1 $(m_2 > m_1)$

Acceleration formula:

Forces on mass m_1 (moving upward):

- Weight: $W_1 = m_1g$ (downward)
- Tension: T (upward)

Forces on mass m_2 (moving downward):

- Weight: $W_2 = m_2g$ (downward)
- Tension: T (upward)

Applying Newton's second law to each body

For m_1 (taking upward as positive): $\Sigma F_1 = m_1a;\ T_1 - m_1g = m_1a$ ... (equation 1)

For m_2 (taking downward as positive): $\Sigma F_2 = m_2a;\ m_2g - T_2 = m_2a$ ... (equation 2)

Solving the simultaneous equations:

Since the pulley is smooth, $T_1 = T_2 = T$

Adding equation 1 and equation 2:

$$(T - m_1g) + (m_2g - T) = m_1a + m_2a$$

$$m_2g - m_1g = m_1a + m_2a$$

$$(m_2 - m_1)g = (m_1 + m_2)a$$

Hence, acceleration is:

$$\mathbf{a = \left(\frac{m_2 - m_1}{m_1 + m_2}\right) g}$$

The final result is our key formula for acceleration in vertical connected bodies.

The formula shows:

- Greater mass difference leads to greater acceleration. If $m_2 >> m_1$ (much heavier), then a approaches g, meaning the system accelerates nearly in free fall.
- Equal masses ($m_2 - m_1 = 0$) means zero acceleration (equilibrium).
- The acceleration cannot exceed the acceleration due to gravity, g.

$$\left(\text{Since } (m_2 - m_1) < (m_1 + m_2), \left(\frac{m_2 - m_1}{m_1 + m_2}\right) < 1;\ \text{hence } a < g\right).$$

Tension formula:

Also from equation 1:

$T_1 = T = m_1g + m_1a$

$T = m_1g + m_1\left(\frac{m_2 - m_1}{m_1 + m_2}\right) g$

$T = m_1g\left(1 + \frac{m_2 - m_1}{m_1 + m_2}\right)$

$T = m_1g\left(\frac{m_1 + m_2 + m_2 - m_1}{m_1 + m_2}\right)$

$T = m_1g\left(\frac{2m_2}{m_1 + m_2}\right)$

Hence, tension is:

$$\mathbf{T = \frac{2m_1m_2g}{m_1 + m_2}}$$

The same result can be obtained by substituting the expression for **a** into equation 2.

It is important to note that *the tension is always* ***greater than the lighter weight*** *and* ***less than the heavier weight****:* $m_1g < T < m_2g$.

Two key insights regarding a vertical connected-body system:

Insight 1: ***Why same acceleration magnitude?***

Because the string is inextensible (does not stretch), any distance m_1 moves up equals the distance m_2 moves down in the same time interval. Therefore, their velocities and accelerations must have the same magnitude, though opposite directions.

Insight 2: ***The smooth pulley assumption***

A smooth pulley means frictionless. This ensures the tension in the string is the same on both sides of the pulley. If friction existed, tension would differ on each side, complicating the analysis significantly. For A-level problems, we almost always assume smooth pulleys unless stated otherwise.

For now, let us pause the theory and bring the ideas down to earth with a few carefully chosen worked examples.

BINDER Example 17

Show that, in a vertical connected-body system, if one body is much heavier than the other, the system accelerates at nearly the acceleration due to gravity, g.

Solution

Consider two masses m_1 and m_2 connected by a light inextensible string over a smooth pulley, with $m_2 > m_1$ so that m_2 moves downward.

The acceleration of the vertical connected-body system is given by the following formula:

$$a = \left(\frac{m_2 - m_1}{m_1 + m_2}\right) g$$

If $m_2 >> m_1$ (much heavier), then m_2 is very large compared to m_1. Thus:

$$m_2 - m_1 \approx m_2 \text{ and } m_2 + m_1 \approx m_2$$

Substituting into the acceleration expression:

$$a \approx \left(\frac{m_2}{m_2}\right) g \approx g$$

Hence, when one body is much heavier than the other, the system accelerates at almost g, (almost in free fall).

Making Sense of the Answer: *If the heavier mass is far larger than the lighter one, the lighter mass becomes almost negligible. The heavy mass then behaves nearly like an object falling under gravity with very little "load" to lift, so its acceleration gets very close to g.*

Thinking Like a Physicist: *This is a limiting-case test. If* m_1 *were zero (an "almost nothing" mass), the heavy mass would essentially be falling freely and the formula would give* $a = g$ *exactly. Real systems never reach g because the other mass is never truly zero, but as the mass difference becomes extreme, the system approaches free-fall behaviour.*

BINDER Example 18

Show that, in a vertical connected-body system, the tension, T is always less than the weight of the heavier mass and greater than the weight of the lighter mass.

Solution

Assume a light inextensible string over a smooth pulley with $m_2 > m_1$, so the system accelerates with magnitude $a > 0$, with m_2 downward and m_1 upward.

Applying Newton's second law on each mass:

For m_1 (accelerating upward):

$$T - m_1 g = m_1 a$$

So: $\mathbf{T = m_1 g + m_1 a}$

Since $a > 0, \rightarrow \mathbf{T > m_1 g}$ … …(i)

For m_2 (accelerating downward):

$$m_2 g - T = m_2 a$$

So: $\mathbf{T = m_2 g - m_2 a}$

Since $a > 0, \rightarrow \mathbf{T < m_2 g}$ … …(ii)

Combining (i) and (ii) gives:

$$\mathbf{m_1 g < T < m_2 g}$$

Hence, the tension is always less than the weight of the heavier mass and greater than the weight of the lighter mass.

Making Sense of the Answer: *The heavier mass pulls downward, but part of its weight accelerates the lighter mass upward, so the tension is less than the heavier weight. At the same time, it (tension) must exceed the lighter weight to lift it.*

Thinking Like a Physicist: *If tension equalled either weight, one mass would not accelerate. Since both move, the tension must lie between the two weights.*

BINDER Example 19

Two masses of 4kg and 6kg are connected by a light inextensible string passing over a smooth pulley. The system is released from rest. Take $g = 9.8 \text{ m/s}^2$.

Calculate: (a) the acceleration of each mass (b) the tension in the string

Solution

Using:

$$a = \left(\frac{m_2 - m_1}{m_1 + m_2}\right) g$$

Where:

$$m_2 = 6\text{kg (heavier)}$$

$$m_1 = 4\text{kg (lighter)}$$

Substituting:

$$a = \left(\frac{6\text{kg} - 4\text{kg}}{4\text{kg} + 6\text{kg}}\right) \times 9.8\text{m/s}^2 = 1.96\text{m/s}^2$$

Using:

$$T = \frac{2m_1 m_2 g}{m_1 + m_2}$$

Substituting:

$$T = \frac{2 \times 4\text{kg} \times 6\text{kg} \times 9.8\text{m/s}^2}{4\text{kg} + 6\text{kg}} = 47.04\text{N}$$

(a) The acceleration is 1.96m/s^2.
(b) The tension is 47.04N.

Making Sense of the Answer: *The heavier mass* $(6kg)$ *pulls the system downward, so both masses accelerate together at* $1.96m/s^2$. *The tension* $(47.04N)$ *lies between the two weights: greater than 4g* $(39.2N)$ *but less than* $6g$ $(58.8N)$*, which confirms the result is physically reasonable.*

Thinking Like a Physicist: *In the vertical connected-body system, always identify the heavier mass first to determine the direction of motion. Also as a quick check, the tension must satisfy* $m_1 g < T < m_2 g$*; if it does, the analysis is likely correct.*

BINDER Example 20

Two masses are connected by a string over a smooth pulley. When released from rest, the system accelerates at 1.5 m/s^2. If one of the masses is 4kg and moves upward, determine the mass of the other body. Take $g = 9.8 \text{ m/s}^2$.

Solution

Since the given mass was moving upward, it is the lighter mass. Thus the given mass is m_1 and the unknown mass is m_2 (heavier mass).

Using:

$$a = \left(\frac{m_2 - m_1}{m_1 + m_2}\right) g$$

Substituting:

$$1.5\ \mathrm{m/s^2} = \left(\frac{\mathrm{m_2} - 4\mathrm{kg}}{4\mathrm{kg} + \mathrm{m_2}}\right) \times 9.8\ \mathrm{m/s^2}$$

Solving for $\mathrm{m_2}$ gives: $\mathrm{m_2} = 5.45\mathrm{kg}$

The mass of the other body is 5.45kg.

Making Sense of the Answer: *The unknown mass ($5.45kg$) is moderately heavier than the known mass ($4kg$), producing a modest acceleration ($1.5m/s^2$). If they were equal, acceleration would be zero. If the difference were larger, acceleration would be greater. The result fits the physical expectation perfectly.*

Thinking Like a Physicist: *Always identify the direction of motion first; the mass moving upward must be the lighter one. Also check whether the result makes physical sense: a slightly heavier opposing mass should produce a moderate acceleration, not zero and not close to free fall (g).*

REAL Example 20

Kipanga watches workers installing a new elevator at his uncle's building. He notices a large concrete block hanging on the other side of the pulley.

Kipanga: *"Why is there a heavy block hanging there? Isn't the elevator motor strong enough to lift people by itself?"*

Kipute: *"I think it's a counterweight. It helps balance the elevator so the motor doesn't have to work as hard."*

Mr. Akilikubwa: (joining them) *"Excellent observation, Kipute! Without that counterweight, the motor would need to lift the full weight of the elevator cabin plus passengers. Imagine lifting a 1000kg load straight up! But with a properly sized counterweight, the motor only needs to overcome the difference. If the cabin with passengers weighs 1000kg and the counterweight is 800kg, the motor effectively only lifts 200kg. Much easier, much more energy efficient."*

Kipanga: *"So it's like those connected masses we studied in class?"*

Mr. Akilikubwa: *"Exactly! The elevator and counterweight are connected bodies in vertical motion. When the elevator goes up, the counterweight comes down, just like our pulley system. The tension in the cable does most of the work, and the motor just provides the extra push or pull to accelerate or decelerate the system smoothly. Without the counterweight, your electricity bill would be shocking, and not in a good way!"*

Kipute: (smiling) *"So Physics saves money too?"*

Mr. Akilikubwa: *"Always! Good engineering is just applied physics with a practical goal. Every elevator in the world uses this principle. Next time you ride one, thank Newton's laws and the counterweight quietly working behind the scenes."*

Question: Based on this conversation:

(a) Using the given masses, show quantitatively how the counterweight reduces the force required from the elevator motor compared with lifting the cabin alone.

(b) If an elevator cabin with passengers has a total mass of 900kg and the counterweight has a mass of 700 kg, calculate:

  (i) the acceleration if the motor provides an additional upward force of 2450N to the cabin,

  (ii) the tension in the cable connecting them.

(Take g = 9.8 m/s²)

Solution

(a) Minimum motor force required without counterweight:

Without a counterweight, the motor must provide the entire upward force to lift the cabin and overcome its full weight. This requires at least: $\mathrm{F} = \mathrm{W} = \mathrm{mg}$

For a 900kg cabin: $\mathrm{F_{motor}} = 900\mathrm{kg} \times 9.8\ \mathrm{m/s^2} = 8820\mathrm{N}$

Minimum motor force required with counterweight:

With a counterweight of 700kg, the downward pull of the counterweight partially balances the cabin's weight. The net force the motor must provide becomes much smaller because it only needs to overcome the difference (200kg):

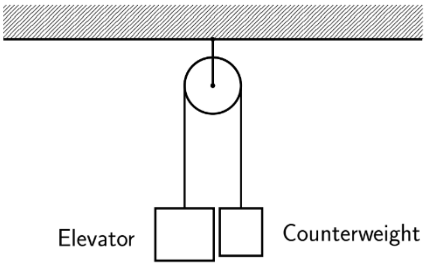
$$F_{motor} = \left(m_{cabin} - m_{counterweight}\right)g = (900 - 700)\text{kg} \times 9.8\text{m/s}^2 = 1960\text{N}$$

This is only 22% of the original force! The counterweight does most of the "heavy lifting" through gravity, and the motor provides just the extra force needed for acceleration and control.

Additionally, when the cabin descends, the counterweight assists by pulling upward, again reducing motor effort.

This bidirectional efficiency explains why nearly all elevators use counterweights as they dramatically reduce energy consumption and allow smaller, cheaper motors.

(b) To fully understand this, consider the following situation diagram and its corresponding free-body diagram.



(a) Elevator cabin and counterweight

(b1) Cabin forces

(b2) Counterweight forces

Forces acting on the elevator cabin (moving upward):

- Weight of cabin (downward): $W_{cabin} = m_1g = 900\text{kg} \times 9.8\ \text{m/s}^2 = 8820\text{N}$
- Tension (upward): T
- Motor force (upward): $F_{motor} = 2450\text{N}$

Total upward forces $= 2450\text{N} + \text{T}$

Total downward forces $= 8820\text{N}$

Applying Newton's second law (upward positive): $\Sigma F = m_1a$

$(2450 + T) - 8820 = 900a$ or

$T - 900a = 6370$ … …(i)

Forces acting on the counterweight (moving downward):

- Weight of counterweight (downward):$W_{counter} = m_2g = 700\text{kg} \times 9.8\ \text{m/s}^2 = 6860\text{N}$
- Tension (upward): T

Again applying Newton's second law (downward positive for counterweight): $\Sigma F = m_2a$

$6860 - T = 700a$ or

$T + 700a = 6860$ … …(ii)

Solving (i) and (ii) simultaneously gives: $a = 0.31\text{m/s}^2, T = 6646\text{N}$

(i) The acceleration is 0.306m/s^2.
(ii) The tension is 6646N.

Making Sense of the Answer: *The counterweight principle dramatically reduces the force requirement. Instead of the motor lifting 900kg, it effectively lifts only the difference (200kg), plus any additional force for acceleration. This is why elevators are energy-efficient despite moving heavy loads many times per day. The tension in the cable is substantial (6646N) but shared between supporting the cabin and the counterweight; the motor just provides the marginal difference.*

Thinking Like a Physicist: *Real engineering applications like elevators demonstrate why understanding connected bodies matters beyond textbooks. Engineers must calculate safe cable tensions, motor power* *requirements, and energy efficiency. A poorly designed counterweight (too light or too heavy) wastes energy or creates unsafe accelerations. Physics isn't just formulas, it's the foundation of technologies we use every day without thinking about the Newton's laws quietly working behind the scenes.*

HOT Example 21

A container of mass 8kg hangs from one end of a light inextensible string that passes over a smooth pulley. The other end of the string is attached to a 12kg block of ice resting on a platform. The system is initially at rest. After the ice block is released and starts moving, it melts at a constant rate of 0.5kg every 10 seconds due to the pulley mechanism warming the string.

(a) Calculate the initial acceleration of the system immediately after release.
(b) Determine the acceleration of the system after 20 seconds.
(c) Explain clearly what happens to the system's motion as the ice continues to melt.

Take $g = 9.8\ \mathrm{m/s^2}$

Solution

(a) Initial masses:
$m_1 = 8\mathrm{kg}$ (container, moving upward)
$m_2 = 12\mathrm{kg}$ (ice block, moving downward initially)

Using:

$$a = \left(\frac{m_2 - m_1}{m_1 + m_2}\right) g$$

Substituting:

$$a = \left(\frac{12\mathrm{kg} - 8\mathrm{kg}}{8\mathrm{kg} + 12\mathrm{kg}}\right) \times 9.8\mathrm{m/s^2} = 1.96\mathrm{m/s^2}$$

The initial acceleration is $1.96\mathrm{m/s^2}$.

(b) Calculating mass of ice after melting:

Melting rate $=$ 0.5kg per 10 seconds

Time elapsed $=$ 20 seconds

Mass melted $= \frac{0.5\ \mathrm{kg}}{10\mathrm{s}} \times 20\mathrm{s} = 1\mathrm{kg}$

New mass of ice block: $m_2' = 12 - 1 = 11\mathrm{kg}$

Again using the acceleration formula:

$$a' = \left(\frac{m_2' - m_1}{m_1 + m_2'}\right) g$$

Substituting:

$$a' = \left(\frac{11\mathrm{kg} - 8\mathrm{kg}}{8\mathrm{kg} + 11\mathrm{kg}}\right) \times 9.8\mathrm{m/s^2} = 1.55\mathrm{m/s^2}$$

The acceleration after 20 seconds is $1.55\mathrm{m/s^2}$.

(c) As ice melts, the mass difference decreases, causing acceleration to decrease. At 80 seconds, masses become equal (both 8kg) and acceleration becomes zero, making the system to move at constant velocity. If melting continues beyond this point, the container becomes heavier, causing acceleration to reverses direction. Eventually, if all the ice melts, the container is left unsupported (the system is no longer connected bodies) and will undergo free fall with acceleration $9.8\mathrm{m/s^2}$ until it hits the ground or another obstacle.

The worked examples have filled the plate nicely; now it is time to enjoy the next subtopic and see what new ideas it brings to the table.

Connected Bodies on Horizontal Plane

In the previous subtopic, we analysed bodies connected by a string over a pulley where both masses hung vertically. Now we shift to a different but equally common configuration: one or more bodies moving horizontally on a surface, connected by a string or rigid bar. You will recognise this arrangement in everyday situations such as a car towing a trailer, a locomotive pulling carriages, or a person dragging a chain of boxes across a floor. Although the connection between bodies remains similar, the source of motion is no longer the same.

The key difference lies in what drives the motion. In the vertical system, the driving force came from the **difference in weights** of the two hanging masses. On a horizontal plane, however, the situation changes slightly: a block resting on a table cannot use its weight to produce horizontal motion. Instead, movement must be provided by an **external pull**, which then transmits motion through the connecting string or bar.

When analysing connected bodies on a horizontal surface, two distinct situations naturally arise depending on the nature of the surface. In the first case, the **surface is smooth** (frictionless), so the only horizontal forces are the applied pull and the tensions in the connecting string or bar, making the motion relatively straightforward to analyse. In the second case, the **surface is rough**, meaning friction acts alongside the applied pull and tension. This additional resistive force modifies the acceleration and must be carefully included when applying Newton's laws. These two cases form the foundation for understanding horizontal connected-body systems.

The general setup:

Consider two bodies of masses m_1 and m_2 connected by a light inextensible string on a horizontal surface. A horizontal force F is applied to mass m_1,which then pulls mass m_2 through the connecting string. Since the surface is horizontal, the motion is entirely in the horizontal direction and both masses share the same acceleration, a.

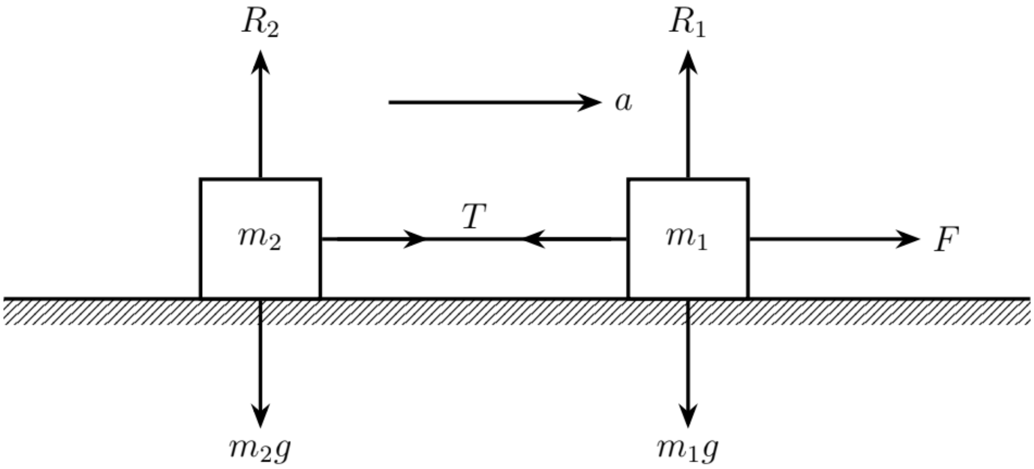


Figure: *A diagram showing two masses m_1 and m_2 connected by a light inextensible string on a horizontal surface.*

First case: Smooth horizontal surface (no friction)

Forces acting on m_1 (being pulled by F):

- Applied force: F (horizontal, forward)
- Tension: T (horizontal, backward: string pulls m_1 back toward m_2)
- Weight: m_1g (downward)
- Normal reaction: $R_1 = m_1g$ (upward)

Since no vertical motion (vertical forces balance), only horizontal forces (forward and backward forces) will be considered.

Applying Newton's second law:

$$F - T = m_1a \ldots \text{(equation 1)}$$

Forces acting on m_2 (being pulled by string):

- Tension: T (horizontal, forward: string pulls m_2 forward)
- Weight: m_2g (downward)

- Normal reaction: $R_2 = m_2g$ (upward)

Again, only horizontal forces will be considered. Thus:

$$T = m_2a \ldots \text{(equation 2)}$$

Adding equations 1 and 2:

$$F - T + T = m_1a + m_2a$$

$$F = (m_1 + m_2)a$$

Therefore, acceleration is:

$$\mathbf{a = \frac{F}{m_1 + m_2}}$$

Substituting the acceleration expression into equation (2):

$$T = m_2 \times \frac{F}{m_1 + m_2}$$

Thus, the tension is:

$$\mathbf{T = \frac{m_2F}{m_1 + m_2}}$$

What physical meaning does each formula convey?

- The acceleration formula shows the entire applied force, F, accelerates the combined total mass $(m_1 + m_2)$ of the system, just as if it were pushing a single object.
- The tension formula shows the following:
  - ✓ Tension, T in the string is only responsible for accelerating m_2 alone $(T = m_2a)$. It does not need to accelerate m_1 because m_1 is already driven by the external force F.
  - ✓ Tension (T) is always less than applied force (F) (since $m_2 < m_1 + m_2$, $\frac{m_2}{m_1 + m_2} < 1$), meaning the string never experiences the full applied force.

Second case: Rough horizontal surface (with friction)

When the surfaces are rough, friction opposes the motion of each body. Friction acts backward (opposing forward motion) on both masses.

Let μ be the coefficient of **kinetic** friction between each mass and surface.

Then, frictional force (f) acting on each mass will be as follows:

$f_1 = \mu R_1 = \mu m_1g$ (on m_1, opposing its forward motion)

$f_2 = \mu R_2 = \mu m_2g$ (on m_2, opposing its forward motion)

Applying Newton's second law:

For m_1:

$F - T - f_1 = m_1a$

$$F - T - \mu m_1g = m_1a \ldots \text{(equation 1)}$$

For m_2:

$T - f_2 = m_2a$

$$T - \mu m_2g = m_2a \ldots \text{(equation 2)}$$

Adding equations 1 and 2:

$F - \mu m_1g - \mu m_2g = m_1a + m_2a$

$F - \mu(m_1 + m_2)g = (m_1 + m_2)a$

Therefore, acceleration is:

$$\mathbf{a} = \frac{\mathbf{F} - \boldsymbol{\mu}(\mathbf{m_1} + \mathbf{m_2})\mathbf{g}}{\mathbf{m_1} + \mathbf{m_2}}$$

Also from equation (2):

$T = m_2a + \mu m_2g$

$T = m_2(a + \mu g)$

But:

$$a = \frac{F - \mu(m_1 + m_2)g}{m_1 + m_2} = \frac{\mathbf{F}}{\mathbf{m_1} + \mathbf{m_2}} - \boldsymbol{\mu}\mathbf{g}$$

It follows that:

$$T = m_2(a + \mu g) = m_2\left(\frac{F}{m_1 + m_2} - \mu g + \mu g\right) = \frac{m_2F}{m_1 + m_2}$$

Hence, the tension is:

$$\mathbf{T} = \frac{\mathbf{m_2F}}{\mathbf{m_1} + \mathbf{m_2}}$$

(*The same as the **smooth** case*)

What does the acceleration formula reveal?

- For motion to occur at all, the applied force must overcome total friction:

$$F > \mu(m_1 + m_2)g \text{ (where, } \mu(m_1 + m_2)g \text{ is the total friction)}$$

If $F \leq \mu(m_1 + m_2)g$ (which would give $a \leq 0$), the system remains stationary and static friction keeps everything at rest. The derived equations apply only when motion occurs.

- In **this particular arrangement**, friction reduces the acceleration by μg. Thus:

$$\text{Acceleration in rough}\left(\frac{F}{m_1 + m_2} - \mu g\right) = \text{Acceleration in smooth}\left(\frac{F}{m_1 + m_2}\right) - \mu g$$

BINDER Example 22

Two blocks of masses 4kg and 6kg are connected by a light inextensible string on a smooth horizontal table. A horizontal force of 20N is applied to the 6kg block, pulling both blocks along the table. Take $g = 9.8\ m/s^2$.

Calculate: (a) the acceleration of the system(b) the tension in the string connecting the two blocks.

Solution

Identifying the system:

$m_1 = 6kg$ (block with applied force)

$m_2 = 4kg$ (block being pulled by string)

Surface: smooth (no friction).

(a) Since the smooth was surface, the acceleration is given by:

$$a = \frac{F}{m_1 + m_2} = \frac{20N}{(6 + 4)kg} = 2m/s^2$$

The acceleration is $2m/s^2$.

(b) The tension is given by:

$$T = m_2a = 4kg \times 2m/s^2 = 8N$$

The tension in the string is 8N.

Making Sense of the Answer: *The 20N force accelerates a total mass of 10kg, giving 2m/s² which is the same as pushing a single 10kg object with 20N on a smooth surface. The tension (8N) is less than the applied force (20N) because it only needs to accelerate the 4kg block, not the entire system.*

Thinking Like a Physicist: *The tension in a connecting string is usually less than the applied force because it accelerates only part of the system. This explains why tow ropes in vehicles can snap: When a vehicle brakes suddenly, the rope must transmit a large tension to slow the trailer quickly. Understanding tension in connected systems is therefore not just theoretical, it is directly linked to engineering safety.*

BINDER Example 23

Two boxes of masses 3kg and 5kg are connected by a light inextensible string on a rough horizontal floor. A horizontal force of 40N is applied to the 5kg box, pulling both boxes. The coefficient of kinetic friction between each box and the floor is $\mu = 0.3$. Take $g = 9.8\ \mathrm{m/s^2}$.

Calculate:(a) the acceleration of the system(b) the tension in the string.

Solution

Identifying the system:

$m_1 = 5\mathrm{kg}$ (box with applied force)

$m_2 = 3\mathrm{kg}$ (box being pulled by string)

Surface: rough with $\mu = 0.3$.

(a) The acceleration is given by:

$$a_{\mathrm{rough}} = a_{\mathrm{smooth}} - \mu g = \frac{F}{m_1 + m_2} - \mu g = \frac{40\mathrm{N}}{(5+3)\mathrm{kg}} - (0.3 \times 9.8\ \mathrm{m/s^2}) = 2.06\mathrm{m/s^2}$$

The acceleration is $2.06\mathrm{m/s^2}$.

Alternative solution

Treating the whole system as one body:

Total mass, $m_t = m_1 + m_2 = 5\mathrm{kg} + 3\mathrm{kg} = 8\mathrm{kg}$

Total frictional force, $f_t = \mu m_t g = 0.3 \times 8\mathrm{kg} \times 9.8\ \mathrm{m/s^2} = 23.52\mathrm{N}$

Resultant force, $F_R = F - f_t = 40\mathrm{N} - 23.52\mathrm{N} = 16.48\mathrm{N}$

Then from $F = ma$ or $a = \frac{F}{m}$;

The acceleration of the system, $a = \frac{F_R}{m_t} = \frac{16.48\mathrm{N}}{8\mathrm{kg}} = 2.06\mathrm{m/s^2}$

(b) The tension is given by:

$$T = m_2 a_{\mathrm{smooth}} = \frac{m_2 F}{m_1 + m_2} = \frac{3\mathrm{kg} \times 40\mathrm{N}}{(5+3)\mathrm{kg}} = 15\mathrm{N}$$

The tension is 15N.

Making Sense of the Answer: *The acceleration is smaller than it would be on a smooth surface because friction opposes motion.*

Thinking Like a Physicist: *Friction reduces acceleration but does not directly dictate the tension formula, which depends on how the force is shared between the masses.*

REAL Example 24

Tow ropes used to pull trailers sometimes snap when the towing vehicle brakes suddenly. Explain why this happens.

Solution

When the vehicle brakes suddenly, it slows down quickly but the trailer tends to keep moving forward due to inertia. The tow rope then becomes tightly stretched and develops a large tension as it tries to stop the trailer in a short time. If this tension becomes too great, the rope may snap.

Making Sense of the Answer: *The trailer does not stop immediately when the vehicle brakes because it has inertia. The tow rope must therefore provide a large tension to slow it down quickly. If this tension exceeds* *the rope's strength, it snaps. This matches everyday experience where sudden braking produces stronger forces than gradual stopping.*

Thinking Like a Physicist: *Whenever motion changes rapidly, large forces are involved. In connected-body systems, tension is the force that transmits these changes. Sudden deceleration increases tension significantly, which is why engineers design tow ropes and cables with safety limits.*

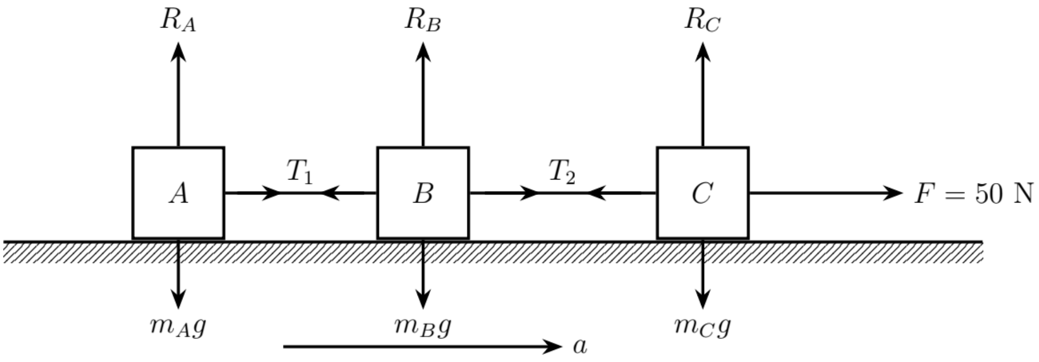
HOT Example 25

Three blocks A, B and C of masses 2kg, 3kg and 5kg respectively are connected in series by two light inextensible strings on a rough horizontal surface. Block A is at the back, connected to block B by string 1, and block B is connected to block C by string 2. A horizontal force F = 50N is applied to block C at the front, pulling the entire system forward. The coefficient of kinetic friction between each block and surface is $\mu = 0.25$.Take $g = 9.8\ \mathrm{m/s^2}$.

(a) Calculate the acceleration of the system.
(b) Find the tension T_1 in the string between A and B.
(c) Find the tension T_2 in the string between B and C.
(d) Show that $T_2 > T_1$ and explain why this must always be the case.

Solution

(a) Consider the following diagram.



Treating the entire system as one body:

Total mass, $m_t = m_A + m_B + m_C = 2\mathrm{kg} + 3\mathrm{kg} + 5\mathrm{kg} = 10\mathrm{kg}$

Total frictional force, $f_t = \mu m_t g = 0.25 \times 10\mathrm{kg} \times 9.8\ \mathrm{m/s^2} = 24.5\mathrm{N}$

Resultant force, $F_R = F - f_t = 50\mathrm{N} - 24.5\mathrm{N} = 25.5\mathrm{N}$

Then $a = \dfrac{F_R}{m_t} = \dfrac{25.5\mathrm{N}}{10\mathrm{kg}} = 2.55\mathrm{m/s^2}$

The acceleration of the system is $2.55\mathrm{m/s^2}$.

(b) Horizontal forces acting on block A:

- Tension: T_1 (forward)
- Frictional force: $f_A = \mu m_A g = 0.25 \times 2\mathrm{kg} \times 9.8\ \mathrm{m/s^2} = 4.9\mathrm{N}$ (backward)

Applying Newton's second law:

$$F_R = T_1 - f_A = m_A a \text{ or } T_1 = m_A a + f_A$$

Substituting:

$$T_1 = 2\mathrm{kg} \times 2.55\mathrm{m/s^2} + 4.9\mathrm{N} = 10\mathrm{N}$$

The tension T_1 is 10N.

(c) Horizontal forces acting on block B:

- Tension: T_1 (backward)

- Tension: T_2 (forward)
- Frictional force: $f_B = \mu m_B g = 0.25 \times 3\text{kg} \times 9.8\ \text{m/s}^2 = 7.35\text{N}$ (backward)

Applying Newton's second law:

$$F_R = T_2 - T_1 - f_B = m_B a \text{ or } T_2 = m_B a + T_1 + f_B$$

Substituting:

$$T_2 = 3\text{kg} \times 2.55\text{m/s}^2 + 10\text{N} + 7.35\text{N} = 25\text{N}$$

The tension T_2 is 25N.

Alternative solution

From the diagram, it is clearly understood that the tension T_2 is responsible for pulling both block A and block B.

So total mass pulled by $T_2 = m_A + m_B = (2+3)\text{kg} = 5\text{kg}$

And total friction (on block A and B) $= \mu mg = 0.25 \times 5\text{kg} \times 9.8\ \text{m/s}^2 = 12.25\text{N}$ (backward)

Applying Newton's second law:

$$F_R = T_2 - f = ma \text{ or } T_2 = ma + f$$

Substituting:

$$T_2 = ma + f = 5\text{kg} \times 2.55\text{m/s}^2 + 12.25\text{N} = 25\text{N}$$

(d) From results in (b) and (c): $T_1 = 10\text{N}, T_2 = 25\text{N}$; hence: $T_2 > T_1$ (shown).

Explanation

T_2 pulls both A and B (total mass 5kg) against greater friction, while T_1 pulls only A (2kg) against less friction; therefore, T_2 must be larger than T_1.

Making Sense of the Answer: *The three tensions in this problem tell a clear story:* $F = 50N$ *(applied to C)* → $T_2 = 25N$ *(transmitted to B and A)* → $T_1 = 10N$ *(transmitted to A only). Each string experiences progressively less tension as it has fewer masses to pull. This cascade of decreasing tension from front to back is a universal feature of any train of connected bodies.*

Thinking Like a Physicist: *Railway engineers exploit this principle when designing train couplings. Each coupling must be rated for the maximum tension it will experience, and the front coupling is always under the greatest tension. When a locomotive accelerates a long train from rest, the front coupling experiences the highest tension; a failure there would split the train. This is also why heavy trucks use reinforced front tow hitches. The front connection always bears the greatest mechanical load in a chain of connected vehicles.*

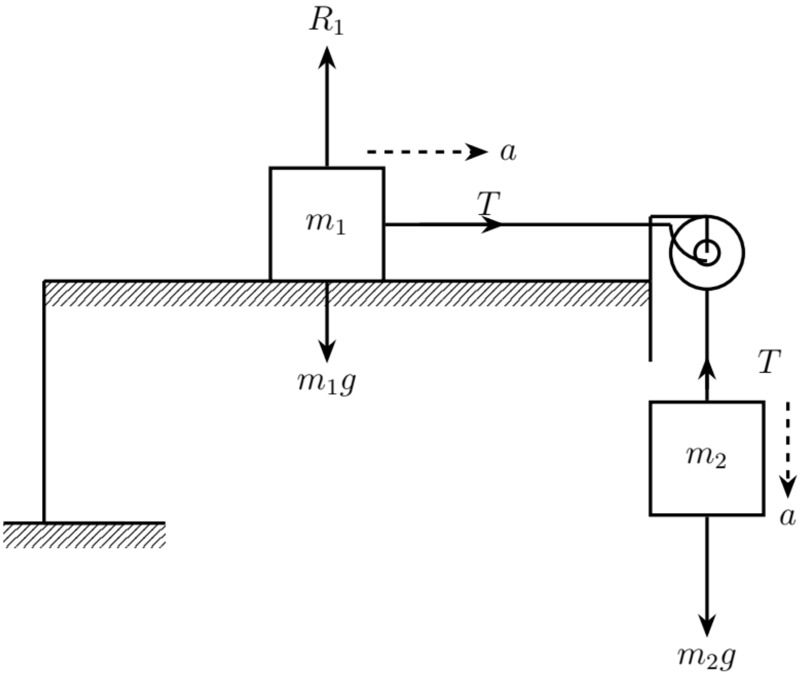
HOT Example 26

A block of mass 4kg rests on a horizontal table. It is connected by a light inextensible string passing over a smooth pulley fixed at the edge of the table to a hanging mass of 3kg. The system is released from rest. Take $g = 9.8\ \text{m/s}^2$.

(a) Assuming the table surface is smooth (frictionless), calculate:
  (i) the acceleration of the system,
  (ii) the tension in the string.
(b) Now assume the table surface is rough with coefficient of kinetic friction $\mu = 0.25$. Calculate:
  (i) the acceleration of the system,
  (ii) the tension in the string.

Solution

Consider the following diagram.



(a) **For smooth surface (No friction):**

Horizontal forces acting on m_1 (on table, moving horizontally, forward):

- Tension, T (horizontal, forward)

Vertical forces acting on m_2 (hanging, moving vertically, downward):

- Weight, m_2g (downward)
- Tension, T (upward)

Since the string is inextensible, both masses have the same acceleration magnitude, a.

For m_1:

$$T = m_1 a \text{ ... (equation 1)}$$

For m_2:

$$m_2 g - T = m_2 a \text{ ... (equation 2)}$$

Adding equations (1) and (2):

$$T + m_2 g - T = m_1 a + m_2 a$$

$$m_2 g = (m_1 + m_2)a$$

Rearrange to make a the subject:

$$a = \frac{m_2 g}{m_1 + m_2}$$

Substituting values:

$$a = \frac{3\text{kg} \times 9.8\text{m/s}^2}{4\text{kg} + 3\text{kg}} = 4.2\text{m/s}^2$$

The acceleration on a smooth surface is $\mathbf{4.2m/s^2}$.

From equation (1):

$$T = m_1 a = 4\text{kg} \times 4.2\text{m/s}^2 = 16.8\text{N}$$

For smooth surface, the tension is $\mathbf{16.8N}$.

A quicker way to calculate acceleration:

With this method, you only need to identify the source of the external pull and then apply Newton's second law of motion, $\boldsymbol{F = ma}$, *directly.*

In this example, the pulling force comes from the weight of the hanging mass, $\mathrm{m_2g}$. This weight provides the force that accelerates both $\mathrm{m_1}$ and $\mathrm{m_2}$.

Thus:

$$\mathrm{F = m_2g, m = m_1 + m_2}$$

It follows that:

$$\mathrm{m_2g = (m_1 + m_2)a \text{ or } \mathbf{a} = \frac{\mathbf{m_2g}}{\mathbf{m_1 + m_2}} = \frac{3kg \times 9.8m/s^2}{4kg + 3kg} = 4.2m/s^2}$$

Making Sense of the Answer (Part a): *The acceleration* $(4.2m/s^2)$ *is less than g because the 4kg block on the table resists the downward pull of the 3kg hanging mass (the hanging mass also accelerate the block horizontally). The tension* $(16.8N)$ *is less than the weight of the hanging mass* $(29.4N)$ *because the hanging mass is accelerating downward; if it were stationary, the tension would equal its full weight.*

(b) **For smooth surface (No friction):**

In this case, the pulling force $(\mathrm{m_2g})$ has to overcome frictional force $(\mathrm{\mu m_1g})$ before accelerating both $\mathrm{m_1}$ and $\mathrm{m_2}$ with acceleration a.

So the resultant force, $\mathrm{F_R = F - f = m_2g - \mu m_1g}$

Applying Newton's second law of motion:

$$\mathrm{m_2g - \mu m_1g = (m_1 + m_2)a}$$

From which:

$$\mathbf{a} = \frac{\mathbf{m_2g - \mu m_1g}}{\mathbf{m_1 + m_2}} = \frac{\mathrm{(3kg \times 9.8m/s^2) - (0.25 \times 4kg \times 9.8m/s^2)}}{\mathrm{(4 + 3)kg}} = 2.8\ \mathrm{m/s^2}$$

The acceleration on a rough surface is $\mathbf{2.8m/s^2}$.

Considering the hanging mass, $\mathrm{m_2}$.

$$\mathrm{m_2g - T = m_2a \text{ or } T = m_2g - m_2a = m_2(g - a) = 3kg(9.8 - 2.8)m/s^2 = 21N}$$

For rough surface, the tension is $\mathbf{21N}$.

Making Sense of the Answer (Part b): *The friction force* $(9.8N)$ *significantly reduces the acceleration from* $4.2m/s^2$ *to* $2.8m/s^2$*. This makes physical sense: friction opposes the forward motion of the block, requiring part of the hanging mass's weight to overcome friction rather than producing acceleration. Interestingly,* ***the tension increases from*** $\boldsymbol{16.8N}$ ***to*** $\boldsymbol{21N}$***. Why?*** *Because now the string must not only accelerate the block but also overcome friction acting on it. The string works harder in the rough case.*

Thinking Like a Physicist: *This problem illustrates a fundamental principle: adding friction to any part of a connected system reduces overall acceleration but may increase forces in connecting elements. This has practical implications: in elevator systems, increased cable friction means the motor must work harder (higher cable tension) even though the elevator accelerates more slowly. Similarly, towing a vehicle on a muddy road (high friction as it has greater* μ *value) puts greater tension on the tow cable despite slower acceleration. Engineers must design cables and connections to withstand these higher tensions, not just the smooth-surface ideal case.*

Connected Bodies on an Inclined Plane

You have now mastered two connected-body configurations: masses hanging vertically over a pulley, and masses moving horizontally on surfaces. Both involved motion in a single direction: pure vertical or pure horizontal. Now we combine these ideas: one mass moves along an incline while another hangs vertically.

This is perhaps the most common connected-body scenario in real engineering such as mountain railway systems, mine hoists, cable cars, and even simple construction pulleys lifting materials up slopes.

The general setup:

Consider a mass m_1 rests on an inclined plane making angle θ with the horizontal. It is connected by a light inextensible string passing over a smooth pulley fixed at the top of the incline to a mass m_2 hanging freely on the vertical side.

When released, the system moves in a direction determined by the relative magnitudes of m_1, m_2, and angle θ. Because the string is inextensible, if m_1 moves distance d up the slope, then m_2 descends distance d vertically. Therefore, both masses share the same acceleration magnitude a, though they move in different directions (one along the incline, another vertically).

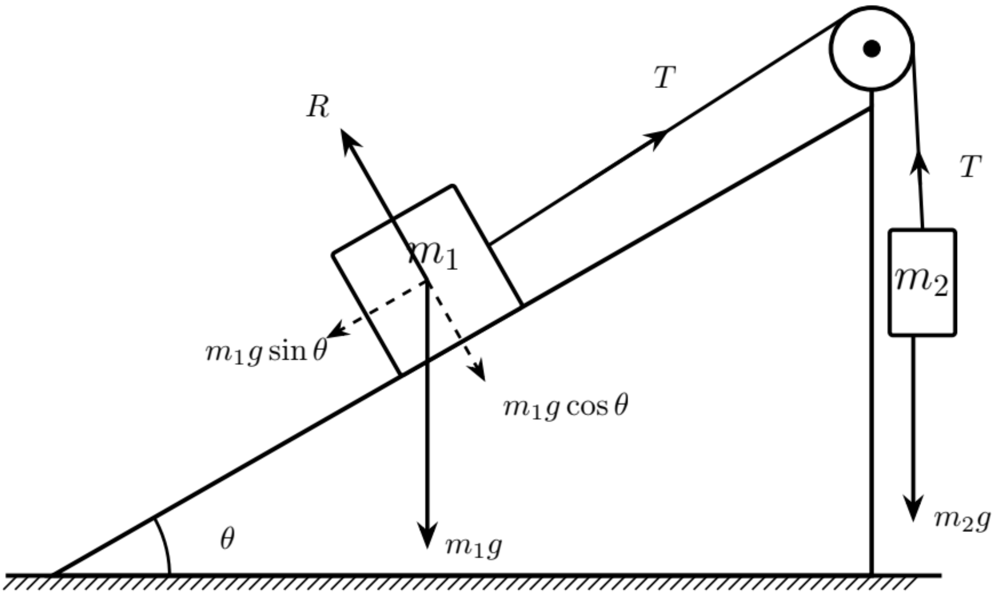


Figure: *Connected-body system on an inclined plane.*

First case: Smooth incline (no friction)

Forces acting on m_1:

- Weight: m_1g (vertically downward)
  - ✓ Component parallel to incline: $m_1g\sin\theta$ (down the slope)
  - ✓ Component perpendicular to incline: $m_1g\cos\theta$ (into the incline)
- Tension: T (up the slope, parallel to incline)
- Normal reaction: $R = m_1g\cos\theta$ (perpendicular to slope, outward)

Forces acting on m_2 (being pulled by string):

- Weight: m_2g (downward)
- Tension: T (upward)

Identifying forces trying to move the system:

- Weight of hanging mass, m_2: $W = m_2g$ (try to move m_1 up the slope while m_2 is moving vertically downward).
- Weight component of m_1 parallel to incline: $m_1g\sin\theta$ (try to move m_2 vertically upward while m_1 is moving down the slope).

So two possibilities exist on direction of motion:

1) If $\mathbf{m_2g > m_1g\sin\theta}$: Mass m_2 is heavier (or the incline is gentle as θ is small), so m_2 descends and m_1 moves up the slope.
2) If $\mathbf{m_2g < m_1g\sin\theta}$: Weight component of m_1 dominates, so m_1 slides down and m_2 ascends.

For analysis, assume case (1): m_2 descends, m_1 ascends.

Applying Newton's second law:

For m_1:

$T - m_1 g\sin\theta = m_1 a$ ... (equation 1)

For m_2:

$m_2 g - T = m_2 a$ ... (equation 2)

Finding acceleration:

Adding equations (1) and (2):

$T - m_1 g\sin\theta + m_2 g - T = m_1 a + m_2 a$

$m_2 g - m_1 g\sin\theta = (m_1 + m_2)a$

Making a the subject:

$a = \dfrac{m_2 g - m_1 g\sin\theta}{m_1 + m_2}$

Hence:

$$\mathbf{a = \frac{(m_2 - m_1\sin\theta)g}{m_1 + m_2}}$$

Alternatively; the pulling force ($m_2 g$) must overcome weight component of m_1 parallel to incline ($m_1 g\sin\theta$) before accelerating both m_1 and m_2 with acceleration a. Thus:

$$m_2 g - m_1 g\sin\theta = (m_1 + m_2)a$$

From which:

$$a = \frac{m_2 g - m_1 g\sin\theta}{m_1 + m_2} = \mathbf{\frac{(m_2 - m_1\sin\theta)g}{m_1 + m_2}}$$

Finding tension:

From equation (1):

$T = m_1 a + m_1 g\sin\theta = m_1(a + g\sin\theta)$

Substituting expression for a:

$$T = m_1\left(\frac{(m_2 - m_1\sin\theta)g}{m_1 + m_2} + g\sin\theta\right)$$

$$T = m_1 g\left(\frac{m_2 - m_1\sin\theta + m_1\sin\theta + m_2\sin\theta}{m_1 + m_2}\right)$$

$$T = m_1 g\left(\frac{m_2(1 + \sin\theta)}{m_1 + m_2}\right)$$

Hence:

$$\mathbf{T = \frac{m_1 m_2 g(1 + \sin\theta)}{m_1 + m_2}}$$

Interesting physical interpretation

From the acceleration and tension equations, we can deduce that:

- If $\theta = 0°$ (horizontal surface): $\sin\theta = 0$, so $a = m_2 g/(m_1 + m_2)$, $T = m_1 m_2 g/(m_1 + m_2)$ which matches the horizontal case we studied earlier.
- If $\theta = 90°$ (vertical pulley system): $\sin\theta = 1$, therefore $a = (m_2 - m_1)g/(m_1 + m_2)$, and $T = 2m_1 m_2 g/(m_1 + m_2)$ which matches the vertical case (Atwood machine).

These limiting cases confirm that our general results are consistent with the familiar horizontal and vertical configurations. Let us now extend the analysis further by introducing friction, leading us to the more realistic case of motion on a rough incline.

Second case: Rough incline (With friction)

When the incline surface is rough with coefficient of kinetic friction μ, friction opposes the motion of m_1 along the slope.

Friction force: $f = \mu R = \mu m_1 g\cos\theta$

The direction of friction depends on the direction of motion of m_1.

- If m_1 moves up the slope: Friction acts down the slope (opposing upward motion).
- If m_1 moves down the slope: Friction acts up the slope (opposing downward motion).

Now, let us return to our assumed scenario where m_2 descends, m_1 ascends.

For this case, the pulling force ($m_2 g$) must overcome both weight component of m_1 parallel to incline ($m_1 g\sin\theta$) and frictional force ($\mu m_1 g\cos\theta$) before accelerating both m_1 and m_2 with acceleration a. Thus:

$$m_2 g - m_1 g\sin\theta - \mu m_1 g\cos\theta = (m_1 + m_2)a$$

From which:

$$a = \frac{m_2 g - m_1 g\sin\theta - \mu m_1 g\cos\theta}{m_1 + m_2} = \frac{m_2 g - (m_1 g\sin\theta + \mu m_1 g\cos\theta)}{m_1 + m_2}$$

$$a = \frac{m_2 g - m_1 g(\sin\theta + \mu\cos\theta)}{m_1 + m_2}$$

Hence:

$$\mathbf{a = \frac{\left(m_2 - m_1(\sin\theta + \mu\cos\theta)\right)g}{m_1 + m_2}}$$

Once the acceleration has been determined, the tension can be found by considering forces acting on the hanging mass.

That is:

$$m_2 g - T = m_2 a$$

From which:

$$\mathbf{T = m_2 g - m_2 a}$$

Never overlook this important condition!

For motion to begin from rest, the driving force must first overcome static friction. In this system, the weight of the hanging mass must exceed the combined resisting forces along the incline:

$$m_2 g > m_1 g\sin\theta + \mu_s m_1 g\cos\theta$$

Where μ_s is the coefficient of static friction. If this condition is not met, the system remains stationary.

The ideas have finished cooking; now let us sit down and enjoy them through a few worked examples.

BINDER Example 27

A block of mass 6kg rests on a smooth plane inclined at 30° to the horizontal. It is connected by a light inextensible string passing over a smooth pulley at the top of the plane to a hanging mass of 4kg. The system is released from rest. Take $g = 9.8\text{m/s}^2$.

(a) Determine the acceleration of the system.
(b) Determine the tension in the string.

Solution

Identifying the system:

$m_1 = 6\text{kg}$ (on incline)

$m_2 = 4\text{kg}$ (hanging)

$\theta = 30°$

Surface: smooth (no friction)

Comparing m_2g and $m_1gsin\theta$ to determine the direction of motion:

$m_2g = 4\text{kg} \times 9.8\text{m/s}^2 = 39.2\text{N}$ (pulls m_1 up)

$m_1g\sin\theta = 6\text{kg} \times 9.8\text{m/s}^2 \times \sin30° = 29.4\text{N}$ (pulls m_1 down)

Since $m_2g\,(39.2\text{N}) > m_1g\sin\theta\,(29.4\text{N})$, the hanging mass moves downward and pulls the block up the plane.

(a) The weight of hanging mass (m_2g) must overcome weight component of m_1 parallel to incline ($m_1g\sin\theta$) before accelerating both m_1 and m_2 with acceleration a. Thus:

$$m_2g - m_1g\sin\theta = (m_1 + m_2)a$$

From which:

$$a = \frac{m_2g - m_1g\sin\theta}{m_1 + m_2} = \frac{9.8\text{m/s}^2(4\text{kg} - 6\text{kg}\sin30°)}{6\text{kg} + 4\text{kg}} = 0.98\text{m/s}^2$$

The acceleration is 0.98m/s^2.

(b) Considering forces acting on the hanging mass, m_2.

$$m_2g - T = m_2a \text{ or } T = m_2g - m_2a$$

Substituting:

$$T = (4\text{kg} \times 9.8\text{m/s}^2) - (4\text{kg} \times 0.98\text{m/s}^2) = 33.32\text{N}$$

The tension is 35.28N.

Making Sense of the Answer: *The hanging side is stronger (39.2N) than the block's down-slope pull (29.4N), so the system accelerates but only gently because both masses must be moved together. The tension is slightly less than 39.2N (since the hanging mass is accelerating downward) and greater than 29.4N (since the block is being pulled up the plane), which fits $T = 35.28N$.*

Thinking Like a Physicist: *On inclined connected-body problems, always compare m_2g with $m_1gsin\theta$ first to predict the direction. That single comparison prevents wrong friction directions later and saves you from sign confusion. A quick check is that the tension should lie between the two opposing pulls along the string.*

BINDER Example 28

A block of mass 9kg rests on a rough plane inclined at 40° to the horizontal. It is connected by a light inextensible string passing over a smooth pulley at the top of the plane to a hanging mass of 4kg. The coefficient of friction between the block and the plane is $\mu = 0.20$. The system is released from rest. Take $g = 9.8\text{m/s}^2$.

Calculate: (a) the acceleration of the system, (b) the tension in the string.

Solution

Identifying the system:

$m_1 = 9\text{kg}$ (on incline)

$m_2 = 4\text{kg}$ (hanging)

$\theta = 40°$

Surface: rough with $\mu = 0.20$

Determining the direction motion:

In this case, the driving force must overcome both the opposing pull from the other mass and the frictional force. Therefore, the comparison is either between m_2g and $(m_1g\sin\theta + f)$, or between $m_1g\sin\theta$ and $(m_2g + f)$, where:

- $m_2g = 4kg \times 9.8m/s^2 = 39.2N$
- $m_1g\sin\theta = 9kg \times 9.8m/s^2 \times \sin 40° = 56.7N$
- $f = \mu m_1 g\cos\theta = 0.2 \times 9kg \times 9.8m/s^2 \times \cos 40° = 13.5N$

So $m_1g\sin\theta$ $(56.7N)$ is greater than $m_1g\sin\theta + f$ $(52.7N)$. Hence, the block moves down the plane and the hanging mass moves upward.

(a) The driving force $(m_1g\sin\theta)$ must overcome all resistive forces so as to accelerate both m_1 and m_2 with acceleration a. Thus:

$$m_1g\sin\theta - m_2g - f = (m_1 + m_2)a \text{ or } a = \frac{m_1g\sin\theta - m_2g - f}{m_1 + m_2}$$

Substituting:

$$a = \frac{(56.7 - 39.2 - 13.5)N}{(9 + 4)kg} = 0.31m/s^2$$

The acceleration is $0.31m/s^2$.

(b) Considering forces acting on the hanging mass, m_2.

$$T - m_2g = m_2a \text{ or } T = m_2g + m_2a$$

Substituting:

$$T = 39.2N + (4kg \times 0.31m/s^2) = 40.44N$$

The tension is 40.44N.

Making Sense of the Answer: *The block's down-slope pull (56.7N) is slightly larger than the combined resistance from the hanging weight and friction (52.7N), so motion occurs but slowly. The tension lies between the opposing pulls, which confirms the result is physically reasonable.*

Thinking Like a Physicist: *Never assume the hanging mass must move downward. Always compare the possible driving force with its corresponding resistive forces. That quick comparison tells you the direction before any algebra and saves many sign mistakes.*

REAL Example 29

During road construction, a small machine is lowered down a sloping surface using a rope connected over a pulley to a counterweight. Workers observe that when the slope is made steeper, the machine begins to move downward even though the counterweight remains unchanged. Account for this observation.

Solution

As the slope becomes steeper, the component of the machine's weight acting down the slope increases. This increases the driving pull downward along the rope. If this down-slope pull becomes greater than the upward pull provided by the counterweight (together with friction), the machine will begin to move downward. Therefore, increasing the slope angle changed the balance of forces and altered the direction of motion even though the masses remained the same.

Making Sense of the Answer: *A steeper slope (greater value of $\sin\theta$) means a stronger downward pull along the ramp. Even without changing the counterweight, the balance can shift simply because geometry ($\sin\theta$) changes the effective force along the slope.*

Thinking Like a Physicist: *Always remember: on an inclined plane, the angle controls how much of the weight acts along the slope. Changing the angle changes the driving force even if the mass stays constant.*

HOT Example 30

A block of mass 10kg rests on a rough plane inclined at 30° to the horizontal. It is connected by a light inextensible string passing over a smooth pulley at the top of the plane to a hanging mass of 4kg. The coefficient of static friction between the block and the plane is $\mu_s = 0.25$, and the coefficient of kinetic friction is $\mu_k = 0.2$. The system is released from rest. Take $g = 9.8 m/s^2$.

(a) Determine whether the system moves. If it moves, state the direction of motion.
(b) If it moves, calculate the acceleration. If it does not move, determine the frictional force and the tension in the string.

Solution

(a) *Calculating key forces in the system:*

For the 10kg block:

Weight component parallel to the plane: $m_1 g\sin30° = 10 \times 9.8 \times 0.5 = 49N$

Limiting friction: $f_{max} = \mu_s R = \mu m_1 g\cos30° = 0.25 \times 10kg \times 9.8m/s^2 \times \sin30° = 21.22N$

For the hanging mass:

Weight: $m_2 g = 4kg \times 9.8 \, m/s^2 = 39.2N$

Testing possible directions:

First possibility: Suppose the hanging mass goes down (block moves up the plane).

The weight of hanging mass must overcome: $m_1 g\sin30° + f_{max} = (49 + 21.22)N = 70.22N$

But the weight, $m_2 g = 39.2N < 70.22N(m_1 g\sin30° + f_{max})$. So motion in this direction is impossible.

Second possibility: Suppose the block goes down the plane (hanging mass moves upward).

The weight acting along the plane must overcome: $m_2 g + f_{max} = (39.2 + 21.22)N = 60.42N$

But the weight acting along the plane, $m_1 g\sin30° = 49N < 60.42N(m_2 g + f_{max})$. So motion in this direction is impossible too.

Conclusion: The system does **not** move from rest. Static friction holds it in equilibrium.

(b) Finding frictional force and tension (since $a = 0$).

Since $m_1 g\sin30°(49N) > m_2 g\,(39.2N)$, the block on the plane attempts to move down the plane.

To move in this direction, the weight component has to overcome the weight of hanging mass and static friction. Thus for the block along the plane in **equilibrium:**

$$m_1 g\sin30° - m_2 g - f = 0 \text{ or } f = m_1 g\sin30° - m_2 g$$

Substituting:

$$f = (49 - 39.2)N = 9.8N$$

The frictional force is 9.8N.

Considering forces acting on the hanging mass in equilibrium, m_2.

$$T - m_2 g = 0 \text{ or } T = m_2 g = 39.2N$$

The tension is 39.2N.

Making Sense of the Answer: *Even though the block has a strong down-slope pull (49N), the hanging mass provides a 39.2N counter-pull through the tension. Only 9.8N of friction is needed to stop motion, and since the surface can provide up to 21.22N, the system stays comfortably at rest. Technically, the system is "locked" by friction.*

Thinking Like a Physicist: *In real engineering, this "locking" effect is both useful and dangerous. Useful: prevents cable cars from sliding backward when motors are off. Dangerous: can trap loads on slopes, requiring extra force to start motion. Engineers must calculate whether static friction will hold loads safely OR whether it will prevent necessary motion. Sometimes "stuck" is good; sometimes it is catastrophic.*

As the worked examples quietly leave the table, the next subtopic arrives; not to overwhelm us, but to be understood and enjoyed.

Motion in a Lift (Elevator)

You step into a lift on the ground floor of a tall building. As the lift starts moving upward, you feel slightly heavier as your legs feel the extra pressure. When the lift reaches the top floor and begins to stop, you feel momentarily lighter, almost as if you are floating. Yet your mass has not changed at all.

This sensation is one of the most direct experiences of Newton's laws in daily life. The lift is a connected-body system: you (the passenger) and the lift cabin move together with the same acceleration. The normal reaction from the lift floor (what you feel as your "weight") changes depending on whether the lift accelerates upward, downward, or moves at constant velocity.

Understanding lift motion is not just strengthening your grasp of tension, weight, acceleration, and equilibrium. It also governs elevator design, astronaut training (simulating weightlessness), amusement park rides, and even the sensation pilots experience during takeoff and landing.

Note on classification: While motion in a lift differs from traditional connected body problems (where we analyse forces between two separate masses connected by strings), we include it here because: (1) the person and lift move together with the same acceleration, similar to connected systems, (2) the analysis uses the same Newton's second law approach we have developed in this section, and (3) it demonstrates practical applications of forces in accelerating systems. Strictly speaking, this is an example of motion in an accelerating reference frame, where we analyse forces on a single body (the person) rather than interactions between connected bodies.

The physical setup:

Consider a person of mass **m** standing on a weighing scale inside a lift (elevator).

The weighing scale reads the **normal reaction R** between the person and the lift floor. It is sometimes called **apparent weight**. So:

$$\textbf{Apparent weight} = \textbf{Normal reaction}\ (\mathbf{R})$$

This may differ from the actual weight depending on the lift's motion.

Forces acting on the person:

- Weight: $W = mg$ (downward, always constant)
- Normal reaction, R from the lift floor (upward, not constant)

The **resultant force** on the person determines the acceleration according to Newton's second law.

With this setup in mind, we can now examine how the lift's motion affects the normal reaction and therefore the apparent weight by considering the different possible cases of motion.

Case 1: Lift at rest or moving with constant velocity

When the lift is stationary or moving at constant velocity (either upward or downward), the acceleration, $\mathbf{a = 0}$. This is equilibrium in vertical motion.

Applying Newton's second law: $R - mg = 0$ or $\mathbf{R = mg}$

Conclusion: *The normal reaction equals the actual weight. The person feels their normal weight.*

Case 2: Lift accelerating upward

When the lift accelerates upward with acceleration a, the person must also accelerate upward (they move together with the lift).

In this case, the resultant force on a person must be upward to produce upward acceleration.

Applying Newton's second law:

$$R - mg = ma \text{ or } R = mg + ma$$

Therefore: $\mathbf{R = m(g + a)}$

Conclusion: *The normal reaction,* $\boldsymbol{R > mg}$. *The person feels* ***heavier*** *than usual.*

Physical interpretation: *The lift floor must push upward with extra force to accelerate the person upward. This extra push is what makes you feel heavier when a lift starts moving upward.*

Case 3: Lift accelerating downward

When the lift accelerates downward with acceleration a, the person also accelerates downward.

In this case, the resultant force on a person must be downward to produce downward acceleration.

Applying Newton's second law:

$$\mathrm{mg} - \mathrm{R} = \mathrm{ma} \text{ or } \mathrm{R} = \mathrm{mg} - \mathrm{ma}$$

Therefore: $\mathbf{R} = \mathbf{m}(\mathbf{g} - \mathbf{a})$

Conclusion: *The normal reaction,* $R < mg$*. The person feels lighter than usual.*

Physical interpretation: *The lift floor does not need to push as hard because gravity is helping to accelerate the person downward. This reduced push makes you feel lighter when a lift slows down while moving upward, or speeds up while moving downward.*

Case 4: Free fall (cable breaks)

If the lift cable breaks and the lift falls freely under gravity alone, the acceleration equals the acceleration due to gravity: $\mathbf{a} = \mathbf{g}$ (downward).

Applying Newton's second law:

$$\mathrm{mg} - \mathrm{R} = \mathrm{ma}$$

From which (as shown earlier):

$$\mathrm{R} = \mathrm{m}(\mathrm{g} - \mathrm{a})$$

Substituting $\mathrm{a} = \mathrm{g}$

$$\mathrm{R} = \mathrm{m}(\mathrm{g} - \mathrm{g}) = 0$$

Conclusion: *The normal reaction,* $\boldsymbol{R} = \mathbf{0}$*. The person experiences* ***weightlessness****.*

Physical interpretation: *Both the person and the lift fall together at the same rate. The lift floor no longer pushes against the person's feet (they do not "press" against each other).* This is exactly how astronauts experience weightlessness in orbit. Fortunately, real lifts have strong safety systems (including emergency brakes and shock absorbers) preventing true free fall.

Summary Table: Apparent Weight in Different Lift Conditions

Lift Motion	**Acceleration**	**Normal reaction**	**Apparent Weight R**	**Sensation**
At rest	$a = 0$	$R = mg$	Normal	Normal weight
Constant velocity (up or down)	$a = 0$	$R = mg$	Normal	Normal weight
Accelerating upward	a (upward)	$R = m(g + a)$	Increased	Feel heavier
Decelerating upward	a(downward)	$R = m(g - a)$	Decreased	Feel lighter
Accelerating downward	a(downward)	$R = m(g - a)$	Decreased	Feel lighter
Decelerating downward	a (upward)	$R = m(g + a)$	Increased	Feel heavier
Free fall	a=g(downward)	$R = 0$	Zero	Weightless

A crucial distinction to remember:

- Accelerating upward means either starting to move upward **or slowing down while moving downward.**
- Accelerating downward means either starting to move downward **or slowing down while moving upward**.

Always focus on the direction of acceleration, not the direction of motion.

With the ideas now simmering nicely, let us serve them properly through a few worked examples and enjoy the flavour of physics in action.

BINDER Example 31

A person of mass 70kg stands on a weighing scale inside a lift. Take $\mathrm{g} = 9.8\,\mathrm{m/s^2}$. Determine the scale reading (in kg):

(a) When the lift is at rest.
(b) When the lift accelerates upward at $3\mathrm{m/s^2}$?
(c) When the lift accelerates downward at $2.5\mathrm{m/s^2}$?

Solution

(a) When at rest, acceleration $\mathrm{a} = 0$.

$$\mathrm{R} = \mathrm{mg} = 70\mathrm{kg} \times 9.8\mathrm{m/s^2} = 686\mathrm{N}$$

Scale reading in kilograms:

$$\text{Reading} = \frac{\mathrm{R}}{\mathrm{g}} = \frac{686\mathrm{N}}{9.8\mathrm{m/s^2}} = 70\mathrm{kg}$$

The scale reads 70kg.

(b) When lift is accelerating upward:

$$\mathrm{R} = \mathrm{m(g + a)} = 70\mathrm{kg}(9.8 + 3)\mathrm{m/s^2} = 896\mathrm{N}$$

Scale reading in kilograms:

$$\text{Reading} = \frac{896\mathrm{N}}{9.8\mathrm{m/s^2}} = 91.4\mathrm{kg}$$

The scale reads 91.4kg.

(c) When lift is accelerating downward:

$$\mathrm{R} = \mathrm{m(g - a)} = 70\mathrm{kg}(9.8 + 2.5)\mathrm{m/s^2} = 511\mathrm{N}$$

Scale reading in kilograms:

$$\text{Reading} = \frac{511\mathrm{N}}{9.8\mathrm{m/s^2}} = 52.1\mathrm{kg}$$

The scale reads 52.1kg.

Making Sense of the Answer: *The actual weight (686N) never changes, that is determined by mass and gravity. What changes is the normal reaction (what you feel), which depends on acceleration. When accelerating upward, the floor must push harder (896N) to accelerate you. When accelerating downward, the floor pushes less (511N) because gravity does some of the work.*

Thinking Like a Physicist: *The percentage change in apparent weight depends on the ratio a/g. For* $a = 3m/s^2$*, the change is* $\frac{896N - 686N}{686N} = \frac{3m/s^2}{9.8m/s^2} \approx 31\%$*. Similarly for* $a = 2.5m/s^2$*, it is 26%. Fast elevators in skyscrapers typically limit acceleration to about* $2m/s^2$ *for passenger comfort since larger accelerations would be uncomfortable and potentially dangerous.*

REAL Example 32

Kipute and Kipanga visit their aunt who lives on the 20th floor of a tall building in Dar es Salaam. As they ride the elevator up, Kipanga places his school bag (mass 8kg) on a weighing scale on the elevator floor.

Kipanga: *"Look, Kipute! When the lift started moving, the scale showed 10kg for a moment. But my bag is only 8 kg!"*

Kipute: *"That's because the lift was accelerating upward. The scale had to push harder against the bag to make it accelerate. What does it show now?"*

Kipanga: *"Now it shows exactly 8kg. We must be moving at constant velocity."*

Kipute: *"Correct! And watch what happens when we approach the 20th floor..."*

(The lift begins to slow down)

Kipanga: *"Wow! Now it shows only 6kg! Did my bag lose weight?"*

Kipute: (laughing) *"No, silly! The lift is decelerating, it's slowing down while moving upward, which means it's accelerating downward. So the scale pushes less against the bag."*

Question:

(a) Calculate the upward acceleration of the lift when the scale read 10kg.
(b) Calculate the downward acceleration when the scale read 6kg.
(c) If the lift cable were to break, what would the scale read?

$$\text{Take g} = 9.8\ \text{m/s}^2.$$

Solution

(a) The scale reading represents the normal reaction R.

$$\text{R} = 10\text{kg} \times 9.8\text{m/s}^2 = 98\text{N}$$

For upward acceleration:

$$\text{R} = \text{m}(\text{g} + \text{a})$$

Substituting:

$$98\text{N} = 8\text{kg}(9.8\text{m/s}^2 + \text{a});\ \text{a} = 2.45\ \text{m/s}^2$$

The lift was accelerating upward at $2.45\ \text{m/s}^2$.

(b) When scale reads 6 kg:

$$\text{R} = 6\text{kg} \times 9.8\text{m/s}^2 = 58.8\text{N}$$

For downward acceleration (deceleration while moving up):

$$\text{R} = \text{m}(\text{g} - \text{a})$$

$$58.8\text{N} = 8\text{kg}(9.8\text{m/s}^2 - \text{a});\ \text{a} = 2.45\ \text{m/s}^2$$

The lift was accelerating downward at $2.45\ \text{m/s}^2$.

(c) In free fall: $\text{a} = \text{g}$.

$$\text{R} = \text{m}(\text{g} - \text{g}) = 0$$

The scale would read 0kg.

Making Sense of the Answer: *Interestingly, the magnitude of acceleration is the same* ($2.45m/s^2$) *whether speeding up or slowing down. This is typical of well-designed elevator that maintains constant acceleration/deceleration rates for smooth rides. The scale reading changed by* $\pm 2kg$ *(from 8 to 10, or 8 to 6), which represents a* $\pm 25\%$ *change. This corresponds to* $a/g = 2.45/9.8 = 0.25\ or\ 25\%$.

Thinking Like a Physicist: *Elevator designers must balance several competing factors: velocity (to move people quickly), acceleration (affects comfort and safety), and cable strength (must support apparent weight during upward acceleration, which exceeds actual weight). Modern high-speed elevators use sophisticated control systems to minimize jerky motion and thus they gradually increase acceleration at the start and gradually decrease it before stopping.*

HOT Example 33

A person of mass 60kg stands in a lift and holds a 5kg bag suspended by a light string.

(a) Calculate the tension in the string when the lift moves upward with constant velocity of 2m/s.
(b) Calculate the tension in the string when the lift accelerates upward at $2.5\ \text{m/s}^2$.
(c) Calculate the tension in the string when the lift accelerates downward at 3m/s^2.
(d) The person feels they can comfortably hold a tension of up to 70N. What is the maximum acceleration the lift can have without the person dropping the bag?
(e) Calculate the total force that the lift floor exerts on the person for case (b). Explain why this differs from the person's weight.

$$\text{Take g} = 9.8\ \text{m/s}^2.$$

Solution

(a) Forces acting on the bag:

- Tension: T (upward)
- Weight: $W = mg$ (downward)

Applying Newton's second law:

$$T - mg = ma$$

But at constant velocity, acceleration $a = 0$.

$$T - mg = 0 \text{ or } T = mg = 5\text{kg} \times 9.8\text{m/s}^2 = 49\text{N}$$

The tension is 49N.

(b) If the lift accelerating upward at 2.5 m/s^2, the bag must also accelerate upward at 2.5 m/s^2.

Again using:

$$T - mg = ma \text{ or } T = mg + ma = m(g + a) = 5\text{kg}(9.8 + 2.5)\text{m/s}^2 = 61.5\text{N}$$

The tension is 61.5N.

(c) If the lift is accelerating downward:

$$mg - T = ma \text{ or } T = mg - ma = m(g - a) = 5\text{kg}(9.8 - 3)\text{m/s}^2 = 34\text{N}$$

The tension is 34N.

(d) When the lift accelerates downward, the tension decreases; when it accelerates upward, the tension increases. Therefore, for the tension to rise to the limiting value of 70N, the lift must be accelerating upward.

For upward acceleration:

$$T - mg = ma \text{ or } T = mg + ma$$

From which:

$$a = \frac{T - mg}{m}$$

It follows that:

$$a_{\max} = \frac{T_{\lim} - mg}{m} = \frac{70\text{N} - 5\text{kg} \times 9.8\text{m/s}^2}{5\text{kg}} = 4.2\text{m/s}^2$$

Hence, the maximum acceleration is 4.2m/s^2 **upward**.

Understand that: *Above this acceleration, the required tension would exceed 70N and the person would drop the bag.*

(e) Forces acting on a person (holding a bag):

- Normal reaction: R (upward)
- Tension: T (downward)
- Weight of the person: mg

Applying Newton's second law (for upward acceleration):

$$R - T - mg = ma \text{ or } R = ma + T + mg$$

Substituting:

$$R = 60\text{kg} \times 2.5\ \text{m/s}^2 + 61.5\text{N} + 60\text{kg} \times 9.8\text{m/s}^2 = 799.5\text{N}$$

The total force that the lift floor exerts on the person is 799.5N.

Alternative solution

Treating both a person and a bag as one object: $m_t = (60 + 5)\text{kg} = 65\text{kg}$

Applying Newton's second law:

$$R - m_t g = m_t a \text{ or } T = m_t g + m_t a = m_t(g + a) = 65\text{kg}(9.8 + 2.5)\text{m/s}^2 = 799.5\text{N}$$

The total force is 799.5N.

Explanation:

This differs from the person's actual weight (588 N) because the person is accelerating upward, which requires an additional upward force. In addition, the bag being held must also be accelerated upward. Consequently, the floor must provide enough force to accelerate the entire system (person plus bag).

Making Sense of the Answer: *Constant velocity means zero acceleration, so the tension equals the bag's weight (49N). Upward acceleration increases the required tension, while downward acceleration reduces it. The floor force on the person is larger than their weight because it must both accelerate the person upward and counter the downward pull from the bag.*

Thinking Like a Physicist: *Only acceleration changes forces, not velocity. Decide the direction of acceleration first, then apply Newton's second law. Remember: upward acceleration increases apparent weight and tension; downward acceleration decreases them.*

That brings our subtopic-by-subtopic worked examples to a satisfying close. The plates are cleared! Now let us enjoy the full buffet, where all the ideas of this topic come together in miscellaneous worked examples.

MISCELLANEOUS WORKED EXAMPLES ON EQUILIBRANT FORCES

Example 34

(a) Explain why a tow rope becomes very tight when a vehicle suddenly accelerates while pulling a trailer.

(b) A picture frame of weight 15N hangs from a single nail by a string. The string makes an angle of 30° with the vertical on each side of the nail.

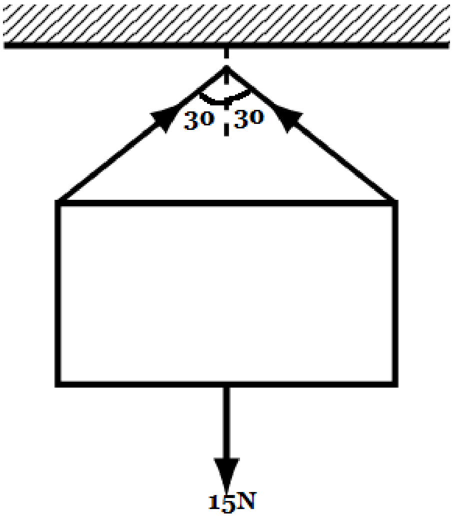
  (i) Calculate the tension in the string.

  (ii) Calculate the horizontal force exerted by the nail on each side of the string.

Solution

(a) When the vehicle accelerates suddenly, the trailer tends to remain in its original state of motion due to inertia. The tow rope must then provide a large force (tension) to accelerate the trailer in a short time. This sudden increase in required force makes the rope to tighten sharply.

(b) Each side of the string makes **30° with the vertical**, the system is symmetrical and hence tensions on both sides are equal (**T**).
The frame is in equilibrium as illustrated in the following diagram:



(i) Resolving vertically:

Vertical component in each string = $T\cos 30°$ (both upward)

$$\sum F_y = T\cos 30° + T\cos 30° - W = 0 \text{ or } 2T\cos 30° = W \text{ or } T = \frac{W}{2\cos 30°}$$

Substituting:

$$T = \frac{15\text{N}}{2\cos 30°} = 8.66\text{N}$$

The tension T is 8.66N.

(ii) Resolving horizontally:

Horizontal component in each string $=$ $\mathrm{Tsin30°}$ (one left side, another right side)

Substituting:

$$8.66\mathrm{Nsin30°} = 4.33\mathrm{N}$$

The horizontal force exerted by the nail is 4.33N on each side of the string.

Example 35

(a) A block rests on a rough inclined plane without sliding. Explain the role of friction in maintaining equilibrium and describe what would happen if the angle of the incline were gradually increased.

(b) A wooden crate of mass 50kg is kept on a ramp inclined at 30° to the horizontal. The coefficient of static friction (μ_s) between the crate and ramp is 0.45 and that of kinetic friction (μ_k) is 0.3. Take $g = 9.8\ \mathrm{m/s^2}$.

(i) Calculate the maximum angle at which the crate would remain in equilibrium without sliding.

(ii) Explain whether the crate is on the verge of sliding or if it could withstand a slight increase in the angle.

(iii) Calculate the frictional force acting on the crate.

(iv) Find the acceleration of the crate.

Solution

(a) When the block rests on the inclined plane, the component of its weight down the plane (mgsinθ) tends to make it slide. Static friction acts up the plane to oppose this tendency and maintain equilibrium. As the angle of the incline increases, mgsinθ increases while the normal reaction (mgcosθ) decreases, thereby reducing the maximum static friction available. Eventually, the limiting friction is reached where $\tan\theta = \mu_s$, and beyond this angle the block slides down the plane.

(b) The solution of each part is as follows:

(i) The maximum angle is found when the system is at limiting equilibrium.

At limiting equilibrium:

$$f_{max} = \mu_s R = \mu_s \mathrm{mgcos}\theta = \text{driving force} = \mathrm{mgsins}\theta$$

Taking:

$$\mu_s \mathrm{mgcos}\theta = \mathrm{mgsins}\theta;$$

$$\mu_s \cos\theta = \mathrm{sins}\theta;$$

$$\mu_s = \frac{\mathrm{sins}\theta}{\cos\theta} = \tan\theta;$$

$$\theta = \tan^{-1}\mu_s = \tan^{-1}0.45 = 24.2°$$

The maximum angle is 24.2°.

(ii) Compare:

$$\tan30° = 0.577 > 0.45(\mu_s)$$

Since $\tan30° > \mu_s$, the static friction is not enough to hold the crate.

So at 30°, the crate will slide (it is beyond the equilibrium limit). It cannot withstand any increase as it is already slipping.

(iii) Because it is sliding, friction is kinetic:

$$f_k = \mu_k R = \mu_k \mathrm{mgcos30°} = 0.3 \times 50\mathrm{kg} \times 9.8\mathrm{m/s^2} \times \cos30° = 127\mathrm{N}$$

The frictional force is 127N (up the plane).

(iv) Applying Newton's second law:

$$\mathrm{mgsins30°} - f_k = \mathrm{ma}$$

$$\mathrm{mgsin30°} - \mu_k \mathrm{mgcos30°} = \mathrm{ma}$$

$$a = \frac{mg\sin 30° - \mu_k mg\cos 30°}{m} = g\sin 30° - \mu_k g\cos 30° = g(\sin 30° - \mu_k\cos 30°)$$

Substituting:

$$a = 9.8\text{m/s}^2(\sin 30° - 0.3\cos 30°) = 2.35\text{m/s}^2$$

The acceleration is 2.35m/s^2 (down the plane).

Example 36

(a) Explain why the tension in a string connecting two blocks being pulled across a horizontal surface is not equal to the applied force, even when the surface is smooth (frictionless).

(b) Two blocks of masses $m_1 = 4$kg and $m_2 = 6$kg are connected by a light inextensible string on a smooth horizontal table. A horizontal force $F = 30$N is applied to the 4kg block, pulling both blocks to the right. Take $g = 9.8\ \text{m/s}^2$.

- (i) Calculate the acceleration of the system.
- (ii) Calculate the tension in the connecting string.
- (iii) If the force F were instead applied to the 6kg block (pulling both blocks), how would the tension change? Calculate the new tension and explain why it differs from part (ii).

Solution

(a) Even with no friction, the applied force is used to accelerate both blocks. However, the tension only needs to accelerate one of the blocks (whichever block is not directly receiving the applied force). Since part of the applied force is spent in accelerating the pulled block itself, only the remaining part is transmitted through the string. Therefore, tension is generally less than applied force.

(b) Solution for each par is as follows

Identifying the system:

$m_1 = 4$kg (block with applied force)

$m_2 = 6$kg (block being pulled by string)

Surface: smooth (no friction).

(i) Using Newton's second law for the whole system:

$$F = (m_1 + m_2)a \text{ or } a = \frac{F}{m_1 + m_2} = \frac{30\text{N}}{(6+4)\text{kg}} = 3\text{m/s}^2$$

The acceleration is 3m/s^2.

(ii) Consider the 6kg block alone (it only experiences tension):

$$T = m_2 a = 6\text{kg} \times 3\text{m/s}^2 = 18\text{N}$$

The tension in the string is 18N.

(iii) If the force (30N) is applied to the 6kg block instead; the acceleration remains the same (same total force, same total mass). But now tension must accelerate the 4kg block (not the 6kg block):

$$T = m_1 a = 4\text{kg} \times 3\text{m/s}^2 = 12\text{N}$$

The tension would decrease to 12N.

Explanation of difference:

New tension (12N) is less than the previous one (18N) because tension now accelerates the lighter 4kg (smaller mass) block instead of the heavier 6kg (larger mass) block.

Example 37

(a) A block of mass m rests on a rough plane inclined at angle θ to the horizontal. A force F is applied up the plane. The coefficient of static friction is μ_s. Prove that the block can remain in equilibrium only if:

$$mg(\sin\theta - \mu_s\cos\theta) \le F \le mg(\sin\theta + \mu_s\cos\theta)$$

(b) A package of mass 30kg rests on a rough plane inclined at 15° to the horizontal. A force of 220N is applied up the plane. If the coefficients of friction are $\mu_s = 0.40$ and $\mu_k = 0.30$ for static and kinetic friction respectively, determine:

- (i) whether the package remains at rest or moves,

(ii) its acceleration.

Take $g = 9.8\text{m/s}^2$.

Solution

(a) Proof:

Normal reaction: $R = mg\cos\theta$

Maximum static friction: $f_{max} = \mu_{sR} = \mu_s mg\cos\theta$

Static friction adjusts in magnitude (0 to f_{max}) and acts opposite to the tendency of motion.

Lower limit of F (tendency of motion: down the plane):

If F is small ($F < mg\sin\theta$), the block tends to slide down the plane, so friction acts up the plane (in the same direction as F but opposite to $mg\sin\theta$).

For equilibrium at the limiting case (just about to slide down):

$$F + f_{max} = mg\sin\theta$$

So:

$$F = mg\sin\theta - \mu_s mg\cos\theta$$

$$F = mg(\sin\theta - \mu_s\cos\theta) = \text{Minimum F to prevent downward sliding}$$

Hence, for equilibrium:

$$\mathbf{F \geq mg(\sin\theta - \mu_s\cos\theta)}$$

Upper limit of F (tendency of motion: up the plane):

If F is large ($F > mg\sin\theta$), the block tends to move up the plane, so friction acts down the plane (in the same direction as $mg\sin\theta$ but opposite to F).

For equilibrium at the limiting case (just about to move up):

$$F = mg\sin\theta + f_{max}$$

So:

$$F = mg\sin\theta + \mu_s mg\cos\theta$$

$$F = mg(\sin\theta + \mu_s\cos\theta) = \text{Maximum F just before upward sliding}$$

Hence, for equilibrium:

$$\mathbf{F \leq mg(\sin\theta + \mu_s\cos\theta)}$$

Combining the two conditions gives:

$$\mathbf{mg(\sin\theta - \mu_s\cos\theta) \leq F \leq mg(\sin\theta + \mu_s\cos\theta)}$$

(b) Solution for each part is as follows:

(i) Comparing F and $mg\sin\theta$ to determine the tendency of motion:

$$mg\sin\theta = 30\text{kg} \times 9.8\text{m/s}^2 \times \sin15° = 76.1\text{N} < 220\text{N (applied force F)}$$

Since the applied force is greater than the weight component acting along the plane, the tendency of motion is up the plane. So the applied force has upper limit for achieving limiting equilibrium and it is given by (from (a)):

$$F = mg(\sin\theta + \mu_s\cos\theta) = F_{maximum}$$

Substituting:

$$F_{maximum} = 30\text{kg} \times 9.8\text{m/s}^2(\sin15° + 0.4\cos15°) = 189.7\text{N} < 220\text{N (applied force)}$$

Since the applied upward force is greater than opposing (resistive) forces, the package **moves** up the plane.

(ii) As the package moves up the plane, the **kinetic** frictional force acts down the plane.

But the kinetic friction is given by:

$$f_k = \mu_k R = \mu_k mg\cos\theta = 0.3 \times 30\text{kg} \times 9.8\text{m/s}^2\cos15° = 85.2\text{N}$$

Total resistive forces $= mg\sin\theta + f_k = 76.1\text{N} + 85.2\text{N} = 161.3\text{N}$

Applying Newton's second law:

$$\text{Applied force} - \text{Resistive forces} = \text{ma or a} = \frac{\text{Applied force} - \text{Resistive forces}}{\text{m}}$$

Substituting:

$$a = \frac{(220 - 161.3)\text{N}}{30\text{kg}} = 1.96\text{m/s}^2$$

The acceleration is 1.96m/s^2.

Example 38

(a) Explain why does a hanging signboard remain at rest even though several forces act on it?

(b) A block of mass 6kg rests on a rough horizontal table with coefficient of static friction, $\mu_s = 0.4$ and coefficient of kinetic friction $\mu_k = 0.3$. It is connected by a light inextensible string passing over a smooth pulley at the edge of the table to a hanging mass of 2kg. A horizontal force F = 50N is applied to the 6kg block pulling it away from the pulley. The system is released from rest.

(i) Determine whether the system will move and, if so, in which direction.

(ii) Calculate the acceleration of the system and the tension in the string.

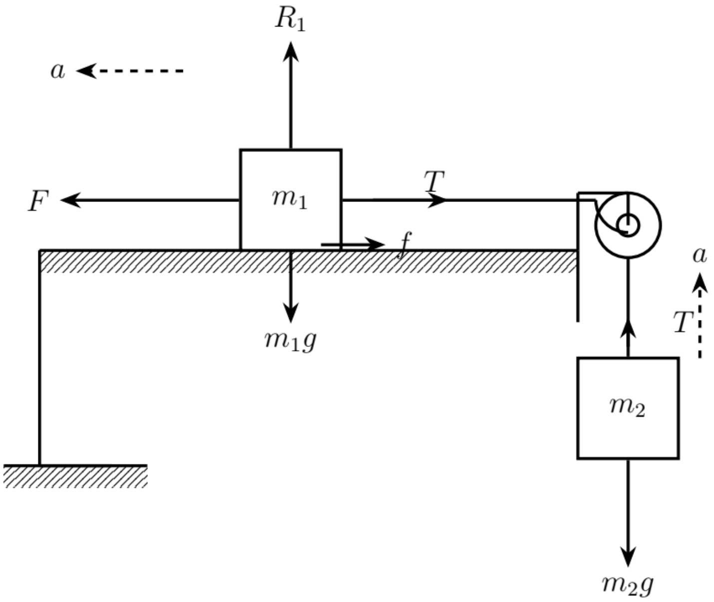
(iii) What is the minimum value of the applied force, F, required to prevent the hanging mass from descending?

Take $g = 9.8\ \text{m/s}^2$.

Solution

(a) The signboard remains at rest because the forces acting on it are balanced. The upward components of the tensions in the supporting strings add up to equal the weight of the signboard, while the horizontal components cancel each other. Therefore, the resultant force is zero and hence there is no acceleration.

(b) Consider the following diagram.



(i) Identifying forces trying to move the system:

- Driving force: $F = 50\text{N}$ (try to move the system **away** from pulley, leftward).
- Weight of hanging mass, m_2: $W = m_2g = 2\text{kg} \times 9.8\ \text{m/s}^2 = 19.6\text{N}$ (try to move the system **toward** pulley, rightward).

So: $\text{Driving force}(50\text{N}) > \text{Weight of hanging mass}(19.6\text{N})$. From this alone we can conclude that the system tries to move away from pulley.

For actual movement to occur, the driving force must overcome **all** resisting forces (static friction and weight of hanging mass).

The static friction is maximum when:$f_{max} = \mu_s R_1 = \mu_s m_1 g = 0.4 \times 6\text{kg} \times 9.8\ \text{m/s}^2 = 23.52\text{N}$

So total resistive forces, the driving force has to overcome for actual motion to occur is:

$$(19.6 + 23.52)\text{N} = 43.12\text{N} < 50\text{N}(\text{Driving force})$$

Since the driving force is greater than total resistive forces, the system move.

Conclusion: Yes, the system moves with m_1 going leftward (away from pulley) and m_2 ascending. This is because the applied force F = 50N exceeds the combined resistance from the weight of hanging mass and static friction.

(ii) Once the system moves, the static friction is no longer functioning and the kinetic friction takes place. Thus:

$$\text{Friction force}, f_k = \mu_k m_1 g = 0.3 \times 6\text{kg} \times 9.8\ \text{m/s}^2 = 17.64\text{N}$$

So total resistive forces during the motion $= W_{m_2} + f_k = (19.6 + 17.64)\text{N} = 37.24$

The net driving force, will accelerate both m_1 and m_2. Therefore:

$$\text{Driving force} - \text{Resistive force} = (m_1 + m_2)a$$

Substituting:

$$50\text{N} - 37.24\text{N} = (6\text{kg} + 2\text{kg})a; a = 1.595\text{m/s}^2$$

The acceleration is $\mathbf{1.595 m/s^2}$.

For hanging mass:

$$T - m_2 g = m_2 a \text{ or } T = m_2 a + m_2 g = m_2(a + g) = 2\text{kg}(1.595 + 9.8)\text{m/s}^2 = 22.79\text{N}$$

The tension is $\mathbf{22.79N}$.

(iii) For m_2 not to descend, the system must either be at rest or moving leftward. The critical condition is when m_2 is on the verge of descending (*limiting equilibrium with* $a = 0$).

At limiting equilibrium:

- Friction force is equal to the maximum static friction = 23.52N.
- Total resistive forces $= f_{max} + W_{m_2} = (23.52 + 19.6)\text{N} = 43.12\text{N}$

So:

- If driving force, $F < 43.12\text{N}$, the hanging mass descends (pulls m_1 rightward).
- If $F \geq 43.12\text{N}$, the hanging mass is not descending.

Hence, the minimum value of the applied force required to prevent the hanging mass from descending is 43.12N.

Example 39

(a) Why do passengers sometimes feel heavier when a lift starts moving upward?

(b) Two bodies of masses 3kg and 5kg are connected by a light inextensible string passing over a smooth pulley and are released from rest while hanging freely. After the system has been moving for 1.5s, the string suddenly snaps just above the 5kg body. Take $g = 9.8\text{m/s}^2$.
Calculate:

(i) The time taken for the 5kg body to reach the ground after the string snaps, given that at the instant the string snaps it is 2m above the ground.
(ii) The maximum height reached by the 3kg body above its initial position if the two bodies were initially at the same level.
(iii) The total time taken by the 3kg body to reach the ground.

Solution

(a) When the lift accelerates upward, the floor must push up on passengers with a force greater than their weight in order to accelerate them upward. This increased normal reaction is what make them feel heavier.

(b) Before the string snapped, the system was undergoing motion similar to an Atwood machine.

For the Atwood machine:

$$a = \left(\frac{m_2 - m_1}{m_1 + m_2}\right) g$$

Where:

$$m_2 = 5\text{kg (heavier)}$$

$$m_1 = 3\text{kg (lighter)}$$

Substituting:

$$a = \left(\frac{5\text{kg} - 3\text{kg}}{3\text{kg} + 5\text{kg}}\right) \times 9.8\text{m/s}^2 = 2.45\text{m/s}^2$$

Velocity at the instant of snapping:

$$v = at = 2.45\text{m/s}^2 \times 1.5\text{s} = 3.675\text{m/s}$$

Distance moved in 1.5s (each body):

$$s = ut + 0.5at^2 = 0\text{m/s} \times 1.5\text{s} + 0.5 \times 2.45\text{m/s}^2 \times (1.5\text{s})^2 = 2.756\text{m}$$

So, at the instant the string snaps:

- The 5kg mass is moving downward at 3.675m/s.
- The 3kg mass is moving upward at 3.675m/s.
- Each body moved through a vertical distance of 2.756m.

(i) After snapping, the 5kg body moves under gravity with initial downward velocity 3.675m/s.

Using:

$$s = ut + 0.5gt^2$$

Substituting:

$$2 = 3.675t + 0.5(9.8)t^2 \text{ or } 4.9t^2 + 3.675t - 2 = 0$$

Solving the quadratic equation, gives the practical value of t which is:

$$t = 0.37\text{s}$$

The time taken for the 5kg body to reach the ground is 0.37s.

(ii) After snapping, the 3kg mass continues to rise with a deceleration of 9.8m/s^2 until its velocity becomes zero at the highest point.

Using:

$$v^2 = u^2 + 2as; \text{with } v = 0, u = 3.675\text{m/s}, a = -9.8\text{m/s}^2, s = h$$

Substituting:

$$0^2 = (3.675\text{m/s})^2 - 2(9.8\text{m/s}^2)\text{h};\ \text{h} = 0.689\text{m}$$

Height gained before snapping was $\text{s} = 2.756\text{m}$, so maximum height above initial position:

$$\text{H}_{\text{max}} = \text{s} + \text{h} = 2.756\text{m} + 0.689\text{m} = 3.445\text{m}$$

The maximum height is 3.445m.

(iii) At snapping, the 5kg body is 2m above ground, and it has descended $\text{s} = 2.756\text{m}$ since release, so its initial height above ground was:

$$\text{H}_0 = 2\text{m} + 2.756\text{m} = 4.756\text{m}$$

The 3kg body started at the same level, so it also started 4.756m above ground.

It rose by $\text{s} = 2.756\text{m}$ before snapping, so its height above ground at snapping was:

$$\text{H}_{\text{snap}} = 4.756\text{m} + 2.756\text{m} = 7.512\text{m}$$

The time to reach the ground from the snapping time can be found by using:

$$\text{s} = \text{ut} + 0.5\text{at}^2;\ \text{with s} = -7.512\text{m}, \text{u} = 3.675\text{m/s}, \text{a} = -9.8\text{m/s}^2$$

Substituting:

$$-7.512 = 3.675\text{t} - 0.5(9.8)\text{t}^2 \text{ or } 4.9\text{t}^2 - 3.675\text{t} - 7.512 = 0$$

Solving the quadratic equation, gives the practical value of t which is:

$$\text{t} = 1.67\text{s}$$

Total time from release to ground for the 3kg body = 1.5 + t = 1.5s + 1.67s = 3.17s

The total time is 3.17s.

If the ideas now feel familiar and connected, you are ready! Let us understand and enjoy them even more in the Digging Deeper Exercise in the next page.

DIGGING DEEPER EXERCISE 4

EXERCISE 4A: BINDER QUESTIONS

Question 1

Two ropes are tied to a ring and pulled in different directions. A third rope must be attached to keep the ring in equilibrium. Explain why the direction of the third rope is uniquely determined by the first two ropes.

Question 2

In an Atwood machine with two unequal masses, the lighter mass accelerates upward. Explain why the tension in the string is **not** equal to the weight of either mass.

Question 3

A block on an inclined plane experiences a normal reaction perpendicular to the surface. Explain why the normal reaction is less than the block's weight.

Question 4

When you stand in a lift that accelerates upward, you feel heavier. Explain what actually changes and what remains constant.

Question 5

Two blocks are connected by a string on a horizontal surface. When pulled by a force F, they accelerate together. Explain why the string tension is less than the applied force F.

Question 6

A block rests on a rough inclined plane at angle θ below the critical angle. Explain why friction acts up the slope even though no other force is pulling the block upward.

Question 7

In a pulley system, the string is described as "light and inextensible." Explain the physical significance of each of these terms.

Question 8

A car tows a trailer on a horizontal road. The tension in the tow-bar is less when traveling at constant velocity than when accelerating. Explain why.

Question 9

A student writes: *"Tension always equals weight for a hanging object."* Give a counter-example using a system in equilibrium and explain why tension can differ from weight.

Question 10

Two strings hold a load at a junction. One string is closer to horizontal than the other. Without calculation, explain which string must have the larger tension and why.

EXERCISE 4B: REAL QUESTIONS

Question 11

A market vendor in Kariakoo hangs a 20 kg bag of rice from a ceiling hook using a single rope. The rope makes a V-shape with both sides going up to the same hook. Explain why the tension in each side of the rope is greater than half the weight of the bag.

Question 12

A bus traveling from Dar es Salaam to Mwanza carries luggage on the roof rack. The ropes tying down the luggage must be tightened periodically during the journey. Explain why the ropes become loose even without any luggage shifting position.

Question 13

Kipute and Kipanga are helping load maize sacks onto a pickup truck using a wooden plank as a ramp.

Kipanga: *"Let's make the ramp steeper; it's shorter, so we'll move the sacks faster!"*

Kipute: *"But won't a steeper ramp make it harder to push the sacks up?"*

Mr. Akilikubwa: *"Kipute has a good point. Let me ask you both: what happens to the force needed to push a sack up the ramp as we increase the angle?"*

Explain why a steeper ramp requires more force to push the same sack at constant velocity, even though the distance is shorter.

Question 14

A traditional water well uses a rope over a pulley to raise a bucket. An experienced well user knows to pull the rope smoothly and steadily rather than in quick jerks. Explain why jerking the rope increases the risk of the rope breaking.

Question 15

A daladala climbs a steep road while carrying many passengers. When it stops on the slope, the driver says, *"If the handbrake is weak, the bus will slide even if the engine is off."* Explain what force must act to prevent sliding and why it is required even when the bus is at rest.

Question 16

Kipanga leans a wooden plank against a rough wall. Sometimes it slips down, but when he pushes the bottom of the plank harder into the floor, the plank stops slipping. Explain why pressing harder can help prevent slipping.

Question 17

A lorry carrying sand is tied down using a rope that slopes downward toward the trailer. The driver claims: *"The rope helps in two ways: it stops the load from sliding sideways and also presses it down more firmly."* Explain how one rope can do both jobs at the same time.

Question 18

A signboard is fixed to a wall using a hinge and supported by a cable at an angle. When the cable is removed, the signboard falls. Explain why the hinge alone cannot keep the signboard in equilibrium.

EXERCISE 4C: HOT QUESTIONS

Take g = 9.8 m/s²

Question 19

A picture frame of mass 2.5kg is suspended from a nail by a string. The string makes an angle of 35° with the vertical on each side of the nail. Calculate:

(a) The tension in the string

(b) The horizontal force exerted by the nail on the string.

Question 20

An Atwood machine consists of masses 4kg and 7kg connected by a light inextensible string over a smooth pulley. The system is released from rest. Calculate:

(a) The acceleration of the system.

(b) The tension in the string.

(c) The distance traveled by the 7kg mass in the first 2 seconds.

Question 21

Two blocks of masses 6kg and 4kg are connected by a light string on a smooth horizontal table. A horizontal force of 35N is applied to the 6kg block.

(a) Calculate the acceleration of the system.

(b) Calculate the tension in the connecting string.

(c) If the surface had coefficient of kinetic friction 0.3, what would be the new acceleration?

Question 22

A block of mass 8 kg rests on an inclined plane at 25° to the horizontal. The coefficient of static friction is 0.45 and the coefficient of kinetic friction is 0.35.

(a) Determine whether the block remains at rest or slides down.

(b) If a force of 40N is applied parallel to and up the incline, calculate the acceleration of the block.

Question 23

An Atwood machine has masses $m_1 = 2$ kg and m_2. When released, the system accelerates at 2.5m/s² and m_2 moves downward.

(a) Calculate the mass m_2.

(b) The tension in the string.

(c) If the initial separation between the masses is 3m, how long until they are at the same height?

Question 24

A lift of mass 800 kg carries 4 passengers each of mass 70kg. Calculate:

(a) The tension in the lift cable when accelerating upward at 1.5m/s²

(b) The tension when moving upward at constant velocity of 3m/s

(c) The tension when decelerating at 2m/s² while moving upward.

Question 25

Two blocks **A** (5kg) and **B** (3kg) are connected by a string. Block **A** is on a smooth horizontal table and block **B** hangs over the edge via a pulley. Block **B** is initially held at rest 2m above the ground. When released:

(a) Calculate the acceleration of the system.

(b) Calculate the velocity of **B** just before it hits the ground.

(c) How far does **A** travel before **B** hits the ground?

Question 26

Three blocks of masses 2kg, 3kg, and 4kg are connected by light inextensible strings on a smooth horizontal surface. A force of 45N is applied to the 2kg block. Calculate:

(a) The acceleration of the system.

(b) The tension in the string between the 2kg and 3kg blocks.

(c) The tension in the string between the 3kg and 4kg blocks.

Question 27

Two blocks lie on smooth opposite inclines and are connected by a light string over a smooth pulley. Block A has mass 10kg on a 35° incline. Block B has mass 8kg on a 25° incline. An additional force P is applied to block A parallel to the plane to keep the system in equilibrium. Find:

(a) the tension in the string,

(b) the magnitude and direction of P (state whether it acts up or down the plane on A).

Question 28

A 8kg block rests on a smooth plane inclined at 30° and is connected by a light string over a smooth pulley to a hanging 6kg mass. The system is in equilibrium due to friction between the block and the plane. Find:

(a) the tension in the string,

(b) the frictional force on the block (magnitude and direction along the plane),

(c) the minimum coefficient of friction required.

Question 29

A load of 120 N is suspended from a junction by two light strings. The string on the left makes an angle of 30° with the horizontal, while the string on the right makes an angle of 45° with the horizontal.

(a) Without calculation, state which string has the greater tension, and why this is expected.

(b) Calculate the tension in each string.

Question 30

A person stands on a weighing scale in a lift. The scale reading is 20% greater than the person's true weight.

(a) Determine the acceleration of the lift and state its direction.

(b) If the person's mass is 75kg, calculate the force exerted by the scale on the person.

ANSWERS

EXERCISE 4A

1. For equilibrium, the vector sum of all forces must be zero. The first two ropes create a resultant force (found by vector addition). The third rope must provide a force equal in magnitude but opposite in direction to this resultant. Therefore, the third rope's direction is uniquely determined as opposite to the resultant of the first two forces.

2. The tension is not equal to either weight because both masses are accelerating. For the heavier mass (accelerating downward), tension must be less than its weight to allow net downward force. For the lighter mass (accelerating upward), tension must be greater than its weight to allow net upward force. The tension has a single value throughout the string that is intermediate between the two weights, allowing both masses to accelerate at the same rate in opposite directions.

3. The normal reaction equals the component of weight perpendicular to the incline surface, which is mgcosθ. This is less than the full weight mg because the weight vector is resolved into two components: one perpendicular to the surface (mgcosθ) and one parallel to the surface (mgsinθ). The perpendicular component determines the normal reaction. As the angle increases, cosθ decreases, so the normal reaction becomes even smaller.

4. Your actual weight (gravitational force = mg) remains constant because your mass and Earth's gravity do not change. What changes is your apparent weight (the normal reaction from the lift floor). When the lift accelerates upward, the floor must push you upward with force greater than your weight to accelerate you. This increased normal reaction is what you feel as "feeling heavier." The sensation of weight comes from normal reaction, not from gravitational force.

5. The applied force F must accelerate both blocks (total mass $m_1 + m_2$). However, the string tension only needs to accelerate one block (the one not directly receiving force F). Since tension accelerates less mass than F does, the tension is smaller. Specifically, if F pulls the first block, tension $= (m_2/(m_1 + m_2)) \times F$, which is always less than F.

6. Friction opposes the tendency of motion, not actual motion. The weight component mgsinθ acts down the slope, creating a tendency for the block to slide downward. Static friction opposes this tendency by acting up the slope. Friction adjusts its magnitude (up to maximum $\mu_s$mgcosθ) to exactly balance the downward component, preventing motion. No external upward force is needed because friction provides the balancing force.

7. "Light" means the string has negligible mass compared to the objects connected. This ensures the string does not require force to accelerate itself, so tension is the same throughout its length. "Inextensible" means the string does not stretch, so when one end moves a certain distance, the other end moves the same distance. This ensures connected objects have the same magnitude of acceleration (though possibly in different directions). Both assumptions simplify analysis by ensuring uniform tension and equal acceleration magnitudes.

8. At constant velocity (zero acceleration), the tension only needs to overcome the resistive forces on the trailer (friction, air resistance). When accelerating, the tension must provide both the force to overcome resistance **and** the force to accelerate the trailer's mass (T = ma + resistive forces). Since the acceleration term adds to the required tension, it is greater during acceleration than at constant velocity.

9. Counter-example: a load supported by two strings at angles. Each tension is not equal to weight because only the vertical components add to balance the weight.

10. The string closer to horizontal must have the larger tension because it has a smaller vertical component per unit tension. To supply enough vertical support, it must be pulled harder.

EXERCISE 4B

11. Each side of the rope has both a vertical and horizontal component. Only the vertical components support the bag's weight. Since each rope is at an angle (not vertical), its vertical component is less than the full tension (vertical component = Tcosθ).

To provide enough vertical support to balance the weight, the tension must be greater than simply half the weight. The more the rope deviates from vertical, the greater the tension required.

12. Vibrations from the road cause the rope fibers to settle and compress slightly, and the knots to tighten, effectively making the rope shorter between fixed points. Additionally, temperature changes (the rope heats during day, cools at night) cause expansion and contraction. Even small stretching under initial load redistributes tension. These effects accumulate to create slack in the rope, requiring periodic re-tightening to maintain secure load.

13. As the ramp angle increases, the component of the sack's weight parallel to the ramp ($mg\sin\theta$) increases. This component acts down the ramp and must be overcome to push the sack up. At steeper angles, $\sin\theta$ is larger, so more force is needed to balance this component and any friction. While the distance is shorter, the required force is greater, making the work (force × distance) roughly the same but the task more difficult due to the larger force requirement.

14. When the rope is jerked suddenly, the bucket must accelerate rapidly from rest or from a lower velocity to a higher velocity. By Newton's second law ($F = ma$), rapid acceleration requires a large force, which means very high tension in the rope. This tension can exceed the rope's breaking strength. Pulling smoothly means smaller acceleration, which requires less force (tension), keeping the rope safely below its breaking point while still raising the bucket efficiently.

15. Static friction must act up the slope to balance the component of weight down the slope. Even at rest, that component tends to cause sliding, so friction is required to maintain equilibrium.

16. Pressing the bottom harder increases the normal reactions at the contacts, which increases the maximum possible static friction. With larger available friction, the plank can stay in equilibrium without slipping.

17. The rope tension has components in different directions. Horizontal component opposes sideways slipping, while the downward vertical component increases the normal reaction, which increases friction and thus helps prevent sliding.

18. The hinge provides a reaction but cannot supply the necessary balancing effect on its own. The cable provides an additional force with suitable components that help balance the weight and keep the signboard in equilibrium; without it, the forces cannot balance and the signboard rotates/falls.

EXERCISE 4C

19. (a) $T = 15N$(b) $F = 17.2N$

20. (a) $a = 2.67\ m/s^2$(b) $T = 49.9N$(c) $s = 5.34m$

21. (a) $a = 3.5\ m/s^2$(b) $T = 14N$(c) $a = 0.56m/s^2$

22. (a) Block slides down ($mg\sin\theta = 33.1N > f_{max} = 31.9N$ (b) $a = 0m/s^2$

23. (a) $m_2 = 3.37kg$(b) $T = 24.6N$(c) $t = 1.1s$

24. (a) 12204N (b) 10584N (c) 8424N

25. (a) $a = 3.68m/s^2$ (b) $v = 3.83m/s$ (c) $s = 2m$

26. (a) $a = 5m/s^2$ (b) $T = 35N$ (c) $T = 20N$

27. (a) $T=33.1N$ (b) $P=23N$ acting up the plane on A.

28. (a) $T=58.8N$ (b) $f=19.6N$ down the plane (c) $\mu=0.289$

29. (a) The string making **45°** with the horizontal has the greater tension, because it is more steeply inclined and must provide a larger vertical component to support the load. (b) Tension in 30° string = 87.8N, Tension in 45° string = 107.6N.

30. (a) $a = 1.96m/s^2$ upward (b) $R = 882N$