MISCELLANEOUS WORKED EXAMPLES ON PROJECTILE MOTION

Example 24

(a) Two projectiles are launched simultaneously from the same point with the same velocity but at complementary angles. Explain why they land at the same horizontal distance.
(b) A goalkeeper kicks a ball at 20m/s at 35° above horizontal. Taking g = 9.8 m/s²:
(i) Calculate the maximum height reached by the ball.
(ii) Calculate the time the ball stays in the air.
(iii) Calculate the horizontal distance the ball travels.
(iv) Explain why goalkeepers often prefer angles around 30−35° rather than the theoretical optimum of 45°.

Solution

(a) A projectile launched at the smaller angle, has greater horizontal velocity and thus moves forward faster but stays in the air for a shorter time**.** A projectile launched at the larger complementary angle moves forward more slowly but remains in the air for a longer time**.** These effects balance, so both land at the same horizontal distance**.**

This is supported by range formula $R = (u^2 \sin 2\theta)/g$ which depends on sin 2θ. For complementary angles θ_1 and θ_2 where $\theta_1 + \theta_2 = 90°$, we have $2\theta_1 + 2\theta_2 = 180°$. This means $2\theta_2 = 180° - 2\theta_1$, so $\sin 2\theta_2 = \sin(180° - 2\theta_1) = \sin 2\theta_1$. Since both angles produce the same sin 2θ value, they give identical ranges.

(b) The solution of each part is as follows:
(i) Maximum height:

$$H = \frac{u^2 \sin^2\theta}{2g} = \frac{(20\text{m/s})^2 \times \sin^2 35°}{2 \times 9.8\text{m/s}^2} = 6.71 \text{ m}$$

The maximum height is 6.71m.

(ii) Time of flight:

$$T = \frac{2u\sin\theta}{g} = \frac{2 \times 20\text{m/s} \times \sin 35°}{9.8\text{m/s}^2} = 2.34 \text{ s}$$

The ball stays in the air for 2.34s.

(iii) Horizontal range:

$$R = \frac{u^2 \sin 2\theta}{g} = \frac{(20\text{m/s})^2 \times \sin 70°}{9.8\text{m/s}^2} = 38.4\text{m}$$

The ball travels 38.4m horizontally.

(iv) Although 45° gives maximum range mathematically, goalkeepers prefer 30° −35° for practical reasons. Such lower trajectories have smaller maximum height and shorter time of flight, allowing the ball to reach teammates faster. As a result, opponents have less time to intercept, and teammates can more easily judge and control the descending ball with moderate downward velocity (higher trajectories lead to very high downward velocity of the ball). Therefore, the slightly reduced range as result of choosing slight smaller angle is acceptable for the tactical advantages gained.

Example 25

(a) A ball is thrown from the edge of a cliff. Explain why throwing it at an angle above horizontal can produce greater horizontal range than throwing it horizontally with the same velocity.
(b) From the top of a 20m building, stone A is thrown horizontally at 15m/s while stone B is thrown at 15m/s at 30° above horizontal. Taking g = 9.8 m/s²:
(i) Calculate the time taken for stone A to hit the ground.
(ii) Calculate the time taken for stone B to hit the ground.

(iii) Determine which stone lands farther and by how much.

Solution

(a) When thrown at a **moderate** angle above horizontal from a height, the projectile rises first before falling, which increases total flight time compared to horizontal throw. Although the horizontal velocity component is reduced ($u\cos\theta < u$), the increased flight time outweighs the decreased horizontal velocity. So the projectile stays in the air longer, allowing it to travel farther horizontally despite moving horizontally at a slower rate.

(b) The solution of each part is as follows:

(i) Time of flight for horizontal projection at height is given by:

$$t = \sqrt{\frac{2h}{g}} = \sqrt{\frac{2 \times 20\text{m}}{9.8\text{m/s}^2}} = 2.02\text{s}$$

Stone A takes 2.02s to hit the ground.

(ii) Time of flight from height at an angle is given by:

$$t = \frac{u\sin\theta + \sqrt{u^2\sin^2\theta + 2gh}}{g} = \frac{15\sin 30° + \sqrt{(15\sin 30°)^2 + 2 \times 9.8 \times 20}}{9.8} = 2.93\text{s}$$

Stone B takes 2.93s to hit the ground.

(iii) Range of stone A:
$R_A = ut = 15\text{m/s} \times 2.02\text{s} = 30.3\text{m}$
Range of stone B:

$R_B = (u\cos\theta)t = (15\text{m/s} \times \cos 30°) \times 2.93\text{s} = 38.1\text{m}$

Difference $= R_B - R_A = (38.1 - 30.3)\text{m} = 7.8\text{m}$

Stone B lands $\mathbf{7.8m}$ farther than stone A.

Example 26

(a) A projectile is launched at angle θ from ground level. Explain why the projectile's speed at any height h during ascent equals its speed at the same height h during descent.

(b) A tennis ball is served from height 2.4m with velocity 25m/s horizontally. The net is 12m away and stands 0.9m high. Taking $g = 9.8\ \text{m/s}^2$:
   (i) Determine whether the ball clears the net, and if so, by what margin.
   (ii) Calculate the total horizontal distance the ball travels before hitting the ground.

Solution

(a) By energy conservation, at any height h, the gravitational potential energy is the same whether the projectile is ascending or descending. Since total mechanical energy remains constant (no air resistance), the kinetic energy at height h must be identical in both cases. Equal kinetic energy means equal speed (since $KE = \frac{1}{2}mv^2$).

(b) The solution for each part is as follows:

Calculating time to reach net's horizontal position:

$$t = \frac{x}{u} = \frac{12\text{m}}{25\text{m/s}} = 0.48\text{s}$$

Time to reach net is 0.48s.

Calculating the ball's height when it reaches the net:

Decrease in height: $s_y = \frac{1}{2}gt^2 = \frac{1}{2} \times 9.8\text{m/s}^2 \times (0.48\text{s})^2 = 1.13\text{m}$

Height at net $= h_o - s_y = 2.4m - 1.13m = 1.27m$ above ground.

The ball is at height of 1.27m when it reaches the net.

Comparing the ball's height versus the net's height:

$\text{Net height} = 0.9m < \text{Ball height } (1.27m)$ (the ball will clear the net)

Clearance $= 1.27m - 0.9m = 0.37m$

(i) Hence, the ball clears the net by margin of 0.37m.

The horizontal range is given by:

$$R = ut = u \times \sqrt{\frac{2h}{g}} = 25m/s \times \sqrt{\frac{2 \times 2.4m}{\frac{9.8m}{s^2}}} = 17.5m$$

(ii) The total horizontal distance the ball travels before hitting the ground is 17.5m.

Example 27

(a) Explain the difference between speed and velocity for a projectile, using the example of a ball at its maximum height.

(b) A cricket ball is thrown at 22m/s at 40° above horizontal from ground level. Taking g=9.8m/s²:

(i) Calculate the ball's speed when it is 8m above ground on its way up.

(ii) Calculate the ball's speed when it is 8m above ground on its way down.

(iii) Calculate the ball's velocity (magnitude and direction) at maximum height.

(iv) Explain why the velocities at 8m (ascending vs descending) are different even though speeds are equal.

Solution

(a) Speed is a scalar quantity (magnitude only) measuring how fast an object moves. Velocity is a vector quantity (magnitude and direction) describing both speed and direction. At maximum height, a projectile has zero vertical velocity component but non-zero horizontal velocity component, so it still has speed (equals horizontal component ucosθ) even though vertical velocity is zero. The velocity vector is purely horizontal at this point.

(b) Initial components:

$u_y = u\sin\theta = 22m/s \times \sin 40° = 14.15\ m/s$

$u_x = u\cos\theta = 22m/s \times \cos 40° = 16.85\ m/s$ (constant)

(i) At h = 8 m (ascending), using $v^2 = u^2 - 2gh$:

$v_y^2 = (14.15m/s)^2 - 2 \times 9.89.8m/s^2 \times 8m;\ v_y = 6.59m/s$ (upward)

Total speed: $v = \sqrt{(v_x)^2 + (v_y)^2} = \sqrt{(16.85m/s)^2 + (6.59m/s)^2} = 18.1m/s$

Speed at 8m going up is 18.1m/s.

(ii) At h = 8 m (descending):
$v_y = -6.59m/s$ (downward, same magnitude)
$v_x = 16.85m/s$ (unchanged)

Speed: $v = \sqrt{(16.85m/s)^2 + (-6.59m/s)^2} = 18.1m/s$

Speed at 8m coming down is 18.1m/s.

(iii) At maximum height: $v_y = 0, v_x = 16.85m/s$

Velocity magnitude $= 16.85\ m/s$, direction = horizontal

At maximum height: velocity is $16.85 m/s$ horizontally.

(iv) Although speeds are equal (18.1m/s in both cases), velocities differ because velocity includes direction. Ascending: vertical component points upward; descending: vertical component points downward. The horizontal component remains the same, but the different vertical directions make the velocity vectors different even though their magnitudes (speeds) are identical.

Example 28

(a) A package dropped from a moving aircraft follows a curved path as seen from the ground. Explain why the pilot sees it fall straight down.
(b) A ball is thrown at 30m/s at 50° from ground level. Taking g = 9.8 m/s²: Calculate the time interval during which the ball is below a height of 20m.

Solution

(a) This is because the motion is observed from different reference frames. From the ground observer's viewpoint, the package has horizontal velocity (inherited from the aircraft) and vertical velocity (from gravity), creating a curved path. From the pilot's viewpoint, both aircraft and package share the same horizontal velocity, so relative to the pilot, the package has zero horizontal motion and only falls vertically.
(b) Calculating the time of flight

$$T = \frac{2u\sin\theta}{g} = \frac{2 \times 30m/s \times \sin 50°}{9.8} = 4.69s$$

Total time of flight is 4.69s.

Calculating the times at which the ball is at height 20m

$s_y = (u\sin\theta)t - ½gt^2$

$20 = (30\sin 50)t - ½(9.8)t^2$

$20 = 23.0t - 4.9t^2$

$4.9t^2 - 23t + 20 = 0$

$t_1 = 1.15s$ (smaller time, ascending)

$t_2 = 3.54s$ (larger time, descending)

Calculating the time interval during which the ball is above 20m

Time interval: $\Delta t = t_2 - t_1 = 3.54s - 1.15s = 2.39s$

The ball remains above 20m for 2.39s.

Calculating the time of flight

$$T = \frac{2u\sin\theta}{g} = \frac{2 \times 30m/s \times \sin 50°}{9.8} = 4.69s$$

Total time of flight is 4.69s.

Finding difference between the time of flight and the time interval above 20m

$\text{Time interval below 20m} = \text{Total flight time} - \text{Time interval above 20m}$

$$= T - \Delta t = 4.69s - 2.39s = 2.3s$$

The time interval during which the ball is below a height of 20m is 2.3s.

Example 29

(a) A stone is thrown downward at an angle below horizontal from a cliff. Explain how this affects the range compared to throwing it horizontally.

(b) During a match, a goalkeeper attempts to pass the ball directly to a teammate positioned 50m away on level ground. However, an opposing defender stands directly between the goalkeeper and the teammate, positioned 1.5m in front of the teammate. The goalkeeper can kick the ball with a maximum velocity of 24m/s. Take 9.8m/s²:

(i) Determine whether the goalkeeper can reach the teammate with a direct kick on level ground.

(ii) Calculate the two possible launch angles that would make the ball land exactly at the teammate's position.

(iii) For each angle found in (ii), calculate the maximum height reached by the ball.

(iv) If the defender is 1.8m tall and remains standing upright without jumping, determine which of the angles found in (ii) would allow the ball to pass over the defender's head before reaching the teammate.

Solution

(a) Throwing the stone downward below the horizontal gives it an initial downward velocity, so it reaches the ground in a shorter time (reduced time of flight) than if it were thrown horizontally. At the same time, its horizontal component of velocity is reduced to ucosθ, which is less than u. Therefore, the stone moves forward more slowly and for a shorter time, so its horizontal range is smaller than for a horizontal throw.

(b) The solution for each part is as follows:

(i) Finding maximum possible range on the level ground (at 45°):

$$R_{max} = \frac{u^2}{g} = \frac{(24\text{m/s})^2}{9.8\text{m/s}^2} = 58.8\text{m} > 50\text{m}$$

Since the maximum horizontal distance travelled by the ball (58.8m) exceeds the horizontal separation between the goalkeeper and the teammate (50m), the goalkeeper can reach the teammate with a direct kick.

(ii) For level ground projection:

$$R = \frac{u^2 \sin 2\theta}{g}$$

$$\sin 2\theta = \frac{Rg}{u^2} = \frac{50\text{m} \times 9.8\text{m/s}^2}{(24\text{m/s})^2} = 0.8507$$

$$2\theta = \sin^{-1}(0.8507) = 58.25° \text{ or } 180° - 58.3° = 121.75°$$

Hence, $\theta = 29.1°$ or $60.9°$

The two possible angles are $29.1°$ and $60.9°$.

(iii) For $\theta = 29.1°$:

$$H_1 = \frac{u^2 \sin^2\theta}{2g} = \frac{(24\text{m/s})^2 \times \sin^2 29.1°}{2 \times 9.8\text{m/s}^2} = 6.93\text{m}$$

At $29.1°$, maximum height is 6.93m.

For $\theta = 60.1°$:

$$H_2 = \frac{u^2 \sin^2\theta}{2g} = \frac{(24\text{m/s})^2 \times \sin^2 60.9°}{2 \times 9.8\text{m/s}^2} = 22.4\text{m}$$

At $60.9°$, maximum height is 22.4m.

(iv) Since the opposing defender is 1m in front of the teammate and between the goalkeeper and the teammate, the distance from goalkeeper to defender is $50\text{m} - 1.5\text{m} = 48.5\text{m}$.

Using $t = \frac{x}{u\cos\theta}$ (From $x = (u\cos\theta)t$)

Time taken by the ball to reach the defender when $\theta = 29.1°$:

$$t = \frac{48.5\text{m}}{24\text{m/s} \times \cos 29.1°} = 2.31\text{s}$$

Vertical displacement for $\theta = 29.1°$:

$$s_y = h = u_y - \frac{1}{2}gt^2 = (24\text{m/s} \times \sin 29.1°)2.31\text{s} - \frac{1}{2} \times 9.8\text{m/s}^2 \times (2.31\text{s})^2 = 0.81\text{m} < 1.8\text{m}$$

Since the height of the ball is less than the height of the defender, the ball will not pass over the defender's head.

Time taken by the ball to reach the defender when $\theta = 60.9°$:

$$t = \frac{48.5\text{m}}{24\text{m/s} \times \cos 60.9°} = 4.16\text{s}$$

Vertical displacement for $\theta = 60.9°$:

$$s_y = h = u_y - \frac{1}{2}gt^2 = (24\text{m/s} \times \sin 60.9°)4.16\text{s} - \frac{1}{2} \times 9.8\text{m/s}^2 \times (4.16\text{s})^2 = 2.52\text{m} > 1.8\text{m}$$

Since the height of the ball is greater than the height of the defender, the ball will pass over the defender's head.

Hence, the suitable angle is $60.9°$.

Alternative solution for (b)(iv)

By using trajectory equation:

$$y = x\tan\theta - \left(\frac{g}{2u^2\cos^2\theta}\right)x^2$$

For $\theta = 29.1°$:

$$y = 48.5\tan 29.1° - \left(\frac{9.8}{2 \times 24^2\cos^2 29.1}\right)48.5^2 = 0.82\text{m} \approx 0.81\text{m as before.}$$

For $\theta = 60.9°$:

$$y = 48.5\tan 60.9° - \left(\frac{9.8}{2 \times 24^2\cos^2 60.9}\right)48.5^2 = 2.57\text{m} \approx 2.52\text{m as before.}$$

Example 30

(a) A projectile is launched with a certain velocity from the ground. Explain why increasing the launch angle beyond 45° reduces the horizontal range on level ground.

(b) Ball A is thrown from ground level with velocity of 20 m/s at angle 60° above horizontal. One second later, ball B is thrown from the same point at angle 45° above horizontal. Find the initial velocity u_B so that both balls land at the same time. Take $g = 9.8$ m/s².

Solution

(a) Increasing the angle beyond 45° directs too much velocity upward (vertical component) and not enough forward (horizontal component). Although a larger launch angle results in a greater maximum height and hence a longer time of flight, the reduced horizontal velocity means it covers less horizontal distance during that time. The optimal 45° balances vertical and horizontal components to maximize range.

(b) Time of light of Ball A (launched at t = 0):

$$T_A = \frac{2u_A \sin\theta_A}{g} = \frac{2 \times 20\text{m/s} \times \sin 60°}{9.8\text{m/s}^2} = 3.54\text{s}$$

Ball A lands at $t = 3.54\text{s}$.

For ball B to land at the same time as ball A, it must land at absolute time t = 3.54s.

Since ball B is launched at t = 1s, (1s delay) its flight time is:

$$T_B = 3.54\text{s} - 1\text{s} = 2.54\text{s}$$

Again, using time of flight formula:

$$T_B = \frac{2u_B \sin\theta_B}{g}$$

From which:

$$u_B = \frac{gT_B}{2\sin\theta_B} = \frac{9.8\text{m/s}^2 \times 2.54\text{s}}{2 \times \sin 45°} = 17.6\text{m/s}$$

For both balls to land simultaneously, the initial velocity u_B must be 17.6m/s.

Example 31

(a) On an inclined plane, explain why the optimal launch angle for maximum range is **not** 45° but shifts depending on the plane's slope.

(b) A ball is thrown at 18m/s from the bottom of a hill sloping upward at 25°. Take $g = 9.8$ m/s²:
  (i) Calculate the optimal launch angle (from horizontal) for maximum range up the slope.
  (ii) Calculate this maximum range along the slope.
  (iii) Calculate how high above the launch point the ball lands.

Solution

(a) On level ground, 45° balances horizontal and vertical components optimally. On an upward slope at angle β, the ground rises to meet the projectile, shortening effective flight time. To compensate, the launch must be steeper to gain more vertical velocity and extend time aloft despite the rising ground. Thus the optimal angle increases by half the slope angle:

$$\theta_{optimum} = 45° + \frac{\beta}{2}$$

Conversely, on a downward slope (β negative), the ground falls away, allowing flatter trajectories, and the optimal angle decreases by half the slope angle:

$$\theta_{optimum} = 45° - \frac{|\beta|}{2}$$

Hence, the slope breaks the symmetry that makes 45° optimal on level ground.

(b) For upward slope β = 25°:

(i) $\theta_{optimal} = 45° + \frac{\beta}{2} = 45° + 12.5° = 57.5°$

Optimal angle is 57.5° above horizontal.

(ii) Maximum range is found from range formula along the incline:

$$L = \frac{u^2}{g\cos^2\beta}(\sin(2\theta - \beta) - \sin\beta)$$

For optimal angle, ($\theta = 57.5°$), L becomes maximum.

Substituting values:

$$L_{max} = \frac{(18m/s)^2}{9.8m/s^2 \times \cos^2(25°)}(\sin(2 \times 57.5° - 25°) - \sin 25°) = 23.24m$$

The maximum range along the slope is 23.24m.

(iii) **Vertical height gain *h*** is found from:

$$\mathbf{h = L\sin\beta} \qquad \left(\text{since } \sin\beta = \frac{h}{L}\right)$$

Substituting values:

$$h = 23.24m \times \sin 25° = 9.82m$$

The ball lands 9.82m above launch point.

Example 32

(a) A projectile's trajectory is parabolic and symmetric. Explain why the projectile spends equal time ascending and descending when launched from and landing on level ground.

(b) Two projectiles are launched simultaneously from the same point at the same velocity 20m/s: projectile A on level ground at 45°, and projectile B down a 30° slope at optimal angle for that slope. Taking g = 9.8 m/s^2:

(i) Calculate the range and flight time for projectile A on level ground.

(ii) Calculate the optimal angle and range for projectile B on the downslope.

(iii) Calculate the flight time for projectile B.

(iv) When projectile A lands, determine how far projectile B has travelled along the slope from the launch point. Hence calculate the additional distance along the slope that projectile B will travel before it lands.

Solution

(a) By symmetry, the upward path mirrors the downward path. Starting from ground with vertical velocity usinθ (upward), the ball decelerates uniformly at rate *g* until $v_y = 0$ at maximum height. During descent from maximum height back to ground level, it accelerates uniformly at rate *g* from $v_y = 0$ back to usinθ (downward). Since both phases involve the same change in velocity (usinθ) and the same acceleration (g), they take equal time.

(b) The solution for each part is as follows:

(i) At 45°:

$$R_A = \frac{u^2}{g} = \frac{(20m/s)^2}{9.8m/s^2} = 40.8 \text{ m}$$

$$T_A = \frac{2u\sin\theta}{g} = \frac{2 \times 20m/s \times \sin 45°}{9.8m/s^2} = 2.89s$$

For projectile A:

Horizontal range is 40.8m,

Flight time is 2.89s.

(ii) Projectile B (30° downslope, $\beta = -30°$):

Optimal angle:

$$\theta_{optimal} = 45° + \frac{\beta}{2} = 45° - \frac{30°}{2} = 30°$$

Using:

$$L = \frac{u^2}{g\cos^2\beta}(\sin(2\theta - \beta) - \sin\beta)$$

Substituting values:

$$L = \frac{(20\text{m/s})^2}{9.8\text{m/s}^2 \times \cos^2(-30°)}(\sin(2 \times 30° + 30°) - \sin(-30°)) = 81.63\text{m}$$

For projectile B:

Optimal angle is 30°,

Horizontal range is 81.63m along slope.

(iii) Flight time of projectile B:

$$T_B = \frac{2u\sin(\theta - \beta)}{g\cos\beta} = \frac{2 \times 20\text{m/s} \times \sin(30° + 30°)}{9.8\text{m/s}^2 \times \cos(-30°)} = 4.08s$$

Flight time of projectile B is 4.08s.

(iv) Projectile A lands at 2.89s. At this time, projectile B is still in flight since its time of flight (4.08s) is greater than 2.89s.

Horizontal position of B at t = 2.89s:

$$x = (u\cos\theta)t = (20\text{m/s} \times \cos 30°) \times 2.89\text{s} = 50.05\text{m}$$

Displacement along the slope, l:

From:

$$\cos\theta = \frac{x}{l}; l = \frac{x}{\cos\theta} = \frac{50.05\text{m}}{\cos 30°} = 57.8\text{m}$$

When A lands, B has travelled 57.8m along slope.

Remaining distance B will travel $= 81.63\text{m} - 57.8\text{m} = 23.8\text{m}$

B will travel another 23.8m along the slope before landing.

Example 33

(a) In long jump, athletes try to jump forward as far as possible. Does jumping very high help increase the horizontal distance? Explain.

(b) A military aircraft is flying horizontally at a constant velocity of 60m/s at a height of 200m above a defended region. At the instant the aircraft passes directly above a defence base, it releases a bomb. At the same instant, the air-defence system launches an interceptor projectile from the ground with a velocity of 90m/s at an angle **θ** above the horizontal in order to destroy the bomb before it reaches the ground. Neglect air resistance and take g = 9.8 m/s²; determine:
   (i) The angle of projection **θ** required for the interceptor to collide with the bomb in mid-air.
   (ii) The time after release when the interception occurs.
   (iii) The position where the collision occurs.

Solution

(a) The effect of jumping very high on the achieved horizontal distance depends on the reason behind that high jump. If it is caused by a greater take-off (initial) velocity at the same angle, it helps to achieve greater horizontal distance. This can be seen from the relation: R = 4Hcotθ, from which if the **angle θ is fixed**, then **cotθ is constant**. Therefore, **R ∝ H**. So in this case, a higher jump means a longer jump.
However, if the athlete jumps higher by using a larger take-off angle, the horizontal component of velocity is reduced, and the horizontal distance (range) may decrease.

Therefore, jumping higher helps only when it comes from greater take-off velocity without losing too much horizontal velocity.

(b) The solution for each part is as follows:

(i) Horizontal positions must be equal at collision.

For bomb, b (released from aircraft): Bomb velocity = Aircraft velocity = 60m/s (horizontally):

$$x_b = 60t$$

For interceptor, i:

$$x_i = (90\cos\theta)t$$

At collision: $x_b = x_i$

$$60t = (90\cos\theta)t$$

$$\cos\theta = \frac{60}{90} = 0.6667$$

$$\theta = \cos^{-1}(0.6667) = 48.19°$$

The required angle of projection is $48.19°$.

(ii) The vertical positions must also be equal. Therefore, if the bomb has descended through a vertical distance **b**, the interceptor must have risen through a vertical distance **200 − b**.

For bomb: $u_y = 0\text{m/s}$ (aircraft was flying horizontally), $s_y = -\mathbf{b}$ (downward, negative)

Using $s_y = -\tfrac{1}{2}gt^2$ (for $u_y = 0\text{m/s}$)

Substituting values:

$-b = -\tfrac{1}{2} \times 9.8 \times t^2$

$b = 4.9t^2$ ... (i)

For interceptor:

Using $s_y = u_y t - \tfrac{1}{2}gt^2$

Substituting values:

$200 - b = (90\sin 48.19°)t - \tfrac{1}{2} \times 9.8 \times t^2$

$b = 200 + 4.9t^2 - (90\sin 48.19°)t$ ... (ii)

At collision:

Equating (i) and (ii):

$4.9t^2 = 200 + 4.9t^2 - (90\sin 48.19°)t$

$$t = \frac{200}{90\sin 48.19°} = 2.99\text{s}$$

The interception occurs after 2.99s.

Alternative solution for (b)(ii)

The quicker method of solving (b)(ii) is by using relative motion concept:

Relative vertical acceleration, $a_R = a_i - a_b = -9.8\text{m/s}^2 - (-9.8\text{m/s}^2) = 0$

And;

Relative initial vertical velocity, $u_R = u_i - u_b = 90\sin 48.19° - 0\text{m/s} = 90\sin 48.19°$

Then, the equation $s_y = u_y t + ½a_y t^2$ becomes:

$$s_y = u_R t + \frac{1}{2}a_R t^2$$

Substituting values:

$200 = (90 \sin 48.19°)t + 0$

$$\mathbf{t} = \frac{200}{90 \sin 48.19°} = \mathbf{2.99s}$$

(iii) At $t = 2.99s$:

Horizontal position:

$x = 60m/s \times 2.99s = 179.4m$

Vertical position:

$y = 200 - b = 200 - (½)(9.8)(2.99)^2 = 156.2m$

They meet at position (179.4m, 156.2m); that is, **179.4m horizontally from the defence base and 156.2m above the ground**.

Having enjoyed the full combination of ideas, it is time to sharpen our thinking; the Digging Deeper Exercise is ready in the next page.