

GRAVITATION

INTRODUCTION

It was a quiet afternoon at Miono Secondary School. The physics class had just finished Chapter 7 on Circular Motion, and the students were packing their books. **Kipanga**, still buzzing with excitement about satellites and orbits, turned to **Mr. Akilikubwa** with a question that had been bothering him all lesson.

“Sir, you said that gravity pulls everything toward the Earth. And you said the Moon goes around the Earth in a circle. But if gravity is pulling the Moon downward, why doesn’t it just fall and crash into us?”

The class went quiet. Even **Kipute**, who usually had an answer ready, paused.

Mr. Akilikubwa smiled and picked up a tennis ball from his desk. *“Kipanga, if I throw this ball gently sideways, what happens?”*

“It curves downward and hits the ground nearby,” **Kipanga** answered.

“And if I throw it harder?”

“It goes further before hitting the ground.”

“And if I throw it harder still?”

“Even further.”

“Now,” said **Mr. Akilikubwa**, his voice dropping as though sharing a secret, *“imagine I could throw it so hard that as it falls, the Earth’s surface curves away beneath it at exactly the same rate. The ball keeps falling toward the Earth, but the ground keeps curving away. The ball falls forever but never lands. What would you call that?”*

Kipanga’s eyes widened. *“An orbit!”*

“Exactly.” **Mr. Akilikubwa** set the ball down. *“The Moon is falling toward the Earth! It has been falling for over four billion years. It just keeps missing. And the force that makes it fall, is the same force that pulls this tennis ball, is the force that holds you in your chair, the force that keeps the Earth circling the Sun; is the oldest, weakest, and most far-reaching force in the universe. Newton called it gravity. And he had the audacity to claim that it was the same force that made an apple fall from a tree.”*

Kipute leaned forward. *“The same force? An apple and the Moon?”*

“The same law,” **Mr. Akilikubwa** confirmed. *“One equation that describes both. That is the power of what you are about to learn.”*

Kipanga grinned. *“So Newton figured out the Moon is basically a very fast apple?”*

Mr. Akilikubwa laughed. *“That, Kipanga, might be the best summary of gravitation anyone has ever given.”*

In Chapter 7, we asked: *what real force provides the centripetal force for circular motion?* We found different answers for different situations: friction for cars on roads, tension for stones on strings, the normal reaction for banked curves. But we left one answer untouched, the most profound one of all: **gravity**.

Gravity is the force that holds the Moon in orbit around the Earth, the Earth in orbit around the Sun, and the Sun in orbit around the centre of the Milky Way galaxy. It is the force that shaped the solar system, controls the tides, and determines the fate of the universe itself. Yet it obeys a single, elegant law that Newton published in 1687, a law so simple that you will derive it yourself before this chapter is over.

In this chapter, we begin with the observations that led to our understanding of planetary motion: **Kepler’s three laws**. We then meet **Newton’s Law of Universal Gravitation** and use it to explain why planets move the way Kepler described. From there, we explore how gravity varies with altitude, depth, and latitude, and we introduce the concepts of **gravitational field strength** and **gravitational potential**. Finally, we apply everything to **satellites, orbits, and escape velocity**; the physics that makes space travel possible.

The mathematics builds directly on what you mastered in Chapter 7. The centripetal force equation $\frac{mv^2}{r}$ returns, but now the force supplying it is gravity: $\frac{GMm}{r^2}$. Setting these equal will unlock nearly every result in this chapter. If you understood circular motion, gravitation will feel like a natural continuation with the same physics, played on the grandest stage imaginable.

KEPLER'S LAWS OF PLANETARY MOTION

Before Newton ever thought about gravity, a German mathematician named **Johannes Kepler** spent years staring at numbers. Not just any numbers, but the most precise astronomical measurements ever collected, recorded by the Danish astronomer **Tycho Brahe**, a man so dedicated to watching the sky that he built an entire observatory on an island. Brahe gathered mountains of data on planetary positions but died in 1601 before he could make sense of it all. Kepler inherited the data and, after years of painstaking calculation (no computers, no calculators! Just pen, paper, and stubbornness), he extracted three laws that described exactly how planets move around the Sun.

Kepler did not know *why* planets obeyed these laws. He simply showed that they did. The “why” would have to wait for Newton, who arrived on the scene about 70 years later and explained all three laws using a single equation. But before we meet Newton’s explanation, we must first understand what Kepler discovered.

Kepler’s First Law (The Law of Orbits)

The law states that:

Each planet moves in an elliptical orbit with the Sun at one focus.

Before Kepler, everyone assumed planetary orbits were perfect circles (or combinations of circles). Kepler showed that orbits are actually **ellipses**, slightly squashed circles. An ellipse has two special points called **foci** (singular: focus). The Sun sits at one focus, not at the centre.

For most planets in our solar system, the orbits are very close to circular. The Earth’s orbit, for example, deviates from a perfect circle by less than 2%. But the deviation matters: it is the reason why the Earth is slightly closer to the Sun in January than in July (this fact often surprises students who think the seasons are caused by changes in distance from the Sun. In reality, the seasons are caused by the tilt of the Earth’s axis, not the distance).

However, we will often approximate planetary and satellite orbits as circular, because this simplifies the mathematics while remaining accurate for most practical calculations.

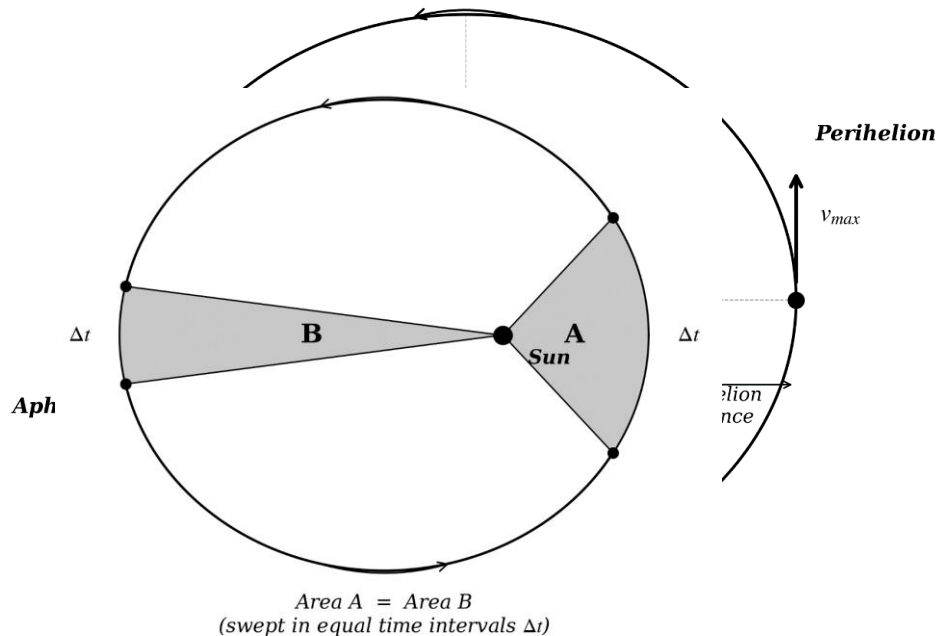


Figure: Kepler's first law. A planet moves in an elliptical orbit with the Sun at one focus. The planet moves fastest at perihelion and slowest at aphelion.

Key terms:

Perihelion: the point in the orbit closest to the Sun. The planet moves fastest (v_{\max}) here.

Aphelion: the point farthest from the Sun. The planet moves slowest here (v_{\min}).

For orbits around the Earth (such as satellite orbits), the equivalent terms are **perigee** (closest) and **apogee** (farthest).

Kepler's Second Law (The Law of Areas)

The law states that:

A line drawn from the Sun to a planet sweeps out equal areas in equal intervals of time.

This law tells us something profound about how the speed of a planet changes as it moves along its orbit. When the planet is close to the Sun (near perihelion), the line from the Sun to the planet is short. To sweep the same area in the same time, the planet must move faster. When the planet is far from the Sun (near aphelion), the line is longer, so the planet can sweep the same area while moving more slowly.

In short: **planets speed up when closer to the Sun and slow down when farther away.**

This is not a coincidence or a design choice. It is a consequence of the **conservation of angular momentum** (this will be discussed in detail in **Chapter 9: Rotation of Rigid Bodies**). Because gravity acts along the line joining the planet and the Sun, it produces no torque about the Sun. With zero torque, angular momentum ($L = mvr$) is conserved. When r decreases, v must increase to keep L constant, and vice versa.

For simplified circular orbits (which we will mostly use in this topic), the speed is constant and the second law is automatically satisfied. This is because, with constant speed, the planet sweeps out equal angles in equal times, and since the radius is constant, equal angles means equal areas, agreeing with Kepler's second law.

Figure: Kepler's second law. A line drawn from the Sun to the planet sweeps out equal areas in equal time intervals Δt . Sector A (near perihelion) spans a wide arc because the planet is close to the Sun and moves fast.

Sector B (near aphelion) spans a narrow arc because the planet is far from the Sun and moves slowly. Despite their different shapes, both areas are equal.

Kepler's Third Law (The Law of Periods)

The law states that:

The square of the period of revolution of a planet is directly proportional to the cube of its mean distance from the Sun.

Mathematically:

$$T^2 \propto r^3$$

$$T^2 = kr^3$$

Where T is the orbital period, r is the mean orbital radius, and k is a constant that is the same for all planets orbiting the same central body.

If two planets orbit the same star, their periods and radii are related by:

$$\frac{T_1^2}{T_2^2} = \frac{r_1^3}{r_2^3}$$

This is an extraordinarily powerful equation. It means that if you know the period and radius of one planet's orbit, you can find the radius of any other planet's orbit just by measuring its period. Kepler used this to map the entire solar system.

What the third law reveals

The third law tells us that planets farther from the Sun take longer to complete one orbit not just because they have a longer path, but also because they move more slowly. Both effects combine to make the period increase steeply with distance. For example, the Earth orbits the Sun in 1 year at a mean distance of 1AU (astronomical unit). Jupiter, at about 5.2AU, takes about 11.9 years. Neptune, at 30AU, takes about 165 years.

Later in this chapter, we will derive Kepler's Third Law from Newton's Law of Universal Gravitation and show that the constant $k = \frac{4\pi^2}{GM}$, where M is the mass of the central body. This derivation is one of the most beautiful results in all of physics: three empirical laws, discovered by painstaking observation, all explained by a single force law.

With Kepler's three laws now understood, let us practise applying them.

BINDER Example 1

- A planet moves faster at certain points in its orbit and slower at others. Using Kepler's laws, explain why this happens and identify the positions where the planet is fastest and slowest.
- The mean distance of the Earth from the Sun is 1.5×10^{11} m and its period of revolution is 3.156×10^7 s (1 year). Calculate the constant k in Kepler's third law.

Solution

- Kepler's first law tells us that a planet's orbit is an ellipse with the Sun at one focus. This means the planet is not always at the same distance from the Sun; it is closer at one point (perihelion) and farther at another (aphelion).

Kepler's second law tells us that a line from the Sun to the planet sweeps equal areas in equal times. When the planet is near perihelion, the line from the Sun is short, so the planet must move through a larger arc to sweep the required area, and thus it moves faster. When the planet is near aphelion, the line is long, so a smaller arc is sufficient, and hence it moves slower.

Therefore, the planet is fastest at perihelion and slowest at aphelion.

(b) Using $T^2 = kr^3$:

$$k = \frac{T^2}{r^3} = \frac{(3.156 \times 10^7 \text{ s})^2}{(1.5 \times 10^{11} \text{ m})^3} = 2.95 \times 10^{-19} \text{ s}^2/\text{m}^3$$

Making Sense of the Answer: This constant is the same for every planet in the solar system. Mars, Jupiter, Saturn; they all give the same value of k . This universality is what makes Kepler's Third Law so powerful, and what hinted to Newton that a single force law governs all planetary motion.

Think Like a Physicist: The constant k depends on the mass of the central body (the Sun, in this case). Planets orbiting a different star would have a different k , because the star has a different mass.

REAL Example 2

Mars has a mean orbital radius of $2.28 \times 10^{11} \text{ m}$. **Kipanga** claims that since Mars is about 1.5 times farther from the Sun than Earth, its year must be about 1.5 times longer than Earth's year.

Explain why Kipanga's reasoning is incorrect, and find the actual period of Mars.

Solution

Kipanga's reasoning is wrong because the period does not increase in direct proportion to the distance. Kepler's third law states $T^2 \propto r^3$, not $T \propto r$.

Using the ratio form:

$$\frac{T_{\text{Mars}}^2}{T_{\text{Earth}}^2} = \frac{r_{\text{Mars}}^3}{r_{\text{Earth}}^3}$$

$$\frac{T_{\text{Mars}}^2}{(1 \text{ year})^2} = \left(\frac{2.28 \times 10^{11} \text{ m}}{1.5 \times 10^{11} \text{ m}} \right)^3 = (1.52)^3 = 3.51$$

$$T_{\text{Mars}} = \sqrt{3.51} \text{ years} = 1.87 \text{ years}$$

Mars takes 1.87 Earth years to orbit the Sun (not 1.5 years as Kipanga predicted).

Making Sense of the Answer: A planet 1.5 times farther out has a longer path and moves more slowly. Both effects combine through the $T^2 \propto r^3$ relationship, making the period nearly twice the Earth's, not just 1.5 times.

Think Like a Physicist: The ratio form of Kepler's third law is extremely useful because it avoids the need to know the constant k or the mass of the Sun. You only need the period and radius of one known orbit to find the other.

HOT Example 3

A geostationary satellite orbits the Earth with a period of exactly 24 hours. The Moon orbits the Earth with a period of 27.3 days at a mean distance of $3.84 \times 10^8 \text{ m}$ from the Earth's centre.

Calculate the orbital radius of the geostationary satellite.

Solution

Both the satellite and the Moon orbit the Earth, so the same constant k applies. Using the ratio form:

$$\frac{T_{\text{sat}}^2}{T_{\text{Moon}}^2} = \frac{r_{\text{sat}}^3}{r_{\text{Moon}}^3}$$

Making r_{sat} the subject:

$$r_{\text{sat}}^3 = r_{\text{Moon}}^3 \times \frac{T_{\text{sat}}^2}{T_{\text{Moon}}^2}$$

Where: $T_{\text{sat}} = 24 \text{ h} = 24 \times 3600 \text{ s} = 86400 \text{ s}$, $T_{\text{Moon}} = 27.3 \text{ days} = 27.3 \times 24 \times 3600 \text{ s} = 2.359 \times 10^6 \text{ s}$,
 $r_{\text{Moon}} = 3.84 \times 10^8 \text{ m}$

$$r_{\text{sat}}^3 = (3.84 \times 10^8 \text{m})^3 \times \frac{(86400 \text{s})^2}{(2.359 \times 10^6 \text{s})^2} = 7.6 \times 10^{22} \text{m}^3$$
$$r_{\text{sat}} = \sqrt[3]{7.6 \times 10^{22} \text{m}^3} = 4.24 \times 10^7 \text{m}$$

The orbital radius of the geostationary satellite is $4.24 \times 10^7 \text{m}$ (about 42400km from the Earth's centre, or about 36,000km above the surface).

Making Sense of the Answer: *The geostationary orbit is about one-ninth of the way to the Moon. Its period (1 day) is much shorter than the Moon's (27.3 days), so it must be much closer, and Kepler's third law gives us exactly how close.*

Think Like a Physicist: *This problem uses the Moon as a reference orbit to find the geostationary radius, without needing the mass of the Earth or the gravitational constant G . The ratio method is powerful precisely because the unknown constants cancel.*

Kepler told us *how* planets move. Three elegant laws, extracted from decades of data. But he could not answer the deeper question: *why* do they move this way? *What force compels a planet to follow an ellipse? Why does it sweep equal areas? Why does T^2 relate to r^3 and not to r^2 or r^4 ?*

The answer came from Isaac Newton, who showed that all three laws are consequences of a single universal force. In the next subtopic, we meet that force.