MODULE 21

CONNECTED BODIES ON HORIZONTAL PLANE

In the previous section, we analysed bodies connected by a string over a pulley where both masses hung vertically. Now we shift to a different but equally common configuration: one or more bodies moving horizontally on a surface, connected by a string or rigid bar. You will recognise this arrangement in everyday situations such as a car towing a trailer, a locomotive pulling carriages, or a person dragging a chain of boxes across a floor. Although the connection between bodies remains similar, the source of motion is no longer the same.

The key difference lies in what drives the motion. In the vertical system, the driving force came from the **difference in weights** of the two hanging masses. On a horizontal plane, however, the situation changes slightly: a block resting on a table cannot use its weight to produce horizontal motion. Instead, movement must be provided by an **external pull**, which then transmits motion through the connecting string or bar.

When analysing connected bodies on a horizontal surface, two distinct situations naturally arise depending on the nature of the surface. In the first case, the **surface is smooth** (frictionless), so the only horizontal forces are the applied pull and the tensions in the connecting string or bar, making the motion relatively straightforward to analyse. In the second case, the **surface is rough**, meaning friction acts alongside the applied pull and tension. This additional resistive force modifies the acceleration and must be carefully included when applying Newton's laws. These two cases form the foundation for understanding horizontal connected-body systems.

The general setup:

Consider two bodies of masses m_1 and m_2 connected by a light inextensible string on a horizontal surface. A horizontal force F is applied to mass m_1,which then pulls mass m_2 through the connecting string. Since the surface is horizontal, the motion is entirely in the horizontal direction and both masses share the same acceleration, a.

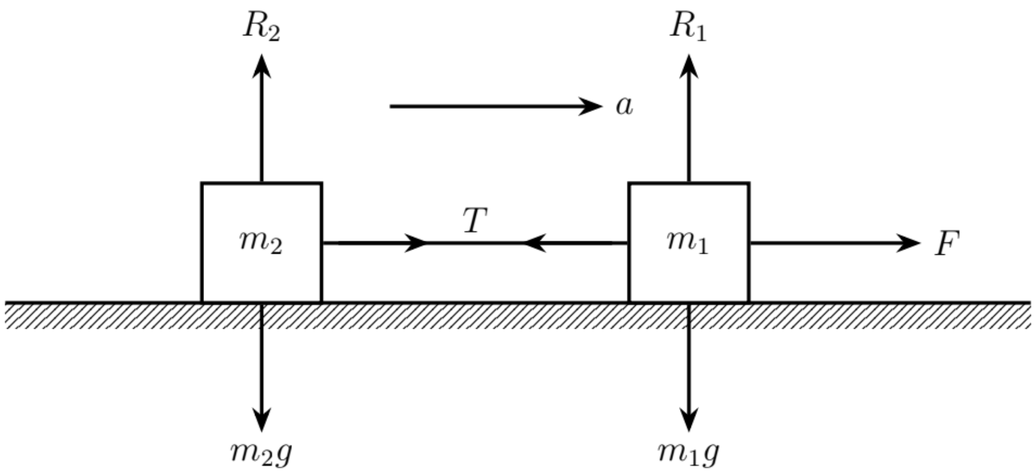


Figure: *A diagram showing two masses m_1 and m_2 connected by a light inextensible string on a horizontal surface.*

First case: Smooth horizontal surface (no friction)

Forces acting on m_1 (being pulled by F):

- Applied force: F (horizontal, forward)
- Tension: T (horizontal, backward: string pulls m_1 back toward m_2)
- Weight: m_1g (downward)
- Normal reaction: $R_1 = m_1g$ (upward)

Since no vertical motion (vertical forces balance), only horizontal forces (forward and backward forces) will be considered.

Applying Newton's second law:

$$F - T = m_1a \ldots \text{(equation 1)}$$

Forces acting on m_2 (being pulled by string):

- Tension: T (horizontal, forward: string pulls m_2 forward)
- Weight: m_2g (downward)
- Normal reaction: $R_2 = m_2g$ (upward)

Again, only horizontal forces will be considered. Thus:

$$T = m_2a \ldots \text{(equation 2)}$$

Adding equations 1 and 2:

$$F - T + T = m_1a + m_2a$$

$$F = (m_1 + m_2)a$$

Therefore, acceleration is:

$$\mathbf{a = \frac{F}{m_1 + m_2}}$$

Substituting the acceleration expression into equation (2):

$$T = m_2 \times \frac{F}{m_1 + m_2}$$

Thus, the tension is:

$$\mathbf{T = \frac{m_2F}{m_1 + m_2}}$$

What physical meaning does each formula convey?

- The acceleration formula shows the entire applied force, F, accelerates the combined total mass $(m_1 + m_2)$ of the system, just as if it were pushing a single object.
- The tension formula shows the following:
  - ✓ Tension, T in the string is only responsible for accelerating m_2 alone ($T = m_2a$). It does not need to accelerate m_1 because m_1 is already driven by the external force F.
  - ✓ Tension (T) is always less than applied force (F) (since $m_2 < m_1 + m_2$, $\frac{m_2}{m_1 + m_2} < 1$), meaning the string never experiences the full applied force.

Second case: Rough horizontal surface (with friction)

When the surfaces are rough, friction opposes the motion of each body. Friction acts backward (opposing forward motion) on both masses.

Let µ be the coefficient of **kinetic** friction between each mass and surface.

Then, frictional force (f) acting on each mass will be as follows:

$f_1 = \mu R_1 = \mu m_1 g$ (on m_1, opposing its forward motion)

$f_2 = \mu R_2 = \mu m_2 g$ (on m_2, opposing its forward motion)

Applying Newton's second law:

For m_1:

$F - T - f_1 = m_1a$

$$F - T - \mu m_1 g = m_1 a \ldots \text{(equation 1)}$$

For m_2:

$T - f_2 = m_2a$

$$T - \mu m_2 g = m_2 a \ldots \text{(equation 2)}$$

Adding equations 1 and 2:

$F - \mu m_1 g - \mu m_2 g = m_1 a + m_2 a$

$F - \mu(m_1 + m_2)g = (m_1 + m_2)a$

Therefore, acceleration is:

$$\mathbf{a = \frac{F - \mu(m_1 + m_2)g}{m_1 + m_2}}$$

Also from equation (2):

$T = m_2 a + \mu m_2 g$

$T = m_2(a + \mu g)$

But:

$$a = \frac{F - \mu(m_1 + m_2)g}{m_1 + m_2} = \mathbf{\frac{F}{m_1 + m_2} - \mu g}$$

It follows that:

$$T = m_2(a + \mu g) = m_2\left(\frac{F}{m_1 + m_2} - \mu g + \mu g\right) = \frac{m_2 F}{m_1 + m_2}$$

Hence, the tension is:

$$\mathbf{T = \frac{m_2 F}{m_1 + m_2}}$$

(*The same as the **smooth** case*)

What does the acceleration formula reveal?

- For motion to occur at all, the applied force must overcome total friction:

$$F > \mu(m_1 + m_2)g \text{ (where, } \mu(m_1 + m_2)g \text{ is the total friction)}$$

If $F \leq \mu(m_1 + m_2)g$ (which would give $a \leq 0$), the system remains stationary and static friction keeps everything at rest. The derived equations apply only when motion occurs.

- In **this particular arrangement**, friction reduces the acceleration by μg. Thus:

$$\text{Acceleration in rough}\left(\frac{F}{m_1 + m_2} - \mu g\right) = \text{Acceleration in smooth}\left(\frac{F}{m_1 + m_2}\right) - \mu g$$

BINDER Example 22

Two blocks of masses 4kg and 6kg are connected by a light inextensible string on a smooth horizontal table. A horizontal force of 20N is applied to the 6kg block, pulling both blocks along the table. Take $g = 9.8\ m/s^2$.

Calculate: (a) the acceleration of the system(b) the tension in the string connecting the two blocks.

Solution

Identifying the system:

$m_1 = 6$kg (block with applied force)

$m_2 = 4$kg (block being pulled by string)

Surface: smooth (no friction).

(a) Since the smooth was surface, the acceleration is given by:

$$a = \frac{F}{m_1 + m_2} = \frac{20N}{(6 + 4)kg} = 2m/s^2$$

The acceleration is $2m/s^2$.

(b) The tension is given by:

$$T = m_2 a = 4kg \times 2m/s^2 = 8N$$

The tension in the string is 8N.

Making Sense of the Answer: *The 20N force accelerates a total mass of 10kg, giving 2m/s² which is the same as pushing a single 10kg object with 20N on a smooth surface. The tension (8N) is less* *than the applied force (20N) because it only needs to accelerate the 4kg block, not the entire system.*

Thinking Like a Physicist: *The tension in a connecting string is usually less than the applied force because it accelerates only part of the system. This explains why tow ropes in vehicles can snap: When a vehicle brakes suddenly, the rope must transmit a large tension to slow the trailer quickly. Understanding tension in connected systems is therefore not just theoretical, it is directly linked to engineering safety.*

BINDER Example 23

Two boxes of masses 3kg and 5kg are connected by a light inextensible string on a rough horizontal floor. A horizontal force of 40N is applied to the 5kg box, pulling both boxes. The coefficient of kinetic friction between each box and the floor is $\mu = 0.3$. Take $g = 9.8\ m/s^2$.

Calculate:(a) the acceleration of the system(b) the tension in the string.

Solution

Identifying the system:

$m_1 = 5kg$ (box with applied force)

$m_2 = 3kg$ (box being pulled by string)

Surface: rough with $\mu = 0.3$.

(a) The acceleration is given by:

$$a_{rough} = a_{smooth} - \mu g = \frac{F}{m_1 + m_2} - \mu g = \frac{40N}{(5+3)kg} - (0.3 \times 9.8\ m/s^2) = 2.06 m/s^2$$

The acceleration is $2.06 m/s^2$.

Alternative solution

Treating the whole system as one body:

Total mass, $m_t = m_1 + m_2 = 5kg + 3kg = 8kg$

Total frictional force, $f_t = \mu m_t g = 0.3 \times 8kg \times 9.8\ m/s^2 = 23.52N$

Resultant force, $F_R = F - f_t = 40N - 23.52N = 16.48N$

Then from $F = ma$ or $a = \frac{F}{m}$;

The acceleration of the system, $a = \frac{F_R}{m_t} = \frac{16.48\text{N}}{8\text{kg}} = 2.06\text{m/s}^2$

(b) The tension is given by:

$$T = m_2 a_{smooth} = \frac{m_2 F}{m_1 + m_2} = \frac{3\text{kg} \times 40\text{N}}{(5+3)\text{kg}} = 15\text{N}$$

The tension is 15N.

Making Sense of the Answer: *The acceleration is smaller than it would be on a smooth surface because friction opposes motion.*

Thinking Like a Physicist: *Friction reduces acceleration but does not directly dictate the tension formula, which depends on how the force is shared between the masses.*

REAL Example 24

Tow ropes used to pull trailers sometimes snap when the towing vehicle brakes suddenly. Explain why this happens.

Solution

When the vehicle brakes suddenly, it slows down quickly but the trailer tends to keep moving forward due to inertia. The tow rope then becomes tightly stretched and develops a large tension as it tries to stop the trailer in a short time. If this tension becomes too great, the rope may snap.

HOT Example 25

Three blocks A, B and C of masses 2kg, 3kg and 5kg respectively are connected in series by two light inextensible strings on a rough horizontal surface. Block A is at the back, connected to block B by string 1, and block B is connected to block C by string 2. A horizontal force F = 50N is applied to block C at the front, pulling the entire system forward. The coefficient of kinetic friction between each block and surface is $\mu = 0.25$.Take $g = 9.8\ \text{m/s}^2$.

(a) Calculate the acceleration of the system.

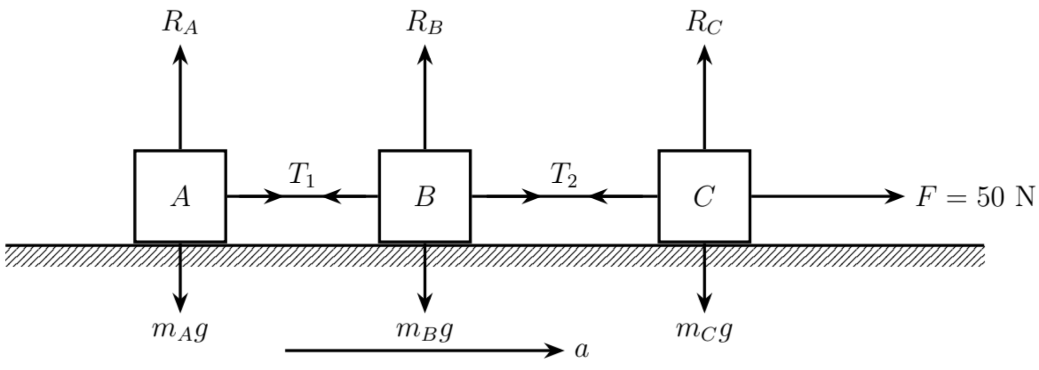
(b) Find the tension T_1 in the string between A and B.

(c) Find the tension T_2 in the string between B and C.

(d) Show that $T_2 > T_1$ and explain why this must always be the case.

Solution

(a) Consider the following diagram.



Treating the entire system as one body:

Total mass, $m_t = m_A + m_B + m_C = 2kg + 3kg + 5kg = 10kg$

Total frictional force, $f_t = \mu m_t g = 0.25 \times 10kg \times 9.8\ m/s^2 = 24.5N$

Resultant force, $F_R = F - f_t = 50N - 24.5N = 25.5N$

Then $a = \frac{F_R}{m_t} = \frac{25.5N}{10kg} = 2.55m/s^2$

The acceleration of the system is $2.55m/s^2$.

(b) Horizontal forces acting on block A:

- Tension: T_1 (forward)
- Frictional force: $f_A = \mu m_A g = 0.25 \times 2kg \times 9.8\ m/s^2 = 4.9N$ (backward)

Applying Newton's second law:

$$F_R = T_1 - f_A = m_A a \text{ or } T_1 = m_A a + f_A$$

Substituting:

$$T_1 = 2kg \times 2.55m/s^2 + 4.9N = 10N$$

The tension T_1 is 10N.

(c) Horizontal forces acting on block B:

- Tension: T_1 (backward)
- Tension: T_2 (forward)
- Frictional force: $f_B = \mu m_B g = 0.25 \times 3kg \times 9.8\ m/s^2 = 7.35N$ (backward)

Applying Newton's second law:

$$F_R = T_2 - T_1 - f_B = m_B a \text{ or } T_2 = m_B a + T_1 + f_B$$

Substituting:

$$T_2 = 3kg \times 2.55m/s^2 + 10N + 7.35N = 25N$$

The tension T_2 is $25N$.

Alternative solution

From the diagram, it is clearly understood that the tension T_2 is responsible for pulling both block A and block B.

So total mass pulled by $T_2 = m_A + m_B = (2 + 3)kg = 5kg$

And total friction (on block A and B) $= \mu mg = 0.25 \times 5kg \times 9.8\ m/s^2 = 12.25N$ (backward)

Applying Newton's second law:

$$F_R = T_2 - f = ma \text{ or } T_2 = ma + f$$

Substituting:

$$T_2 = ma + f = 5kg \times 2.55m/s^2 + 12.25N = 25N$$

(d) From results in (b) and (c): $T_1 = 10N, T_2 = 25N$; hence: $T_2 > T_1$ (shown).

Explanation

T_2 pulls both A and B (total mass 5kg) against greater friction, while T_1 pulls only A (2kg) against less friction; therefore, T_2 must be larger than T_1.

Making Sense of the Answer: *The three tensions in this problem tell a clear story:* $F = 50N$ *(applied to C)* $\to T_2 = 25N$ *(transmitted to B and A)* $\to T_1 = 10N$ *(transmitted to A only). Each string experiences progressively less tension as it has fewer masses to pull. This cascade of decreasing tension from front to back is a universal feature of any train of connected bodies.*

Thinking Like a Physicist: *Railway engineers exploit this principle when designing train couplings. Each coupling must be rated for the maximum tension it will experience, and the front coupling is always under the greatest tension. When a locomotive accelerates a long train from rest, the front coupling experiences the highest tension; a failure there would split the train. This is also why heavy trucks use reinforced front tow hitches. The front connection always bears the greatest mechanical load in a chain of connected vehicles.*

HOT Example 26

A block of mass 4kg rests on a horizontal table. It is connected by a light inextensible string passing over a smooth pulley fixed at the edge of the table to a hanging mass of 3kg. The system is released from rest. Take $g = 9.8 \, m/s^2$.

(a) Assuming the table surface is smooth (frictionless), calculate:

(i) the acceleration of the system,

(ii) the tension in the string.

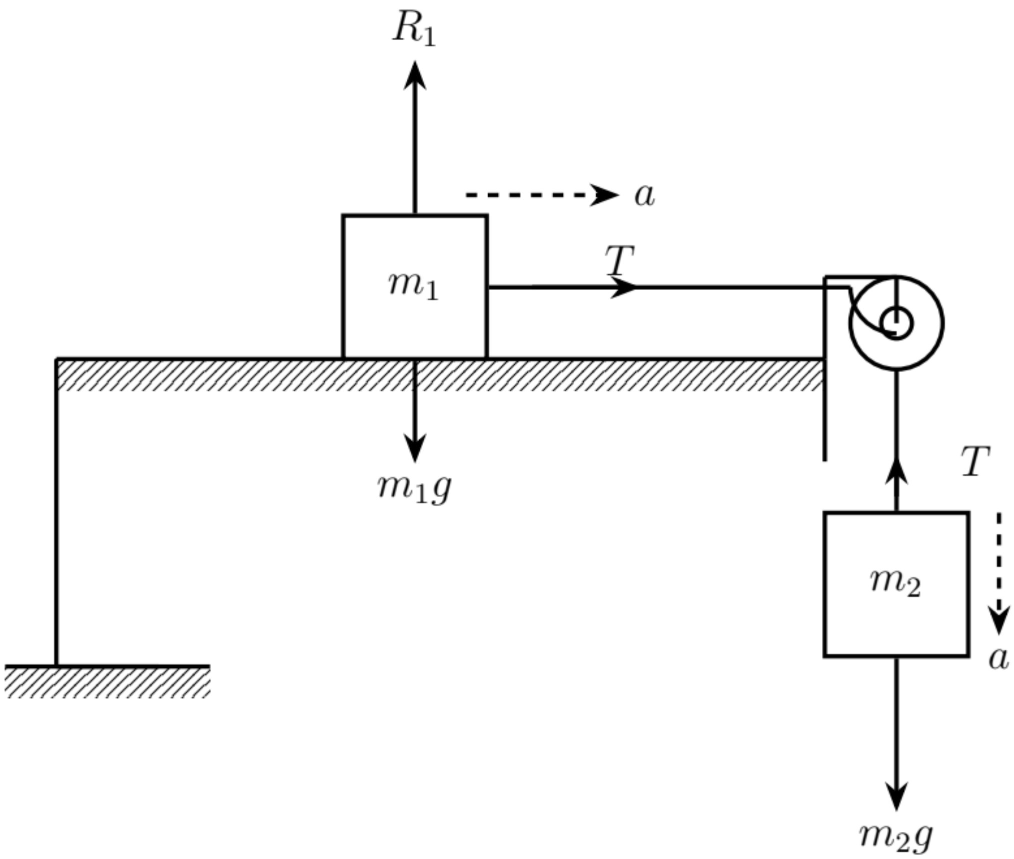
(b) Now assume the table surface is rough with coefficient of kinetic friction $\mu = 0.25$. Calculate:

(i) the acceleration of the system,

(ii) the tension in the string.

Solution

Consider the following diagram.



(a) **For smooth surface (No friction):**

Horizontal forces acting on m_1 (on table, moving horizontally, forward):

- Tension, T (horizontal, forward)

Vertical forces acting on m_2 (hanging, moving vertically, downward):

- Weight, m_2g (downward)
- Tension, T (upward)

Since the string is inextensible, both masses have the same acceleration magnitude, a.

For m_1:

$$T = m_1 a \text{ ... (equation 1)}$$

For m_2:

$$m_2g - T = m_2a \text{ ... (equation 2)}$$

Adding equations (1) and (2):

$$T + m_2g - T = m_1a + m_2a$$

$$m_2g = (m_1 + m_2)a$$

Rearrange to make a the subject:

$$a = \frac{m_2g}{m_1 + m_2}$$

Substituting values:

$$a = \frac{3kg \times 9.8m/s^2}{4kg + 3kg} = 4.2m/s^2$$

The acceleration on a smooth surface is $\mathbf{4.2m/s^2}$.

From equation (1):

$$T = m_1a = 4kg \times 4.2m/s^2 = 16.8N$$

For smooth surface, the tension is $\mathbf{16.8N}$.

A quicker way to calculate acceleration:

With this method, you only need to identify the source of the external pull and then apply Newton's second law of motion, $\boldsymbol{F = ma}$, directly.

In this example, the pulling force comes from the weight of the hanging mass, m_2g. This weight provides the force that accelerates both m_1 and m_2.

Thus:

$$F = m_2g, m = m_1 + m_2$$

It follows that:

$$m_2g = (m_1 + m_2)a \text{ or } \mathbf{a} = \frac{\mathbf{m_2g}}{\mathbf{m_1 + m_2}} = \frac{3kg \times 9.8m/s^2}{4kg + 3kg} = 4.2m/s^2$$

Making Sense of the Answer (Part a): *The acceleration ($4.2m/s^2$) is less than g because the 4kg block on the table resists the downward pull of the 3kg hanging mass (the hanging mass also* *accelerate the block horizontally). The tension* $(16.8N)$ *is less than the weight of the hanging mass* $(29.4N)$ *because the hanging mass is accelerating downward; if it were stationary, the tension would equal its full weight.*

(b) **For smooth surface (No friction):**

In this case, the pulling force (m_2g) has to overcome frictional force $(\mu m_1 g)$ before accelerating m_1 and m_2.

So the resultant force, $F_R = F - f = m_2g - \mu m_1 g$

Applying Newton's second law of motion:

$$m_2g - \mu m_1 g = (m_1 + m_2)a$$

From which:

$$\mathbf{a} = \frac{\mathbf{m_2 g - \mu m_1 g}}{\mathbf{m_1 + m_2}} = \frac{(3\text{kg} \times 9.8\text{m/s}^2) - (0.25 \times 4\text{kg} \times 9.8\text{m/s}^2)}{(4+3)\text{kg}} = 2.8\ \text{m/s}^2$$

The acceleration on a rough surface is $\mathbf{2.8m/s^2}$.

Considering the hanging mass, m_2.

$$m_2g - T = m_2a \text{ or } T = m_2g - m_2a = m_2(g-a) = 3\text{kg}(9.8-2.8)\text{m/s}^2 = 21\text{N}$$

For rough surface, the tension is 21N.

Making Sense of the Answer (Part b): *The friction force* $(9.8N)$ *significantly reduces the acceleration from* $4.2m/s^2$ *to* $2.8m/s^2$*. This makes physical sense: friction opposes the forward motion of the block, requiring part of the hanging mass's weight to overcome friction rather than producing acceleration. Interestingly,* ***the tension increases from*** $\mathbf{16.8}N$ ***to*** $\mathbf{21}N$***. Why?*** *Because now the string must not only accelerate the block but also overcome friction acting on it. The string works harder in the rough case.*

Thinking Like a Physicist: *This problem illustrates a fundamental principle: adding friction to any part of a connected system reduces overall acceleration but may increase forces in connecting elements. This has practical implications: in elevator systems, increased cable friction means the motor must work harder (higher cable tension) even though the elevator accelerates more slowly. Similarly, towing a vehicle on a muddy road (high friction as it has greater μ value) puts greater tension on the tow cable despite slower acceleration. Engineers must design cables and connections to withstand these higher tensions, not just the smooth-surface ideal case.*