MOMENTUM AND COLLISIONS

INTRODUCTION

Imagine this scene: It is a busy afternoon at Kariakoo market in Dar es Salaam. A heavily loaded *mkokoteni* (handcart), stacked high with sacks of rice, is being pushed steadily by a strong young man. From the opposite direction, a small boy on a bicycle riding with no hands and showing off to his friends comes speeding down the narrow road between the small shops.

Everyone sees what is coming. The boy's mother shouts. Vendors scatter. And then... CRASH!

When the dust settles, the bicycle has stopped completely and is now moving backwards. The mkokoteni has barely slowed down and continues forward, now with a bent wheel and a very embarrassed boy stuck in the sacks of rice.

Why did the light bicycle bounce backward while the heavy mkokoteni barely noticed the collision? Why could the boy's small mass not stop the massive cart, no matter how fast he was going? And most importantly, why did the market women immediately start laughing and taking photos?

Welcome to the fascinating world of momentum and collisions where size and velocity combine to create effects that can't be explained by force or mass alone. You will discover why a slow-moving truck is harder to stop than a speeding motorcycle, why airbags save lives, how rockets work in the vacuum of space (where there is nothing to "push against"), and why relaxing your body during a fall can actually reduce injury.

By the end of this chapter, you will understand the fundamental principle that governs everything from billiard balls to planetary motion, from car crashes to the recoil of a gun. You will even understand why that boy on the bicycle should have been paying attention to physics class instead of showing off!

MOMENTUM

In **Chapter 2**, we learned about velocity, how fast an object moves and in which direction. In **Chapter 3**, we discovered Newton's laws and learned that force causes changes in motion. We even encountered Newton's second law in its most general form:

$$F = \frac{dp}{dt}$$

Where p represents a quantity called **momentum**. This tells us that force does not simply change velocity; it changes momentum. To understand why this distinction matters, imagine the following two situations:

Situation 1: A 10kg stone falls from a second-floor window onto your toe. Ouch! That hurts.

Situation 2: The same 10kg stone is handed gently to you. You catch it comfortably.

In both cases, the stone has the same mass (10kg) and experiences the same gravitational force. Yet the effects are dramatically different. What is missing from our description? The answer is velocity. The falling stone is moving rapidly when it reaches you, while the handed stone has almost no motion. To fully describe such situations, we need a quantity that combines both mass and velocity. This combined effect of mass and velocity is what we call **momentum**.

Concisely, **momentum** *is the product of an object's mass and its velocity.*

$$p = mv$$

Where:

p = momentum (measured in kgm/s)

m = mass (kg)

v = velocity (m/s)

Key insights about momentum

Now that we have defined momentum, it is useful to highlight what this definition really tells us. The following points summarise its most important features:

1. **Momentum is a vector quantity:** Since velocity is a vector, momentum is also a vector. This means momentum has both magnitude and direction, and its direction is always the same as that of the velocity.
2. **Momentum depends on both mass and velocity:**
   Momentum exists only when both mass and velocity are present. An object at rest (v = 0) has zero momentum, no matter how large its mass may be.
3. **Momentum represents the "quantity of motion":** A heavy object moving slowly can have the same momentum as a lighter object moving faster. In both cases, the overall amount of motion being carried is the same.

Impulse and the Impulse-Momentum Theorem

We have learned that Newton's second law in its most general form tells us that **force changes momentum**. But *how exactly does force change momentum? How much momentum change results from a given force? And does it matter whether the force acts for a long time or a short time?*

These questions lead us to one of the most practical and powerful concepts in mechanics: **impulse**.

Before exploring impulse in depth, let us first understand a closely related concept known as **impulsive forces**. To build clear intuition, consider the following everyday situations:

Situation 1: A cricket batsman hits a ball. The bat touches the ball for only about 0.01 seconds, yet the ball's momentum changes dramatically from moving toward the batsman to flying away at high speed.

Situation 2: When you catch a fast-moving ball, your hands experience a large force for a brief moment.

Situation 3: During a car collision, the passengers experience enormous forces, but only for a fraction of a second.

In all these cases, forces act for very short times. Such forces (*short-duration forces*) are difficult to measure directly because they start at zero, shoot up to very large values, then drop back to zero, all within milliseconds. We call these, **impulsive forces**.

How can we analyse motion when forces change so rapidly? The answer is surprisingly elegant: instead of tracking the force at every instant, we focus on the **combined effect** of force and time. This combined effect is called **impulse**.

Understanding impulse

When a force F acts on an object for a time interval Δt, the impulse J delivered to the object is defined as:

$$J = F\Delta t$$

Where:

J = impulse (measured in Ns or kgm/s)

F = force applied (N)

Δt = time interval during which the force acts (s)

So in a simple physical meaning, **impulse** *is the product of force and the time duration over which that force acts.*

The following insights will help you grasp the concept of impulse more clearly and see how it operates in real physical situations.

1. **Impulse is a vector quantity:** It has the same direction as the force that produces it.
2. **Same impulse, different ways:** The same impulse can be delivered by:

- A **large force** acting for a **short time**, or
- A **small force** acting for a **long time.**

3. **Impulse measures the "effort" of the force:** A strong force acting briefly can have the same effect as a weak force acting for a long time.

The Impulse-Momentum Theorem

Now comes the beautiful connection. Starting from Newton's second law:

$$F = \frac{dp}{dt}$$

We can rearrange this to:

$$Fdt = dp$$

Integrate over a time interval:

$$\int_{t_1}^{t_2} Fdt\,Fdt = \int_{p_1}^{p_2} dp$$

If F is **constant** (or F means **average force**), then the above integral becomes;

$$\mathbf{F(t_2 - t_1) = p_2 - p_1}$$

But:

$$t_2 - t_1 = \Delta t \text{ and } p_2 - p_1 = \Delta p$$

It follows that:

$$\mathbf{F\Delta t = \Delta p}$$

Remember $F\Delta t$ is exactly what we defined as impulse (J)!

Hence:

$$\mathbf{J = \Delta p}$$

This equation is called the **Impulse-Momentum Theorem**. It states that:

Impulse delivered to an object is equal to the change in its momentum.

This theorem is extraordinarily useful because:

1. **It connects force, time, and momentum change:** You don't need to know how the force varies with time, only the average force and total time.
2. **It works for impulsive forces:** Even when forces change rapidly (like during collisions), we can still calculate momentum change without knowing the detailed force variation.
3. **It provides practical insights:** It explains many safety features and sports techniques we encounter daily.

Variable Forces and Average Force

In real situations, forces rarely stay constant. During a collision, the force might start at zero, increase to a maximum, then decrease back to zero. *How do we handle such situations?*

The answer is to use the **average force**, $\mathbf{F_{avg}}$. The impulse-momentum theorem still works:

$$\mathbf{F_{avg} \times \Delta t = \Delta p}$$

The **average force** *is the constant force that, if it acted over the same time interval, would produce the same change in momentum.*

This idea becomes even clearer when we represent the interaction graphically. If we plot force against time, the area under the force–time curve represents the impulse. This is because impulse is defined as the product of force and time. When the force varies with time, each small time interval contributes a small impulse. Adding all these contributions together gives the total impulse, which is represented graphically by the area under the force–time curve.

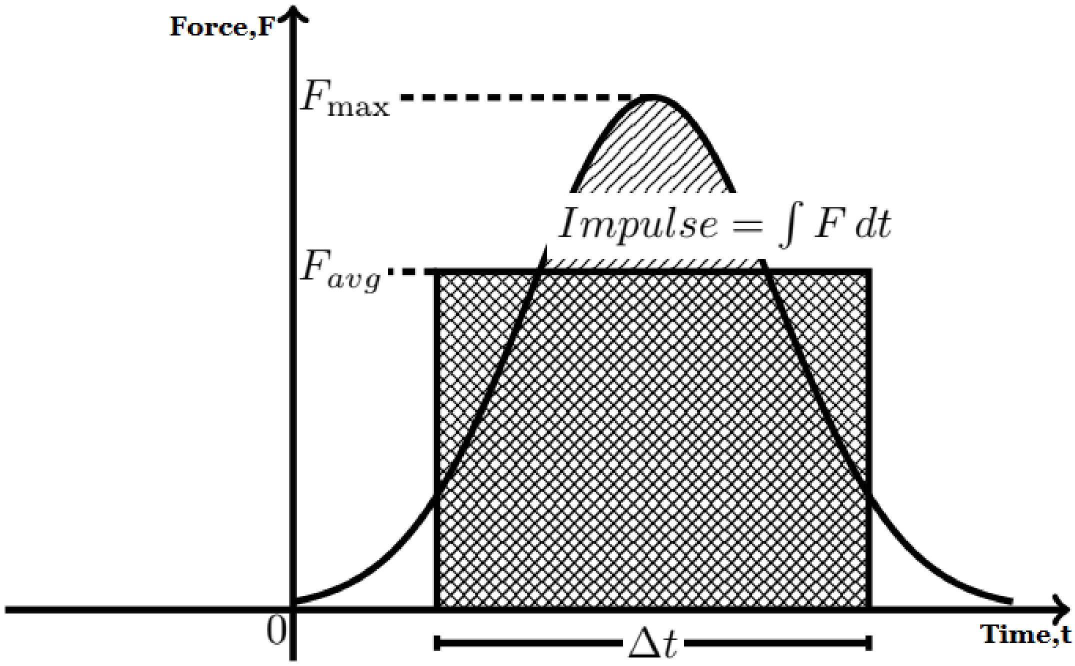

Figure: *Force–time graph for a variable force during a collision. The shaded area under the curve represents the impulse, which is equal to the change in momentum. The rectangle shows the constant average force that produces the same impulse over the same time interval (Δt).*

Practical Applications of Impulse-Momentum Theorem

The impulse-momentum theorem explains many real-life phenomena:

1. Why airbags save lives

An airbag increases the time Δt over which the momentum changes during a crash. Same Δp, but larger Δt means smaller average force F_{avg} on the body.

2. Why athletes "follow through"

In cricket or baseball, following through increases the time the bat contacts the ball, delivering more impulse and thus more momentum change to the ball.

3. Why you bend your knees when landing

Bending your knees increases the time Δt over which your momentum changes, thereby reducing the force on your legs.

4. Why a karate expert can break boards

By striking very quickly (small Δt), the expert delivers the required impulse with a very large force F.

5. Why it hurts more to catch a fast ball with stiff arms

Stiff arms mean short Δt, leading to large F. Relaxed, "giving" arms increase Δt, therefore reduce the force experienced.

With these foundational ideas firmly in place, let us now enjoy some carefully crafted worked examples that bring momentum to life.

BINDER Example 1

A tennis ball of mass 0.06kg moving at 20m/s is hit by a racket and rebounds in the opposite direction at 30m/s. The ball is in contact with the racket for 0.008s. Calculate:

(a) The change in momentum of the ball.
(b) The impulse delivered to the ball.
(c) The average force exerted by the racket on the ball.

Solution

Interpreting the data:

$m = 0.06\text{kg}$

Initial velocity: $u = 20\text{m/s}$ (let us take this as positive direction)

Final velocity: $v = -30\text{m/s}$ (opposite direction, hence negative)

Contact time: $\Delta t = 0.008\text{s}$

(a) Initial momentum:

$$p_1 = mu = 0.06\text{kg} \times 20\text{m/s} = 1.2\text{kgm/s}$$

Final momentum:

$$p_2 = mv = 0.06\text{kg} \times (-30\text{m/s}) = -1.8\text{kgm/s}$$

Change in momentum:

$$\Delta p = p_2 - p_1 = (-1.8 - 1.2)\text{kgm/s} = -3\text{kgm/s}$$

The negative sign indicates the change is in the opposite direction to the initial motion.

The change in momentum is -3kgm/s (or 3kgm/s in the opposite direction)

(b) Using the impulse-momentum theorem:

$$J = \Delta p = -3\text{Ns}$$

The impulse is 3Ns.

(c) Using:

$$J = F_{avg} \times \Delta t$$

From which:

$$F_{avg} = \frac{J}{\Delta t} = \frac{-3\text{Ns}}{0.008\text{s}} = -375\text{N}$$

The average force is -375N (or 375N in the direction opposite to initial motion).

Making Sense of the Answer: *The force is quite large (375N, about 38kg-weight!) even though the ball is light. This happens because the momentum change is large (ball reversed direction) and the contact time is very short (0.008s). The negative sign means the force acts opposite to the ball's initial direction, which makes sense, since the racket pushed the ball backward.*

Think Like a Physicist: *When an object reverses direction, the momentum change is especially large because you must first stop the object (eliminate its initial momentum) then accelerate it in the opposite direction (add momentum in new direction). This is why hitting a ball back requires more force than simply catching it.*

BINDER Example 2

A force of 50N acts on a body of mass 5kg initially at rest for 4 seconds. Calculate:

(a) The impulse delivered to the body.

(b) The final velocity of the body.

Solution

(a) Impulse is given by:

$$J = F\Delta t = 50N \times 4s = 200Ns$$

The impulse is 200Ns.

(b) Using impulse-momentum theorem:

$$J = \Delta p = p_2 - p_1$$

Since the body starts from rest ($p_1 = 0$):

$$J = p_2 = mv$$

It follows that:

$$v = \frac{J}{m} = \frac{200kgm/s}{5kg} = 40m/s$$

The final velocity is 40m/s.

Making Sense of the Answer: *A 50N force acting for 4 seconds delivers substantial impulse (200Ns). Starting from rest, this impulse accelerates the 5kg body to 40m/s. We can verify this using $F = ma$: acceleration $a = F/m = 50/5 = 10m/s^2$, and $v = u + at = 0 + 10 \times 4 = 40m/s$ ✓.*

Think Like a Physicist: *The impulse-momentum approach ($J = \Delta p$) gave us the answer directly without needing to find acceleration first. This is the beauty of impulse as it connects force, time, and velocity change in one step.*

REAL Example 3

One afternoon, **Kipanga** and **Kipute** are playing catch with a cricket ball. Kipanga throws the ball hard to Kipute. She catches it two different ways and notices something interesting.

First catch: Kipute keeps her hands stiff and catches the ball quickly. "Ouch! That hurt!"

Second catch: Kipute relaxes her arms and lets them move backward as she catches the ball. "Ah, much better, that didn't hurt at all."

Mr. Akilikubwa, watching nearby, asks: *"Why did the same ball, thrown with the same speed, feel so different in the two catches?"*

Explain the physics behind Kipute's observation.

Solution

Explanation:

The same momentum change occurred in both catches, requiring the same impulse. Stiff hands mean short contact time (small Δt), resulting in large force that causes pain. Relaxed, "giving" hands increase the contact time (large Δt), resulting in much smaller force that feels comfortable.

Demonstration:

This demonstrates that impulse can be delivered by large force over short time or small force over long time.

Making Sense of the Answer: *Experienced cricket players and goalkeepers instinctively understand this principle. They always "give" with the ball rather than resisting it rigidly. This is not weakness, it is physics!*

Think Like a Physicist: *This same principle explains why car crumple zones save lives (increase Δt, decrease F), why parachutes work (increase Δt for landing, decrease F on the body), and why landing on soft sand hurts less than landing on concrete (sand increases Δt).*

REAL Example 4

At a physics demonstration, Mr. Akilikubwa places a raw egg on a table and challenges his students:

Mr. Akilikubwa: *"I will drop this hammer onto the egg, and the egg will not break. Who wants to bet against me?"*

Kipanga (confident): *"That's impossible, sir! A hammer will definitely break an egg!"*

Mr. Akilikubwa places a thick foam pad between the hammer and the egg. He drops the hammer from a height. The hammer hits the foam, compresses it slowly, and the egg remains intact.

Kipute: *"Oh! I see what happened. But Kipanga, what if sir had dropped the hammer directly onto the egg without the foam?"*

Kipanga: *"The egg would break immediately!"*

Mr. Akilikubwa: *"Exactly! Now, explain why the foam saved the egg using impulse ideas."*

Explain the physics principle Mr. Akilikubwa is demonstrating.

Solution

Explanation:

The foam increases the time interval Δt over which the hammer's momentum changes to zero. Since impulse J = Δp must be the same (same hammer, same fall), increasing Δt dramatically decreases the average force F_{avg}. The reduced force is small enough that the eggshell can withstand it.

Demonstration:

This demonstrates that the same impulse can be delivered with large force over short time (egg breaks) or small force over long time (egg safe).

Making Sense of the Answer: *This is exactly how packaging materials work. They increase the stopping time during impacts, reducing forces on fragile items. It is also why modern cars have crumple zones that deform during crashes, protecting passengers by extending the collision time.*

Think Like a Physicist: *Safety devices like airbags, seatbelts, helmets with foam padding, and gymnastics mats all work on this principle: increase Δt to decrease F for the same momentum change. Nature uses this too; for example, cats spread their bodies when landing to increase contact time and reduce impact forces!*

HOT Example 5

A ball of mass 0.2kg moving horizontally at 10m/s strikes a vertical wall and rebounds horizontally at 8m/s. The ball is in contact with the wall for 0.05s. Calculate:

(a) The impulse received by the ball from the wall.
(b) The average force exerted by the wall on the ball.
(c) The average force exerted by the ball on the wall.
(d) If the wall were softer and the contact time increased to 0.15s with the same rebound velocity, calculate the new average force.

Solution

Interpreting the data:

$m = 0.2\text{kg}$

Initial velocity: $u = 10\text{m/s}$ (toward wall, take as positive direction)

Final velocity: $v = -8\text{m/s}$ (away from wall, negative)

Contact time: $\Delta t = 0.05\text{s}$

(a) Initial momentum:

$$p_1 = mu = 0.2\text{kg} \times 10\text{m/s} = 2\text{kgm/s}$$

Final momentum:

$$p_2 = mv = 0.2\text{kg} \times (-8\text{m/s}) = -1.6\text{kgm/s}$$

Impulse (change in momentum):

$$\Delta p = p_2 - p_1 = (-1.6 - 2)\text{kgm/s} = -3.6\text{kgm/s}$$

The negative sign indicates the change is in the opposite direction to the initial motion.

The impulse is -3.6Ns (or 3.6Ns away from the wall)

(b) Using:

$$J = F_{avg} \times \Delta t$$

From which:

$$F_{avg} = \frac{J}{\Delta t} = \frac{-3.6\text{Ns}}{0.05\text{s}} = -72\text{N}$$

The average force is -72N (or 72N directed away from the wall).

(c) By Newton's third law, the ball exerts an equal and opposite force on the wall:

$$F_{\text{on wall}} = -F_{\text{on ball}} = -(-72) = +72\text{N}$$

The average force is $+72\text{N}$ (or 72N directed into the wall).

(d) The impulse remains the same ($J = -3.6\text{Ns}$) because the momentum change is the same (same initial and final velocities).

With new contact time $\Delta t = 0.15\text{s}$:

$$F_{avg} = \frac{J}{\Delta t} = \frac{-3.6\text{Ns}}{0.15\text{s}} = -24\text{N}$$

The new average force is -24N (or 24N directed away from the wall).

(The force decreased from 72N to 24N which is equivalent to a reduction to one-third of the original value because the contact time tripled).

Making Sense of the Answer: *The negative signs indicate opposite directions of force: the wall exerts a force on the ball away from the wall, while the ball exerts an equal and opposite force on the wall toward it (Newton's third law). If the wall is softer, the contact time is longer, so the same change in momentum occurs over a greater time, reducing the force. This is why soft-surfaced walls in sports facilities help reduce the risk of injury.*

Think Like a Physicist: *Notice that the rebound velocity (8m/s) is less than the approach velocity (10m/s). This means some kinetic energy was lost, converted to heat and sound during collision. We will explore this more when we study collisions later in this chapter. Also, observe that Newton's third law applies throughout the collision: at every instant, the force on the ball equals the force on the wall but in opposite direction.*

The worked examples have beautifully illustrated the power of impulse and the impulse-momentum theorem. Our understanding is now deep and practical. We are ready to welcome the next subtopic with confidence and curiosity.