VERTICAL CIRCULAR MOTION

Understanding Vertical Circular Motion

In horizontal circular motion, gravity stayed in the background. It held objects down, balanced normal reactions, and determined the weight. But it never directly helped or opposed the centripetal force. The centripetal force came from friction, tension, or the horizontal component of a normal reaction, while gravity quietly maintained vertical equilibrium.

In vertical circular motion, everything changes. The circular path is now oriented vertically, and gravity acts along the same line as the centripetal direction at certain points. At the top of the circle, gravity pulls the object **toward the centre** and helps maintain the circular path. At the bottom, gravity pulls the object **away from the centre** and must be overcome. At the sides, gravity acts perpendicular to the radius and contributes nothing to the centripetal force.

This means the forces on the object change continuously as it moves around the loop. The tension in a string, or the normal reaction from a track, is different at every position. The speed also changes: the object slows down as it rises and speeds up as it falls, because gravity does work on it throughout the motion. Vertical circular motion is therefore **not uniform**: the speed is not constant.

Despite this complexity, the method remains exactly the same as in horizontal circular motion. At any instant, we apply Newton's second law in the radial direction (toward the centre) and set the net inward force equal to $\frac{mv^2}{r}$, where v is the speed at that particular point. The only difference is that the forces contributing to the net inward force change with position.

Forces at different positions in the vertical circle

To understand vertical circular motion, we must examine what happens at three key positions: the **bottom**, the **top**, and the **side** (at the level of the centre). Consider an object of mass m attached to a string of length r and swung in a vertical circle. At each position, we identify the forces and apply Newton's second law toward the centre.

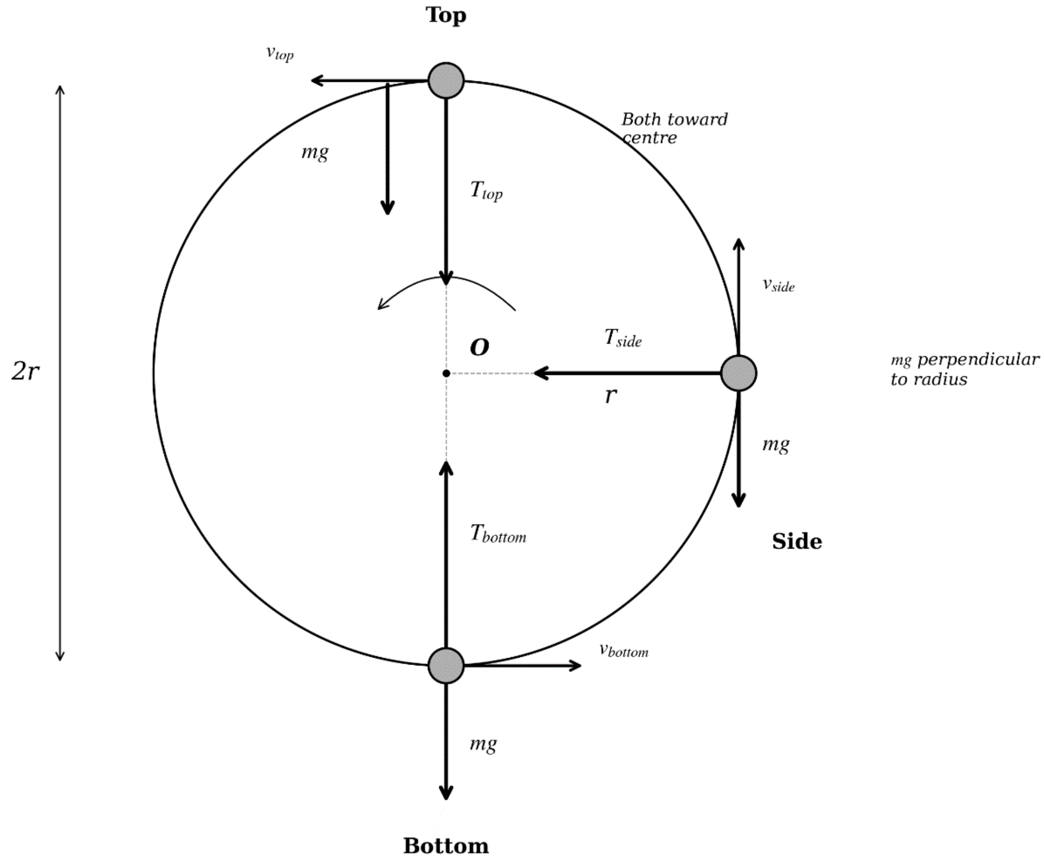

Figure: ***Forces on an object at three key positions in a vertical circle.*** *At the bottom, tension acts toward the centre (upward) and weight acts away from it (downward). At the top, both tension and weight act toward the centre (downward). At the side, tension acts toward the centre (horizontally) while weight acts perpendicular to the radius (downward). The velocity is tangent to the circle at each position.*

At the bottom of the circle

At the lowest point, the centre of the circle is directly above the object. The forces are:

- Tension T_{bottom} acts upward (toward the centre).
- Weight mg acts downward (away from the centre).

Applying Newton's second law toward the centre (upward at this point):

$$T_{bottom} - mg = \frac{mv_{bottom}^2}{r}$$

From which:

$$\mathbf{T_{bottom} = mg + \frac{mv_{bottom}^2}{r}}$$

The tension at the bottom is **greater** than the weight. The string must support the weight against gravity and simultaneously provide the centripetal force. *This is the position where the tension is maximum and the string is most likely to break.*

At the top of the circle

At the highest point, the centre is directly below the object. The forces are:

- Tension T_{top} acts downward (toward the centre).
- Weight mg acts downward (also toward the centre).

Both forces point in the same direction: toward the centre. Applying Newton's second law toward the centre (downward at this point):

$$T_{top} + mg = \frac{mv_{top}^2}{r}$$

From which:

$$\mathbf{T_{top}} = \frac{\mathbf{mv_{top}^2}}{\mathbf{r}} - \mathbf{mg}$$

The tension at the top is **less** than at the bottom. Gravity now assists the centripetal force, so the string does not need to work as hard. *This is the position where the tension is minimum*. *If the speed is too low, the tension becomes zero or the formula gives a negative value, meaning the string goes slack and the object falls out of the circular path.*

At the side of the circle (level with the centre)

At the position level with the centre, the forces are:

- Tension T_{side} acts horizontally toward the centre.
- Weight mg acts vertically downward (perpendicular to the radius at this point).

Since weight is perpendicular to the radial direction, it does not contribute to the centripetal force. Applying Newton's second law toward the centre:

$$T_{side} = \frac{mv_{side}^2}{r}$$

At this position, the tension provides the entire centripetal force by itself, and gravity has no radial effect. However, gravity does cause the speed to change as the object moves through this position.

Summary of tension at three key positions

$$T_{bottom} = mg + \frac{mv_{bottom}^2}{r} \qquad \text{(maximum)}$$

$$T_{side} = \frac{mv_{side}^2}{r}$$

$$T_{top} = \frac{mv_{top}^2}{r} - mg \qquad \text{(minimum)}$$

Tension at a general angle

The bottom, top, and side are three special positions. *But what if you need the tension when the string makes an angle θ measured from the lowest point (the bottom)?*

At a general angle θ from the bottom, the object is at a height $h = r - r\cos\theta = r(1 - \cos\theta)$ above the lowest point. The string points from the object toward the centre, and the weight mg acts vertically downward.

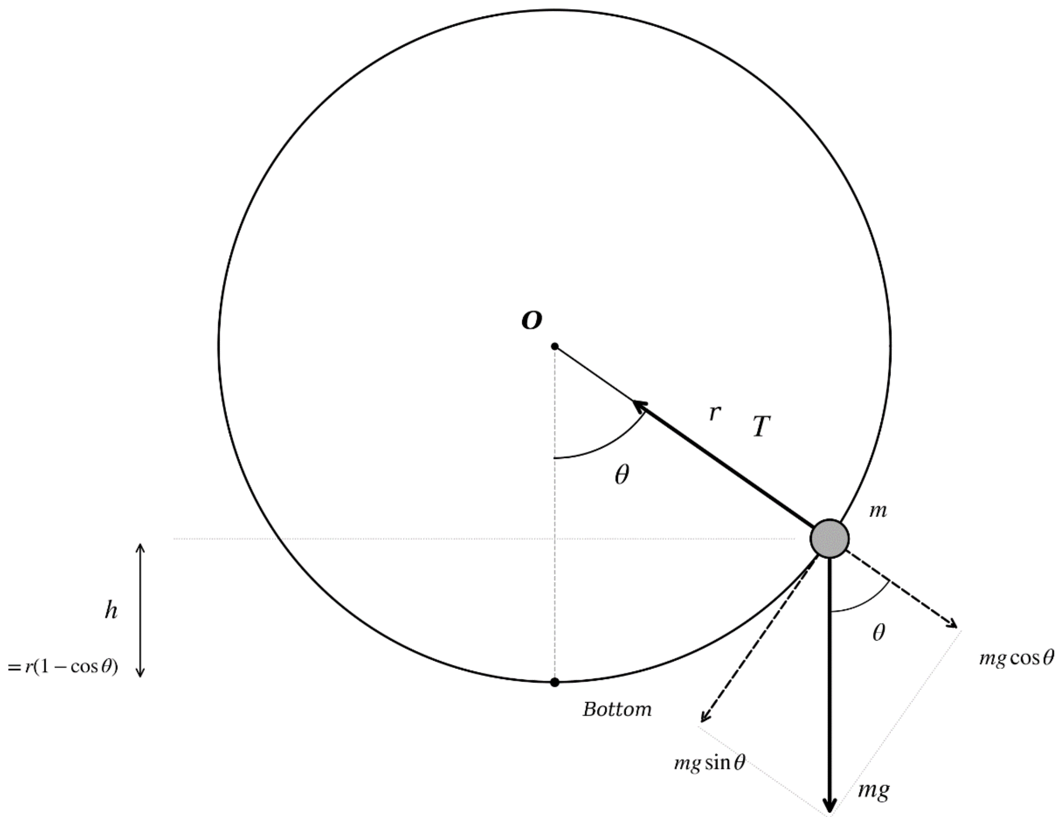

Figure: ***An object at a general angle θ from the bottom of a vertical circle.*** *Tension T acts along the string toward the centre O. Weight mg acts vertically downward and is resolved into two components: mgcosθ along the radius (away from the centre) and mgsinθ along the tangent. The angle between mg and the outward radial direction equals θ. The height above the bottom is* $h = r(1 - cos\theta)$.

The component of the weight along the radius (toward or away from the centre) is $mg\cos\theta$. At angle θ from the bottom, this component acts away from the centre (opposing the centripetal direction).

Applying Newton's second law toward the centre:

$$T - mg\cos\theta = \frac{mv^2}{r}$$

From which:

$$\mathbf{T = mg\cos\theta + \frac{mv^2}{r}}$$

This is the **general expression for tension at any point** in the vertical circle. Let us verify that it gives our earlier results at the three special positions:

At the **bottom** ($\theta = 0°$): $\cos 0° = 1$, so $T = mg + \frac{mv^2}{r}$. Correct.

At the **side** ($\theta = 90°$): $\cos 90° = 0$, so $T = \frac{mv^2}{r}$. Correct.

At the **top** ($\theta = 180°$): $\cos 180° = -1$, so $T = -mg + \frac{mv^2}{r} = \frac{mv^2}{r} - mg$. Correct.

All three special cases emerge naturally from the general formula. This confirms that the general expression is consistent.

Finding speed at any angle using energy conservation

To use the general tension formula, we need the speed v at angle θ. Using conservation of energy between the bottom and the position at angle θ:

$$\frac{1}{2}mv_{bottom}^2 = \frac{1}{2}mv^2 + mgh$$

$$\frac{1}{2}v_{bottom}^2 = \frac{1}{2}v^2 + gh$$

$$v^2 = v_{bottom}^2 - gh$$

Where $h = r(1 - \cos\theta)$ (you can easily deduce it from diagram by using geometry):

$$v^2 = v_{bottom}^2 - 2gr(1 - \cos\theta)$$

Substituting into the general tension formula:

$$T = mg\cos\theta + \frac{m}{r}\left(v_{bottom}^2 - 2gr(1 - \cos\theta)\right)$$

$$T = mg\cos\theta + \frac{mv_{bottom}^2}{r} - 2mg(1 - \cos\theta)$$

$$\mathbf{T = \frac{mv_{bottom}^2}{r} - 2mg + 3mg\cos\theta}$$

This powerful expression gives the tension at any angle θ in terms of the speed at the bottom alone. It shows clearly that:

- Tension decreases as θ increases from 0° to 180° (because cosθ decreases), confirming that the bottom has maximum tension and the top has minimum tension.
- The tension depends on cosθ, so it changes smoothly and continuously around the circle.

It is important for you to understand that:

The speed v is different at each position. As the object rises from bottom to top, it loses kinetic energy and gains gravitational potential energy, so it slows down. As it descends from top to bottom, it speeds up. This means $v_{bottom} > v_{side} > v_{top}$ for an object that completes the full circle.

To find the relationship between speeds at different positions, we use **conservation of energy**, which we will apply in the worked examples and in the next subtopic.

The difference between the tension at the bottom and at the top

A useful result can be obtained by subtracting the tension at the top from the tension at the bottom:

$$T_{bottom} - T_{top} = \left(mg + \frac{mv_{bottom}^2}{r}\right) - \left(\frac{mv_{top}^2}{r} - mg\right)$$

$$T_{bottom} - T_{top} = 2mg + \frac{m}{r}\left(v_{bottom}^2 - v_{top}^2\right)$$

Using conservation of energy between the bottom and top of the circle:

$$\frac{1}{2}mv_{bottom}^2 = \frac{1}{2}mv_{top}^2 + mg(2r)$$

From which:

$$v_{bottom}^2 - v_{top}^2 = 4gr$$

Substituting:

$$T_{bottom} - T_{top} = 2mg + \frac{m}{r} \times 4gr = 2mg + 4mg$$

$$\mathbf{T_{bottom} - T_{top} = 6mg}$$

This is an elegant result: regardless of the speed, the difference in tension between the bottom and the top is always exactly 6mg. This result holds for any object completing a full vertical circle on a string, provided no energy is lost to friction or air resistance.

With the framework established, let us practise through worked examples.

BINDER Example 31

A ball of mass 0.4 kg is attached to a string of length 0.5 m and swung in a vertical circle. At the bottom of the circle, the speed is 6 m/s. Take $g = 9.8\ m/s^2$.

(a) Calculate the tension in the string at the bottom of the circle.

(b) State whether this is the maximum or minimum tension in the string during the motion. Explain briefly.

Solution

(a) Using:

$$T_{bottom} = mg + \frac{mv_{bottom}^2}{r}$$

Where: $m = 0.4$ kg, $g = 9.8\ m/s^2$, $v_{bottom} = 6$ m/s, $r = 0.5$ m

Substituting:

$$T_{bottom} = 0.4kg \times 9.8\ m/s^2 + \frac{0.4kg \times (6\ m/s^2)^2}{0.5m} = 32.7N$$

The tension at the bottom is 32.7N.

(b) This is the **maximum** tension. At the bottom, the string must support the weight and provide centripetal force simultaneously, making the tension greatest at this position.

Making Sense of the Answer: *The weight is only 3.92N, but the tension is 32.7N, more than eight times the weight. The centripetal force demand (28.8N) dominates because the speed is high and the radius is small. This is why strings and ropes can snap at the bottom of a vertical swing even when the object is light.*

Think Like a Physicist: *At the bottom, tension and weight pull in opposite directions, so the net inward force is* $T - mg$. *At the top, they pull in the same direction, so the net inward force is* $T + mg$. *Always draw the forces first and identify which direction is "toward the centre" at each position.*

BINDER Example 32

A ball of mass 0.5kg is attached to a string of length 0.8m and swung in a vertical circle. At the top of the circle, the tension in the string is 5N. Take $g = 9.8\ m/s^2$.

Determine the tension in the string at the bottom of the circle.

Solution

This problem can be solved directly using the result $T_{bottom} - T_{top} = 6mg$:

$$T_{bottom} = T_{top} + 6mg$$

Substituting:

$$T_{bottom} = 5N + 6 \times 0.5kg \times 9.8\ m/s^2 = 34.4N$$

The tension at the bottom is 34.4N.

Making Sense of the Answer: *The tension increases by exactly $6mg = 29.4\ N$ from top to bottom. This is a fixed difference that depends only on the mass and gravity, not on the speed. Even if the ball were moving faster or slower (as long as it completes the circle), this difference would be the same. The result is powerful because it allows us to find one tension from the other without knowing the speed at either point.*

Think Like a Physicist: *The result $T_{bottom} - T_{top} = 6mg$ is worth memorising. It provides a shortcut in many vertical circular motion problems. However, remember that it applies only when energy is conserved (no friction or air resistance) and only for motion on a string where tension provides the centripetal force.*

REAL Example 33

Kipanga swings a stone tied to a string in a vertical circle. He notices that the string feels much tighter when the stone passes through the bottom of the circle than when it passes through the top. He says to **Kipute**: *"The stone feels heavier at the bottom. Does its weight change during the swing?"*

Help Kipute answer Kipanga's question.

Solution

The weight of the stone (mg) does not change; gravity pulls it downward with the same force at every point in the circle. What changes is the **tension** in the string.

At the bottom, the centre of the circle is above the stone. The string must pull upward with enough force to both support the weight and provide the centripetal force toward the centre. So the tension at the bottom is $T_{bottom} = mg + \frac{mv^2}{r}$, which is greater than the weight.

At the top, the centre is below the stone. Both gravity and tension pull toward the centre (downward). Gravity already provides part of the centripetal force, so the string needs to supply less. The tension at the top is $T_{top} = \frac{mv^2}{r} - mg$, which is less than at the bottom.

What Kipanga feels as "heavier" is not a change in weight but an increase in tension. The string must work hardest at the bottom and easiest at the top. The difference is always exactly 6mg.

Making Sense of the Answer: *It is the same illusion as feeling heavier when a lift accelerates upward. Your weight has not changed, but the support force (tension or normal reaction) has increased. At the bottom of a vertical circle, the string provides an upward acceleration (centripetal), making it feel like the object is heavier.*

Think Like a Physicist: *The feeling of heaviness or lightness is determined by the contact force (tension or normal reaction), not by the actual weight.*

HOT Example 34

A ball of mass 0.25kg is attached to a string of length 0.6m and set in motion in a vertical circle. The speed at the bottom of the circle is 4.5m/s. Take $g = 9.8\ m/s^2$.

Determine the angle from the bottom at which the string goes slack.

Solution

The string goes slack when the tension becomes zero. Using the general expression for tension at angle θ from the bottom:

$$T = \frac{mv_{bottom}^2}{r} - 2mg + 3mg\cos\theta$$

Setting $T = 0$:

$$0 = \frac{mv_{bottom}^2}{r} - 2mg + 3mg\cos\theta$$

Making $\cos\theta$ the subject:

$$3mg\cos\theta = 2mg - \frac{mv_{bottom}^2}{r}$$

$$\cos\theta = \frac{2}{3} - \frac{v_{bottom}^2}{3gr}$$

Where: $v_{bottom} = 4.5$ m/s, $g = 9.8$ m/s^2, $r = 0.6$ m

Substituting:

$$\cos\theta = \frac{2}{3} - \frac{(4.5\text{ m/s})^2}{3 \times 9.8\text{ m/s}^2 \times 0.6\text{ m}} = -0.481$$

$$\theta = \cos^{-1}(-0.481) = 118.8°$$

The string goes slack at 118.8° from the bottom.

Making Sense of the Answer: *The angle is between 90° (the side) and 180° (the top). This means the ball passes through the side position successfully but loses tension before reaching the top. At 118.8° from the bottom, the ball is well above the centre of the circle. After the string goes slack, the ball follows a projectile path (parabola) rather than continuing along the circle. If the speed at the bottom were greater, the string would go slack at a larger angle (closer to the top), or not at all if the speed is sufficient to complete the full circle.*

Think Like a Physicist: *The formula* $\cos\theta = \frac{2}{3} - \frac{v_{bottom}^2}{3gr}$ *tells us directly whether the ball completes the circle. If the right side is less than* -1 *(meaning* $\cos\theta < -1$*, which is impossible), the tension never reaches zero and the ball completes the full circle. This happens when* $v_{bottom}^2 > 5gr$*, a result we will derive in the next subtopic.*

The ideas are now in place. In the next subtopic, we examine the most important question in vertical circular motion: *what is the minimum speed needed to maintain a complete vertical circle?* This is where strings go slack, water falls from buckets, and roller coasters must be carefully engineered.

Minimum Speed and Critical Condition of Object on a String

In the previous subtopic, we derived the tension at the top of the circle as:

$$T_{top} = \frac{mv_{top}^2}{r} - mg$$

An important question arises from this equation: *what happens if the speed at the top is very small?*

As v_{top} decreases, the term $\frac{mv_{top}^2}{r}$ becomes smaller and the tension decreases. At some critical speed, the tension reaches zero. If the speed drops below this critical value, the formula gives a negative tension, but a string cannot push; it can only pull. So a negative tension is physically meaningless. It means the string has gone slack and the object has left the circular path.

Finding the minimum speed at the top

At the critical condition, the string is on the verge of going slack:

$$T_{top} = 0$$

Substituting into the equation for tension at the top:

$$0 = \frac{mv_{min,top}^2}{r} - mg$$

$$\frac{mv_{min,top}^2}{r} = mg$$

Making $v_{min,top}$ the subject:

$$\mathbf{v_{min,top} = \sqrt{gr}}$$

At this critical speed, gravity alone provides exactly the centripetal force needed at the top of the circle. If the speed becomes any smaller, gravity would be more than enough for the circular motion at that speed. However, this would require a circular path of a smaller radius while the object cannot actually move in that smaller circle because the string fixes the radius. As a result, the object leaves the circular path and begins to fall under gravity.

Again, **you have to understand this:** The minimum speed at the top does not depend on mass. A heavy ball and a light ball on the same string both need the same minimum speed $\sqrt{gr}$ at the top to complete the circle. This is because both the centripetal force requirement ($\frac{mv^2}{r}$) and the gravitational force (mg) are proportional to mass, and mass cancels.

Finding the minimum speed at the bottom for a complete circle

Knowing the minimum speed at the top is useful, but in practice we control the speed at the bottom (where we launch or swing the object). The question becomes: *what minimum speed must the object have at the bottom so that it still has at least $\sqrt{gr}$ at the top?*

We use conservation of energy between the bottom and the top. The height difference is 2r (the full diameter of the circle).

$$\frac{1}{2}mv_{bottom}^2 = \frac{1}{2}mv_{top}^2 + mg(2r)$$

At the critical condition, $v_{top} = \sqrt{gr}$, so $v_{top}^2 = gr$:

$$\frac{1}{2}mv_{min,bottom}^2 = \frac{1}{2}m(gr) + mg(2r)$$

$$\frac{1}{2}v_{min,bottom}^2 = \frac{1}{2}gr + 2gr = \frac{5}{2}gr$$

$$\mathbf{v_{min,bottom} = \sqrt{5gr}}$$

This is a key result in vertical circular motion. To complete a full vertical circle on a string, the speed at the bottom must be at least $\sqrt{5gr}$. Below this speed, the string goes slack somewhere before reaching the top.

Connecting to the general angle result

In the previous subtopic, the Think Like a Physicist section for Example 34 noted that the ball completes the circle when $v_{bottom}^2 > 5gr$. We have now derived this condition formally. The two results are consistent: $v_{min,bottom} = \sqrt{5gr}$ is the boundary between completing the circle and falling out of it.

Summary of critical speeds for a vertical circle on a string:

$$v_{min,top} = \sqrt{gr}$$

$$v_{min,bottom} = \sqrt{5gr}$$

Tension at the bottom when the ball just completes the circle

When $v_{bottom} = \sqrt{5gr}$:

$$T_{bottom} = mg + \frac{mv_{bottom}^2}{r} = mg + \frac{m(5gr)}{r} = mg + 5mg = 6mg$$

At the critical condition, the tension at the bottom is exactly 6mg. This is a useful benchmark: if the string can withstand a tension of at least 6mg, the object can just complete the vertical circle.

With the critical conditions established, let us put them to work.

BINDER Example 35

A ball of mass 0.3kg is attached to a string of length 0.5m and swung in a vertical circle. Take $g = 9.8\text{m/s}^2$.

(a) Calculate the minimum speed at the top for the ball to complete the circle.

(b) Calculate the minimum speed at the bottom for the ball to complete the circle.

(c) Find the tension in the string at the bottom when the ball has exactly the minimum speed to complete the circle.

Solution

(a) Using:

$$v_{\text{min,top}} = \sqrt{gr}$$

Where: $g = 9.8\text{m/s}^2$, $r = 0.5\text{m}$

Substituting:

$$v_{\text{min,top}} = \sqrt{9.8\text{m/s}^2 \times 0.5\text{m}} = \sqrt{4.9\text{m}^2/\text{s}^2} = 2.21\text{m/s}$$

The minimum speed at the top is 2.21m/s.

(b) Using:

$$v_{\text{min,bottom}} = \sqrt{5gr}$$

Substituting:

$$v_{\text{min,bottom}} = \sqrt{5 \times 9.8\text{m/s}^2 \times 0.5\text{m}} = \sqrt{24.5\text{m}^2/\text{s}^2} = 4.95\text{m/s}$$

The minimum speed at the bottom is 4.95m/s.

(c) Using:

$$T_{\text{bottom}} = 6mg$$

Substituting:

$$T_{\text{bottom}} = 6 \times 0.3\text{kg} \times 9.8\text{m/s}^2 = 17.64\text{N}$$

The tension at the bottom is 17.64N.

Making Sense of the Answer: *The minimum speed at the bottom (4.95m/s) is more than double the minimum speed at the top (2.21m/s). This is because the ball must have enough kinetic energy at the bottom to climb a height of $2r = 1m$ and still arrive at the top with speed 2.21m/s. Most of the kinetic energy at the bottom is consumed by the climb against gravity. The tension at the bottom (17.64N) is six times the weight (2.94N), confirming that the string is under considerable stress even at the minimum speed.*

Think Like a Physicist: *The ratio $\frac{v_{min,bottom}}{v_{min,top}} = \frac{\sqrt{5gr}}{\sqrt{gr}} = \sqrt{5} \approx 2.24$. This ratio is universal: the minimum speed at the bottom is always $\sqrt{5}$ times the minimum speed at the top, regardless of the radius or the mass.*

REAL Example 36

During a school science fair, **Kipanga** fills a bucket with water and swings it in a vertical circle over his head. To everyone's amazement, the water stays inside the bucket even when the bucket is upside down at the top of the circle. **Kipute** watches carefully and then says:

"I bet if you swing it slowly, the water will fall on your head."

Kipanga grins and slows down. Sure enough, as the swing becomes slower, water splashes out of the bucket and drenches him from above. The class erupts in laughter.

Mr. Akilikubwa, wiping tears of laughter from his eyes, says: *"Kipanga has just demonstrated the minimum speed condition. Now, Kipute, explain to the class: why did the water stay in the bucket at high speed, and why did it fall out when Kipanga slowed down?"*

Help Kipute explain to the class.

Solution

When the bucket is at the top of the vertical circle, it is upside down. At this point, the centre of the circle is directly below. Both the weight of the water and any contact force from the bucket act downward (toward the centre).

At high speed, the water needs a large centripetal force ($\frac{mv^2}{r}$) to follow the circular path. Gravity alone provides mg, but this is not enough. The bucket base must push the water downward to supply the extra centripetal force. Since the bucket pushes down on the water, the water pushes up on the bucket (Newton's third law). The water stays in contact with the bucket and remains inside.

When Kipanga slows down, the required centripetal force decreases. At the critical speed $v = \sqrt{gr}$, gravity alone provides exactly the centripetal force needed, and the bucket base exerts zero force on the water. Below this speed, gravity provides more downward force than needed for the circular path. The water wants to fall faster than the bucket, so it separates from the bucket base and falls out, landing on Kipanga's head.

In short: the water stays in when the required centripetal acceleration ($\frac{v^2}{r}$) exceeds g, and falls out when $\frac{v^2}{r} < g$.

Making Sense of the Answer: *The bucket trick works because at high speed, the water's inertia is so large that gravity cannot pull it away from the circular path. The water "wants" to fly off tangentially but the bucket forces it to curve. At low speed, gravity dominates over inertia, and the water simply falls. The critical speed $\sqrt{gr}$ is the balance point between these two regimes.*

Think Like a Physicist: *This is exactly the same physics as the string going slack. Replace "bucket base pushing water" with "string pulling ball" and the analysis is identical. In both cases, the contact force reaches zero at $v = \sqrt{gr}$ at the top. The bucket trick is simply a dramatic demonstration of the minimum speed condition.*

HOT Example 37

A ball on a string of length 1.2m is set in motion in a vertical circle. The speed at the bottom is 6.5m/s. Take $g = 9.8\text{m/s}^2$.

Determine whether the ball completes the full circle, and if not, find the angle from the bottom at which the string goes slack.

Solution

First, check whether the speed at the bottom is sufficient to complete the circle.

The minimum speed required at the bottom is:

$$v_{min,bottom} = \sqrt{5gr} = \sqrt{5 \times 9.8\text{m/s}^2 \times 1.2\text{m}} = \sqrt{58.8\text{m}^2/\text{s}^2} = 7.67\text{m/s}$$

Since the actual speed at the bottom (6.5m/s) is less than the minimum required (7.67m/s), the ball does **not** complete the full circle. The string goes slack before the ball reaches the top.

To find the angle where the string goes slack, set $T = 0$ in the general formula:

$$\cos\theta = \frac{2}{3} - \frac{v_{bottom}^2}{3gr}$$

Substituting:

$$\cos\theta = \frac{2}{3} - \frac{(6.5\text{m/s})^2}{3 \times 9.8\text{m/s}^2 \times 1.2\text{m}} = -0.531$$

$$\theta = \cos^{-1}(-0.531) = 122.1°$$

The string goes slack at $122.1°$ from the bottom.

Making Sense of the Answer: *The ball fails to complete the circle because 6.5m/s is well below the required 7.67m/s. It goes slack at 122.1° from the bottom, which is about 32° past the horizontal (side) position and roughly 58° short of the top. The ball makes it more than two-thirds of the way around but runs out of speed before reaching the top. After the string goes slack, the ball follows a parabolic projectile path.*

Think Like a Physicist: *This problem requires two distinct skills: first, recognising that the speed is insufficient (comparing v_{bottom} with $\sqrt{5gr}$), and second, finding exactly where the string fails (using the general tension formula with $T = 0$). A common mistake is to assume that if the ball cannot reach the top, the string must go slack at the top. In reality, it goes slack at whatever angle the tension first becomes zero, which is generally below the top.*

That brings the critical speed conditions to a close. We now know when a string goes slack and what speed is needed to prevent it. In the next subtopic, we replace the string with a track: an object moves inside a circular loop (like a roller coaster). The physics is similar, but with one important difference: the track pushes rather than pulls, and the critical condition involves the normal reaction rather than tension.

Loop-the-Loop

A roller coaster climbs slowly to the top of a tall slope, pauses for a breathless moment, then plunges downward, gaining terrifying speed. Moments later, it enters a vertical loop and the passengers find themselves upside down, pressed into their seats, screaming with a mixture of terror and delight. The car races around the inside of the loop and emerges safely at the bottom.

How does this work? Why don't the passengers fall when they are upside down? And how do engineers decide how tall the initial slope must be?

The answers come from the same physics we developed for the ball on a string, with one crucial difference. A string pulls. A track pushes. The string exerts tension directed inward along its length. The track exerts a **normal reaction** directed perpendicular to its surface. For a circular loop, the inner surface pushes the object toward the centre at every point. So in the loop-the-loop, normal reaction replaces tension as the force that works alongside gravity to maintain circular motion.

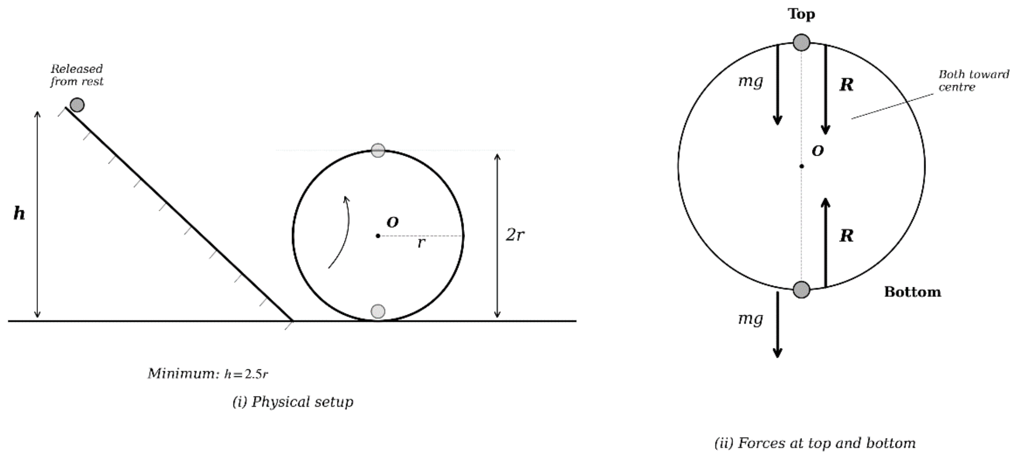

Figure: ***Loop-the-loop.*** *(i) A ball is released from rest at height h on a ramp and enters a vertical circular loop of radius r. The minimum release height for a complete loop is h = 2.5r. (ii) Forces on the ball at the top and bottom of the inside loop. At the top, both the normal reaction R and weight mg act downward toward the centre. At the bottom, R acts upward toward the centre while mg acts downward away from it.*

Forces at the top of an inside loop

At the top of a loop, the object is on the **inside** of the track. The centre of the circle is directly below. Both forces act downward (toward the centre):

- Weight mg acts downward.
- Normal reaction R from the track acts downward (the track is above the object and pushes it toward the centre).

Applying Newton's second law toward the centre (downward):

$$mg + R = \frac{mv_{top}^2}{r}$$

From which:

$$\mathbf{R = \frac{mv_{top}^2}{r} - mg}$$

Compare this carefully with the string result: $T_{top} = \frac{mv_{top}^2}{r} - mg$. The formula is identical in structure. The only change is the symbol: R instead of T.

Forces at the bottom of the loop

At the bottom, the object is on the inside of the track. The centre is directly above. The forces oppose each other:

- Weight mg acts downward (away from centre).
- Normal reaction R acts upward (toward the centre).

Applying Newton's second law toward the centre (upward):

$$R - mg = \frac{mv_{bottom}^2}{r}$$

$$\mathbf{R = mg + \frac{mv_{bottom}^2}{r}}$$

Again, identical in structure to $T_{bottom} = mg + \frac{mv_{bottom}^2}{r}$.

The critical condition: losing contact

For the string, the critical condition was $T = 0$ (string goes slack). For the track, the critical condition is $R = 0$ (object loses contact with the track surface).

Setting $R = 0$ at the top:

$$0 = \frac{mv_{min,top}^2}{r} - mg$$

$$\mathbf{v_{min,top} = \sqrt{gr}}$$

The same result as for the string. At this critical speed, gravity alone provides the centripetal force at the top, and the track exerts no force on the object. Below this speed, the object separates from the track.

There is, however, a subtle but important difference. When a string goes slack, the object falls inward (toward the centre) because nothing holds it outward. When an object loses contact with the **inside** of a loop, it also falls away from the track; but now that means falling inward as well (the track is on the outside of the object's position). In both cases, the object leaves the circular path and follows a projectile trajectory.

Minimum release height for a complete loop

Roller coasters and toy loop-the-loop tracks launch the object from a height, converting gravitational potential energy into kinetic energy. The practical question is: *from what minimum height must the object be released to complete the loop?*

Let the loop have radius r, and let the object be released from rest at height h above the bottom of the loop.

Using conservation of energy between the release point and the top of the loop:

$$mgh = \frac{1}{2}mv_{top}^2 + mg(2r)$$

At the critical condition, $v_{top}^2 = gr$:

$$mgh_{min} = \frac{1}{2}m(gr) + mg(2r)$$

$$gh_{min} = \frac{1}{2}gr + 2gr = \frac{5}{2}gr$$

$$\mathbf{h_{min} = \frac{5}{2}r = 2.5r}$$

The object must be released from a height of at least 2.5r above the bottom of the loop. For a loop of radius 2m, the minimum release height is 5m. Engineers always design the initial slope to be taller than this to provide a safety margin.

The worked examples that follow will bring these concepts to life and make them concrete.

BINDER Example 38

A small ball rolls without friction along a track that includes a vertical circular loop of radius 0.4m. The ball is released from rest at a height h above the bottom of the loop. Take $g = 9.8\text{m/s}^2$.

(a) Find the minimum height from which the ball must be released to complete the loop.

(b) At this minimum release height, find the speed of the ball at the top of the loop.

(c) Find the normal reaction on the ball at the top of the loop at the minimum speed.

Solution

(a) Using:

$$h_{min} = \frac{5}{2}r = \frac{5}{2} \times 0.4m = 1m$$

The minimum release height is 1m above the bottom of the loop.

(b) At the minimum condition, the speed at the top is:

$$v_{min,top} = \sqrt{gr} = \sqrt{9.8m/s^2 \times 0.4m} = \sqrt{3.92m^2/s^2} = 1.98m/s$$

The speed at the top is 1.98m/s.

(c) At the minimum speed, the normal reaction at the top is:

$$R = 0N$$

At the critical condition (minimum speed), gravity provides all the centripetal force and the track exerts no force on the ball.

Making Sense of the Answer: *The release height (1m) is 2.5 times the loop radius (0.4m). The ball arrives at the top of the loop with just enough speed (1.98m/s) for gravity to maintain the circular path alone. The track barely touches the ball at this point. Any lower release height and the ball falls away from the track before reaching the top.*

Think Like a Physicist: *Part (c) is deceptively simple but tests deep understanding. Many students calculate R using the full formula and get zero, then doubt their answer. Trust the physics: the minimum speed condition is defined as the speed at which R = 0. If you set up the problem correctly and get R = 0 at the minimum speed, that confirms your working, not a mistake.*

REAL Example 39

A toy car track includes a vertical loop. A child releases the toy car from the top of a ramp. On the first attempt, the car enters the loop, climbs partway up the inside, then falls away from the track before reaching the top. On the second attempt, the child places the car higher on the ramp, and this time it goes all the way around.

Explain why increasing the release height allows the car to complete the loop.

Solution

When the car is released from rest on the ramp, its gravitational potential energy converts into kinetic energy as it descends. A higher release point means more potential energy is available, so the car arrives at the bottom of the loop with greater speed.

As the car climbs inside the loop, it loses kinetic energy and gains potential energy. At the top of the loop, the car must still have enough speed for gravity to provide the centripetal force needed to maintain contact with the track (at minimum, $v_{top} = \sqrt{gr}$).

On the first attempt, the release height was too low. The car arrived at the bottom with insufficient speed, and its kinetic energy was exhausted before reaching the top. The car slowed to the point where gravity could no longer maintain the circular path, and it separated from the track.

On the second attempt, the higher release point provided more kinetic energy. The car reached the top of the loop with speed at or above $\sqrt{gr}$, so the track maintained contact and the car completed the loop.

The minimum release height is 2.5r above the bottom of the loop.

Making Sense of the Answer: *Every loop-the-loop toy teaches this lesson: if the ramp is too short, the car falls. The child learns by trial and error what physics calculates precisely. The factor of 2.5 accounts for the height of the loop (2r) plus the extra kinetic energy needed at the top ($\frac{1}{2}r$ worth of height).*

Think Like a Physicist: *The minimum height of $2.5r$ is measured from the bottom of the loop, not from the ground. If the ramp connects smoothly to the loop at ground level, then $2.5r$ is the height above the ground. But if the loop is elevated, the total height above the ground would be $2.5r$ plus the height of the platform.*

HOT Example 40

A ball of mass 0.2kg is released from rest at the top of a frictionless track. The track descends and enters a vertical circular loop of radius 0.5m. The release point is 2m above the bottom of the loop. Take $g = 9.8\text{m/s}^2$.

(a) Show that the ball completes the loop.

(b) Determine the speed and the normal reaction on the ball at the top of the loop.

Solution

(a) The minimum release height for a complete loop is:

$$h_{min} = 2.5r = 2.5 \times 0.5\text{m} = 1.25\text{m}$$

Since the actual release height (2m) exceeds the minimum (1.25m), the ball completes the loop.

(b) Using conservation of energy between the release point (height $h = 2\text{m}$ above the bottom) and the top of the loop (height $2r = 1\text{m}$ above the bottom):

$$mgh = \frac{1}{2}mv_{top}^2 + mg(2r)$$

Making v_{top} the subject:

$$v_{top}^2 = 2g(h - 2r)$$

$$v_{top} = \sqrt{2g(h - 2r)}$$

Substituting:

$$v_{top} = \sqrt{2 \times 9.8\text{m/s}^2 \times (2\text{m} - 1\text{m})} = 4.43\text{m/s}$$

The speed at the top is 4.43m/s.

Normal reaction at the top:

$$R = \frac{mv_{top}^2}{r} - mg = \frac{0.2\text{kg} \times 19.6\text{m}^2/\text{s}^2}{0.5\text{m}} - 0.2\text{kg} \times 9.8\text{m/s}^2 = 5.88\text{N}$$

The normal reaction at the top is 5.88N.

Making Sense of the Answer: *The release height is $2m$, well above the minimum of $1.25m$. The ball arrives at the top with $4.43m/s$, which is more than double the minimum of $\sqrt{gr} = 2.21m/s$. As a result, the normal reaction is $5.88N$, which is about three times the weight ($1.96N$). The ball presses firmly against the track at the top. The extra height translates directly into extra speed and extra contact force.*

Think Like a Physicist: *Part (a) requires a quick comparison (h versus $2.5r$), not a lengthy calculation. Recognising when a simple check answers the question saves time and shows confidence. Part (b) combines energy conservation (to find v_{top}) with Newton's second law (to find R). These two tools, used in sequence, solve almost every loop-the-loop problem.*

With the loop-the-loop understood, one final scenario remains. *What happens when a vehicle travels over the top of a hill?* The circular path is now on the **outside** of the curve rather than the inside, and the physics flips in a surprising way. Instead of needing a minimum speed to stay on the track, there is a **maximum** speed beyond which the vehicle leaves the road. Let us see why.

Vehicle over a Hill or Hump

Every situation we have studied so far, namely the ball on a string and the loop-the-loop, involves motion on the **inside** of a circular path. The object presses against the inner surface, and the contact force points toward the centre.

But what happens when an object moves over the ***outside*** *of a curved surface?* A car cresting a hill, a motorcycle riding over a hump, or a roller coaster passing over the top of a rise; in all these cases, the object is on the outside of the curve, and the physics reverses in an important way.

On the inside of a loop, the critical question was: *what is the minimum speed to maintain contact?* Too slow, and the object falls away inward.

On the outside of a curve, the critical question flips: *what is the maximum speed before the object leaves the surface?* Too fast, and the object lifts off.

Forces at the top of a hill

Consider a vehicle of mass m travelling at speed v over the top of a circular hill of radius r. At the highest point, the centre of the circular path is directly below the vehicle.

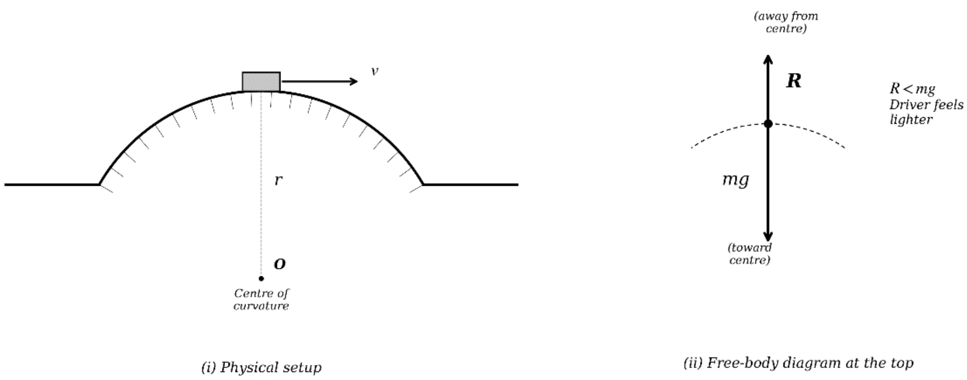

Figure: ***A vehicle at the top of a circular hill.*** *(i) The centre of curvature O is below the road surface. The car travels at speed v over the crest. (ii) Free-body diagram at the top: weight mg acts downward toward the centre, while the normal reaction R acts upward away from the centre. Since R < mg, the driver feels lighter than normal.*

The forces are:

- Weight mg acts downward (toward the centre).
- Normal reaction R acts upward (away from the centre).

This is the key difference from the inside of a loop. Here, weight acts toward the centre and the normal reaction acts away from it. They oppose each other.

Applying Newton's second law toward the centre (downward at the top of the hill):

$$\mathrm{mg} - \mathrm{R} = \frac{\mathrm{mv}^2}{\mathrm{r}}$$

From which:

$$\mathbf{R} = \mathbf{mg} - \frac{\mathbf{mv}^2}{\mathbf{r}}$$

Compare this with the top of an inside loop: $R = \frac{mv^2}{r} - mg$. The terms are reversed. On a hill, increasing speed **decreases** the normal reaction; inside a loop, increasing speed **increases** it.

Apparent weight and the sensation of lightness

The normal reaction R is what the driver feels as their "weight." At the top of a hill:

$$R = mg - \frac{mv^2}{r} = m\left(g - \frac{v^2}{r}\right)$$

Since $\frac{v^2}{r}$ is subtracted from g, the apparent weight is less than the true weight. The driver feels lighter at the top of the hill. This is the familiar stomach-dropping sensation you feel when a car goes over a bump at high speed.

The faster the car moves, the lighter the driver feels, because $\frac{v^2}{r}$ grows larger and R becomes smaller.

The critical speed: leaving the road

As speed increases, R decreases. At some critical speed, R reaches zero. The road no longer pushes the car, and the car is on the verge of lifting off the surface.

Setting $R = 0$:

$$0 = mg - \frac{mv_{max}^2}{r}$$

$$\frac{mv_{max}^2}{r} = mg$$

$$\mathbf{v_{max} = \sqrt{gr}}$$

At this speed, gravity alone provides exactly the centripetal force needed. The car follows the curved hilltop without any help from the road. Beyond this speed, the car would need more centripetal force than gravity can provide, so it lifts off and becomes briefly airborne.

Notice that $v_{max} = \sqrt{gr}$ is the same expression as $v_{min,top}$ for the inside loop. The critical speed has the same magnitude, but the meaning is opposite: for the inside loop it is a **minimum** (below which contact is lost), and for the hilltop it is a **maximum** (above which contact is lost).

Weightlessness at the critical speed

When $R = 0$, the driver has zero apparent weight. This is the condition of **weightlessness**. The driver is not truly weightless (gravity still acts), but there is no contact force, so they feel as though they are floating. This is the same sensation astronauts experience in orbit, where $R = 0$ and gravity provides all the centripetal force.

The worked examples bring all of this to life.

BINDER Example 41

A car of mass 1200kg travels over a hill whose crest has a radius of curvature of 50m. The speed at the top is 15m/s. Take $g = 9.8 m/s^2$.

(a) Calculate the normal reaction on the car at the top of the hill.

(b) Express this as a percentage of the car's actual weight.

Solution

(a) Using:

$$R = mg - \frac{mv^2}{r}$$

Where: $m = 1200kg$, $g = 9.8m/s^2$, $v = 15m/s$, $r = 50m$

Substituting:

$$R = 1200kg \times 9.8m/s^2 - \frac{1200kg \times (15m/s)^2}{50m} = 11760N - 5400N = 6360N$$

The normal reaction at the top is $6360N$.

(b) The actual weight is $W = mg = 11760N$.

$$\frac{R}{W} \times 100\% = \frac{6360N}{11760N} \times 100\% = 54\%$$

The apparent weight is 54% of the actual weight.

Making Sense of the Answer: *At 15m/s over a 50m-radius hill, the car loses nearly half its apparent weight. Passengers would definitely feel the "stomach drop." If the speed were higher, the effect would be more dramatic. At about 22m/s, the apparent weight would reach zero.*

Think Like a Physicist: *The fraction* $\frac{R}{W} = 1 - \frac{v^2}{gr}$ *tells you immediately how "light" the driver feels. When* $\frac{v^2}{gr} = 0.5$*, the driver feels half their weight. When* $\frac{v^2}{gr} = 1$*, the driver feels weightless. This ratio is a quick diagnostic tool.*

REAL Example 42

On a family road trip, **Kipanga** notices that every time the car goes over a small hill at high speed, he feels his body lift slightly off the seat. He grabs the door handle in alarm. **Kipute**, sitting next to him, laughs: *"Relax, Kipanga. You are not flying. The road just stopped pushing you as hard."*

Explain what Kipute means, and why Kipanga feels lighter at the top of the hill.

Solution

When the car is on level ground, the road pushes upward on the car with a normal reaction equal to the weight: $R = mg$. Kipanga feels his full weight pressing him into the seat.

At the top of a hill, the car follows a curved path, and the centre of curvature is below the road. Part of gravity is "used" to provide the centripetal force for this curved motion. The remaining gravity is balanced by the normal reaction, which is now less than the weight: $R = mg - \frac{mv^2}{r}$.

Since the seat pushes Kipanga upward with less force than his weight, he feels lighter. His body does not actually leave the seat (unless the speed is extremely high), but the reduced contact force creates the sensation of lifting. This is what Kipute means: the road has not disappeared; it is simply pushing with less force because some of gravity's pull is being redirected into centripetal acceleration.

Making Sense of the Answer: *The same physics explains why roller coasters are thrilling. The hills are designed so that passengers feel light (or even weightless) at the top. The sensation is entirely due to the reduced normal reaction, not any change in gravity itself.*

Think Like a Physicist: *What you feel as "weight" is never gravity itself; it is the contact force (normal reaction, tension, or seat force) acting on you. Gravity cannot be felt directly. This is why astronauts in orbit feel weightless even though gravity is still pulling them: there is no contact force because both the astronaut and the spacecraft are in free fall together.*

HOT Example 43

A car travels along a road that passes over a circular hump of radius 40m. Take $g = 9.8m/s^2$.

At what speed does the car lose contact with the road at the top of the hump? Explain what happens to the car immediately after it loses contact.

Solution

The car loses contact when $R = 0$:

$$v_{max} = \sqrt{gr} = \sqrt{9.8\text{m/s}^2 \times 40\text{m}} = \sqrt{392\text{m}^2/\text{s}^2} = 19.8\text{m/s}$$

The car loses contact at 19.8m/s (about 71.3km/h).

At this speed, gravity provides exactly the centripetal force needed to follow the curved road surface. Beyond this speed, gravity cannot supply enough centripetal force for the road's curvature. The car cannot follow the tight curve of the hilltop and instead follows a gentler parabolic path (projectile motion). The car becomes briefly airborne, rising above the road surface before gravity brings it back down.

The car does not fly upward dramatically. It simply follows a trajectory with less curvature than the road, so a gap opens between the car and the road surface. When gravity eventually curves the car's path enough to meet the road again (on the downward slope), the car lands (often with a bump).

Making Sense of the Answer: *The critical speed of about 71km/h is surprisingly achievable on ordinary roads. A sharp hump with 40m radius is not unusual in hilly terrain. This is why speed bumps are effective: even a gentle hump at moderate speed creates the light sensation, and a sharper hump at higher speed can genuinely cause the car to leave the ground. Road engineers design hill crests with large radii specifically to prevent this at legal speeds.*

Think Like a Physicist: *After losing contact, the car is a projectile. Its horizontal velocity continues (ignoring air resistance) and gravity acts downward. The parabolic path it follows has less curvature than the circular road, so it rises above the road at the top and rejoins the road further down the slope. The landing point depends on the car's speed and the road's geometry; a problem that combines circular motion with projectile motion.*

With this final subtopic, vertical circular motion is complete. From the ball on a string to the loop-the-loop and now the hilltop, we have seen how gravity and contact forces interact differently depending on whether the object is on the inside or outside of the circular path. The critical speed $\sqrt{gr}$ appears in every case, but its meaning depends on the geometry: minimum speed for the inside, maximum speed for the outside.

Throughout this chapter, we have built circular motion from first principles and applied it to idealised situations: smooth tracks, uniform strings, frictionless loops. But circular motion is not confined to textbook problems. It governs the orbits of satellites, the design of centrifuges, the banking of aircraft in flight, the spin cycle of a washing machine, and the motion of charged particles in magnetic fields. In the next subtopic, we step outside the classroom and see how the physics we have mastered finds its way into the real world.

APPLICATIONS OF CIRCULAR MOTION

The equations are powerful, but they were never meant to live only on paper. Every formula we derived in this chapter was born from nature and returns to nature. In this subtopic, we leave the idealised world of smooth strings and frictionless tracks, and see circular motion at work in the machines, vehicles, and systems that shape everyday life and advanced technology alike.

1. Satellites in Orbit

A satellite orbiting the Earth is in continuous free fall. It falls toward the Earth every moment, but because it moves sideways fast enough, the Earth's surface curves away beneath it at the same rate. The result is a circular (or nearly circular) orbit.

For a satellite at height h above the Earth's surface, the gravitational force provides the centripetal force:

$$\frac{GMm}{(R_E + h)^2} = \frac{mv^2}{R_E + h}$$

From which the orbital speed is:

$$v = \sqrt{\frac{GM}{R_E + h}}$$

Where G is the gravitational constant, M is the mass of the Earth, and R_E is the Earth's radius. The mass of the satellite cancels, confirming that all satellites at the same altitude orbit at the same speed, regardless of their mass.

For a low-Earth orbit (h much smaller than R_E), the orbital speed is approximately 7.9km/s and the period is about 90 minutes. The International Space Station orbits at this speed, completing roughly 16 orbits every day.

A special case is the **geostationary orbit**, where the satellite's period matches the Earth's rotation period (24 hours). At this altitude (about 35,786km above the equator), the satellite appears stationary relative to the ground. Communication satellites and weather satellites are placed in geostationary orbits for this reason.

2. Centrifuges

A centrifuge is a machine that spins samples at high angular velocity to separate substances of different densities. In a hospital laboratory, blood samples are spun in a centrifuge to separate red blood cells (heavier) from plasma (lighter). In industrial settings, centrifuges separate cream from milk, uranium isotopes from one another, and sediment from wastewater.

The principle relies on density difference. When the centrifuge spins, the surrounding fluid creates a pressure gradient that supplies centripetal force proportional to the **fluid's** density. A particle denser than the fluid requires more centripetal force than the fluid can supply at that radius, so it drifts outward. A particle less dense than the fluid drifts inward. This is how separation occurs.

The **relative centrifugal force (RCF)** compares the centripetal acceleration to gravitational acceleration:

$$RCF = \frac{\omega^2 r}{g}$$

A laboratory centrifuge spinning at 3000rpm with a radius of 0.15m produces an RCF of about 1500. This means particles experience an effective "gravitational pull" 1500 times stronger than normal gravity, causing separation to occur in minutes rather than the hours or days it would take under gravity alone.

3. Banking of Aircraft

When an aircraft turns in flight, there is no road to provide friction. The centripetal force must come entirely from the aircraft itself. The pilot banks (tilts) the aircraft so that the lift force, which is always perpendicular to the wings, has a horizontal component pointing toward the centre of the turn.

The analysis is identical to the banked road with no friction:

$$\tan\theta = \frac{v^2}{rg}$$

Where θ is the bank angle, v is the aircraft's speed, and r is the radius of the turn. A faster turn or a tighter radius requires a steeper bank angle. Commercial aircraft typically bank at angles up to about 25° during normal turns, while fighter jets may bank at 60° or more during sharp manoeuvres.

At a bank angle of 60°, the passengers experience an apparent weight of $\frac{mg}{\cos 60°} = 2mg$, meaning they feel twice as heavy. This is described as pulling "2g." Fighter pilots undergo special training to withstand accelerations of up to 9g during extreme turns.

4. Washing Machine Spin Cycle

During the spin cycle of a washing machine, the drum rotates at high speed. Clothes are pressed against the inner wall of the drum, and the drum wall provides the normal reaction that acts as the centripetal force:

$$R = m\omega^2 r$$

But water in the clothes is not held by the fabric strongly enough to maintain this circular motion. When the required centripetal force exceeds the weak adhesive forces holding the water to the fabric, the water separates from the clothes and moves outward through the holes in the drum.

This is not "centrifugal force throwing water outward." What actually happens is that the drum wall pushes the clothes inward (centripetal force), but the water, lacking sufficient inward force, cannot follow the circular path. It continues tangentially through the drum holes, following Newton's first law. The effect is the same as the Kilimani Hill bus: when the available inward force is insufficient, the object (water, in this case) leaves the circular path.

5. Circular Motion of Charged Particles in Magnetic Fields

When a charged particle moves through a magnetic field, it experiences a force perpendicular to its velocity. This force does not change the particle's speed (because it acts perpendicular to the motion), but it continuously changes the direction of motion. The result is circular motion.

For a particle of charge q and mass m moving at speed v perpendicular to a uniform magnetic field of strength B, the magnetic force provides the centripetal force:

$$qvB = \frac{mv^2}{r}$$

From which the radius of the circular path is:

$$r = \frac{mv}{qB}$$

This principle is used in **cyclotrons** (particle accelerators that accelerate charged particles along spiral paths), **mass spectrometers** (which separate ions by mass based on their circular path radii), and the large particle accelerators at research centres such as CERN, where protons travel in circular paths of several kilometres radius at speeds close to the speed of light.

6. Artificial Gravity in Space Stations

In the weightless environment of space, astronauts' bones weaken and muscles deteriorate over time. One proposed solution is to build a rotating space station. As the station spins, the floor (the outer wall) pushes astronauts inward, providing centripetal force. This inward push feels exactly like gravity to the astronauts standing on the inside of the outer wall.

For the artificial gravity to match Earth's gravity:

$$\omega^2 r = g$$

$$\omega = \sqrt{\frac{g}{r}}$$

A station of radius 100m would need to rotate at $\omega = \sqrt{\frac{9.8}{100}} = 0.313$rad/s, completing about one revolution every 20 seconds. This is slow enough to be comfortable for the inhabitants.

This concept has appeared in many science fiction films, but the physics is real and straightforward: it is nothing more than the centripetal acceleration $\omega^2 r$ mimicking the acceleration due to gravity g.

7. Speed Governors

The conical pendulum, which we have studied, has a direct engineering application: the **centrifugal governor**, for controlling steam engine speed. Two heavy balls are attached to a rotating shaft by hinged arms. As the shaft spins faster, the balls rise outward (increasing θ), just as in the conical pendulum. This outward movement is linked mechanically to a valve that reduces the steam supply, slowing the engine. When the engine slows, the balls drop, opening the valve again.

The system is self-regulating: the angle of the balls automatically adjusts to maintain a constant engine speed. The relationship $\tan\theta = \frac{v^2}{rg}$ ensures that each angle corresponds to a specific rotation speed, providing precise feedback control.

This was one of the earliest examples of **automatic feedback control** in engineering, and it is a beautiful demonstration of the conical pendulum at work outside the physics classroom.

From satellites silently circling the Earth to the washing machine noisily spinning in the kitchen, from the blood centrifuge in Muhimbili hospital to the International Space Station floating above our heads, circular motion is everywhere. The equations we derived in this chapter are not abstract mathematics; they are the operating instructions of the universe.

And speaking of operating instructions, it is time to put everything together. In the next section, the concepts of this entire chapter: centripetal acceleration, horizontal motion, vertical motion, energy conservation, and critical conditions will stop being polite and start working together in miscellaneous worked examples. If the individual subtopics were the ingredients, the miscellaneous examples are the full meal. Sharpen your appetite, and don’t forget a calculator!