SATELLITES AND ORBITAL MOTION

Everything we have built in this chapter now comes together. Newton's law of gravitation provides the force. Circular motion from Chapter 7 provides the framework. Gravitational potential from the previous section provides the energy tools. In this section, we combine all three to understand how satellites orbit, what determines their speed and period, how much energy they carry, and what makes the geostationary orbit so special.

A **satellite** *is any object that orbits another object under the influence of gravity*. The Moon is a natural satellite of the Earth. The Earth is a natural satellite of the Sun. **Artificial satellites** *are man-made objects placed into orbit around the Earth (or other bodies) for communication, navigation, weather monitoring, scientific research, and military purposes.*

Orbital Velocity

Consider a satellite of mass m in a circular orbit of radius R around the Earth (mass $\mathrm{M_E}$). The orbital radius R is measured from the centre of the Earth, so $\mathrm{R = r_E + h}$, where h is the height above the surface.

The satellite is in circular motion. From Chapter 7, circular motion requires a centripetal force directed toward the centre:

$$\mathrm{F_{centripetal} = \frac{mv^2}{R}}$$

This centripetal force is provided entirely by gravity:

$$\mathrm{F_{gravity} = \frac{GM_E m}{R^2}}$$

Setting them equal:

$$\mathrm{\frac{mv^2}{R} = \frac{GM_E m}{R^2}}$$

The mass of the satellite m cancels from both sides. This is a result of deep significance: **the orbital velocity does not depend on the mass of the satellite.** A communication satellite of 500kg and a space station of 400000kg at the same altitude orbit at exactly the same speed.

Solving for v:

$$\mathrm{v^2 = \frac{GM_E}{R}}$$

$$\mathbf{v = \sqrt{\frac{GM_E}{R}}}$$

Since $\mathrm{R = r_E + h}$, the orbital velocity decreases as the height increases. A satellite in a low orbit moves faster than one in a high orbit. This makes physical sense: *a low satellite is deeper in the gravitational field, where the pull is stronger, so it needs a higher speed to avoid falling inward.*

For a satellite very close to the surface ($\mathrm{h \ll r_E}$, so $\mathrm{R \approx r_E}$), and using $\mathrm{GM_E = gr_E^2}$:

$$\mathbf{v} = \sqrt{\frac{\mathrm{gr_E^2}}{\mathrm{r_E}}} = \sqrt{\mathbf{gr_E}} = \sqrt{9.8\mathrm{m/s^2} \times 6.4 \times 10^6\mathrm{m}} = \mathbf{7920m/s} \approx \mathbf{7.9km/s}$$

This is the minimum orbital velocity for any satellite near the Earth's surface: about 7.9km/s, or roughly 28500km/h. Faster than a bullet, faster than any aircraft, and roughly 23 times the speed of sound.

Orbital Period

The period T is the time for one complete orbit. Since the satellite travels a distance of $2\pi R$ (the circumference) at speed v:

$$T = \frac{2\pi R}{v}$$

Substituting $v = \sqrt{\frac{GM_E}{R}}$:

$$T = \frac{2\pi R}{\sqrt{\frac{GM_E}{R}}} = 2\pi R \times \sqrt{\frac{R}{GM_E}} = 2\pi\sqrt{\frac{R^3}{GM_E}}$$

$$\mathbf{T = 2\pi\sqrt{\frac{R^3}{GM_E}}}$$

Squaring both sides gives $T^2 = \frac{4\pi^2}{GM_E}R^3$, which is Kepler's third law. We have now come full circle (no pun intended): the law Kepler discovered empirically follows directly from Newton's gravitation combined with circular motion.

The period increases with orbital radius. Low satellites orbit quickly; high satellites orbit slowly. For a satellite near the surface ($R \approx r_E$): Again using $GM_E = gr_E^2$:

$$T = 2\pi\sqrt{\frac{(r_E)^3}{gr_E^2}} = 2\pi\sqrt{\frac{r_E}{g}} = 2 \times 3.14 \times \sqrt{\frac{6.4 \times 10^6 m}{9.8 m/s^2}} = 5074 s \approx 85 min$$

The shortest possible orbital period for satellite near the Earth's surface is about 85 minutes. Any orbit with a longer period must be at a greater height.

Energy of a Satellite in Orbit

A satellite in circular orbit possesses two forms of energy: kinetic energy (because it moves) and gravitational potential energy (because it is in a gravitational field). Understanding the relationship between these energies reveals something beautiful and surprising.

Kinetic energy (for simplicity, we will use **K** to denote it and not the usual **KE):**

$$K = \frac{1}{2}mv^2$$

From the orbital velocity result,

$$v^2 = \frac{GM_E}{R}$$

Thus:

$$\mathbf{K} = \frac{1}{2}m \times \frac{GM_E}{R} = \mathbf{\frac{GM_E m}{2R}}$$

Notice that KE is **positive** (as it must be, since kinetic energy is always positive).

Gravitational potential energy:

We have derived that:

$$U = -\frac{GM_E m}{R}$$

Notice that U is **negative** (the satellite is attracted to the Earth).

Total mechanical energy:

$$E = K + U = \frac{GM_E m}{2R} - \frac{GM_E m}{R} = -\frac{GM_E m}{2R}$$

$$\mathbf{E = -\frac{GM_E m}{2R}}$$

The total energy is **negative**. This is the hallmark of a bound (attraction) system. A satellite with negative total energy cannot escape to infinity; it is gravitationally attracted (bound) to the Earth. To free the satellite completely (send it to infinity with zero residual speed), you would need to supply energy equal to:

$$|E| = \frac{GM_E m}{2R}$$

The remarkable relationships:

Looking at the three energies together:

$$\mathbf{K = \frac{GM_E m}{2R}}, \qquad \mathbf{U = -\frac{GM_E m}{R}}, \qquad \mathbf{E = -\frac{GM_E m}{2R}}$$

Several elegant patterns emerge:

1. **The kinetic energy is exactly half the magnitude of the potential energy:** $K = -\frac{1}{2}U$
2. **The total energy equals the negative of the kinetic energy:** $E = -K$
3. **The total energy equals half the potential energy:** $E = \frac{1}{2}U$

These are not coincidences. They are direct consequences of the inverse square nature of gravity combined with circular motion. They hold for any circular orbit, around any central body.

What happens when a satellite gains or loses energy?

If a satellite gains energy (for example, by firing its engines), the total energy E becomes less negative. Since $E = -\frac{GM_E m}{2R}$, a less negative E means a larger R. The satellite moves to a **higher orbit**. Counterintuitively, it also moves **slower** (since $v = \sqrt{\frac{GM_E}{R}}$ and R has increased).

If a satellite loses energy (for example, through atmospheric drag), E becomes more negative, R decreases, and the satellite drops to a **lower orbit** where it moves **faster**. This is the paradox of orbital mechanics: air resistance (drag) slows a satellite down in one sense (it loses energy), but speeds it up in another sense (it moves to a lower, faster orbit). Eventually, if enough energy is lost, the satellite spirals inward and burns up in the atmosphere.

The Geostationary Orbit

Among all possible orbits, one is particularly useful: the orbit where the satellite's period exactly matches the Earth's rotation period of 24 hours. A satellite in this orbit appears to remain stationary above a fixed point on the equator, as seen from the ground. This is called a **geostationary orbit** (also known as a **geosynchronous equatorial orbit** or a **parking orbit**).

For a satellite to be geostationary, three conditions must be met:

Condition 1: The period must be exactly 24 hours (86400s).

Condition 2: The orbit must be circular (otherwise the satellite would appear to drift back and forth during the day).

Condition 3: The orbit must lie in the equatorial plane (otherwise the satellite would appear to move north and south during the day).

The radius of the geostationary orbit can be found from Kepler's third law as follows:

$$\mathrm{T}^2 = \frac{4\pi^2}{\mathrm{GM_E}}\mathrm{R}^3$$

Making R the subject:

$$\mathrm{R}^3 = \frac{\mathrm{GM_E T^2}}{4\pi^2}$$

$$\mathrm{R} = \sqrt[3]{\left(\frac{\mathrm{GM_E T^2}}{4\pi^2}\right)}$$

Substituting $\mathrm{G} = 6.67 \times 10^{-11}\mathrm{Nm^2kg^{-2}}$, $\mathrm{M_E} = 6.0 \times 10^{24}\mathrm{kg}$, and $\mathrm{T} = 86400\mathrm{s}$:

$$\mathrm{R} = \sqrt[3]{\left(\frac{6.67 \times 10^{-11} \times 6.0 \times 10^{24} \times (86400)^2}{4 \times \pi^2}\right)} = 4.23 \times 10^7\mathrm{m} = 42300\mathrm{km}$$

The height above the surface:

$$\mathrm{h} = \mathrm{R} - \mathrm{r_E} = 42300\mathrm{km} - 6400\mathrm{km} = 35900\mathrm{km} \approx 36000\mathrm{km}$$

Every geostationary satellite orbits at approximately $36000\mathrm{km}$ above the equator. There is no choice in this; the altitude is fixed by the physics. This is why geostationary orbit positions are valuable and strictly regulated internationally.

The orbital speed in this orbit:

$$\mathrm{v} = \frac{2\pi\mathrm{R}}{\mathrm{T}} = \frac{2 \times 3.14 \times 4.23 \times 10^7\mathrm{m}}{86400\mathrm{s}} = 3074\mathrm{m/s} \approx 3.1\mathrm{km/s}$$

This is much slower than the 7.9km/s of a low-orbit satellite, consistent with the rule that higher orbits have lower speeds.

Why geostationary satellites are so useful:

Because a geostationary satellite appears fixed above one point on the equator, a ground antenna can be pointed at it permanently without tracking. This makes it ideal for television broadcasting, weather observation, and telecommunications. The satellite dishes you see on rooftops across Tanzania are all pointed at geostationary satellites hovering above the equator.

Energy Required to Launch a Satellite

To place a satellite into orbit, we must do two things:

First: lifting it from the surface to the orbital height (increasing its potential energy), and

Second: accelerating it to the orbital speed (giving it kinetic energy).

The total energy required *is the difference between the total energy in orbit and the total energy at rest on the surface.*

On the surface (at rest):

$$\mathrm{E_{surface}} = \mathrm{K_{surface}} + \mathrm{U_{surface}} = 0 + \left(-\frac{\mathrm{GM_E m}}{\mathrm{r_E}}\right) = -\frac{\mathrm{GM_E m}}{\mathrm{r_E}}$$

In orbit (at radius R):

$$E_{orbit} = -\frac{GM_E m}{2R}$$

The energy that must be supplied is:

$$\Delta E = E_{orbit} - E_{surface} = -\frac{GM_E m}{2R} - \left(-\frac{GM_E m}{r_E}\right)$$

$$\boldsymbol{\Delta E = GM_E m\left(\frac{1}{r_E} - \frac{1}{2R}\right)}$$

This is the minimum energy required, ignoring air resistance and other losses. In practice, much more energy is needed because rockets must also lift the fuel itself, overcome air resistance, and account for inefficiencies.

Let us now put all of these ideas to work.

BINDER Example 18

A satellite orbits the Earth at a height of 400km above the surface (approximately the altitude of the International Space Station).

(a) Calculate its orbital speed.

(b) Calculate its orbital period in minutes.

Take $M_E = 6.0 \times 10^{24}\text{kg}$, $r_E = 6.4 \times 10^{6}\text{m}$, and $G = 6.67 \times 10^{-11}\text{Nm}^2\text{kg}^{-2}$.

Solution

(a) The orbital radius:

$$R = r_E + h = 6.4 \times 10^6\text{m} + 400 \times 10^3\text{m} = 6.8 \times 10^6\text{m}$$

The orbital speed:

$$v = \sqrt{\frac{GM_E}{R}} = \sqrt{\frac{6.67 \times 10^{-11}\text{Nm}^2\text{kg}^{-2} \times 6.0 \times 10^{24}\text{kg}}{6.8 \times 10^6\text{m}}} = 7672\text{m/s}$$

The orbital speed is 7672m/s (about 7.67km/s).

(b) The orbital period:

$$T = \frac{2\pi R}{v} = \frac{2 \times 3.14 \times 6.8 \times 10^6\text{m}}{7672\text{m/s}} = 5567\text{s}$$

Converting to minutes:

$$T = \frac{5567\text{s}}{60} = 92.8\text{min}$$

The period is about 92.8 minutes.

The ISS completes roughly 15.5 orbits every day, experiencing 15 sunrises and 15 sunsets in 24 hours.

Making Sense of the Answer: *The speed (7.67km/s) and period (93 minutes) match the well-known values for the ISS. At this speed, the station crosses the entire width of Tanzania in about 2 minutes!*

Think Like a Physicist: *Compare with the minimum values: 7.9km/s and 85 minutes near the surface. At 400km, the speed is slightly lower and the period slightly longer, exactly as the formulas predict. The differences are small because 400km is only about 6% of the Earth's radius.*

REAL Example 19

Kipanga asks **Mr. Akilikubwa**: *"Sir, if a satellite is always falling toward the Earth, why doesn't it lose energy and crash?"*

Mr. Akilikubwa replies: *"Kipanga, it* **is** *losing energy in a sense. But look at what happens when it does."*

Explain what Mr. Akilikubwa means. Describe what happens to the speed, altitude, and period of a satellite when it gradually loses energy due to atmospheric drag.

Solution

When a satellite loses energy through atmospheric drag, its total energy $E = -\frac{GM_Em}{2R}$ becomes more negative. A more negative E corresponds to a smaller orbital radius R, so the satellite drops to a lower orbit.

At this lower orbit, the orbital speed $v = \sqrt{\frac{GM_E}{R}}$ is higher (because R is smaller), and the period $T = 2\pi\sqrt{\frac{R^3}{GM_E}}$ is shorter.

This is the orbital paradox that Mr. Akilikubwa is hinting at: drag removes energy from the satellite, yet the satellite speeds up. The satellite spirals inward into ever-lower, ever-faster orbits, until it eventually enters the dense atmosphere and burns up.

The paradox resolves when you realise that the satellite gains more kinetic energy from falling to a lower orbit than it loses to drag. The potential energy lost exceeds the kinetic energy gained plus the energy lost to drag.

Making Sense of the Answer: *A satellite slowed by drag does not simply slow down and stop, the way a car does when you take your foot off the accelerator. Instead, it trades altitude for speed, spiralling inward faster and faster until it meets its fiery end in the atmosphere.*

Think Like a Physicist: *The relationships* $K = -\frac{1}{2}U$ *and* $E = -K$ *are the key. When the satellite drops to a lower orbit,* $|U|$ *increases,* K *increases (to half of* $|U|$*), and* E *decreases (becomes more negative). The satellite has less total energy but more kinetic energy.*

HOT Example 20

(a) Calculate the kinetic energy, potential energy, and total energy of a 200kg satellite orbiting the Earth at a height of 600km.

(b) How much energy must be supplied to move this satellite from its current orbit to a new orbit at 1200km above the surface?

Take $M_E = 6.0 \times 10^{24}\text{kg}$, $r_E = 6.4 \times 10^6\text{m}$, and $G = 6.67 \times 10^{-11}\text{Nm}^2\text{kg}^{-2}$.

Solution

(a) Orbital radius at 600km:

$$R_1 = r_E + h_1 = 6.4 \times 10^6\text{m} + 600 \times 10^3\text{m} = 7 \times 10^6\text{m}$$

Kinetic energy:

$$K = \frac{GM_Em}{2R_1} = \frac{6.67 \times 10^{-11}\text{Nm}^2\text{kg}^{-2} \times 6 \times 10^{24}\text{kg} \times 200\text{kg}}{2 \times 7 \times 10^6\text{m}} = 5.717 \times 10^9\text{J} = 5.72\text{GJ}$$

Potential energy:

$$U = -\frac{GM_Em}{R_1} = -\frac{6.67 \times 10^{-11}\text{Nm}^2\text{kg}^{-2} \times 6 \times 10^{24}\text{kg} \times 200\text{kg}}{7 \times 10^6\text{m}} = -1.143 \times 10^{10} = -11.43\text{GJ}$$

Total energy:

$$E_1 = K + U = 5.72\text{GJ} + (-11.43\text{GJ}) = -5.72\text{GJ}$$

You can easily check the correctness of the answers:

Check: $K = -\frac{1}{2}U? \frac{11.43}{2} = 5.72$. Confirmed.

Check: $E = -K? -5.72 = -(5.72)$. Confirmed.

(b) Orbital radius at 1200km:

$$R_2 = 6.4 \times 10^6\text{m} + 1200 \times 10^3\text{m} = 7.6 \times 10^6\text{m}$$

Total energy in the new orbit:

$$E_2 = -\frac{GM_Em}{2R_2} = -\frac{6.67 \times 10^{-11}\text{Nm}^2\text{kg}^{-2} \times 6 \times 10^{24}\text{kg} \times 200\text{kg}}{2 \times 7.6 \times 10^6\text{m}} = -5.266 \times 10^9\text{J} = -5.27\text{GJ}$$

Energy that must be supplied:

$$\Delta E = E_2 - E_1 = (-5.27\text{GJ}) - (-5.72\text{GJ}) = 0.45\text{GJ} = 4.5 \times 10^8\text{J}$$

The satellite needs 4.5×10^8J (about 450MJ) of additional energy to move from the 600km orbit to the 1200km orbit.

Making Sense of the Answer: *The higher orbit has less negative total energy (it is less tightly bound). The difference, 450MJ, is the energy the satellite's engines must provide. Notice that moving 600km higher requires far less energy than the initial launch from the surface, because the satellite already has most of the energy it needs.*

Think Like a Physicist: *The explicit checks (*$K = -\frac{1}{2}U$ *and* $E = -K$*) are not just bookkeeping. They confirm that the orbit is genuinely circular. If these relationships did not hold, it would mean the orbit is elliptical or the calculation contains an error. Always verify these relationships when working with orbital energies.*

HOT Example 21

Calculate the minimum energy required to launch a 1000kg satellite from the Earth's surface into a circular orbit at a height of 300km. Ignore air resistance and the Earth's rotation.

Take $M_E = 6.0 \times 10^{24}$kg, $r_E = 6.4 \times 10^6$m, and $G = 6.67 \times 10^{-11}\text{Nm}^2\text{kg}^{-2}$.

Solution

The satellite starts at rest on the surface. Its initial energy:

$$E_{\text{surface}} = 0 + \left(-\frac{GM_Em}{r_E}\right) = -\frac{6.67 \times 10^{-11} \times 6.0 \times 10^{24} \times 1000}{6.4 \times 10^6}$$

$$E_{\text{surface}} = -\frac{4.002 \times 10^{17}\text{J}}{6.4 \times 10^6} = -6.253 \times 10^{10}\text{J}$$

The orbital radius:

$$R = 6.4 \times 10^6\text{m} + 300 \times 10^3\text{m} = 6.7 \times 10^6\text{m}$$

The total energy in orbit:

$$E_{\text{orbit}} = -\frac{GM_Em}{2R} = -\frac{4.002 \times 10^{17}\text{J}}{2 \times 6.7 \times 10^6\text{m}} = -\frac{4.002 \times 10^{17}}{1.34 \times 10^7} = -2.987 \times 10^{10}\text{J}$$

Minimum energy required:

$$\Delta E = E_{\text{orbit}} - E_{\text{surface}} = (-2.987 \times 10^{10}\text{J}) - (-6.253 \times 10^{10}\text{J})$$

$$\Delta E = 3.266 \times 10^{10}\text{J} = 32.7\text{GJ}$$

The minimum energy required is 32.7GJ.

Making Sense of the Answer: $32.7GJ$ *for a* $1000kg$ *satellite is an enormous amount of energy. For comparison, a litre of rocket fuel (kerosene) releases about* $34MJ$*, so this launch would need at least* 960 *litres of fuel for the payload alone. In reality, the fuel-to-payload ratio for orbital launches is typically* $20:1$ *or higher, because the rocket must also lift the fuel, the engines, and the structure.*

Think Like a Physicist: *Most of the launch energy goes into kinetic energy, not potential energy. The potential energy change is* $\mathrm{GM_Em}\left(\frac{1}{\mathrm{r_E}}-\frac{1}{\mathrm{R}}\right) \approx 2.93\mathrm{GJ}$*, while the kinetic energy needed is* $\frac{\mathrm{GM_Em}}{2\mathrm{R}} \approx 29.9\mathrm{GJ}$*. About* 91% *of the energy goes into making the satellite move fast enough to stay in orbit, and only* 9% *goes into lifting it to the correct height. This is why reaching orbit is fundamentally about speed, not altitude.*

With the physics of satellites and orbits now complete, one question remains: *what if we want to escape the Earth's gravity entirely?* Not orbit, but leave. That is the subject of the next and penultimate section of this chapter.