MISCELLANEOUS WORKED EXAMPLES ON MOMENTUM AND COLLISIONS

Example 32

(a) Kipanga says: *"When a moving car brakes to a stop, its momentum is destroyed."* Is Kipanga correct? Explain briefly.

(b) A cricket ball of mass 160g is bowled horizontally at 25m/s toward a batsman. The batsman hits it straight back toward the bowler at 30m/s. The ball is in contact with the bat for 0.01s. Calculate:
   (i) the change in momentum of the ball,
   (ii) the average force exerted by the bat on the ball,
   (iii) the change in kinetic energy of the ball.
   (iv) Using your answers in (i), (ii), and (iii), explain whether the collision between bat and ball is elastic or inelastic, giving clear reasoning.

Solution

(a) Kipanga is not correct.

Explanation:

Momentum is not destroyed. When a car brakes, friction between the tyres and the road exerts a force that reduces the car's momentum. At the same time, the Earth gains an equal amount of momentum in the opposite direction. Thus, the total momentum of the car–Earth system remains constant.

(b) The solution of each part is as follows:
(i) Taking initial direction as positive:

$$\Delta p = m(v - u) = 0.16\text{kg}(-30 - 25)\text{m/s} = -8.8\text{kgm/s}$$

The change in momentum is -8.8kgm/s (negative indicates change is opposite to initial direction).

(ii) The average force is given by:

$$F_{avg} = \frac{\Delta p}{\Delta t} = -\frac{8.8\text{kgm/s}}{0.01\text{s}} = -880\text{N}$$

The average force is 880N (directed opposite to the ball's initial motion).

(iii) Calculating initial and final kinetic energy:

Initial KE $= ½mu^2 = ½(0.16\text{kg})(25\text{m/s})^2 = 50\text{J}$

Final KE $= ½mv^2 = ½(0.16\text{kg})(30\text{m/s})^2 = 72\text{J}$

Change in KE = Final KE − Initial KE $= (72 – 50)\text{J} = 22\text{J}$

The kinetic energy increased by 22J.

(iv) Collision type: Inelastic.

Reasoning:

Although the kinetic energy increased (rather than decreased), this does not make the collision elastic. In an elastic collision, kinetic energy is conserved ($\Delta KE = 0$). Here, $\Delta KE = +22\text{J}$, meaning energy was added to the system. The batsman's muscles did work on the ball through the bat, adding 22J of energy. This external energy input means the collision is inelastic. The collision would only be elastic if the ball rebounded with kinetic energy equal to its initial kinetic energy with no external energy added.

Example 33

(a) Explain why a rocket can accelerate in empty space even though there is nothing to "push against."

(b) A rocket of mass 1500kg (including 1000kg fuel) is in deep space, initially at rest. The engines eject exhaust at 2500m/s relative to the rocket.

(i) Calculate the thrust force if fuel burns at 8kg/s.
(ii) Calculate the rocket's final velocity after all fuel is burned.
(iii) If instead the same rocket were launched vertically from Earth's surface with the same thrust, explain whether its final velocity would be greater, smaller, or the same as in part (ii), and why. Take $g = 9.8\text{m/s}^2$.

Solution

(a) The rocket works by conservation of momentum, not by "pushing." When exhaust gases are ejected backward, they carry backward momentum. Since total momentum must remain zero (no external forces in space), the rocket gains equal forward momentum. The rocket and exhaust push on each other (Newton's third law), not on external objects.
(b) The solution of each part is as follows:
(i) The thrust is given by:

$$F_{thrust} = u_{rel}\left(\frac{dm}{dt}\right) = 2500\text{m/s} \times 8\text{kg/s} = 20000\text{N}$$

The thrust is 20000N.

(ii) Using the integral rocket equation with $v_i = 0$:

$$v_f = u_{rel}\ln\left(\frac{m_i}{m_f}\right)$$

Substituting values:

$$v_f = 2500\text{m/s} \times \ln\left(\frac{1500\text{kg}}{500\text{kg}}\right) = 2747.5\text{m/s}$$

The final velocity is 2747.5m/s.

(iii) Final velocity: Smaller

Reasoning:

On Earth, the rocket must overcome gravity. Part of the thrust is used just to support the rocket's weight ($F = mg$), leaving less thrust available for acceleration. Additionally, during the burn time ($t = 1000/8 = 125\text{s}$), gravity continuously acts downward, causing "gravity losses." The rocket loses velocity $\Delta v \approx g \times t_{burn} \approx 9.8 \times 125 \approx 1225\text{m/s}$ due to gravity. Therefore, the final velocity would be significantly smaller than the 2747.5 m/s achieved in space.

Example 34

(a) Two objects collide and stick together. Explain whether the momentum of the system is conserved, and whether the kinetic energy is conserved.
(b) A rocket of mass 2000kg (including 1400kg fuel) is moving at 100m/s in deep space. The engines are fired, ejecting exhaust at 2800m/s relative to the rocket. Take $g = 9.8\text{ m/s}^2$.
(i) Calculate the thrust force if fuel burns at 12kg/s.
(ii) Calculate the rocket's velocity after all fuel is burned.
(iii) If instead the same rocket started from rest (with the same thrust and exhaust velocity), calculate its final velocity.
(iv) Explain why the velocity gain in part (ii) is the same as in part (iii), even though the rocket started with different initial velocities.

Solution

(a) Momentum: Conserved.
Kinetic energy: Not conserved.

Explanation:

When objects stick together (perfectly inelastic collision), momentum is conserved because momentum conservation depends only on absence of external forces, not on the collision type. However, kinetic energy is not conserved; some converts to heat, sound, and deformation during the collision.

(b) The solution of each part is as follows:

(i) The thrust is given by:

$$F_{thrust} = u_{rel}\left(\frac{dm}{dt}\right) = 2800\text{m/s} \times 12\text{kg/s} = 33600\text{N}$$

The thrust is 33600N.

(ii) Using the integral rocket equation:

$$v_f = v_i + u_{rel}\ln\left(\frac{m_i}{m_f}\right)$$

Substituting values:

$$v_f = 100\text{m/s} + 2800\text{m/s} \times \ln\left(\frac{2000\text{kg}}{600\text{kg}}\right) = 3471\text{m/s}$$

The final velocity is 3471m/s.

(iii) Starting from rest ($v_i = 0$):

$$v_f = 0\text{m/s} + 2800\text{m/s} \times \ln\left(\frac{2000\text{kg}}{600\text{kg}}\right) = 3371\text{m/s}$$

The final velocity is 3371m/s.

(iv) The velocity gain (Δv) in both cases is the same: 3371 m/s. In part (ii), the rocket went from 100m/s to 3471m/s (gain = 3371 m/s). In part (iii), it went from 0 to 3371 m/s (gain = 3371m/s). The velocity change depends only on the exhaust velocity and mass ratio ($\Delta v = u_{rel} \times \ln(m_i/m_f)$), not on the initial velocity. This is because the rocket equation describes the change in velocity, which is independent of the reference frame.

Example 35

(a) A ball bounces on the floor and rises to 80% of its original height. Explain what happened to the "missing" 20% of the ball's energy.

(b) A rubber ball is dropped from rest from a height $h_1 = 2.5\text{m}$ onto a hard floor. After bouncing, it rises to a maximum height $h_2 = 1.6\text{m}$. Take $g = 9.8\text{m/s}^2$.

- (i) Calculate the coefficient of restitution between ball and floor.
- (ii) Calculate the percentage of mechanical energy lost during the bounce.
- (iii) The ball is now dropped from a new height H such that after bouncing it rises to exactly 2.5m. Calculate H.
- (iv) Explain whether the percentage of energy lost in part (iii) is greater than, less than, or equal to the percentage lost in part (ii).

Solution

(a) The "missing" 20% of energy was converted to heat (warming the ball and floor slightly), sound (the bounce noise), and permanent deformation of the ball material. Energy was not destroyed but was transformed from organized mechanical energy (gravitational potential energy and kinetic energy) into disorganized thermal energy and other non-recoverable forms.

(b) The solution of each part is as follows:

(i) Using $v^2 = u^2 + 2as$ for free fall from rest with h_1 as initial drop height:
$v^2 = 0^2 + 2gh_1$ or $v = \sqrt{2gh_1} = u_{ball}$ (downward)

Again, using $v^2 = u^2 + 2as$ for upward motion to reach height h_2, with $a = -g, v = 0m/s$.

$0^2 = u^2 - 2gh_2$ or $u = \sqrt{2gh_2} = v_{ball}$ (upward)

$$\boldsymbol{e} = \frac{v_{ball} - v_{floor}}{u_{floor} - u_{ball}} = \frac{\sqrt{2gh_2} - 0}{\left(0 - (-\sqrt{2gh_1})\right)} = \frac{\sqrt{2gh_2}}{\sqrt{2gh_1}} = \sqrt{\frac{\mathbf{h_2}}{\mathbf{h_1}}} = \sqrt{\frac{1.6m}{2.5m}} = 0.8$$

The coefficient of restitution is 0.8.

(ii) Kinetic energy before collision = $½mv^2 = ½m(6.26^2) = 19.6m$

Kinetic energy after collision = $½mu^2 = ½m(4.85^2) = 11.76m$

Percentage of mechanical energy lost

$$= \frac{\text{Initial PE} - \text{Final PE}}{\text{Initial PE}} \times 100\%$$

$$= \frac{mgh_1 - mgh_2}{mgh_1} \times 100\% = \frac{mg(h_1 - h_2)}{mgh_1} \times 100\%$$

$$= \frac{\mathbf{h_1} - \mathbf{h_2}}{\mathbf{h_1}} \times \mathbf{100}\% = \frac{(2.5 - 1.6)m}{2.5m} \times 100\% = 36\%$$

The percentage of mechanical energy lost is 36%.

(iii) Using:

$$e = \sqrt{\frac{h_2}{h_1}}$$

Substituting:

$$0.8 = \sqrt{\frac{2.5m}{H}}$$

Solving gives: $H = 3.91m$

The required drop height is 3.91m.

(iv) Percentage lost: **Equal to part (ii)** (also 36%).

Reasoning:

The coefficient of restitution *e* is a property of the materials (ball and floor) and remains constant regardless of drop height. Since $e = 0.8$ is constant, the ratio $h_2/h_1 = e^2 = 0.64$ is always constant. This means the ball always rises to 64% of its drop height, losing 36% of its energy in every bounce, regardless of the initial height. Therefore, the percentage energy loss is the same in both cases.

Example 36

(a) Kipanga says: *"In a perfectly elastic collision between two objects, they must bounce apart with high velocity."* Is this statement always true? Explain.

(b) A 0.04kg bullet is fired horizontally at $500m/s$ into a 4kg wooden block suspended by a string of length 2m. The bullet embeds in the block and the block swings upward.
Take $g = 9.8m/s^2$.
  (i) Calculate the velocity of the block immediately after the bullet embeds.

(ii) Calculate the maximum height the block rises above its initial position.
(iii) Calculate the maximum angle the string makes with the vertical.
(iv) If the string length were doubled to 4m but everything else remained the same, explain whether the maximum angle would be greater, smaller, or the same.

Solution

(a) Statement: Not always true.

Explanation:

In a perfectly elastic collision where one object has much greater mass than the other (e.g., tennis ball hitting a wall), the lighter object may bounce back at high velocity while the heavier object experiences negligible motion.

Also, if two objects of equal mass collide elastically and one is initially at rest, the moving object stops completely while the stationary object moves at the original velocity and thus the first object has zero final velocity, not "high velocity."

In conclusion, an elastic collision means that kinetic energy is conserved; it does not necessarily imply that the final velocity must be large.

(b) The solution of each part is as follows:
(i) Let m and M denote the masses of the bullet and the block respectively.

By conservation of linear momentum:

Momentum before collision = Momentum after collision

$mu_{bullet} = (m + M)v$

From which:

$$v = \frac{mu_{bullet}}{m + M} = \frac{0.04kg \times 500m/s}{(0.04 + 4)kg} = 4.95m/s$$

The velocity immediately after embedding is $4.95m/s$.

(ii) By conservation of mechanical energy:

Total mechanical energy at the bottom = Total mechanical energy at maximum height

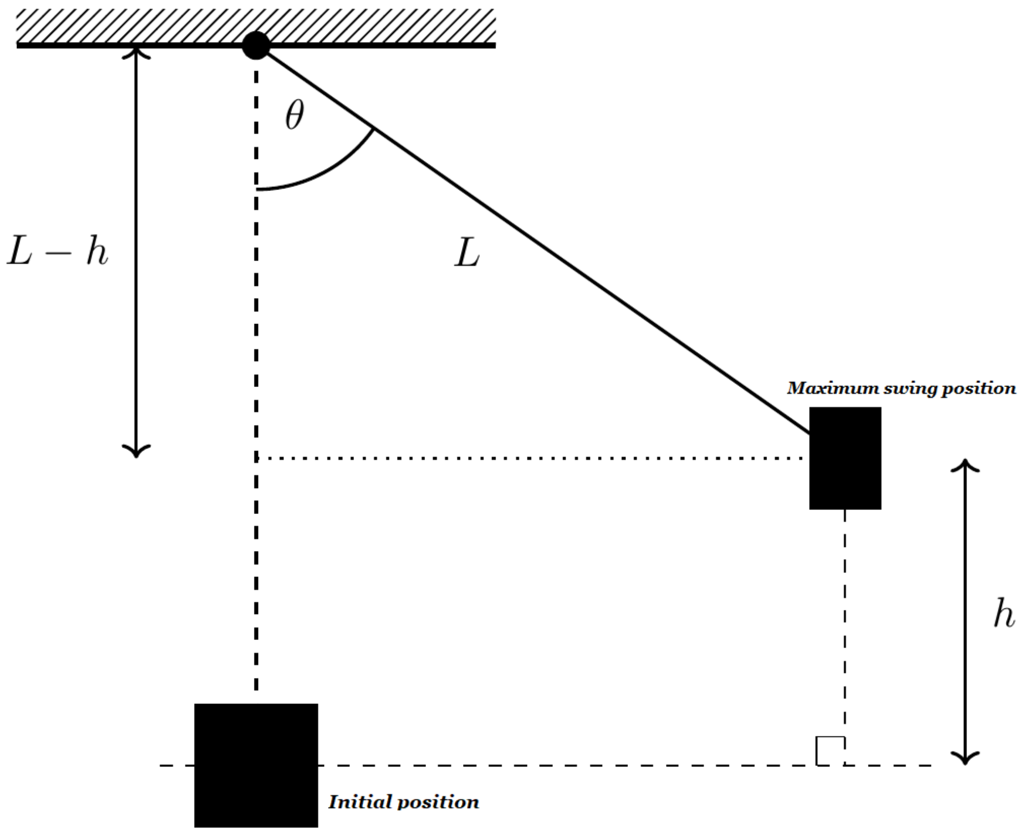
$\frac{1}{2}(m + M)v^2 = (m + M)gh$

Cancelling $m + M$:

$$v^2 = 2gh \text{ or } h = \frac{v^2}{2g} = \frac{(4.95m/s)^2}{2 \times 9.8\ m/s^2} = 1.25m$$

The maximum height is $1.25m$.

(iii) Consider the following diagram:



From geometry of the right-angled triangle:

$$\cos\theta = \frac{L - h}{L} = \frac{(2 - 1.25)m}{2m} = 0.375$$

$\theta = \cos^{-1}(0.375) = 68°$

The maximum angle is approximately 68° from vertical.

(iv) Maximum angle: Smaller.

Reasoning:

The maximum height h depends only on the velocity after embedding, which is determined by momentum conservation and remains constant at $h = 1.25m$ regardless of string length.

With doubled string length $L = 4m$, $\cos\theta = \frac{4-1.25}{4} = 0.6875;\ \theta = 47°$

Therefore, the maximum angle decreases from 68° to 47° because the same vertical height corresponds to a smaller angle when the string is longer.

Example 37

(a) Kipute says: *"When two objects collide, the lighter object always experiences greater acceleration than the heavier object."* Is Kipute's statement correct? Explain.

(b) A 0.05kg bullet is fired horizontally into a 3kg wooden block hanging by a string of length 1m. The bullet embeds in the block, and the block swings to a maximum angle of 60° from the vertical. Take $g = 9.8\ m/s^2$.

Calculate:

(i) The height the block rises.
(ii) The velocity of the block immediately after the bullet embeds.
(iii) The initial velocity of the bullet.

Solution

(a) Statement: Correct.

Explanation:

During collision, both objects experience equal magnitude forces (Newton's third law). Since F=ma, for equal forces, acceleration is inversely proportional to mass ($a = F/m$). The lighter object has smaller mass; therefore, it experiences greater acceleration. This is always true during any collision.

(b) The solution of each part is as follows:
(i) Again from:

$$\cos\theta = \frac{L - h}{L};$$

Making h the subject:

$$\mathbf{h} = L - L\cos\theta = \mathbf{L(1 - \cos\theta)}$$

Substituting values:

$$h = 1m(1 - \cos 60°) = 0.5m$$

The height is 0.5m.

(ii) By conservation of mechanical energy:

Total mechanical energy at the bottom = Total mechanical energy at maximum height

$½(m + M)v^2 = (m + M)gh$

Cancelling $m + M$:

$v^2 = 2gh$ or $v = \sqrt{2gh} = \sqrt{2 \times 9.8\ m/s^2 \times 0.5m} = 3.13m/s$

The velocity is $3.13m/s$.

(iii) By conservation of linear momentum:

Momentum before collision = Momentum after collision

$mu_{bullet} = (m + M)v$

From which:

$u_{bullet} = \frac{(m + M)v}{m} = \frac{(0.05 + 3)kg \times 3.13m/s}{0.05kg} = 191m/s$

The initial velocity of bullet is $191m/s$.

Example 38

(a) When a gun fires a bullet, both the bullet and the gun experience the same magnitude of force. Explain why the bullet accelerates much more than the gun.
(b) A 5kg rifle fires a 0.025kg bullet horizontally. The bullet leaves the barrel at $400m/s$.
 (i) Calculate the recoil velocity of the rifle.
 (ii) Calculate the ratio of kinetic energies (bullet : rifle).

(iii) If the rifle is brought to rest in $0.08s$ by the shooter's shoulder, calculate the average force exerted by the shoulder.

(iv) Explain why most of the chemical energy from the gunpowder goes into the bullet's kinetic energy rather than the rifle's kinetic energy, even though they have equal momenta.

Solution

(a) Both experience equal forces (Newton's third law), but the bullet has much smaller mass than the gun. Since $F = ma$, for the same force F, acceleration a is inversely proportional to mass, m. The bullet's much smaller mass results in much greater acceleration ($a = F/m$), while the gun's large mass results in small acceleration.

(b) The solution of each part is as follows:

(i) By conservation of momentum:

$$0 = m_{bullet}v_{bullet} + m_{rifle}v_{recoil}$$

From which:

$$v_{recoil} = \frac{-m_{bullet}v_{bullet}}{m_{rifle}} = \frac{-0.025\text{kg} \times 400\text{m/s}}{5\text{kg}} = -2\text{m/s}$$

The recoil velocity is 2m/s backward.

(ii) The ratio is given by:

$$\text{Ratio} = \frac{\text{KE of bullet}}{\text{KE of rifle}} = \frac{\frac{1}{2}m_b v_b^2}{\frac{1}{2}m_r v_r^2} = \frac{½(0.025\text{kg})(400\text{m/s})^2}{½(5\text{kg})(2\text{m/s})^2} = 200$$

The ratio of kinetic energies is $200:1$ (bullet has 200 times more kinetic energy).

Alternative solution:

$$KE = \frac{1}{2}mv^2 = \frac{1}{2}mv^2 \times \left(\frac{m}{m}\right) = \frac{\frac{1}{2}m^2v^2}{m} = \frac{(mv)^2}{2m} = \frac{p^2}{2m}$$

So kinetic energy and momentum are related by the following equation:

$$\mathbf{KE = \frac{p^2}{2m}}$$

Since both bullet and rifle have the same momentum:

$$\frac{(\mathbf{KE})_\mathbf{b}}{(\mathbf{KE})_\mathbf{r}} = \frac{p^2}{2m_b} \div \frac{p^2}{2m_r} = \frac{p^2}{2m_b} \times \frac{2m_r}{p^2} = \frac{\mathbf{m_r}}{\mathbf{m_b}} = \frac{5\text{kg}}{0.025\text{kg}} = 200$$

(iii) Using the impulse-momentum theorem:

Impulse = change in momentum

$$F \times \Delta t = 0 - m_{rifle}v_{recoil};\ F = \frac{-m_{rifle}v_{recoil}}{\Delta t} = \frac{-5\text{kg} \times (-2\text{m/s})}{0.08\text{s}} = 125\text{N}$$

The average force exerted by the shoulder is 125N (opposing the recoil).

(iv) Although bullet and rifle have equal magnitude momenta ($p = mv$), kinetic energy depends on both mass and velocity squared $\left(KE = ½mv^2 = \frac{p^2}{2m}\right)$. For equal momentum p, kinetic energy is inversely proportional to mass. Since the bullet has much smaller mass (200 times smaller), it receives much more kinetic energy (200 times more) for the same momentum. The rifle's large mass means most of its momentum comes from small velocity, resulting in small kinetic energy. Therefore, the bullet carries most of the chemical energy despite equal momenta.

Example 39

(a) Rain falls vertically onto a moving car. Explain why the rain exerts a horizontal force on the car even though the rain falls vertically.

(b) Rain falls vertically onto a plane roof, 2m square, which is inclined to the horizontal at an angle of 40°. The raindrops strike the roof with a vertical velocity of 4m/s, and a volume of $3 \times 10^{-2}\text{m}^3$ of water is collected from the roof in one minute. Assuming that the rain conditions are steady and that the velocity of the raindrops after impact is zero, calculate:

(i) the mass of water striking the roof per second,
(ii) the vertical force exerted on the roof by the impact of the falling rain,
(iii) the component of this force normal to the roof surface,
(iv) the pressure exerted normal to the roof due to the impact of the rain.

(The density of water is 1000kg/m^3)

Solution

(a) The rain gains horizontal momentum equal to the car's velocity when it lands, requiring the car to exert a horizontal force on the rain, and by Newton's third law, the rain exerts an equal horizontal force on the car.

(b) Interpreting the data:

Roof area: $A = 2\text{m} \times 2\text{m} = 4\text{m}^2$

Angle of inclination: $\theta = 40°$

Rain velocity: $v = -4\text{m/s}$ (vertical, downward)

Volume collected: $V = 3.0 \times 10^{-2}\text{m}^3$ in $t = 60\text{s}$

Final velocity of rain: 0 (rain sticks to roof)

Density of water: $\rho = 1000\text{kg/m}^3$

(i) Mass per second

$$= \frac{\text{Total mass collected}}{\text{Time taken in s}} = \frac{\rho V}{t} = \frac{10001000\text{kg/m}^3 \times 3 \times 10^{-2}\text{m}^3}{60s} = 0.5\text{kg/s}$$

The mass of water striking the roof per second is $0.5\text{kg/}s$.

(ii) The rain moves downward with velocity v = 4m/s.

$$F_{\text{vertical}} = \Delta v\left(\frac{dm}{dt}\right) = \left(0 - (-4\text{m/s})\right) \times 0.5\text{kg/s} = +2\text{N}$$

It is **positive** because the roof exerts an **upward force on the rain** to stop its downward motion.

By Newton's third law, the rain exerts an equal magnitude of downward force on the roof.

Hence, the vertical force on the roof by rain is 2N (downward).

(iii) The vertical force can be resolved into components parallel and perpendicular to the roof.

$$F_{\text{normal}} = F_{\text{vertical}} \times \cos\theta = 2\text{N} \times \cos 40° = 1.53\text{N}$$

The force component normal to the roof is 1.53N.

(iv) Pressure normal to roof is given by:

$$P = \frac{F_{\text{normal}}}{A} = \frac{1.53\text{N}}{4\text{m}^2} = 0.38\text{N/m}^2$$

The pressure exerted normal to the roof is 0.38N/m^2.

Example 40

(a) A stone is thrown horizontally from a cliff. Explain why the stone's horizontal momentum is conserved during its flight, but its vertical momentum is not conserved.
(b) A ball of mass 0.5kg is thrown vertically upward from ground level with initial velocity 20m/s. Take $g = 9.8\text{m/s}^2$ and upward as positive direction.
   (i) Calculate the momentum of the ball at the instant it is thrown.
   (ii) Calculate the momentum of the ball when it reaches its maximum height.
   (iii) Calculate the change in momentum from launch to maximum height.
   (iv) The ball returns to ground level and is caught by the thrower. Calculate the total change in momentum from the moment of throw to the moment of catch.

Solution

(a) Momentum is conserved only when no net external force acts. Horizontally, no forces act (neglecting air resistance), so horizontal momentum remains constant throughout flight. Vertically, gravity acts continuously downward, exerting an external force that changes the vertical momentum.
(b) The solution of each part is as follows:
(i) $p_{\text{initial}} = mu = 0.5\text{kg} \times 20\text{m/s} = 10\text{kgm/s}$

The initial momentum is $+10$ kgm/s.

(ii) At maximum height, v = 0:

$p_{\text{max}} = mv = 0.5\text{kg} \times 0\text{m/s} = 0\text{kgm/s}$

The momentum at maximum height is zero.

(iii) $\Delta p = p_{\text{max}} - p_{\text{initial}} = (0 - 10)\text{kgm/s} = -10\text{kgm/s}$

The change in momentum is -10kgm/s.

(iv) When the ball returns to ground level, by symmetry of vertical motion, it has velocity of $v = -20\text{m/s}$ (downward):

$p_{\text{final}} = mv = 0.5\text{kg} \times (-20\text{m/s}) = -10\text{kgm/s}$

Total change in momentum:

$\Delta p_{\text{total}} = p_{\text{final}} - p_{\text{initial}} = -10\text{kgm/s} - (+10\text{kgm/s}) = -20\ \text{kgm/s}$

The total change in momentum from throw to catch is -20 kgm/s.

Having enjoyed the full combination of ideas, it is time to sharpen our thinking; the Digging Deeper Exercise is ready in the next Module.