# PROJECTILE MOTION

## INTRODUCTION

It was inter-house sports day at Miono Secondary School, and Mr. Akilikubwa stood at the edge of the football field, clipboard in hand, recording student performances for the shot put competition. Kipanga stepped into the throwing circle, gripping a 4 kg shot put with determination. *"Sir,"* he called out confidently, *"I've been practicing! Watch me break the school record!"*

**Mr. Akilikubwa** smiled. *"Remember, Kipanga! It's not just about how hard you throw, but also the angle."*

*"Angle?"* **Kipanga** snorted. *"Sir, I'm strong! I'll just throw it as hard as I can straight forward!"* With a mighty grunt, he launched the shot put at nearly zero degrees above horizontal. The metal ball rocketed forward... and crashed into the ground barely three metres away, creating a small crater in the sandy field.

**Kipute**, waiting her turn, could not help but giggle. "*Kipanga, you threw it into the ground! That's not how shot put works!"*

*"Fine!"* **Kipanga** huffed, retrieving the ball. *"If throwing straight doesn't work, I'll throw it straight up!"* On his second attempt, he launched the shot put almost vertically. It soared magnificently into the sky, climbing higher and higher and higher... then came straight back down, landing half a metre from where he had thrown it. The students scattered as the heavy ball thudded back to earth.

**Mr. Akilikubwa** shook his head, trying not to laugh. *"Kipanga, you've just demonstrated the two extreme failures of projectile motion! Too flat, and gravity defeats you immediately. Too steep, and you waste all your energy fighting gravity instead of moving forward." He turned to Kipute. "Your turn. Show him how physics works."*

Kipute stepped up, closed her eyes briefly to estimate, and released the shot put at approximately 45 degrees. The ball traced a beautiful arc through the air, landing smoothly 8 metres away, a respectable throw!

*"But how did you know the right angle?"* **Kipanga** demanded, genuinely puzzled.

**Mr. Akilikubwa** grinned. *"Because unlike you, Kipute paid attention in Chapter 2 when we studied vertical motion under gravity. Now she's applying it to motion in two dimensions. Welcome to Chapter 6, everyone, where we learn that every thrown ball, launched javelin, or kicked football becomes a slave to the same beautiful mathematics. Master projectile motion, and you'll understand why goalkeepers kick at certain angles, why water fountains arc gracefully, and why Kipanga just lost five metres of his throwing distance by defying physics!"*

*"Sir,"* **Kipanga** protested weakly, *"I was just... testing different approaches..."*

*"Testing?"* **Kipute** teased. *"Is that what we're calling it now?"*

By the end of this chapter, even Kipanga will understand exactly how to calculate the perfect angle for maximum distance. The mathematics that governs his embarrassing shot put attempts is the same mathematics that describes rockets, bullets, and water spraying from hoses. Let us explore how motion in two dimensions emerges from the simpler concepts we mastered in Chapter 2.

## FUNDAMENTAL EQUATIONS OF PROJECTILE MOTION

**A projectile** *is any object that, once launched with an initial velocity, moves under the influence of gravity alone.* Whether it is a shot put released by Kipanga, a stone thrown by hand, a javelin launched by an athlete, or water spraying from a hose, the motion follows the same fundamental principles. Once the object leaves contact with whatever launched it, the only force acting upon it (when air resistance is negligible) is the downward pull of gravity. *The path traced by a projectile through space* is called its **trajectory**.

**Connecting to chapter 2:** *Do you remember when we studied vertical motion under gravity in Chapter 2?* We learned that any object either thrown upward or dropped from rest experiences constant downward acceleration, $g = 9.8\text{m/s}^2$. We also learned three equations of motion ($v = u + at$, $s = ut + \frac{1}{2}at^2$, and $v^2 = u^2 + 2as$) that perfectly describe this one-dimensional vertical motion. The beautiful insight of projectile motion is that these same principles still apply; we simply extend them to two dimensions.

The key to understanding projectile motion lies in recognising that it represents motion in two dimensions simultaneously, yet these two components of motion are completely independent of one another. A projectile exhibits uniform motion in the horizontal direction while simultaneously experiencing uniformly accelerated motion in the vertical direction due to gravity, exactly the same accelerated motion we studied in Chapter 2. This independence means that *what happens horizontally has no effect on what happens vertically, and vice versa*. This fundamental principle, known as **the independence of perpendicular components of motion**, allows us to analyse each direction separately using the equations we developed in Chapter 2 for one-dimensional motion.

Consider Kipanga's disastrous shot put throws from our introduction. From the moment the shot left his hand, it possessed both horizontal and vertical components of velocity. Horizontally, it moved forward at constant velocity as there is no horizontal forces to slow it or speed it up (ignoring air resistance). Vertically, however, gravity continuously acted downward, first slowing any upward motion, bringing it momentarily to rest at its highest point, then accelerating it downward (exactly as we learned in Chapter 2 for vertical motion). These two motions occurred simultaneously and independently, creating the parabolic arc.

To analyse projectile motion systematically, we establish a coordinate system with the x-axis along the horizontal direction and the y-axis pointing vertically upward. The origin is typically placed at the **point of projection** which *is the location from which the projectile is launched*. By **convention**, we take **upward** as the **positive y-direction**, which means that the acceleration due to gravity, acting downward, has a negative value: $\mathbf{a_y = -g = -9.8\ m/s^2}$. In the horizontal direction, where no forces act (again neglecting air resistance), the acceleration is zero: $a_x = 0$.

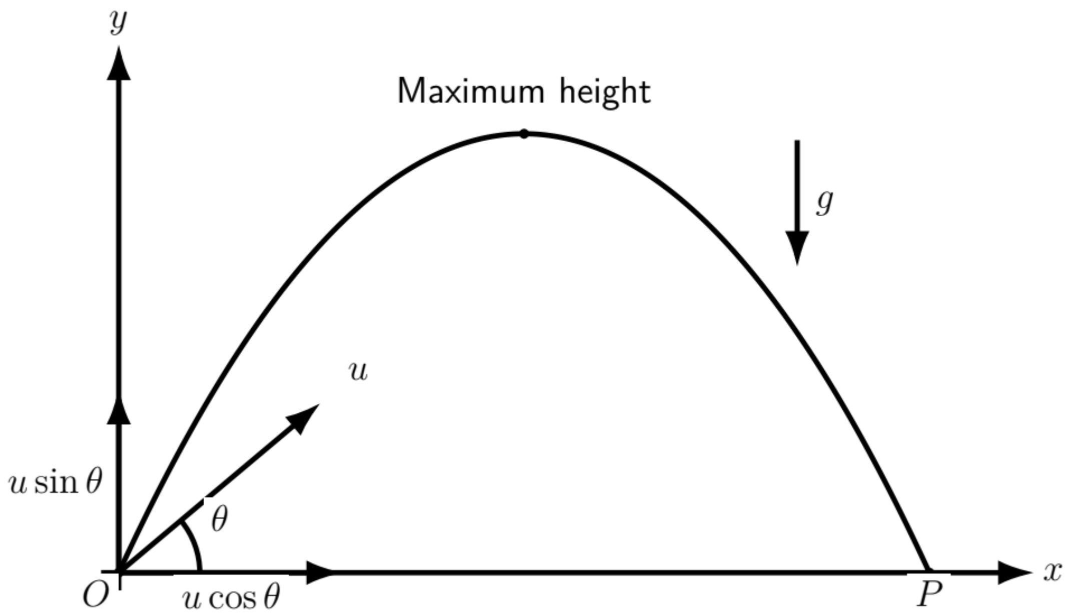


**Figure:** *Projectile launched from the origin O with initial velocity, u at an angle θ above the horizontal. The trajectory is parabolic. The initial velocity is resolved into horizontal component ucosθ and vertical component usinθ. The highest point represents the maximum height, and the projectile lands at point P on the x-axis while the acceleration due to gravity g acts vertically downward.*

**Resolving initial velocity into components**

When a projectile is launched from point O with initial velocity u at an angle θ above the horizontal, this velocity vector can be resolved into two perpendicular components using basic trigonometry. The horizontal component, $u_x$, represents the portion of the initial velocity directed along the x-axis, while the vertical component, $u_y$, represents the portion directed along the y-axis. From the geometry of the velocity vector triangle, these components are given by:

$$\mathbf{u_x = ucos\theta}$$

$$\mathbf{u_y = usin\theta}$$

The initial velocity u forms the hypotenuse of a right triangle, with $u_x$ and $u_y$ as the two perpendicular sides. The horizontal component uses the cosine function because it lies adjacent to the angle θ, while the vertical component uses the sine function because it lies opposite to the angle. These trigonometric relationships ensure that the vector sum of the components equals the original velocity vector in both magnitude and direction.

**Velocity as a function of time**

Since the horizontal acceleration is zero, the horizontal component of velocity remains constant throughout the motion. Thus, at **any time t** after launch, the horizontal velocity is simply:

$$\mathbf{v_x = ucos\theta = u_x}$$

This constancy reflects the fact that, in the absence of horizontal forces, there is nothing to change the projectile's horizontal velocity. *The projectile continues to move forward at the same horizontal rate from launch until landing.*

The vertical component of velocity, however, changes continuously under the influence of gravity. Applying the first equation of motion ($v = u + at$) to the vertical direction, with initial velocity $u_y = usin\theta$ and acceleration $a_y = -g$, we obtain:

$$\mathbf{v_y = usin\theta - gt}$$

The negative sign before g reflects gravity's downward action. Initially, when $t = 0$, the vertical velocity equals $u\sin\theta$, directed upward. As time progresses, gravity steadily reduces this upward velocity. At some point, $v_y$ becomes zero; this marks the instant when the projectile reaches its maximum height and momentarily stops rising. Beyond this point, $v_y$ becomes negative, indicating downward motion as the projectile begins to fall.

**Displacement as a function of time**

The **horizontal displacement** ($\mathbf{s_x}$) at any time t follows from the second equation of motion, which is: $s = ut + ½at^2$. With zero horizontal acceleration, the $½at^2$ term vanishes, leaving:

$$\mathbf{s_x = (u\cos\theta)t}$$

This linear relationship between horizontal displacement and time shows that the projectile covers equal horizontal distances in equal time intervals, which is a characteristic of uniform motion. Thus, the horizontal distance increases steadily for as long as the projectile remains in the air.

For **vertical displacement**, we again apply $s = ut + ½at^2$ to the vertical direction, substituting $u_y = u\sin\theta$ and $a_y = -g$:

$$\mathbf{s_y = (u\sin\theta)t - ½gt^2}$$

This equation contains two competing terms. *The first term, (usinθ)t, represents how high the projectile would rise if gravity did not act* (it increases linearly with time). *The second term,* $-½gt^2$*, represents how far gravity pulls the projectile downward* (it increases quadratically with time). The net vertical displacement at any instant results from these two opposing effects. Initially, the linear term dominates and the projectile rises. Eventually, the quadratic term overtakes the linear term, and the projectile begins to descend.

**The trajectory equation**

The equations for horizontal and vertical displacement both contain time as a parameter. By eliminating time between these two equations, we can obtain a direct relationship between $s_x$ and $s_y$ that describes the shape of the trajectory itself.

From $s_x = (u\cos\theta)t$, we can express time as:

$$t = \frac{s_x}{u\cos\theta}$$

Substituting this expression for t into the equation for vertical displacement:

$$s_y = (u\sin\theta)\left(\frac{s_x}{u\cos\theta}\right) - ½g\left(\frac{s_x}{u\cos\theta}\right)^2$$

Simplifying the first term:

$$\mathbf{s_y = s_x\tan\theta - \left(\frac{g}{2u^2\cos^2\theta}\right)s_x^2}$$

This is the trajectory equation.

We may take $s_y$ as *y*, and $s_x$ as *x*; then the trajectory equation becomes:

$$\boldsymbol{y = x\tan\theta - \left(\frac{g}{2u^2\cos^2\theta}\right)x^2}$$

With this form, we can clearly see that the equation is in the standard form of a parabola equation:

$$\boldsymbol{y = bx + ax^2}$$

Where:

b (coefficient of $s_x$) = $\tan\theta$

a (coefficient of $s_x^2$) = $-\left(\frac{g}{2u^2\cos^2\theta}\right)$ (negative)

So the trajectory equation has the mathematical form of a parabola that opens downward.

Also it is important to note that both coefficients are constants for a given projectile launched with particular initial conditions. This reveals that *regardless of the initial velocity or launch angle, the path of a projectile is always parabolic* when air resistance is negligible.

**Magnitude and direction of velocity at any instant**

Although we analyse the horizontal and vertical components of velocity separately, the actual velocity of the projectile at any instant is a single vector pointing along the instantaneous direction of motion. The magnitude of this velocity vector can be found by combining its perpendicular components using the Pythagorean theorem:

$$\mathbf{v} = \sqrt{(\mathbf{v_x})^2 + \left(\mathbf{v_y}\right)^2} = \sqrt{(\mathbf{ucos\theta})^2 + (\mathbf{usin\theta\, g - t})^2}$$

The direction of the velocity vector at time t, measured as an angle α **from the horizontal**, is given by:

$$\tan\alpha = \frac{v_y}{v_x}$$

$$\boldsymbol{\alpha = \tan^{-1}\left(\frac{v_y}{v_x}\right)}$$

As the projectile moves along its trajectory, both the magnitude and direction of its velocity change continuously. The horizontal component remains constant, but the changing vertical component causes both the velocity and direction to vary from moment to moment.

**The Principle of Independence**

Students often find it puzzling that we can treat horizontal and vertical motions as if they were completely separate, when clearly the projectile moves as a single object through space. This separation is justified by a fundamental principle of mechanics: ***perpendicular components of motion are independent.*** *What happens in one direction has no influence on what happens in a perpendicular direction, provided no forces act to couple the two directions.*

To understand this more clearly, imagine an observer on a train moving at constant velocity who drops a ball. Relative to the observer, the ball falls straight downward, taking time t to reach the floor. The fact that the train is moving horizontally does not alter the time required for the ball to fall (gravity acts only vertically and is unrelated to the train's horizontal motion). An observer standing beside the track sees the ball follow a parabolic path, but even from this external perspective, the time for the ball to fall remains t. Thus, the horizontal motion is superimposed on the vertical motion without affecting it.

This independence extends to all projectile motion. The time a football remains in the air depends only on its initial vertical velocity and the action of gravity; it does not matter whether the ball was kicked gently or powerfully in the horizontal direction. Similarly, the horizontal distance traveled depends on how long the projectile stays in the air (determined by vertical motion) and how fast it moves horizontally (determined by the horizontal component of initial velocity). The two components of motion proceed simultaneously and independently, yet together they produce the observed parabolic trajectory.

**Addressing Common Misconceptions**

Several persistent misconceptions about projectile motion deserve explicit attention. Let us address them directly:

**Misconception 1:** "Heavier objects follow different trajectories than lighter objects."

**The truth:** *Mass does not appear anywhere in our projectile equations!* A heavy stone and a light pebble, if thrown with the same initial velocity and angle, will follow identical paths. Examination of all the equations we have derived reveals no dependence on mass whatsoever.

***Why students believe this:*** In everyday experience, air resistance affects lighter objects more severely than heavier ones, making them appear to follow different paths. But the fundamental projectile motion equations (which assume negligible air resistance) are completely independent of mass.

**Misconception 2:** "At the highest point, the velocity is zero."

**The truth:** At the maximum height, **only** the vertical component of velocity ($v_y$) becomes zero. The horizontal component remains unchanged at $v_x = u\cos\theta$. *The projectile never stops moving completely, even at the peak of its trajectory, it continues to travel forward.*

***Why this matters:*** If the velocity were truly zero at the highest point, the projectile would simply drop straight downward from there. Instead, it continues its parabolic arc because horizontal motion persists throughout the flight.

**Misconception 3:** "Gravity acts differently during ascent than during descent."

**The truth:** Throughout the entire motion; rising, at the peak, and falling, *the acceleration due to gravity remains constant and always acts downward ($a = -g = -9.8m/s^2$ always)*, exactly as we learned in Chapter 2. This never changes.

***The apparent difference:*** On the way up, this downward acceleration opposes and reduces the upward velocity. On the way down, the same downward acceleration increases the downward velocity. The acceleration itself remains identical; only its effect on the velocity appears different depending on the direction of motion.

**Misconception 4**: "Throwing harder changes the shape of the trajectory."

**The truth:** *The trajectory is always parabolic, regardless of how hard you throw.* Throwing harder increases the horizontal distance and maximum height, creating a larger parabola, but the shape remains fundamentally the same (it is still a parabola). Throwing gently produces a smaller parabola, but still a parabola.

***Relating to Kipanga's two attempts:*** In Kipanga's throwing attempts: one nearly horizontal, another nearly vertical, both failed. In the first case, he created a very flat parabola that hit the ground almost immediately. In the second, he created a very tall, narrow parabola that barely traveled forward. Kipute's 45-degree throw created a parabola with optimal proportions for maximum horizontal distance.

**Application to Kipanga's shot put disaster**

Returning to the opening scenario, we can now analyse what went wrong with Kipanga's two attempts and why Kipute succeeded.

**Kipanga's first throw (nearly horizontal, say θ ≈ 5°):**

If he threw with velocity $u = 10m/s$ at angle $\theta = 5°$:

- $u_x = 10\cos5° \approx 9.96m/s$ (almost all horizontal)
- $u_y = 10\sin5° \approx 0.87m/s$ (very little vertical)

*The small vertical component meant the shot barely rose above his release height before gravity pulled it down. With such **minimal time in the air**, even the large horizontal component could not carry it far.*

**Kipanga's second throw (nearly vertical, say θ ≈ 86°):**

If he threw with the same velocity at $\theta = 86°$:

- $u_x = 10\cos 86° \approx 0.70m/s$ (barely any horizontal)
- $u_y = 10\sin 86° \approx 9.98m/s$ (almost all vertical)

*The large vertical component sent the shot high into the air, giving it plenty of time before landing. But with such minimal horizontal velocity, it barely moved forward during all that airtime!*

**Kipute's optimal throw (θ = 45°):**

With $u = 10m/s$ at $\theta = 45°$:

- $u_x = 10\cos 45° \approx 7.07\text{m/s}$ (balanced horizontal)
- $u_y = 10\sin 45° \approx 7.07\text{m/s}$ (balanced vertical)

This *balance between horizontal and vertical components gives maximum horizontal distance*. It allowed the shot to stay in the air long enough while also traveling forward effectively. The result: 8 metres compared to Kipanga's embarrassing 3 metres and 0.5 metres!

The mathematics of projectile motion transforms shot put from pure strength into applied science. Understanding these principles applies equally to javelin throws, long jumps, basketball shots, and yes, even Mr. Akilikubwa's favorite example, the perfect arc of water from a garden hose!

With these fundamental concepts now simmering nicely, let us serve them properly through a few worked examples and enjoy the flavour of physics in action!

**BINDER Example 1**

A stone is thrown with an initial velocity of $20\text{m/s}$ at an angle of $30°$ above the horizontal. Take $g = 9.8\ \text{m/s}^2$. Calculate:

(a) the horizontal and vertical components of the initial velocity,
(b) the horizontal and vertical components of velocity after 1.5s.

**Solution**

(a) Horizontal component:

$$u_x = u\cos\theta = 20\text{m/s} \times \cos 30° = \mathbf{17.3m/s}$$

Vertical component:

$$u_y = u\sin\theta = 20\text{m/s} \times \sin 30° = \mathbf{10m/s}$$

(b) Horizontal velocity (remains constant):

$$v_x = u_x = \mathbf{17.3m/s}$$

Vertical velocity (changes due to gravity):

$$v_y = u_y - gt = 10\text{m/s} - (9.8\text{m/}s^2)(1.5\text{s}) = -4.7\ \text{m/s}$$

The negative sign indicates the stone is now moving downward.

So the vertical velocity is $-4.7\ \text{m/s}$ (or $4.7\ \text{m/s}$ downward)

**Making Sense of the Answer:** *After 1.5 seconds, the horizontal velocity remains unchanged at 17.3m/s because there's no horizontal force. The vertical velocity changed from +10m/s (upward) to −4.7 m/s (downward), meaning the stone has passed its maximum height and is now falling. The stone initially rose with vertical velocity 10m/s, slowed to zero at the peak, then began falling, reaching 4.7m/s downward after total time of 1.5s.*

**Think Like a Physicist:** *Notice that we could immediately tell the stone had passed its highest point because $v_y$ became negative. At the exact moment of maximum height, $v_y = 0$. We can calculate when this occurred: $0 = 10 - 9.8t$, giving $t = 1.02s$. So the stone reached maximum height at $t = 1.02s$, and by $t = 1.5s$ it was already descending. This demonstrates how the vertical component alone determines when the projectile reaches its peak.*

**BINDER Example 2**

A ball is kicked horizontally from the top of a cliff with initial velocity $15\text{m/s}$. Calculate the position and velocity of the ball after 2 seconds. Take $g = 9.8\ \text{m/s}^2$.

**Solution**

In this case, $u_x = 15\text{m/s}, u_y = 0$ (horizontal launch), Horizontal displacement:

$$s_x = u_x t = 15\text{m}/\times 2\text{s} = 30\text{m}$$

Vertical displacement:

$$s_y = u_y t - ½gt^2 = 0 - ½(9.8\text{m/s}^2)(2\text{s})^2 = -19.6\text{m}$$

The negative sign indicates displacement downward from the launch point.

Hence**, the position of the ball is $30\text{m}$ horizontally from the cliff and $19.6\text{m}$ below the launch point.**

Horizontal velocity:

$$v_x = u_x = 15\text{m/s}$$

Vertical velocity:

$$v_y = u_y - gt = 0 - (9.8\text{m/s}^2)(2\text{s}) = -19.6\text{m/s}$$

Magnitude of velocity:

$$v = \sqrt{(v_x)^2 + \left(v_y\right)^2} = \sqrt{(15\text{m/s})^2 + (-19.6\text{m/s})^2} = 24.7\text{m/s}$$

Direction (below horizontal since the motion is downward):

$$\alpha = \tan^{-1}\left(\frac{v_y}{v_x}\right) = \tan^{-1}\left(\frac{-19.6\text{m/s}}{15\text{m/s}}\right) = -52.6°$$

The negative sign means the angle was measured clockwise from the horizontal, and hence the direction of motion is below the horizontal.

Hence, **the velocity of the ball is $24.7\text{m/s}$ directed at $52.6°$ below horizontal.**

**Making Sense of the Answer:** *Even though the ball was kicked horizontally (no initial vertical velocity), after 2 seconds it has fallen 19.6m and gained significant downward velocity (19.6m/s). The horizontal velocity remained constant at 15m/s throughout. The final velocity (24.7m/s) is greater than the initial velocity (15m/s) because gravity has added a vertical component. The steep angle (52.6° below horizontal) shows the ball is now falling quite rapidly.*

**Think Like a Physicist:** *This example illustrates a special case of projectile motion: horizontal launch ($\theta$ = 0°). Even though there was no initial upward motion, the ball still follows a parabolic path as it falls. This is exactly what happens when you roll a ball off a table; it continues forward at constant horizontal velocity while simultaneously falling under gravity. Both motions combine to create the characteristic parabolic trajectory, even without an initial upward component.*

**REAL Example 3**

At Kariakoo market, vendors selling oranges often demonstrate the quality of their fruit by tossing an orange to potential customers standing several metres away. An experienced vendor can consistently land the orange gently in a customer's hands, while inexperienced vendors either throw too hard (orange sails over the customer) or too soft (orange falls short).

(a) Explain why the vendor must adjust both the velocity and angle of throw depending on how far away the customer stands.
(b) Explain what happens if the vendor uses the correct velocity but wrong angle.

**Solution**

(a) The horizontal distance travelled by the orange (the range) depends on both the initial velocity and the angle of projection. For a given distance to the customer, there are actually multiple combinations of velocity and angle that could work: a fast throw at a low angle, or a slower throw at a higher angle, can both reach the same spot. However, experienced vendors prefer a moderate velocity with optimal angle because:

- **High velocity at low angle:** The orange arrives with high horizontal velocity, making it difficult to catch and potentially bruising the fruit on impact
- **Low velocity at high angle:** The orange spends too long in the air, during which wind could deflect it.

The vendor instinctively calculates (through experience) the combination of velocity and angle that makes the orange arrive gently with low velocity, making it easy to catch.

(b) If the vendor uses correct velocity but wrong angle, two problems arise:

**Case 1 - Angle too large (too steep):**

Too much initial velocity goes into the vertical component, less into horizontal. The orange rises high, spending long time in the air, but does not travel far enough horizontally. It falls short of the customer, landing on the ground in front of them. (This is similar to Kipanga's second shot put attempt.)

**Case 2 - Angle too small (too flat):**

Too much initial velocity goes into horizontal component, too little into vertical. The orange does not rise high enough or stay in the air long enough. Despite moving forward quickly, it drops to the ground before reaching the customer. (This is similar to Kipanga's first attempt.)

**Making Sense of the Answer:** *This example demonstrates that projectile motion is not just abstract physics; it is embedded in everyday activities. Market vendors, though they have never studied our equations, understand projectile motion intuitively through experience. They know that "how hard" (initial velocity) and "which direction" (angle) are both crucial and must work together. This intuitive understanding is precisely what our mathematical framework makes explicit and quantifiable.*

**Think Like a Physicist:** *Notice how real-world projectile motion involves optimization: not just reaching a target, but reaching it in a way that minimizes certain undesirable effects (impact force, time in air, trajectory height, etc.). In sports, warfare, firefighting, and countless other applications, understanding the mathematics allows us to optimize these factors systematically rather than relying purely on trial and error. The vendor's experience represents thousands of unconscious calculations of which our equations let us perform those calculations consciously and precisely.*

**HOT Example 4**

A projectile is launched with initial velocity $25\mathrm{m/s}$ at an angle of $53°$ above the horizontal from ground level. Taking $g = 9.8\ \mathrm{m/s^2}$ and $\sin 53° = 0.8$, $\cos 53° = 0.6$:

(a) Derive the equation of the trajectory for this projectile.
(b) Calculate the height of the projectile when it is at horizontal distance 15m from the launch point.
(c) Calculate the horizontal distance at which the projectile returns to ground level.

**Solution**

(a) The general trajectory equation is:

$$s_y = s_x \tan\theta - \left(\frac{g}{2u^2\cos^2\theta}\right) s_x^2$$

Substituting given values:

$$\tan 53° = \frac{\sin 53°}{\cos 53°} = \frac{0.8}{0.6} = 1.333$$

$$\frac{g}{2u^2\cos^2\theta} = \frac{9.8}{(2(25)^2(0.6)^2)} = 0.0218$$

Therefore:

$$s_y = 1.333s_x - 0.0218s_x^2$$

Thus the trajectory equation for this particular projectile is:

$$s_y = 1.333s_x - 0.0218s_x^2$$

(b) At $s_x = 15\text{m}$:

$$s_y = 1.333(15) - 0.0218(15)^2 = 15.1\text{ m}$$

The projectile is at height of $\mathbf{15.1m}$ when it is 15m horizontally from launch.

(c) The projectile returns to ground level when $s_y = 0$:

$$0 = 1.333s_x - 0.0218s_x^2$$

$$0 = s_x(1.333 - 0.0218s_x)$$

This gives two solutions: $s_x = 0$ (the launch point); or

$1.333 - 0.0218s_x = 0$; $s_x = \frac{1.333}{0.0218} = 61.1\text{m}$

Hence, the projectile lands at horizontal distance 61.1m.

**Making Sense of the Answer:** *The trajectory equation, $s_y = 1.333s_x - 0.0218s_x^2$ is indeed a parabola (it has the form $y = ax + bx^2$ where b is negative). At $x = 15m$, the height is $15.1m$ which is still quite high, indicating the projectile is probably near its peak. The horizontal distance of $61.1m$ means the projectile travels this far before returning to launch height. Notice that the coefficient $1.333$ ($= \tan 53°$) determines the initial slope of the trajectory, while $0.0218$ determines how quickly gravity curves it downward.*

**Think Like a Physicist:** *The trajectory equation is powerful because it relates position to position directly without time as an intermediate variable. This is useful when we care about "where" rather than "when." For instance, if there's an obstacle at a certain location, we can immediately check if the projectile will clear it by substituting that horizontal position and checking if $s_y$ exceeds the obstacle height. The fact that we found two solutions when $s_y = 0$ (namely $s_x = 0$ and $s_x = 61.1$) makes physical sense: the projectile is at ground level at both launch ($x = 0$) and landing ($x = 61.1m$). The parabola crosses the x-axis at exactly two points, as expected for a symmetric trajectory launched from and returning to the same level.*

**Summary of Fundamental Equations**

For convenient reference, we collect here the essential equations governing projectile motion, valid when air resistance is negligible and the acceleration due to gravity is constant:

**Initial velocity components:**

$$u_x = u\cos\theta, u_y = u\sin\theta$$

**Velocity at time any time t:**

$$v_x = u\cos\theta, v_y = u\sin\theta - gt$$

**Displacement at any time t:**

$$s_x = (u\cos\theta)t, s_y = (u\sin\theta)t - \tfrac{1}{2}gt^2$$

**Trajectory equation:**

$$s_y = s_x\tan\theta - \left(\frac{g}{2u^2\cos^2\theta}\right)s_x^2$$

**Magnitude of velocity at any instant:**

$$v = \sqrt{(v_x)^2 + \left(v_y\right)^2}$$

**Direction of velocity at any instant:**

$$\alpha = \tan^{-1}\left(\frac{v_y}{v_x}\right)$$

These equations form the mathematical foundation for analysing any projectile motion problem. In the following subtopic, we shall use them to derive expressions for important physical quantities such as time of flight, maximum height, and horizontal range.