MODULE 19

STANDARD FORCE MODELS

In equilibrium problems, correctly identifying the forces acting on a body is often more important than the mathematics that follows. Fortunately, many physical situations involve a small number of common force types that appear repeatedly. Recognising these standard force models allows you to draw accurate free-body diagrams quickly and apply the equilibrium equations with confidence. Among the most frequently encountered forces are **weight (W)**, **normal reaction (R), tension (T)**, and **friction**. Each has characteristic origins, directions, and physical interpretations, and mastering them forms an essential foundation for solving problems in Particle Mechanics.

Understanding these force models not only improves diagram accuracy but also prevents common mistakes such as inventing forces, misdirecting forces, or overlooking important interactions. Once these forces are correctly identified, the equilibrium equations: $\Sigma F_x = 0$ and $\Sigma F_y = 0$ become straightforward to apply.

And just as a well-prepared free-body diagram was compared to the essential spices in **pilau**, these standard force models are the actual ingredients themselves. Without recognising them properly, you may still attempt the analysis, but the final result can feel as disappointing as pilau cooked without the right spices; it may fill the stomach, but it certainly does not excite the taste buds, much like solving Physics problems without properly identifying the forces.

Weight, W

Weight is the gravitational force exerted on a body due to the Earth's gravitational field. It is one of the most fundamental forces encountered in Particle Mechanics because it acts on virtually every object near the Earth's surface.

The magnitude of the weight is given by: $W = mg$

Where:

m = mass of the body

g = acceleration due to gravity (approximately 9.8 m/s^2 near the Earth's surface).

Recognising weight correctly in free-body diagrams is essential. It is often the first force to identify, and errors in its direction or magnitude can lead to incorrect equilibrium analysis.

It is very important to understand that:

Weight always acts *vertically downward*, towards the centre of the Earth, regardless of the body's orientation or motion. *Even when an object rests on an inclined plane, hangs from a string, or moves through the air, its weight **still acts vertically downward**.*

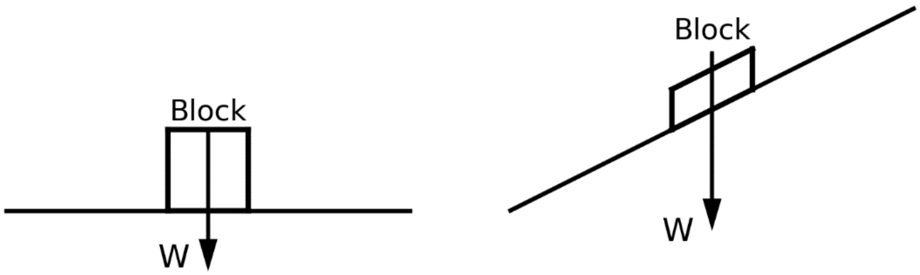


Figure: *The weight of a body always acts vertically downward towards the centre of the Earth, regardless of whether the body rests on a horizontal surface, an inclined plane, or is suspended in space. Surface orientation does not affect the direction of weight.*

Components of Weight Parallel and Normal to an Inclined Plane

When a body rests on an inclined plane, its weight still acts vertically downward toward the centre of the Earth. However, because the surface is tilted, it is often more convenient to analyse this single gravitational force by resolving it into two components: one **parallel to the plane** and one **perpendicular (normal) to the plane**. This approach simplifies equilibrium analysis because each component directly relates to the forces (friction and normal reaction) that usually oppose them: friction acts parallel to the surface (opposite to the weight's parallel component), while the normal reaction acts perpendicular to it (opposite to the weight's perpendicular component).

To have better understanding of this, consider a body of weight, **W = mg** placed on a plane inclined at an angle **θ to the horizontal**. Resolving the weight into components relative to the plane gives:

- **Component parallel to the plane** $= W\sin\theta = mg\sin\theta$
- **Component perpendicular to the plane** $= W\cos\theta = mg\cos\theta$

These expressions arise from simple right-triangle geometry formed when the vertical weight vector is resolved relative to the inclined surface as shown in the figure.

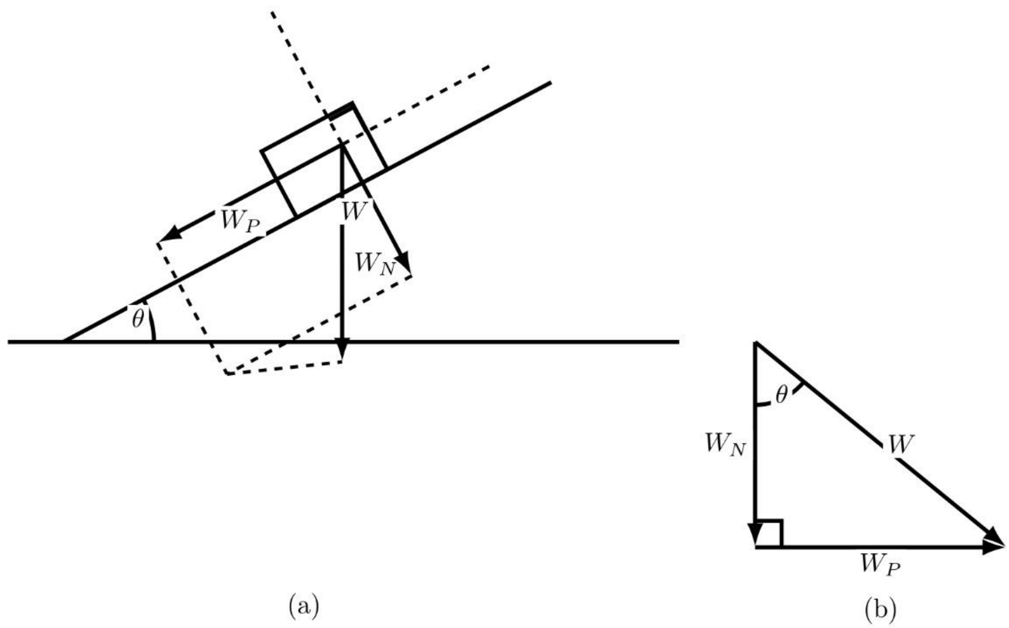


Figure: ***(a)*** *Weight (W) of a body on an inclined plane resolved into components parallel to the plane (W_P) and normal (perpendicular) to the plane (W_N).* ***(b)*** *Component triangle showing the geometric relationship between the weight and its parallel and normal components.*

From the figure it is clearly understood that when a body rests on an inclined plane, its weight $W = mg$ acts vertically downward. For analysis relative to the plane, we resolved this single force into two mutually perpendicular components:

1) W_P: component parallel to the plane (down the slope).

2) W_N: component normal (perpendicular) to the plane (into the plane).

From diagram (a), the three vectors form a right-angled triangle: the hypotenuse is W, and other sides are W_N and W_P.

Before we proceed, let us first familiarise ourselves with the key geometric features of the diagram.

1. The plane makes an angle θ with the horizontal.
2. The normal line is perpendicular to the plane, so the normal is tilted by θ from the vertical.
3. Since W is vertical, the angle between W and the normal direction is θ.
   So, in the component triangle (diagram b), θ is the angle between W and W_N.

Now, in the tringle:

Hypotenuse $= W$, Adjacent to $\theta = W_N$, Opposite to $\theta = W_P$

Therefore:

$$\cos\theta = \frac{\text{adjacent}}{\text{hypotenuse}} = \frac{W_N}{W}$$

$$\sin\theta = \frac{\text{opposite}}{\text{hypotenuse}} = \frac{W_P}{W}$$

Hence:

$$\mathbf{W_N = W\cos\theta = mg\cos\theta = Normal\ (perpendicular)\ component\ of\ the\ weight}$$

$$\mathbf{W_P = W\sin\theta = mg\sin\theta = Parallel\ component\ of\ the\ weight}$$

An important conceptual point is that resolving weight does **not create new forces**. The weight remains a single gravitational force acting vertically downward. The components are simply a convenient way of describing how that force influences motion relative to the plane.

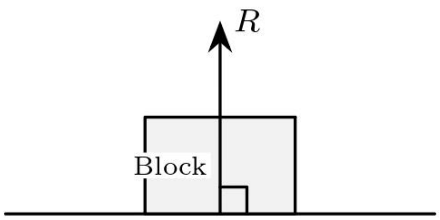
Understanding these components is essential in Particle Mechanics because many real systems: vehicles on slopes, ladders against walls, cables supporting loads, or objects resting on ramps; depend on correctly analysing forces relative to inclined surfaces. Once the weight is resolved appropriately, applying the equilibrium equations: $\mathbf{\Sigma F_{parallel} = 0}$ and $\mathbf{\Sigma F_{perpendicular} = 0}$ becomes straightforward and systematic.

In practice, *whenever an inclined plane appears in a problem, resolving the weight parallel and perpendicular to the plane is usually the first and most important step toward a clear and accurate solution.*

Normal Reaction, R (or R_n)

The **normal reaction** *is the force exerted by a surface on a body in contact with it*. It arises due to the interaction between the surfaces and acts to prevent the two bodies from passing through each other and thus they do not occupy the same space. In equilibrium problems, the normal reaction is one of the most frequently encountered contact forces.

The direction of the normal reaction is always perpendicular (**normal**) to the surface of contact. This is an important point: *the normal reaction is not necessarily vertical*. It becomes vertical **only** when the surface itself is horizontal. When a body rests on an inclined plane, for example, the normal reaction acts perpendicular to that plane rather than vertically upward.



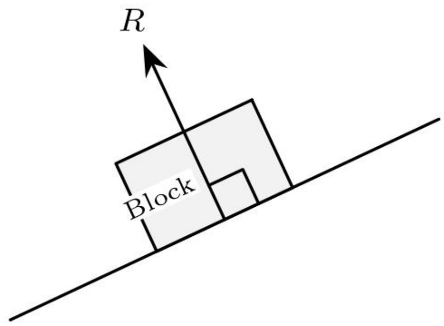


Figure: *The normal reaction always acts perpendicular to the surface of contact. Its direction depends on the orientation of the surface and is not always vertical.*

Correct identification of the normal reaction in free-body diagrams is essential for accurate equilibrium analysis. Confusing its direction with that of weight is a common mistake, especially in problems involving inclined surfaces.

It should also be remembered that the normal reaction is a responsive force. It adjusts its magnitude according to the situation. In some cases, it may equal the weight of the body, but this is not a general rule; additional forces acting on the body can increase or decrease its value. **For example,** *if a downward push is applied in addition to weight, the normal reaction increases; if a lifting force acts upward, the normal reaction decreases.*

The following points summarise the key features of the normal reaction:

1. It always acts perpendicular to the surface of contact.

2. It exists only when there is physical contact between the body and the surface.

3. Its magnitude is not necessarily equal to the weight of the body. Equality occurs only in special cases, such as when a body rests on a horizontal surface with no other vertical forces acting.

4. For a body of mass, m resting on an inclined plane at an angle θ to the horizontal, **the normal reaction is $R = mg\cos\theta$** (normal component of weight) provided that no other forces act in that perpendicular direction.

Tension, T

In Particle Mechanics, **tension** is one of the most common forces in equilibrium problems. Tension appears whenever a body is pulled, supported, or suspended by a string, rope, cable, chain, or wire.

From hanging signboards and ceiling lamps to suspension bridges, cranes, elevators, and even the strap of your school bag, tension forces are quietly working to maintain balance.

Concisely, tension *is the **pulling force** transmitted through a **stretched** string, rope, cable, or wire.*

A key feature of tension is its direction. Unlike the normal reaction, which pushes perpendicular to a surface, tension always acts **along the length** of the string or cable and pulls **away from the body** to which it is attached. Therefore, when representing tension in a free-body diagram, always draw the arrow pointing **away from (NOT toward) the body along the string or cable** (see the figure). This reflects the physical fact that a string can only pull, not push. *Drawing the arrow toward the body may suggest compression rather than tension and can lead to **incorrect** interpretation of the forces acting on the system.*

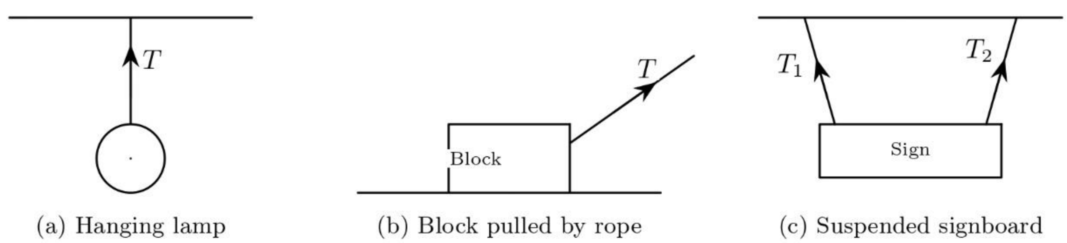


Figure: *In all cases, tension acts along the string or cable and always pulls away from the body.*

Modelling assumptions used in A-Level mechanics

To make problems solvable, we often use idealised models:

- **Light (massless) string:**

  The string's weight is negligible, so we ignore its weight in the analysis.

- **Inextensible string:**

  The string does not stretch, so its length remains constant.

- **Smooth pulley (if present):**

  No friction between string and pulley, so the tension remains the same on both sides of the pulley.

Under these ideal conditions, the tension has the same magnitude throughout a continuous string.

Important clarifications about tension

- **Tension is not automatically equal to weight.**

For a hanging object at rest, tension equals weight only if the object is supported by a single vertical string and no other vertical forces act.

- **More than one string means tensions can share the load.**

For example, if a body is supported by two strings at angles, each string provides a tension that contributes a **vertical component** supporting the weight.

- **Tension acts at the point of attachment.**

In particle mechanics we often draw it as acting on the particle, but the physical meaning is **"the string pulls the body at the attachment point."**

- **Tension exists only when the string is taut**

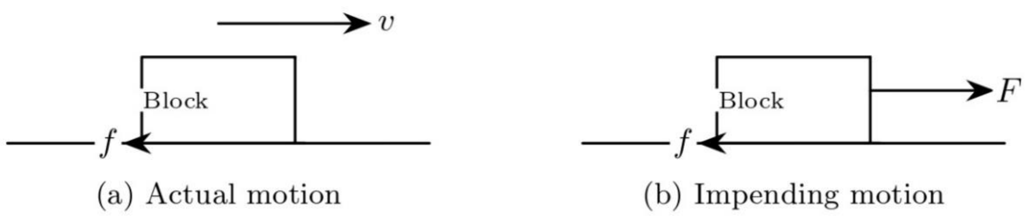
If the string becomes slack, it can no longer exert a pulling force; therefore, the tension effectively becomes zero.

Friction Force, f

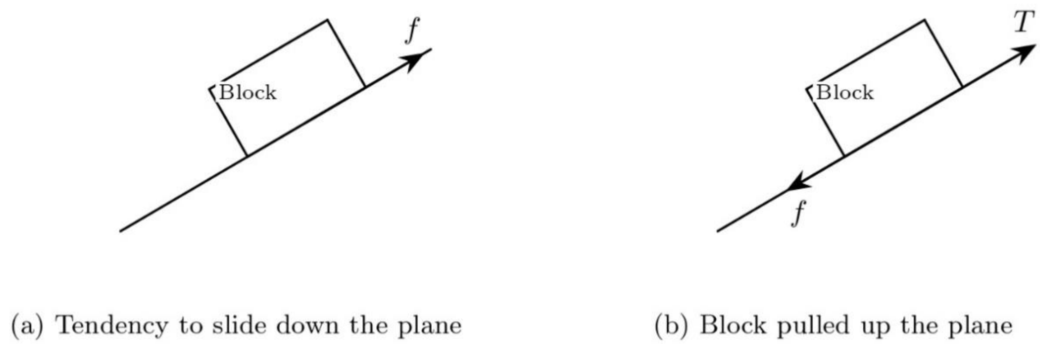
In Particle Mechanics, **friction** is another very common force encountered in many physical situations, including both equilibrium and motion. From walking without slipping and vehicles braking safely to ladders resting against walls and objects remaining stationary on inclined surfaces, friction plays an important role in maintaining stability, control, and balance in everyday life.

Friction is a contact force that arises when two surfaces touch and there is relative motion, or a tendency for relative motion, between them. It acts along the surface of contact and always opposes the direction in which motion occurs or would occur.

A crucial feature of friction is its direction. **Friction always acts parallel to the surface and opposite to the direction of relative motion or the tendency to move.** Therefore, when representing friction in a free-body diagram, always draw the friction force parallel to the surface and opposite to the direction of motion or impending motion (see the figure).



Friction on a horizontal surface: *Friction acts along the surface of contact and always opposes either the actual motion of the body or its tendency to move.*



Friction on an inclined plane: *Friction acts parallel to the plane and opposes the tendency of motion, acting up the plane if the body tends to slide down and down the plane if the body is pulled upward.*

Two main types of friction are commonly considered in Particle Mechanics:

1. Static friction

Static friction acts when two surfaces are in contact but there is **no relative motion** between them. Its role is to prevent motion from starting.

An important property of static friction that its magnitude adjusts itself as needed (up to a limiting value) to prevent motion. This is why a gently pushed object may remain at rest as friction increases just enough to maintain equilibrium.

The adjustment obeys the following condition:

$$\mathbf{f \leq \mu R}$$

Where:

f is frictional force.

μ is coefficient of static friction.

R is normal reaction between the surfaces.

It is worth to understand that when static friction reaches its maximum possible value just before motion begins, it is called **limiting friction** and the body is said to be at **limiting equilibrium**.

At this point:

$$\mathbf{f = limiting\ friction = \mu R}$$

Beyond this point, motion starts and static friction is no longer existing.

2. Kinetic (sliding) friction

Once motion begins, static friction no longer operates because the surfaces are no longer at rest relative to each other. However, a resisting force still exists that opposes the motion. This force is known as **kinetic (**or **sliding) friction**. Its magnitude is usually slightly smaller than the maximum static friction (limiting friction), which explains why an object often moves more easily once it has started sliding.

Important clarifications about friction

1. **Friction always acts parallel to the contact surface and never perpendicular to it**
   Although friction is related to the normal reaction (the perpendicular contact force), friction itself does not act perpendicular to the surface. Instead, it always acts parallel to the surface of contact.

2. **Friction is not always present**
   In many mechanics models, surfaces may be treated as smooth (frictionless). In such cases, friction is assumed to be zero to simplify analysis.

3. **Friction does not always equal μR**
   The relation f = μR applies only at limiting equilibrium. In many situations, friction is less than μR.

4. **Friction opposes motion or attempted motion**
   It does not necessarily oppose an applied force directly; it opposes the resulting motion tendency.

5. **Friction exists only when surfaces are in contact**

If contact is lost, friction immediately becomes zero.

Do not invent friction: A conceptual warning

Friction should never be invented simply to "make forces balance." It must have a physical cause: contact between surfaces with a tendency to slide. If no such tendency exists, friction is zero. For example: *A book resting on a horizontal table experiences only its weight downward and the normal reaction upward. Since there is no horizontal force tending to cause motion, no friction is required, and the frictional force is therefore zero.*

Combining All Four Forces: W, R, T, and f

Up to this point, we have examined the standard forces of Particle Mechanics one at a time:

- **Weight (W)** acting vertically downward due to gravity,
- **Normal reaction (R)** arising from surface contact and acting perpendicular to that surface,
- **Tension (T)** transmitted through strings, ropes, or cables and always pulling away from the body, and
- **Friction (f)** acting parallel to the surface and opposing relative motion or the tendency of motion.

Individually, each force is straightforward. But real physical situations rarely present forces one by one. A block may rest on a rough inclined plane while attached to a string, a ladder may lean against a wall with friction preventing slipping, or a suspended load may simultaneously experience tension, weight, and contact forces. Understanding equilibrium therefore requires seeing **how these forces work together**.

This integration marks an important step: you are no longer just recognising forces; you are analysing complete physical systems.

A unified view of equilibrium

When all four forces appear together, three fundamental direction rules must always be remembered:

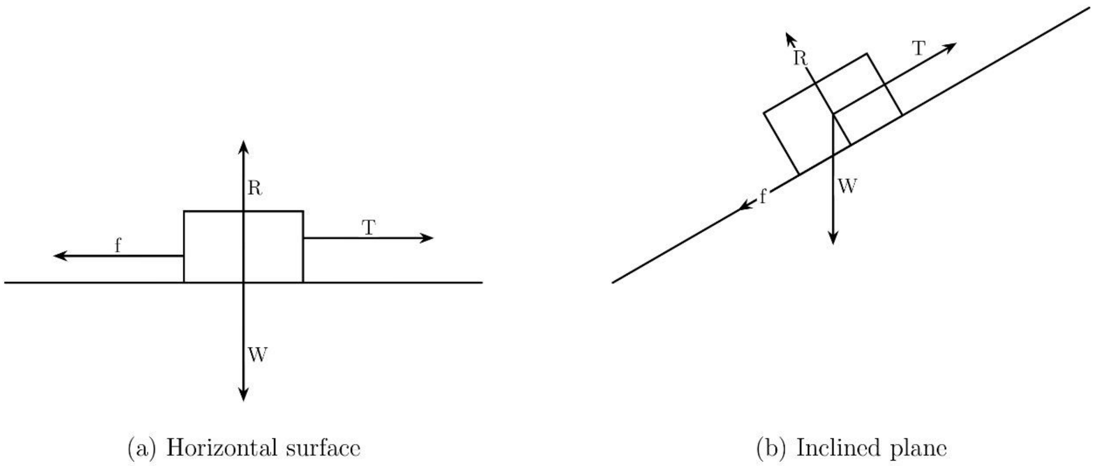
Rule 1: ***Weight (W)*** *always acts vertically downward, regardless of the surface orientation.*

Rule 2: ***Normal reaction (R)*** *always acts perpendicular to the contact surface.*

Rule 3: ***Tension (T)*** *always acts along the string or cable, pulling away from the body.*

Rule 4: ***Friction (f)*** *always acts parallel to the surface and opposes actual or impending motion.*

These directional rules are not optional; they come directly from the physical origin of each force. Correct free-body diagrams depend on respecting these directions.



Forces on a block on horizontal and inclined surfaces: *Weight (W), normal reaction (R), tension (T), and friction (f) acting on a body.*

Have you noticed this interesting fact?

Weight comes from gravity,

Reaction comes from contact,

Tension comes from connection,

Friction comes from surface interaction.

Different physical origins; yet all cooperate to maintain equilibrium.

Physics often looks complicated only until the forces are identified correctly. Once the forces are clear, equilibrium analysis becomes systematic rather than mysterious.

Why this integration matters?

Combining W, R, T, and f allows us to analyse many practical situations, including:

1. Bodies on rough inclined planes supported by strings,

2. Ladders resting against walls,
3. Suspended structures in contact with surfaces,
4. Engineering systems where stability depends on multiple interacting forces.

These are not artificial textbook problems; they reflect how structures remain safe and stable in everyday life.

Congratulation!

You have now completed one of the most important sections in Particle Mechanics. Mastery of free-body diagrams, the two equilibrium equations ($\Sigma F_x = 0$ and $\Sigma F_y = 0$), and the standard force models (weight, normal reaction, tension, and friction) gives you powerful tools for analysing forces with confidence. These ideas will not only make Particle Mechanics clearer and more enjoyable, but will also support your understanding across many other areas of Physics where balance, motion, and interaction are involved.

From this stage onward, equilibrium problems become less about memorising formulas and more about thinking clearly. And that, ultimately, is the goal of Advanced Physics: not just solving problems, but understanding why systems remain balanced in the first place.

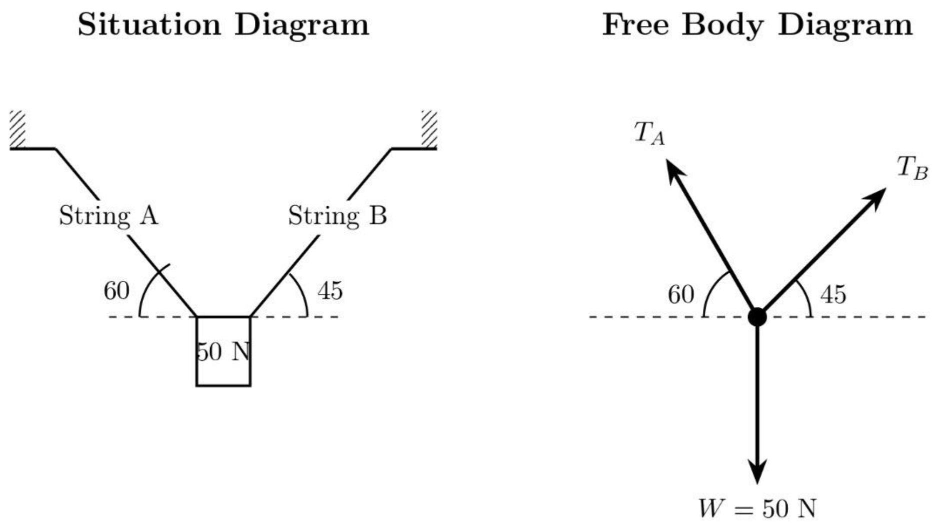
With the ideas now simmering nicely, let us serve them properly through a few worked examples and enjoy the flavour of physics in action.

BINDER Example 9

A 50N weight is suspended at rest from a junction by two light strings. **String A** is inclined at 60° to the horizontal and **string B** is inclined at 45° to the horizontal (both above the horizontal). Calculate the tension in each string.

Solution

To be at rest (equilibrium), the two strings must pull upward and outward from opposite sides of the load as shown in the **situation diagram**.



Resolving each force to horizontal and vertical component (refer to the free body diagram):

For string A:

$$\theta = \text{angle measured from positive x} - \text{axis} = 180° - 60° = 120°$$

Horizontal component $= (T_A)_x = T_A\cos120° = -0.5T_A$

Vertical component $= (T_A)_y = T_A\sin120°$

For string B:

$$\theta = \text{angle measured from positive x} - \text{axis} = 45°$$

Horizontal component $= (T_B)_x = T_B\cos45°$

Vertical component $= (T_B)_y = T_B\sin45°$

For weight:

Weight always acts vertically downward; therefore, its horizontal component is zero, while its vertical component is equal in magnitude to the weight but directed downward ($-\mathbf{50N}$).

This can also be shown using the **standard resolution method** as follows:

$$\theta = \text{angle measured from positive x} - \text{axis} = 270°$$

Horizontal component $= W_x = W\cos270° = 0$

Vertical component $= W_y = W\sin 270° = -W = -50N$

From equilibrium equations: : $\Sigma F_x = 0$ and $\Sigma F_y = 0$

Then;

$$\Sigma F_x = -0.5T_A + T_B\cos 45° + 0 = 0 \text{ or}$$

$-0.5T_A + T_B\cos 45° = 0 \ldots\ldots$ (i)

And;

$$\Sigma F_y = T_A\sin 120° + T_B\sin 45° - 50 = 0 \text{ or}$$

$T_A\sin 120° + T_B\sin 45 = 50 \ldots\ldots$ (ii)

Solving (i) and (ii) simultaneously gives: $T_A = 36.6N$; $T_B = 25.9N$

Tension in string A is 36.6N.

Tension in string B is 25.9N.

Making Sense of the Answer: *The tensions are unequal because the strings are inclined at different angles. The string at 60° (string A) provides a larger vertical component, so it carries more of the weight. Their vertical components balance the 50N load, while the horizontal components cancel maintaining equilibrium.*

Thinking Like a Physicist: *Instead of guessing signs,* ***measure angles consistently from the positive x-axis****. For example, using 120° rather than 60° automatically gives the correct horizontal and vertical components. This systematic approach reduces sign errors and keeps equilibrium analysis clear.*

Method Insight: *In this first example, the weight was resolved explicitly using the general vector method to illustrate the procedure clearly. In most later problems, since weight acts vertically downward, we will usually write its components directly without repeating the full resolution each time.*

BINDER Example 10

A 5kg block rests on a horizontal table. A horizontal force of 12N is applied to the block. The coefficient of static friction between block and table is $\mu = 0.4$.

(a) Determine whether the block remains in equilibrium.

(b) Find the magnitude of the frictional force acting on the block.

$$\text{Take g} = 9.8\text{m}/s^2$$

Solution

Normal reaction: $R = mg = 5\text{kg} \times 9.8\text{N/kg} = 49\text{N}$

Limiting friction: $f = \mu R = 0.4 \times 49\text{N} = 19.6\text{N}$

But the applied force, $F = 12\text{N}$ which is smaller than the limiting friction.

(a) Since the applied force (12N) is smaller than the limiting friction (19.6N), the static friction is sufficient to prevent motion and hence the block remains in equilibrium.

(b) When the block is at equilibrium, frictional force equals the applied force (12N) and hence the frictional force acting on the block is 12N.

Making Sense of the Answer: *Static friction adjusts itself to oppose the applied force up to its maximum value. Since the applied force (12N) is less than the maximum possible static friction (19.6N), the block does not move, and friction simply matches the applied force to maintain equilibrium.*

Thinking Like a Physicist: *Do not assume friction is always μR. That expression gives only the maximum static friction. Always compare the applied force with this maximum first; if it is smaller, the actual friction equals the applied force, not μR.*

REAL Example 11

Kipanga is pulling his overloaded school bag along the ground using a strap. He can pull horizontally or at an angle upward. Explain why pulling at an angle upward might make it easier to move the bag, even though some of the force is "wasted" going upward instead of forward.

Solution

Pulling the bag at an upward angle reduces the **normal reaction (R)** between the bag and the ground. When Kipanga pulls partly upward, that upward component of the pulling force slightly lifts the bag, so the ground presses on it less strongly.

Since friction depends on the normal reaction (f = μR), a smaller normal reaction means **less frictional force resisting motion**. Although part of the pulling force is directed upward rather than forward, the reduction in friction usually outweighs this "loss," making the bag easier to move.

Making Sense of the Answer: *Physics often involves trade-offs. Sacrificing some forward force to reduce friction can make pulling easier overall. Efficiency is not always about pushing hardest in one direction.*

Thinking Like a Physicist: *Real-world situations often involve balancing competing effects. The best pulling angle can be analysed mathematically at advanced levels, but the key idea is simple: reducing resistance can be more effective than simply increasing effort.*

REAL Example 12

During a practical lesson, **Kipute** places her Physics textbook on a desk that is tilted at a small angle. The book stays in place.

Kipanga: *"The book must be glued to the desk! Otherwise gravity would pull it down."*

Kipute: *"Don't be silly! There's no glue. But Kipanga has a point, Mr. Akilikubwa; why doesn't the book slide?"*

Mr. Akilikubwa: (smiling) *"Ah! You have discovered friction, the invisible guardian. No glue needed. The desk surface and the book touch, and friction acts like tiny invisible hands holding the book in place, opposing the tendency to slide."*

Kipanga: *"So friction is always there?"*

Mr. Akilikubwa: *"Not quite! Friction only appears when there is a tendency to move. Place the same book on a perfectly horizontal desk……, no tendency to slide, so friction takes a break and remains zero. Tilt the desk slightly, and friction wakes up just enough to prevent motion. Tilt it more, and friction increases up to its maximum strength. Tilt it too much, and friction loses the battle; the book slides. Friction is a responsive force, not a permanent one."*

Kipute: *"So it's like a lazy security guard who only works when needed?"*

Mr. Akilikubwa: (laughing) *"Exactly! And just like a lazy guard, once the thief (motion) escapes and the book starts sliding, the guard (static friction) stops trying and switches to a different mode called kinetic friction, which is usually weaker."*

Question: Based on this dialogue, explain:

(a) Why friction does not act on a book resting on a horizontal table.

(b) Why friction increases as the desk tilt increases (up to a limit)

(c) What happens when the book finally starts sliding.

In (b) and (c), justify your explanation with relevant mathematical equations.

Solution

(a) When the book rests on a horizontal table, weight acts vertically downward and is balanced by the normal reaction acting vertically upward. There is no force component parallel to the surface that would tend to cause motion. Since friction opposes attempted motion, and there is no such attempt, friction is zero.

(b) As the desk tilts, weight develops a component parallel to the surface (down the slope). This component tries to pull the book downward. Static friction responds by acting up the slope with exactly the right magnitude to prevent motion. The steeper the tilt, the larger the parallel component of weight, so friction must increase to maintain equilibrium up to its maximum possible value (limiting friction, $f = \mu R$).

Mathematical justification:

As the desk tilts by angle θ:

- Weight develops a component parallel to surface: $W\sin\theta$ (down the slope)
- This parallel component tries to pull the book downward.
- Static friction responds by acting up the slope: $f = W\sin\theta$ (upward)
- The steeper the tilt (larger value of $\sin\theta$), the larger $W\sin\theta$ becomes, so friction must increase to maintain equilibrium.

However, friction has a maximum value: $f_{max} = \mu R = \mu W\cos\theta$

Friction can increase up to this limit. Beyond this, equilibrium breaks.

(c) When tilt becomes too steep, the parallel component of weight exceeds the maximum static friction. At this point, static friction can no longer prevent motion, and the book begins to slide. Once motion starts, kinetic (sliding) friction takes over, which is typically smaller than the maximum static friction and thus the book continues to slide easily.

Mathematical justification:

When tilt becomes too steep: $\mathbf{W\sin\theta > \mu W\cos\theta}$ or $\mathbf{\tan\theta > \mu}$ (by dividing $W\cos\theta$ both sides)

At this point:

- Static friction reaches its maximum but can no longer prevent motion. So the book begins to slide.
- Static friction no longer operates (surfaces are moving relative to each other).
- Kinetic (sliding) friction takes over: $f_k = \mu_k R$

As $\mu_k < \mu$ (kinetic friction is weaker than maximum static friction), once the book starts sliding, it continues more easily.

Making Sense of the Answer: *Friction is not a fixed force; it adjusts itself as needed (up to a limit). Understanding this adaptive behavior prevents the common mistake of assuming friction always equals μR. The equation f = μR applies only at limiting equilibrium, just before motion begins.*

Thinking Like a Physicist: *Mr. Akilikubwa's "lazy security guard" analogy captures an important truth: friction is a passive, responsive force. It doesn't initiate anything; it merely reacts to prevent motion. This is fundamentally different from active forces like applied pushes or gravity, which act regardless of the situation.*

BINDER Example 13

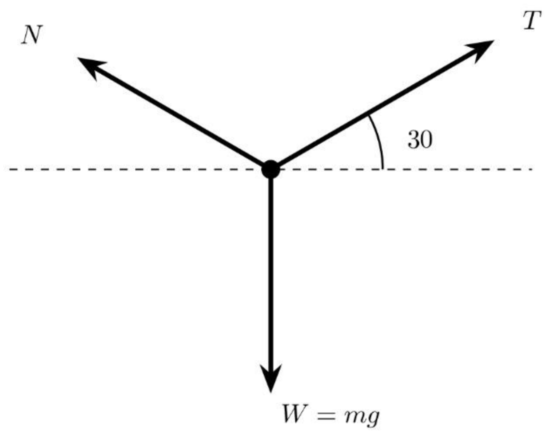
A 10kg block rests on a smooth (frictionless) inclined plane that makes an angle of 30° with the horizontal. The block is held in place by a string parallel to the plane. Take $g = 9.8\text{m/s}^2$.

Calculate:

(a) the normal reaction from the plane and (b) the tension in the string

Solution

Forces acting on the block are shown in the following free body diagram.



Resolving weight into components:

Component parallel to plane (down):

$$W_{parallel} = mg\sin\theta = 10 \times 9.8\sin 30° = 49N$$

Component perpendicular to plane (into surface):

$$W_{perp} = mg\cos\theta = 10 \times 9.8\cos 30° = 98N\cos 30°$$

Forces acting perpendicular to plane:

- Normal reaction (**away** from surface), R.
- Perpendicular component of weight (**into** surface), $98N\cos 30°$.

Since the two forces act in opposite directions and the block is at equilibrium (held in place):

$$R = 98N\cos 30° = 84.9N$$

Forces acting parallel to plane:

- Tension (upward), T.
- Parallel component of weight (downward), $W_{parallel} = 49N$

Again, the two forces act in opposite directions and the block is still at equilibrium. So:

$$T = 49N$$

(a) The normal reaction is 84.9N.

(b) The tension in string is 49N.

Making Sense of the Answer: *The normal reaction (84.9N) is less than the weight (98N) because the plane is tilted. Only the component of weight perpendicular to the plane is balanced by R. The weight component parallel to the plane (49N) tries to pull the block down, so the string must provide equal tension upward along the plane to maintain equilibrium.*

Thinking Like a Physicist: *On an incline, always resolve weight into components parallel and perpendicular to the surface. This transforms a two-dimensional problem into two separate one-dimensional problem. The steeper the incline, the larger the parallel component (harder to hold) and the smaller the perpendicular component (smaller normal reaction).*

BINDER Example 14

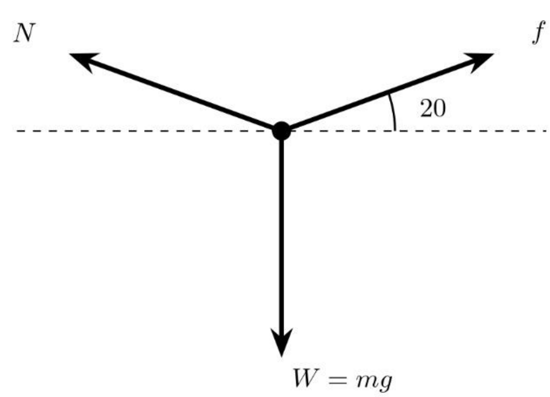
A block of mass 20kg rests on a rough inclined plane inclined at $20°$ to the horizontal. The block is on the verge of sliding down the plane. The coefficient of static friction between the block and the plane is $\mu = 0.364$. Take $g = 9.8 m/s^2$.

Calculate:

(a) The normal reaction between the block and the plane.
(b) The frictional force acting on the block.
(c) Verify that the block is in limiting equilibrium.

Solution

Forces acting on the block are shown in the following free body diagram.



Resolving weight into components:

Component parallel to plane (downward):

$$W_{parallel} = mgsin\theta = 20 \times 9.8sin20° = 196Nsin20°$$

Component perpendicular to plane (into surface):

$$W_{perp} = mgcos\theta = 20 \times 9.8cos20° = 196Ncos20°$$

Forces acting perpendicular to plane:

- Normal reaction (**away** from surface), R.
- Perpendicular component of weight (**into** surface), 196Ncos20°.

Since the two forces act in opposite directions and the block is at equilibrium (it is just on verge of sliding, not yet moving):

$$R = 196Ncos20° = 184.15N$$

(a) The normal reaction is 184.15N.

Forces acting parallel to plane:

- Friction force (upward), f.
- Parallel component of weight (downward), $W_{parallel} = 196Nsin20$.

Again, the two forces act in opposite directions and the block is still at equilibrium. So:

$$f = 196Nsin20 = 67.04N$$

(b) The frictional force is 67.04N.

Limiting equilibrium is found when: $f_{max} = \mu R$

Substituting $f_{max} = 0.364 \times 184.15N = 67.03N \approx 67.04N = f$ (found in (b))

(c) Since the actual frictional force is equal to the limiting friction, the block is in limiting equilibrium.

Alternative verification

Using the condition $\boldsymbol{\mu = tan\theta}$ at limiting equilibrium.

$\tan 20° = 0.364$

Given $\mu = 0.364$

Since μ ≈ tanθ, limiting equilibrium is confirmed.

Making Sense of the Answer: *At limiting equilibrium, friction reaches its maximum value, which exactly balances the down-slope component of weight. Any slight increase in angle or decrease in μ would cause sliding. This is why* $\mu = tan\theta$ *is a useful relationship.*

Thinking Like a Physicist: *Limiting equilibrium represents the boundary between static equilibrium and motion. Understanding this threshold is crucial in engineering: roads must have sufficient friction (high μ) or low slope (small θ) to prevent vehicles from sliding, especially in rain when μ decreases.*

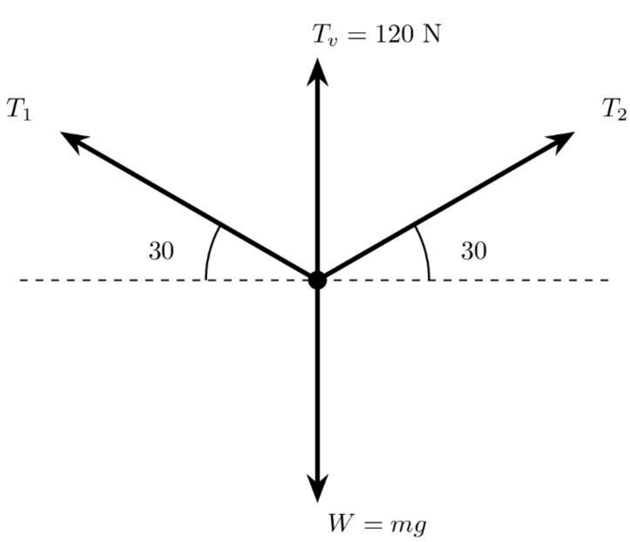
HOT Example 15

A 20kg signboard hangs from a vertical cable attached to its center and two side cables. Each side cable is attached to the edge of the signboard and makes 30° with the horizontal, pulling outward to wall supports. If the vertical cable has tension 120N, calculate the tension in each side cable.

$$\text{Take } g = 9.8\text{m/s}^2.$$

Solution

All forces acting on the signboard are shown in the following free body diagram.



Resolving each force to horizontal and vertical component:

For T_1:

$$\theta = \text{angle measured from positive x} - \text{axis} = 180° - 30° = 150°$$

Horizontal component $= T_1 \cos 150° = -T_1 \cos 30°$

Vertical component $= T_1 \sin 150° = T_1 \sin 30°$

For T_v:

Horizontal component $= 0$

Vertical component $= 120N$

For T_2:

$$\theta = \text{angle measured from positive x} - \text{axis} = 30°$$

Horizontal component $= T_2 \cos 30°$

Vertical component $= T_2 \sin 30°$

For weight:

Horizontal component $= 0$

Vertical component $= -mg = -20 \times 9.8N = -196N$

From equilibrium equations: : $\Sigma F_x = 0$ and $\Sigma F_y = 0$

Then;

$\Sigma F_x = -T_1 \cos 30° + T_2 \cos 30° = 0$ or

$T_2 \cos 30° = T_1 \cos 30°$; $\mathbf{T_2 = T_1}$

And;

$\Sigma F_y = T_1 \sin 30° + 120 + T_2 \sin 30° - 196 = 0$ or

$T_1 \sin 30° + T_2 \sin 30° = 76$

But $T_2 = T_1$

$T_1 \sin 30° + T_1 \sin 30° = 76$; $2T_1 \sin 30° = 76$ or $T_1 = 76N$

Tension in each side cable is 76N.

Making Sense of the Answer: *The signboard weighs 196N downward, with 120N already supported by the vertical cable. The remaining 76N is shared by the two side cables through their vertical components. The side cables also provide horizontal stabilization, preventing the signboard from swinging.*

Thinking Like a Physicist: *Real engineering often involves over-constraining systems for safety. A signboard could hang from a single cable, but using three distributes the load and provides backup support. If one side cable fails, the board may tilt, but it does not fall immediately.*

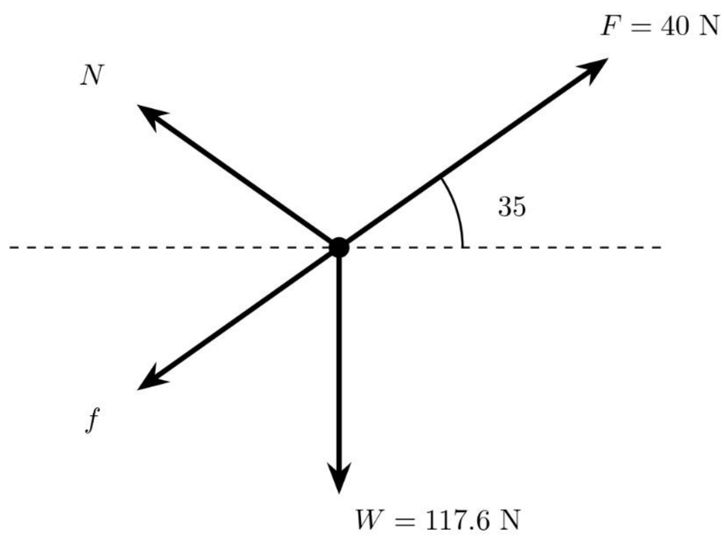
HOT Example 16

A 12kg block rests on a rough plane inclined at 35° to the horizontal. A force of 40N acts on the block parallel to and up the plane. The block is in equilibrium. The coefficient of static friction between block and plane is $\mu = 0.6$. Take $g = 9.8 m/s^2$.

(a) Calculate the normal reaction.
(b) Determine the magnitude and direction of the frictional force.
(c) Verify that the block is not at limiting equilibrium.

Solution

All forces acting on the block are shown in the following free body diagram.



(a) Forces acting perpendicular to plane are only normal reaction, **R** and the perpendicular component of weight.

Since the two forces act in opposite directions;

$$R = mg\cos\theta = 12 \times 9.8 \times \cos 35° = 96.3N$$

The normal reaction is 96.3N.

(b) Forces acting parallel to the plane.

- Parallel weight component (downward): $W_P = -mg\sin\theta = -117.6\sin 35 = -67.4N$
- Applied force (upward): $F = 50N$
- Frictional force (unknown direction): f

Applying equilibrium equation parallel to the plane:

$$-67.4N + 40N + f = 0; f = 27.4N \text{ (upward as it is positive)}$$

Direction determination: since weight component down the plane (67.4N) is larger than the applied force (40N) up the plane, the tendency is to slide **down** the plane, so friction must act **up the plane** to prevent the downward motion.

The frictional force is 27.4N up the plane.

(c) Limiting friction: $f_{max} = \mu R = 0.6 \times 96.3 = 57.8N$

But actual friction required for equilibrium is 27.4N, which is less than 57.8N.

Since the actual frictional force is less than the limiting friction, the block is not at limiting equilibrium.

Making Sense of the Answer: *The applied force* $(40N)$ *helps support the block but isn't strong enough to overcome the down-slope pull* $(67.4N)$*. Friction makes up the difference* $(27.4N)$*. Together, the applied force and friction provide* $67.4N$ *up the slope, exactly balancing the down-slope weight component. The large safety margin* $(57.8N$ *maximum vs* $27.4N$ *actual) means the block is very stable.*

Thinking Like a Physicist: *This problem illustrates an important principle:* ***friction direction depends on motion tendency, not on which forces are present****. Always first determine which way the block would move if friction were absent, then friction acts opposite to that tendency. In this case, removing friction would cause downward motion* $(67.4N > 40N)$*, so friction acts upward.*

With these worked examples now neatly packed away, let us move on and meet the next subtopic; it has been waiting patiently to join the conversation.