COLLISIONS

We have now developed a complete understanding of momentum and its conservation. We have learned that when objects interact in an isolated system, the total momentum before equals the total momentum after, regardless of how violent or gentle the interaction.

But everyday experience shows us that collisions come in many varieties:

- When two billiard balls collide, they bounce apart crisply with sharp clicking sounds.
- When two lumps of clay collide, they stick together and move as one.
- When a tennis ball bounces off a wall, it returns at nearly the same speed.
- When a ball of dough hits a wall, it sticks and does not bounce at all.

All these collisions obey conservation of momentum but they look and feel completely different. *So what distinguishes one type of collision from another?*

The answer lies in what happens to **kinetic energy** during the collision. But before going to that, we have to understand first what is collision. But before we explore how kinetic energy behaves, we must first ask a more fundamental question:

What exactly is a collision?

A **collision** *is an event in which two or more objects come into contact with each other for a short time, exerting relatively large forces on each other*.

A collision is typically identified by the following characteristics:

1. Brief duration

Collisions typically last from microseconds (atomic collisions) to milliseconds (e.g. vehicle collisions). This short time is crucial.

2. Large forces

During collision, the forces between objects are much larger than any external forces (like friction or gravity). This is why we can often treat momentum as conserved even when small external forces exist.

3. Localised interaction

Objects are in direct contact (or very close, in the case of atomic/molecular collisions).

Now, ***why do we study collisions?***

Collisions are fundamental to understanding:

- Vehicle safety design (crumple zones, airbags).
- Sports (hitting balls, tackling in rugby).
- Particle physics (understanding matter at atomic scale).
- Astronomy (formation of planets, asteroid impacts).
- Engineering (impact testing, materials science).

Classification of Collisions

Collisions are classified according to what happens to **kinetic energy** during the interaction. In an isolated system, momentum is **always** conserved. However, kinetic energy may or may not be conserved. This difference in the behaviour of kinetic energy is what distinguishes one type of collision from another.

On this basis, collisions are commonly discussed in three forms:

1) Elastic collisions
2) Inelastic collisions
3) Perfectly inelastic collisions *(a special case of **inelastic collisions** in which the bodies stick together)*

Elastic Collisions

An **elastic collision** *is one in which both momentum and kinetic energy are conserved.* It is characterised by the following features:

- Objects bounce apart after collision.
- No energy is converted to heat, sound, or permanent deformation.
- Total kinetic energy before = Total kinetic energy after.
- Objects may exchange kinetic energy, but the total remains constant.

Examples of approximately elastic collision include:

1. **Steel ball bearings colliding:** Very little energy lost to deformation.
2. **Billiard balls:** Slight energy loss to sound and deformation, but close to elastic.
3. **Atomic and molecular collisions in gases:** Perfectly elastic at atomic scale.
4. **Super balls bouncing:** High elasticity materials.

However, the reader should note that perfectly elastic collisions are only idealisation. In reality, nor collision is perfect elastic; all collisions lose some energy to heat, sound, or deformation. Nevertheless, many collisions are "nearly elastic" where energy loss is small enough to ignore.

Mathematical conditions for elastic collisions:

Conservation of momentum:

$m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2$ ... (equation 1)

Conservation of kinetic energy:

$\frac{1}{2}m_1u_1^2 + \frac{1}{2}m_2u_2^2 = \frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2$... (equation 2)

These two equations can be solved simultaneously to find the final velocities v_1 and v_2.

Also rearranging equation 1:

$$m_1u_1 - m_1v_1 = m_2v_2 - m_2u_2$$

From which:

$$m_1(u_1 - v_1) = m_2(v_2 - u_2) \text{ ... (equation 3)}$$

From equation 2 (multiplying both sides by 2):

$$m_1u_1^2 + m_2u_2^2 = m_1v_1^2 + m_2v_2^2$$

Rearranging:

$$m_1u_1^2 - m_1v_1^2 = m_2v_2^2 - m_2u_2^2$$

$$m_1(u_1^2 - v_1^2) = m_2(v_2^2 - u_2^2)$$

Factorising:

$$m_1(u_1 - v_1)(u_1 + v_1) = m_2(v_2 - u_2)(v_2 + u_2) \text{ ... (equation 4)}$$

Dividing equation (4) by equation (3) gives:

$$u_1 + v_1 = v_2 + u_2$$

Rearranging:

$$\mathbf{u}_1 - \mathbf{u}_2 = -(\mathbf{v}_1 - \mathbf{v}_2) \text{ or } \mathbf{u}_1 - \mathbf{u}_2 = \mathbf{v}_2 - \mathbf{v}_1$$

In words: *The relative velocity of approach before collision equals the relative velocity of separation after collision (but in opposite direction).*

Inelastic Collisions

An **inelastic collision** *is one in which momentum is conserved but kinetic energy is* ***not*** *conserved.* It is characterised by the following features:

- Some kinetic energy is converted to other forms (heat, sound, deformation, vibrations).
- Total kinetic energy after collision < Total kinetic energy before collision.
- Momentum is conserved (as always).
- Objects may or may not stick together.

Examples of inelastic collisions include:

1. **Car crashes:** Crumpling metal, breaking glass, heat, sound (a lot of kinetic energy converted to other forms).
2. **Dropping a ball of dough:** Sticks to floor, does not bounce.
3. **Bullet embedding in wood:** Kinetic energy is converted to heat and deformation.
4. **Tackling in rugby:** Players move together after collision.

Mathematical conditions for elastic collisions:

Conservation of momentum (still applies):

$$m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2$$

Kinetic energy is not conserved:

$$½m_1u_1^2 + ½m_2u_2^2 > ½m_1v_1^2 + ½m_2v_2^2$$

So in an inelastic collision, the total kinetic energy decreases. This happens because part of the initial kinetic energy is converted into:

1. **Heat:** Friction and deformation generate thermal energy.
2. **Sound:** The "thud" or "crash" of collision carries energy as sound waves.
3. **Permanent deformation:** Bending, crumpling, breaking of objects.
4. **Internal vibrations:** Objects vibrate after impact.

Perfectly Inelastic Collisions

A **perfectly inelastic collision** *is a special type of inelastic collision in which* ***objects stick together after collision*** *and move with a common final velocity after impact.* This situation results in the maximum possible loss of kinetic energy while still conserving momentum.

It is therefore important to distinguish between a general inelastic collision and a perfectly inelastic one:

- In an **inelastic collision**, the objects may either separate after impact or stick together, but some kinetic energy is always lost.
- In a **perfectly inelastic collision**, the objects must stick together and move as a single body. This represents the greatest possible loss of kinetic energy for the given initial conditions.

Examples of perfectly inelastic collisions include:

1. A **bullet embedding in a block of wood**, where the bullet remains lodged in the block after impact.
2. **Railway wagons coupling together**, moving as a single unit after they collide.
3. **Clay balls colliding and sticking**, forming one combined mass.
4. A **meteorite striking and merging with a celestial body**, becoming part of it.
5. **Catching a moving ball and continuing to move with it**, where the ball and catcher momentarily share a common velocity.

Mathematical conditions for perfectly inelastic collisions:

After collision: $\mathbf{v}_1 = \mathbf{v}_2 = \mathbf{v}$ (common velocity)

Conservation of momentum:

$$m_1u_1 + m_2u_2 = (m_1 + m_2)v$$

Kinetic energy is not conserved (maximum loss in K.E):

$$½m_1u_1^2 + ½m_2u_2^2 > ½(m_1 + m_2)v^2$$

Coefficient of Restitution

We have classified collisions as either elastic or inelastic. However, in real life, most collisions are not perfectly elastic and not perfectly inelastic either. They fall somewhere in between.

Some collisions bounce strongly. Others barely rebound at all. So instead of asking only *"Is this collision elastic or inelastic?"*, we should ask a better question:

How elastic is the collision?

To answer this, we introduce a numerical measure called the **coefficient of restitution**, represented by the symbol ***e***.

The coefficient of restitution tells us how much "bounciness" a collision has. It allows us to measure elasticity on a scale, rather than simply labeling a collision as one type or the other.

To understand the coefficient of restitution more precisely, we focus on what happens during the collision itself.

Before the collision, the two objects move toward each other with a certain **relative velocity of approach**. After the collision, if they separate, they move away from each other with a certain **relative velocity of separation**.

The coefficient of restitution (***e***.), is defined as:

The ratio of the relative velocity of separation to the relative velocity of approach.

In mathematical form:

$$\boldsymbol{e} = \frac{\textbf{relative velocity of separation}}{\textbf{relative velocity of approach}}$$

The coefficient of restitution, *e*, tells us what fraction of the relative speed of approach is recovered as relative speed of separation after the collision. In other words, it measures how much of the "closing speed" before impact reappears as "rebounding speed" after impact.

The value of *e* therefore reveals the nature of the collision. Specifically, if:

- If $\boldsymbol{e} = \mathbf{1}$, relative velocity of separation = relative velocity of approach. The collision is **perfectly elastic**.
- If $\boldsymbol{e} = \mathbf{0}$, relative velocity of separation $= 0$ (there is no separation after collision as the objects move together). The collision is **perfectly inelastic**.
- If $\mathbf{0} < \boldsymbol{e} < \mathbf{1}$, relative velocity of separation $<$ relative velocity of approach. The collision is **inelastic (partially elastic).**

Understand how to apply *e* in calculations

When calculating the value of e, we use the magnitudes of the relative velocities. This ensures that the coefficient of restitution is always a positive quantity.

For **one-dimensional** collisions:

Magnitude of relative velocity of separation $= v_2 - v_1$

Magnitude of relative velocity of approach $= u_1 - u_2$

Therefore:

$$\boldsymbol{e} = \frac{v_2 - v_1}{u_1 - u_2}$$

With the restitution equation now introduced, we have three fundamental equations available for analysing collisions:

1) **Conservation of momentum (always):**

$$m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2$$

2) **Coefficient of restitution:**

$$\boldsymbol{e} = \frac{v_2 - v_1}{u_1 - u_2}$$

3) **Conservation of energy (only if e = 1):**

$$½m_1u_1^2 + ½m_2u_2^2 = ½m_1v_1^2 + ½m_2v_2^2.$$

For elastic collisions ($e = 1$), we can use equations 1 and 3, or equations 1 and 2.

For inelastic collisions ($e < 1$), we must use equations 1 and 2.

Warning!

The relation $e = (v_2 - v_1) / (u_1 - u_2)$ is valid **only for one-dimensional collisions**, where all velocities lie along the same straight line (the line of impact). It must **not** be applied directly in two-dimensional collisions. In such cases, the coefficient of restitution applies only to the components of velocity **along the line of impact**, not to the total velocities.

Summary of Collision Types

Type	**Momentum Conserved?**	**KE Conserved?**	**Coefficient *e***	**After Collision**
Elastic	Yes	Yes	$e = 1$	Objects separate
Inelastic	Yes	No (Partial loss)	$0 < e < 1$	Objects may separate
Perfectly Inelastic	Yes	No (Maximum loss)	$e = 0$	Objects stick together

With this comprehensive theoretical foundation firmly established, we are now ready to bring these concepts to life through carefully crafted worked examples that demonstrate how to apply collision principles to solve real problems.

BINDER Example 12

Two balls A and B collide head-on. Ball A has mass 2kg and moves at 6m/s. Ball B has mass 3kg and moves at 4m/s in the opposite direction. After collision, ball A rebounds with a velocity of 2m/s.

(a) Calculate the velocity of ball B after collision.
(b) Determine whether the collision is elastic or inelastic.

Solution

Interpreting the data:

Mass of A: $m_A = 2kg$

Mass of B: $m_B = 3kg$

Initial velocity of A: $u_A = 6m/s$ (take as positive direction)

Initial velocity of B: $u_B = -4m/s$ (opposite direction, negative)

Final velocity of A: $v_A = -2m/s$ (rebounds, negative)

Final velocity of B: v_B (to find)

(a) Applying conservation of momentum:

$$m_A u_A + m_B u_B = m_A v_A + m_B v_B$$

Substituting values:

$$2kg \times 6m/s + 3kg \times -4m/s = 2kg \times -2m/s + 3kg \times v_B$$

On solving gives; $v_B = 1.33m/s$

(b) Comparing initial and final kinetic energy:

Initial kinetic energy:

$$(K.E)_{\text{before collision}} = ½m_A u_A^2 + ½m_B u_B^2 = ½(2kg)(6m/s)^2 + ½(3kg)(-4m/s)^2 = 60J$$

Final kinetic energy:

$$(K.E)_{\text{after collision}} = ½m_A v_A^2 + ½m_B v_B^2 = ½(2kg)(-2m/s)^2 + ½(3kg)(1.33m/s)^2 = 6.7J$$

Since Initial kinetic energy $(60J) \neq$ Final kinetic energy $(6.7J)$, kinetic energy is not conserved and hence, the collision is inelastic.

Making Sense of the Answer: *Ball A reversed direction (from +6 m/s to −2m/s) while ball B also reversed direction (from −4 m/s to +1.33m/s). Both balls bounced backward from their original directions, which is typical in head-on collisions. The large energy loss (89% of initial KE) indicates that the collision is very inelastic (like collision of two soft rubber balls or balls covered in felt).*

Think Like a Physicist: *Never assume a collision is elastic just because objects bounce apart. Always verify by comparing kinetic energies. Momentum is conserved in all collisions (it must be), but kinetic energy is only conserved in elastic collisions.*

BINDER Example 13

A ball of mass 0.5kg moving at 10m/s collides elastically with a stationary ball of mass 1.5kg. Calculate the velocities of both balls after collision.

Solution

Interpreting the data:

Mass of ball 1: $m_1 = 0.5kg$

Mass of ball 2: $m_2 = 1.5kg$

Initial velocity of ball 1: $u_1 = 10m/s$

Initial velocity of ball 2: $u_2 = 0\ m/s$ (at rest)

$e = 1$ (balls collided **elastically**)

Final velocities: v_1, v_2 (to find)

Applying conservation of momentum:

$m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2$

$0.5 \times 10 + 1.5 \times 0 = 0.5v_1 + 1.5v_2$

$0.5v_1 + 1.5v_2 = 5 \ldots (1)$

Using coefficient restitution equation:

$$e = 1 = \frac{v_2 - v_1}{u_1 - u_2}$$

Thus:

$v_2 - v_1 = u_1 - u_2$

$v_2 - v_1 = 10 - 0$

$v_2 - v_1 = 10 \ldots (2)$

Solving equation (1) and (2) simultaneously gives: $v_1 = -5\text{m/s}, v_2 = 5\text{m/s}$

After collision, ball 1 moves at 5m/s backward (opposite to original direction), and ball 2 moves at 5m/s forward (in ball 1's original direction).

Making Sense of the Answer: *Ball 1 (lighter, moving) hit ball 2 (heavier, stationary) and bounced backward at half its original velocity. Ball 2 moved forward at half of ball 1's original velocity. The heavier ball "won" the collision, sending the lighter ball backward. Despite this, kinetic energy was perfectly conserved (the energy was transferred from ball 1 to ball 2).*

Think Like a Physicist: *When a light object collides elastically with a heavier stationary object, the lighter object typically rebounds. As the mass of the second object becomes much larger, the rebound becomes more pronounced. In the limiting case where the second mass is effectively infinite, such as a ball striking a rigid wall, the lighter object bounces back with nearly the same velocity but in the opposite direction.*

REAL Example 14

A 60kg student running at 5m/s jumps onto a 20kg stationary skateboard and continues moving together with it after landing.

(a) Calculate the velocity of the student–skateboard system immediately after landing.
(b) Calculate the kinetic energy lost during the collision.
(c) Where did the "lost" kinetic energy go? Explain.

Solution

(a) Conservation of momentum:

$$m_s u_s + m_b u_b = (m_s + m_b)v$$

Where: *s* stands for student and *b* stands for skateboard.

Substituting given values:

$$60\text{kg} \times 5\text{m/s} + 20\text{kg} \times 0\text{m/s} = (60\text{kg} + 20\text{kg})v; v = 3.75\text{m/s}$$

The velocity is 3.75m/s.

(b) Kinetic energy lost is given by the following equation:

$$(\text{K.E})_{\text{lost}} = ½m_1u_1^2 + ½m_2u_2^2 - ½(m_1 + m_2)v^2$$

Substituting:

$$(\text{K.E})_{\text{lost}} = ½(60\text{kg})(5\text{m/s})^2 + 0 - ½(60 + 20)\text{kg} \times (3.75\text{m/s})^2 = 187.5\text{J}$$

The kinetic energy lost is 187.5J.

(c) The 187.5J of "lost" kinetic energy was converted into:

1. **Heat:** Friction between the student's feet and skateboard surface generated thermal energy.
2. **Sound:** The "thud" of landing produced sound waves carrying energy away.
3. **Deformation:** Small compression of skateboard deck and student's shoe soles.
4. **Vibrations:** The skateboard vibrated after impact, eventually dissipating as heat through internal friction.

Making Sense of the Answer: *The student slowed from 5m/s to 3.75m/s (25% reduction) by sharing momentum with the skateboard. This 25% speed reduction resulted in a 25% energy loss (187.5J out of 750J). This is typical for perfectly inelastic collisions; significant kinetic energy loss even though momentum is conserved.*

Think Like a Physicist: *Perfectly inelastic collisions always have the maximum possible energy loss while still conserving momentum. If the student had jumped onto a moving skateboard going in the opposite direction, the energy loss would be even greater. This is why crashes between vehicles moving in opposite directions are more dangerous as they dissipate more energy.*

REAL Example 15

During a physics demonstration, Mr. Akilikubwa sets up a **Newton's cradle**: five identical steel balls suspended by strings so they can swing and collide.

He pulls back one ball and releases it. It swings down and hits the row of stationary balls. Surprisingly, only the ball at the far end swings up, while the other balls remain almost stationary.

Kipanga: *"I don't understand! Why didn't all the balls move a little bit? Wouldn't that share the momentum?"*

Kipute: *"But if all balls moved together, that would be perfectly inelastic. These are **steel balls**, they bounce elastically!"*

Mr. Akilikubwa: *"Excellent thinking, Kipute! Since the balls collide elastically, let's analyse this with physics."*

Setup:

- 5 identical steel balls, each mass m
- Ball 1 swings in at velocity u
- Balls 2, 3, 4, 5 initially at rest
- After collision, ball 5 swings out at velocity v
- Balls 1, 2, 3, 4 remain nearly at rest

Mr. Akilikubwa asks: *"What must be the velocity of ball 5?"*

Now, assist Kipute and Mr. Akilikubwa in answering the question.

Solution

Conservation of momentum:

$$\text{Total initial momentum} = \text{Total final momentum}$$

$m \times u + m \times 0 + m \times 0 + m \times 0 + m \times 0 = m \times 0 + m \times 0 + m \times 0 + m \times 0 + m \times v$

From which: $mu = mv$

Therefore: $\mathbf{v = u}$

Conservation of kinetic energy:

Since the collision is elastic:

$$\text{Initial kinetic energy} = \text{Final kinetic energy}$$

$\frac{1}{2}mu^2 = \frac{1}{2}mv^2$

This also gives: $\mathbf{v = u}$

Both conservation laws give the same answer: ball 5 must leave with exactly the same velocity that ball 1 arrived with.

Hence, Ball 5 swings out at velocity $v = u$ (same velocity as ball 1 came in).

Making Sense of the Answer: *Newton's cradle demonstrates that elastic collisions between equal masses produce a unique result: complete energy and momentum transfer. The incoming ball stops; the outgoing ball moves at the incoming velocity. The middle balls act as perfect transmitters, passing the energy along without keeping any. This only works because the balls are identical and the collision is elastic.*

Think Like a Physicist: *Newton's cradle demonstrates a powerful principle: for elastic collisions between identical masses, applying **both** conservation laws (momentum and energy) simultaneously forces one unique outcome whereby the moving ball stops completely and the stationary ball moves at the original velocity. If momentum alone were considered, we could imagine all five balls moving together slowly (momentum conserved), but this would violate energy conservation. If only energy mattered, we could imagine the first ball continuing while others moved differently (energy conserved), but this would violate momentum conservation.*

Only the one-in-one-out pattern satisfies both laws, and this is why Newton's cradle is such a popular physics demonstration; it makes conservation laws visible!

HOT Example 16

A ball is dropped from a height of 2m onto a hard floor. It rebounds to a height of 1.2m. Calculate:

(a) The velocity just before hitting the floor.
(b) The velocity just after leaving the floor.
(c) The coefficient of restitution between the ball and floor.
(d) The percentage of kinetic energy lost in the collision

Take g = 9.8m/s²

Solution

(i) Using $v^2 = u^2 + 2as$ for free fall from rest with h_1 as initial drop height:
$v^2 = 0^2 + 2gh_1$ or $v = \sqrt{2gh_1} = \sqrt{2 \times 9.8\text{m/s}^2 \times 2\text{m}} = 6.26\text{m/s}$ (downward)

The velocity is 6.26m/s (downward)

(ii) Using $v^2 = u^2 + 2as$ for upward motion to reach height h_2, with $a = -g, v = 0\text{m/s}$.

$0^2 = u^2 - 2gh_2$ or $u = \sqrt{2gh_2} = \sqrt{2 \times 9.8\text{m/s}^2 \times 1.2\text{m}} = 4.85\text{m/s}$ (upward)

The velocity is 4.85m/s (upward).

(iii) Taking upward as positive:

Before collision: $u_{\text{ball}} = -6.26\text{m/s}$ (downward)

Before collision: $u_{\text{floor}} = 0\text{m/s}$ (floor is stationary)

After collision: $v_{\text{ball}} = +4.85\text{m/s}$ (upward)

After collision: $v_{\text{floor}} = 0\text{m/s}$ (floor still stationary)

$$e = \frac{v_{\text{ball}} - v_{\text{floor}}}{u_{\text{floor}} - u_{\text{ball}}} = \frac{(4.85 - 0)\text{m/s}}{(0 - (-6.26))\text{m/s}} = 0.77$$

The coefficient of restitution is 0.77.

(iv) Kinetic energy before collision = $½mv^2 = ½m(6.26^2) = 19.6m$

Kinetic energy after collision = $½mu^2 = ½m(4.85^2) = 11.76m$

Percentage of kinetic energy lost = $\frac{\text{Kinetic energy lost}}{\text{Kinetic energy before collision}} \times 100\%$

$$= \frac{(19.6m - 11.76m)}{19.6m} \times 100\% = 40\%$$

The percentage is 40%.

Making Sense of the Answer:

The ball rebounded to 60% of its original height ($1.2m/2m = 0.6$*). Since height is proportional to mechanical energy, the rebound to 60% of the original height means the ball retains 60% of its energy and loses* ***40%.*** *However, velocity is proportional to the square root of height, which is why the coefficient of restitution is* $\boldsymbol{e} = \sqrt{\mathbf{0.6}} = \mathbf{0.77}$*, not 0.6.*

Think Like a Physicist: *The "lost" 40% of kinetic energy went into heat (ball and floor warmed slightly), sound (the bounce made noise), and small permanent deformations. If we dropped the ball repeatedly, it would bounce* *lower each time until eventually stopping when all its initial gravitational potential energy would have been converted to heat and other non-mechanical forms. That is why bouncing balls do not bounce forever!*

HOT Example 17

Two smooth spheres can move freely on a horizontal surface. A sphere of mass 2kg moves with a velocity of 6m/s in a straight line and collides with a stationary sphere of mass 3kg. After the collision, the 2kg sphere is observed to move off with a velocity of 4m/s at an angle of 30° above its original direction of motion. A student claims that the collision was perfectly elastic.

(a) Determine magnitude and direction of the velocity of the 3kg sphere immediately after the collision.

(b) Hence determine whether the student's claim is physically valid. Justify your answer clearly.

Solution

(a) *Conserving momentum horizontally:*

$$\sum (p_x)_{initial} = \sum (p_x)_{final}$$

$m_1(u_1)_x + m_2(u_2)_x = m_1(v_1)_x + m_2(v_2)_x$

Where:

$(u_1)_x = 6m/s$ (Initially 2kg sphere was moving horizontally)

$(u_2)_x = 0$ (At rest)

$(v_1)_x = 4\cos 30°$

$m_1 = 2kg, m_2 = 3kg$

Substituting:

$$2 \times 6 + 3 \times 0 = 2 \times 4\cos 30° + 3(v_2)_x$$

Solving gives: $(v_2)_x = 1.69m/s$

Conserving momentum vertically:

$$\sum (p_y)_{initial} = \sum (p_y)_{final}$$

$m_1(u_1)_y + m_2(u_2)_y = m_1(v_1)_y + m_2(v_2)_y$

Where:

$(u_1)_y = 0$ (No initial vertical component of velocity because 2kg sphere was moving horizontally)

$(u_2)_y = 0$ (At rest)

$(v_1)_y = 4\sin 30°$

Substituting:

$0 = 2 \times 4\sin 30° + 3(v_2)_y$

Solving gives: $(v_2)_y = -1.33m/s$

The negative sign indicates that the velocity component is directed opposite to the chosen positive vertical direction; that is, along the negative y-direction (below the original line of motion).

Magnitude: $v_2 = \sqrt{((v_2)_x)^2 + \left((v_2)_y\right)^2} = \sqrt{(1.69m/s)^2 + (-1.33m/s)^2} = 2.15m/s$

Direction: $\tan\theta = \frac{(v_2)_y}{(v_2)_x} = \frac{-1.33\text{m/s}}{1.69\text{m/s}} = -0.787$

$\theta = \tan^{-1} -0.787 = -38.3°$

The negative angle means that the angle was measured below the horizontal (clockwise from positive x-axis).

Hence, the velocity of the 3kg sphere is 2.15m/s at an angle of 38.3° below the original direction.

(b) Comparing initial and final kinetic energy:

Initial kinetic energy:

$$(K.E)_{\text{before collision}} = ½m_1u_1^2 + 0 = ½(2\text{kg})(6\text{m/s})^2 = 36\text{J}$$

Final kinetic energy:

$$(K.E)_{\text{after collision}} = ½m_1v_1^2 + ½m_2v_2^2 = ½(2\text{kg})(4\text{m/s})^2 + ½(3\text{kg})(2.15\text{m/s})^2 = 22.94\text{J}$$

Since Initial kinetic energy $(36\text{J}) \neq$ Final kinetic energy (22.94J), kinetic energy is not conserved and hence, the collision is inelastic.

Conclusion: student's claim was **invalid**.

Making Sense of the Answer: *In **oblique collisions**, the vector nature of momentum becomes crucial. 2kg sphere deflected upward at 30°, so ball 3kg sphere must deflect downward to conserve y-momentum.*

Think Like a Physicist: *In 2D collisions, always:*

1. Resolve velocities into components

2. Apply momentum conservation separately in x and y directions

3. Remember that momentum is a vector; direction matters.

The negative y-component of velocity of 3kg sphere makes physical sense: since 2kg sphere went upward, 3kg sphere must go downward to keep total y-momentum at zero.

HOT Example 18

Two identical pucks collide on a smooth horizontal air-hockey table. Before the collision, puck A moves with a velocity of 8m/s in a straight line, while puck B is at rest. The collision is perfectly elastic. After impact, puck A moves off at an angle of 60° to its original direction of motion.

Determine:

(a) The velocity of puck A immediately after the collision.
(b) The velocity of puck B immediately after the collision, stating its magnitude and direction relative to the original line of motion.

Solution

(a) *Conserving momentum horizontally:*

$$\sum (p_x)_{\text{initial}} = \sum (p_x)_{\text{final}}$$

$m_A(u_A)_x + m_B(u_B)_x = m_A(v_A)_x + m_B(v_B)_x$

Where:

$(u_A)_x = 8\text{m/s}$, $(u_B)_x = 0$, $(v_A)_x = v_A\cos60° = 0.5v_A$, $(v_B)_x = v_B\cos\theta$, $m_1 = m_2 = m$

Substituting:

$$8m + m \times 0 = 0.5mv_A + mv_B\cos\theta$$

Dividing by m throughout gives:

$$0.5v_A + v_B\cos\theta = 8$$

Making cosθ the subject:

$$\cos\theta = \frac{8 - 0.5v_A}{v_B} \ldots \text{(i)}$$

Conserving momentum vertically:

$$\sum(p_y)_{initial} = \sum(p_y)_{final}$$

$m_A(u_A)_y + m_B(u_B)_y = m_A(v_A)_y + m_B(v_B)_y$

Where:

$(u_A)_y = 0$, $(u_B)_y = 0$, $(v_A)_y = v_A\sin60° = \frac{\sqrt{3}}{2}v_A$, $(v_B)_y = v_B\sin\theta$

Substituting:

$0 = mv_A\sin60° + mv_B\sin\theta$

Again dividing by m throughout gives:

$$\frac{\sqrt{3}}{2}v_A + v_B\sin\theta = 0$$

Making sinθ the subject:

$$\sin\theta = \frac{-\frac{\sqrt{3}}{2}v_A}{v_B} = -\frac{(\sqrt{3})v_A}{2v_B} \ldots \text{(ii)}$$

Conserving kinetic energy:

Since the collision was perfect elastic, Initial K. E = Final K. E:

$$½mu_A^2 + 0 = ½mv_A^2 + ½mv_B^2$$

From which:

$$u_A^2 = v_A^2 + v_B^2$$

Where: $u_A = 8\text{m/s}$

Then:

$$v_A^2 + v_B^2 = 64 \ldots \text{(iii)}$$

Using identity $\sin^2\theta + \cos^2\theta = 1$ to eliminate θ:

From (i):

$$\cos^2\theta = \left(\frac{8 - 0.5v_A}{v_B}\right)^2 = \frac{64 - 8v_A + 0.25v_A^2}{v_B^2}$$

From (ii):

$$\sin^2\theta = \left(-\frac{(\sqrt{3})v_A}{2v_B}\right)^2 = \frac{3v_A^2}{4v_B^2}$$

Thus:

$$\sin^2\theta + \cos^2\theta = \frac{3v_A^2}{4v_B^2} + \frac{64 - 8v_A + 0.25v_A^2}{v_B^2} = \frac{v_A^2 - 8v_A + 64}{v_B^2} = 1$$

From which:

$$v_A^2 - 8v_A - v_B^2 = -64 \ldots \text{(iv)}$$

But from (iii):

$$v_B^2 = 64 - v_A^2$$

Substituting to (iv):

$$v_A^2 - 8v_A - (64 - v_A^2) = -64$$

$$2v_A^2 = 8v_A; v_A = 4\text{m/s}$$

Then:

$$v_B^2 = 64 - 4^2 = 48;\ v_B = 6.93\text{m/s}$$

Also from (i):

$$\cos\theta = \frac{8 - 0.5v_A}{v_B}$$

Substituting:

$$\cos\theta = \frac{8 - 0.5 \times 4}{6.93} = 0.866$$

$$\theta = \cos^{-1} 0.866 = 30°$$

Hence:

(a) The velocity of puck A is 4m/s.
(b) The velocity of puck B is 6.93m/s at an angle of 30° below the original direction of motion.

Alternative solution

For **elastic collision** between **equal masses** where one is initially at **rest**, there is a special property: *the objects move at right angles to each other after collision.*

This means if puck A deflects at 60° above the original direction of motion, puck B must deflect at $(60° - 90°) = -30°$ (or at 30° below x-axis).

(a) *Conserving momentum horizontally:*

$m_A(u_A)_x + m_B(u_B)_x = m_A(v_A)_x + m_B(v_B)_x$

$$8m + m \times 0 = 0.5mv_A + mv_B\cos 30°$$

Dividing by m throughout gives:

$$0.5v_A + v_B\cos 30° = 8 \ldots \text{(i)}$$

Conserving momentum vertically:

$m_A(u_A)_y + m_B(u_B)_y = m_A(v_A)_y + m_B(v_B)_y$

$$0 = mv_A\sin 60° + mv_B\sin(-30°)$$

Again dividing by m throughout gives:

$$v_A\sin 60° - v_B\sin 30° = 0 \ldots \text{(ii)}$$

Solving (i) and (ii) simultaneously (by using calculator) gives: $v_A = 4\text{m/s}, v_B = 6.93\text{m/s}$

So as before, answers are: $v_A = 4m/s$, $v_B = 6.93m/s$, $\theta = 30°$ below the original direction.

Making Sense of the Answer: *For elastic collisions between equal masses (one initially at rest), the two objects always separate at right angles. Puck A deflected 60° upward, so puck B deflected 30° downward (60° + 30° = 90° between their paths). The slower puck (A at 4m/s) went in the more deflected direction, while the faster puck (B at 6.93m/s) went in the less deflected direction.*

Think Like a Physicist: *The right-angle separation for equal-mass elastic collisions is a beautiful consequence of conservation laws. This is why billiard balls (nearly equal mass, nearly elastic) often separate at angles close to 90° after collisions. Professional players use this property intuitively when planning shots!*

HOT Example 19

A bullet of mass 0.02kg is fired horizontally into a wooden block of mass 2kg suspended by a string (forming a ballistic pendulum). The bullet embeds in the block, and the block swings to a maximum height of 0.3m above its initial position. Taking g = 9.8 m/s², calculate:

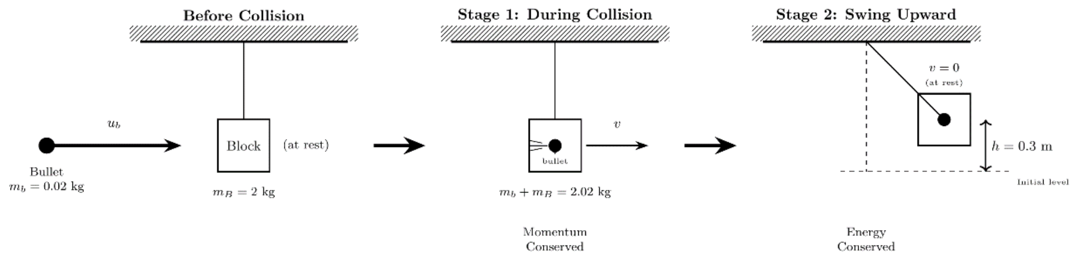
(a) The velocity of the block immediately after the bullet embeds.
(b) The initial velocity of the bullet.
(c) The kinetic energy lost during the collision.
(d) The percentage of the bullet's initial kinetic energy that was lost.

Solution

The motion occurs in two distinct stages:

1) **Collision stage:** Bullet embeds in block in perfectly inelastic collision (conservation of linear momentum).
2) **Swing stage:** Block embedded with bullet swing upward (conservation of mechanical energy).

The following diagram, clarifies these stages.



(a) Considering stage 2: Swing upward (mechanical energy conservation):

Let m and M denote the masses of the bullet and the block respectively.

At the bottom (just after collision):

Kinetic energy $= ½(m + M)v^2$

Potential energy $= 0$ (reference level)

At maximum height:

Kinetic energy $= 0$ (momentarily at rest, $v = 0$)

Potential energy $= (m + M)gh$

By conservation of mechanical energy:

Total mechanical energy at the bottom = Total mechanical energy at maximum height

$½(m + M)v^2 = (m + M)gh$

Cancelling $m + M$:

$v^2 = 2gh$ or $v = \sqrt{2gh}$

Substituting values:

$$v = \sqrt{2 \times 9.8\ m/s^2 \times 0.3m} = 2.42m/s$$

The velocity of the block is $2.42m/s$.

(b) Considering stage 1: Swing upward (momentum conservation):

Before collision:

- Bullet momentum: mu_{bullet}
- Block momentum: 0 (at rest)

After collision (bullet and block move together):

Combined momentum: $(m + M)v$

By conservation of momentum:

Momentum before collision = Momentum after collision

$mu_{bullet} = (m + M)v$

Making u_{bullet} the subject and substituting values:

$$u_{bullet} = \frac{(m + M)v}{m} = \frac{(0.02 + 2)kg \times 2.42m/s}{0.02kg} = 244.4\ m/s$$

The initial velocity of bullet is $244.4\ m/s$.

(c) Kinetic energy lost is given by the following equation:

$$(\text{K.E})_{\text{lost}} = ½mu_{\text{bullet}}^2 - ½(m + M)v^2$$

Substituting:

$(\text{K.E})_{\text{lost}} = ½(0.02\text{kg})(244.4\text{m/s})^2 - ½(0.02 + 2)\text{kg} \times (2.42\text{m/s})^2 = 591.4\text{J}$

The kinetic energy lost is 591.4J.

(d) The percentage of the kinetic energy lost

$$= \frac{\text{Kinetic energy lost}}{\text{Initial kinetic energy}} \times 100\% = \frac{591.4\text{J}}{½(0.02\text{kg})(244.4\text{m/s})^2} \times 100\% = 99\%$$

The percentage is 99%.

Making Sense of the Answer: *The bullet was traveling at an extremely high velocity (244m/s), typical for firearms. When it embedded in the much heavier block (100 times heavier!), almost all its kinetic energy converted to heat, sound, and deformation of wood. Only 1% remained as kinetic energy of the swinging block. This is why bullets cause so much damage, as they deposit almost all their energy into the target.*

Think Like a Physicist: *The ballistic pendulum is a classic device historically used to measure bullet speeds before modern chronographs existed. By measuring how high the pendulum swings (easily observable), you can calculate the bullet's initial velocity (hard to measure directly). This is a beautiful application of conservation principles working together: momentum during collision, then energy during the swing.*

HOT Example 20

A stationary bomb of mass 12kg explodes into three fragments. Fragment A (mass 3kg) flies north at 20m/s. Fragment B (mass 4kg) flies east at 15m/s. Find:

(a) The velocity of fragment C (magnitude and direction).
(b) The kinetic energy released by the explosion.

Take $g = 9.8$ m/s²

Solution

Interpreting the data:

Total mass: $M = 12\text{kg}$

Fragment A: $m_A = 3\text{kg}, v_A = 20\text{m/s}$ north

Fragment B: $m_B = 4\text{kg}, v_B = 15\text{m/s}$ east

Fragment C: $m_C = 12\text{kg} - (3 + 4)\text{kg} = 5\text{kg}, v_C = ?$

(a) Conservation of linear momentum (initially at rest, total momentum = 0):

Taking north as +y direction and east as +x direction.

x-direction:

$0 = m_A(v_A)_x + m_B(v_B)_x + m_C(v_C)_x$

$0 = 3 \times 0 + 4 \times 15 + 5(v_C)_x$

$(v_C)_x = -12\text{m/s}$ (the negative sign along x-axis means **west** direction).

y-direction:

$0 = m_A(v_A)_y + m_B(v_B)_y + m_C(v_C)_y$

$0 = 3 \times 20 + 4 \times 0 + 5(v_C)_y$

$(v_C)_y = -12\text{m/s}$ (the negative sign along y-axis means **south** direction).

Magnitude: $v_C = \sqrt{((v_C)_x)^2 + \left((v_C)_y\right)^2} = \sqrt{((-12m/s)^2 + (-12m/s)^2)} = 16.97m/s$

Direction: $\tan\theta = \frac{(v_C)_y}{(v_C)_x} = \frac{-12m/s}{-12m/s} = 1;\ \theta = \tan^{-1} 1 = 45°$ θ = 45° (but both components are negative, so it is in the third quadrant).

The velocity of fragment C is $16.97m/s$ at 45° south of west (or southwest direction).

(b) Initial KE $= 0$ (bomb at rest)

Final KE (sum of all fragments)

$= ½m_Av_A^2 + ½m_Bv_B^2 + ½m_Cv_C^2$

$= ½(3kg)(20m/s)^2 + ½(4kg)(15m/s)^2 + ½(5kg)(16.97m/s)^2 = 1770J$

Energy released $=$ Final KE $-$ Initial KE $= 1770J - 0 = 1770J$

The energy released is $1770J$.

Making Sense of the Answer: *The bomb started with zero momentum and zero kinetic energy. After explosion, the vector sum of momenta is still zero (each component sums to zero), but kinetic energy increased dramatically. This energy came from chemical energy stored in the explosive. Fragment C had to move southwest to balance the northward motion of A and eastward motion of B.*

Think Like a Physicist: *Explosions are "**reverse collisions.**" In collisions, kinetic energy usually decreases (converted to heat/sound/deformation). In explosions, kinetic energy increases (chemical energy converts to kinetic energy). But momentum is conserved in both cases.*

The worked examples have comprehensively demonstrated all types of collisions and how to analyse them using momentum conservation, energy considerations, and the coefficient of restitution. We can now confidently solve any collision problem by identifying the type of collision and applying the appropriate conservation laws.