DIGGING DEEPER EXERCISE 6

EXERCISE 6A: BINDER QUESTIONS

Question 1

A stone is thrown horizontally from a cliff. Explain why the horizontal velocity remains constant throughout the flight while the vertical velocity continuously increases.

Question 2

Explain why does a projectile trajectory form a parabola rather than any other curve?

Question 3

Explain why a projectile launched from a height travels farther horizontally than the same projectile launched at the same angle and velocity from ground level.

Question 4

At the highest point of a projectile's trajectory, explain why the velocity is not zero even though the vertical velocity component is zero.

Question 5

Explain why throwing a ball downward at an angle from a cliff results in shorter horizontal range than throwing it horizontally at the same velocity.

Question 6

Two identical balls are thrown simultaneously from the same cliff: one horizontally and one at 30° above horizontal, both with the same velocity. Explain which ball hits the ground first and why.

Question 7

A ball is thrown at certain angle from ground level. Explain why increasing the initial velocity increases both the maximum height and the horizontal range.

Question 8

Explain why a projectile's horizontal range on level ground cannot exceed $\frac{u^2}{g}$ regardless of launch angle.

EXERCISE 6B: REAL QUESTIONS

Question 9

Why does water from a fountain follow a curved path?

Question 10

Explain why a fountain designed to spray water to a specific distance on level ground requires choosing between two different nozzle angles.

Question 11

A firefighter directs a water jet from a hose to extinguish flames on the upper floor of a building. The flames are at the same horizontal distance whether the water is aimed at 30° or 60° above horizontal. However, the firefighter chooses the 30° angle. Explain the practical advantages of using the lower angle despite both angles reaching the same distance.

Question 12

During a basketball practice session, Kipanga attempts several three-point shots. After many misses, he becomes frustrated.

"Sir, I'm shooting with good power, but the ball keeps hitting the front rim!" **Kipanga** complains to Mr. Akilikubwa.

Mr. Akilikubwa observes, *"Your shots are too flat; you're using too much horizontal velocity and not enough arc."*

Kipute adds, *"I notice that when you shoot flatter, the ball arrives at the basket while still rising or level. But when I shoot with more arc, the ball is descending when it goes through the hoop."*

"Exactly right, Kipute," says **Mr. Akilikubwa.** *"That's the key difference between a good shot and a miss."*

Explain why a ball arriving at the basket while still rising or traveling horizontally is more likely to miss than a ball that is descending.

Question 13

A gardener uses a hose to water plants at various distances. He notices that tilting the nozzle slightly downward from horizontal reduces the distance the water travels. Explain why aiming downward decreases the range even though it seems like it should help the water "fall" toward the target faster.

Question 14

Mountain rescue teams sometimes need to drop supplies to stranded climbers. When dropping from a hovering helicopter, the package lands some distance away from the helicopter rather than directly below. Explain why this happens and how the pilot can ensure the package lands at the desired location.

Question 15

A stone falls from a tall bridge into a river below. A observer standing on the bridge sees the stone fall straight down. However, a person on a moving boat passing under the bridge at the moment of release sees the stone follow a curved path. Explain how both observers can be correct.

Question 16

In the sport of shot put, athletes must release the shot from within a throwing circle and the shot must land in a marked sector. Why do shot putters release at angles around 35−40° rather than 45°, which gives maximum range in theory?

EXERCISE 6C: HOT QUESTIONS

Take $g = 9.8ms^{-2}$

Question 17

A ball is projected from the ground with velocity $25ms^{-1}$. A wall 15m high stands 40m away.

(a) Derive the expression for the height of the projectile as a function of horizontal distance.
(b) Determine the minimum angle of projection required for the ball to just clear the wall.

Question 18

A projectile is fired with velocity $40ms^{-1}$ toward a target located 50m horizontally away and 30m above the launch point. Determine the possible angles of projection that allow the projectile to hit the target.

Question 19

A projectile is fired from the ground with velocity $25ms^{-1}$. At the same instant, a target located 80m away from the point of projection begins to move directly toward the projectile with a constant velocity of $4ms^{-1}$. Determine the angle of projection required for the projectile to strike the moving target.

Question 20

A projectile is launched with velocity $30ms^{-1}$ at 50° above the horizontal toward a slope descending at 20°. Determine the distance along the slope from the launch point to the point of impact.

Question 21

A projectile is launched with velocity $20ms^{-1}$ at an angle 45° above the horizontal toward a plane inclined at 30° above the horizontal.

(a) Determine the horizontal distance from the point of projection where the projectile strikes the plane.
(b) Hence determine the distance measured along the slope.

Question 22

A ball is dropped from rest from a height of 40m. At the same instant another ball is projected horizontally with velocity $15ms^{-1}$ from a point 30m away.

(a) Determine the time taken for the horizontally projected ball to reach the vertical line beneath the falling ball.
(b) Determine the vertical positions of both balls at that instant.
(c) State whether the two balls collide.

Question 23

Projectile A is fired vertically upward with velocity $25ms^{-1}$. At the same instant projectile B is fired from ground 40m away toward A with velocity $30ms^{-1}$ at 45°. Determine whether the two projectiles collide.

Question 24

A ball is thrown horizontally from a tower 20m high with velocity $15ms^{-1}$. At the same instant another ball is projected upward from the ground directly below the tower with velocity $22m\ s^{-1}$.

(a) Determine whether the two balls collide.
(b) If they collide, determine the height of collision.

Question 25

A cart moves along a straight horizontal track away from the point of projection with a constant velocity of $5ms^{-1}$. At the instant the cart is at a distance x from the launch point, a projectile is fired from the ground with velocity $25ms^{-1}$ at an angle of 45° to the horizontal. Find the distance x from the launch point at which the cart must be located at the instant of projection so that the projectile lands in the cart.

ANSWERS TO DIGGING DEEPER EXERCISE 6

EXERCISE 6A

1. In the horizontal direction, there is no force acting on the stone (ignoring air resistance), so by Newton's first law, the horizontal velocity remains constant. In the vertical direction, gravity acts continuously downward with constant acceleration $g = 9.8\ m/s^2$, causing the vertical velocity to increase steadily.

2. The trajectory is parabolic because horizontal displacement increases linearly with time ($x = ut$) while vertical displacement varies with the square of time ($y = ut - ½gt^2$). When we eliminate time t from these equations, we get $y = x\tan\theta - \frac{gx^2}{2u^2\cos^2\theta}$, which is a quadratic equation in x. Any quadratic relationship between y and x represents a parabola. Thus, the parabolic shape results from the combination of uniform horizontal motion and uniformly accelerated vertical motion.

3. Launching from height h increases the total flight time. The projectile first completes its normal parabolic arc (as if on level ground), then continues falling through the additional height h before landing. This extra descent time allows more horizontal distance to accumulate since horizontal velocity remains constant. This is supported by formula $t = \frac{u\sin\theta + \sqrt{(u\sin\theta)^2 + 2gh}}{g}$, which shows that flight time increases with h, and since R = (ucos θ)t, greater time means greater range (horizontal distance).

4. At maximum height, only the vertical velocity component becomes zero ($v_y = 0$), but the horizontal velocity component remains unchanged throughout flight ($v_x = u\cos\theta$). So the total velocity vector at maximum height is purely horizontal with magnitude $v = u\cos\theta$. Consequently, the projectile continues moving forward even at its peak because no horizontal force acts to stop it; it simply has no upward or downward motion at that instant.

5. Throwing downward adds initial downward velocity, causing faster descent and thus reduced time of flight. Additionally, the horizontal velocity component is reduced to ucos θ < u for any downward angle. Both factors work against range: shorter time of flight and slower horizontal velocity combine to produce less horizontal distance. Horizontal throw maximizes horizontal velocity (all velocity directed forward) and maximizes flight time among, giving maximum horizontal range.

6. The horizontal throw hits first. Both start at the same height, but the angled throw has an upward vertical component that makes it rise first before falling, while the horizontal throw has zero initial vertical velocity and begins falling immediately. Since both experience the same downward acceleration g, the one that starts descending immediately (horizontal throw) reaches ground sooner. The angled throw must first decelerate upward, stop at maximum height, then accelerate downward, taking longer overall.

7. Both maximum height $H = \frac{u^2\sin^2\theta}{2g}$ and range $R = \frac{u^2\sin 2\theta}{g}$ are proportional to u^2, so doubling velocity quadruples their amounts. Physically, greater initial velocity provides more kinetic energy. In the vertical direction, this converts to greater potential energy (peak of greater height). In the horizontal direction, increased initial velocity means greater velocity maintained throughout flight ($v_x = u\cos\theta$ unchanged) combined with longer flight time (from greater height) produces much greater range.

8. Range formula $R = \frac{u^2\sin 2\theta}{g}$ shows R is proportional to sin 2θ. The maximum value of sine function is 1, which is achieved when 2θ = 90°, giving θ = 45°. At this angle, $R = \frac{u^2}{g}$, which is the maximum possible range for given velocity u. Any other angle gives sin 2θ < 1, resulting in $R < \frac{u^2}{g}$. This represents a fundamental limit imposed by the combination of initial kinetic energy and gravitational acceleration; no choice of angle can overcome this limit.

EXERCISE 6B

9. When the water leaves the nozzle, it has an initial horizontal velocity. After leaving the nozzle, gravity acts downward while the horizontal motion continues unchanged (neglecting air resistance). The combination of constant horizontal motion and downward acceleration produces a parabolic path.

10. For any target distance R less than maximum range ($R < \frac{u^2}{g}$), the range equation $R = \frac{u^2\sin 2\theta}{g}$ has two solutions for 2θ: one acute angle, and its supplement. These correspond to two launch angles that are complementary (summing to 90°). One angle is shallow (below 45°), giving a low, fast arc; the other is steep (above 45°), giving a high, slow arc. Both reach the same distance but follow different paths. Fountain designers typically **choose** the steeper angle for aesthetic appeal as the higher arc is more visually dramatic and impressive.

11. Although both angles reach the same horizontal distance (complementary angles), the 30° angle has several practical advantages.

1. The water arrives faster because it travels a lower, shorter arc, allowing quicker response to the fire.
2. The lower trajectory means water arrives with greater horizontal velocity component, providing more impact force to penetrate smoke and reach the base of flames.
3. The flatter arc is easier to aim and control as the firefighter can see the water path more clearly and make quick adjustments.
4. The 30° jet wastes less water on excessive height (water spends less time in the air and thus experiences less loss due to air resistance and dispersion), directing more energy toward forward reach rather than vertical climb.

12. When the ball approaches the rim while rising or moving nearly horizontally, it has little or upward vertical velocity. If it strikes the rim, this vertical motion tends to deflect the ball upward and away from the basket. When the ball approaches while descending, it has a downward vertical velocity. If it hits the rim, this downward momentum helps guide the ball into the basket rather than causing it to bounce away. Additionally, a rising ball that misses the hoop continues upward and away, whereas a descending ball that is close to the rim may still drop through the basket.

13. Aiming downward reduces range because of two compounding effects. First, the downward angle reduces the horizontal component of velocity. Second, adding downward initial velocity makes the water hit the ground faster, reducing flight time. Slower horizontal velocity and shorter time of flight combine to produce much less horizontal distance.

14. When a package is released from a helicopter, it retains the horizontal velocity it had at the moment of release. Even if the helicopter appears to hover, it usually has small horizontal motions due to wind or position adjustments. The package therefore continues with this horizontal motion while gravity pulls it downward, producing a parabolic path that may land away from the release point.

15. Both observers are correct because they describe motion from different reference frames. The bridge observer is stationary relative to the stone's release point. Since the stone is simply dropped (zero horizontal velocity relative to the bridge), it falls straight down under gravity; pure vertical motion.

The boat observer moves horizontally relative to the bridge. From the boat's reference frame, the stone has horizontal velocity equal and opposite to the boat's motion at the moment of release. The boat observer sees the stone combine this horizontal component with vertical acceleration due to gravity, creating a parabolic trajectory.

16. Shot putters release below 45° because they do not launch from ground level; they release from shoulder height. When launching from an elevated position, the optimal angle for maximum range is less than 45°. The additional height provides extra flight time even without much upward velocity, so directing more velocity horizontally (flatter angle) produces greater range than 45° would. Additionally, in real conditions air resistance cannot be neglected and it (air resistance) reduces the horizontal range more strongly for higher angles due to longer time of flight at larger angle. As a result, the optimal angle becomes smaller than 45°.

EXERCISE 6C

17. (a) $y = x\tan\theta - \frac{g x^2}{2u^2\cos^2\theta}$ (b) 36.7°

18. $\theta_1 = 32°$, $\theta_2 = 67°$

19. 39°

20. 74m

21. (a) 28.3m (b) 32.7m

22. (a) 2s (b) $y_1 = 20.4$m, $y_2 = 20.4$m (c) Since both balls have the same horizontal and vertical position, the **balls collide**.

23. Since heights differ, **collision does not occur** (At t =1.89s; y_A (29.8m) $\neq$ y_B(22.6m))

24. Collision occurs at **15.9 m height**.

25. x=45.75m