UNIFORM CIRCULAR MOTION

INTRODUCTION

The Miono Secondary School bus was returning from a geography field trip when disaster struck while negotiating a sharp bend at Kilimani Hill. The driver, perhaps distracted by Kipanga's loud singing from the back seats, entered the curve at excessive speed. Physics cared nothing for good intentions or desperate braking. The tires screeched, lost their grip on the asphalt, and the bus began its inevitable journey off the road; sliding sideways like a reluctant dancer, crashing through the wooden barrier, and finally coming to rest at a precarious angle in the roadside ditch.

Silence. Then chaos. Students screamed, bags tumbled, and someone's food container burst open, sending mandazi flying through the air in parabolic arcs that would have made Chapter 6 proud!

Miraculously, no one was seriously hurt. As Mr. Akilikubwa helped the shaken students get out of the bus, **Kipanga** stood there dusty and untidy. He asked in confusion, *"Sir, the driver was turning the steering wheel. I saw him. Why didn't the bus turn?"*

Mr. Akilikubwa pointed at the dramatic skid marks etched into the road surface. The marks told an unambiguous story: they curved initially, obediently following the road's bend, then suddenly straightened into a ruler-straight line heading directly toward the ditch. *"Kipanga, the driver did turn the steering wheel. But steering alone doesn't bend a vehicle's path. Look at those marks. See how they curve, then suddenly go straight! That's the moment the tires lost their grip."*

Kipute, brushing dirt from her uniform, studied the marks with scientific curiosity. *"So the bus wanted to go straight, but the road wanted it to curve?"*

"Exactly!" **Mr. Akilikubwa's** eyes lit up despite the circumstances. *"Every object moving in a straight line wants to continue in a straight line, remember Newton's first law? To force something into a curved path requires a continuous inward force pulling it toward the center of the curve. For a car on a road, that force comes from friction between tires and asphalt."*

He walked to where the skid marks transitioned from curved to straight. *"But friction has limits. At low speeds on gentle curves, no problem. But this curve is sharp, and we were going too fast. The required inward force exceeded what friction could provide. The instant that happened…"* he snapped his fingers *"…the tires slid instead of gripped, the inward force vanished, and the bus immediately obeyed Newton's first law. Straight line. Straight into the ditch."*

Kipanga looked nervously at the bent barrier. *"So... if we'd gone even faster?"*

"We'd have slid off even earlier in the curve," **Mr. Akilikubwa** said seriously. *"This is why speed limits exist on curves, Kipanga. This is why mountain roads have those warning signs with curved arrows and recommended speeds. Engineers calculate the maximum safe speed based on the curve's radius and the expected friction. Exceed that speed, and physics becomes unforgiving."*

As they waited for the rescue vehicle, **Mr. Akilikubwa** sketched a diagram in his notebook. ***"Circular motion,"*** he announced, *"is nature's most deceptive phenomenon. It looks simple! Just going around in circles. But beneath that apparent simplicity lies a profound truth: maintaining a circular path requires constant work, constant force! Satellites circle Earth at precisely calculated speeds because gravity provides relentless inward pull. Clothes stick to the drum of a spinning washing machine because the drum wall provides inward force."*

Kipanga nodded slowly, understanding dawning. *"So circular motion isn't the natural state. Straight-line motion is natural. Curves require force."*

"Precisely! You've just grasped the central insight of this entire chapter." **Mr. Akilikubwa** smiled. *"And you learned it from skid marks and a ditch instead of a textbook. Perhaps this accident was the universe's way of teaching physics."*

"I would have preferred the textbook, sir," **Kipanga** muttered, picking mandazi crumbs from hair.

Welcome to circular motion, where every curve is a battle against inertia, where every turn demands its tribute of force, and where the difference between a safe journey and a roadside ditch can be calculated with precision. In this chapter, we will discover why the moon does not fall despite Earth's gravity constantly pulling it inward, why racing cars lean dramatically into turns, why your wet clothes cling to the washing machine drum during spin cycle, and why that innocent-looking curve at Kilimani Hill requires respect, attention, and most importantly, the right speed. The bus has already taught us the consequences of getting it wrong. Now let us learn the mathematics of getting it right.

UNDERSTANDING UNIFORM CIRCULAR MOTION

Circular motion *is the movement of an object along a circular path*. You observe it everywhere, for example: a car turning a corner, a satellite orbiting Earth, clothes spinning in a washing machine, a stone whirled on a string, the moon circling our planet, or a ball rolling around the inside of a bowl.

The most important insight about circular motion is this: it is not natural. Left undisturbed, every object moves in a straight line at constant velocity accordance with Newton's first law. To force an object into a curved path requires continuous application of force. Remove that force, and the object immediately returns to straight-line motion, as our bus at Kilimani Hill dramatically demonstrated.

When an object moves along a circular path at constant speed, we call this **uniform circular motion.** The word "uniform" refers to the speed remaining constant; the object covers equal distances in equal times around the circle. A satellite orbiting Earth at constant altitude, a car maintaining steady speed around a circular track, or a DVD spinning at constant rotation rate all exhibit uniform circular motion.

But here lies a subtle trap that catches many students: *constant speed does* ***not*** *mean constant velocity*. This distinction is crucial to understanding circular motion.

Velocity in Circular Motion

Velocity is a vector quantity possessing both magnitude (speed) and direction. In circular motion, even when speed remains constant, the direction of motion continuously changes as the object moves around the circle. The velocity vector always points **tangent to the circle** at the object's current position, that is, perpendicular to the radius at that point.

Imagine a stone tied to a string being whirled in a horizontal circle. At every instant, the stone's velocity points along the tangent to its circular path. As the stone moves around the circle, this direction continuously rotates. The speed might be constant, but the direction changes every fraction of a second, which means the velocity vector is constantly changing.

This is why, when the string breaks, the stone does not continue moving in a circle or fall straight down; it flies off tangentially, following the direction of its velocity at the instant the string snapped. The bus at Kilimani Hill did exactly the same thing: when friction failed to provide the necessary inward force, the bus continued along the tangent to the curve, which happened to point straight into the ditch.

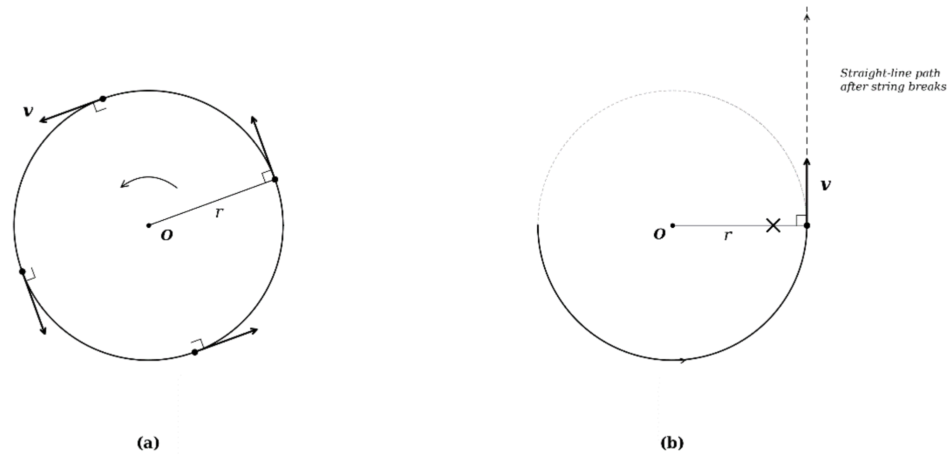

Figure: ***Velocity in circular motion:*** *(a) Velocity is always tangent to the circular path and perpendicular to radius; speed is constant but direction changes continuously. (b) When string breaks, centripetal force vanishes and the object continues along the tangent (Newton's first law).*

Acceleration in Circular Motion

If velocity is changing, acceleration must exist. This conclusion is inescapable. Acceleration is defined as the rate of change of velocity, and since velocity in circular motion continuously changes direction, there must be continuous acceleration.

This acceleration is always directed toward the centre of the circle. We call it **centripetal acceleration** (from Latin "centrum" meaning center and "petere" meaning to seek). The word literally means "center-seeking." Even though the object moves tangentially, the acceleration points radially inward.

This might seem strange. We usually think of acceleration as changing an object's speed, that is making it faster or slower. But acceleration can also change an object's direction while leaving speed unchanged. In uniform circular motion, the centripetal acceleration continuously deflects the velocity vector toward the center, bending the object's path into a circle while maintaining constant speed.

Centripetal Force

Newton's second law tells us that acceleration requires force: $F = ma$. If an object in circular motion experiences centripetal acceleration toward the center, there must be a force causing that acceleration. This force is called **centripetal force** which is *the inward force that maintains circular motion.*

Centripetal force is not a new type of force. It is simply whatever force happens to be pulling the object toward the center:

- **For a satellite orbiting Earth:** ***gravity*** *provides centripetal force.*
- **For a car turning a corner:** ***friction*** *between tires and road provides centripetal force.*
- **For a stone on a string:** ***tension*** *in the string provides centripetal force.*
- **For clothes in a spinning washing machine drum:** *the* ***normal reaction*** *from the drum wall provides centripetal force.*
- For our bus at Kilimani Hill: friction should have provided centripetal force, but the required force exceeded friction's capacity, so the bus slid off

The key insight: *circular motion requires continuous inward force. Remove or reduce that force, and circular motion immediately becomes straight-line motion along the tangent. This is not a flaw in the object or the motion; it is Newton's first law in action. Objects naturally move in straight lines. Only continuous inward force can maintain a curved path.*

With these foundational concepts established, let us explore some worked examples to deepen our understanding before we introduce the mathematical relationships in the next subtopic.

BINDER Example 1

A car moves at constant speed of 20m/s around a circular track. Explain whether the car is accelerating, and if so, in which direction.

Solution

Yes, the car is accelerating even though its speed is constant.

Acceleration is the rate of change of velocity, and velocity is a vector (magnitude and direction). Although the car's speed (magnitude of velocity) remains constant at 20m/s, the direction of the velocity continuously changes as the car moves around the circular track. At every instant, the velocity points tangent to the circle at the car's position. As the car moves, this tangent direction rotates, meaning the velocity vector is constantly changing.

Since velocity is changing, acceleration must exist. This acceleration points toward the centre of the circle; it is centripetal acceleration. The car is therefore continuously accelerating toward the centre of the circular track.

Making Sense of the Answer: *Acceleration does not only mean speeding up or slowing down. It also means changing direction. The car changes direction continuously as it goes around the circle, so it must be accelerating even though its speed stays constant.*

Think Like a Physicist: *Whenever you analyze circular motion, separate speed (scalar) from velocity (vector) in your mind. Constant speed in a circle always means changing velocity, which always means acceleration exists.*

BINDER Example 2

A stone tied to a string is whirled in a horizontal circle at constant speed. When the string suddenly breaks, the stone flies off in a straight line tangent to the circle rather than continuing to move in a circle or falling straight down. Explain why this happens.

Solution

While the string was intact, it provided tension force pulling the stone inward toward the center of the circle. This centripetal force continuously deflected the stone's velocity toward the centre, maintaining the circular path.

The moment the string broke, the centripetal force vanished. With no centripetal force acting on the stone, Newton's first law applies: an object in motion continues in a straight line at constant velocity unless acted upon by a force. The stone's velocity at the instant of breaking pointed tangent to the circle. With no force to deflect it from this direction, the stone continued moving straight along this tangent line.

Hence, the stone does not continue in a circle because circular motion requires continuous centripetal force which disappeared when the string broke. It does not fall straight down because it has horizontal velocity that persists in the absence of horizontal forces.

Making Sense of the Answer: *The tangent is the direction the stone was moving at the instant the string broke. Without the string pulling it inward, nothing deflects it from this straight-line path, so it continues moving straight ahead.*

Think Like a Physicist: *When analyzing what happens when centripetal force disappears, always identify the velocity direction at that instant. The object will continue along that tangent line because that is the direction it was already moving.*

REAL Example 3

A passenger sits in a car taking a sharp bend at speed. Many people feel as if they are being “thrown outward” against the door. Explain what is actually happening using correct physics.

Solution

The passenger is not thrown outward. Instead, the car pushes the passenger inward. This inward push provides the centripetal force needed for circular motion. However, the passenger's body wants to continue moving in a straight line (Newton's first law). So the "outward" sensation is just the passenger's inertia resisting the inward force. There is no real outward force; the real force always points toward the centre of the curve.

Making Sense of the Answer: *We feel "thrown outward" in circular motion because our bodies want to go straight while something (car door, drum wall, etc.) pushes us inward to force us into the curve. The inward push feels like we are being thrown the opposite direction, but the real force is always inward.*

Think Like a Physicist: *When you feel pushed to the outside during circular motion, recognize this as evidence of the inward centripetal force acting on you. Your sensation of being thrown outward is your inertia resisting the inward force and not an actual outward force.*

HOT Example 4

A car travels around a circular roundabout. At point A, the car is heading north. At point B, one-quarter of the way around the circle, the car is heading east. The car maintains constant speed throughout.

(a) Explain why the car's velocity has changed between points A and B even though its speed has not changed.
(b) If the car's velocity at point A is 15m/s north and at point B is 15m/s east, determine the change in velocity between these two points (magnitude and direction).
(c) Explain why this change in velocity requires a force, and identify the force that provides it for a car on a roundabout.
(d) Explain in which direction this force must act.

Solution

(a) Velocity is a vector quantity with both magnitude (speed) and direction. Although the car's speed remains constant at 15 m/s, the direction changes from north at point A to east at point B. Since the direction component of velocity has changed, the velocity vector has changed, even though the magnitude (speed) stayed the same.
(b) *Resolving velocities*

For point A: $(\mathrm{v_A})_\mathrm{x} = 0, (\mathrm{v_A})_\mathrm{y} = 15\mathrm{m/s}$ (north taken as the positive y-direction)

For point B: $(\mathrm{v_B})_\mathrm{x} = 15\mathrm{m/s}, (\mathrm{v_B})_\mathrm{y} = 0$ (east taken as the positive x-direction)

Finding horizontal and vertical change in velocity

$\Delta\mathrm{v_x} = (\mathrm{v_B})_\mathrm{x} - (\mathrm{v_A})_\mathrm{x} = 15\mathrm{m/s} - 0 = 15\mathrm{m/s}$

$\Delta\mathrm{v_y} = (\mathrm{v_B})_\mathrm{y} - (\mathrm{v_A})_\mathrm{y} = 0 - 15\mathrm{m/s} = -15\mathrm{m/s}$

Finding magnitude and direction of change in velocity

Magnitude: $\Delta\mathrm{v} = \sqrt{(\Delta\mathrm{v_x})^2 + \left(\Delta\mathrm{v_y}\right)^2} = \sqrt{(15\mathrm{m/s})^2 + (-15\mathrm{m/s})^2} = 21.2\mathrm{m/s}$

Direction: $\theta = \tan^{-1}\left(\frac{\Delta\mathrm{v_y}}{\Delta\mathrm{v_x}}\right) = \tan^{-1}\left(\frac{-15}{15}\right) = -45°$(The negative sign indicates the direction is **below the positive x-axis**).

Hence, the change in velocity is 21.2m/s directed southeast (45° south of east), pointing toward the center of the roundabout.

(c) Newton's second law states that acceleration requires a net force to act on an object: F = ma. Since the car's velocity changes direction as it moves around the roundabout, the car has centripetal acceleration. Therefore, a force must act on the car. For a car moving around a roundabout, this force is provided by static friction between the tyres and the road. As the car turns, the tyres tend to move sideways relative to the road surface. Static friction prevents this sideways motion and acts toward the centre of the circular path. This frictional force provides the centripetal force required to keep the car moving along the curved path.

(d) The force must act toward the centre of the roundabout. This is because the required acceleration (centripetal acceleration) points toward the centre, and force must act in the same direction as the acceleration it produces ($F = ma$).

Making Sense of the Answer: *The change in velocity points toward the centre because the car's path is being continuously bent inward. Friction must act in that same inward direction to produce this change. Without sufficient friction (wet road, icy conditions), the car cannot generate enough inward force and slides off the curve tangentially; just like our bus at Kilimani Hill.*

Think Like a Physicist: *In circular motion problems, always remember that force and acceleration point toward the centre, while velocity points along the tangent. These are perpendicular to each other at every instant. The centripetal force continuously changes the direction of the tangent velocity vector, bending the path into a circle without changing the speed.*

With these fundamental concepts established, that circular motion requires continuous inward force, that velocity is always tangential while acceleration is always radial, and that constant speed does not mean constant velocity; we are ready to develop the mathematical relationships that allow us to calculate these quantities precisely. The next subtopic introduces angular motion and the quantitative tools we need to solve circular motion problems.