PART ONE
FOUNDATIONS OF MOTION AND MEASUREMENT

Physics begins with motion and measurement. Every moving object; from a passing bus to a falling fruit, reflects principles that can only be understood when described carefully and measured properly. Since no measurement is perfectly exact, ideas such as units, dimensions, precision, and error are essential for reliable scientific work. This part strengthens your understanding of both measurement and motion, providing the foundation needed for deeper understanding of mechanics and the rest of Physics.

Chapter 1

MEASUREMENT

INTRODUCTION

The physics laboratory at Miono Secondary School had seen better decades. Of its twelve wooden benches, three wobbled so badly that students had learned to wedge folded exercise-book covers under the legs before every practical, a ritual so familiar it might as well have been listed as Step 1 in the apparatus setup. Two others had developed a worrying lean, like elderly relatives at a family gathering who had been standing too long.

When the school board finally approved funds for replacement benches, the relief in the science department was enormous. The headmaster, a practical man who believed in involving students in every aspect of school life, asked for volunteers during the July preparation week to measure the laboratory so that new benches could be ordered from a carpenter in Bagamoyo.

Kipanga's hand went up before the sentence was finished. This surprised absolutely no one. Kipanga was the kind of student who volunteered first and thought about the details afterwards, if he thought about them at all. He was tall, energetic, and carried himself with the unshakable confidence of someone who had never met a task he did not believe he could handle, a belief that his actual track record did remarkably little to support.

"I will measure the room, sir," **Kipanga** announced. *"I am very accurate."*

The headmaster, who taught History and had no particular reason to question this claim, handed Kipanga a notebook and a pen. He did not hand him a tape measure, because he assumed, quite reasonably, that a Form Five science student would know to find one. He assumed wrong.

Kipanga entered the empty laboratory, removed his shoes with the theatrical seriousness of a man preparing for surgery, and began walking heel-to-toe from one wall to the opposite wall. He counted each step aloud, tongue pressed between his teeth in concentration. When he reached the far wall, he wrote in the notebook: **LENGTH = 27 FEET**. He then repeated the process across the width, wrote **WIDTH = 14 FEET**, put his shoes back on, and reported to the headmaster with the satisfaction of a job completed to the highest scientific standard.

The measurements were sent to Fundi Baraka, a respected carpenter in Bagamoyo who had been building school furniture for fifteen years and had never received a complaint. Until now.

Three weeks later, a lorry arrived at the school gate carrying twelve brand-new laboratory benches. They were beautiful: sanded smooth, varnished to a warm honey glow, with sturdy legs and perfectly level surfaces. The entire Form Five class gathered to watch the installation. The first bench was carried inside and slid into position along the wall with a satisfying fit. Then the second. Then the third. By the sixth bench, the mood was celebratory. By the ninth, someone noticed a problem. By the eleventh, the problem was undeniable.

The twelfth bench would not fit. There was a gap of nearly fifteen centimetres between the last bench and the far wall on one side, and the benches along the width were packed so tightly that the stools could barely be tucked in.

Fundi Baraka, standing in the doorway with his arms folded, was baffled. He had built the benches exactly as specified: for a room twenty-seven feet long and fourteen feet wide. He had measured twenty-seven of his own feet along the timber for the length pieces, and fourteen for the width. His feet, unfortunately, were a full size and a half smaller than Kipanga's. The length was short. The width was wrong. The furniture, as far as the geometry of the room was concerned, belonged to a laboratory that existed nowhere on Earth.

Kipanga, who had been watching the installation with steadily decreasing confidence, attempted a defence. *"But I measured it! I counted very carefully!"*

This was the scene that greeted **Mr. Akilikubwa** when he walked into the laboratory that Tuesday morning for the first physics lesson of the new term. Twelve gleaming benches. One visible gap. One confused carpenter. One defensive student. And twenty-nine other students trying extremely hard not to laugh.

Mr. Akilikubwa was the kind of teacher who never raised his voice but somehow made every student pay attention, because his lessons had a habit of becoming stories you remembered long after the exam was over. He studied the room for a long moment. He looked at the gap. He looked at Kipanga. He looked at Kipanga's feet. He looked at the carpenter's feet. The sequence of events assembled itself in his mind with the inevitability of a well-constructed proof.

"Kipanga," he said, with the calm of a man who has taught physics for twenty years and has learned that the universe always provides the perfect demonstration exactly when you need it, *"what unit did you use to measure this room?"*

"Feet, sir."

"Whose feet?"

A brief silence settled across the laboratory; the kind of silence that arrives when thirty-six people realise the answer at the same moment and one person still does not.

"My feet," said **Kipanga**. Then, more quietly: *"...obviously."*

Kipute, sitting in the second row with her notebook already open (she was the kind of student who arrived on the first day of term with questions from the Pre-Form Five programme still waiting to be answered), spoke without looking up. *"And the carpenter used his own feet. So you were speaking two different languages and neither of you knew."*

"Exactly." **Mr. Akilikubwa** walked to the front. *"Kipanga counted something. But counting is not measuring. A **measurement** requires two things: a number and a **unit that means exactly the same thing to everyone involved**. Kipanga's feet are a personal invention. They mean something to Kipanga and absolutely nothing to Fundi Baraka. And now the school has beautiful furniture built for a room that does not exist."*

He paused. *"But Kipute has spotted something even deeper. Without a standard unit, we cannot even describe the **error**. We cannot say who was wrong, or by how much, or how to fix it. There is no reference point. A measurement without a standard unit is a private language; it speaks to you alone."*

He turned to face the whole class, and his voice settled into the tone they would come to recognise over the next two years as the sound of something important arriving.

"This is not a lesson about benches. This is a lesson about everything you will study in this book. Every formula you will meet, from Newton's second law to Bernoulli's equation, is only as good as the measurements that feed it. Every number you write must carry a unit. Every unit must belong to an agreed system. And every measurement must be honest about its own uncertainty, because no measurement, no matter how careful, is perfectly exact. This is not a failure of the physicist. It is a property of the physical world itself."

He picked up a metre ruler from one of the new benches (which, despite their dimensional confusion, had perfectly smooth surfaces ideal for resting teaching aids) and held it where the whole class could see.

"Welcome to Chapter 1. Before physics can describe anything, it must first learn how to measure. And that begins right here, in a room that is apparently twenty-seven Kipanga-feet long."

Kipanga sank slightly in his seat. The class laughed. And just like that, physics had begun.

PHYSICAL QUANTITIES: FUNDAMENTAL AND DERIVED

Before a single formula appears, before any equation is written or any graph is plotted, physics must answer one quiet but essential question: *what exactly are we measuring?* The answer to this question is the subject of this subtopic. It is not glamorous. It will not make anyone gasp. But every calculation in the next nine chapters depends on getting it right, so we will take our time and get it right.

What is a Physical Quantity?

Not everything in the world can be measured. Beauty cannot. Happiness cannot. The quality of your mother's cooking cannot (and if you tried, you would be in more trouble than Kipanga). But many things can be measured: the length of a table, the mass of a stone, the time taken for a bus to travel from Dar es Salaam to Dodoma, the temperature of boiling water, the speed of a football after a penalty kick.

A **physical quantity** *is a property of matter or a phenomenon that can be measured and expressed as a number with a unit.* The two parts of this definition are equally important. A physical quantity always has a **numerical value** (*how much*) and a **unit** (*of what*). Without both, the measurement is meaningless.

Consider the statement: "I ran 5." Five what? Five metres? Five kilometres? Five minutes? Without the unit, the number carries no information at all. It is as useless as a key without a lock. Physics insists, without exception, that every measurement must state both the number and the unit. This is not a formality. It is the difference between communication and confusion, as Kipanga and Fundi Baraka discovered with twelve beautiful but useless benches.

Examples of physical quantities include: length, mass, time, temperature, velocity, force, pressure, energy, and electric current. Examples of things that are *not* physical quantities include: beauty, anger, intelligence, bravery, and the likelihood that Kipanga will volunteer for the next task without thinking it through. These cannot be measured on any agreed scale and expressed with a unit.

A common question worth settling now: *is temperature a physical quantity?* Yes, it is. It took humanity centuries to agree on how to measure it (and even today, different countries argue about Celsius versus Fahrenheit), but temperature can be expressed as a number with a unit, measured with a thermometer, and compared from one experiment to another. It qualifies fully.

Fundamental (Base) Physical Quantities

Some physical quantities are independent. They do not need any other quantity to define them. Length is length. Mass is mass. Time is time. You cannot break them down further into simpler physical quantities. These are called **fundamental physical quantities** (also known as **base physical quantities**).

A fundamental physical quantity *is one that cannot be defined in terms of other physical quantities.* It is an irreducible building block of measurement. The internationally agreed system of units recognises exactly seven such quantities:

Table: The Seven SI Base Quantities

No.	Fundamental Quantity	SI Unit	Symbol
1	Length	metre	*m*
2	Mass	kilogram	*kg*
3	Time	second	*s*
4	Electric current	ampere	*A*
5	Temperature	kelvin	*K*
6	Amount of substance	mole	*mol*
7	Luminous intensity	candela	*cd*

For A-level physics, the first five quantities (length, mass, time, electric current, and temperature) are the most frequently used. Amount of substance appears occasionally in thermodynamics and gas laws, while luminous intensity rarely features in the syllabus.

A critical distinction must be understood: “fundamental” refers to the *quantity itself*, not to the unit chosen to measure it. Length is a fundamental quantity whether it is measured in metres, feet, cubits, or Kipanga-feet. The quantity does not change; only the unit does. This distinction will become very important when we reach dimensional analysis later.

Derived Physical Quantities

Most physical quantities in physics are not fundamental. *They are built by combining two or more fundamental quantities through multiplication, division, or both.* These are called **derived physical quantities**.

The idea is simple: start with the building blocks (length, mass, time, ...) and assemble everything else. Watch how the construction works, step by step:

Velocity = length ÷ time

Acceleration = velocity ÷ time = length ÷ time ÷ time = length ÷ time^2

Force = mass × acceleration = mass × length ÷ time^2

Pressure = force ÷ area = mass × length ÷ time^2 ÷ length^2 = mass ÷ (length × time^2)

Energy = force × length = mass × length^2 ÷ time^2

Every derived quantity, no matter how complex, can be traced back to the seven fundamental quantities. The table below summarises the most important derived quantities you will encounter in this book.

Table: Key Derived Quantities in A-Level Physics

Quantity	How it is defined	Named SI Unit	In Base Units
Velocity	length / time	—	ms^{-1}
Acceleration	velocity / time	—	ms^{-2}
Force	mass × acceleration	newton (N)	$kgms^{-2}$
Momentum	mass × velocity	—	$kgms^{-1}$
Pressure	force / area	pascal (Pa)	$kgm^{-1}s^{-2}$
Energy / Work	force × length	joule (J)	kgm^2s^{-2}
Power	energy / time	watt (W)	kgm^2s^{-3}
Torque	force × distance	—	kgm^2s^{-2}
Surface tension	force / length	—	kgs^{-2}
Viscosity	stress / strain rate	—	$kgm^{-1}s^{-1}$

Notice that some derived quantities have special named units (newton, pascal, joule, watt) while others are simply expressed in base units. The named units are shorthand: writing “1 N” is quicker than writing “1 $kgms^{-2}$,” but they mean exactly the same thing. You must be comfortable working in both forms, because dimensional analysis requires the base-unit form.

With the vocabulary now established, let us see whether the ideas hold up when tested. The following examples will check whether the boundary between fundamental and derived is genuinely clear in your mind.

BINDER Example 1

Classify each of the following as a fundamental quantity or a derived quantity. Justify each answer.

(a) Mass (b) Velocity (c) Time (d) Force (e) Temperature (f) Pressure (g) Electric current (h) Energy (i) Acceleration (j) Density

Solution

(a) Mass — Fundamental. It is one of the seven SI base quantities. It cannot be expressed in terms of simpler physical quantities.

(b) Velocity — Derived. Velocity is defined as displacement divided by time. It depends on two fundamental quantities: length and time.

(c) Time — Fundamental. It is one of the seven SI base quantities. It stands on its own.

(d) Force — Derived. Force equals mass times acceleration. Since acceleration itself depends on length and time, force depends on mass, length, and time.

(e) Temperature — Fundamental. It is one of the seven SI base quantities, measured in kelvin.

(f) Pressure — Derived. Pressure equals force divided by area. It depends on mass, length, and time.

(g) Electric current — Fundamental. It is one of the seven SI base quantities, measured in amperes.

(h) Energy — Derived. Energy equals force times displacement. It depends on mass, length, and time.

(i) Acceleration — Derived. Acceleration is the rate of change of velocity, which depends on length and time.

(j) Density — Derived. Density equals mass divided by volume. Since volume depends on length, density depends on mass and length.

Making Sense of the Answer: *The test is simple: can the quantity be broken down into simpler physical quantities? If yes, it is derived. If no, it is fundamental. This test works every time, regardless of how familiar or unfamiliar the quantity seems.*

Think Like a Physicist: *Do not be misled by the fact that a quantity has a named unit. Force is measured in newtons, but it is still a derived quantity because it depends on mass, length, and time. A named unit is a convenience, not a promotion to fundamental status.*

REAL Example 2

During a class discussion on physical quantities, Kipanga raises his hand and says:

Kipanga: *"Sir, I believe love is a physical quantity. When I am in love, my heart beats faster. Heart rate can be measured in beats per minute. Therefore, love has a number and a unit, so it is a physical quantity."*

The class erupts in laughter. But Mr. Akilikubwa does not laugh. He pauses thoughtfully and says:

Mr. Akilikubwa: *"That is a more interesting argument than you realise, Kipanga. Let us examine it carefully."*

Explain, using the definition of a physical quantity, why love does not qualify as a physical quantity, and identify the flaw in Kipanga's reasoning.

Solution

Kipanga's argument confuses the quantity itself with something it *causes*. Love is not the same thing as heart rate. A faster heart rate can also be caused by running, fear, drinking coffee, or seeing a spider. The fact that heart rate increases when a person is in love does not mean that love *is* heart rate.

A physical quantity must be a property that can be measured **directly and consistently** on an agreed scale with an agreed unit. Heart rate is a physical quantity (measured in beats per minute). But love itself cannot be measured directly, cannot be assigned a unique numerical value, and does not have a universally agreed unit. Two people experiencing the same intensity of emotion may have completely different heart rates, and the same person's heart rate changes for many reasons unrelated to love.

Therefore, love is not a physical quantity. Kipanga measured an *effect*, not the thing itself. This is a surprisingly common error in science: confusing a cause with its observable consequences.

Making Sense of the Answer: *Kipanga's reasoning sounds logical on the surface, which is exactly what makes it a valuable teaching moment. Many things in nature produce measurable effects without being measurable themselves. Anger increases blood pressure, but anger is not blood pressure. Fear produces sweat, but fear is not sweat. A physical quantity must be the thing being measured, not merely something that influences a measurable thing.*

Think Like a Physicist: *When someone claims a new physical quantity exists, apply the test rigorously:* ***can it be assigned a numerical value, measured with an instrument, and expressed with an agreed unit?*** *If any part of the test fails, it is not a physical quantity, no matter how real it feels.*

The vocabulary is now in place. Fundamental quantities are the bricks; derived quantities are the buildings constructed from them. Every physical quantity has a number and a unit, and without both, a measurement says nothing.

But knowing that units exist is not enough. The disaster in the laboratory taught us that units must be *standard*: agreed upon by everyone, everywhere, before a measurement can travel from one person to another without losing its meaning. In the next section, we meet the system that solved this problem for the entire world, and learn how to move fluently within it.

THE SI SYSTEM AND UNIT CONVERSION

In the introduction, we watched Kipanga's personal unit of measurement produce benches that belonged to a room that existed nowhere on Earth. The lesson was clear: a measurement can only travel from one person to another if the unit belongs to an **agreed system**. This subtopic introduces that system, explains the prefixes that make it practical for quantities ranging from the size of an atom to the distance between galaxies, and teaches the conversion technique that prevents expensive mistakes.

The International System of Units

Humanity took a surprisingly long time to agree on how to measure things. For most of history, every region used its own units. The cubit in ancient Egypt was the length of the Pharaoh's forearm. The foot in England was based on the King's foot. The problem was obvious: change the king, change the unit. Trade between regions required elaborate conversion tables, and even then, disputes were common. (Kipanga's bench problem was, in a sense, a re-enactment of several thousand years of measurement confusion, compressed into three weeks.)

The solution came in stages. In 1795, France introduced the metric system, based on natural standards rather than the dimensions of any particular ruler's body. In 1960, the international scientific community formalised this into the *Système International d'Unités* (International System of Units), abbreviated **SI**. Today, SI is the standard measurement system used in science, engineering, and commerce in nearly every country on Earth.

SI is built on the seven base quantities and their units that we met earlier. Every other unit in the system is derived from these seven. The strength of SI is that it is **universal:** a kilogram in Dar es Salaam is the same as a kilogram in Tokyo, Moscow, or on the International Space Station. This universality is not a convenience; it is a necessity.

How necessary? Consider the most expensive unit-conversion error in the history of science. In 1999, NASA launched the **Mars Climate Orbiter**, a spacecraft designed to study the Martian atmosphere. The mission cost $327 million. As the spacecraft approached Mars, the navigation team at the Jet Propulsion Laboratory sent course-correction commands based on data received from the spacecraft's builder, Lockheed Martin. There was one problem: Lockheed Martin's software calculated thruster forces in *pound-force seconds* (an imperial unit), while NASA's navigation software expected *newton seconds* (the SI unit). Nobody noticed the mismatch. The spacecraft entered the Martian atmosphere at the wrong angle and was destroyed.

Three hundred and twenty-seven million dollars, years of engineering, and the work of hundreds of scientists, lost because two teams used different units and neither checked. The Mars Climate Orbiter is the most powerful argument ever made for using a single, agreed system of units. That system is SI.

For this entire book, we adopt a simple rule: **always convert all quantities to SI units before substituting into any formula, unless the question explicitly states otherwise.** This single habit will prevent more errors than any other piece of advice in this chapter.

SI Prefixes

Physics deals with quantities that span an extraordinary range. The radius of a hydrogen atom is about 0.000000000053 m. The distance from the Earth to the Sun is about 150,000,000,000 m. Writing these numbers in full is impractical and invites errors. SI solves this problem with **prefixes**: standard multipliers attached to a unit to indicate powers of ten.

Instead of writing 0.000000001 m, we write 1 nm (one nanometre). Instead of writing 1,000,000 W, we write 1 MW (one megawatt). The number stays manageable, and the power of ten is carried by the prefix.

Table: SI Prefixes Most Used in A-Level Physics

Prefix	Symbol	Meaning	Example
tera	T	10^{12}	1 THz = 10^{12} Hz
giga	G	10^{9}	1 GW = 10^{9} W
mega	M	10^{6}	1 MJ = 10^{6} J
kilo	k	10^{3}	1 km = 10^{3} m
centi	c	10^{-2}	1 cm = 10^{-2} m
milli	m	10^{-3}	1 mm = 10^{-3} m
micro	μ	10^{-6}	1 μm = 10^{-6} m
nano	n	10^{-9}	1 nm = 10^{-9} m
pico	p	10^{-12}	1 pF = 10^{-12} F

A critical warning: students consistently confuse micro (μ, 10^{-6}) with milli (m, 10^{-3}). These differ by a factor of 1000. Writing 5 mm when you mean 5 μm means your answer is 1000 times too large. In electronics, capacitances are often given in microfarads (μF) and lengths of wires in millimetres (mm). Mixing them up is one of the most common errors in A-level calculations.

A useful memory aid: arrange the prefixes from largest to smallest as a staircase. Each step down by three means dividing by 1000: kilo (10^{3}) → base unit (10^{0}) → milli (10^{-3}) → micro (10^{-6}) → nano (10^{-9}) → pico (10^{-12}). Every step is exactly a factor of 1000.

Unit Conversion

Unit conversion is the single most practical skill in this chapter. Every worked example in this book begins with converting given values into SI units. A reliable method, used consistently, eliminates conversion errors entirely.

The method is based on a golden rule: **multiply by a conversion factor that equals 1.** Since 1 km = 1000 m, the fraction 1000 m / 1 km equals 1. Multiplying any quantity by this fraction changes the unit without changing the value.

The three-step method:

Step 1: Write the given quantity with its unit.

Step 2: Multiply by one or more conversion factors, arranged so that the unit you want to eliminate appears on the opposite side of the fraction.

Step 3: Cancel units that appear on both top and bottom. The surviving unit is your answer.

For example, converting 72 km/h to m/s:

$$72\,\frac{\text{km}}{\text{h}} \times \frac{1000\text{ m}}{1\text{ km}} \times \frac{1\text{ h}}{3600\text{ s}} = \frac{72 \times 1000}{3600}\frac{\text{m}}{\text{s}} = 20\text{ ms}^{-1}$$

The km cancels with km, the h cancels with h, and only m and s survive.

A critical point that demands special emphasis: area and volume conversions require squaring or cubing the conversion factor. This is where students make some of the most persistent errors in A-level physics.

Since 1 cm = 10^{-2} m, it follows that:

$$1\ \text{cm}^2 = (10^{-2}\ \text{m})^2 = 10^{-4}\ \text{m}^2$$
$$1\ \text{cm}^3 = (10^{-2}\ \text{m})^3 = 10^{-6}\ \text{m}^3$$

Notice: 1 cm² is **not** 10^{-2} m². It is 10^{-4} m². Similarly, 1 cm³ is **not** 10^{-3} m³. It is 10^{-6} m³. The exponent applies to the *entire* conversion factor, not just the number. Forgetting to square or cube is the single most common conversion error in A-level physics, and it appears in every chapter from here to Chapter 10.

Another frequently needed conversion: 1 litre = 1000 cm³ = 10^{-3} m³. This will appear in fluid mechanics, thermodynamics, and in a bill from a water supplier

With the method and the warning firmly in place, let us put the technique to work.

BINDER Example 3

Convert each of the following to SI units. Show full working.

(a) 5 km (b) 3 hours (c) 250 cm² (d) 0.5 litres (e) 72 km/h

Solution

(a)

$$5\ \text{km} = 5 \times 1000\ \text{m} = 5000\ \text{m}$$

(b)

$$3\ \text{h} = 3 \times 3600\ \text{s} = 10\,800\ \text{s}$$

(c)

$$250\ \text{cm}^2 = 250 \times 10^{-4}\ \text{m}^2 = 0.025\ \text{m}^2 = 2.5 \times 10^{-2}\ \text{m}^2$$

(d)

$$0.5\ \text{litres} = 0.5 \times 10^{-3}\ \text{m}^3 = 5 \times 10^{-4}\ \text{m}^3$$

(e)

$$72\ \frac{\text{km}}{\text{h}} = 72 \times \frac{1000\ \text{m}}{3600\ \text{s}} = 20\ \text{ms}^{-1}$$

Making Sense of the Answer: *Part (c) is the one most students get wrong. The temptation is to write 250 × 10^{-2} instead of 250 × 10^{-4}. Always ask: am I converting a length, an area, or a volume? If area, square the factor. If volume, cube it.*

Think Like a Physicist: *The quick conversion 1 km/h = 1/3.6 m/s is useful to memorise. Divide by 3.6 to go from km/h to m/s. Multiply by 3.6 to go back.*

BINDER Example 4

(a) Express 1 year in seconds.

(b) The mass of a proton is 1.67 × 10^{-27} kg. Express this mass in micrograms (μg).

Solution

(a) 1 year = 365 days (ignoring leap years)

$$1\ \text{year} = 365 \times 24 \times 3600\ \text{s} = 31536000\ \text{s} \approx 3.15 \times 10^7\ \text{s}$$

(b) Since 1 μg = 10^{-6} g = $10^{-6} \times 10^{-3}$ kg = 10^{-9} kg:

$$\text{m} = \frac{1.67 \times 10^{-27}\ \text{kg}}{10^{-9}\ \text{kg/μg}} = 1.67 \times 10^{-18}\ \text{μg}$$

Making Sense of the Answer: *A year contains roughly 31.5 million seconds, a number worth remembering for quick estimates. The proton mass in micrograms is an extraordinarily tiny number, which reflects just how small subatomic particles are compared to anything we can see or touch.*

Think Like a Physicist: *When converting between prefixed units, first convert both to the base SI unit, then convert to the target unit. Going directly from one prefix to another (say, from nanometres to micrometres) invites sign errors in the exponent.*

REAL Example 5

Kipanga's family receives their monthly DAWASCO water bill. The bill charges 3,500 Tsh per cubic metre of water used. The household water meter, however, reads in litres. This month, the meter shows that the family used 12,400 litres. Kipanga's mother is furious: she believes the family is being overcharged because "12,400 is a very big number and 3,500 shillings per unit sounds expensive."

Kipanga: *"Mum, I think they are converting litres to cubic metres and that makes the number smaller. Let me calculate."*

Help Kipanga to convert 12,400 litres to cubic metres and hence calculate the total water bill.

Solution

Since 1 litre = 10^{-3} m³:

$$12400 \text{ litres} = 12400 \times 10^{-3} \text{ m}^3 = 12.4 \text{ m}^3$$

Total bill:

$$\text{Bill} = 12.4 \text{ m}^3 \times 3500 \text{ Tsh/m}^3 = 43400 \text{ Tsh}$$

The bill is correct. The family used 12.4 cubic metres of water, not 12,400 cubic metres. The large number in litres becomes a modest number in cubic metres because 1 m³ contains 1000 litres.

Making Sense of the Answer: *This is a real-life example of why unit conversion matters outside the physics classroom. Many billing disputes in Tanzania arise from confusion between litres and cubic metres. A household that uses 12.4 m³ of water per month is typical for a family of five.*

Think Like a Physicist: *Whenever a number looks suspiciously large or small, check the units. A billing system that charges per cubic metre but measures in litres will always show a reading 1000 times larger than the billed volume. The physics is simple. The confusion is common.*

HOT Example 6

A car of mass 1200 kg travels at 90 km/h.

(a) Convert the speed to ms^{-1}.

(b) Calculate the kinetic energy of the car.

(c) Express the kinetic energy in kJ and in MJ.

Solution

(a) Converting the speed:

$$v = 90 \frac{\text{km}}{\text{h}} = \frac{90}{3.6} \text{ ms}^{-1} = 25 \text{ ms}^{-1}$$

(b) Calculating kinetic energy:

Using:

$$\text{KE} = \frac{1}{2}mv^2$$

Substituting:

$$\text{KE} = \frac{1}{2} \times 1200 \text{ kg} \times (25)^2 \text{ m}^2\text{s}^{-2} = 375000 \text{ J}$$

(c) Expressing in kJ and MJ:

$$\text{KE} = \frac{375000 \text{ J}}{10^3 \text{ J/kJ}} = 375 \text{ kJ}$$

$$\text{KE} = \frac{375000\ \text{J}}{10^6\ \text{J/MJ}} = 0.375\ \text{MJ}$$

Making Sense of the Answer: *375 kJ is roughly the energy contained in a small chocolate bar. A car travelling at 90 km/h carries enough kinetic energy to be very dangerous in a collision, which is why speed limits exist. Doubling the speed would quadruple the kinetic energy to 1500 kJ, because kinetic energy depends on the square of velocity.*

Think Like a Physicist: *Notice that the unit conversion was done before substituting into the formula. If you had substituted 90 km/h directly into KE = ½mv², you would have obtained a number in kg·km²/h², which is not joules. Converting first guarantees that the answer comes out in SI units automatically.*

The SI system is now in your hands: seven base units, a clean set of prefixes, and a conversion method that works every time if followed carefully. The area and volume warning has been sounded loudly, and the DAWASCO bill has been settled.

But knowing how to convert units is only part of the story. In the next section, we discover a far more powerful use of units: they can tell you whether a formula is even *possible*. Welcome to dimensional analysis, the detective tool that catches wrong equations before they ever reach a calculator.

DIMENSIONAL ANALYSIS

If the previous subtopic taught us how to convert units, this section teaches us something even more powerful: how to use units to *think*. We are about to meet a tool that can verify whether a formula makes physical sense, derive relationships between quantities without solving a single differential equation, and convert measurements between entirely different unit systems. The tool is called **dimensional analysis**, and it is one of the most elegant techniques in all of physics.

Dimensional analysis *is the method of analysing physical equations and relationships by examining the dimensions (fundamental nature) of the quantities involved*. It is based on the simple but powerful principle that any physically meaningful equation must be consistent in its dimensions.

In this section, we will first establish what dimensions are and how they differ from units. Then we will explore the three uses of dimensional analysis, each with a full worked derivation. Finally, we will state honestly what the method cannot do. By the end, you will have a detective tool that catches wrong equations before they ever reach a calculator.

What are dimensions?

Earlier, we classified physical quantities as fundamental or derived. Then, we learned to express them using SI units. Now we go one level deeper.

Every physical quantity has a **unit** and a **dimension**. These are related but different. The **unit** tells you the *scale* of measurement chosen by humans: metres, feet, nanometres, or light-years for length. The **dimension** tells you the fundamental *nature* of the quantity, independent of any measurement system.

The dimension of a physical quantity *is the expression that shows how the quantity is related to the fundamental (base) quantities.* It describes *what kind of thing* the quantity is, stripped of all numerical values and units.

For example, velocity is defined as displacement divided by time. No matter how you measure it (in m/s, km/h, miles per hour, or any other unit), velocity is always *length per time*. That is its dimension. Change the unit system, and the numerical value changes, but the dimension stays exactly the same.

For A-level physics, we represent the dimensions of the five most important fundamental quantities using standard symbols:

- **M** for mass
- **L** for length
- **T** for time
- **I** for electric current
- **Θ** for temperature

The **dimensional formula** of a quantity *is the expression that shows the powers to which the fundamental dimensions must be raised to represent that quantity.* We write it using **square brackets**. For example, the dimensional formula of velocity is written as:

$$[\text{velocity}] = LT^{-1}$$

This tells us that velocity has the dimension of length (to the power 1) divided by time (to the power 1). The negative exponent indicates division.

How to find the dimensional formula of any derived quantity

The method is always the same: start from the *definition* of the quantity, replace each quantity with its fundamental dimensions, and simplify. Let us walk through several examples to build fluency.

Force is defined as mass × acceleration. Acceleration is velocity ÷ time. Velocity is length ÷ time. So:

$$\text{Force} = \text{mass} \times \text{acceleration} = \text{mass} \times \frac{\text{velocity}}{\text{time}} = \text{mass} \times \frac{\text{length}}{\text{time}^2}$$

$$[\text{force}] = MLT^{-2}$$

Pressure is defined as force per unit area. Force has dimensions MLT^{-2}, and area has dimensions L^2. So:

$$[\text{pressure}] = \frac{MLT^{-2}}{L^2} = ML^{-1}T^{-2}$$

Energy (or work) is defined as force × displacement:

$$[\text{energy}] = MLT^{-2} \times L = ML^2T^{-2}$$

Notice the pattern: every derived quantity, no matter how complex, reduces to a combination of M, L, and T (with I and Θ appearing in electrical and thermal quantities). This reduction is the foundation on which all of dimensional analysis is built.

One more point before we move on. Some quantities are **dimensionless**: they have no dimensions at all. Pure numbers like 2, π, and 1/2 are dimensionless. Other examples are angles (measured in radians), strain (change in length divided by original length), and refractive index. Their dimensional formula is written as:

$$[\text{dimensionless quantity}] = M^0L^0T^0 = 1$$

Dimensionless quantities will play a quiet but important role in what follows, especially when we discuss the limitations of dimensional analysis.

Uses of Dimensional Analysis

Now that we can speak the language of dimensions, it is time to explore what this language can do through four applications of dimensional analysis. We begin with the most direct.

1) Finding the SI unit of a physical quantity

The simplest and most immediate application of dimensional analysis is this: once you know the dimensional formula of a quantity, you can write down its SI unit directly by replacing each dimension symbol with its corresponding SI base unit.

The rule is straightforward:

Replace M with kg, L with m, T with s, I with A, and Θ with K.

For example, we showed earlier that the dimensional formula of force is MLT^{-2}. Replacing the symbols:

SI unit of force = $\text{kg} \cdot \text{m} \cdot \text{s}^{-2} = \text{kgms}^{-2}$

This is exactly the newton (N), expressed in base units. The named unit is a shorthand; the base-unit form is what dimensional analysis produces.

Let us apply this to several quantities:

The dimensional formula of pressure is $ML^{-1}T^{-2}$. Therefore:

SI unit of pressure = $\text{kg} \cdot \text{m}^{-1} \cdot \text{s}^{-2} = \text{kgm}^{-1}\text{s}^{-2}$

This is the pascal (Pa).

The dimensional formula of the coefficient of viscosity is $ML^{-1}T^{-1}$. Therefore:

SI unit of viscosity = $\text{kg} \cdot \text{m}^{-1} \cdot \text{s}^{-1} = \text{kgm}^{-1}\text{s}^{-1}$

This is the pascal-second (Pas), the unit that will appear when we convert from poise in fourth use.

The dimensional formula of surface tension is MT^{-2}. Therefore:

SI unit of surface tension = $kg \cdot s^{-2} = kgs^{-2}$

This is equivalent to N/m (newtons per metre), confirming what we will meet in Chapter 10.

This technique is especially useful when you encounter an unfamiliar quantity in an examination. If the question gives you a formula involving the quantity, you can derive its dimensional formula, then immediately write down its SI unit without memorising it. The dimensional formula is the map; the SI unit is the destination

2) Checking the correctness of a formula

This is the simplest and most frequently used application of dimensional analysis. It is based on a principle so fundamental that it deserves to be stated clearly and remembered permanently.

The principle of dimensional homogeneity states that *in any physically valid equation, **every term must have the same dimensions***. You cannot add metres to seconds, just as you cannot add apples to lorries. If the dimensions of the left side of an equation do not match the dimensions of the right side, the equation is certainly wrong.

The method is straightforward:

Step 1: Write down the equation to be checked.

Step 2: Find the dimensional formula of every term on both sides of the equation.

Step 3: Compare the dimensions of all terms. If they are all identical, the equation is dimensionally homogeneous and **could** be correct. If any term differs, the equation is definitely wrong.

Let us apply this to a well-known kinematic equation. Consider:

$$v^2 = u^2 + 2as$$

where v and u are velocities, a is acceleration, and s is displacement.

Left side: $[v^2] = (LT^{-1})^2 = L^2T^{-2}$

Right side, first term: $[u^2] = L^2T^{-2}$

Right side, second term (2 is dimensionless): $[2as] = (LT^{-2}) \times L = L^2T^{-2}$

All three terms have dimensions L^2T^{-2}. The equation is dimensionally homogeneous.

An important and honest warning: dimensional correctness does **not** guarantee physical correctness. A formula can have the right dimensions but incorrect due to the following reasons:

- the wrong numerical coefficient,
- a missing term, or
- be applied in the wrong context.

For instance, $v^2 = u^2 + 3as$ is also dimensionally homogeneous, but the coefficient 3 is wrong. Dimensional analysis catches equations that are **definitely wrong**; but it cannot confirm that an equation is definitely right.

3) Deriving relationships between physical quantities

This is the most powerful and most impressive application of dimensional analysis. Suppose you know that a certain physical quantity depends on several other quantities, but you do not know the exact formula connecting them. Dimensional analysis can determine the **form** of that formula, up to a dimensionless constant, without solving any differential equation or conducting any experiment.

The method:

Step 1: Identify the quantity you want to find and the quantities it depends on.

Step 2: Write the assumed relationship as a product of the dependent quantities raised to unknown powers:

$$Q = kR^aS^bU^c$$

where k is a dimensionless constant that cannot be found by this method.

Step 3: Replace every quantity with its dimensional formula.

Step 4: Equate the powers of each fundamental dimension (M, L, T) on both sides. This gives simultaneous equations.

Step 5: Solve the simultaneous equations for the unknown powers.

Step 6: Substitute back to obtain the formula.

Classic derivation: the period of a simple pendulum

Suppose we want to find the period T of a simple pendulum. Physical reasoning suggests that T might depend on the length l of the string and the acceleration due to gravity g. (We exclude the mass of the bob for now and will justify this exclusion at the end.)

Step 1: The quantity to find is T. It depends on l and g.

Step 2: Assume:

$$T = kl^a g^b$$

Step 3: The dimensions of each quantity are:

$$[T] = T, \quad [l] = L, \quad [g] = LT^{-2}$$

So the dimensional equation becomes:

$$M^0L^0T^1 = L^a(LT^{-2})^b$$

$$M^0L^0T^1 = M^0L^{(a+b)}T^{-2b}$$

Step 4: Equating powers of each dimension:

For M: 0 = 0 (automatically satisfied)

For L: 0 = a + b ... (i)

For T: 1 = –2b ... (ii)

Step 5: From equation (ii): $b = -\frac{1}{2}$. Substituting into equation (i): $a = \frac{1}{2}$

Step 6: Substituting back:

$$T = kl^{1/2}g^{-1/2} = k\sqrt{\frac{l}{g}}$$

This is the result of dimensional analysis. It tells us the **form** of the formula. A full derivation from Newton's second law (or a careful experiment) reveals that $k = 2\pi$, giving:

$$T = 2\pi\sqrt{\frac{l}{g}}$$

Why was mass excluded? Because the period T has dimension T (time only), and l and g together can produce T without needing M. If we had included the mass m with an unknown power c, the equation for M would give 0 = c, confirming that mass does not affect the period. Dimensional analysis does not just accept our assumption that mass is irrelevant; it *proves* it.

4) Converting units between systems

This is the most practical application of dimensional analysis in the laboratory. While SI is the standard for modern physics, many older reference tables, engineering handbooks, and published data still use the CGS system (centimetre-gram-second). A physicist must be able to convert numerical values between systems quickly and reliably.

The method is based on a simple idea: the **physical quantity itself does not change** when you change the unit system. Only its numerical value and its units change. A length of 5 metres is the same as 500 centimetres. The length has not changed; the number has changed because the unit changed.

Suppose a physical quantity has dimensional formula $M^aL^bT^c$. Let its numerical value be n_1 in System 1 (where the base units are M_1, L_1, T_1) and n_2 in System 2 (where the base units are M_2, L_2, T_2). Since the physical quantity is the same in both systems:

$$n_1 M_1{}^a L_1{}^b T_1{}^c = n_2 M_2{}^a L_2{}^b T_2{}^c$$

Rearranging for the numerical value in System 2:

$$\mathbf{n_2 = n_1 \left(\frac{M_1}{M_2}\right)^a \left(\frac{L_1}{L_2}\right)^b \left(\frac{T_1}{T_2}\right)^c}$$

This is the general conversion formula. The ratios of base units are raised to the powers given by the dimensional formula. Once the dimensions are known, the conversion is entirely mechanical.

Example within the text: converting the coefficient of viscosity

In the CGS system, the unit of viscosity is the **poise** (P), defined as 1 $gcm^{-1}s^{-1}$. We want to find how many SI units ($kgm^{-1}s^{-1}$) one poise equals.

The dimensional formula of viscosity is $ML^{-1}T^{-1}$, so a = 1, b = –1, c = –1.

The base units in each system are:

CGS: $M_1 = 1$ g, $L_1 = 1$ cm, $T_1 = 1$ s

SI: $M_2 = 1$ kg, $L_2 = 1$ m, $T_2 = 1$ s

Substituting into the conversion formula:

$$n_2 = 1 \times \left(\frac{1\text{ g}}{1\text{ kg}}\right)^1 \left(\frac{1\text{ cm}}{1\text{ m}}\right)^{-1} \left(\frac{1\text{ s}}{1\text{ s}}\right)^{-1}$$

$$= 1 \times (10^{-3})^1 \times (10^{-2})^{-1} \times 1$$

$$= 10^{-3} \times 10^2 = 10^{-1} = 0.1$$

Therefore, 1 poise $= 0.1\ kgm^{-1}s^{-1} = 0.1$ Pas (pascal-second).

This result is essential for Chapter 10 (Fluid Mechanics), where viscosity data from older sources is often given in poise or centipoise (1 cP $= 10^{-3}$ Pas). With dimensional analysis, the conversion is systematic and error-free.

Limitations of Dimensional Analysis

Dimensional analysis is powerful, but it has clear boundaries. A responsible physicist uses the tool confidently while knowing exactly where it stops working. There are five important limitations:

1) It cannot determine dimensionless constants

The pendulum derivation gave $T = k\sqrt{\frac{l}{g}}$ but could not find $k = 2\pi$. The constant 2π has no dimensions, so it is invisible to dimensional analysis. Similarly, the kinetic energy formula $KE = \frac{1}{2}mv^2$ cannot be distinguished from $KE = mv^2$ or $KE = 3mv^2$ by dimensions alone. The factor of $\frac{1}{2}$ must come from a full derivation or from experiment.

2) It cannot distinguish between quantities with the same dimensions

Work and torque both have dimensions ML^2T^{-2}, yet they are physically different quantities. Speed and velocity both have dimensions LT^{-1}, yet one is a scalar and the other is a vector. Dimensional analysis treats them as identical because it sees only dimensions, not physical meaning.

3) It cannot handle trigonometric, logarithmic, or exponential functions

The arguments of sin, cos, log, and exponential functions must all be dimensionless. Dimensional analysis cannot discover that a formula involves $\sin(\omega t)$ or $e^{-\lambda t}$ because it has no way of detecting these functions. A formula like $x = A\sin(\omega t + \varphi)$ cannot be derived by dimensional analysis; only the amplitude dependence (A) can be found.

4) It does not guarantee uniqueness

Two wrong formulas can both be dimensionally correct. Dimensional analysis narrows the possibilities but does not always eliminate all incorrect options. A formula that passes the dimensional check is plausible, not proven.

5) It fails when there are too many unknowns

The method produces one equation for each fundamental dimension used (typically three: M, L, T). If the assumed relationship involves more unknown powers than available equations, the system is underdetermined and cannot be solved uniquely.

Despite these five limitations, dimensional analysis remains one of the most elegant and frequently used tools in physics. It catches errors cheaply, guides intuition reliably, and impresses examiners consistently. The honest physicist uses it as a *first check*, not the *final word*.

The theory is now complete: four uses, five limitations, each clearly defined. Let us put the detective to work with a set of worked examples that will build your fluency and confidence.

BINDER Example 7

Derive the dimensional formula for each of the following quantities:

(a) Momentum (b) Work

Solution

(a) Momentum = mass × velocity

$$[\text{momentum}] = \text{M} \times \text{LT}^{-1} = \text{MLT}^{-1}$$

(b) Work = force × displacement

$$[\text{work}] = (\text{ML}^1\text{T}^{-2}) \times \text{L} = \text{ML}^2\text{T}^{-2}$$

Making Sense of the Answer: *Each quantity has its own unique dimensional fingerprint. Momentum (*MLT^{-1}*) and force (*MLT^{-2}*) differ only in the power of T, which makes physical sense: force is the rate of change of momentum, so it has one extra factor of* T^{-1}.

Think Like a Physicist: *Always start from the definition of the quantity, not from a formula you have memorised. Definitions are more reliable starting points because they express exactly what the quantity means physically.*

BINDER Example 8

Check whether the equation $s = ut + ½at^2$ is dimensionally homogeneous. Show full working for every term.

Solution

Left side: $[\text{s}] = \text{L}$

First term on right: $[\text{ut}] = \text{LT}^{-1} \times \text{T} = \text{L}$

Second term on right (½ is dimensionless):

$$[½\,\text{at}^2] = \text{LT}^{-2} \times \text{T}^2 = \text{L}$$

All three terms have dimension L. The equation is dimensionally homogeneous.

Making Sense of the Answer: *Notice how the time dimensions cancel perfectly in each term. In ut, one* T^{-1} *from velocity cancels with T from time. In* $½at^2$*, the* T^{-2} *from acceleration cancels with* T^2 *from time squared. These cancellations are not coincidence; they are required by the physics. Any term where the cancellation fails would signal a wrong formula.*

Think Like a Physicist: *If any single term has different dimensions from the others, the equation is guaranteed wrong. This makes dimensional checking one of the quickest error-detection tools in physics.*

BINDER Example 9

The centripetal force F acting on a body moving in a circle is believed to depend on the mass m of the body, its speed v, and the radius r of the circular path. Use dimensional analysis to derive an expression for F.

Solution

Assume: $F = km^a v^b r^c$, where k is a dimensionless constant.

Dimensions of each quantity:

$$[F] = MLT^{-2}$$

$$[m] = M$$

$$[v] = LT^{-1}$$

$$[r] = L$$

Writing the dimensional equation:

$$MLT^{-2} = M^a(LT^{-1})^b L^c = M^a L^{b+c} T^{-b}$$

Equating powers:

For M: $1 = a$

For T: $-2 = -b \Rightarrow b = 2$

For L: $1 = b + c \Rightarrow c = 1 - 2 = -1$

Therefore:

$$F = km^1 v^2 r^{-1} = k\frac{mv^2}{r}$$

Making Sense of the Answer: *The formula tells us that centripetal force increases with the square of speed (doubling the speed quadruples the required force) and decreases with radius (a tighter curve demands more force).*

Think Like a Physicist: *This is another case where dimensional analysis gives the complete formula with k = 1. Notice that three unknowns (a, b, c) required exactly three equations (from M, L, T). When the number of unknowns matches the number of equations, the method works perfectly.*

REAL Example 10

After watching Mr. Akilikubwa derive the pendulum formula using dimensional analysis, Kipute raises her hand.

Kipute: *"Sir, if dimensional analysis can derive $T = k\sqrt{\frac{l}{g}}$ without solving any differential equation, why do we still need experiments at all? Why not derive every formula this way?"*

Mr. Akilikubwa: *"That is exactly the question every student should ask after seeing this technique for the first time. The answer has three parts."*

Explain, using specific examples, why dimensional analysis alone is not sufficient to replace experiments and full theoretical derivations.

Solution

First, dimensional analysis cannot determine dimensionless constants. It gave $T = k\sqrt{\frac{l}{g}}$ but not $k = 2\pi$. The constant 2π has no dimensions and is therefore invisible to the method. Only a full derivation from Newton's second law or a careful experiment can supply it.

Second, dimensional analysis requires knowing in advance which variables the quantity depends on. For the pendulum, we assumed T depends only on l and g. But how did we know to exclude mass, air resistance, or the angle of swing? That knowledge came from experiment or physical reasoning external to dimensional analysis. If we had omitted a relevant variable, the method would have produced the wrong formula without warning.

Third, dimensional analysis assumes the relationship is a simple power law. Many physical relationships involve trigonometric functions, exponentials, or logarithms. The period of a pendulum at large angles involves a complex infinite series that dimensional analysis cannot predict.

Making Sense of the Answer: *Dimensional analysis is a powerful first step, not the final word. It narrows the possibilities enormously, sometimes to a single formula up to a constant. But it always needs either experiment or theory to finish the job.*

Think Like a Physicist: *The greatest value of dimensional analysis is not replacing experiments but guiding them. If you know the formula must be T $T = k\sqrt{\frac{l}{g}}$, you know exactly what to measure and what to plot. Dimensional analysis tells you where to look; experiment tells you what you find.*

HOT Example 11

The speed v of a transverse wave on a stretched string depends on the tension T (force) and the linear mass density μ (mass per unit length). Use dimensional analysis to derive an expression for v. Verify using the known formula $v = \sqrt{\frac{T}{\mu}}$.

Solution

Assume: $v = kT^a\mu^b$

Dimensions:

$[v] = LT^{-1}$, $[T] = ML^1T^{-2}$ (force), $[\mu] = ML^{-1}$ (mass per length)

Dimensional equation:

$$LT^{-1} = (ML^1T^{-2})^a(ML^{-1})^b = M^{(a+b)}L^{(a-b)}T^{-2a}$$

Equating powers:

For M: 0 = a + b

For L: 1 = a – b

For T: –1 = –2a, so a = 1/2

From a + b = 0: b = –1/2

Therefore:

$$v = k\sqrt{\frac{T}{\mu}}$$

The known formula is $v = \sqrt{\frac{T}{\mu}}$, confirming k = 1.

Making Sense of the Answer: *Greater tension means a faster wave. Greater linear density means a slower wave. Both match intuition. In this rare case, the dimensionless constant is exactly 1.*

Think Like a Physicist: *This formula appears later when studying waves in strings and columns. Knowing its origin from dimensional analysis makes it easier to remember and harder to misapply.*

HOT Example 12

A student claims that the formula for kinetic energy is $E = mv^3$.

(a) Use dimensional analysis to show that this formula is wrong.

(b) Determine the correct power of v that kinetic energy must contain.

(c) State clearly what dimensional analysis alone cannot tell you about the formula for kinetic energy.

Solution

(a) Checking the student's claim:

Left side: $[E] = ML^2T^{-2}$

Right side: $[mv^3] = M(LT^{-1})^3 = ML^3T^{-3}$

Since $ML^2T^{-2} \neq ML^3T^{-3}$, the formula $E = mv^3$ is dimensionally inconsistent and therefore definitely wrong.

(b) Let $E = mv^n$ where n is unknown:

$$ML^2T^{-2} = M(LT^{-1})^n = ML^nT^{-n}$$

Equating powers of L: 2 = n. Equating powers of T: –2 = –n, so n = 2.

Kinetic energy depends on v^2, not v^3.

(c) Dimensional analysis tells us $E = kmv^2$ for some dimensionless constant k. It cannot determine that $k = \frac{1}{2}$. The constant must come from the work-energy theorem or experiment.

Making Sense of the Answer: *The student's formula fails on two counts: the power of L is wrong (3 instead of 2) and the power of T is wrong (–3 instead of –2). Either alone is enough to reject it.*

Think Like a Physicist: *This example shows both the power and the limitation of dimensional analysis in one problem. It proves a formula wrong, finds the correct power, but cannot find the coefficient. The most efficient first check in physics.*

HOT Example 13

The gravitational constant G appears in Newton's law of universal gravitation, $F = \frac{Gm_1m_2}{r^2}$.

(a) Show that the dimensional formula of G is $M^{-1}L^3T^{-2}$.

(b) In the CGS system, the value of G is $6.67 \times 10^{-8}\ g^{-1}cm^3s^{-2}$. Use the method of dimensions to convert this value to SI units.

(c) Express the SI unit of G in terms of newtons.

Solution

(a) From Newton's law of gravitation:

$$G = \frac{Fr^2}{m_1m_2}$$

Writing dimensions:

$$[G] = \frac{[F][r^2]}{[m_1][m_2]} = \frac{MLT^{-2} \times L^2}{M \times M} = \frac{ML^3T^{-2}}{M^2} = M^{-1}L^3T^{-2}$$

Hence:

$$[G] = M^{-1}L^3T^{-2}$$

(b) In CGS: $G = 6.67 \times 10^{-8}\ g^{-1}cm^3s^{-2}$

The dimensions of G are $M^{-1}L^3T^{-2}$, so the unit of G transforms as:

$$G_{SI} = G_{CGS} \times \left(\frac{M_{CGS}}{M_{SI}}\right)^{-1} \times \left(\frac{L_{CGS}}{L_{SI}}\right)^{3} \times \left(\frac{T_{CGS}}{T_{SI}}\right)^{-2}$$

The conversion factors are: $1g = 10^{-3}kg$, $1cm = 10^{-2}m$, $1s = 1s$.

$$G_{SI} = 6.67 \times 10^{-8} \times \left(\frac{1g}{1kg}\right)^{-1} \times \left(\frac{1cm}{1m}\right)^{3} \times \left(\frac{1s}{1s}\right)^{-2}$$

$$G_{SI} = 6.67 \times 10^{-8} \times (10^{-3})^{-1} \times (10^{-2})^3 \times (1)^{-2}$$

$$G_{SI} = 6.67 \times 10^{-11}\ kg^{-1}m^3s^{-2}$$

Therefore, $G = 6.67 \times 10^{-11}\ kg^{-1}m^3s^{-2}$ in SI units.

(c) The SI unit $kg^{-1}m^3s^{-2}$ can be rewritten using the definition of the newton: $1N = 1kgms^{-2}$.

$$kg^{-1}m^3s^{-2} = \frac{m^3}{kgs^2} = \frac{m^3}{kgs^2} \times \frac{kg}{kg} = \frac{kgm^3}{kg^2s^2} = \frac{kgms^{-2} \times m^2}{kg^2} = \frac{Nm^2}{kg^2}$$

Therefore, the SI unit of G is $\mathrm{Nm^2kg^{-2}}$.

Making Sense of the Answer: *The numerical value of G changed from 6.67×10^{-8} (CGS) to 6.67×10^{-11} (SI), a factor of 10^{-3} smaller. This makes sense: the SI unit of mass (kg) is 10^3 times larger than the gram, and since G has M^{-1} in its dimensions, switching to a larger mass unit decreases the number. The L^3 factor contributes 10^{-6} (since $1cm = 10^{-2}m$ and the cube gives 10^{-6}), and combined with 10^3 from the mass, the net factor is 10^{-3}. As a quick cross-check: the well-known SI value of G is indeed $6.67 \times 10^{-11} Nm^2kg^{-2}$, confirming the conversion is correct.*

Think Like a Physicist: *When converting between unit systems, always start by writing the dimensional formula. Each base dimension (M, L, T) carries its own conversion factor raised to the appropriate power. This systematic approach prevents sign errors in the exponents, which is where most students make mistakes. Notice that a negative exponent in the dimensions (like M^{-1}) flips the conversion factor: switching to a larger unit of mass makes the number smaller, not larger.*

Dimensional analysis is now in your toolkit: four uses and five limitations, each understood through worked examples rather than memorised from a list. You can check formulas, derive relationships, and convert between systems. You also know, honestly, where the method stops and experiment must begin.

The next section takes us from the structure of equations to the honesty of numbers. Every measurement carries uncertainty, and pretending otherwise is not physics. Welcome to errors and uncertainties, where we learn to say what we know and admit what we do not.

ERRORS AND UNCERTAINTIES

Suppose Kipanga had used a proper metre ruler instead of his feet to measure the laboratory. Would his measurement have been perfect? Not a chance. The ruler itself has markings only to the nearest millimetre. His eye would have judged the position of the wall slightly differently each time he looked. The temperature in the room would have caused the ruler (and the wall) to expand or contract by tiny amounts. Even the most careful measurement, made with the best instrument, by the most skilled physicist, carries some degree of uncertainty.

This is not a failure. It is a fact of nature. And a physicist who ignores it is more dangerous than one who makes a large error, because at least the second physicist knows something went wrong. This section teaches you to measure honestly: to state what you know, to quantify what you do not, and to propagate uncertainty through calculations so that your final answer tells the truth about its own reliability.

Why Every Measurement Has Uncertainty

An **uncertainty** (also called an **error** in the technical sense) *is an **unavoidable** deviation of a measured value from the true value of the quantity being measured.* It arises not from carelessness but from the fundamental nature of the measurement process itself.

Three terms appear constantly in this section, and students confuse them so frequently that we must distinguish them right at the start:

An **error** (in the physics sense) is an unavoidable deviation from the true value, caused by the *limitations of the instrument* or *the measurement process.* It is present in every measurement, no matter how carefully performed. When a physicist says "the measurement has an error of 0.2 mm," they are not confessing incompetence. They are reporting, with professional honesty, the range within which the true value is believed to lie.

A **mistake** *is simply doing something wrong: misreading a scale, recording the wrong number,* writing 2.54 when the scale clearly shows 2.45, or *forgetting to subtract the zero reading.* Mistakes are avoidable. They can and should be eliminated by *careful technique, double-checking readings, and attentive record-keeping.* They are not errors in the physics sense.

A **blunder** (sometimes called a **gross error** in older textbooks, though the name is misleading since it is avoidable) *is a severe mistake that produces a reading wildly different from all the others in a set.* For example, if five measurements of a pendulum period give 2.04, 2.06, 2.08, 2.05, and 3.71 seconds, the last value is almost certainly a blunder, perhaps caused by miscounting oscillations or starting the stopwatch at the wrong moment. *Blunders are identified by inspection: if one reading is dramatically different from the rest, **it is discarded from the dataset before calculating the mean.*** A blunder is not a type of experimental error; it is a human failure that should not contaminate the analysis.

With these three terms clearly separated, we can now focus entirely on genuine errors, the unavoidable uncertainties that remain even when every mistake and blunder has been eliminated.

Uncertainty arises from two fundamental sources:

1) **The instrument itself:** every measuring device has a smallest division, called its least count or resolution. A ruler marked in millimetres cannot reliably distinguish between 12.3 mm and 12.4 mm. A stopwatch that reads to 0.01 s cannot detect events shorter than that interval. The resolution sets a floor below which the instrument simply cannot see.
2) **The measurement process:** human judgement, environmental fluctuations (temperature, vibration, air currents), timing reflexes, and parallax all introduce variability that no instrument can remove entirely.

Types of Errors

Not all errors behave the same way. Understanding the difference between the two types is essential, because the strategy for dealing with each is completely different.

1. Systematic errors

A systematic error *is an error that shifts all measurements in the same direction (always too high or always too low) by the same amount or by the same proportion*. It is consistent and repeatable, which is precisely what makes it dangerous: the data looks reliable even though every value is wrong.

Common sources of systematic errors include:

Zero error: The instrument does not read zero when it should. A ruler whose end is worn down always gives readings that are slightly too short. A spring balance that reads 0.3 N when nothing is hanging from it adds 0.3 N to every subsequent measurement. A voltmeter that shows 0.02 V with no circuit connected shifts every voltage reading upward by that amount.

Calibration error: The instrument's scale is uniformly wrong. A thermometer that reads 102°C in steam at standard atmospheric pressure will give every subsequent temperature reading 2°C too high. The readings will be perfectly consistent with each other, perfectly precise, and all equally wrong.

Parallax error: Reading a scale from an angle rather than from directly in front. When the eye is not perpendicular to the scale, the apparent position of the pointer shifts because of the gap between the pointer and the scale markings. If you consistently read a mercury thermometer from slightly above, your eye sees the mercury level against a higher mark than the true reading. The error is always in the same direction.

To understand why parallax occurs, consider a pointer suspended a small distance above a ruler. Viewed from directly in front, the pointer appears to line up with the correct mark. Viewed from an angle, the pointer appears to align with a different mark because your line of sight passes through the pointer at an angle and hits the scale at the wrong position. The cure is simple: always *position your eye so that your line of sight is perpendicular to the scale.*

Reaction time: When timing with a stopwatch, the delay between seeing an event and pressing the button is roughly the same every time (typically 0.1 to 0.3 seconds for a human). This adds a consistent bias to every timing measurement. In a **pendulum experiment**, the error *appears twice, once when starting and once when stopping the watch, and the two delays may partially cancel or add depending on the situation.*

The effect on data: systematic errors shift the entire dataset in one direction. The readings may cluster tightly together (appearing precise), but their average is displaced from the true value. The data looks good but is inaccurate.

How to minimise: calibrate instruments before use, check for zero error and correct for it, use proper technique (eye level for meniscus readings, perpendicular viewing for scales), use instruments with higher resolution, and compare results with known standard values where possible. The most important point: **repetition does not reduce systematic errors.** If the ruler is 2 mm too short, measuring a hundred times gives a hundred wrong readings that all agree with each other.

2. Random errors

A random error is an error that varies unpredictably in both magnitude and direction from one measurement to the next. Sometimes the reading is slightly too high, sometimes slightly too low, with no consistent pattern.

Common sources of random errors include:

Environmental fluctuations: temperature drafts, vibrations from traffic or nearby machinery, air currents that deflect a pendulum bob slightly differently on each swing, and voltage fluctuations in the power supply during electrical measurements.

Human inconsistency: slight variations in how you align your eye with the scale, how you judge the exact moment a pendulum passes its rest position, or how firmly you close the jaws of a vernier calliper around an object.

Genuine variability: the quantity itself may fluctuate. Blood pressure changes slightly between heartbeats. The diameter of a wire varies slightly along its length. The period of a real pendulum is affected by tiny, unpredictable air disturbances.

The effect on data: random errors scatter readings above and below the true value. Individual readings are unreliable on their own, but the **mean of many readings** tends toward the true value because the positive and negative deviations tend to cancel.

How to minimise: repeat the measurement many times and calculate the mean. The more repetitions, the more the random errors cancel, and the closer the mean approaches the true value. Timing twenty oscillations of a pendulum instead of one effectively divides the random timing error by twenty.

The Distinction That Must Be Owned Permanently

This single distinction explains why a nurse takes a patient's blood pressure three times (to reduce random error) but also checks the cuff size before starting (to avoid systematic error from using a cuff that is too small or too large). The two strategies are completely different because the two problems are completely different:

Systematic errors **cannot** be reduced by repeating measurements. Every repetition carries the same shift. The cure is recalibration, better technique, or a better instrument.

Random errors **can** be reduced by repeating measurements. Each repetition adds information. The mean of ten readings is more reliable than a single reading, and the mean of a hundred is more reliable still.

A summary table may help anchor this permanently:

Feature	Systematic Error	Random Error
Direction	Always the same	Varies unpredictably
Effect on readings	Shifts all readings in one direction	Scatters readings above and below
Effect on mean	Displaces the mean from the true value	Mean approaches true value with more readings
Reduced by repetition?	No	Yes
Reduced by…?	Recalibration, better technique	More repetitions, averaging
Affects…	Accuracy	Precision

Expressing Errors: Absolute, Relative, and Percentage

Once we accept that every measurement has uncertainty, we need a language to express it precisely. Three quantities do this job, and each tells you something different.

Absolute error

Suppose we measure a quantity a several times and obtain values $a_1, a_2, a_3, \dots, a_n$. The best estimate of the true value is the arithmetic mean:

$$\bar{a} = \frac{a_1 + a_2 + a_3 + \cdots + a_n}{n}$$

The absolute error of any single measurement a_i is the magnitude of its deviation from the mean:

$$\Delta a_i = |a_i - \bar{a}|$$

The mean absolute error is the average of all the individual absolute errors:

$$\Delta\bar{a} = \frac{|\Delta a_1| + |\Delta a_2| + \cdots + |\Delta a_n|}{n}$$

Using the mean as the best estimate and the mean absolute error as the uncertainty determined from the deviations above, the result is expressed as:

$$\mathbf{a = \bar{a} \pm \Delta\bar{a}}$$

This notation communicates two things at once: the best estimate ($\bar{a}$) and the range within which the true value is believed to lie (from $\bar{a} - \Delta\bar{a}$ to $\bar{a} + \Delta\bar{a}$). It is the standard way physicists report measurements.

Relative error and percentage error

Absolute error tells you the size of the uncertainty, but not how significant it is. Consider: a 1 mm error measuring a 2 mm wire diameter is catastrophic (50% of the measured value). The same 1 mm error measuring the 2 km Nyerere Bridge in Dar es Salaam is completely irrelevant (0.00005% of the measured value). The absolute error is identical; the significance is vastly different.

The relative error (also called fractional error) captures this significance:

$$\text{Relative error} = \frac{\Delta\bar{a}}{\bar{a}}$$

It is dimensionless and allows fair comparison between measurements of completely different quantities. A relative error of 0.01 in a length measurement and 0.01 in a mass measurement mean both measurements are equally reliable in proportional terms, even though their absolute errors have different units and magnitudes.

The percentage error is the relative error expressed as a percentage:

$$\text{Percentage error} = \frac{\Delta\bar{a}}{\bar{a}} \times 100\%$$

Errors in Derived Physical Quantities

In physics, we rarely measure the final quantity directly. We measure several quantities and combine them using a formula. For example, we find density by measuring mass and volume separately, then dividing. If each input measurement carries uncertainty, the calculated result also carries uncertainty. The question is: *how much?*

Three rules answer this question. Each rule is derived from first principles below, not merely stated, so that you understand why each rule takes the form it does.

Rule 1: Addition and Subtraction (*absolute errors add*)

If $x = a + b$ or $x = a - b$, then:

$$\Delta x = \Delta a + \Delta b$$

Derivation for addition: If the measured values are $(a \pm \Delta a)$ and $(b \pm \Delta b)$, then in the worst case both errors act in the same direction:

$$x + \Delta x = (a + \Delta a) + (b + \Delta b) = (a + b) + (\Delta a + \Delta b)$$

Since $x = a + b$, the maximum error is $\Delta x = \Delta a + \Delta b$.

Derivation for subtraction: This is the case students find surprising. If $x = a - b$, the worst case occurs when the errors push a and b in opposite directions, making the difference as large (or as small) as possible:

$$x + \Delta x = (a + \Delta a) - (b - \Delta b) = (a - b) + (\Delta a + \Delta b)$$

Since $x = a - b$, the maximum error is again $\Delta x = \Delta a + \Delta b$.

Notice the crucial point: even though the quantities are subtracted, the absolute errors still add. This is because the worst case for a difference is when a is too high and b is too low simultaneously (or the reverse), which pushes the errors in the same direction relative to the result.

Special warning: Subtracting two nearly equal quantities is extremely dangerous. Suppose $a = 50.0 \pm 0.5$ and $b = 49.0 \pm 0.5$. Each measurement has a modest 1% relative error. The difference is $x = 1.0$, but the absolute error is still $\Delta x = 0.5 + 0.5 = 1.0$. The relative error of the difference is $\frac{1.0}{1.0} = 100\%$. Two individually precise measurements have produced a result that is completely useless. This phenomenon, sometimes called catastrophic cancellation, is one of the most common sources of large errors in student experiments. Whenever your experiment involves subtracting two similar numbers, treat the result with extreme suspicion.

What to do about it: whenever possible, redesign the experiment to measure the small difference directly rather than obtaining it by subtracting two large numbers. For example, if you need the temperature rise of water during heating, do not measure the initial temperature and the final temperature with separate thermometers and subtract. Instead, use the same thermometer and read the change on its scale, or use a thermocouple designed to measure temperature differences directly. If subtraction cannot be avoided, use instruments with much higher precision for the individual measurements, so that the absolute errors are small compared to the expected difference. Whenever your experiment involves subtracting two similar numbers, treat the result with extreme suspicion and report its inflated uncertainty honestly.

Rule 2: Multiplication and Division (*relative errors add*)

If $x = ab$ or $x = a/b$, then:

$$\frac{\Delta x}{x} = \frac{\Delta a}{a} + \frac{\Delta b}{b}$$

Derivation for multiplication: For $x = ab$, the measured value with errors is:

$$x + \Delta x = (a + \Delta a)(b + \Delta b) = ab + a\Delta b + b\Delta a + \Delta a \cdot \Delta b$$

Since Δa and Δb are both small compared to a and b, their product $\Delta a \cdot \Delta b$ is negligibly small. Dropping it:

$$\Delta x \approx a\Delta b + b\Delta a$$

Dividing both sides by $x = ab$:

$$\frac{\mathbf{\Delta x}}{\mathbf{x}} \approx \frac{a\Delta b + b\Delta a}{ab} = \frac{\mathbf{\Delta b}}{\mathbf{b}} + \frac{\mathbf{\Delta a}}{\mathbf{a}}$$

Derivation for division: For $x = \frac{a}{b}$, the maximum value of x occurs when a is at its largest and b is at its smallest:

$$x + \Delta x = \frac{a + \Delta a}{b - \Delta b}$$

The error is:

$$\Delta x = \frac{a + \Delta a}{b - \Delta b} - \frac{a}{b}$$

Finding a common denominator:

$$\Delta x = \frac{b(a + \Delta a) - a(b - \Delta b)}{b(b - \Delta b)} = \frac{ab + b\Delta a - ab + a\Delta b}{b(b - \Delta b)} = \frac{b\Delta a + a\Delta b}{b(b - \Delta b)}$$

Since Δb is small compared to b, the denominator $b(b - \Delta b) \approx b^2$:

$$\Delta x \approx \frac{b\Delta a + a\Delta b}{b^2} = \frac{\Delta a}{b} + \frac{a\Delta b}{b^2}$$

Dividing both sides by $x = \frac{a}{b}$:

$$\frac{\mathbf{\Delta x}}{\mathbf{x}} = \frac{\frac{\Delta a}{b}}{\frac{a}{b}} + \frac{\frac{a\Delta b}{b^2}}{\frac{a}{b}} = \frac{\mathbf{\Delta a}}{\mathbf{a}} + \frac{\mathbf{\Delta b}}{\mathbf{b}}$$

The result is the same as for multiplication: the relative errors add. This is why a single rule covers both operations.

Rule 3: Powers (relative error multiplied by the power)

If $x = a^n$, then:

$$\frac{\Delta x}{x} = n \times \frac{\Delta a}{a}$$

This follows directly from Rule 2. If $x = a \cdot a \cdot a$ (three factors of a), then by Rule 2 the relative error is $\frac{\Delta a}{a} + \frac{\Delta a}{a} + \frac{\Delta a}{a} = 3\left(\frac{\Delta a}{a}\right)$. For a general power n, the relative error is $n\left(\frac{\Delta a}{a}\right)$.

This is why squaring a measurement doubles its relative error, and cubing it triples it. A 2% error in measuring a radius becomes a 6% error in the calculated volume (since $V \propto r^3$). Students find this surprising the first time they see it, and important every time they use it afterwards.

The General Formula

For a quantity that depends on multiple variables raised to different powers, the results above combine into a single master formula. If:

$$x = a^n \times b^m$$

then:

$$\frac{\Delta \mathbf{x}}{\mathbf{x}} = \mathbf{n} \times \frac{\Delta \mathbf{a}}{\mathbf{a}} + \mathbf{m} \times \frac{\Delta \mathbf{b}}{\mathbf{b}}$$

where n and m are the magnitudes of the powers (always taken as positive, since we are calculating the maximum possible error). This is the formula you will use most often for error propagation in A-level physics.

Errors from Graphs

In practical examinations, you will often determine a physical quantity by plotting a graph and measuring its gradient. The uncertainty in the gradient must be estimated from the graph itself, using a systematic method.

The best-fit line is drawn so that data points are distributed as evenly as possible above and below the line. It is not forced through every single point (that would amplify random errors rather than averaging them out), and it is not forced through the origin unless there is a physical reason to do so.

Error bars are vertical (and sometimes horizontal) lines drawn at each data point, showing the uncertainty in that reading. A data point plotted at $y = 5.0 \pm 0.3$ has a vertical error bar extending from 4.7 to 5.3. The best-fit line should pass through as many error bars as possible, not necessarily through the points themselves.

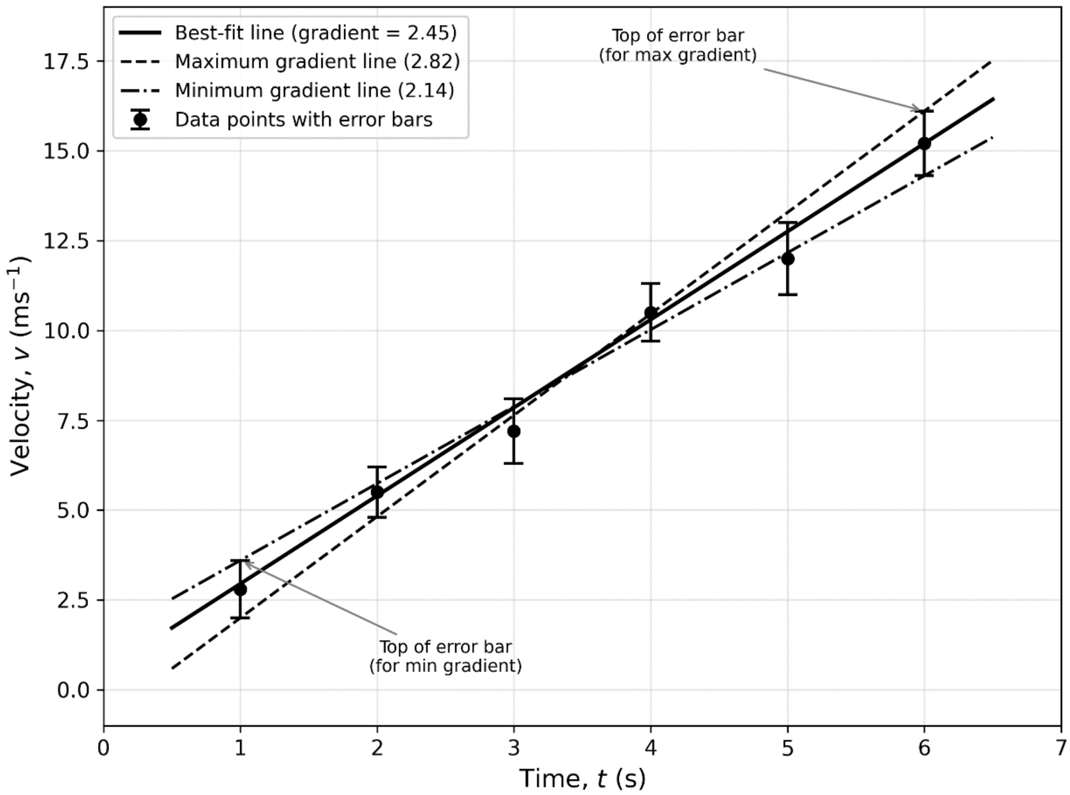

Figure: ***Determining the uncertainty in a gradient from a graph.*** *Six data points are plotted with vertical error bars showing the uncertainty in each reading. The best-fit line (solid) is drawn through the data as evenly as possible. The maximum gradient line (dashed) passes from the bottom of the first error bar to the top of the last, giving the steepest plausible gradient. The minimum gradient line (dash-dot) passes from the top of the first error bar to the bottom of the last, giving the shallowest plausible gradient. The gradient uncertainty is calculated as* $\Delta m = \frac{m_{max} - m_{min}}{2}$.

The maximum gradient line is the steepest straight line that still passes through (or close to) the error bars of the data points. **The minimum gradient line** is the shallowest such line.

The uncertainty in the gradient is:

$$\Delta \mathrm{m} = \frac{\mathrm{m_{max}} - \mathrm{m_{min}}}{2}$$

where $\mathrm{m_{max}}$ and $\mathrm{m_{min}}$ are the gradients of the steepest and shallowest acceptable lines.

The gradient is then reported as:

$$\mathrm{m} = \mathrm{m_{best}} \pm \Delta \mathrm{m}$$

The same approach applies to the y-intercept: read the intercept from each of the three lines and report the best-fit value with its uncertainty.

A practical tip for examinations: when drawing the maximum and minimum gradient lines, anchor one end at the top of the highest error bar and the other at the bottom of the lowest (for maximum gradient), or the reverse (for minimum gradient). This ensures you are using the full range of the data's uncertainty.

With the theory complete, let us build fluency through worked examples that progress from straightforward to demanding.

BINDER Example 14

A student measures the period of a simple pendulum five times and obtains the following values (in seconds): 2.04, 2.08, 2.06, 2.10, 2.02.

Find: (a) the mean value, (b) the mean absolute error, (c) the relative error, (d) the percentage error. Write the final result in standard notation.

Solution

(a) Mean value:

$$\bar{a} = \frac{(2.04 + 2.08 + 2.06 + 2.10 + 2.02)\text{s}}{5} = 2.06 \text{ s}$$

(b) Individual absolute errors:

$|2.04 - 2.06| = 0.02$ s, $|2.08 - 2.06| = 0.02$ s, $|2.06 - 2.06| = 0.00$ s, $|2.10 - 2.06| = 0.04$ s, $|2.02 - 2.06| = 0.04$ s

Mean absolute error:

$$\Delta\bar{a} = \frac{(0.02 + 0.02 + 0.00 + 0.04 + 0.04)\text{s}}{5} = 0.024 \text{ s} \approx 0.02 \text{ s}$$

(c) Relative error:

$$\frac{\Delta\bar{a}}{\bar{a}} = \frac{0.02 \text{ s}}{2.06 \text{ s}} = 0.0097 \approx 0.01$$

(d) Percentage error:

$$0.0097 \times 100\% \approx 1.0\%$$

Final result in standard notation: $T = (2.06 \pm 0.02)$ s

Making Sense of the Answer: *A 1% error in a pendulum period measurement is respectable for a school laboratory. The readings are closely clustered around the mean, indicating good technique and small random errors. The student should now use this period (with its uncertainty) to calculate g and propagate the error forward using the techniques from this subtopic.*

Think Like a Physicist: *Always round the uncertainty to one or two significant figures, then round the mean to match the last significant digit of the uncertainty. Writing $T = 2.06 \pm 0.024357$ s suggests a false precision in the uncertainty itself. The uncertainty is an estimate, not an exact number.*

BINDER Example 15

Calculate the maximum possible error in:

(a) The sum of two lengths measured as (12.5 ± 0.1) cm and (8.3 ± 0.2) cm.

(b) The product of a force (25.0 ± 0.5) N and a displacement (4.0 ± 0.1) m.

(c) The cube of a radius measured as (3.0 ± 0.1) cm.

Solution

(a) Addition (Rule 1: absolute errors add):

$$x = 12.5 \text{ cm} + 8.3 \text{ cm} = 20.8 \text{ cm}$$

$$\Delta x = 0.1 \text{ cm} + 0.2 \text{ cm} = 0.3 \text{ cm}$$

Result: $x = (20.8 \pm 0.3)$ cm

(b) Multiplication (Rule 2: relative errors add):

$$W = 25.0 \text{ N} \times 4.0 \text{ m} = 100 \text{ J}$$

$$\frac{\Delta W}{W} = \frac{0.5 \text{ N}}{25.0 \text{ N}} + \frac{0.1 \text{ m}}{4.0 \text{ m}} = 0.020 + 0.025 = 0.045$$

$$\Delta W = 0.045 \times 100 \text{ J} = 4.5 \text{ J}$$

Result: $W = (100 \pm 5)$ J (rounded)

(c) Powers (Rule 3: relative error multiplied by power):

$$r^3 = (3.0 \text{ cm})^3 = 27 \text{ cm}^3$$

$$\frac{\Delta(r^3)}{r^3} = 3 \times \frac{0.1 \text{ cm}}{3.0 \text{ cm}} = 3 \times 0.033 = 0.10$$

$$\Delta(r^3) = 0.10 \times 27\ \text{cm}^3 = 2.7\ \text{cm}^3$$

Result: $r^3 = (27 \pm 3)\ \text{cm}^3$

Making Sense of the Answer: *In part (c), a 3.3% error in the radius became a 10% error in the cube. The power of 3 tripled the relative error. This is why volume measurements are inherently less precise than length measurements, even when the same ruler is used for both. Any quantity that depends on a high power of a measured variable will amplify that variable's uncertainty.*

Think Like a Physicist: *For addition and subtraction, work with absolute errors. For multiplication, division, and powers, work with relative errors. Mixing them up is one of the most common procedural errors in error propagation. A helpful memory aid: if the operation combines quantities by adding or subtracting, the absolute errors add. If it combines them by multiplying or dividing, the relative errors add. The error rule mirrors the operation.*

BINDER Example 16

The acceleration due to gravity is determined using the formula $g = \frac{4\pi^2 l}{T^2}$. The percentage error in l is 1% and the percentage error in T is 2%. Find the percentage error in g. State which measurement contributes more to the error.

Solution

Since $g = 4\pi^2 l T^{-2}$, and $4\pi^2$ is an exact constant (no error):

$$\frac{\Delta g}{g} = 1 \times \frac{\Delta l}{l} + 2 \times \frac{\Delta T}{T}$$

The power of l is 1, so its percentage error enters with a multiplier of 1. The power of T is 2 (we use the magnitude), so its percentage error enters with a multiplier of 2.

$$\%\ \text{error in g} = 1 \times 1\% + 2 \times 2\% = 1\% + 4\% = 5\%$$

The timing measurement dominates the error: T contributes 4% out of the total 5%, while l contributes only 1%.

Making Sense of the Answer: *Because T is squared in the formula, its error is doubled before it enters the total. A 2% error in T becomes a 4% contribution to g. This is why, in a pendulum experiment, the most effective way to improve the accuracy of g is to improve the timing. Timing 20 oscillations instead of 1 divides the random timing error by 20, dramatically reducing the percentage error in T.*

Think Like a Physicist: *Always identify which measurement contributes the most error to the final result. Then improve that measurement first. Spending time and effort improving the less significant measurement is wasted work. In this case, buying a more precise ruler (to improve l) is far less effective than simply timing more oscillations (to improve T).*

REAL Example 17

A nurse at Bugando Hospital measures a patient's temperature five times in quick succession and gets: 37.1°C, 37.3°C, 37.0°C, 37.2°C, 37.4°C. Kipute, observing during a school hospital visit, asks:

Kipute: *"Is the thermometer broken? It gives a different answer every time."*

Mr. Akilikubwa: *"No, Kipute. The thermometer is working exactly as it should. In fact, a thermometer that gave exactly the same reading every single time would worry me more."*

Explain why the readings differ, why the mean is more reliable than any single reading, and identify a situation where averaging would not improve the result.

Solution

The readings differ because of random errors: tiny fluctuations in blood flow near the skin surface, slight variations in thermometer placement between measurements, and natural biological variation in body temperature between breaths and heartbeats. These cause each reading to deviate slightly and unpredictably from the true body temperature.

The mean (37.2°C) is more reliable than any single reading because random errors are equally likely to push a reading above or below the true value. When many readings are averaged, the positive deviations tend to cancel the negative deviations, and the mean converges toward the true temperature. Five readings give a better estimate than one; twenty would be better still.

However, if the thermometer had a systematic error (for example, if it consistently read 0.5°C too low because of a calibration fault), averaging would not help. Every single reading would be too low by the same amount, so the mean would also be too low by that amount. The nurse would need to recalibrate or replace the thermometer. Repetition cures random error; it cannot cure systematic error.

Making Sense of the Answer: *This is why hospitals calibrate their instruments regularly. A thermometer that consistently reads 0.5°C low could cause a doctor to miss a dangerous fever. The instrument looks precise (consistent readings) but is inaccurate (all readings displaced from the true value). Mr. Akilikubwa's comment that a perfectly consistent thermometer would worry him reflects this: perfect consistency in a real measurement is suspicious, because it may indicate that the instrument's resolution is too low to detect the natural random variation that should be present.*

Think Like a Physicist: *When you see scattered readings, ask yourself two questions. First: is this random error? If so, averaging will help. Second: could there also be a systematic error hiding beneath the scatter? Averaging handles the first problem but completely ignores the second. A good experimentalist addresses both.*

HOT Example 18

The density of a rectangular metal block is to be determined. The following measurements are taken:

Mass: $\mathrm{m} = (50.0 \pm 0.1)\ \mathrm{g}$

Length: $\mathrm{l} = (4.00 \pm 0.01)\ \mathrm{cm}$

Width: $\mathrm{w} = (3.00 \pm 0.01)\ \mathrm{cm}$

Height: $\mathrm{h} = (2.00 \pm 0.01)\ \mathrm{cm}$

Find the density with its absolute and percentage uncertainty. State the dominant source of error.

Solution

Volume:

$$\mathrm{V = l \times w \times h = 4.00\ cm \times 3.00\ cm \times 2.00\ cm = 24.0\ cm^3}$$

Density:

$$\rho = \frac{\mathrm{m}}{\mathrm{V}} = \frac{50.0\ \mathrm{g}}{24.0\ \mathrm{cm^3}} = 2.083\ \mathrm{gcm^{-3}}$$

Error propagation. Since $\rho = \frac{\mathrm{m}}{\mathrm{l \times w \times h}}$, the relative error is:

$$\frac{\Delta\rho}{\rho} = \frac{\Delta \mathrm{m}}{\mathrm{m}} + \frac{\Delta \mathrm{l}}{\mathrm{l}} + \frac{\Delta \mathrm{w}}{\mathrm{w}} + \frac{\Delta \mathrm{h}}{\mathrm{h}}$$

Substituting:

$$\frac{\Delta\rho}{\rho} = \frac{0.1\ \mathrm{g}}{50.0\ \mathrm{g}} + \frac{0.01\ \mathrm{cm}}{4.00\ \mathrm{cm}} + \frac{0.01\ \mathrm{cm}}{3.00\ \mathrm{cm}} + \frac{0.01\ \mathrm{cm}}{2.00\ \mathrm{cm}}$$

$$= 0.002 + 0.0025 + 0.0033 + 0.005 = 0.0128$$

Percentage error: $0.0128 \times 100\% = 1.3\%$

Absolute error in density:

$$\Delta\rho = 0.0128 \times 2.083\ \mathrm{gcm^{-3}} = 0.027\ \mathrm{gcm^{-3}}$$

Final result: $\rho = (2.08 \pm 0.03)\ \mathrm{gcm^{-3}}$

The dominant source of error is the height measurement, which contributes $\frac{0.005}{0.0128} = 39\%$ of the total relative error, despite using the same ruler as the other two dimensions.

This happens because 2.00 cm is the smallest dimension, so the same absolute error (± 0.01 cm) produces the largest relative error (0.5%).

Making Sense of the Answer: *The smallest dimension always dominates the error because relative error equals $\Delta x/x$, and the denominator is smallest for the shortest measurement. If you could improve only one measurement, improve the height: use a micrometer screw gauge instead of a ruler. Improving the length measurement (which already has only 0.25% relative error) would barely change the overall uncertainty.*

Think Like a Physicist: *Identifying and reporting the dominant error source in your final experimental report shows you understand not just how to calculate errors, but which measurement to improve. This is the difference between efficient experimentation and wasted effort.*

HOT Example 19

From a velocity-time graph, a student draws a best-fit line and determines the gradient (acceleration) to be 12.5 ms^{-2}. The maximum gradient line gives 13.8 ms^{-2} and the minimum gradient line gives 11.4 ms^{-2}. Express the acceleration with its uncertainty in standard notation. Calculate the percentage uncertainty.

Solution

Uncertainty in gradient:

$$\Delta m = \frac{m_{max} - m_{min}}{2} = \frac{13.8\ ms^{-2} - 11.4\ ms^{-2}}{2} = 1.2\ ms^{-2}$$

Acceleration in standard notation:

$$a = (12.5 \pm 1.2)\ ms^{-2}$$

Percentage uncertainty:

$$\frac{1.2\ ms^{-2}}{12.5\ ms^{-2}} \times 100\% = 9.6\%$$

Making Sense of the Answer: *A 9.6% uncertainty from a graph is typical for school-level experiments with five or six data points. The gradient uncertainty reflects how much the data points scatter. More data points, measured more carefully, would produce a tighter scatter and a smaller difference between the maximum and minimum gradient lines.*

Think Like a Physicist: *Always draw the maximum and minimum gradient lines through the extremes of the error bars, not through arbitrary points you find convenient. The error bars are the data's honest statement about its own uncertainty; the gradient lines must respect them. If you skip the error bars, you are guessing at the uncertainty instead of measuring it.*

Errors and uncertainties are now part of your vocabulary. You can classify errors as systematic or random, identify and discard blunders, express uncertainty in absolute, relative, and percentage form, propagate errors through any formula, and extract uncertainty from graphs. Every measurement you write from this point forward should carry its uncertainty, not as a confession of failure, but as a badge of scientific honesty.

In the next section, we sharpen two words that students use carelessly and examiners test mercilessly: accuracy and precision. They are not the same thing, and confusing them is the kind of error that costs marks quietly.

ACCURACY AND PRECISION

Two words. Six syllables. And more confusion than any other pair of terms in A-level physics. Students use them interchangeably. Textbooks sometimes blur the distinction. Examiners, however, do not. In every practical report, every error analysis, and every examination question that asks you to evaluate a set of measurements, these two words carry specific, different meanings. This section establishes those meanings clearly and permanently.

The Dartboard Analogy

The easiest way to understand the distinction is through an analogy that has nothing to do with physics and everything to do with darts.

Precision *describes how closely repeated measurements agree with each other.* A set of measurements is precise if the values are tightly clustered, regardless of where they fall relative to the true value.

Accuracy *describes how close the measurements are to the true value.* A set of measurements is accurate if their mean is close to the true value, regardless of how much the individual values scatter.

These are independent properties. A measurement can be one without the other, both, or neither. The four possible combinations are best visualised as a dartboard, where the bullseye represents the true value:

Precise AND accurate: All darts clustered tightly around the bullseye. The measurements agree with each other (precise) and their mean is very close to the true value (accurate). This is the ideal outcome, achieved by good technique, a well-calibrated instrument, and sufficient repetitions.

Precise but NOT accurate: All darts clustered tightly together, but the cluster is far from the bullseye. The measurements agree with each other (precise) but their mean is displaced from the true value (inaccurate). This is the signature of a systematic error. The instrument or technique is consistent but wrong. A thermometer that always reads 2°C too high produces readings that are beautifully precise and completely inaccurate.

Accurate but NOT precise: Darts scattered widely across the board, but their average position is the bullseye. The individual measurements scatter badly (imprecise), but their mean happens to land on or near the true value (accurate). This is the signature of large random errors with no systematic error. Averaging many readings would give a good result, but any single reading is unreliable.

Neither precise nor accurate: Darts scattered widely and not centred on the bullseye. Both random and systematic errors are large. The measurements neither agree with each other nor approach the true value. This is the worst outcome, indicating poor technique, a badly calibrated instrument, or both.

Connecting to Error Types

The dartboard analogy is not just a pretty picture. It connects directly to the error classification we learned earlier:

Systematic errors destroy accuracy. They shift all readings in the same direction, displacing the mean from the true value. The data may look precise (tight cluster) but the cluster is in the wrong place. This is why systematic errors are so dangerous: the data appears trustworthy when it is not.

Random errors destroy precision. They scatter readings above and below the true value, spreading the cluster. If no systematic error is present, the mean of many readings will still approach the true value (the centre of the scatter is correct), but any individual reading may be far off.

The practical consequence is direct:

To improve accuracy, you must identify and eliminate systematic errors: recalibrate instruments, correct for zero errors, use proper technique.

To improve precision, you must reduce random errors: take more readings, use instruments with finer resolution, control environmental conditions.

A measurement that is precise but inaccurate cannot be fixed by taking more readings. A hundred readings from a miscalibrated thermometer give a very precise wrong answer. Only recalibration fixes it.

A measurement that is accurate but imprecise can be improved by taking more readings. The scatter is random, so averaging reduces it. More data points give a tighter estimate of the true mean.

Resolution and Sensitivity

Two further terms appear in examination questions about measurement quality. They are related to precision but are not the same thing.

Resolution (also called **least count**) *is the smallest change in a quantity that an instrument can detect.* It equals the smallest division on the instrument's scale. A ruler marked in millimetres has a resolution of 1 mm. A micrometer screw gauge has a resolution of 0.01 mm. A digital stopwatch reading to 0.01 s has a resolution of 0.01 s.

Resolution sets the **minimum** uncertainty in a single reading. *You cannot claim an uncertainty smaller than the instrument's resolution, no matter how carefully you read it.*

Sensitivity *is how much the instrument's output changes per unit change in the quantity being measured.* A thermometer in which the mercury column moves 2 cm for every 1°C change is more sensitive than one in which it moves 0.5 cm per °C. A galvanometer that deflects 10 divisions per milliamp is more sensitive than one that deflects 2 divisions per milliamp.

An important distinction: a sensitive instrument is not necessarily an accurate one. A highly sensitive thermometer may detect tiny temperature changes (good sensitivity) but still read 2°C too high on every reading (poor accuracy due to systematic error). Sensitivity tells you how finely the instrument responds; accuracy tells you whether it responds correctly.

With the vocabulary now sharp, let us test it.

BINDER Example 20

A student measures a length known to be exactly 25.0 cm. Six measurements are recorded: 24.8, 24.9, 25.1, 24.7, 25.0, 24.9 (all in cm).

A second student measures the same length and records: 26.3, 26.4, 26.3, 26.5, 26.4, 26.3 (all in cm).

For each set of measurements, determine whether the measurements are precise, accurate, both, or neither. Justify each conclusion.

Solution

First student:

$$\text{Mean} = \frac{(24.8 + 24.9 + 25.1 + 24.7 + 25.0 + 24.9)\text{ cm}}{6} = 24.9\text{ cm}$$

The readings scatter from 24.7 to 25.1 cm, a spread of 0.4 cm. This is a moderate spread, so the precision is reasonable but not exceptional.

The mean (24.9 cm) is very close to the true value (25.0 cm), differing by only 0.1 cm. The measurements are accurate.

Conclusion: Student 1's measurements are **accurate but only moderately precise**. The scatter suggests random errors are present but no significant systematic error, since the mean is close to the true value.

Second student:

$$\text{Mean} = \frac{(26.3 + 26.4 + 26.3 + 26.5 + 26.4 + 26.3)\text{ cm}}{6} = 26.37\text{ cm}$$

The readings scatter from 26.3 to 26.5 cm, a spread of only 0.2 cm. This is a tight cluster, indicating high precision.

However, the mean (26.37 cm) is 1.37 cm above the true value (25.0 cm). The measurements are inaccurate.

Conclusion: Second student's measurements are **precise but not accurate**. The tight clustering indicates small random errors, but the large displacement from the true value indicates a systematic error. The ruler may have a damaged or missing zero end, or the student is consistently misaligning the starting point.

Making Sense of the Answer: *Second student's data looks better at first glance because the readings are so consistent. But consistency without correctness is misleading. A set of tightly clustered wrong answers is more dangerous than a set of scattered readings whose mean is correct, because the tight clustering creates a false sense of confidence.*

Think Like a Physicist: *You cannot judge accuracy without knowing the true value. If the true value is unknown (as it usually is in a real experiment), precision is all you can assess from the data alone. This is why calibration against known standards is so important: it is the only way to check for systematic errors that precision alone cannot reveal.*

REAL Example 21

During a football practice session, Kipanga takes five penalty kicks, aiming for the top-right corner of the goal. All five shots land in the bottom-left corner. Kipute, watching from the sideline, makes an observation:

Kipute: *"You are very precise, Kipanga. All five shots went to exactly the same place."*

Kipanga: (offended) *"What do you mean, precise? I missed every single one!"*

Kipute: *"Exactly. You are precise but not accurate. Every shot is consistent with the others, but none of them went where you aimed."*

Mr. Akilikubwa, who has been watching quietly with a cup of tea, walks over.

Mr. Akilikubwa: *"Kipute is right. And this is not just about football. This is exactly what happens when a measuring instrument has a systematic error. Imagine Kipanga is a thermometer. He consistently gives the same reading (bottom-left corner), and that reading is always wrong by the same amount and in the same direction (displaced from the top-right corner). Repeating the measurement does not help, because every repetition lands in the same wrong place."*

He pauses. *"Now, if Kipanga's shots had scattered randomly all over the goal, with some going left, some right, some high, some low, but the average landing position was the top-right corner, he would be accurate but imprecise. The individual shots would be unreliable, but the overall pattern would be centred on the target."*

Kipanga: *"So how do I fix my kicks?"*

Mr. Akilikubwa: *"The same way you fix a systematic error. You do not take more shots hoping the problem will go away, because it will not. You change your technique. You re-aim. You recalibrate."*

Explain, using this analogy, why repeating a measurement cannot correct a systematic error, and what must be done instead.

Solution

Kipanga's five identical shots to the bottom-left corner represent a systematic error: every measurement is displaced from the true value by the same amount in the same direction. The displacement is consistent and repeatable.

Taking more penalty kicks (repeating the measurement) will not help, because every additional shot will also land in the bottom-left corner. The average of a hundred bottom-left shots is still the bottom-left corner. Repetition reduces random scatter, but there is no random scatter here. The problem is a consistent bias in technique.

To correct the error, Kipanga must change his technique: re-aim, adjust his body position, or alter his kicking angle. In measurement terms, this corresponds to recalibrating the instrument, correcting for zero error, or changing the experimental method. The cause of the systematic shift must be identified and removed. Only then will future measurements (or penalty kicks) land where they should.

Making Sense of the Answer: *The football analogy makes the abstract distinction between systematic and random errors concrete and memorable. Kipanga's consistent miss is a systematic error. If his shots had scattered randomly around the target, the problem would be random error, fixable by averaging. The cure must match the disease: averaging cures scatter, recalibration cures shift.*

Think Like a Physicist: *In a real experiment, the hardest part is recognising that a systematic error exists, because the data looks precise and therefore trustworthy. The only way to detect a systematic error is to compare your result with a known standard or with an independent measurement using a different method. If both methods agree, you can be more confident that systematic errors are small. If they disagree, at least one method has a systematic problem.*

Accuracy and precision are now permanently separated in your mind. Precision is about agreement between readings. Accuracy is about agreement with truth. Systematic errors destroy accuracy. Random errors destroy precision. And Kipanga's penalty kicks, while consistent, will need some serious recalibration before the inter-school tournament.

In the next section, we step briefly outside the laboratory to see why everything we have learned in this chapter matters in the real world, from the design of bridges to the dosing of medicines to the most expensive measurement error in the history of space exploration.

APPLICATIONS OF MEASUREMENT

Physics does not exist only inside textbooks. The principles of measurement, uncertainty, and error analysis that fill this chapter are at work every day in hospitals, construction sites, factories, and space agencies around the world. This section assembles a set of real-world applications that show why precision in measurement is not an academic exercise but a matter of safety, money, and sometimes survival.

1. **Engineering and construction**

Every structure you walk into, every bridge you cross, and every road you drive on was built from measurements. The consequences of getting those measurements wrong range from inconvenient to catastrophic.

In building construction, a small angular error in the foundation can amplify as the structure rises. A 1 mm misalignment at the base of a ten-storey building can become a 10 cm lean at the top. This is why surveyors use instruments with resolutions measured in fractions of a millimetre, and why every measurement is repeated and cross-checked.

In manufacturing, engine parts must fit together with tolerances measured in hundredths of a millimetre. A piston that is 0.1 mm too wide will seize inside the cylinder and destroy the engine. A piston that is 0.1 mm too narrow allows combustion gases to leak past, reducing power and efficiency. The entire performance of the engine depends on measurements that are invisible to the naked eye.

In Tanzania, the construction of major infrastructure projects such as the Standard Gauge Railway and the Julius Nyerere Hydropower Station required GPS-based measurement systems accurate to within centimetres over distances of hundreds of kilometres. Traditional surveying methods, while adequate for smaller projects, could not achieve the precision needed for structures of this scale.

2. Medicine and pharmacy

In medicine, measurement errors can be the difference between treatment and tragedy.

Drug dosage depends on precise measurement of the drug's mass or concentration and the patient's body weight. For drugs with a narrow therapeutic window (where the effective dose is close to the toxic dose), a 10% error can push the dose from therapeutic to dangerous. Digoxin, lithium, and warfarin are examples of drugs where dosage precision is critical. A pharmacist who measures carelessly risks poisoning the patient.

Blood pressure measurement is affected by systematic errors if the wrong cuff size is used. A cuff that is too small compresses the arm insufficiently, giving a falsely high reading. A doctor relying on this reading may prescribe unnecessary medication. The solution is simple: use the correct cuff size. But recognising the systematic error requires understanding what systematic errors are and how they behave, exactly the knowledge this chapter provides.

In laboratory diagnosis, clinical decisions depend on precise measurement of blood glucose levels, electrolyte concentrations, and cell counts. An error of a few percent in a blood glucose reading could lead to an incorrect diagnosis of diabetes or a missed case of hypoglycaemia.

3. Space exploration and the most expensive measurement error in history

Earlier, we briefly encountered the Mars Climate Orbiter. The story deserves its full telling here, because it is the most powerful argument for measurement discipline ever produced by human failure.

In December 1998, NASA launched the Mars Climate Orbiter, a spacecraft designed to study the Martian atmosphere and serve as a communications relay for future Mars missions. The mission cost 327 million US dollars. The spacecraft travelled for nine months across 670 million kilometres of space, functioning perfectly.

On 23 September 1999, the spacecraft began its approach to Mars. The navigation team at NASA's Jet Propulsion Laboratory sent course-correction commands based on trajectory data. The data was correct. The commands were correct. But there was one problem: the software written by the spacecraft's builder, Lockheed Martin, calculated thruster impulses in pound-force seconds (an imperial unit), while NASA's navigation software expected the data in newton seconds (the SI unit). One pound-force second equals approximately 4.45 newton seconds. Nobody checked. Nobody noticed the mismatch. For nine months.

The spacecraft entered the Martian atmosphere at an altitude of 57 km instead of the planned 226 km. At 57 km, the atmospheric friction was far too great. The spacecraft broke apart and was destroyed.

Three hundred and twenty-seven million dollars. Nine months of travel. Years of engineering. The work of hundreds of scientists and engineers. All lost because two teams used different units and neither performed a simple conversion check. The investigation board concluded that the root cause was a failure to verify units at the interface between two software systems. It was not a failure of physics. It was not a failure of engineering. It was a failure of measurement discipline.

The Mars Climate Orbiter has become the single most cited example in physics education of why units matter, why standard systems exist, and why every number must carry its unit. The lesson is simple: the physics was perfect, and the spacecraft was destroyed anyway, because someone assumed the units would take care of themselves.

4. Scientific Research

At the frontier of physics, the margin between discovery and noise is often a matter of measurement precision.

The discovery of the Higgs boson at CERN in 2012 required measurements accurate to parts per billion. The signal that confirmed the particle's existence was only five standard deviations above background noise. Without rigorous error analysis, the signal would have been indistinguishable from a statistical fluctuation. The entire discovery, one of the most important in the history of physics, rested on the ability to measure, quantify uncertainty, and distinguish real effects from measurement noise.

In climate science, global temperature records depend on measurements taken at thousands of stations around the world over more than a century. A systematic error of even 0.1°C in how temperatures were recorded at certain stations can distort the apparent warming trend when data is accumulated over decades. Climate scientists spend enormous effort identifying and correcting these systematic biases, because the conclusions about global warming must rest on measurements that are both precise and accurate.

Measurement is not the most dramatic part of physics. It does not have the visual appeal of a rocket launch or the conceptual excitement of black holes. But it is the foundation on which every other part of physics stands. A formula is only as good as the measurements that feed it. A theory is only as convincing as the data that tests it. And a physicist who cannot measure honestly is not a physicist at all.

You now have the vocabulary, the tools, and (unlike Kipanga) the correct units. What remains is practice. The following miscellaneous worked examples will test everything from this chapter, combining concepts in the way examinations do. Pick up your pen, prepare your calculator, and let us begin.

MISCELLANEOUS WORKED EXAMPLES ON MEASUREMENT

Example 22

(a) The viscous drag force on a sphere moving through a fluid is given by $F = 6\pi\eta rv$ where η is the coefficient of viscosity, r is the radius of the sphere, and v is its velocity. Use this equation to find the dimensional formula and SI unit of η.

(b) A student claims that the formula for the period of a spring-mass system is $T = \frac{2\pi m}{k}$ where m is mass and k is the spring constant (dimensions MT^{-2}). Use dimensional analysis to show that this formula is incorrect, and find the correct form.

Solution

(a) From $F = 6\pi\eta rv$, making η the subject: $\eta = \frac{F}{6\pi rv}$

$$[\eta] = \frac{[F]}{[r][v]} = \frac{MLT^{-2}}{L \times LT^{-1}} = \frac{MLT^{-2}}{L^2T^{-1}} = ML^{-1}T^{-1}$$

SI unit: $kgm^{-1}s^{-1}$ (also called Pas).

(b) Checking the student's formula:

$$[T] = T$$

$$\left[\frac{m}{k}\right] = \frac{M}{MT^{-2}} = T^2$$

Since $T \neq T^2$, the formula $T = \frac{2\pi m}{k}$ is dimensionally incorrect.

For the correct form, let $T = 2\pi m^a k^b$:

$$T = M^a(MT^{-2})^b = M^{a+b}T^{-2b}$$

For M: $0 = a + b$

For T: $1 = -2b$, giving $b = -1/2$ and $a = 1/2$

$$T = 2\pi\sqrt{\frac{m}{k}}$$

Example 23

(a) The density of mercury is 13.6gcm^{-3}. A student converts this to SI by writing $13.6 \times 10^{-3}\text{kgm}^{-3}$. Without performing the full conversion, explain why this answer must be wrong.

(b) Perform the correct conversion and express the density in SI units.

Solution

(a) Mercury is one of the densest liquids known. It is so heavy that iron floats on it. The density of air is approximately 1.2kgm^{-3}. The student's answer, 0.0136kgm^{-3}, is about 100 times less dense than air. This is physically absurd. A liquid that is less dense than air would float upward and disperse, not sit in a thermometer. The conversion must be wrong.

(b) Density has dimensions ML^{-3}. Converting each unit:

$$13.6\text{gcm}^{-3} = 13.6 \times \frac{10^{-3}\text{kg}}{(10^{-2}\text{m})^3} = 13.6 \times \frac{10^{-3}\text{kg}}{10^{-6}\text{m}^3} = 13.6 \times 10^3\text{kgm}^{-3} = 13600\text{kgm}^{-3}$$

Example 24

(a) The escape velocity from a planet depends on the planet's mass M, its radius R, and the gravitational constant G (dimensions $\text{M}^{-1}\text{L}^3\text{T}^{-2}$). Use dimensional analysis to derive an expression for the escape velocity.

(b) The actual formula is $v = \sqrt{\frac{2GM}{R}}$. State what dimensional analysis was unable to determine.

Solution

(a) Assume: $v = kG^aM^bR^c$

Dimensions: $[v] = \text{LT}^{-1}$, $[G] = \text{M}^{-1}\text{L}^3\text{T}^{-2}$, $[M] = \text{M}$, $[R] = \text{L}$

$$\text{LT}^{-1} = (\text{M}^{-1}\text{L}^3\text{T}^{-2})^a(\text{M})^b(\text{L})^c = \text{M}^{-a+b}\text{L}^{3a+c}\text{T}^{-2a}$$

For M: $0 = -a + b$, so $b = a$

For T: $-1 = -2a$, giving $a = 1/2$

For L: $1 = 3a + c = 3/2 + c$, giving $c = -1/2$

Therefore $b = 1/2$ and:

$$v = k\sqrt{\frac{GM}{R}}$$

(b) Dimensional analysis could not determine the **dimensionless** constant k. The actual value is $k = \sqrt{2}$. This constant comes from the full derivation (using conservation of energy). Dimensional analysis tells us the structure; theory or experiment supplies the constant.

Example 25

A student measures the refractive index of a glass block seven times and obtains: 1.44, 1.50, 1.48, 1.45, 1.97, 1.52, 1.46.

(a) Examine the data carefully. Calculate the mean value, mean absolute error, relative error, and percentage error. Write the result in standard notation.

(b) Explain your treatment of any suspect reading.

Solution

(a) Inspecting the data: six of the seven readings cluster between 1.44 and 1.52. The reading 1.97 is far outside this range. It differs from its neighbours by nearly 0.5, while the spread among the other six is only 0.08. This reading is almost certainly a blunder (perhaps caused by misreading the protractor or miscalculating the angle). It is **discarded** before further analysis.

Using the remaining six readings: 1.44, 1.50, 1.48, 1.45, 1.52, 1.46

Mean:

$$\bar{n} = \frac{1.44 + 1.50 + 1.48 + 1.45 + 1.52 + 1.46}{6} = 1.475$$

Individual absolute errors: $|1.44 - 1.475| = 0.035$, $|1.50 - 1.475| = 0.025$, $|1.48 - 1.475| = 0.005$, $|1.45 - 1.475| = 0.025$, $|1.52 - 1.475| = 0.045$, $|1.46 - 1.475| = 0.015$

Mean absolute error:

$$\Delta\bar{n} = \frac{0.035 + 0.025 + 0.005 + 0.025 + 0.045 + 0.015}{6} = 0.025$$

Relative error: $\frac{0.025}{1.475} = 0.017$

Percentage error: $0.017 \times 100\% = 1.7\%$

Result: $n = 1.48 \pm 0.03$

(b) The reading 1.97 was discarded as a blunder. A blunder is a severe mistake that produces a reading wildly different from the rest of the dataset. Including it would have shifted the mean to 1.546 and inflated the error to 0.11, contaminating the result with a single human mistake rather than reflecting the genuine measurement uncertainty. Blunders are identified by inspection and removed before averaging.

Example 26

(a) In the formula $g = \frac{4\pi^2 l}{T^2}$, explain why the percentage error in T contributes more to the total error than the percentage error in l, even when both are measured with equal precision.

(b) In an experiment, $l = (0.800 \pm 0.002)\text{m}$ and $T = (1.796 \pm 0.005)\text{s}$. Calculate g with its absolute and percentage uncertainty. Identify the dominant source of error and suggest one improvement.

Solution

(a) In $g = 4\pi^2 l T^{-2}$, the power of l is 1 and the power of T is 2. The percentage error in g is $\frac{\Delta g}{g} = 1 \times \frac{\Delta l}{l} + 2 \times \frac{\Delta T}{T}$. The factor of 2 multiplying $\frac{\Delta T}{T}$ doubles its contribution. Even with identical percentage precision in l and T, the timing error contributes twice as much because T is squared.

(b)

$$g = \frac{4\pi^2 \times 0.800\text{m}}{(1.796\text{s})^2} = 9.791\text{ms}^{-2}$$

Percentage errors:

$$\frac{\Delta l}{l} = \frac{0.002\text{m}}{0.800\text{m}} = 0.25\%$$

$$\frac{\Delta T}{T} = \frac{0.005\text{s}}{1.796\text{s}} = 0.28\%$$

Total: $\frac{\Delta g}{g} = 1 \times 0.25\% + 2 \times 0.28\% = 0.25\% + 0.56\% = 0.81\%$

Absolute uncertainty: $\Delta g = 0.0081 \times 9.791\text{ms}^{-2} = 0.08\text{ms}^{-2}$

Result: $g = (9.79 \pm 0.08)\ \text{ms}^{-2}$

The dominant source is T (contributing 0.56% out of 0.81%).

To improve: time greater number of oscillations (e.g. 50 oscillations instead of 20), which divides more the timing error (by a further factor of 2.5 for 50 oscillations).

Example 27

The coefficient of viscosity η of a liquid is determined using Stokes' law:

$$\eta = \frac{2r^2(\rho_s - \rho_f)g}{9v}$$

The following measurements are taken: $r = (1.00 \pm 0.05)\text{mm}$, $\rho_s = (7800 \pm 50)\text{kgm}^{-3}$, $\rho_f = (1260 \pm 10)\text{kgm}^{-3}$, $v = (0.069 \pm 0.003)\text{ms}^{-1}$, $g = 9.8\text{ms}^{-2}$ (exact).

(a) Calculate η.

(b) Find the percentage error in η, showing the contribution from each measurement.

(c) Identify the dominant source of error and suggest how to reduce it.

Solution

(a)

$$\rho_s - \rho_f = 7800\text{kgm}^{-3} - 1260\text{kgm}^{-3} = 6540\text{kgm}^{-3}$$

$$\eta = \frac{2 \times (1.00 \times 10^{-3}\text{m})^2 \times 6540\text{kgm}^{-3} \times 9.8\text{ms}^{-2}}{9 \times 0.069\text{ms}^{-1}} = 0.206\text{kgm}^{-1}\text{s}^{-1}$$

(b) This formula combines all three error rules:

Step 1 — Powers: r appears as r^2, so: relative error from $r = 2 \times \frac{0.05\text{mm}}{1.00\text{mm}} = 0.1 = 10\%$

Step 2 — Subtraction: $(\rho_s - \rho_f)$ uses Rule 1. Absolute error: $\Delta(\rho_s - \rho_f) = 50 + 10 = 60\text{kgm}^{-3}$

Relative error: $\frac{60\text{kgm}^{-3}}{6540\text{kgm}^{-3}} = 0.92\%$

Step 3 — Division: combine all relative errors:

$$\frac{\boldsymbol{\Delta\eta}}{\boldsymbol{\eta}} = 10\% + 0.92\% + 4.35\% = \mathbf{15.3}\%$$

where the contribution from v: $\frac{0.003\text{ms}^{-1}}{0.069\text{ms}^{-1}} = 4.35\%$

Individual contributions: r: 10%, v: 4.35%, $(\rho_s - \rho_f)$: 0.92%.

(c) The radius r dominates overwhelmingly (10% out of 15.3%). This happens because r is squared, doubling its relative error, **and** because the ball is small (1 mm), making any measurement uncertainty a large fraction of the value. To reduce it: use a larger ball (larger r means the same absolute error gives a smaller relative error) or use a micrometer screw gauge instead of a ruler to measure the diameter more precisely.

Example 28

Two forces act on an object in opposite directions: $F_1 = (25.0 \pm 0.5)\text{N}$ and $F_2 = (23.0 \pm 0.5)\text{N}$. The net force accelerates a mass $m = (2.0 \pm 0.1)\text{kg}$.

(a) Calculate the acceleration with its percentage uncertainty and give the final result.

(b) Explain why the result has such a large uncertainty even though each individual measurement seems precise.

Solution

(a) Net force: $F = F_1 - F_2 = 25.0\text{N} - 23.0\text{N} = 2.0\text{N}$

Absolute error in F (Rule 1 — subtraction): $\Delta F = 0.5\text{N} + 0.5\text{N} = 1\text{N}$

Percentage error in F: $\frac{1\text{N}}{2\text{N}} \times 100\% = 50\%$

Acceleration: $a = \frac{F}{m} = \frac{2.0\text{N}}{2.0\text{kg}} = 1\text{ms}^{-2}$

Percentage error in a (Rule 2 — division):

$$\frac{\Delta a}{a} = \frac{\Delta F}{F} + \frac{\Delta m}{m} = 50\% + \frac{0.1\text{kg}}{2.0\text{kg}} \times 100\% = 50\% + 5\% = 55\%$$

Absolute uncertainty: $\Delta a = 0.55 \times 1ms^{-2} = 0.6ms^{-2}$

Result: $a = (1 \pm 0.6)\ ms^{-2}$

(b) Each force measurement has only 2% relative error ($0.5/25.0$ and $0.5/23.0$). Individually, these are precise. But when two nearly equal quantities are subtracted, the result is small while the absolute errors add. The relative error of the difference (50%) is enormously larger than the relative errors of the individual forces (2%). This is catastrophic cancellation: the subtraction destroyed the precision of the original measurements. To improve this experiment, either use a force meter with much higher precision or redesign so that the net force is measured directly.

Example 29

(a) A student plots a graph of T^2 (vertical axis) against l (horizontal axis) for a simple pendulum experiment. The best-fit gradient is $4.05s^2m^{-1}$, the maximum gradient is $4.35s^2m^{-1}$, and the minimum gradient is $3.78s^2m^{-1}$.

Given that $T^2 = \left(\frac{4\pi^2}{g}\right) l$, determine g with its absolute uncertainty and percentage uncertainty.

(b) Explain why plotting T^2 against l (rather than T against l) is preferred in this experiment.

Solution

(a) Gradient $m = \frac{4\pi^2}{g}$, so $g = \frac{4\pi^2}{m}$.

Best estimate: $g = \frac{4\pi^2}{4.05s^2m^{-1}} = 9.75ms^{-2}$

From maximum gradient: $g_{min} = \frac{4\pi^2}{4.35s^2m^{-1}} = 9.08ms^{-2}$

From minimum gradient: $g_{max} = \frac{4\pi^2}{3.78s^2m^{-1}} = 10.45ms^{-2}$

Uncertainty: $\Delta g = \frac{g_{max} - g_{min}}{2} = \frac{(10.45ms^{-2} - 9.08ms^{-2})}{2} = 0.69ms^{-2} \approx 0.7ms^{-2}$

Percentage uncertainty: $\left(\frac{0.7ms^{-2}}{9.75ms^{-2}}\right) \times 100\% = 7.2\%$

Result: $g = (9.8 \pm 0.7)\ ms^{-2}$

(b) The relationship $T = 2\pi\sqrt{\frac{l}{g}}$ is not linear: T is proportional to $\sqrt{l}$, not to l. Plotting T against l produces a curve, and fitting a straight line to a curve is unreliable. Squaring both sides gives $T^2 = \left(\frac{4\pi^2}{g}\right) l$, which is linear. A straight-line graph is much easier to draw accurately, and the gradient can be measured precisely using two well-separated points.

Example 30

During a break between lessons, Kipanga announces to the class:

Kipanga: *"I have used dimensional analysis to prove that the amount of trouble I am in has dimensions ML^2T^{-2}. Trouble equals the energy I have wasted. Therefore, trouble is a physical quantity."*

Kipute: *"That is not how it works, Kipanga."*

Mr. Akilikubwa: (overhearing from the doorway) *"Go on, Kipanga. Show us your derivation."*

Kipanga: *"The amount of trouble depends on three things: the mass of the object I broke (M), the distance it fell (L), and the time my mother takes to find out (T). So trouble $= kM^aL^bT^c$. Since trouble is like energy, it has dimensions ML^2T^{-2}. Therefore $a = 1$, $b = 2$, $c = -2$. Trouble $= kMd^2t^{-2}$."*

(a) Identify the fundamental flaw in Kipanga's argument.

(b) Assuming the formula were valid, calculate the "trouble" when he drops a 2 kg vase from 1.5 m and his mother finds out in 3 s. Take $k = 1$.

(c) Mr. Akilikubwa points out that the formula predicts the trouble decreases if Kipanga's mother takes longer to find out. Explain whether this makes the formula more or less believable.

Solution

(a) Kipanga assumes the conclusion before carrying out the analysis. He first declares that trouble has the dimensions of energy and then determines the powers that yield those dimensions. The argument therefore assumes what it is trying to prove and is logically circular. Dimensional analysis can relate the dimensions of known physical quantities, but it cannot establish that a non-physical quantity has particular dimensions. Trouble is not a physical quantity because it cannot be measured objectively on any agreed scale with an agreed unit.

(b) $\text{Trouble} = 1 \times 2\text{kg} \times (1.5\text{m})^2 \times (3\text{s})^{-2} = \frac{2\text{kg}\times 2.25\text{m}^2}{9\text{s}^2} = 0.5\text{J}$

(c) The formula predicts that the longer Kipanga's mother takes to discover the broken vase, the less trouble he is in (since trouble $\propto 1/t^2$). Whether delay increases or decreases the trouble depends on the mother, the vase, and the day of the week, but no one would seriously claim it follows a precise inverse-square law. The formula produces a prediction that cannot be tested on any agreed scale, illustrating that a dimensionally correct formula can be physically absurd. The algebra does not know about mothers.

If you followed all miscellaneous worked examples without breaking a sweat, you are ready for what comes next. The Digging Deeper exercise that follows will not be gentle. It will test every idea in this chapter, from the definition of a physical quantity to the propagation of errors through formulas that combine all three rules at once. Some questions will build your confidence. Others will stretch it. That is the point.

DIGGING DEEPER EXERCISE 1

EXERCISE 1A: BINDER QUESTIONS

Question 1

(a) Explain why a physical quantity must have both a number and a unit. What information does a measurement carry without a unit?

(b) Give one real-life example in which a missing or incorrect unit caused a practical problem.

Question 2

A student says: *"Length is fundamental and velocity is derived, but I can measure both directly with instruments. So what makes one fundamental and the other derived?"*

Explain clearly what determines whether a physical quantity is fundamental or derived.

Question 3

A student checks the equation $v^2 = u^2 + 2as$, finds it dimensionally correct, and concludes the equation must therefore be physically correct. Identify the flaw in this reasoning and support your answer with an example.

Question 4

State three uses of dimensional analysis. For each, give one specific example showing the method in action.

Question 5

State three limitations of dimensional analysis. For each, give one specific example demonstrating where the method fails.

Question 6

(a) Distinguish between systematic errors and random errors.

(b) Explain why the strategy for reducing each type is completely different. Give a practical example of each strategy.

Question 7

Explain what happens to the percentage error when two nearly equal measured quantities are subtracted.

Question 8

A student measures a standard mass known to be exactly 5.50 g. The readings are: 5.01, 5.02, 5.01, 5.02, 5.01 (all in grams).

(a) Are these measurements precise? Justify.

(b) Are they accurate? Justify.

(c) What type of error is likely present?

Question 9

Explain why timing 20 complete oscillations of a pendulum and dividing by 20 gives a more precise value for the period than timing a single oscillation directly. Include a numerical estimate to support your argument.

Question 10

A student claims: *"My ruler has millimetre markings, so the uncertainty in every length I measure is exactly 1 mm and nothing more."*

Identify two things wrong with this claim.

EXERCISE 1B: REAL QUESTIONS

Question 11

A tailor in Kariakoo market uses a tape measure that has been stretched from years of heavy use. The first 5 cm of the tape has expanded and now covers what should be 5.3 cm.

(a) What type of error does this introduce into every measurement the tailor makes?

(b) If the tailor measures each customer three times and averages the readings, will the error reduce? Explain.

Question 12

A doctor at Mloganzila Hospital takes a patient's blood pressure three times in quick succession and records slightly different values each time. Explain why the readings differ and why the doctor averages them rather than trusting any single reading.

Question 13

Kipute and Kipanga both measure the length of the same laboratory table. Kipute gets 1.235 m on every attempt. Kipanga gets 1.20 m, 1.27 m, 1.22 m, 1.25 m, and 1.24 m, giving a mean of 1.236 m.

Kipanga: *"We got the same answer, so our measurements are equally good."*

Kipute: *"I disagree."*

(a) Who is more precise? Justify.

(b) Can you say with certainty which set is more accurate? Explain what additional information (if any) would be needed.

(c) Explain why Kipanga is wrong to claim the measurements are equally good.

Question 14

In manufacturing, engineers specify tolerances (for example, a piston diameter of 10.000 ± 0.005 cm) rather than demanding exact dimensions.

(a) Explain why demanding an exact dimension is impossible.

(b) Explain why it is also unnecessary, provided the tolerance is met.

Question 15

A 5% error in a medicine dose can be life-threatening, but a 5% error in measuring the length of a classroom is harmless. Both are 5% errors. Explain what makes one dangerous and the other irrelevant.

Question 16

A student measures the period of a pendulum using a wristwatch.

(a) Identify two possible sources of systematic error in this experiment.

(b) Identify two possible sources of random error in this experiment.

Question 17

Explain why the relative error is more meaningful than the absolute error when comparing the quality of two different measurements. Support your answer with a specific example involving two measurements of different quantities.

Question 18

Kipanga measures his body temperature with a thermometer and gets 37.0°C on five consecutive attempts.

Kipanga: *"Five identical readings. My thermometer is both accurate and precise."*

Kipute: *"You can be sure about one of those. For the other, you need more information."*

(a) Which property (accuracy or precision) can Kipanga confirm from his data alone? Explain.

(b) What additional information or test would be needed to check the other property?

Question 19

A map of Tanzania is drawn to a scale of 1:1,000,000. A student measures the distance between Dar es Salaam and Dodoma on the map and makes an error of 1 mm.

(a) What is the corresponding error in the real-world distance?

(b) What does this tell you about the importance of precision in cartography?

EXERCISE 1C: HOT QUESTIONS

Question 20

Express the SI unit of each of the following in terms of base units only:

(a) Pressure

(b) Energy

(c) Coefficient of viscosity (defined by $F = \eta A\, dv/dx$)

(d) Surface tension (defined as force per unit length)

Question 21

The terminal velocity of a sphere falling through a viscous fluid is given by:

$$v = \frac{2r^2(\rho_s - \rho_f)g}{9\eta}$$

where r is the radius of the sphere, ρ_s and ρ_f are the densities of the sphere and fluid, g is the acceleration due to gravity, and η is the coefficient of viscosity (dimensions $ML^{-1}T^{-1}$).

Verify that this equation is dimensionally homogeneous.

Question 22

The height h to which a liquid rises in a capillary tube depends on the surface tension γ (dimensions MT^{-2}), the density ρ of the liquid, the radius r of the tube, and the acceleration due to gravity g. Use dimensional analysis to derive an expression for h in terms of these quantities.

Question 23

A student uses dimensional analysis to derive the formula for gravitational potential energy and obtains $E = mgh^2$.

(a) Use dimensional analysis to show that this formula is wrong.

(b) Find the correct power of h.

(c) State clearly what dimensional analysis alone cannot tell you about the correct formula for gravitational potential energy.

Question 24

A ball is dropped from rest at height h and takes time t to reach the ground. Given that t depends only on h and g:

(a) Use dimensional analysis to express t in terms of h and g.

(b) A student measures $h = 5.0$ m and $t = 1.01$ s. Use these values and $g = 9.8 ms^{-2}$ to determine the dimensionless constant in the formula and hence write the full formula.

Question 25

The density of a cylindrical wire is determined using $\rho = 4m/(\pi d^2 l)$. The measurements are: mass $m = (2.50 \pm 0.01)$ g, length $l = (50.0 \pm 0.1)$ cm, diameter $d = (0.80 \pm 0.01)$ mm.

(a) Convert all measurements to SI units and calculate ρ.

(b) Find the percentage error contribution from each of the three measurements.

(c) Find the total percentage error in ρ.

(d) Explain why the diameter contributes the most error despite having the smallest absolute uncertainty.

(e) The student wants the total percentage error below 3%. Which single measurement should they improve, and to what precision?

Question 26

The focal length of a converging lens is determined using $f = \frac{uv}{u-v}$ where $u = (30.0 \pm 0.5)$ cm and $v = (20.0 \pm 0.5)$ cm.

(a) Calculate f.

(b) Explain why this method produces such a large uncertainty in f even though u and v are each measured with only about 2% error.

Question 27

In an Atwood machine experiment, g is determined from:

$$g = \frac{2s(m_1 + m_2)}{(m_1 - m_2)t^2}$$

where $m_1 = (150.0 \pm 0.5)$ g, $m_2 = (130.0 \pm 0.5)$ g, $s = (0.800 \pm 0.002)$ m, and $t = (1.50 \pm 0.02)$ s.

(a) Find the total percentage error in g.

(b) Calculate g.

(c) Explain why this method gives a less precise value of g than the simple pendulum method, even though all individual measurements appear precise.

Question 28

From a straight-line graph of distance s against time squared t^2, the best-fit gradient is $4.85ms^{-2}$, the maximum gradient is $5.20ms^{-2}$, and the minimum gradient is $4.55ms^{-2}$. The relationship is $s = \frac{1}{2}at^2$.

(a) Determine the acceleration a from the best-fit gradient.

(b) Find the uncertainty in a and express the result in standard notation.

(c) Calculate the percentage uncertainty.

Question 29

Design a complete experiment to determine the density of an irregular stone using only a ruler, a measuring cylinder, water, and a balance. In your design, specify:

(a) All measurements to be taken.

(b) The instruments and their least counts.

(c) Sources of error and how to minimise them.

(d) The formula for density and how to estimate its uncertainty.

Question 30

A physics teacher wants students to determine g using a simple pendulum with a total percentage error below 1%. The pendulum length is $l = 1.000$ m, measured with a metre ruler (least count 1 mm). The total timing uncertainty from reaction time is 0.2 s.

(a) Write an expression for the total percentage error in g in terms of $\Delta l/l$ and $\Delta T/T$, using the formula $g = 4\pi^2 l/T^2$.

(b) Calculate the percentage error in l.

(c) Hence determine the maximum allowable percentage error in T.

(d) Determine the minimum number of oscillations that should be timed in a single run to achieve this target.

Chapter 2

MOTION IN STRAIGHT LINE

INTRODUCTION

A daladala moves, stops, jerks forward, and moves again. A boda-boda swerves, a football flies through the air, rain falls from the sky, and you walk from the school gate to your classroom. None of these events look like equations. Yet Physics insists that they all speak the same language; **motion**.

Most students meet motion as a collection of formulas. But your body already understands motion. You feel it when a bus starts suddenly. You predict it when you throw a ball. You fear it when a bicycle rushes downhill. Long before you meet formulas, you already live inside motion.

In this part, we rebuild the foundations you once met at O-level. Not to repeat them, but to understand them properly. We will move from everyday meaning to precise description, and only then to mathematics. By the time we reach formulas, they will feel like summaries of ideas you already own.

MOTION AND REFERENCE FRAMES

Is a person sitting in a bus at rest or in motion? The answer depends on who is watching. To another passenger, the person is at rest. To a person standing by the road, the same person is moving at the speed of the bus. Motion is therefore relative.

Physics resolves this confusion by introducing a **reference frame** which *is a chosen point of view from which motion is described.* Without stating the reference frame, statements about motion are incomplete. Whenever you describe motion, always ask: *From whose point of view is it being observed?*

For now, let us pause the theory and bring the ideas down to earth with a few carefully chosen worked examples.

BINDER Example 1

A student sits quietly inside a moving train, deeply absorbed in a book. One may argue that the student is both *at rest* and *in motion* at the same time. Justify this statement.

Solution

Since motion is not absolute but depends on the reference frame chosen, the student is **at rest relative to the train** because there is no change in position with respect to the seat, the floor, or other passengers. However, the student is **in motion relative to the ground** because the train itself is moving along the track.

Making Sense of the Answer: *The student looks still to someone inside the train, but to someone outside, the student is moving with the train. The difference comes from* ***who is observing****.*

Think Like a Physicist: *Before deciding whether something is at rest or in motion, always ask:* ***"Relative to what?"***

REAL Example 2

Mr. Akilikubwa is teaching Motion and Reference Frames. While standing by the roadside with **Kipute** and **Kipanga**, they watch a daladala moving past them. At that moment, a passenger inside the daladala accidentally drops a coin.

Kipute says, *"From here, the coin seems to follow a curved path."*

But Kipanga insists, *"To the passenger inside, the coin must fall straight down."*

Explain why Kipute and Kipanga can both be correct.

Solution

Kipute and Kipanga are both correct because they are describing the motion of the coin from **different reference frames**.

- Kipute is standing on the ground, so she observes the coin as it falls while the daladala continues moving forward. To her, the coin has **two motions at the same time**:

- ✓ a forward motion (same as the daladala), and
- ✓ a downward motion (due to gravity).

The combination of these motions makes the coin appear to follow a curved path.

- The passenger inside the daladala is moving with the vehicle, so in that reference frame the coin has **no forward motion relative to the passenger.** It only moves downward due to gravity, so it appears to fall straight down.

Making Sense of the Answer: *The coin does the same motion, but it looks different depending on where you are observing from outside or inside the moving daladala.*

Think Like a Physicist: *Whenever an object is released from a moving vehicle, it still keeps the* ***horizontal velocity*** *of the vehicle at the moment of release.*

With these worked examples complete, we are ready to welcome the next subtopic and see how the story continues.

DISTANCE AND DISPLACEMENT

Distance answers the question: *How much ground did you cover in total?* It depends on the entire path taken and it ignores direction, so it is always a **scalar quantity** and can never be negative.

Displacement answers a deeper question: *Where is your final position relative to your starting point?* It depends only on the starting and ending points (not the path), and it carries direction, so it is a **vector quantity**. Displacement can be **positive, negative, or zero**, depending on the direction chosen.

Because of this, two journeys can have the **same distance** but **different displacements**, or even have a **large distance** with **zero displacement** (when you return to your starting point). In short, *distance measures the length of the journey, while displacement measures the actual change in position.*

To truly understand and enjoy these ideas, let us serve them in the form of worked examples.

BINDER Example 3

A student walks 60m east from the dormitory to the library, then 20m west to the water tap. Find:

(a) Distance travelled

(b) Displacement

Solution

(a) Distance $= 60 + 20 = 80$m
(b) Displacement $= 60 - 20 = 40$m east

Making Sense of the Answer: *The student walked 80m, but ended only 40m east of the starting point.*

Think Like a Physicist: *Distance is always* ***greater than or equal to*** *the magnitude of displacement, because distance counts the whole path.*

REAL Example 4

After school, **Kipanga** runs one complete lap around the circular football field and stops exactly where he started.

Determine his:

(a) Distance travelled

(b) Displacement

Solution

(a) Distance travelled = the **circumference** of the circular field (one complete lap).

(b) Displacement $= \mathbf{0}$, because Kipanga ends at the same point where he started.

Making Sense of the Answer: *Kipanga ran a long distance, but his final position did not change. Distance counts the whole journey, while displacement depends only on the start and end points.*

Think Like a Physicist: *Whenever an object returns to its starting point, its displacement is zero, even though the distance travelled is not zero.*

Having given the worked examples their full say, we can now welcome the next subtopic and see what fresh ideas it brings along.

SPEED AND VELOCITY

Speed tells *how fast* an object moves. It is the rate of covering distance, so it is a scalar quantity and it is always positive or zero.

Velocity tells *how fast and in which direction* an object moves. It is the rate of change of displacement, so it is a vector quantity. Velocity can be positive, negative, or zero depending on the chosen direction.

Two cars may have the same **speed** but different **velocities** if they move in opposite directions. This is why speed is just a number, but velocity is a complete description of motion as it tells the *magnitude* and the *direction*

In Physics, direction is never a decoration. It can change the final answer, especially in problems involving **vectors**, **relative motion**, and **acceleration**.

Before the ideas drift away, let us anchor them using a few worked examples.

BINDER Example 5

Two cars are moving along the same straight road. Car A moves at **20m/s north**, while Car B moves at **20m/s south**. Compare the **speeds** and **velocities** of the two cars.

Solution

- The **speeds** of both cars are the **same** because speed depends only on how fast they move:

$$\text{Speed} = 20\text{m/s}$$

- The **velocities** are **different** because velocity includes both magnitude and direction:

  ✓ Car A: $\mathbf{20m/s\ north}$
  ✓ Car B: $\mathbf{20m/s\ south}$

Making Sense of the Answer: *Speed ignores direction, but velocity depends on direction, so opposite directions give different velocities even if the speed is the same.*

Think Like a Physicist: *If two objects move with the same speed in opposite directions, their velocities have the same magnitude but opposite signs (if one direction is chosen as positive).*

REAL Example 6

One rainy afternoon, **Kipanga** stands still at the school gate, watching the rain fall **straight down**. A moment later, he remembers he left his exercise books at home, so he starts running fast toward home. Suddenly, the rain begins to hit his face as if it is coming **from the front**, not from above. Explain why the rain appears to fall **slanted** when Kipanga starts running.

Solution

When Kipanga is standing still, the rain has only a **vertical downward velocity** due to gravity, so it appears to fall straight down.

When he starts running forward, Kipanga gains a **forward velocity**. The rain is still falling downward, but relative to Kipanga, the rain now seems to have a **horizontal component** opposite to his motion. Combining the rain's downward motion and Kipanga's forward motion makes the rain appear to move **diagonally**, so it feels like it is coming from the front and hitting his face.

Making Sense of the Answer: *The rain has not changed its motion, but Kipanga's motion changes how he observes it; running forward makes the rain seem to "chase" him from the front.*

Think Like a Physicist: *Whenever two motions happen at the same time (forward motion of the observer and downward motion of rain), the observed direction becomes the* ***resultant*** *of the two velocities.*

The worked examples have said their piece; now the acceleration subtopic clears its throat and steps forward.

ACCELERATION

Many students think acceleration means "moving fast." Others think it means "speeding up." Both ideas are close, but not complete. In Physics, acceleration does not describe how fast you are moving; it describes *how quickly your velocity changes.*

To feel acceleration, you do not need a laboratory. You only need a daladala.

When the daladala is moving at a steady speed, your body feels normal. But when it suddenly starts, you feel pushed backward. When it suddenly stops, you feel pushed forward. That feeling is not speed, it is **acceleration**.

What Acceleration Really Means?

Acceleration is the *rate of change of velocity with time.*

This means acceleration happens when:

- velocity increases (speeding up)
- velocity decreases (slowing down)
- direction changes (even at constant speed!)

So, an object can have acceleration even if its speed stays the same as long as its direction is changing.

Acceleration is given by:

$$a = \frac{\Delta v}{\Delta t}$$

Where:

a = acceleration (m/s^2)

Δv = change in velocity (m/s)

Δt = time taken (s)

For now, take a short breath and set theory aside, let us warm up with some carefully chosen worked examples.

BINDER Example 7

A car increases its velocity from 5m/s to 25m/s in 4s. Find its acceleration.

Solution

$$a = \frac{v - u}{t}$$

$$a = \frac{(25 - 5)m/s}{4s} = 5m/s^2$$

Therefore, the acceleration is $5m/s^2$.

Making Sense of the Answer: *The car's velocity increases by 5m/s every second.*

Think Like a Physicist: *Acceleration tells you "how much velocity changes per second," not how fast the object is moving.*

REAL Example 8

Mr. Akilikubwa asks **Kipute** and **Kipanga** to observe a boda-boda leaving the school gate. At first the boda-boda moves slowly, then within a few seconds it becomes very fast. What does the change in the boda-boda's motion tell you about its acceleration?

Solution
Since the boda-boda's velocity is **increasing with time**, its acceleration is **positive**.

Making Sense of the Answer: *The boda-boda is not only moving; it is **changing its velocity**, and that is what acceleration means.*

Think Like a Physicist: *If velocity increases with time, acceleration is positive; if velocity decreases, acceleration is negative.*

HOT Example 9

A bus is moving at 18m/s. The driver applies brakes and the bus comes to rest in 6s. Find the acceleration.

Solution

Given: $u = 18\text{m/s}, v = 0\text{m/s (at rest)}, t = 6s$

$$a = \frac{v - u}{t}$$

$$a = \frac{(0 - 18)\text{m/s}}{6\text{s}} = -3\text{m/s}^2$$

Therefore, the acceleration is -3m/s^2.

Making Sense of the Answer: *The negative sign shows the bus is slowing down. Its velocity decreases by 3m/s every second.*

Think Like a Physicist: *A negative acceleration does not mean "bad acceleration." It simply means the acceleration is opposite to the direction of motion.*

It is important for you to understand that:

In real life, acceleration can change every moment. A driver may press the accelerator gently, then strongly, then release it. But in many Physics problems, we simplify motion by assuming the acceleration is **constant**. This is called **uniform acceleration**.

Uniform acceleration does not mean the object is moving at a constant velocity. It means *the acceleration is constant, so the velocity changes by the same amount every second.*

That brings our worked examples to a satisfying close. The example meal has been enjoyed; now let us prepare our taste buds, because the next subtopic is about to serve its own delicious ideas.

EQUATIONS OF MOTION (SUVAT Equations)

Once you understand acceleration, Physics becomes more predictable. Why? Because acceleration connects three things that always matter in motion:

1) *How fast you started moving (initial velocity, u)?*
2) *How fast you ended up moving (final velocity, v)?*
3) *How long the change took (time, t)?*

When acceleration is constant (uniform acceleration), these quantities are linked by powerful formulas called the **equations of motion** (or **SUVAT equations**).

Meaning of SUVAT

These letters remind you of the five variables used:

1) **S** = displacement (m)
2) **U** = initial velocity (m/s)
3) **V** = final velocity (m/s)
4) **A** = acceleration (m/s^2)
5) **T** = time (s)

The Three Main Equations of Motion

When acceleration is constant, the following equations apply:

1. **First equation:**

$$v = u + at$$

2. **Second equation:**

$$s = ut + 1/2at^2$$

3. **Third equation:**

$$v^2 = u^2 + 2as$$

Understand These Simple Tactics!

- If the question involves time, think of either: $v = u + at$ or $s = ut + 1/2at^2$
- If the question does not include time, use: $v^2 = u^2 + 2as$
- If it asks about **average velocity**, remember:

$$\text{Average velocity} = \frac{u + v}{2}$$

And thus:

$$s = \left(\frac{u + v}{2}\right)t$$

Quick Reminder: Always pick the equation that matches the quantities given in the question.

Before the equations start arguing among themselves, let us invite a few worked examples to settle the matter!

BINDER Example 10

A car starts from rest and accelerates uniformly at 3m/s² for 5s. Find its final velocity.

Solution

"Starts from rest" means initial velocity is zero. Thus u = 0 m/s

Using the first equation of motion: $v = u + at$

Where: u = 0m/s, a = 3m/s², t = 5s

Substituting $v = (0\text{m/s}) + (3\text{m/s})(5\text{s}) = 15\text{m/s}$

The final velocity of the car is 15m/s.

Making Sense of the Answer: *Each second, the car gains **3m/s**. After 5 seconds it has gained **15m/s**.* **Think Like a Physicist:** *If you start from rest, the final velocity after time t depends only on a and t; that is $v = at$.*

REAL Example 11

Kipute is late for class. She runs from the school gate with an initial velocity of 2m/s and accelerates uniformly at 1.5m/s² for 6s. Find her final velocity.

Solution

Again using: $v = u + at$

Where: u = 2m/s, a = 1.5m/s², t = 6s

Substituting $v = (2\text{ m/s}) + (1.5\text{m/s})(6\text{ s}) = 11\text{m/s}$

Kipute's final velocity is 11m/s.

Making Sense of the Answer: *Kipute already had some velocity (2m/s), then she added another 9m/s due to acceleration.*

Think Like a Physicist: *Never ignore the initial velocity; it can change the entire answer.*

HOT Example 12

A bus moving at 25m/s slows down uniformly and comes to rest after travelling 100m. Find its acceleration.

Solution

"Comes to rest" means the final velocity is zero. Thus v = 0m/s.

Using the third equation of motion: $v^2 = u^2 + 2as$.

Where: u = 25m/s, s = 100m, v = 0m/s.

Substituting $(0\text{m/s})^2 = (25\text{m/s})^2 + 2(\text{a})(100\text{ m}); \text{a} = -3.125\text{ m/s}^2$

The acceleration is -3.125m/s^2.

Making Sense of the Answer: *The negative sign shows deceleration. The bus loses velocity steadily until it stops.*

Think Like a Physicist: *When time is missing, don't panic, use the equation that eliminates time.*

As the worked examples quietly leave the table, vertical motion under gravity arrives; not to overwhelm us, but to be understood and enjoyed!

VERTICAL MOTION UNDER GRAVITY

Motion can look very simple, yet it hides one of the most powerful ideas in Physics: near the Earth's surface, every object released in the air experiences the same downward pull. Whether you drop a mango from a tree, toss a stone upward, or jump off a step, the motion is controlled by a constant downward acceleration called the **acceleration due to gravity**. This is why a falling object speeds up as it descends and why an upward-moving object slows down until it briefly stops at the highest point before returning downward. Understanding this vertical motion is essential because it forms the entire foundation of **projectile motion**, where vertical and horizontal motions combine to produce curved paths.

Vertical motion under gravity is the motion of an object along a straight vertical line in which the only acceleration acting is due to gravity (air resistance is ignored).

This is going to be very helpful to you!

In dealing with motion under gravity it is important to understand the following facts and conventions about sign:

- ✓ The acceleration due to gravity (g) is always directed downward. Its value is 9.8m/s^2 which means that: each second, the vertical velocity changes by 9.8m/s.
- ✓ If an upward-thrown body is moving upward, the velocity and displacement will be positive while the acceleration due to gravity will be negative ($\text{a} = -\text{g}$).
- ✓ If the same body is moving downward, the velocity and displacement will be negative while the acceleration due to gravity will remain negative (g always acts downward).
- ✓ When an object is thrown upward, gravity acts downward (gravity opposes the motion), so the object loses velocity as it rises until it becomes momentarily at rest at highest point just before start returning downward. Thus, velocity=0m/s at the highest point.
- ✓ If a body returns to the same level it was thrown from, the motion is symmetric and hence:
    - Time going up = time coming down.
    - Speed at the same height going up = speed at the same height coming down.

The ingredients are ready; now let us see how they come together in worked examples.

BINDER Example 13

A stone is dropped from rest from the top of a building. Neglect air resistance.

(a) What is its initial velocity?

(b) What is its acceleration?

Solution

Tactical idea: "dropped from rest" ⇒ u=0m/s

(a) Initial velocity: u=0m/s ("dropped from rest" means initial velocity is 0m/s).

(b) Acceleration: $\text{a} = -\text{g} = -9.8\text{m/s}^2$ (downward, negative)

Making Sense of the Answer: *The stone starts with no velocity, but gravity immediately makes it speed up downward.*

Think Like a Physicist: *In free fall, every second adds more velocity because gravity acts continuously.*

REAL Example 14

Kipute drops a small coin from the laboratory balcony. **Kipanga** notices the coin moves slowly at first, then faster and faster until it hits the ground. Explain why.

Solution

The coin speeds up because gravity provides a constant downward acceleration, g. As time increases, the downward velocity increases, so the coin falls faster and faster.

Making Sense of the Answer: *The coin is not being pushed by hands, but gravity is pulling it every moment.*

Think Like a Physicist: *Falling objects accelerate because the acceleration due to gravity acts downward, producing velocity changes each second.*

BINDER Example 15

A stone is dropped from rest and falls for 3s. Take $g = 10m/s^2$. Find its final velocity.

Solution

Using $v = u + at$

Where: $u = 0m/s, t = 3s, a = -10m/s^2$

Substituting $v = (0m/s) + (-10m/s^2)(3s) = v = -30m/s$ (negative means the stone is moving downward.

The final velocity is $30m/s$ (downward).

Making Sense of the Answer: *Each second the stone gains 10m/s, so after 3s it gains 30m/s.* **Think Like a Physicist:** *If you start from rest, velocity after time t is simply gt.*

BINDER Example 16

A stone is dropped from rest from a height of 45m. Take $g = 10m/s^2$. Find the time taken to reach the ground.

Solution

Using $s = ut + 1/2at^2$

Where: $u = 0m/s, s = -45m, a = -10m/s^2$

Substituting $-45m = (0m/s)t - \frac{1}{2}(10m/s)t^2$

From which; $t = 3s$

The time taken is 3s.

Making Sense of the Answer: *The stone does not fall at a constant velocity. It starts from rest and gains velocity by 10m/s every second, so in the first second it covers a small distance, but in later seconds it covers much more. That is why it reaches the ground in 3s.*

Think Like a Physicist: *In vertical fall from rest, displacement depends on t^2, so the fall accelerates quickly.*

HOT Example 17

A ball is thrown vertically upward from the ground with initial velocity $19.6m/s$. Take $g = 9.8m/s^2$. Find the time taken to reach the highest point.

Solution

At the highest point, $v = 0m/s$

Using $v = u + at$

Where: $u = 19.6m/s, v = 0m/s, a = -9.8m/s^2$

Substituting $0m/s = 19.6m/s + (-9.8m/s^2)t$

From which; $t = 2s$

The time to the highest point is 2s.

Making Sense of the Answer: *Gravity removes* $9.8m/s$ *from the upward velocity every second, so it takes 2 seconds to reduce 19.6m/s* $(2 \times 9.8m/s)$ *to zero.*

Think Like a Physicist: *Time to the top is equal to* $\frac{u}{g}$.

HOT Example 18

A ball is thrown vertically upward from the ground with initial velocity 24.5m/s. Take $\text{g} = 9.8\text{m/s}^2$. Find the maximum height reached.

Solution

Again, at the highest point, $\text{v} = 0\text{m/s}$.

Using $\text{v}^2 = \text{u}^2 + 2\text{as}$

Where $\text{u} = 24.5\text{m/s}, \text{v} = 0\text{m/s}, \text{a} = -9.8\text{m/s}^2$ and $\text{s} = \text{H}$ (maximum height)

Substituting $(0\text{m/s})^2 = (24.5\text{m/s})^2 + 2(-9.8\text{m/s}^2)\text{H}$

From which; $\text{H} = 30.625\text{m}$

The maximum height is 30.625m.

Making Sense of the Answer: *The stone rises until gravity removes all its upward velocity at* $30.625m$ *above the ground.*

Think Like a Physicist: *Maximum height depends on the square of the initial velocity; that is* $H = \frac{u^2}{2g}$.

HOT Example 19

A ball is thrown vertically upward from the ground with initial velocity 19.6m/s. Take $\text{g} = 9.8\text{m/s}^2$. Find the total time of flight before it returns to the ground.

Solution

Time to reach the highest point was found using $\text{v} = 0\text{m/s}$.

And by using $\text{v} = \text{u} + \text{at}$

Where: $\text{u} = 19.6\text{m/s}, \ \text{a} = -9.8\text{m/s}^2$

Then, by substituting $0\text{m/s} = 19.6\text{m/s} + (-9.8\text{m/s}^2)\text{t}; \text{t} = 2\text{s}$

The time to the highest point is 2s.

Since the motion is symmetric, the total time of flight, $\text{T} = 2\text{t} = 2 \times 2\text{s} = 4\text{s}$.

The time of flight is 4s.

Making Sense of the Answer: *It takes 2 seconds to rise and 2 seconds to fall back to the ground, making a total of 4s.*

Think Like a Physicist: *The total time of flight is equal to* $\frac{2u}{g}$.

HOT Example 20

A ball is thrown vertically upward from the ground with initial velocity 20m/s. Take $\text{g} = 9.8\text{m/s}^2$. Find the time when the ball is at a height of 5m on its way downward.

Solution

A given height is reached twice: once going up and once coming down. The larger time corresponds to the downward path.

Using $\text{s} = \text{ut} + 1/2\text{at}^2$

Where: $\text{u} = 20\text{m/s}, \text{a} = -9.8\text{m/s}^2, \text{s} = 5\text{m}$

Substituting $5\text{m} = (20\text{m/s})\text{t} + \frac{1}{2}(-9.8\text{m/s}^2)\text{t}^2$

Rearrange to the quadratic equation:

$$4.9t^2 - 20t + 5 = 0$$

From which; $t = 0.27s$ or $3.81s$

Since the ball is on its way downward, the suitable time is the larger one which is $3.81s$.

The time at 5m on the way down is $3.81s$.

Making Sense of the Answer: *The ball passes 5m quickly when rising, then returns to 5m much later while falling.*

Think Like a Physicist: *One displacement can correspond to two different times in vertical motion.*

That brings our subtopic-by-subtopic worked examples to a satisfying close. The plates are cleared! Now let us enjoy the full buffet, where all the ideas of this topic come together in miscellaneous worked examples.

MISCELLANEOUS WORKED EXAMPLES ON MOTION IN STRAIGHT LINE

Example 21

(a) A daladala moves east. Kipanga walks inside it. Explain clearly what "Kipanga's velocity relative to the ground" means, and why it depends on direction.
(b) A daladala moves at 15m/s. Kipanga inside the daladala walks forward at 2m/s relative to the daladala.
  (i) Find Kipanga's speed relative to the ground.
  (ii) If he instead walks toward the back at 2m/s, find his speed relative to the ground.

Solution

(a) "Velocity relative to the ground" means **the speed and direction at which Kipanga's position changes as measured by an observer standing on the ground**. It depends on direction because if Kipanga walks in the **same direction** as the daladala, his ground velocity increases (the velocities add). If he walks in the **opposite direction**, his ground velocity decreases (the velocities subtract). So direction changes the final velocity even if the walking speed is the same.
(b) **For the same direction:**

$$\text{Kipanga's speed relative to ground} = \text{bus speed} + \text{Kipanga's speed relative to bus}$$

$$15\text{m/s} + 2\text{m/s} = 17\text{m/s}$$

  (i) Kipanga's speed relative to the ground is 17m/s.

For the opposite direction:

$$\text{Kipanga's speed relative to ground} = \text{bus speed} - \text{Kipanga's speed relative to bus}$$

$$15\text{m/s} - 2\text{m/s} = 13\text{m/s}$$

  (ii) Kipanga's speed relative to the ground is 13m/s.

Example 22

(a) Explain why distance can be large even when displacement is zero.
(b) A daladala cruised past a roadside checkpoint at a steady 25m/s. The officer signaled it to stop, but the driver kept going. The officer watched it pull away for 2s, then jumped onto a police bike. Starting from rest, he accelerated uniformly at 4m/s^2 , determined to catch the daladala, which continued at a constant 25m/s.
  (i) How many seconds after the daladala passed the checkpoint does the police catch it?
  (ii) How far from the checkpoint does the catch occur?

Solution

(a) Distance measures the total path covered, while displacement depends only on the starting and final positions. So if you return to the starting point, displacement becomes zero even though distance is not.
(b) Let ***t*** be the time (in seconds) measured from the instant the daladala passes the checkpoint to the moment it is caught by the police.

Daladala motion

The daladala moves at a constant velocity of 25m/s.

And for the body moving with constant velocity; displacement $=$ velocity $\times$ time

So after time t, its distance from the checkpoint is: $s_d = 25t$

Police motion

The police starts chasing 2s later.

So by time t, the police has been moving for: $t_p = t - 2$

The police starts from rest and accelerates uniformly at 4m/s^2.

Distance covered by the police in time t_p is given by: $s = ut + \frac{1}{2}at^2$

So $s_p = \frac{1}{2}a(t_p)^2$ $(u = 0 \text{ and thus } ut_p = 0)$

$s_p = 1/2 \times 4(t - 2)^2$ (Substituting $t_p = t - 2$)

$s_p = 2(t - 2)^2$

Catching condition

The catch occurs when both are at the same position:

$$s_d = s_p$$

So:

$$2(t - 2)^2 = 25t$$

From which;

$$2t^2 - 33t + 8 = 0$$

Solving gives: $t = 16.25\text{s}$

(i) The police catches the daladala 12.5s after it passed the checkpoint.

Distance of the catch

$$s_d = 25t = 25\text{m/s} \times 16.25\text{s} = 406.3\text{m}$$

(ii) The catch occurs 406.3m from the checkpoint.

Be careful!

Many students mistakenly write $t_p = t + 2$ instead of $t_p = t - 2$.

That is wrong because t is measured from the moment the daladala passes the checkpoint. The police starts **2 seconds later**, so by time, t, the police has been moving for **less time**, not more.

Therefore:

$$\text{Police time} = \text{Total time} - \text{Delay} = t - 2$$

Hence, using $t + 2$ would mean the police started **before** the daladala passed the checkpoint, which contradicts the story and leads to a wrong model.

Example 23

(a) A car moves at constant speed in a circle. Explain why it is accelerating even though the speed is constant.

(b) Car A moves east at 18m/s, Car B moves west at 12m/s.
   (i) Find their closing speed.
   (ii) If they are 450m apart, find time to meet.

Solution

(a) Velocity changes when direction changes. In circular motion the direction changes continuously, so velocity changes and acceleration exists.

(b) The two cars were moving in the opposite directions (one east and another west) and thus:

$$\text{Closing speed} = \textbf{relative speed of approach} = 18\text{m/s} + 12\text{m/s} = 30\text{m/s}$$

(i) The closing speed is 30m/s.

If the distance (in metres) travelled by car A up to a meeting point is x, then the distance travelled by car B to the same point will be $450 - x$.

Then, using: $\text{distance} = \text{speed} \times \text{time}$;

$$x = 18t \ldots\ldots (i)$$

And;

$$450 - x = 12t \ldots\ldots (ii)$$

Substituting (i) to (ii) gives;

$$450 - 18t = 12t; t = 15\text{s}$$

Alternative solution

The time to the meeting point can be found more directly by using the fact that:

$$\text{t} = \frac{\text{Distance}}{\text{Closing speed}} = \frac{450\text{m}}{30\text{m/s}} = 15\text{s}.$$

The time to the meeting point is 15s.

Example 24

(a) Explain why in real life average speed is not always the mean of the starting and final speeds.
(b) A test driver sends a car straight forward along a road for 400m in 30s. He then immediately reverses along the same line and travels 300m back in 50s. Calculate:
  (i) the average **speed** of the car,
  (ii) the average **velocity** of the car.

Solution

(a) Average speed is defined as total distance divided by total time. It depends on how the speed changes during the journey, not only on the starting and final speeds. The mean of the starting and final speeds, $(\text{u} + \text{v})/2$, gives the average speed **only when the acceleration is uniform (constant) and the motion is along a straight line with no change of direction**. In most real motions the acceleration is not constant (or the motion involves stopping, changing direction, or different road conditions), so the speed does not increase or decrease evenly with time. In such cases, $(\text{u} + \text{v})/2$ is not reliable, and the correct average speed must be found from total distance and total time.

(b) **Total distance travelled**
The car moves 400m forward and 300m backward.

$$\text{Total distance} = 400\text{m} + 300\text{m} = 700\text{m}$$

Total time taken

$$\text{Total time} = 30\text{s} + 50\text{s} = 80\text{s}$$

(i) Average speed

$$\text{Average speed} = \text{total distance} \div \text{total time}$$

$$\text{Average speed} = \frac{700\text{m}}{80\text{s}} = 8.75\text{m/s}$$

The average speed is 8.75m/s.

(ii) Average velocity

Forward displacement $= +400\text{m}$
Backward displacement $= -300\text{m}$

Net displacement $= 400\text{m} - 300\text{m} = 100\text{m}$ forward

$$\text{Average velocity} = \text{displacement} \div \text{total time}$$

$$\text{Average velocity} = \frac{100\text{m}}{80\text{s}} = 1.25\text{m/s}$$

The average velocity is 1.25m/s forward.

Interesting!
For the same motion, the magnitude of the average velocity is much smaller than the average speed. This is because the car covered a long total distance (700m), so its average speed is fairly large. However, it reversed for most of the journey, so its net displacement is only 100m from the starting point. Since average velocity depends on displacement (not total distance), its magnitude becomes much smaller.

Example 25

(a) Kipute argues: *"At the highest point, the ball stops, so acceleration must be zero."* Explain why this argument is incorrect.
(b) A ball is projected vertically upward from the ground with initial velocity u and reaches a maximum height H.
   (i) Show that the time to reach the highest point is $t_{up} = \frac{u}{g}$.
   (ii) Show that the maximum height is $H = \frac{u^2}{2g}$.
   (iii) For $u = 24.5\text{m/s}$, find H. Take $g = 9.8\text{m/s}^2$.

Solution

(a) The argument is incorrect because stopping does not mean acceleration is zero. At the highest point the velocity becomes 0m/s momentarily, but gravity is still acting downward. Since gravity acts continuously, the acceleration remains downward with magnitude g.
(b)
   (i) At the highest point, $v = 0\text{m/s}$.

   Using**:** $v = u + at$, where: $a = -g$:

$$0 = u + (-g)t_{up}$$

   So $gt_{up} = u$

   From which; $\mathbf{t_{up}} = \frac{\mathbf{u}}{\mathbf{g}}$.

(ii) Using: $v^2 = u^2 + 2as$, with $v = 0\text{m/s}, a = -g$, and displacement to the top, $s = H$:

$$0 = u^2 + 2(-g)H$$
$$0 = u^2 - 2gH$$

So $2gH = u^2$

From which; $\mathbf{H} = \frac{\mathbf{u^2}}{\mathbf{2g}}$.

(iii) Substituting $u = 24.5\text{m/s}, g = 9.8\text{m/s}^2$:

$$H = \frac{(24.5\text{m/s})^2}{2 \times 9.8\text{m/s}^2} = 30.625\text{m}$$

Thus, $H = 30.625\text{m}$.

Example 26

(a) Explain why two stones of different masses fall with the same acceleration when air resistance is neglected.
(b) Kipute says: *"A ball thrown vertically upward with initial velocity 14.7m/s reaches the highest point after 3s. I used the value of g as $9.8m/s^2$."* Without reworking the whole problem, show that Kipute's claim is incorrect.

Solution

(a) Gravity gives the same acceleration, g to all bodies because acceleration due to gravity is independent of mass (when air resistance is ignored).
(b) This can be done by calculating the value of g (constant) by using Kipute's claimed values then comparing it with the given standard value.
   The time to reach the highest point is given by $t_{up} = \frac{u}{g}$.

   From which;

$$g = \frac{u}{t_{up}}$$

Substituting Kipute's claimed values;

$$g = \frac{14.7\text{m/s}}{3\text{s}} = 4.9\text{m/s}^2 \neq 9.8\text{m/s}^2$$

Since the calculated value of g by using Kipute's claimed values is not equal to the known value of acceleration due to gravity, Kipute's claim is incorrect.

Example 27

(a) Explain why time going up equals time coming down when an object returns to the same height (air resistance neglected).

(b) Ball A is thrown vertically upward from the ground with initial velocity of 24.5m/s. At the same instant, Ball B is dropped from rest from a point 40m above the ground. Take $g = 9.8\text{m/s}^2$.

(i) Find the time when they meet.

(ii) Find the height above the ground where they meet.

Solution

(a) The motion is symmetric because the magnitude of acceleration due to gravity is constant. Consequently, the object loses velocity at the same rate while rising as it gains velocity while falling back to the same level.

(b) Using: $s = ut + \frac{1}{2}at^2$

For ball A (thrown upward from ground):

$u = 24.5\text{m/s}, s = h_A, a = -g = -9.8\text{m/s}^2$

Substituting $h_A = 24.5t - \frac{1}{2} \times 9.8t^2$

From which:

$$h_A = 24.5t - 4.9t^2 \ldots\ldots \text{(i)}$$

For ball B (dropped from 40m):

If ball A has moved the distance of h_A, then at the meeting point, ball B will move the distance of $40 - h_A$.

So: $u = 0\text{m/s}, s = -(40 - h_A), a = -g = -9.8\text{m/s}^2$

Substituting $-(40 - h_A) = 0 \times t - \frac{1}{2} \times 9.8t^2$

From which:

$$40 - h_A = 4.9t^2 \text{ or } h_A = 40 - 4.9t^2 \ldots\ldots \text{(ii)}$$

Substituting (ii) to (i) gives:

$$40 - 4.9t^2 = 24.5t - 4.9t^2; t = 1.63\text{s}$$

(i) The time of meeting is 1.63s.

Substituting the value of t in (ii) gives:

$$h_A = 40 - 4.9(1.63)^2 = 26.98\text{m}$$

The height above the ground where they meet is 26.98m.

Alternative solution

The question can be solved by using the **relative motion method** which is shorter.

Relative acceleration, $a_R = a_A - a_B = -9.8\text{m/s}^2 - (-9.8\text{m/s}^2) = 0$

And;

Relative initial velocity, $u_R = u_A - u_B = 24.5\text{m/s} - 0\text{m/s} = 24.5\text{m/s}$

Then, the equation $s = ut + \frac{1}{2}at^2$ becomes:

$$s = u_Rt + \frac{1}{2}a_Rt^2$$

Where: $u_R = 24.5\text{m/s}, s = 40\text{m}, a_R = 0$

Substituting $40 = 24.5t$; $\mathbf{t = 1.63s}$

Also for ball A; $s = ut + \frac{1}{2}at^2$

Substituting $\mathbf{h_A} = 24.5 \times 1.63 - \frac{1}{2} \times 9.8(1.63)^2 = \mathbf{26.9m}$

Example 28

(a) Two cars move toward each other. Explain why their relative speed is the sum of their speeds.
(b) Ball A is thrown vertically upward from the ground with initial velocity of 24.5m/s. One second later, ball B is thrown upward from the same point with initial velocity of 34.3m/s.
Take $g = 9.8\text{m/s}^2$.
 (i) Find the time (measured from A's launch) when Ball B catches Ball A.
 (ii) Find the height where they meet.

Solution

(a) If two cars move in opposite directions, the separation between them reduces by both distances each second, so relative speed adds.
(b) Using: $s = ut + 1/2at^2$

For ball A:

$u = 24.5\text{m/s}, s = h_A, t_A = t, a = -g = -9.8\text{m/s}^2$

Substituting $h_A = 24.5t - \frac{1}{2} \times 9.8t^2$

From which:

$$h_A = 24.5t - 4.9t^2 \;\ldots\ldots\text{(i)}$$

For ball B: thrown 1s later.

$u = 34.3\text{m/s}, s = h_B, t_B = t - 1, a = -g = -9.8\text{m/s}^2$

Substituting $h_B = 34.3(t-1) - \frac{1}{2} \times 9.8(t-1)^2$

From which:

$$h_B = -4.9t^2 + 44.1t - 39.2 \;\ldots\ldots\text{(ii)}$$

But at the catching point; $h_A = h_B$

It follows that:

$$24.5t - 4.9t^2 = -4.9t^2 + 44.1t - 39.2 \text{ or } 19.6t = 39.2; t = 2\text{s}$$

(i) The time from A's launch is 2s *(So B catches A exactly 1 second after B is launched.)*
(ii) Using $h_A = 24.5t - 4.9t^2$ (from (i))

Substituting $h_A = (24.5 \times 2 - 4.9 \times 2^2)\text{m} = 29.4\text{m}$

The height of meeting is 29.4m

Example 29

(a) State one reason why distance is always greater than or equal to the magnitude of displacement.
(b) Ball A is thrown vertically upward from the ground with initial velocity of 19.6m/s. One second later, ball B is thrown upward from the same point with velocity u_B. Find u_B so that both balls land at the same time. Take $g = 9.8\text{m/s}^2$

Solution

(a) Distance counts the full path length, while displacement is the straight-line change from start to end, so distance cannot be smaller than displacement magnitude.

(b) For a vertical throw returning to the same level, time of flight is $T = \frac{2u}{g}$.

So ball A time of flight:

$$T_A = \frac{2(19.6m/s)}{9.8m/s^2} = 4s$$

Ball B is thrown **one second later**, so for both to land together, ball B must land at $t = 4s$ from A's launch. Therefore, ball B's time in the air must be $(4 - 1)s$ or $3s$.

For ball B: $T_B = \frac{2u_B}{g}$

Substituting $\frac{2u_B}{9.8m/s^2} = 3s$

From which: $u_B = 14.7m/s$

Example 30

(a) A ball is thrown vertically upward. At some moment it is still moving upward but slowing down. Explain how this is possible.

(b) A boy wants to throw a ball from a ground so that it just reaches a roof $25m$ above the ground.
   (i) Find the required initial velocity.
   (ii) Find the time to reach the roof.

Solution

(a) The ball continues moving upward because it has an initial upward velocity. However, gravity acts downward throughout the motion, opposing the upward motion. As a result, the upward velocity decreases each second until it becomes zero at the highest point.

(b) "Just reaches" means it arrives at the top point of its motion (maximum height) at the roof, so $v = 0m/s$ there.

But the maximum height is given by the following equation:

$$H = \frac{u^2}{2g}$$

From which:

$$u = \sqrt{(2gH)} = \sqrt{(2 \times 9.8 \times 25)} = 22.1m/s$$

(i) The required initial velocity is $22.1m/s$.

$$t_{up} = \frac{u}{g} = \frac{22.1m/s}{9.8m/s^2} = 2.26s$$

(ii) The time to reach the roof is $2.26s$.

Example 31

(a) In real life, does time going up always equal time coming down for vertical throw? Explain briefly.

(b) A ball is thrown vertically upward with $u = 19.6m/s$.
   (i) Find ideal time of flight.
   (ii) State whether the real time of flight with air resistance is likely to be greater, smaller, or the same, and why.

Solution

(a) Not always.

Reason

With **air resistance** in real life, symmetry breaks (time coming down becomes greater than time going up) because resistive force opposes motion both upward and downward, changing the acceleration magnitude differently in each phase.

(b)

(i) Ideal time of flight: $T = \frac{2u}{g} = \frac{2(19.6)}{9.8} = \mathbf{4s}$.

(ii) The real time of flight is more likely to be **greater than 4s** because air resistance reduces downward velocity, increasing the time of descent by a greater amount than it reduces the time of ascent.

Example 32

(a) A mango and a stone are dropped at the same time from the same height. Many students expect the heavier stone to land first. In reality, they hit the ground together. Explain why.

(b) A ball is dropped from 20m and rebounds to 12.8m. Take $g = 9.8m/s^2$.
   (i) Find velocity just before impact.
   (ii) Find velocity just after rebound.
   (iii) Find total time from release until it reaches the top of the rebound.

Solution

(a) Near the Earth's surface, gravity gives the same acceleration to all objects regardless of their mass (ignoring air resistance). Although the stone is heavier, it does not receive a greater acceleration. Both objects start from rest and gain velocity at the same rate, so they reach the ground at the same time.

(b) Using: $v^2 = u^2 + 2as$, with $u = 0, a = -9.8m/s^2$, and, $s = h = -20m$ (downward):

Substituting $v^2 = 0^2 + 2 \times (-9.8m/s^2) \times (-20m)$
From which: $v = \sqrt{(2 \times 9.8m/s^2 \times 20m)} = 19.8m/s$

(i) The velocity just before impact is $19.8m/s$.

After the rebound, the rebound velocity will be initial velocity for ascending to the maximum height (H) of 12.8m.

Thus using:

$$H = \frac{u^2}{2g}$$

From which:

$$u = \sqrt{(2gH)} = \sqrt{(2 \times 9.8m/s^2 \times 12.8m)} = 15.8m/s$$

(ii) The velocity just after rebound is $15.8m/s$.

Time to reach the ground can be found by using:

$$s = ut + \frac{1}{2}at^2$$

Where: $u = 0, a = -g = 9.8m/s^2, s = -h = -20m$

Substituting $-20 = 0 - \frac{1}{2} \times 9.8t^2; \mathbf{t = 2.02s}$

After rebound, time to reach the maximum height, H (12.8m) can be found by using:

$$\mathbf{t_{up}} = \frac{u}{g} = \frac{15.8m/s}{9.8m/s^2} = \mathbf{1.61s}$$

Total time $= 2.02s + 1.61s = 3.63s$

(iii) The total time from release until it reaches the top of the rebound is $3.63s$.

Example 33

(a) Explain why the acceleration of a ball thrown upward is still downward even when the ball is moving upward.

(b) **Kipanga** was solving a vertical-motion problem. He calculated the time when a ball is at a height of 5m **on its way downward** and obtained 4.32s. When he presented his work, **Mr. Akilikubwa** marked the answer wrong and explained that Kipanga's mistake was taking the displacement as $s = -5m$ instead of $s = +5m$. Mr. Akilikubwa added that Kipanga had **correctly** used the acceleration due to gravity as $a = -9.8m/s^2$.

(i) Explain clearly why taking $s = -5m$ was inappropriate for a point that is 5m above the point of projection, even if the ball is moving downward at that moment.

(ii) Help Kipanga to calculate the correct time when the ball is at 5m on its way downward.

(iii) What does the time 4.32s really represent?

Solution

(a) Gravity always acts downward, so the acceleration remains downward throughout the motion. The body rises only because it was given an initial upward velocity, not because it has an upward acceleration. As it moves upward, gravity steadily reduces the upward velocity until the velocity becomes zero at the highest point.

(b)

(i) Displacement s depends on the **position of the ball relative to the starting point**, not on the direction the ball is moving at that instant. Since the ball is 5m above the point of projection, its displacement must be s = +5m whether it is rising or falling.
The fact that the ball is moving downward is shown by the **sign of velocity**, not by changing the sign of displacement.

(ii)

Obtaining value of u from the incorrectly determined value of t

Using: $s = ut + 1/2at^2$

Where: $s = -5m$, (As incorrectly used by Kipanga), $t = 4.32s$, $a = -9.8m/s^2$

Substituting Kipanga's values:

$$-5m = u(4.32s) + 1/2(-9.8m/s^2)(4.32s)^2$$

From which; $u = 20.01m/s$

So Kipanga's working implies the ball was thrown with initial velocity about $20.01m/s$.

Determining correct value of t

Because the ball is still 5m above the point of projection: $s = +5m$

Again using $s = ut + 1/2at^2$

Where: $u = 20.01m/s, a = -9.8m/s^2, s = 5m$

Substituting $5m = (20.01m/s)t + 1/2(-9.8m/s^2)t^2$

Rearrange to the quadratic equation:

$$4.9t^2 - 20.01t + 5 = 0$$

From which; $t = 0.27s$ or $3.82s$

Since the ball is on its way downward, the suitable time is the larger one which is 3.82s.

The correct time is 3.82s.

(iii) It represents the moment when the ball is **5m below** the point of projection on its way downward.

With these miscellaneous worked examples now complete, the picture should feel nicely connected. It is time to stretch our understanding a little further. Welcome to the Digging Deeper Exercise!

DIGGING DEEPER EXERCISE 2

EXERCISE 2A: BINDER QUESTIONS

Question 1

A boda-boda is "at rest" at the bus stand. State the values of:

(a) its velocity
(b) its acceleration

Give a brief explanation.

Question 2

A student says: "If acceleration is zero, the object must be at rest." Is the statement correct? Explain.

Question 3

In straight-line motion, clearly distinguish **distance** from **displacement** using one example where distance is not equal to the magnitude of displacement.

Question 4

A car moves along a straight road and its displacement is negative. Explain what "negative displacement" means physically.

Question 5

A vehicle moves in a straight line with **constant velocity** of $12\mathrm{m/s}$. State:

(a) its acceleration
(b) the net force on it.

Question 6

A body has a constant acceleration of $-3\mathrm{m/s^2}$. Explain what the negative sign indicates. Give two motion situations that fit.

Question 7

State when **speed** equals the magnitude of **velocity**. State when speed can differ from the magnitude of **velocity**.

Question 8

An object moves in a straight line and its velocity changes uniformly from $4\mathrm{m/s}$ to $10\mathrm{m/s}$ in $3\mathrm{s}$.

(a) State the sign of the acceleration.
(b) Find the acceleration.

Question 9

A body moves in a straight line, comes momentarily to rest, then continues moving in the same direction.

(a) State its velocity at that instant.
(b) Is the acceleration necessarily zero at that instant? Explain.

Question 10

A car moves along a straight line. Its velocity is $20\mathrm{m/s}$ at the start of a time interval and $20\mathrm{m/s}$ at the end of the interval.

(a) State the change in velocity.
(b) What can you conclude about the *average acceleration* during that interval?

Question 11

An object moves 40m east, then 15m west along the same straight road.

(a) Find the distance travelled.
(b) Find the displacement.

Question 12

A runner starts from rest and reaches 8m/s in 4s with uniform acceleration.

(a) Find the acceleration.
(b) Interpret your answer in one sentence.

EXERCISE 2B: REAL QUESTIONS

Question 13

Passengers in a bus notice that roadside trees appear to move backward as the bus moves forward. Explain why stationary objects outside appear to move.

Question 14

A student walks 200m east along a straight road to a shop, then walks 200m west back home. Her phone pedometer shows 400m, but her map app shows she ended where she started. Explain this difference.

Question 15

A boda-boda rider makes a straight out-and-back trip from a stand to a customer and returns to the same stand. He says, *"My average speed was not zero, but my average velocity for the whole trip was zero."* Explain why both statements can be true.

Question 16

Two daladala drivers travel the same straight route. One drives steadily; the other repeatedly speeds up and slows down due to frequent stops. Passengers describe the second trip as *"more jerky,"* even if both arrive in similar time. Explain what this suggests about their motions.

Question 17

From a bridge, Kipute drops one stone and throws another stone straight downward at the same time. She observes the thrown stone reaches the ground first. Explain why (ignore air resistance).

Question 18

In real life, a dry leaf falls more slowly than a coin. Many people conclude that heavier objects fall faster. Explain what physics says about falling under gravity and why everyday observations can mislead.

Question 19

Students notice that a falling object gets faster and faster, yet physics says the acceleration due to gravity remains constant. Explain how constant acceleration can still produce increasing velocity.

Question 20

A student measures motion along a straight road and reports a negative velocity. The car was not reversing; it was moving forward on the road. Explain how velocity can be negative even though the car is moving "forward" in ordinary language.

Question 21

A ball is thrown straight upward and later comes back down. Observers notice that it slows down on the way up and speeds up on the way down. Explain how the direction of velocity and the direction of acceleration compare during the upward and downward motion.

Question 22

A student says: *"At the highest point of its motion, the ball has no velocity, so it has no acceleration."*. Was the student correct? Explain.

EXERCISE 2C: HOT QUESTIONS

Question 23

A car moves along a straight road. It starts from rest, accelerates uniformly at 2.0m/s^2 for 8.0s, then continues at constant velocity for 10s, then decelerates uniformly at 4.0m/s^2 until it stops. Find:
(a) the maximum velocity reached

(b) the total distance travelled

(c) the total time taken.

Question 24

Two bodabodas start from the same point at the same time. Bodaboda A moves with constant velocity 12m/s. Bodaboda B starts from rest and accelerates uniformly at 1.5m/s^2. Find:

(a) the time when B first catches A

(b) the distance from the starting point where they meet.

Question 25

A ball is thrown vertically upward with velocity 24m/s. Take $\text{g} = 9.8\text{m/s}^2$. Find:

(a) the time to reach the highest point

(b) the maximum height reached

(c) the total time of flight

(d) Why the upward and downward times are equal.

Question 26

From a bridge, a stone is dropped. Exactly 1.5s later, a second stone is thrown vertically downward with velocity 20m/s from the same point. Take $\text{g} = 9.8\text{m/s}^2$. Find the time (measured from the first release) when the second stone catches the first.

Question 27

A car's velocity changes from $+18\text{m/s}$ to -12m/s in 5.0s with uniform acceleration. Find:

(a) the acceleration

(b) the displacement during the interval

(c) the distance travelled

(d) Why distance is not equal to the magnitude of displacement in this motion.

Question 28

A ball is thrown vertically upward from a balcony 30m above the ground and hits the ground after 4.0s. Take $\text{g} = 9.8\text{m/s}^2$. Find:

(a) the initial velocity of the ball

(b) the velocity just before impact.

Question 29

A stone is thrown vertically upward from the ground with velocity 25m/s. Take $\text{g} = 9.8\text{m/s}^2$. Find:

(a) the two times when the stone is at a height of 20m

(b) which time corresponds to upward motion

(c) How velocity is used to select the correct time.

Question 30

A stone is dropped from a height of 80m at the same instant another stone is thrown vertically upward from the ground with velocity 30m/s. Take $\text{g} = 9.8\text{m/s}^2$. Find:

(a) the time when they meet

(b) the height above the ground where they meet

(c) the velocity of each stone at the meeting instant.

Question 31

A ball is thrown vertically upward from a balcony 45m above the ground with velocity 10m/s. Take $g = 9.8\text{m/s}^2$. Find:

(a) the time taken to reach the ground

(b) the velocity just before impact.

Question 32

Ball A is thrown vertically upward from the ground with velocity 20m/s at the same instant Ball B is dropped from rest from a height of 30m. Take $g = 9.8\text{m/s}^2$. Find:

(a) the time when they are at the same height

(b) the height at which they meet.

Question 33

Ball A is thrown vertically upward with velocity 18m/s. Exactly 1.0s later, Ball B is thrown vertically upward from the same point with velocity 24m/s. Take $g = 9.8\text{m/s}^2$. Find:

(a) the time (from A's launch) when B catches A

(b) the height at which they meet

(c) the velocity of each ball at the meeting instant.

Question 34

A ball is thrown vertically upward from a roof 25m above the ground with velocity 15m/s. Take $g = 9.8\text{m/s}^2$. Find:

(a) the maximum height above the ground

(b) the time taken to hit the ground

(c) the velocity just before impact

(d) If a student obtained a negative value for time, explain why this result is not physically acceptable.

Question 35

Stone A is dropped from a bridge at $t = 0$. Stone B is dropped from the same point at $t = 2.0\text{s}$. Take $g = 9.8\text{m/s}^2$. At $t = 6.0\text{s}$ (from the first release), find:

(a) the separation between the stones

(b) the relative velocity of A with respect to B

(c) Explain why the separation is increasing at this instant at constant rate.

Chapter 3

NEWTON'S LAWS OF MOTION

INTRODUCTION

If you have ever tried to push a stalled motorcycle, drag a heavy desk across the floor, or suddenly brake a fast-moving bicycle, then you have already met Newton's Laws of Motion even if you did not know their names.

Every day, we interact with objects that move and objects that stubbornly refuse to move. Some start moving easily, others resist. Some stop quickly, others seem unwilling to stop at all. Yet we often describe these experiences using casual words such as *"heavy"*, *"fast"*, or *"hard to push"*. These words are useful in daily conversation, but they are not good enough for physics. Physics enjoys asking uncomfortable questions.

After completing **Chapter 2,** a curious student should naturally begin to ask deeper questions such as:

- *Why does a body at rest require effort to start moving?*
- *Why does the same push affect different objects differently?*
- *Why do we feel thrown forward when a bus suddenly stops?*

These questions are no longer about ***how*** objects move, but about ***why*** they move the way they do. The answers are found in a set of simple but powerful ideas known as **Newton's Laws of Motion**.

Newton's Laws do not describe rare or special situations. They describe ordinary motion; walking, pushing, braking, jumping, and falling. Yet from these everyday actions, they explain an astonishing range of physical phenomena, from the motion of vehicles on the road to the motion of rockets in space.

Because of this, Newton's Laws form the **foundation of mechanics**. Everything that follows in A-Level Physics builds upon them. A weak understanding here makes later topics unnecessarily difficult.

There is, however, a common danger. Many students memorize Newton's Laws as statements, but struggle to use them correctly. This happens when the laws are learned as words rather than as tools for thinking.

To avoid this, this chapter will not rush directly into formulas or laws. Instead, it will focus first on the central idea that gives meaning to all three laws. That idea is **force**.

If Newton's Laws were a complete meal, then force would be the ingredient that gives the meal its taste. Once force is understood clearly, Newton's Laws become natural and logical rather than mysterious.

With this in mind, we now turn our attention to force.

FORCE

In Chapter 2, we described motion using quantities such as displacement, velocity, and acceleration. We answered questions like *how fast*, *how far*, and *how motion changes with time*. However, one important question was left unanswered: ***Why does motion change in the first place?***

For example:

- *Why does a body at rest start moving when pushed?*
- *Why does a moving object sometimes slow down even when no one is touching it?*
- *Why does the same push affect a light object more than a heavy one?*

To answer such questions, physics introduces the idea of **force**.

Why Do We Need the Idea of Force?

A ball lying on the ground remains at rest for a long time. When kicked, it suddenly starts moving. After some distance, it slows down and eventually stops.

Describing this motion using velocity and acceleration is possible, but those quantities alone do not explain the **cause** of the motion or its change. Objects do not begin to move, stop, or change direction by themselves.

Experience tells us that motion changes only when something acts on the object. This "something" is what physics calls a **force**.

So at this stage, it is important not to treat force as an abstract formula. Instead, it should be understood as the **physical reason** behind changes in motion.

At this point, a simple but very important question arises: ***What a force is, and what it is not?***

A force can be described as a **push or a pull** resulting from the interaction between two bodies.

However, this description must be used carefully.

A force:

- is not a property that an object possesses,
- cannot exist without another body involved,
- is always associated with an interaction.

For example:

- When you push a wall, the force exists because your hand and the wall interact.
- When the Earth attracts a stone, the force exists because of the interaction between the stone and the Earth.

An isolated object, completely alone, cannot experience a force. This idea will become important later when discussing interacting systems and Newton's third law.

Avoid This Misconception

A very common misconception is that force is required to keep an object moving. Everyday experience seems to support this idea: when a person stops pushing a box, the box slows down and stops.

However, this observation does not tell the full story, how?

From Chapter 2, we learned that:

- Acceleration is the rate of change of velocity.
- Motion with constant velocity corresponds to zero acceleration.

Now we connect this to force as follows:

When the velocity of an object changes; either in magnitude or direction, the object is accelerating. Such a change does not occur without a cause. That cause is the presence of a **net force** acting on the object.

If the net force acting on a body is zero, then its acceleration is zero. In that case, the body either remains at rest or continues to move with constant velocity. This does not mean that no forces act on the body. It means that the forces acting on it **balance each other**.

Therefore:

- Force is linked to **change of motion**, not motion itself.
- An object can move with constant velocity even while forces are acting on it.

This idea is central to Newton's laws and must be clearly understood before moving forward.

Multiple Forces Acting on a Body

In real situations, a body is rarely acted upon by only one force. Usually, several forces act at the same time. For example:

- A book resting on a table is pulled downward by the Earth and pushed upward by the table.
- A moving vehicle experiences a driving force forward and resistive forces backward.

What determines the motion of the body is not any single force, but the **combined effect of all forces acting on it**. This combined effect is called the **net force** or **resultant force**.

- If the net force is zero, the motion does not change which implies that acceleration is zero.
- If the net force is not zero, the motion changes and acceleration occurs in the direction of the net force.

This idea will appear repeatedly throughout this chapter and must be kept in mind at all times.

With the ideas now simmering nicely, let us serve them properly through a few worked examples and enjoy the flavour of physics in action.

BINDER Example 1

A stone rests on the ground and remains at rest for a long time. Explain, using force ideas, why the stone does not start moving on its own.

Solution
The stone is acted upon by two main forces: its weight acting downward and the support force from the ground acting upward. These two forces act in opposite directions and balance each other. Since the resultant force on the stone is zero, there is no acceleration. With zero acceleration, the stone remains at rest.

Making Sense of the Answer: *If one force were stronger than the other, the stone would start moving upward or downward. Since it stays at rest, the forces must be balanced.*

Think Like a Physicist: *Rest or constant velocity means the resultant force on the body is zero.*

REAL Example 2

Kipute pushes a heavy wooden crate along a straight path. At first, the crate speeds up. After some time, she continues pushing with the same effort, but the crate moves at constant velocity. Explain this observation using force ideas.

Solution
At the beginning, Kipute's pushing force is greater than the forces opposing the motion, so the net force is forward. This produces acceleration, causing the crate to speed up.
Later, when the crate moves at constant velocity, its acceleration is zero. Zero acceleration means the resultant force is zero. Therefore, the resistive forces must have increased until they balance Kipute's pushing force.

Making Sense of the Answer: *It feels harder to start moving a heavy object than to keep it moving steadily, which agrees with the idea that acceleration requires a non-zero resultant force, but constant velocity does not.*

Think Like a Physicist: *Constant velocity means forces are balanced, not that no force is applied.*

HOT Example 3

Three forces act on a body moving along a straight line. Two of the forces have magnitudes of 18N and 6N and act in the same direction. The third force has magnitude 10N and acts in the opposite direction.

(a) Determine the net force acting on the body.
(b) State the effect of this net force on the motion of the body.

Solution
(a) Forces acting in one direction: 18N + 6N = 24N

Opposing force: 10N

Net force = 24N − 10N = 14N, acting in the direction of the 18N and 6N forces.

(b) Since the net force is not zero, the body accelerates in the direction of the net force. Its velocity increases in that direction.

Making Sense of the Answer: *The larger combined force must dominate, but part of it is cancelled by the opposing force, leaving a smaller resultant force of 14N.*

Think Like a Physicist: *You can decide whether motion changes by finding the net force, even before knowing the mass.*

The worked examples have done their part. Now, with curiosity switched on, let us enjoy the next subtopic as it unfolds.

INERTIA AND NEWTON'S FIRST LAW OF MOTION

Inertia

So far, we are familiar with the idea of force as the physical reason behind changes in motion. We also understand that is the **resultant force** that matters, not individual forces acting alone.

We now take a crucial step forward by examining what happens **when the resultant force on a body is zero**. Surprisingly, this simple situation explains a large part of everyday motion and introduces one of the most fundamental ideas in physics: **inertia**.

To understand inertia, consider the following observations:

First observation: Consider a book resting on a table. It remains at rest unless someone pushes it.

Second observation: Consider the same book sliding smoothly on a polished surface. It continues moving unless something stops it.

These observations suggest a common idea: *Bodies resist changes in their state of motion.*

This resistance to change is what we call **inertia**. Inertia *is the tendency of a body to remain at rest if it is at rest, or continue moving with constant velocity if it is already in motion, unless acted upon by a* ***resultant force***. It is not a force. It is a property of matter.

Mass as a Measure of Inertia

Not all bodies resist change in the same way. A light object is easy to start moving or stop. A heavy object is much harder to do so.

This tells us that inertia depends on the **mass** of the body. Therefore, we can conclude that:

- A body with large mass has large inertia.
- A body with small mass has small inertia.

So, *mass is a measure of inertia.*

This idea explains why the same force produces different accelerations on different bodies.

Newton's First Law of Motion

The ideas of inertia and resultant force are formally stated in **Newton's first law of motion** as follows:

A body remains at rest or continues to move with constant velocity in a straight line unless acted upon by a resultant force.

This law does **not** say that forces are absent. It says that **the resultant force is zero**. Therefore:

- zero resultant force implies that acceleration is zero,
- no acceleration implies that velocity remains constant (or zero).

Newton's first law is sometimes called the **law of inertia** because it describes how inertia governs motion when forces balance. The law is important because:

- It corrects the common belief that force is needed to maintain motion.
- It explains what happens when forces balance.
- It provides the reference case for all other force situations.

For now, take a short breath and set theory aside, let us warm up with some carefully chosen worked examples.

BINDER Example 4

A puck slides along a smooth horizontal surface with constant velocity. Explain what this motion tells you about the forces acting on the puck.

Solution
Constant velocity means zero acceleration. Zero acceleration means the resultant force on the puck is zero. This implies that any forces acting on the puck balance each other, so there is no unbalanced force causing a change in velocity.

Making Sense of the Answer: *If there were a forward or backward resultant force, the puck would speed up or slow down. Since its velocity remains constant, no such force exists.*

Think Like a Physicist: *Constant velocity always means zero resultant force.*

REAL Example 5

Kipanga is standing inside a bus that is moving with constant velocity along a straight road. Suddenly, the bus stops abruptly, and Kipanga is thrown forward. Explain this observation using inertia.

Solution

Before the bus stops, Kipanga and the bus are moving together with the same velocity. When the bus stops, an external force acts on the bus, but Kipanga's body tends to continue moving forward due to inertia. Since no immediate force stops his body at the same time, he is thrown forward.

Making Sense of the Answer: *Kipanga's body tries to keep its original motion even though the bus has stopped.*

Think Like a Physicist: *Inertia makes a body resist changes in velocity.*

HOT Example 6

A block rests on a horizontal surface. Several forces act on it, but the block remains at rest. What can you conclude about the resultant force acting on the block? Explain.

Solution

Resultant force acting on block is zero.

Explanation

Since the block remains at rest, its velocity is constant and equal to zero. Constant velocity means zero acceleration. According to Newton's first law, this implies that the resultant force acting on the block is zero. Therefore, the forces acting on the block must balance.

Making Sense of the Answer: *If the forces did not balance, the block would start moving in the direction of the unbalanced force.*

Think Like a Physicist: *Rest is simply constant velocity equal to zero.*

That brings our worked examples to a comfortable stop; the second law is already peeking around the corner, curious to be explored!

NEWTON'S SECOND LAW OF MOTION

From Balanced Forces to Unbalanced Forces

In Newton's first law of motion, we examined situations in which forces balance. We saw that when the resultant force on a body is zero, the body either remains at rest or continues moving with constant velocity. In simple terms:

$$\text{If resultant force} = 0\text{N, then acceleration} = 0\text{m/s}^2.$$

We now take the next step. Most real-life motion is not perfectly balanced. Vehicles start, stop, and turn. Balls speed up or slow down. A crate begins to move when pushed hard enough. All of these happen because the forces are not balancing.

This leads to a more general and more interesting case: *When the resultant force is not zero, what exactly happens to its motion?*

Newton's second law answers this question with both meaning and mathematics. It can be stated as follows:

The acceleration of a body is directly proportional to the resultant force acting on it and inversely proportional to its mass, and occurs in the direction of the resultant force.

This statement contains four key ideas:

1) Resultant force causes acceleration.
2) For a fixed mass, a greater resultant force produces greater acceleration.
3) For a fixed resultant force, a larger mass experiences smaller acceleration.

4) The direction of acceleration is the same as the direction of the resultant force.

These ideas form the foundation of all force–motion analysis in mechanics.

Mathematical Form of the Law

For motion involving constant mass, Newton's second law can be written in the simple mathematical form:

$$F = ma$$

Where:

F is the resultant force acting on the body in N,

m is the mass of the body in kg,

a is the acceleration produced in m/s^2.

This equation does not introduce a new idea; it simply expresses, in mathematical form, the physical meaning of the law. This form is the special case (when the mass of the body remains constant) of general form of Newton's second law which can be stated as follows:

The resultant force acting on a body is equal to the rate of change of its momentum and acts in the same direction as that change.

Mathematically:

$$F = \frac{dp}{dt}$$

Where:

F is the resultant force in N.

p is the momentum of the body in kgm/s,

t is time in s.

This general form is more powerful than F = ma because it is valid even in cases where *the mass changes* or where *force acts over a short time interval*. In other words; F = ma is contained in $F = dp/dt$ as shown below:

Using $p = mv$

It follows that:

$$F = \frac{dp}{dt} = \frac{d(mv)}{dt} = v\frac{dm}{dt} + m\frac{dv}{dt}$$

So mathematically, the second law can be written as:

$$F = v\frac{dm}{dt} + m\frac{dv}{dt}$$

If mass is constant:

$$\frac{dm}{dt} = 0$$

The above equation reduces to:

$$F = m\frac{dv}{dt}$$

But;

$$\frac{dv}{dt} = a$$

Hence;

$$F = ma$$

Be careful!

Newton's second law does **not** say that force creates motion. Instead, it says that force creates **changes in motion**. In other words, it says that force produces acceleration (change in velocity), not velocity.

In closing the discussion of Newton's second law, we emphasize that *the direction of a body's acceleration is always the same as the direction of the resultant force acting on it.* From this, the following three important conclusions can be drawn:

1) If the resultant force acts in the same direction as the velocity of the body, the body accelerates and its velocity increases.
2) If the resultant force acts in the opposite direction to the velocity of the body, the body decelerates and its velocity decreases.
3) If the resultant force acts perpendicular to the velocity of the body, the velocity of the body remains constant but its direction of motion changes.

This final case prepares the ground for motion in two dimensions, particularly circular motion, where acceleration is always perpendicular to velocity.

To truly understand and enjoy these ideas, let us serve them in the form of worked examples.

BINDER Example 7

A body of mass 3kg moves along a straight line. Two forces act on it: a driving force of 18N forward and a resistive force of 6N backward. At a certain instant its velocity is 4m/s forward.

(a) Find the acceleration of the body.

(b) Find its velocity after 5s.

Solution
$\text{Resultant force} = 18\text{N} - 6\text{N} = 12\text{N}$

Using $\text{F} = \text{ma}$

$$12\text{N} = 3\text{kg} \times \text{a};\ \text{a} = 4\text{m/s}^2$$

Using $\text{v} = \text{u} + \text{at}$

$$\text{v} = 4\text{m/s} + (4\text{m/s}^2)(5\text{s});\ \text{v} = 24\text{m/s}$$

(a) The acceleration is 4m/s^2 forward.
(b) The velocity after 5s is 24m/s forward.

Making Sense of the Answer: *A forward resultant force should increase velocity. A large increase is expected after 5s of steady acceleration.*

Think Like a Physicist: *Always find the resultant force first; then acceleration follows from F/m.*

BINDER Example 8

A body of mass 2kg moves along a straight line. A driving force of 10N acts forward while a frictional force of 2N acts backward. Its velocity changes from 3m/s to 13m/s in a time t. Find the value of t.

Solution
$\text{Resultant force} = 10\text{N} - 2\text{N} = 8\text{N}$

Using $\text{F} = \text{ma}$

$$8\text{N} = 2\text{kg} \times \text{a};\ \text{a} = 4\text{m/s}^2$$

Using $\text{v} = \text{u} + \text{at}$

$$13\text{m/s} = 3\text{m/s} + (4\text{m/s}^2)\text{t}$$
$$10\text{m/s} = (4\text{m/s}^2)\text{t};\ \text{t} = 2.5\text{s}$$

The time taken is 2.5s.

Making Sense of the Answer: *An acceleration of 4m/s^2 adds 4m/s each second, so gaining 10m/s should take 2.5s.*

Think Like a Physicist: *If resultant force is constant, acceleration is constant, so velocity changes linearly with time.*

BINDER Example 9

A body of mass 5kg moves along a straight line with constant velocity. Two forces act along the line: a driving force forward and a resistive force backward. State the relationship between the two forces and explain.

Solution

The relationship:

The driving force is equal in magnitude to the resistive force.

Explanation:
Constant velocity means acceleration is $0 m/s^2$. By Newton's second law, resultant force = mass × acceleration, so resultant force = 0N. Therefore, the forces along the line must balance, meaning the forward and backward forces are equal in magnitude.

Making Sense of the Answer: *If one force were larger, the velocity would not remain constant; it would increase or decrease.*

Think Like a Physicist: *Constant velocity does not mean no forces; it means zero resultant force.*

REAL Example 10

Kipute rides a bicycle along a straight road. While she pedals, the bicycle gains velocity. When she stops pedalling, the bicycle continues moving but its velocity gradually decreases until it stops. Use Newton's second law to explain both phases of motion.

Solution

During pedalling, the driving force from the chain and wheel-ground interaction exceeds resistive forces (friction and air resistance), giving a forward resultant force. A forward resultant force produces forward acceleration, so velocity increases. When she stops pedalling, the driving force drops greatly while friction and air resistance remain backward. The resultant force becomes backward, giving backward acceleration (deceleration), so velocity decreases.

Making Sense of the Answer: *This matches everyday riding: you gain velocity when pushing forward, and you lose velocity when the forward push disappears.*

Think Like a Physicist: *Speeding up or slowing down depends on whether the resultant force is in the same direction as, or opposite to, the velocity.*

REAL Example 11

Mr. Akilikubwa pushes a loaded trolley with a nearly steady push, but the trolley's velocity increases slowly at first and then increases more noticeably after it has been moving for some time. Explain why the acceleration may increase even when the applied force is nearly constant.

Solution

Reason:
The resistive force can decrease after motion begins, increasing the resultant force and therefore increasing acceleration.

Explanation:
At the start, frictional effects (especially static friction or higher starting resistance) can be larger. Once the trolley is moving, the resistive force may reduce slightly (rolling becomes smoother, wheels align, surfaces settle). If the applied force stays nearly constant but the resistive force decreases, the resultant force increases. By Newton's second law, a larger resultant force produces a larger acceleration.

Making Sense of the Answer: *Many objects feel hardest to start moving; once rolling smoothly, the same push produces more noticeable change in motion.*

Think Like a Physicist: *Acceleration changes when the resultant force changes, even if the applied force remains the same.*

HOT Example 12

A body of mass 4kg moves along a straight line with velocity 12m/s. A constant resistive force of 10N acts opposite to its motion. At the same time, a driving force of 26N acts in the direction of motion for 3s, after which the driving force is removed but the resistive force continues.

(a) Find the acceleration during the first 3s.

(b) Find the velocity after 3s.

(c) After the driving force is removed, find the additional time taken for the body to come to rest.

Solution

(a) **First 3s:**

$\text{Resultant force} = 26\text{N} - 10\text{N} = 16\text{N}$
$\text{Using } F = ma$

$$16\text{N} = 4\text{kg} \times a, a = 4\text{m/s}^2$$

The acceleration during the first 3s is 4m/s^2.

(b) $\text{Using } v = u + at$

$$v = 12\text{m/s} + (4\text{m/s}^2)(3\text{s}), v = 24\text{m/s}$$

The velocity after 3s is 24m/s.

(c) After 3s, only resistive force acts opposite to the motion direction. So resultant force = −10N.
$\text{Using } F = ma$

$$-10\text{N} = 4\text{kg} \times a;\ a = -2.5\text{m/s}^2$$

$\text{Now use } v = u + at \text{ with } v = 0\text{m/s},\ u = 24\text{m/s}$

$$0\text{m/s} = 24\text{m/s} + (-2.5\text{m/s}^2)t$$
$$2.5t = 24;\ t = 9.6\text{s}$$

The additional time to come to rest is 9.6s.

Making Sense of the Answer: *While the driving force acts, resultant force is forward so velocity increases. After it is removed, only motion resistance remains so the body must slow down to rest.*

Think Like a Physicist: *When forces change in stages, analyze the motion in stages as well, using the final velocity of one stage as the initial velocity of the next.*

HOT Example 13

A body of mass 2kg moves along a straight line. A constant driving force of 14N acts forward. The resistive force is not constant; it increases with velocity and is given by R = 2v, where R is in N and v is in m/s. At a certain instant, the velocity is 4m/s.

(a) Find the resistive force at that instant.

(b) Find the resultant force at that instant.

(c) Find the acceleration at that instant.

Solution
(a) $R = 2v = 2(4\text{m/s}) = 8\text{N}$

(b) $\text{Resultant force} = 14\text{N} - 8\text{N} = 6\text{N}$

(c) $\text{Using } F = ma$

$$6\text{N} = 2\text{kg} \times a;\ a = 3\text{m/s}^2$$

The acceleration is 3m/s^2.

Making Sense of the Answer: Higher velocity gives higher motion resistance, so the resultant force becomes smaller than the driving force, giving a moderate acceleration.

Think Like a Physicist: *If resistive force depends on velocity, acceleration is not constant; evaluate forces at the instant asked.*

HOT Example 14

A body moves along a straight line with constant velocity 15m/s. A constant resultant force of 12N then acts opposite to its motion. After 5s, the body starts to reverse its direction of motion. Find the mass of the body.

Solution
Since velocity varies continuously, the instant at which the body reverses direction is the instant at which its velocity becomes zero. Therefore, the body's velocity is momentarily zero at t = 5s.

Using $v = u + at$; where: $v = 0m/s, u = 15m/s, t = 5s$

$$0m/s = 15m/s + a(5s); a = -3m/s^2$$

Now use $F = ma$; where: $F = -12N, a = -3m/s^2$

$$-12N = m(-3m/s^2); m = 4kg$$

The mass of the body is 4kg.

Making Sense of the Answer: *A larger mass would resist the change more and would take longer to stop and reverse under the same force, so 4kg is reasonable.*

Think Like a Physicist: *"Reverses direction" usually means the velocity becomes zero at the turning instant, then changes sign.*

HOT Example 15

A force acts on a body and changes its momentum from 6kgm/s to 18kgm/s in 0.4s along a straight line. Assume the force is constant during this time.

(a) Find the resultant force.

(b) If the mass of the body is 3kg, find the change in velocity during this time.

Solution
(a) Using $F = \frac{\Delta p}{\Delta t}$
Change in momentum, $\Delta p = 18kgm/s - 6kgm/s = 12kgm/s$
Time, $\Delta t = 0.4s$

$$F = \frac{\Delta p}{\Delta t} = \frac{12kgm/s}{0.4s}; F = 30N$$

The resultant force is 30N.

(b) Using $p = mv$, change in momentum $\Delta p = m\Delta v$

$$12kgm/s = 3kg \times \Delta v; \Delta v = 4m/s$$

The change in velocity is 4m/s.

Making Sense of the Answer: *Even when a force acts for a very short time, a sufficiently large force can produce a significant change in momentum.*

Think Like a Physicist: *When a force acts for a short time, it is often more efficient to use the momentum form of Newton's second law rather than F= ma.*

HOT Example 16

A body of mass 6kg moves along a straight line with velocity 18m/s. A constant resistive force of 9N acts opposite to the motion until the body comes to rest.

(a) Find the acceleration of the body.

(b) Find the time taken to come to rest.

(c) Find the distance travelled before coming to rest.

Solution

(a) Resultant force $= -9\text{N}$
Using $F = ma$

$$-9\text{N} = 6\text{kg} \times a;\ a = -1.5\text{m/s}^2$$

The acceleration is -1.5m/s^2.

(b) Using $v = u + at$; where: $v = 0\text{m/s}, u = 18\text{m/s}, a = -1.5\text{m/s}^2$

$$0\text{m/s} = 18\text{m/s} + (-1.5\text{m/s}^2)t$$
$$1.5t = 18;\ t = 12\text{s}$$

The time taken to come to rest is 12s.

(c) Using $s = ut + \frac{1}{2}at^2$

$$s = (18\text{m/s})(12\text{s}) + 1/2(-1.5\text{m/s}^2)(12\text{s})^2$$
$$s = 216\text{m} - 0.75(144)\text{m}$$
$$s = 216\text{m} - 108\text{m}$$
$$s = 108\text{m}$$

The stopping distance is 108m.

Making Sense of the Answer: *A small resistive force gives a small deceleration, so the body takes a long time and a long distance to stop.*

Think Like a Physicist: *Once you have acceleration, use kinematics to connect velocity change to time and distance.*

The worked examples have filled the plate nicely; now it is time to enjoy the third law and see what new ideas it brings to the table.

NEWTON'S THIRD LAW OF MOTION

Newton's second law tells us what a resultant force does to a body: it produces acceleration and changes velocity. But it does not answer a deeper question: *Where do forces come from?*

In real life, a force never appears from nowhere. Whenever a force acts, it comes from an interaction between two bodies. Newton's third law tells us what is always true about forces that arise from interactions. It states that:

When two bodies interact, each body exerts a force on the other. These two forces are equal in magnitude, opposite in direction, and act on different bodies.

This is often written in short form as:

To every action there is an equal and opposite reaction.

Be careful with that short form. The key idea is not the words "action" and "reaction." The key idea is: *The pair of forces act on different bodies.*

Avoid the Most Common Misconception

Many students think Newton's third law means forces always cancel. That is not correct.

Newton's third law pairs do not cancel because they act on different bodies. Forces cancel only when they act on the same body and are opposite.

So always separate these two ideas:

1) Forces **on one body** can cancel (balanced forces).
2) Action–reaction forces do not cancel because they act on two different bodies.

How to Recognise a Third Law Pair

A Newton's third law force pair always has the following essential properties:

1. They are the same type of force (for example, both gravitational, both normal, or both frictional).
2. They have equal magnitudes.
3. They act in opposite directions.
4. They act on different bodies (never on the same object).
5. They arise from the same interaction between the two bodies.

The most reliable way to identify a third law pair is to describe the forces using clear, complete language:

- Force on A by B
- Force on B by A

These two forces form a Newton's third law pair. For example:

- Force on the book by the table (upward)
- Force on the table by the book (downward)

Newton's Third Law and Motion

Newton's third law does not directly tell you whether a body accelerates. Acceleration depends on the resultant force on that body (second law). Third law tells you how interaction forces come in pairs.

A body can still accelerate even though it is part of a third law pair, because the partner force acts on the other body, not on it. This is one reason Newton's third law becomes very powerful when studying systems such as:

1. Walking and running.
2. Vehicles moving on a road.
3. Pushing objects.
4. Recoil.
5. Two bodies connected by a string.
6. Collisions.

Later, when we study momentum and collisions, Newton's third law becomes the bridge between forces and changes in momentum during interactions. *Since the forces during an interaction are equal in magnitude and opposite in direction, the momentum changes of the two interacting bodies occur in opposite directions.* We will examine this relationship in detail when we study collisions.

For now, take a short breath and set momentum and collisions aside, let us warm up with some carefully chosen worked examples.

BINDER Example 17

A student presses a palm strongly against a wall. The hand feels pain, even though the wall does not move. Explain why the hand feels a force from the wall.

Solution

Reason:
The wall exerts a force on the hand equal in magnitude and opposite in direction to the force the hand exerts on the wall.

Explanation:
The hand and the wall interact. When the hand pushes the wall, the wall pushes back on the hand. By Newton's third law, these forces are equal and opposite and act on different bodies. The force from the wall acts on the hand, producing pressure on the skin, which is why pain is felt.

Making Sense of the Answer: *If the wall did not push back, the hand would go through the wall, which is not realistic for a rigid wall.*

Think Like a Physicist: *If you can name "force on A by B," the partner force is automatically "force on B by A."*

REAL Example 18

Kipanga is running on a dry road. Each time his foot pushes backward on the ground, his body moves forward. Explain how Newton's third law helps to explain forward motion in running.

Solution

Kipanga moves forward because the ground pushes him forward.

Explanation:
When Kipanga's foot pushes backward on the ground, the ground exerts an equal and opposite force forward on his foot. That forward force is transmitted to his body, producing a forward resultant force and therefore forward acceleration. Without the forward push from the ground, his velocity could not increase forward.

Making Sense of the Answer: *On a slippery surface, the ground cannot provide a strong forward force, so it becomes hard to run forward effectively.*

Think Like a Physicist: *You do not "move forward by pushing yourself forward"; you move forward by pushing something backward and receiving an equal forward force.*

HOT Example 19

Two students, Kipute and Kipanga, stand facing each other on a smooth surface. Kipute exerts a horizontal force of 60N on Kipanga.

(a) State the magnitude and direction of the force exerted by Kipanga on Kipute.
(b) If Kipute has mass 50kg and Kipanga has mass 75kg, find their accelerations immediately after the push.

Solution

(a) By Newton's third law, the force exerted by Kipanga on Kipute is equal in magnitude and opposite in direction to the force exerted by Kipute on Kipanga.
So *the force on Kipute by Kipanga is 60N opposite to the direction of the force on Kipanga by Kipute.*
(b) For Kipute: resultant force = 60N (in her backward direction)
Using $F = ma$

$$60\text{N} = 50\text{kg} \times a;\ a = 1.2\text{m/s}^2$$

For Kipanga: resultant force = 60N (in his forward direction)
Using $F = ma$

$$60\text{N} = 75\text{kg} \times a;\ a = 0.8\text{m/s}^2$$

Thus Kipute's acceleration is 1.2m/s^2 and Kipanga's acceleration is 0.8m/s^2, in opposite directions.

Making Sense of the Answer: *The forces are equal, but the accelerations are different because the masses are different. The smaller mass gets the larger acceleration.*

Think Like a Physicist: *Equal and opposite forces do not mean equal accelerations; acceleration depends on mass.*

With the regular worked examples now complete, we can step back and enjoy the bigger picture; a set of miscellaneous worked examples where all the ideas meet at the same table.

MISCELLANEOUS WORKED EXAMPLES ON NEWTON'S LAWS OF MOTION

Example 20

(a) A cyclist moving at constant velocity on a level road suddenly stops pedalling. Why the cyclist begins to slow down.
(b) A 0.45kg ball is initially at rest. A player kicks it so that it leaves with velocity18m/s. The contact time is 0.030s. Calculate:
  (i) the change in momentum of the ball,
  (ii) the average resultant force on the ball during contact.

Solution

(a) When pedalling stops, the driving force reduces greatly (or becomes zero) while resistive forces (air resistance and friction) still act backward. The resultant force becomes backward, so the cyclist decelerates.
(b) The solution of each part is as follows:

(i) $\Delta p = m(v - u) = 0.45kg(18m/s - 0m/s) = 8.1kgm/s$

The change in momentum is $8.1kgm/s$.

(ii) Average resultant force $F = \frac{\Delta p}{t} = \frac{8.1kgm/s}{0.03s} = 270N$

The average resultant force is $270N$.

Example 21

(a) Kipute says: *"If I throw a ball upward, there must be an upward force acting on it as it rises."*

Is Kipute correct? Explain.

(b) A crate of mass 40kg is on a rough floor. The maximum frictional force available is 180N. A worker pulls the crate with a horizontal force of 220N. When slipping begins, the frictional force becomes constant at 160N.

(i) Explain why the crate is able to move despite the presence of a large frictional force.
(ii) Calculate the acceleration immediately after it starts moving.
(iii) Calculate the velocity after 2.5s from the start of motion.

Solution

(a) Kipute is **not** correct.

Explanation

After the ball leaves the hand, there is no longer any upward push from the hand. The only significant force is weight acting downward due to gravity. The ball continues rising due to its upward velocity, but it has downward acceleration because the resultant force is downward.

(b) The solution of each part is as follows:
(i)

Reason:
The applied pulling force exceeds the maximum frictional force available.

Explanation:
Before the crate starts moving, friction is static and can increase only up to its maximum value of 180N. Since the applied horizontal force is 220N, which is greater than the maximum frictional force, the forces cannot balance. A forward resultant force therefore acts on the crate, causing it to start moving. Once slipping begins, the frictional force reduces to a constant value of 160N, allowing continued motion.

(ii) Once moving, friction$= 160N$, so resultant force $F = 220N - 160N = 60N$ forward

$$a = \frac{F}{m} = \frac{60N}{40kg} = 1.5m/s^2$$

The acceleration is $1.5m/s^2$.

(iii) $v = u + at = 0m/s + (1.5m/s^2)(2.5s) = 3.75m/s$

The velocity is $3.75m/s$.

Example 22

(a) A bus moving forward at constant velocity suddenly enters a muddy section of road and begins to slow down, even though the driver has not pressed the brakes. Explain why this does not contradict Newton's first law.

(b) A 1200 kg car is moving forward at 24m/s on a straight level road. At the instant the driver removes the foot from the accelerator, the driving force becomes zero. The total resistive force on the car depends on velocity and is given by:

$$R = 600 + 40v$$

(where R is in Newton when v is in m/s)

Assume that during this motion the car continues in a straight line and the only horizontal force on the car is this resistive force acting opposite to the motion.

Find:

(i) The initial deceleration at v = 24 m/s.

(ii) The velocity after 5.0 s (treat the deceleration as constant and equal to the initial value).

(iii) The distance covered in the 5.0 s

(iv) Explain briefly why treating the deceleration as constant is an approximation, and state whether the true velocity after 5.0 s would be greater or smaller than your answer.

Solution

(a) Newton's First Law states that if the resultant force on a body is zero, its velocity remains constant. In the muddy section of road, the resistive forces (friction and drag) increase, so the resultant force on the bus is no longer zero. The backward resultant force produces a backward acceleration (deceleration), causing the bus to slow down. This behaviour therefore agrees with Newton's First law rather than contradicting it.

(b)

Given:

$$\text{Resistive force: } R = 600 + 40v$$

(i) Initial deceleration at v = 24 m/s

$$R = 600 + 40(24)$$
$$R = 600 + 960$$
$$R = 1560\text{ N (acting backward)}$$

Since the resistive force oppose the motion, it is negative.

Thus, the resultant force on the car is: $F = -1560N$

Using Newton's second law: $F = ma$

$$a = F / m$$

$$a = \frac{-1560N}{1200kg}\ ;\ a = -1.30\ m/s^2$$

So the initial deceleration is 1.30 m/s^2. (Negative sign is omitted in the final answer because it has been contained in the word deceleration).

(ii) Velocity after 5.0 s

Using: $v = u + at$

$$v = 24m/s + (-1.3m/s^2)(5s)$$
$$v = 17.5m/s$$

The velocity is 17.5 m/s.

(iii) Distance covered in 5.0 s

Using: $s = ut + ½at^2$

$$s = 24m/s(5s) + ½(-1.3m/s^2)(5s)^2$$
$$s = 103.75m$$

The distance covered is 103.75m.

(iv) The deceleration is not truly constant because the resistive force depends on velocity. As the car slows down, v decreases, so the resistive force becomes smaller and the deceleration decreases with time. Treating the deceleration as constant therefore overestimates the slowing effect. Hence, the true velocity after 5.0 s would be **greater than 17.5 m/s**.

Example 23

(a) A passenger standing in a daladala feels as if a "force pushes them forward" when the driver suddenly applies the brakes. Explain why this feeling occurs, and state the actual direction of the resultant force on the passenger at the beginning of braking.

(b) A 1500kg car is moving forward with velocity 28m/s on a straight level road. The driver applies the brakes and the car comes to rest in 7.0 s. During braking, the total resistive force on the car may be treated as constant. Find:

(i) the acceleration of the car,
(ii) the resultant force acting on the car,
(iii) the distance travelled while braking,
(iv) the change in momentum of the car.
(v) Use your results to explain why doubling the braking time reduces the average braking force.

Solution

(a)

Reason:

The feeling occurs because of **inertia**. When the daladala begins to slow down, the passenger's body tends to maintain its original forward velocity.

Explanation:
At the instant braking starts, the daladala experiences a backward resultant force and begins to decelerate. The passenger, however, initially continues moving forward at the original velocity due to inertia. Relative to the decelerating daladala, this makes the passenger appear to move forward, creating the sensation of being "pushed" forward. In reality, no forward force acts on the passenger; instead, a **backward resultant force** must act on the passenger to reduce their forward velocity and bring them to rest with the vehicle.

(b) The solution of each part is as follows:
(i) Acceleration

$$a = \frac{v - u}{t}$$

Substituting given values:

$$a = \frac{(0\ \mathrm{m/s} - 28\ \mathrm{m/s})}{(7.0\ \mathrm{s})} = -4.0\ \mathrm{m/s^2}$$

So the car's acceleration is $-4.0\ \mathrm{m/s^2}$.

(ii) Resultant force

$$F = ma$$

Substituting:

$$F = (1500\ \mathrm{kg})(-4.0\ \mathrm{m/s^2}) = -6000\mathrm{N}$$

So the resultant force is 6000N backward.

(iii) Braking distance

$$s = ut + \tfrac{1}{2}at^2$$

Substituting:

$$s = (28\ \mathrm{m/s})(7.0\ \mathrm{s}) + \tfrac{1}{2}(-4.0\ \mathrm{m/s^2})(7.0\ \mathrm{s})^2 = 98\mathrm{m}$$

The braking distance is 98m.

(iv) Change in momentum

$$\Delta p = m(v - u)$$

$$\Delta p = (1500\ \mathrm{kg})(0\ \mathrm{m/s} - 28\ \mathrm{m/s})$$
$$\Delta p = (1500\ \mathrm{kg})(-28\ \mathrm{m/s}) = -42000\ \mathrm{kgm/s}$$

So the momentum **decreases** by 42000 kgm/s.

Alternative solution

Resultant force can be given by:

$$F = \frac{\Delta p}{\Delta t}$$

From which, the change in momentum can be given as:

$$\Delta p = F\Delta t$$

Where: $F = -6000N,\ \Delta t = 7s$

Substituting:

$\Delta p = -6000N \times 7s = -42000Ns = -42000\ kgm/s$ (the negative sign implies the decrease in momentum)

(v) Explanation about doubling braking time:

The change in momentum needed to stop the car is fixed: it is $\Delta p\ =\ -42000\ kg\ m/s$. The average resultant braking force, however, depends on how quickly this momentum change occurs:

$$\text{Average force}\ = \frac{\text{change in momentum}}{\text{time}}$$

If the stopping time is doubled while the same momentum change is required, then average force becomes half, because the same Δp is spread over a larger time. This is why increasing braking time reduces the average force experienced.

Example 24

(a) A driver says: *"Even when my engine is very powerful, the car does not accelerate much at high velocity, and may eventually continue moving with constant velocity."* Explain why this statement is physically reasonable.

(b) A car of mass 1000kg moves along a straight, level road. The engine provides a constant driving force of 2500N forward. The total resistive force opposing the motion depends on the velocity and is given by:

R = 500 + 40v

(where R is in Newton when v is in m/s)

Assume that the motion remains in a straight line and that these are the only horizontal forces acting on the car. Find:

(i) the acceleration of the car at v = 10m/s,
(ii) the resistive force when the acceleration becomes zero,
(iii) the velocity at which the car moves with constant velocity.

Solution

(a)

Reason:

Because the acceleration of the car depends on the **resultant force**, not on the engine force alone.

Explanation:
As the car's velocity increases, resistive forces such as air resistance increase. Although the engine continues to provide a large forward force, the increasing resistive force reduces the resultant forward force acting on the car. According to Newton's second law, a smaller resultant force produces a smaller acceleration.

At sufficiently high velocity, the resistive force may become equal to the engine force, making the resultant force zero and causing the acceleration to fall to zero, leading to the constant velocity.

(b) The solution of each part is as follows:
(i) Using: $\text{Resultant force F}\ =\ \text{Driving force}\ -\ \text{Resistive force}$

Where:

$\text{Driving force} = 2500N$ (forward)

$\text{Resistive force R} = 500 + 40v$ (backward); with $v = 10m/s, R = 500 + 40 \times 10 = 900N$

It follows that:

$$F = 2500N - 900N = 1600N \text{ (forward)}$$

Then using: $F = ma$ or $a = \frac{F}{m}$

Substituting:

$$a = \frac{1600N}{1000kg}$$
$$a = 1.6m/s^2$$

So the acceleration when $v = 10m/s$ is $1.6m/s^2$.

(ii) The acceleration becomes zero, when the resultant force is zero.

Now from: Resultant force $F =$ Driving force $-$ Resistive force

When $F = 0$; Resistive force $=$ Driving force

But: Driving force $= 2500N$

Hence, the resistive force when the acceleration becomes zero is $2500N$.

(iii) Constant velocity implies zero acceleration and from (ii) above, acceleration is zero when resistive force, R is $2500N$.

Using the resistive force expression:

$$R = 500 + 40v$$

Substituting:

$$2500 = 500 + 40v$$

From which: $v = 50m/s$

So the constant velocity is $50m/s$.

Example 25

(a) A driver says: *"If my car is moving forward, the resultant force on it must be forward."* Explain why this statement is physically incorrect.

(b) Mr. Akilikubwa is driving a 1500kg car to school along a straight, level road at a steady velocity of 22m/s. As he approaches the school gate, a goat suddenly starts crossing the road ahead. However, being briefly absorbed in thought about the Physics lesson he is about to teach, he delays applying the brakes for 0.60s.
During this reaction time, the car continues to move forward with the same velocity. After the delay, he applies the brakes, and from that moment the resultant force acting on the car is constant and equal to 4500N, acting backward, until the car comes to rest. Find:
   (i) the deceleration during braking,
   (ii) the time taken to stop after braking begins,
   (iii) the braking distance,
   (iv) the total stopping distance from the moment Mr. Akilikubwa first sees the goat.
   (v) Explain, using your results, why reaction time can be as important as braking strength.

Solution

(a)

Reason:
Because the direction of the resultant force is the direction of **acceleration**, not necessarily the direction of **velocity**.

Explanation:
A body can move forward while slowing down. In that case, its velocity is forward but its acceleration is backward. Since the resultant force is in the direction of acceleration (Newton's second law), the resultant force can be backward even while the car is still moving forward. Therefore, forward motion does not require a forward resultant force.

(b) The solution of each part is as follows:

(i) Using Newton's second law:

$$F = ma \text{ or } a = \frac{F}{m}$$

Substituting:

$$a = \frac{-4500N}{1500kg}$$
$$a = -3.0m/s^2$$

So the deceleration is $3.0m/s^2$.

(ii) During braking: $u = 22m/s, v = 0m/s, a = -3.0m/s^2$

Using: $v = u + at$

$$0m/s = 22m/s + (-3.0m/s^2)t$$

From which: $t = 7.33s$

So the braking time is 7.33s.

(iii) Using: $s = ut + ½at^2$

$$s = (22m/s)(7.33s) + ½(-3.0m/s^2)(7.33s)^2 = 80.66m$$

The braking distance is 80.66m.

(iv) $\text{Total stopping distance} = \text{Reaction distance} + \text{Braking distance}$

But during reaction, acceleration is zero, so velocity remains 22m/s.

So $\text{reaction distance} = \text{velocity} \times \text{reaction time} = (22m/s) \times (0.60s) = 13.2m$

$\text{Total stopping distance} = 13.2m + 80.66m = 93.86m$

So the total stopping distance is 93.86m.

(v) From the results, the car travels **13.2m** before braking even begins. This distance depends only on reaction time and initial velocity, not on braking force. Even if the brakes are very strong, a long reaction time adds a significant distance to the stopping distance. Therefore, improving reaction time (alertness, attention) can reduce total stopping distance just as meaningfully as improving braking strength.

Example 26

(a) Why is it safer to land on sand than on concrete when falling from the same height, even though the person comes to rest in both cases?

(b) A 70 kg person running forward at 6.0m/s falls and is brought to rest.
Case I: stopping time is 0.10s.
Case II: stopping time is 0.40s.

Calculate:

(i) the change in momentum in each case,
(ii) the average resultant force magnitude in each case,
(iii) the ratio of the forces.
(iv) Comment on what the results imply about injury risk.

Solution

(a) The momentum change needed to come to rest is the same in both cases. However, soft ground deforms during impact, increasing the stopping time. Since the average resultant force equals the change in momentum divided by the stopping time, a longer stopping time results into a smaller force during impact.

(b) The solution of each part is as follows:

(i) $\Delta p = m(v - u) = 70(0 - 6.0) = -420\ kgm/s$
So in each case, the momentum decreases by 420kgm/s.

(ii) Using average force, $F = \frac{\Delta p}{\Delta t}$

$$\textbf{Case I}: F_1 = \frac{420\text{kgm/s}}{0.10\text{s}} = 4200\text{N}$$

$$\textbf{Case II}: F_2 = \frac{420\text{kgm/s}}{0.40\text{s}} = 1050\text{N}$$

(iii) $\frac{F_1}{F_2} = \frac{4200}{1050} = 4$

(iv) The shorter stopping time produces four times the average force, increasing injury risk.

Example 27

(a) **Kipute** and **Kipanga** are revising Newton's laws of motion after class. Kipanga says:

"If an object is moving in a circle at constant speed, then there is no acceleration, because acceleration only occurs when speed changes."

However, Kipute disagrees**.** Who is correct? Explain why.

(b) A ball of mass 0.30kg is moving horizontally at 8.0m/s. It strikes a wall and rebounds along the same line with velocity 6.0m/s in the opposite direction. The contact time is 0.040s. Calculate:

(i) the change in momentum of the ball,

(ii) the average resultant force on the ball during contact and its direction.

(iii) Using your answers in (i) and (ii), explain why the force on the ball during contact is opposite to its initial direction of motion, even though the ball rebounds with a smaller velocity than it had before impact.

Solution

(a) **Correct student:** Kipute

Reason:
Acceleration depends on the change of **velocity**, not only on the change of speed.

Explanation:
Velocity is a vector quantity, so it changes when either its magnitude (speed) or its direction changes. In circular motion, even though the speed remains constant, the direction of motion changes continuously. Because the direction of velocity is changing at every instant, the velocity is changing, and therefore the object has acceleration.

(b) The solution of each part is as follows:

(i) Take the initial direction as positive: $u = +8.0\text{m/s}, v = -6.0\text{m/s}$

It follows that:

$$\Delta p = m(v - u) = 0.3\text{kg}(-6\text{m/s} - 8\text{m/s}) = 0.3\text{kg}(-14\text{m/s}) = -4.2\text{kgm/s}$$

The change in momentum is -4.2kgm/s; (the negative sign shows that the change in momentum is directed opposite to the original direction of motion of the ball).

(ii) Average force $F = \frac{\Delta p}{t} = \frac{-4.2\text{kgm/s}}{0.04\text{s}} = -105\text{N}$

The magnitude of the average resultant force is 105N, and its direction is opposite to the ball's initial direction of motion.

(iii)

Reason:
The direction of the resultant force depends on the direction of the change in momentum, not on the final velocity alone.

Explanation:
Before impact, the ball's momentum is in the direction of its initial motion. After rebounding, its momentum is in the opposite direction. Therefore, the change in momentum is directed opposite to the initial motion. Since the average resultant force acts in the direction of the change in momentum, the force during contact must be opposite to the initial direction of motion, even though the rebound velocity is smaller.

If the ideas now feel familiar and connected, you are ready! Let us understand and enjoy them even more in the Digging Deeper Exercise in the next page

DIGGING DEEPER EXERCISE 3

EXERCISE 3A: BINDER QUESTIONS

Question 1

A body is observed to move in a straight line with constant velocity for several seconds. Explain what must be true about the vector sum of forces during this interval, and why this conclusion does not depend on the magnitude of the velocity.

Question 2

Two bodies move along the same straight line:

Body A is at rest.

Body B moves with constant velocity 8m/s.

Explain why physics imposes the same force-condition on both situations, even though one involves motion and the other does not.

Question 3

Everyday experience suggests "motion needs force." Identify precisely which physical quantity actually needs a resultant force to change, and explain why the everyday statement seems true on Earth.

Question 4

Give a careful explanation of why "no force acts" and "resultant force is zero" are not the same statement, and explain which one matches real motion on Earth more often.

Question 5

A student says: *"Since Newton's third law suggests that forces come in equal and opposite pairs, acceleration should be impossible."* Explain the exact error in this reasoning without using any numerical example.

Question 6

Two students push each other on a smooth surface and move apart. Without using action–reaction language as a shortcut, explain why both can accelerate even though they interact with equal and opposite forces.

Question 7

A body moves upward while slowing down. Many learners say *"the acceleration is upward because it is moving upward."* Explain why that statement is wrong and how to decide the acceleration direction correctly.

Question 8

A student insists that *"heavier objects always fall faster because they have larger weight*." Explain the missing idea that repairs the argument and show how Newton's second-law thinking resolves the confusion.

Question 9

On a rough horizontal surface, a box is pushed and moves at constant velocity. Explain what this implies about the horizontal forces, and why this does not imply that friction is absent.

Question 10

A student treats the normal reaction as an "automatic equal and opposite" force to weight. Explain why this is not generally correct and describe one condition that must be true for $N = mg$ to hold.

Question 11

Two objects collide and exert forces on each other for the same time interval. Explain how they can experience equal and opposite forces yet undergo different accelerations and different changes in velocity.

Question 12

In a braking event, explain why increasing stopping time reduces injury risk, and identify which Newton-law idea links force size to how rapidly velocity changes.

EXERCISE 3B: REAL QUESTIONS

Question 13

Kipute is standing in a daladala. The driver brakes suddenly and she lurches forward even though nobody pushes her. Explain this observation.

Question 14

Kipanga rides a bicycle on a straight road. When he stops pedalling, the bicycle does not stop immediately but gradually slows down. Explain what this reveals about the forces acting during the motion.

Question 15

Mr. Akilikubwa places a phone on the dashboard of a moving car. When the car accelerates forward, the phone sometimes slides backward. Explain why the phone does not always move together with the car.

Question 16

Kipute pushes a heavy desk across a classroom floor. At first it does not move, then it suddenly starts moving. Explain why the behaviour changes.

Question 17

A bag of cement is loaded in the back of a pickup. When the pickup speeds up, the bag tends to slide backward relative to the vehicle. Explain why this relative motion occurs.

Question 18

Kipute is inside a lift. As the lift starts moving upward, she feels heavier for a moment. As it slows down near the top, she feels lighter. Explain these sensations.

Question 19

A bodaboda rider carries a passenger and tries to accelerate away from rest. Compared with riding alone, the motorcycle picks up speed more slowly. Explain why.

Question 20

A student jumps down from a low platform and bends their knees deeply on landing. Explain why this reduces the risk of injury.

Question 21

A book rests on a table. Kipanga says: *"Because the book is not moving, there are no forces acting on it."* Explain why this statement is incorrect.

Question 22

Kipanga rides a bicycle through a muddy path and then on a smooth tarmac road. He notices that the same pedalling effort gives different accelerations. Explain this difference.

Question 23

A bodaboda suddenly stops but the passenger may fall if not holding tightly. Explain this effect.

Question 24

Two identical plastic bottles are pushed along the floor with the same initial velocity. One is empty and one is filled with sand. The filled bottle travels a shorter distance before stopping. Explain why this happens in real conditions.

Question 25

A car hits a pothole and passengers feel a sudden upward jolt. Explain why the forces on the passengers change suddenly.

EXERCISE 3C: HOT QUESTIONS

Question 26

A lift moves vertically. At a certain instant, the lift is moving upward but slowing down at $2.0m/s^2$. A passenger of mass 70kg stands on a scale.

(a) Determine the scale reading at that instant.
(b) State whether the passenger feels heavier, lighter, or normal, and justify using the result.

(Take $g=9.8m/s^2$)

Question 27

A block of mass 5.0kg rests on a rough horizontal surface. The coefficient of static friction is 0.40 and the coefficient of kinetic friction is 0.30.

A horizontal force P is applied and gradually increased.

(a) Determine the maximum value of P for which the block remains at rest.
(b) Immediately after the block just begins to move, determine its acceleration if P remains unchanged.

(Take $g=9.8m/s^2$)

Question 28

A car of mass 1000kg travels on a straight horizontal road at 25m/s. The driver applies the brakes, producing a constant braking force of magnitude F. The car comes to rest in 100m.

(a) Determine F.
(b) Determine the stopping time.
(c) State whether increasing the mass of the car (with the same braking force) would increase, decrease, or leave unchanged the stopping distance. Explain.

Question 29

Kipute and Kipanga stand on frictionless ice facing each other. Kipute has mass 50kg and Kipanga has mass 75kg. They push each other apart and Kipute moves away at 4m/s.

(a) Determine Kipanga's velocity immediately after the push.
(b) Determine the ratio of their accelerations during the push.

Question 30

A ball of mass 0.20kg strikes a wall normally with velocity 6.0m/s and rebounds with velocity 4.0m/s. The contact time with the wall is 0.020s.

(a) Determine the average force exerted by the wall on the ball.
(b) State the direction of this force clearly.

ANSWERS TO DIGGING DEEPER EXERCISES

EXERCISE 1A

1. (a) A physical quantity is a property that can be measured and expressed as a number with a unit. The number tells you how much; the unit tells you of what. Without a unit, the number carries no information. "I ran 5" could mean 5 metres, 5 kilometres, or 5 hours. The measurement is meaningless until the unit is specified.

(b) In 1999, the Mars Climate Orbiter spacecraft was destroyed because one engineering team reported thrust data in pound-force seconds while NASA's navigation software expected newton seconds. The number was correct; the missing unit conversion cost 327 million US dollars.

2. A fundamental quantity is one that cannot be expressed in terms of simpler physical quantities. Length is fundamental because no combination of other quantities defines what length is. Velocity is derived because it can be expressed as length divided by time. The classification depends on whether the quantity can be decomposed into simpler quantities, not on whether it can be measured directly with an instrument.

3. Dimensional correctness is necessary but not sufficient for physical correctness. For example, $v^2 = u^2 + 3as$ is dimensionally correct (every term has dimensions L^2T^{-2}), but the coefficient 3 is wrong. The correct equation has coefficient 2. Dimensional analysis catches equations that are definitely wrong; it cannot confirm that an equation is definitely right.

4. (1) Checking formulas: verify that $s = ut + \frac{1}{2}at^2$ is dimensionally homogeneous by confirming that all terms have dimensions L.

(2) Deriving relationships: show that the period of a simple pendulum must have the form $T = k\sqrt{l/g}$ by equating powers of M, L, and T in the assumed relationship $T = kl^a g^b$.

(3) Converting units between systems: convert the gravitational constant G from CGS ($6.67 \times 10^{-8} g^{-1} cm^3 s^{-2}$) to SI ($6.67 \times 10^{-11} kg^{-1} m^3 s^{-2}$) by raising each base-unit conversion factor to the power given by the dimensional formula $M^{-1}L^3T^{-2}$.

5. (1) Cannot determine dimensionless constants: dimensional analysis gives $T = k\sqrt{l/g}$ but cannot find $k = 2\pi$. The constant must come from a full derivation or experiment.

(2) Cannot distinguish quantities with the same dimensions: work and torque both have dimensions ML^2T^{-2}, yet they are physically different quantities. Dimensional analysis treats them as identical.

(3) Cannot handle trigonometric, logarithmic, or exponential functions: the formula $x = A\sin(\omega t + \phi)$ cannot be derived by dimensional analysis because the sine function requires a dimensionless argument that is invisible to the method.

6. (a) A systematic error shifts all readings in the same direction by the same amount or proportion. It is consistent and repeatable. A random error varies unpredictably in both magnitude and direction from one reading to the next.

(b) Systematic errors are not reduced by repetition. Example: a balance with a zero error of +0.2 g adds 0.2 g to every reading. Averaging 50 readings still gives a mean that is 0.2 g too high. The cure is recalibration.

Random errors are reduced by repetition and averaging. Example: slight variations in pendulum timing scatter readings above and below the true value. The mean of 20 readings is closer to the true period than any single reading because the high and low deviations tend to cancel.

7. When two nearly equal quantities are subtracted, the absolute errors add, but the result is much smaller than either original measurement. The relative error of the difference is therefore much larger than the relative error of either individual measurement.

8. (a) Yes, the measurements are highly precise. They cluster tightly between 5.01 g and 5.02 g, a spread of only 0.01 g.

(b) No, they are not accurate. The mean is 5.014 g, which is displaced from the true value (5.50 g) by 0.486 g.

(c) A systematic error is present, most likely a calibration error or zero error in the balance. The instrument consistently gives readings that are approximately 0.5 g too low.

9. When timing a single oscillation of period roughly 2 s, the total timing uncertainty comes from reaction time at start and stop, roughly 0.2 s each in the worst case, giving about 0.2 s total uncertainty. The percentage error is approximately $0.2/2 = 10\%$.

When timing 20 oscillations, the same reaction time uncertainty (about 0.2 s) applies to the total time of about 40 s. The percentage error in the total time is $0.2/40 = 0.5\%$. Dividing by 20 gives the period with the same percentage error of 0.5%.

The percentage error drops from 10% to 0.5%, an improvement by a factor of 20, simply by timing more oscillations in a single run.

10. First thing: The uncertainty from a ruler is not necessarily exactly 1 mm. When measuring a length, there is an uncertainty at both ends (where the object starts and where it ends). If each end has an uncertainty of 0.5 mm (half the smallest division), the total uncertainty from the ruler alone is up to 1 mm. But if one end is carefully aligned at the zero mark, the uncertainty at that end may be smaller, making the total less than 1 mm.

Second thing: The total uncertainty includes more than just the instrument resolution. Parallax, the object not lying perfectly straight, temperature expansion, and human judgement all add additional uncertainty. The actual uncertainty in the measurement may be larger than 1 mm even though the ruler's resolution is 1 mm.

EXERCISE 1B

11. (a) A systematic error. Every measurement is consistently too short, because the stretched section makes the first 5 cm of the tape cover more than 5 cm of actual length. All readings are shifted in the same direction by the same proportion.

(b) No. Averaging cannot reduce a systematic error. Every repetition carries the same shift, so the mean of three wrong readings is still wrong by the same amount. The tailor needs a new tape measure or must calibrate the old one against a known standard length.

12. The readings differ because of **random errors**: slight variations in cuff pressure, the patient's blood flow between heartbeats, small differences in arm positioning, and the doctor's judgement in reading the gauge. These fluctuations are unpredictable and vary from one reading to the next. Averaging reduces their effect because the deviations above the true value tend to cancel those below it. The mean of three readings is closer to the true blood pressure than any single reading.

13. (a) Kipute is more precise. Her readings are perfectly consistent (zero spread), while Kipanga's scatter over a range of 0.07 m.

(b) Accuracy cannot be determined without knowing the true length of the table. Without the true value, we can assess precision but not accuracy.

(c) Kipanga is wrong because the quality of a set of measurements depends on both precision and accuracy, not just the mean value. Kipute's measurements are far more precise: any single reading from Kipute is reliable, while any single reading from Kipanga could be anywhere from 1.20 m to 1.27 m. In an experiment where individual readings matter (for example, plotting a graph), Kipute's data is far superior.

14. (a) No manufacturing process can produce a part with zero uncertainty. Every cutting tool, lathe, and mould introduces some variation. Demanding exactly 10.000 cm would require infinite precision, which is physically impossible.

(b) A piston does not need to be exactly the right size; it needs to be close enough to function correctly. A piston within the tolerance range 10.000 ± 0.005 cm will fit properly, seal against combustion gases, and move freely. Any dimension within this range gives acceptable performance, so exact perfection is unnecessary.

15. The danger depends on the consequences of the error, not just its percentage. A 5% error in a drug dose for a medication with a narrow therapeutic window can push the dose from effective to toxic, risking organ damage or death. A 5% error in a classroom length (say 0.5 m in a 10 m room) has no safety, medical, or structural consequence. The same percentage error carries completely different stakes depending on what is being measured and what depends on it.

16. (a) Systematic errors: (1) The wristwatch may run consistently fast or slow, adding the same bias to every timing. (2) The student's reaction time at the start is roughly the same on every trial, creating a consistent delay.

(b) Random errors: (1) The student's reaction time varies slightly from one trial to the next, sometimes pressing the button a fraction early, sometimes late. (2) Small air currents cause the pendulum's amplitude and period to fluctuate unpredictably between swings.

17. Absolute error has units and tells you the size of the uncertainty, but not its significance. Relative error is dimensionless and expresses the uncertainty as a fraction of the measured value, allowing fair comparison between different measurements.

Example: Measurement A has an absolute error of 0.1 mm on a 2 mm wire diameter (relative error 5%). Measurement B has an absolute error of 10 m on a 2 km bridge length (relative error 0.5%). Measurement B has a much larger absolute error but is far more precise in relative terms. Relative error reveals this difference; absolute error hides it.

18. (a) Kipanga can confirm precision. His five readings are identical (37.0°C every time), so the spread is zero and the measurements are highly precise.

(b) To check accuracy (the other property), Kipanga would need to compare his thermometer against a known standard. For example, he could measure the temperature of pure melting ice (should read 0°C) or steam above boiling water at standard atmospheric pressure (should read 100°C). If the thermometer reads correctly at these reference points, it is accurate. If not, it has a systematic error, and all his 37.0°C readings are displaced from the true temperature by the same amount.

19. (a) At a scale of 1:1,000,000, each millimetre on the map represents $1{,}000{,}000\ \text{mm} = 1000\ \text{m} = 1\ \text{km}$ in reality. So an error of 1 mm on the map corresponds to an error of 1 km on the ground.

(b) In cartography, even tiny measurement errors are amplified enormously by the scale factor. A slight wobble of a pen or a 1 mm misplacement of a point translates to a kilometre-scale error in the real world. This is why cartographers work with extremely high precision and why modern maps rely on GPS systems that measure to within centimetres.

EXERCISE 1C

20 (a) $\text{kgm}^{-1}\text{s}^{-2}$ (b) $\text{kgm}^{2}\text{s}^{-2}$(c) $\text{kgm}^{-1}\text{s}^{-1}$ (d) kgs^{-2}

21.

Left side: $[\text{v}] = \text{LT}^{-1}$

Right side:

$$\frac{[r^2][\rho_s - \rho_f][g]}{[\eta]} = \frac{L^2 \times ML^{-3} \times LT^{-2}}{ML^{-1}T^{-1}} = \frac{ML^0T^{-2}}{ML^{-1}T^{-1}} = LT^{-1}$$

Both sides have dimensions LT^{-1}. The equation is dimensionally homogeneous.

22. Assume $h = k\gamma^a\rho^b r^c g^d$

Dimensions: $[h] = L$, $[\gamma] = MT^{-2}$, $[\rho] = ML^{-3}$, $[r] = L$, $[g] = LT^{-2}$

$$L = (MT^{-2})^a(ML^{-3})^b(L)^c(LT^{-2})^d = M^{a+b}L^{-3b+c+d}T^{-2a-2d}$$

For M: $0 = a + b$ … (i)

For T: $0 = -2a - 2d$, so $d = -a$ … (ii)

For L: $1 = -3b + c + d$ … (iii)

From (i): $b = -a$. Substituting into (iii): $1 = 3a + c - a = 2a + c$

Four unknowns but three equations: the system is underdetermined. Taking $a = 1$ (simplest case): $b = -1$, $d = -1$, $c = 1 - 2(1) = -1$.

$$h = k\frac{\gamma}{\rho r g}$$

23.(a) $[E] = ML^2T^{-2}$

$$[mgh^2] = M \times LT^{-2} \times L^2 = ML^3T^{-2}$$

Since $ML^2T^{-2} \neq ML^3T^{-2}$, the formula is dimensionally incorrect.

(b) Let $E = mgh^n$: $ML^2T^{-2} = M \times LT^{-2} \times L^n = ML^{1+n}T^{-2}$

Equating powers of L: $2 = 1 + n$, giving $n = 1$.

(d) Dimensional analysis gives $E = kmgh$ but cannot determine the dimensionless constant k.

24. (a) $t = k\sqrt{\frac{h}{g}}$ (b) $k = \sqrt{2}$; The full formula is $t = \sqrt{\frac{2h}{g}}$.

25. (a) $9950 kgm^{-3}$ (b) From m: 0.4%. From l: 0.2%. From d: 2.5%. (c) 3.1%

(d) The diameter is the smallest measured quantity (0.80 mm), so even a small absolute error (0.01 mm) gives a large relative error (1.25%). The power of 2 then doubles this to 2.5%. The smallest measurement, raised to the highest power, always dominates.

(e) The diameter contributes 2.5% out of 3.1%. To bring the total below 3%, its contribution must drop below about 2.4%. Since $2 \times \Delta d/d < 2.4\%$, we need $\Delta d < 0.0096$ mm. A micrometer with 0.005 mm resolution would achieve this, reducing the diameter contribution to $2 \times 0.005/0.80 = 1.25\%$ and the total to $0.4\% + 0.2\% + 1.25\% = 1.85\%$.

26. (a) $f = (60 \pm 9)$ cm

(b) Each measurement has only about 2% error. But the denominator $(u - v) = 10$ cm is much smaller than either u or v, while its absolute error (1.0 cm) is the sum of both individual errors. This gives 10% relative error in the denominator alone. Subtracting two similar quantities destroys the precision of the original measurements.

27. (a) 8.3%, $g = (10.0 \pm 0.8) ms^{-2}$

(b) The simple pendulum formula $g = 4\pi^2 l/T^2$ contains no subtraction of nearly equal quantities. Its total error is typically below 1%. The Atwood machine formula requires $(m_1 - m_2)$, which contributes 5% error from the subtraction alone, even when each mass is measured to only 0.3%. This catastrophic cancellation is an inherent weakness of the Atwood method when the two masses are close in value.

28. (a) $9.70 ms^{-2}$ (b) $\Delta a = 0.65 ms^{-2} \approx 0.7 ms^{-2}$, $a = 9.7 \pm 0.7 ms^{-2}$ (c) 6.7%

29.

(a) Measurements: mass of stone m (using balance), volume of stone V (by displacement, measure water level in measuring cylinder before and after submerging the stone).

(b) Balance (least count 0.1 g or 0.01 g), measuring cylinder (least count 1 ml or 0.5 ml).

(c) Sources of error:

(1) Systematic error: trapped air bubbles on the stone's surface increase the apparent volume. Minimise by tilting the stone as it enters the water and tapping the cylinder.

(2) Systematic error: the stone may absorb water, increasing the apparent mass if massed after submersion. Minimise by measuring mass of the dry stone before submersion.

(3) Random error: reading the meniscus level varies slightly between attempts. Minimise by reading at eye level, perpendicular to the scale, and repeating three times.

(d) $\rho = \frac{m}{V}$. The percentage uncertainty is $\frac{\Delta\rho}{\rho} = \frac{\Delta m}{m} + \frac{\Delta V}{V}$, where $\Delta V = \Delta V_1 + \Delta V_2$ (the uncertainties in the two water level readings add because volume is found by subtraction).

30. (a) From $g = 4\pi^2 l T^{-2}$:

$$\frac{\Delta g}{g} = \frac{\Delta l}{l} + 2\frac{\Delta T}{T}$$

(b) $\frac{\Delta l}{l} = \frac{1\text{mm}}{1000\text{mm}} = 0.1\%$

(c) The total must be below 1%:

$$0.1\% + \frac{2\Delta T}{T} \leq 1\%$$

$$\frac{2\Delta T}{T} \leq 0.9\%$$

$$\frac{\Delta T}{T} \leq 0.45\%$$

The maximum allowable percentage error in T is 0.45%.

(d) For $l = 1.000$ m: $T \approx 2\pi\sqrt{\frac{1.000}{9.8}} = 2.007$ s

If the student times n oscillations, the total time is nT and the percentage error in T is:

$$\frac{\Delta T}{T} = \frac{0.2\text{s}}{n \times 2.007\text{s}} \leq 0.45\%$$

$$n \geq \frac{0.2}{0.0045 \times 2.007} = 22.1$$

The student should time at least 23 oscillations.

EXERCISE 2A

1. (a) $v = 0\text{m/s}$ (b) $a = 0\text{m/s}^2$

Explanation

At rest means the velocity is zero. If the velocity does not change with time, then acceleration (rate of change of velocity) is zero too.

2. Not correct.

Explanation

$a = 0\text{m/s}^2$ means velocity is constant; it can be 0m/s or non-zero.

3. Distance is the total path length while displacement is the change in position with direction.

Example: moving 6m east then 6m west means distance $= 12\text{m}$, while displacement $= 0\text{m}$.

4. The final position is opposite to the chosen positive direction relative to the origin.

5. (a) $a = 0\text{m/s}^2$ (b) Net force $= 0\text{N}$

6. The acceleration is in the negative direction (opposite to the chosen positive direction).

Examples:

First situation: moving forward and slowing down (velocity positive but decreasing in magnitude).

Second situation: moving backward and speeding up (velocity negative and increasing in magnitude).

7.

- Equal when motion is in one direction only.
- Different when motion involves a change of direction over the interval.

8. (a) positive (b) 2m/s^2

9. (a) $v = 0\text{m/s}$ (b) No, the acceleration is not necessarily zero.

Explanation:

Acceleration depends on the **rate of change of velocity**, not on the value of velocity itself. At the instant the body is momentarily at rest, the velocity is 0m/s, but it may still be **changing direction or magnitude**. Therefore, the acceleration can be non-zero even when velocity is zero.

10. (a) 0m/s (b) $a = 0\text{m/s}^2$ during that interval.

11. (a) 55m (b) 25m east

12. (a) 2m/s^2 (b) The velocity increases by 2m/s every 1s.

EXERCISE 2B

13. Motion is relative. Relative to the moving bus, stationary trees have velocity in the opposite direction, so they appear to move backward.

14. Distance measures total path length (400m), while displacement depends only on initial and final positions. Returning home makes displacement $= 0\text{m}$.

15. Average speed depends on total distance and time, so it is non-zero. Average velocity depends on displacement and time; for an out-and-back trip, displacement $= 0\text{m}$, so average velocity $= 0\text{m/s}$.

16. The steady driver has nearly uniform motion, while the other has non-uniform motion with changing velocity. Repeated speeding up and slowing down causes the "jerky" feeling.

17. Both stones experience the same downward acceleration due to gravity, but the thrown stone starts with a non-zero initial downward velocity, so it reaches the ground first.

18. Ignoring air resistance, all objects fall with the same acceleration due to gravity, g, regardless of mass. In air, the effect of air resistance depends on how large the resistive force is **compared to the object's weight**. For a leaf, air resistance is large compared with its small weight, so it greatly reduces the acceleration and velocity. For a coin, the weight is much larger compared with air resistance, so the motion is only slightly affected.

19. Acceleration is the rate of change of velocity. With constant acceleration due to gravity, velocity increases by equal amounts in equal times, so velocity increases even though acceleration remains constant.

20. Velocity depends on the chosen reference direction. If the chosen positive direction is opposite to the car's motion, the velocity is negative even though the car is moving forward.

21. On the way up, velocity is upward while acceleration is downward. On the way down, both velocity and acceleration are downward.

22. No, the student was not correct.

Explanation

At the highest point, the velocity of the ball is momentarily zero, but acceleration depends on the rate of change of velocity, not on the value of velocity itself. Gravity continues to act downward, so the ball still has a downward acceleration even when its velocity is zero.

EXERCISE 2C

23.
(a) 16m/s
(b) 256m
(c) 22s

24.
(a) 16s
(b) 192m

25.
(a) 2.45s
(b) 29.4m
(c) 4.90s
(d) Motion is symmetric under constant gravitational acceleration.

26. 3.58s

27.
(a) -6.0m/s^2
(b) 15m
(c) 39m
(d) Direction reversal increases distance without increasing displacement.

28.
(a) +12.1m/s
(b) 27.1m/s

29.
(a) 0.993s and 4.11s
(b) 0.993s
(c) Velocity is positive at the earlier (smaller) time and negative at the later (larger) time.

30.
(a) 2.67s
(b) 45.2m
(c) Thrown-up stone:+3.87m/s; dropped stone: −26.1m/s

31.
(a) 4.22s
(b) 31.3m/s

32.
(a) 1.50s
(b) 19.0m

33.
(a) 1.83s
(b) 16.5m
(c) +0.08m/s and +15.9m/s

34.
(a) 36.5m
(b) 4.26s
(c) 26.7m/s
(d) Negative time would correspond to an event before

the chosen start instant and is therefore rejected as unphysical.

35.
(a) 98.0m
(b) 19.6m/s

(c) After Stone B is released, both stones have the same acceleration, so their relative acceleration is zero. Since Stone A has been accelerating for longer, it already has a larger downward velocity; therefore, the separation increases at a constant rate.

EXERCISE 3A

1. Since the velocity is constant, acceleration is zero. By Newton's first law, a body with zero acceleration must have zero resultant force. This conclusion depends only on the absence of velocity change because acceleration measures change of velocity, not its magnitude.

2. In both cases the acceleration is zero. By Newton's first law, zero acceleration requires zero resultant force whether the velocity is zero (rest) or a non-zero constant value.

3. A resultant force is required to change velocity, not to maintain it. By Newton's second law, the net force is proportional to acceleration, not velocity.

The everyday idea seems true because presence of resistive forces on Earth oppose motion, so an applied force is needed to balance them.

4. "No force acts" means the body is completely isolated. "Resultant force is zero" means forces act but cancel.

Real motion on Earth usually involves balanced forces, not absence of forces because there is always interaction between bodies.

5. The error is assuming action–reaction forces act on the same body. By Newton's third law, action–reaction forces act on different bodies and therefore cannot cancel to prevent acceleration of a single body.

6. Each student experiences a force due to interaction with the other. **Because their masses may differ**, the same force produces different accelerations according to $a = F/m$.

7. Acceleration depends on the direction of the resultant force, not on the direction of motion (velocity direction).

Slowing down while moving upward means acceleration is downward.

8. Although heavier objects have larger weight (W), they also have proportionally larger mass. By Newton's Second Law, $a = W/m$, so gravitational acceleration is independent of mass when resistive forces are negligible.

9. Constant velocity implies zero acceleration which in turn means zero resultant force in accordance with Newton's first law.

This implies that the applied force balances friction exactly; so friction is present, not absent.

10. The normal reaction can differ from weight if there are vertical components of other forces or vertical acceleration. $N = mg$ only when acceleration perpendicular to the surface is zero and no other perpendicular forces act.

11. By Newton's third law, forces during collision are equal and opposite. By Newton's second law, different masses experience different accelerations, leading to different velocity changes over the same time interval.

12. Increasing stopping time reduces acceleration. By Newton's second law, smaller acceleration produces smaller force, reducing injury risk during braking.

EXERCISE 3B

13. Kipute's body continues with its forward velocity because of inertia. By Newton's first law, her motion will not change unless a resultant force acts on her. When the daladala brakes, her feet are slowed by contact with the floor, but her upper body tends to keep moving forward briefly, so she lurches forward relative to the vehicle.

14. When Kipanga stops pedalling, the bicycle is no longer receiving a driving force, but it keeps moving due to inertia. It slows down because there is a backward resultant force due to resistive forces such as air resistance and friction. By Newton's second law, a backward resultant force produces a backward acceleration, so the velocity decreases gradually.

15. For the phone to move with the car, a forward frictional force must act on the phone to give it the same acceleration as the dashboard. If the friction available is not large enough, the phone cannot gain the car's acceleration, so it slips and appears to move backward relative to the accelerating car.

16. Before the desk moves, static friction adjusts to match the applied push, keeping the resultant force zero so the desk stays at rest, consistent with Newton's first law. When the push becomes large enough to exceed the maximum possible static friction, the desk starts moving. Once it is moving, friction becomes kinetic friction, which is usually smaller than the maximum static friction, so motion becomes easier to maintain.

17. When the pickup accelerates forward, the bag tends to maintain its original state of motion due to inertia. If friction between the bag and the pickup bed is not sufficient to provide the needed forward acceleration to the bag, the bag accelerates less than the pickup and therefore slides backward relative to the vehicle.

18. Feeling heavier means that the normal reaction on Kipute is greater than her weight, while feeling lighter means that the normal reaction is smaller than her weight. When the lift accelerates upward, Kipute must have an upward resultant force, so by Newton's second law the normal reaction must be greater than her weight. When the lift accelerates downward, the resultant force is downward, so the normal reaction is less than her weight.

19. Carrying a passenger increases the total mass of the system being accelerated. If the driving force is roughly unchanged, Newton's second law shows that a larger mass experiences a smaller acceleration, so the motorcycle speeds up more slowly.

20. Bending the knees increases the time over which momentum is reduced to zero. For the same change in momentum, Newton's second law shows that increasing stopping time reduces the average force, lowering injury risk.

21. The book is at rest, so its acceleration is zero and the resultant force is zero. The forces acting are its weight downward and the normal reaction upward, which balance. Therefore, the book is not force-free; it is in equilibrium because forces act and cancel, not because no forces act.

22. On muddy ground, resistive forces are larger, reducing the resultant forward force. On smooth tarmac, resistive forces are smaller, so the same driving effort produces a larger acceleration according to Newton's second law.

23. When the bodaboda stops suddenly, the passenger tends to continue moving forward due to inertia. A backward resultant force from contact with the seat or rider is required to stop the passenger safely.

24. Although the filled bottle has greater inertia, its larger weight increases the normal reaction and hence friction. The larger resistive force produces a larger deceleration, allowing it to stop sooner in real conditions.

25. When the car hits a pothole, the vertical motion of the car changes very rapidly. This produces a sudden change in the passengers' acceleration. By Newton's second law, a sudden change in acceleration requires a sudden change in the resultant force, so the normal reaction on the passengers increases sharply for a short time, producing the strong upward jolt.

EXERCISE 3C

26. (a) 546N (b) The passenger feels lighter. **Justification:** This is because although the lift is moving upward, it is slowing down, so the acceleration (and hence the resultant force) is downward. This makes the normal reaction smaller than the passenger's weight.

27. (a) 19.6N (b) 0.98m/s^2

28. (a) 3125N (b) 8.0s (c) Stopping distance increases. **Explanation:** With the same braking force, increasing the mass reduces the deceleration of the car according to Newton's second law. Since the car still has to reduce its velocity from the same initial value to zero, a smaller deceleration means the car takes longer and travels a greater distance before stopping.

29. (a) 2.67m/s (b) 3:2

30. (a) 100N (b) Opposite to initial motion