

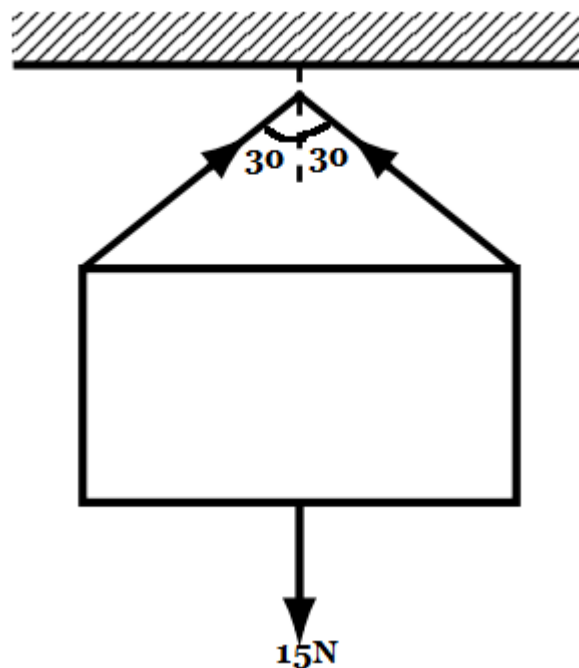
MISCELLANEOUS WORKED EXAMPLES ON EQUILIBRANT FORCES

Example 34

- (a) Explain why a tow rope becomes very tight when a vehicle suddenly accelerates while pulling a trailer.
- (b) A picture frame of weight 15N hangs from a single nail by a string. The string makes an angle of 30° with the vertical on each side of the nail.
- Calculate the tension in the string.
 - Calculate the horizontal force exerted by the nail on each side of the string.

Solution

- (a) When the vehicle accelerates suddenly, the trailer tends to remain in its original state of motion due to inertia. The tow rope must then provide a large force (tension) to accelerate the trailer in a short time. This sudden increase in required force makes the rope to tighten sharply.
- (b) Each side of the string makes 30° with the vertical, the system is symmetrical and hence tensions on both sides are equal (T).
The frame is in equilibrium as illustrated in the following diagram:



- (i) Resolving vertically:

Vertical component in each string = $T\cos 30^\circ$ (both upward)

$$\sum F_y = T\cos 30^\circ + T\cos 30^\circ - W = 0 \text{ or } 2T\cos 30^\circ = W \text{ or } T = \frac{W}{2\cos 30^\circ}$$

Substituting:

$$T = \frac{15\text{N}}{2\cos 30^\circ} = 8.66\text{N}$$

The tension T is 8.66N.

(ii) Resolving horizontally:

Horizontal component in each string = $T\sin 30^\circ$ (one left side, another right side)

Substituting:

$$8.66N\sin 30^\circ = 4.33N$$

The horizontal force exerted by the nail is 4.33N on each side of the string.

Example 35

- (a) A block rests on a rough inclined plane without sliding. Explain the role of friction in maintaining equilibrium and describe what would happen if the angle of the incline were gradually increased.
- (b) A wooden crate of mass 50kg is kept on a ramp inclined at 30° to the horizontal. The coefficient of static friction (μ_s) between the crate and ramp is 0.45 and that of kinetic friction (μ_k) is 0.3. Take $g = 9.8 \text{ m/s}^2$.
- Calculate the maximum angle at which the crate would remain in equilibrium without sliding.
 - Explain whether the crate is on the verge of sliding or if it could withstand a slight increase in the angle.
 - Calculate the frictional force acting on the crate.
 - Find the acceleration of the crate.

Solution

- (a) When the block rests on the inclined plane, the component of its weight down the plane ($mg\sin\theta$) tends to make it slide. Static friction acts up the plane to oppose this tendency and maintain equilibrium. As the angle of the incline increases, $mg\sin\theta$ increases while the normal reaction ($mg\cos\theta$) decreases, thereby reducing the maximum static friction available. Eventually, the limiting friction is reached where $\tan\theta = \mu_s$, and beyond this angle the block slides down the plane.
- (b) The solution of each part is as follows:
- The maximum angle is found when the system is at limiting equilibrium.

At limiting equilibrium:

$$f_{\max} = \mu_s R = \mu_s mg\cos\theta = \text{driving force} = mg\sin\theta$$

Taking:

$$\mu_s mg\cos\theta = mg\sin\theta;$$

$$\mu_s \cos\theta = \sin\theta;$$

$$\mu_s = \frac{\sin\theta}{\cos\theta} = \tan\theta;$$

$$\theta = \tan^{-1} \mu_s = \tan^{-1} 0.45 = 24.2^\circ$$

The maximum angle is 24.2° .

(ii) Compare:

$$\tan 30^\circ = 0.577 > 0.45 (\mu_s)$$

Since $\tan 30^\circ > \mu_s$, the static friction is not enough to hold the crate.

So at 30° , the crate will slide (it is beyond the equilibrium limit). It cannot withstand any increase as it is already slipping.

(iii) Because it is sliding, friction is kinetic:

$$f_k = \mu_k R = \mu_k mg\cos 30^\circ = 0.3 \times 50\text{kg} \times 9.8\text{m/s}^2 \times \cos 30^\circ = 127\text{N}$$

The frictional force is 127N (up the plane).

(iv) Applying Newton's second law:

$$\begin{aligned}
 mgs\sin 30^\circ - f_k &= ma \\
 mgs\sin 30^\circ - \mu_k mg\cos 30^\circ &= ma \\
 a = \frac{mgs\sin 30^\circ - \mu_k mg\cos 30^\circ}{m} &= g\sin 30^\circ - \mu_k g\cos 30^\circ = g(\sin 30^\circ - \mu_k \cos 30^\circ)
 \end{aligned}$$

Substituting:

$$a = 9.8\text{m/s}^2(\sin 30^\circ - 0.3\cos 30^\circ) = 2.35\text{m/s}^2$$

The acceleration is 2.35m/s^2 (down the plane).

Example 36

- (a) Explain why the tension in a string connecting two blocks being pulled across a horizontal surface is not equal to the applied force, even when the surface is smooth (frictionless).
- (b) Two blocks of masses $m_1 = 4\text{kg}$ and $m_2 = 6\text{kg}$ are connected by a light inextensible string on a smooth horizontal table. A horizontal force $F = 30\text{N}$ is applied to the 4kg block, pulling both blocks to the right. Take $g = 9.8\text{ m/s}^2$.
- Calculate the acceleration of the system.
 - Calculate the tension in the connecting string.
 - If the force F were instead applied to the 6kg block (pulling both blocks), how would the tension change? Calculate the new tension and explain why it differs from part (ii).

Solution

- (a) Even with no friction, the applied force is used to accelerate both blocks. However, the tension only needs to accelerate one of the blocks (whichever block is not directly receiving the applied force). Since part of the applied force is spent in accelerating the pulled block itself, only the remaining part is transmitted through the string. Therefore, tension is generally less than applied force.
- (b) Solution for each part is as follows

Identifying the system:

$m_1 = 4\text{kg}$ (block with applied force)

$m_2 = 6\text{kg}$ (block being pulled by string)

Surface: smooth (no friction).

- (i) Using Newton's second law for the whole system:

$$F = (m_1 + m_2)a \text{ or } a = \frac{F}{m_1 + m_2} = \frac{30\text{N}}{(6+4)\text{kg}} = 3\text{m/s}^2$$

The acceleration is 3m/s^2 .

- (ii) Consider the 6kg block alone (it only experiences tension):

$$T = m_2a = 6\text{kg} \times 3\text{m/s}^2 = 18\text{N}$$

The tension in the string is 18N .

- (iii) If the force (30N) is applied to the 6kg block instead; the acceleration remains the same (same total force, same total mass). But now tension must accelerate the 4kg block (not the 6kg block):

$$T = m_1a = 4\text{kg} \times 3\text{m/s}^2 = 12\text{N}$$

The tension would decrease to 12N .

Explanation of difference:

New tension (12N) is less than the previous one (18N) because tension now accelerates the lighter 4kg (smaller mass) block instead of the heavier 6kg (larger mass) block.

Example 37

- (a) A block of mass m rests on a rough plane inclined at angle θ to the horizontal. A force F is applied up the plane. The coefficient of static friction is μ_s . Prove that the block can remain in equilibrium only if:
- $$mg(\sin\theta - \mu_s \cos\theta) \leq F \leq mg(\sin\theta + \mu_s \cos\theta)$$
- (b) A package of mass 30kg rests on a rough plane inclined at 15° to the horizontal. A force of 220N is applied up the plane. If the coefficients of friction are $\mu_s = 0.40$ and $\mu_k = 0.30$ for static and kinetic friction respectively, determine:
- whether the package remains at rest or moves,
 - its acceleration.

$$\text{Take } g = 9.8\text{m/s}^2.$$

Solution

- (a) Proof:

$$\text{Normal reaction: } R = mg\cos\theta$$

$$\text{Maximum static friction: } f_{\max} = \mu_{sR} = \mu_s mg\cos\theta$$

Static friction adjusts in magnitude (0 to f_{\max}) and acts opposite to the tendency of motion.

Lower limit of F (tendency of motion: down the plane):

If F is small ($F < mg\sin\theta$), the block tends to slide down the plane, so friction acts up the plane (in the same direction as F but opposite to $mg\sin\theta$).

For equilibrium at the limiting case (just about to slide down):

$$F + f_{\max} = mg\sin\theta$$

So:

$$F = mg\sin\theta - \mu_s mg\cos\theta$$

$$F = mg(\sin\theta - \mu_s \cos\theta) = \text{Minimum } F \text{ to prevent downward sliding}$$

Hence, for equilibrium:

$$\mathbf{F \geq mg(\sin\theta - \mu_s \cos\theta)}$$

Upper limit of F (tendency of motion: up the plane):

If F is large ($F > mg\sin\theta$), the block tends to move up the plane, so friction acts down the plane (in the same direction as $mg\sin\theta$ but opposite to F).

For equilibrium at the limiting case (just about to move up):

$$F = mg\sin\theta + f_{\max}$$

So:

$$F = mg\sin\theta + \mu_s mg\cos\theta$$

$$F = mg(\sin\theta + \mu_s \cos\theta) = \text{Maximum } F \text{ just before upward sliding}$$

Hence, for equilibrium:

$$\mathbf{F \leq mg(\sin\theta + \mu_s \cos\theta)}$$

Combining the two conditions gives:

$$\mathbf{mg(\sin\theta - \mu_s \cos\theta) \leq F \leq mg(\sin\theta + \mu_s \cos\theta)}$$

- (b) Solution for each part is as follows:

- Comparing F and $mg\sin\theta$ to determine the tendency of motion:

$$mg\sin\theta = 30\text{kg} \times 9.8\text{m/s}^2 \times \sin 15^\circ = 76.1\text{N} < 220\text{N (applied force F)}$$

Since the applied force is greater than the weight component acting along the plane, the tendency of motion is up the plane. So the applied force has upper limit for achieving limiting equilibrium and it is given by (from (a)):

$$F = mg(\sin\theta + \mu_s \cos\theta) = F_{\text{maximum}}$$

Substituting:

$$F_{\text{maximum}} = 30\text{kg} \times 9.8\text{m/s}^2(\sin 15^\circ + 0.4\cos 15^\circ) = 189.7\text{N} < 220\text{N (applied force)}$$

Since the applied upward force is greater than opposing (resistive) forces, the package **moves** up the plane.

(ii) As the package moves up the plane, the **kinetic** frictional force acts down the plane.

But the kinetic friction is given by:

$$f_k = \mu_k R = \mu_k mg\cos\theta = 0.3 \times 30\text{kg} \times 9.8\text{m/s}^2 \cos 15^\circ = 85.2\text{N}$$

$$\text{Total resistive forces} = mg\sin\theta + f_k = 76.1\text{N} + 85.2\text{N} = 161.3\text{N}$$

Applying Newton's second law:

$$\text{Applied force} - \text{Resistive forces} = ma \text{ or } a = \frac{\text{Applied force} - \text{Resistive forces}}{m}$$

Substituting:

$$a = \frac{(220 - 161.3)\text{N}}{30\text{kg}} = 1.96\text{m/s}^2$$

The acceleration is 1.96m/s^2 .

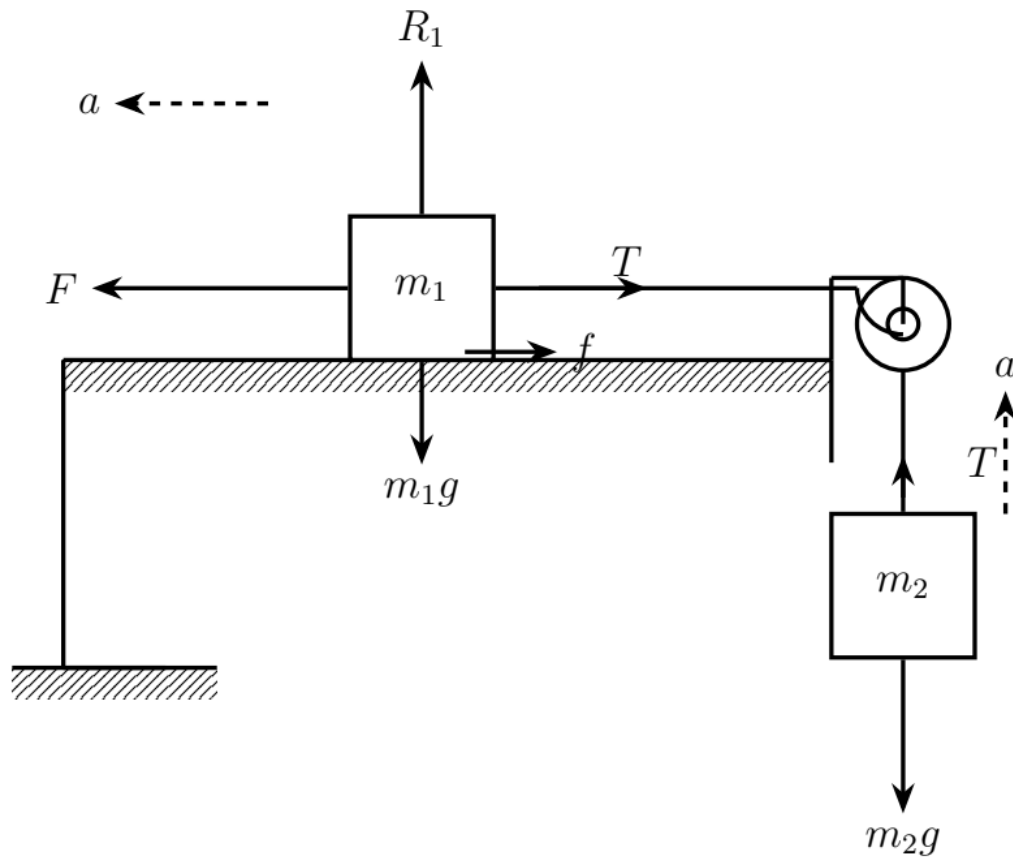
Example 38

- (a) Explain why does a hanging signboard remain at rest even though several forces act on it?
- (b) A block of mass 6kg rests on a rough horizontal table with coefficient of static friction, $\mu_s = 0.4$ and coefficient of kinetic friction $\mu_k = 0.3$. It is connected by a light inextensible string passing over a smooth pulley at the edge of the table to a hanging mass of 2kg . A horizontal force $F = 50\text{N}$ is applied to the 6kg block pulling it away from the pulley. The system is released from rest.
- Determine whether the system will move and, if so, in which direction.
 - Calculate the acceleration of the system and the tension in the string.
 - What is the minimum value of the applied force, F , required to prevent the hanging mass from descending?

$$\text{Take } g = 9.8 \text{ m/s}^2.$$

Solution

- (a) The signboard remains at rest because the forces acting on it are balanced. The upward components of the tensions in the supporting strings add up to equal the weight of the signboard, while the horizontal components cancel each other. Therefore, the resultant force is zero and hence there is no acceleration.
- (b) Consider the following diagram.



(i) Identifying forces trying to move the system:

- Driving force: $F = 50\text{N}$ (try to move the system **away** from pulley, leftward).
- Weight of hanging mass, m_2 : $W = m_2g = 2\text{kg} \times 9.8\text{ m/s}^2 = 19.6\text{N}$ (try to move the system **toward** pulley, rightward).

So: Driving force(50N) > Weight of hanging mass(19.6N). From this alone we can conclude that the system tries to move away from pulley.

For actual movement to occur, the driving force must overcome **all** resisting forces (static friction and weight of hanging mass).

The static friction is maximum when: $f_{\text{max}} = \mu_s R_1 = \mu_s m_1 g = 0.4 \times 6\text{kg} \times 9.8\text{ m/s}^2 = 23.52\text{N}$

So total resistive forces, the driving force has to overcome for actual motion to occur is:

$$(19.6 + 23.52)\text{N} = 43.12\text{N} < 50\text{N}(\text{Driving force})$$

Since the driving force is greater than total resistive forces, the system move.

Conclusion: Yes, the system moves with m_1 going leftward (away from pulley) and m_2 ascending. This is because the applied force $F = 50\text{N}$ exceeds the combined resistance from the weight of hanging mass and static friction.

(ii) Once the system moves, the static friction is no longer functioning and the kinetic friction takes place. Thus:

$$\text{Friction force, } f_k = \mu_k m_1 g = 0.3 \times 6\text{kg} \times 9.8\text{ m/s}^2 = 17.64\text{N}$$

So total resistive forces during the motion = $W_{m_2} + f_k = (19.6 + 17.64)\text{N} = 37.24$

The net driving force, will accelerate both m_1 and m_2 . Therefore:

$$\text{Driving force} - \text{Resistive force} = (m_1 + m_2)a$$

Substituting:

$$50\text{N} - 37.24\text{N} = (6\text{kg} + 2\text{kg})a; a = 1.595\text{m/s}^2$$

The acceleration is 1.595m/s^2 .

For hanging mass:

$$T - m_2g = m_2a \text{ or } T = m_2a + m_2g = m_2(a + g) = 2\text{kg}(1.595 + 9.8)\text{m/s}^2 = 22.79\text{N}$$

The tension is 22.79N .

- (iii) For m_2 not to descend, the system must either be at rest or moving leftward. The critical condition is when m_2 is on the verge of descending (*limiting equilibrium with $a = 0$*).

At limiting equilibrium:

- Friction force is equal to the maximum static friction = 23.52N .
- Total resistive forces = $f_{\text{max}} + W_{m_2} = (23.52 + 19.6)\text{N} = 43.12\text{N}$

So:

- If driving force, $F < 43.12\text{N}$, the hanging mass descends (pulls m_1 rightward).
- If $F \geq 43.12\text{N}$, the hanging mass is not descending.

Hence, the minimum value of the applied force required to prevent the hanging mass from descending is 43.12N .

Example 39

- (a) Why do passengers sometimes feel heavier when a lift starts moving upward?
 (b) Two bodies of masses 3kg and 5kg are connected by a light inextensible string passing over a smooth pulley and are released from rest while hanging freely. After the system has been moving for 1.5s , the string suddenly snaps just above the 5kg body. Take $g = 9.8\text{m/s}^2$.

Calculate:

- The time taken for the 5kg body to reach the ground after the string snaps, given that at the instant the string snaps it is 2m above the ground.
- The maximum height reached by the 3kg body above its initial position if the two bodies were initially at the same level.
- The total time taken by the 3kg body to reach the ground.

Solution

- (a) When the lift accelerates upward, the floor must push up on passengers with a force greater than their weight in order to accelerate them upward. This increased normal reaction is what make them feel heavier.
 (b) Before the string snapped, the system was undergoing motion similar to an Atwood machine.

For the Atwood machine:

$$a = \left(\frac{m_2 - m_1}{m_1 + m_2} \right) g$$

Where:

$$m_2 = 5\text{kg (heavier)}$$

$$m_1 = 3\text{kg (lighter)}$$

Substituting:

$$a = \left(\frac{5\text{kg} - 3\text{kg}}{3\text{kg} + 5\text{kg}} \right) \times 9.8\text{m/s}^2 = 2.45\text{m/s}^2$$

Velocity at the instant of snapping:

$$v = at = 2.45\text{m/s}^2 \times 1.5\text{s} = 3.675\text{m/s}$$

Distance moved in 1.5s (each body):

$$s = ut + 0.5at^2 = 0\text{m/s} \times 1.5\text{s} + 0.5 \times 2.45\text{m/s}^2 \times (1.5\text{s})^2 = 2.756\text{m}$$

So, at the instant the string snaps:

- The 5kg mass is moving downward at 3.675m/s.
- The 3kg mass is moving upward at 3.675m/s.
- Each body moved through a vertical distance of 2.756m.

(i) After snapping, the 5kg body moves under gravity with initial downward velocity 3.675m/s.

Using:

$$s = ut + 0.5gt^2$$

Substituting:

$$2 = 3.675t + 0.5(9.8)t^2 \text{ or } 4.9t^2 + 3.675t - 2 = 0$$

Solving the quadratic equation, gives the practical value of t which is:

$$t = 0.37\text{s}$$

The time taken for the 5kg body to reach the ground is 0.37s.

(ii) After snapping, the 3kg mass continues to rise with a deceleration of 9.8m/s^2 until its velocity becomes zero at the highest point.

Using:

$$v^2 = u^2 + 2as; \text{ with } v = 0, u = 3.675\text{m/s}, a = -9.8\text{m/s}^2, s = h$$

Substituting:

$$0^2 = (3.675\text{m/s})^2 - 2(9.8\text{m/s}^2)h; h = 0.689\text{m}$$

Height gained before snapping was $s = 2.756\text{m}$, so maximum height above initial position:

$$H_{\text{max}} = s + h = 2.756\text{m} + 0.689\text{m} = 3.445\text{m}$$

The maximum height is 3.445m.

- (iii) At snapping, the 5kg body is 2m above ground, and it has descended $s = 2.756\text{m}$ since release, so its initial height above ground was:

$$H_0 = 2\text{m} + 2.756\text{m} = 4.756\text{m}$$

The 3kg body started at the same level, so it also started 4.756m above ground.

It rose by $s = 2.756\text{m}$ before snapping, so its height above ground at snapping was:

$$H_{\text{snap}} = 4.756\text{m} + 2.756\text{m} = 7.512\text{m}$$

The time to reach the ground from the snapping time can be found by using:

$$s = ut + 0.5at^2; \text{ with } s = -7.512\text{m}, u = 3.675\text{m/s}, a = -9.8\text{m/s}^2$$

Substituting:

$$-7.512 = 3.675t - 0.5(9.8)t^2 \text{ or } 4.9t^2 - 3.675t - 7.512 = 0$$

Solving the quadratic equation, gives the practical value of t which is:

$$t = 1.67\text{s}$$

Total time from release to ground for the 3kg body = $1.5 + t = 1.5\text{s} + 1.67\text{s} = 3.17\text{s}$

The total time is 3.17s.

If the ideas now feel familiar and connected, you are ready! Let us understand and enjoy them even more in the Digging Deeper Exercise in the next Module.