

DIGGING DEEPER EXERCISE 7

EXERCISE 7A: BINDER QUESTIONS

Question 1

A body moves in a circular path at constant speed. State whether each of the following is zero, constant, or changing:

- (a) speed
- (b) velocity
- (c) centripetal acceleration
- (d) centripetal force

Question 2

A body moves in a circular path at constant speed.

- (a) Explain why the body is accelerating even though its speed does not change.
- (b) State the direction of this acceleration.

Question 3

A stone tied to a string is whirled in a horizontal circle on a smooth table. The string suddenly breaks.

- (a) Describe the path of the stone immediately after the string breaks.
- (b) Explain your answer.

Question 4

Distinguish between angular velocity and linear velocity of an object in circular motion. State how they are related.

Question 5

A body moves in a circle of radius r with constant angular velocity ω .

- (a) Write the centripetal acceleration in terms of v and r .
- (b) Write the centripetal acceleration in terms of ω and r .
- (c) Explain why both expressions give the same result.

Question 6

State the real force that provides the centripetal force in each of the following situations:

- (a) a car rounding a level curve
- (b) a satellite orbiting the Earth
- (c) a ball on a string swung in a vertical circle
- (d) a car on a banked curve with no friction
- (e) a charged particle moving in a magnetic field

Question 7

Explain why the maximum safe speed of a car on a level circular curve does not depend on the mass of the car.

Question 8

In vertical circular motion on a string:

- (a) State the position where the tension in the string is maximum. Explain why.
- (b) State the position where the tension in the string is minimum. Explain why.

Question 9

Explain what is meant by the “design speed” of a banked curve, and describe what happens when a vehicle travels (a) faster than the design speed, and (b) slower than the design speed.

Question 10

A car moves over the top of a hill. Explain why the driver feels lighter at the top, and state the condition under which the car would lose contact with the road.

Question 11

Compare the critical speed condition at the top of a vertical loop for two cases:

- (a) an object on the inside of a circular loop (e.g. a ball inside a loop-the-loop track)
- (b) an object on the outside of a circular curve (e.g. a car at the top of a hill)

State how the critical speed differs in meaning between the two cases.

EXERCISE 7B: REAL QUESTIONS**Question 12**

Passengers in a bus travelling along a straight road suddenly feel thrown to the left when the bus turns sharply to the right. Explain why this happens, and state whether any force actually pushes the passengers to the left.

Question 13

On the Dar es Salaam–Morogoro highway, sharp curves have road signs showing a recommended speed limit. Explain, using physics, how the recommended speed for a curve is determined and why different curves have different recommended speeds.

Question 14

During a motorcycle race, riders lean their bodies sharply toward the inside of a bend while turning. Explain why leaning is necessary and what would happen if a rider tried to turn without leaning.

Question 15

On a school trip, the bus crosses a bridge that has a gentle hump at the centre. As the bus goes over the hump at speed, **Kipanga** grabs his seat and shouts: “*My stomach just dropped! I think we are falling!*”

Kipute laughs and says: “*We are not falling, Kipanga. The road just stopped pushing you as hard.*”

Explain what Kipute means. Why does Kipanga feel the sensation of his stomach dropping?

Question 16

Explain why a wet road is more dangerous at a curve than on a straight section of road.

Question 17

A washing machine removes water from clothes during the spin cycle by rotating the drum at high speed. Explain how the water is separated from the clothes.

Question 18

Tall vehicles such as buses and lorries are more likely to overturn on sharp curves than low vehicles such as saloon cars. Explain why.

Question 19

Kipanga watches a news report about a satellite orbiting the Earth. He turns to **Mr. Akilikubwa** and asks: “*If gravity pulls the satellite toward the Earth, why doesn’t it fall down and crash?*”

Mr. Akilikubwa smiles and says: “*It is falling, Kipanga. It just keeps missing the ground.*”

Explain what Mr. Akilikubwa means.

Question 20

Engineers build roads with banked curves rather than relying entirely on friction. Explain the advantage of banking, and state why banking alone is not sufficient for all speeds.

Question 21

A student observes that astronauts inside the International Space Station float as if there is no gravity. The student concludes that there is no gravity in space. Explain why this conclusion is incorrect.

EXERCISE 7C: HOT QUESTIONS**Question 22**

A car of mass 1200kg rounds a level curve of radius 80m at 25m/s. The coefficient of static friction is 0.60. Take $g = 9.8\text{m/s}^2$.

- Calculate the centripetal force required.
- Calculate the maximum friction force available.
- Determine whether the car can safely negotiate the curve at this speed.

Question 23

A conical pendulum has a string of length 1.2m and the bob has mass 0.4kg. The bob moves in a horizontal circle with a period of 2s. Take $g = 9.8\text{m/s}^2$.

- Calculate the angle the string makes with the vertical.
- Calculate the tension in the string.
- Calculate the radius of the circular path and the speed of the bob.

Question 24

A student solves the following problem: “*A ball on a string moves in a vertical circle of radius 0.5m. The speed at the bottom is 4m/s. Find the tension at the top.*”

The student writes: $T_{\text{top}} = \frac{mv_{\text{bottom}}^2}{r} - mg$, using the speed at the bottom directly in the tension formula at the top.

- Explain the error in the student’s approach.
- For a ball of mass 0.3kg, carry out the correct calculation. If the ball does not reach the top, determine the angle from the bottom at which the string goes slack. Take $g = 9.8\text{m/s}^2$.

Question 25

A curved road of radius 100m is banked at 20° . The coefficient of static friction between the tyres and the road is 0.30. Take $g = 9.8\text{m/s}^2$.

- Calculate the design speed of the curve.
- Determine the safe speed range.

Question 26

A ball of mass 0.2kg is attached to a string of length 0.7m and swung in a vertical circle. The speed at the bottom is 6m/s . Take $g = 9.8\text{m/s}^2$.

- (a) Calculate the tension at the bottom.
- (b) Calculate the speed and the tension at the top.
- (c) Find the angle from the bottom at which the tension equals 8N .

Question 27

A small ball rolls along a frictionless track and enters a vertical circular loop of radius 0.5m . The ball is released from rest at a height $h = 1.5\text{m}$ above the bottom of the loop. Take $g = 9.8\text{m/s}^2$.

- (a) Show that the ball completes the loop.
- (b) Calculate the speed and the normal reaction at the top of the loop for a ball of mass 0.1kg .
- (c) Calculate the normal reaction at the bottom of the loop.

Question 28

An engineer designs a level circular curve of radius 70m for a road where the expected traffic speed is 60km/h . The road surface has coefficient of static friction 0.55 . Take $g = 9.8\text{m/s}^2$.

- (a) Calculate the maximum safe speed on this curve.
- (b) Determine whether the curve is safe for the expected traffic speed.
- (c) If the engineer decides to bank the road instead, calculate the banking angle required for the expected speed to be the design speed.
- (d) With this banking angle and the same friction, calculate the new maximum safe speed.

Question 29

A car of mass 1500kg travels at 20m/s over a road that has a valley (dip) of radius 50m . Take $g = 9.8\text{m/s}^2$.

- (a) Calculate the normal reaction on the car at the bottom of the valley.
- (b) Express the apparent weight of the driver (mass 75kg) as a multiple of the driver's actual weight.
- (c) Calculate the speed at which the apparent weight would be twice the actual weight.

Question 30

A stone of mass 0.3kg is whirled in a vertical circle of radius 0.9m . At the top of the circle, the speed is 4m/s . Take $g = 9.8\text{m/s}^2$.

- (a) Calculate the tension at the top.
- (b) Find the tension at the bottom.
- (c) Calculate the speed at the bottom.

Question 31

A student claims that a heavier ball needs a greater minimum speed at the bottom to complete a vertical circle than a lighter ball on the same string.

- (a) Calculate the minimum speed at the bottom for a 0.2kg ball and a 0.5kg ball, each on a string of length 0.8m . Take $g = 9.8\text{m/s}^2$.
- (b) Comment on the student's claim.

Question 32

A small ball of mass 0.1kg is suspended by a light string from the roof of a car. The car travels at constant speed around a horizontal circular track of radius 20m . The string makes an angle of 30° with the vertical. Take $g = 9.8\text{m/s}^2$.

- (a) Calculate the speed of the car.
- (b) Calculate the tension in the string.
- (c) If the speed doubles, calculate the new angle the string makes with the vertical.

Question 33

A car has wheels 1.5m apart and its centre of gravity is 0.9m above the ground. It rounds a level curve of radius 40m . The coefficient of static friction is 0.70 . Take $g = 9.8\text{m/s}^2$.

- (a) Calculate the maximum speed before overturning.
- (b) Calculate the maximum speed before skidding.
- (c) Determine which happens first.
- (d) Calculate the normal reactions on the inner and outer wheels when the car travels at 15m/s .

Question 34

A ball on a string of length 1m is set in motion in a vertical circle. The speed at the bottom is 5.5m/s . Take $g = 9.8\text{m/s}^2$.

- (a) Determine whether the ball completes the full circle.
- (b) If not, find the angle from the bottom at which the string goes slack.
- (c) Find the speed at that angle.

Question 35

An engineer designs a roller coaster loop of radius 6m. The loaded car has mass 600kg. Safety regulations require that the normal reaction on the car at the top of the loop must be at least $0.5mg$. Take $g = 9.8\text{m/s}^2$.

- Calculate the minimum speed at the top to meet the safety requirement.
- Calculate the minimum height of the launch ramp above the bottom of the loop.
- Calculate the normal reaction at the bottom of the loop when launched from this minimum height. Assess whether the structure must withstand more than $6mg$.

Question 36

A small ball sits on top of a smooth hemispherical dome of radius 3m. It is given a tiny push and slides down the surface. Take $g = 9.8\text{m/s}^2$.

- Find the angle from the vertical and the speed at the point where the ball leaves the dome.
- Determine the height above the ground at which the ball leaves the dome.
- After leaving the dome, the ball becomes a projectile. At the point of leaving, determine the horizontal and vertical components of the velocity.

ANSWERS ON DIGGING DEEPER EXERCISE 7**EXERCISE 7A**

1. (a) Constant. (b) Changing (direction changes continuously). (c) Changing (magnitude is constant but direction changes continuously, always pointing toward the centre). (d) Changing (same reason as centripetal acceleration — constant magnitude but changing direction).

2. (a) Acceleration is the rate of change of velocity, which is a vector. Although the speed (magnitude) is constant, the direction of the velocity changes continuously as the body moves around the circle. A change in direction of velocity constitutes an acceleration. (b) The acceleration is directed toward the centre of the circular path (centripetal).

3. (a) The stone moves in a straight line along the tangent to the circle at the point where the string broke. (b) When the string breaks, the centripetal force vanishes. With no net force acting on it, the stone continues moving at constant velocity in the direction it was travelling at the instant of the break, in accordance with Newton's first law.

4. Angular velocity (ω) is the rate of change of angular displacement, measured in rad/s. Linear velocity (v) is the rate of change of linear displacement along the circular path, measured in m/s. They are related by $v = \omega r$, where r is the radius of the circular path.

5. (a) $a = \frac{v^2}{r}$ (b) $a = \omega^2 r$ (c) Since $v = \omega r$, substituting into $\frac{v^2}{r}$ gives $\frac{(\omega r)^2}{r} = \omega^2 r$. Hence, both are equivalent forms of the same physical quantity expressed using different variables.

6. (a) Static friction between the tyres and the road. (b) Gravitational force of the Earth on the satellite. (c) The component of tension directed toward the centre (assisted or opposed by gravity depending on position). (d) The horizontal component of the normal reaction from the banked road surface. (e) The magnetic force on the moving charged particle.

7. The required centripetal force is $\frac{mv^2}{r}$ and the maximum friction available is μmg . Both are proportional to mass m . When equated, the mass cancels, giving $v_{\max} = \sqrt{\mu gr}$, which is independent of mass. A heavier car needs more centripetal force but also has proportionally more friction available.

8. (a) At the bottom. The tension must support the weight (acting away from centre) and provide the centripetal force (toward centre) simultaneously, so $T_{\text{bottom}} = mg + \frac{mv^2}{r}$, which is the largest value.

(b) At the top. Both tension and weight act toward the centre, so gravity assists the centripetal force and the string needs to supply less: $T_{\text{top}} = \frac{mv^2}{r} - mg$, which is the smallest value.

9. The design speed is the speed at which a vehicle can negotiate the banked curve without any friction. At this speed, the horizontal component of the normal reaction alone provides the required centripetal force.

(a) Faster than design speed: the banking alone cannot provide enough centripetal force. Friction must act down the slope (inward) to supplement the centripetal force. Without sufficient friction, the car slides outward (up the slope).

(b) Slower than design speed: the banking provides more centripetal force than needed. Friction must act up the slope (outward) to prevent the car from sliding inward (down the slope). Without friction, the car slides inward.

10. At the top of a hill, the centripetal acceleration is directed downward (toward the centre of curvature below the road). Part of the driver's weight is "used" to provide this centripetal acceleration, so the normal reaction (which the driver feels as weight) is reduced: $R = mg - \frac{mv^2}{r}$. The driver feels lighter because the support force is less than the actual weight.

The car loses contact with the road when $R = 0$, which occurs at speed $v = \sqrt{gr}$.

11. (a) Inside a loop: the critical speed at the top is a **minimum** speed. Below $v = \sqrt{gr}$, the object loses contact with the track and falls away from the circular path. The normal reaction is zero at the critical speed.

(b) Outside a curve (hill): the critical speed at the top is a **maximum** speed. Above $v = \sqrt{gr}$, the object loses contact with the road and becomes airborne. The normal reaction is zero at the critical speed.

In both cases, the critical speed has the same magnitude \sqrt{gr} , but for the inside loop it is a minimum to maintain contact, while for the outside curve it is a maximum before contact is lost.

EXERCISE 7B

12. When the bus turns right, friction between the seats and the passengers provides the centripetal force needed to follow the curved path. However, the passengers' bodies, due to inertia, tend to continue in a straight line (Newton's first law). This makes them feel as though they are being pushed to the left. No real force pushes them to the left; they simply resist the change in direction while the bus turns beneath them.

13. The recommended speed is based on the maximum speed at which a vehicle can safely negotiate the curve without sliding. This depends on the radius of the curve and the coefficient of friction between the tyres and the road: $v_{\max} = \sqrt{\mu gr}$. A sharp curve (small radius) has a lower safe speed than a gentle curve (large radius). Different curves have different radii, so their recommended speeds differ.

14. When a motorcycle turns, friction at the contact point provides the centripetal force horizontally, while the weight acts vertically through the centre of mass high above the ground. If the rider does not lean, these two forces create a net torque (turning effect of a force) that would topple the motorcycle outward. Leaning inward shifts the centre of mass so that the resultant of weight and friction passes through the contact point, eliminating the toppling torque. Without leaning, the motorcycle would tip outward and crash.

15. At the top of the hump, the bus follows a curved path whose centre of curvature is below the road. Part of the gravitational force is used to provide the centripetal acceleration downward, so the normal reaction from the seat on Kipanga is reduced: $R = mg - \frac{mv^2}{r}$. Kipanga feels lighter because the support force is less than his actual weight. The "stomach dropping" sensation is caused by this reduced contact force.

16. On a straight road, friction is mainly needed for braking and accelerating. On a curve, friction must provide the centripetal force to keep the vehicle on the circular path. When the road is wet, the coefficient of friction decreases. On a straight road, this mainly affects stopping distance. On a curve, it directly reduces the maximum safe speed ($v_{\max} = \sqrt{\mu gr}$), and a vehicle travelling at a speed that was safe on a dry road may now exceed the reduced safe speed and slide off the curve.

17. During the spin cycle, the drum wall provides the centripetal force that keeps the clothes moving in a circle. The water trapped in the fabric requires the same centripetal force to follow the circular path. However, the adhesive forces holding the water to the fabric are too weak to supply the required centripetal force at high speed. The water, unable to maintain circular motion, moves tangentially outward through the holes in the drum, following Newton's first law. The clothes, pressed firmly against the drum wall, remain inside.

18. When a vehicle rounds a curve, friction at ground level provides the centripetal force, while the centre of gravity is above the ground. This creates a torque that tends to tip the vehicle outward about the outer wheels. A tall vehicle has a high centre of gravity, which increases the arm of the overturning torque. A low, wide vehicle has a low centre of gravity and large wheel separation, which increases the restoring torque resisting overturning. The maximum speed before overturning is $v = \sqrt{\frac{arg}{2h}}$, which is smaller for tall vehicles (large h , small a) and larger for low, wide vehicles (small h , large a).

19. The satellite is indeed falling toward the Earth due to gravity. However, it also has a large horizontal velocity. As it falls toward the Earth, it moves forward fast enough that the Earth's surface curves away beneath it at the same rate. The satellite continuously falls toward the Earth but never reaches it because the curvature of its path matches the curvature of the Earth. Gravity provides the centripetal force that bends the satellite's path into a circle. Mr. Akilikubwa's description is accurate: orbiting is continuous free fall where the falling object keeps "missing" the ground.

20. On a banked curve, the road surface is tilted so that the normal reaction has a horizontal component directed toward the centre of the curve. This component provides centripetal force without relying on friction. The advantage is that the vehicle can negotiate the curve safely even on a wet or icy surface, as long as it travels near the design speed.

However, banking alone is sufficient only at the design speed ($v = \sqrt{rg \tan \theta}$). At higher speeds, the banking cannot supply enough centripetal force, and friction is needed to make up the difference. At lower speeds, the banking provides too much inward force, and friction is needed to prevent the vehicle from sliding inward. Therefore, banking reduces dependence on friction but cannot eliminate it at all speeds.

21. Gravity does act on the astronauts and the space station. The astronauts appear weightless not because gravity is absent, but because both the astronauts and the station are in free fall together. Gravity provides the centripetal force for the orbital motion, and since everything inside the station accelerates toward the Earth at the same rate, there is no relative force between the astronauts and the station walls. With no contact force (normal reaction), the astronauts feel weightless. Weightlessness is the absence of a contact force, not the absence of gravity.

22. (a) 9375N (b) 7056N (c) Since the required force (9375N) exceeds the maximum friction (7056N), the car **cannot** safely negotiate the curve. It will skid outward.

23. (a) 34.1° (b) 4.73N (c) $r = 0.673\text{m}$, $v = 2.11\text{m/s}$

24. (a) The speed is different at different positions in a vertical circle. The student used v_{bottom} in the formula for the top, but the correct speed at the top (v_{top}) must first be found using energy conservation.

(b) Since v_{top}^2 is negative, the ball **never reaches the top**. (b) 114.9

25. (a) 18.9m/s (b) $7.52\text{m/s} \leq v \leq 27\text{m/s}$

26. (a) 12.25N (b) $v_{\text{top}} = 2.93\text{m/s}$, $T_{\text{top}} = 0.49\text{N}$ (c) 73.9°

27. (a) Since $h(1.5\text{m}) > h_{\text{min}}(1.25\text{m})$, the ball completes the loop. (b) $v_{\text{top}} = 3.13\text{m/s}$, $R_{\text{top}} = 0.98\text{N}$ (c) $R_{\text{bottom}} = 6.86\text{N}$

28. (a) 19.4m/s

(b) Since expected speed (16.7m/s) < maximum speed (19.4m/s), the curve is safe for the expected traffic speed on a dry road.

(c) 22.1° (d) 29.1m/s

29. (a) 26700N (b) 1.82 (c) 22.1m/s

30. (a) 2.39N (b) 20.03N (c) 7.16m/s

31. (a) For 0.2kg ball: 6.26m/s, For 0.5kg ball: 6.26m/s

(b) The student's claim is **wrong**. The minimum speed does not depend on mass. The formula $v_{\text{min}} = \sqrt{5gr}$ contains no mass term. This is because both the centripetal force requirement and the gravitational force increase in equal proportion with mass, so mass cancels.

32. (a) 10.6m/s (b) 1.13N (c) 66.4°

33. (a) 18.1m/s (b) 16.6m/s (c) the car **skids first** (Hint: compare speeds or coefficient of friction and $\frac{a}{2h}$)

(d) Inner wheel: 1.525m, outer wheel: 8.275m; where m is the mass of the car in kg.

34. (a) Since $v_{\text{bottom}}(5.5\text{m/s}) < v_{\text{min, bottom}}(7\text{m/s})$, the ball does **not** complete the circle. (b) 111.3° (c) 1.88m/s

35. (a) 9.39m/s (b) 16.5m (c) $\frac{R_{\text{bottom}}}{mg} = 6.5$. The structure must withstand 6.5mg at the bottom, which is greater than 6mg. Hence, the engineer must design for forces exceeding 6mg.

36. (a) $\theta = 48.2^\circ$, $v = 4.43\text{m/s}$ (b) 2m (c) Horizontal component: 2.95m/s, Vertical component: 3.3m/s

