

MODULE 11

MASTERING NEWTON'S SECOND LAW

From Balanced Forces to Unbalanced Forces

In Newton's first law of motion, we examined situations in which forces balance. We saw that when the resultant force on a body is zero, the body either remains at rest or continues moving with constant velocity. In simple terms:

If resultant force = 0N, then acceleration = 0m/s².

We now take the next step. Most real-life motion is not perfectly balanced. Vehicles start, stop, and turn. Balls speed up or slow down. A crate begins to move when pushed hard enough. All of these happen because the forces are not balancing.

This leads to a more general and more interesting case: *When the resultant force is not zero, what exactly happens to its motion?*

Newton's second law answers this question with both meaning and mathematics. It can be stated as follows:

The acceleration of a body is directly proportional to the resultant force acting on it and inversely proportional to its mass, and occurs in the direction of the resultant force.

This statement contains four key ideas:

- 1) Resultant force causes acceleration.
- 2) For a fixed mass, a greater resultant force produces greater acceleration.
- 3) For a fixed resultant force, a larger mass experiences smaller acceleration.
- 4) The direction of acceleration is the same as the direction of the resultant force.

These ideas form the foundation of all force–motion analysis in mechanics.

Mathematical Form of the Law

For motion involving constant mass, Newton's second law can be written in the simple mathematical form:

$$F = ma$$

Where:

F is the resultant force acting on the body in N,

m is the mass of the body in kg,

a is the acceleration produced in m/s^2 .

This equation does not introduce a new idea; it simply expresses, in mathematical form, the physical meaning of the law. This form is the special case (when the mass of the body remains constant) of general form of Newton's second law which can be stated as follows:

The resultant force acting on a body is equal to the rate of change of its momentum and acts in the same direction as that change.

Mathematically:

$$F = \frac{dp}{dt}$$

Where:

F is the resultant force in N.

p is the momentum of the body in kgm/s ,

t is time in s.

This general form is more powerful than $F = ma$ because it is valid even in cases where *the mass changes* or where *force acts over a short time interval*. In other words; $F = ma$ is contained in $F = dp/dt$ as shown below:

Using $p = mv$

It follows that:

$$F = \frac{dp}{dt} = \frac{d(mv)}{dt} = v \frac{dm}{dt} + m \frac{dv}{dt}$$

So mathematically, the second law can be written as:

$$F = v \frac{dm}{dt} + m \frac{dv}{dt}$$

If mass is constant:

$$\frac{dm}{dt} = 0$$

The above equation reduces to:

$$F = m \frac{dv}{dt}$$

But;

$$\frac{dv}{dt} = a$$

Hence;

$$F = ma$$

Be careful!

Newton's second law does **not** say that force creates motion. Instead, it says that force creates **changes in motion**. In other words, it says that force produces acceleration (change in velocity), not velocity.

In closing the discussion of Newton's second law, we emphasize that *the direction of a body's acceleration is always the same as the direction of the resultant force acting on it*. From this, the following three important conclusions can be drawn:

- 1) If the resultant force acts in the same direction as the velocity of the body, the body accelerates and its speed increases.
- 2) If the resultant force acts in the opposite direction to the velocity of the body, the body decelerates and its speed decreases.
- 3) If the resultant force acts perpendicular to the velocity of the body, the speed of the body remains constant but its direction of motion changes.

This final case prepares the ground for motion in two dimensions, particularly circular motion, where acceleration is always perpendicular to velocity.

To truly understand and enjoy these ideas, let us serve them in the form of worked examples.

BINDER Example 7

A body of mass 3kg moves along a straight line. Two forces act on it: a driving force of 18N forward and a resistive force of 6N backward. At a certain instant its velocity is 4m/s forward.

- (a) Find the acceleration of the body.
- (b) Find its velocity after 5s.

Solution

$$\text{Resultant force} = 18\text{N} - 6\text{N} = 12\text{N}$$

$$\text{Using } F = ma$$

$$12\text{N} = 3\text{kg} \times a; a = 4\text{m/s}^2$$

$$\text{Using } v = u + at$$

$$v = 4\text{m/s} + (4\text{m/s}^2)(5\text{s}); v = 24\text{m/s}$$

(a) The acceleration is 4m/s^2 forward.

(b) The velocity after 5s is 24m/s forward.

Making Sense of the Answer: *A forward resultant force should increase velocity. A large increase is expected after 5s of steady acceleration.*

Think Like a Physicist: *Always find the resultant force first; then acceleration follows from F/m .*

BINDER Example 8

A body of mass 2kg moves along a straight line. A driving force of 10N acts forward while a frictional force of 2N acts backward. Its velocity changes from 3m/s to 13m/s in a time t . Find the value of t .

Solution

$$\text{Resultant force} = 10\text{N} - 2\text{N} = 8\text{N}$$

$$\text{Using } F = ma$$

$$8\text{N} = 2\text{kg} \times a; a = 4\text{m/s}^2$$

$$\text{Using } v = u + at$$

$$13\text{m/s} = 3\text{m/s} + (4\text{m/s}^2)t$$

$$10\text{m/s} = (4\text{m/s}^2)t; t = 2.5\text{s}$$

The time taken is 2.5s.

Making Sense of the Answer: *An acceleration of 4m/s^2 adds 4m/s each second, so gaining 10m/s should take 2.5s.*

Think Like a Physicist: *If resultant force is constant, acceleration is constant, so velocity changes linearly with time.*

BINDER Example 9

A body of mass 5kg moves along a straight line with constant velocity. Two forces act along the line: a driving force forward and a resistive force backward. State the relationship between the two forces and explain.

Solution

The relationship:

The driving force is equal in magnitude to the resistive force.

Explanation:

Constant velocity means acceleration is 0m/s^2 . By Newton's second law, resultant force = mass \times acceleration, so resultant force = 0N . Therefore, the forces along the line must balance, meaning the forward and backward forces are equal in magnitude.

Making Sense of the Answer: *If one force were larger, the velocity would not remain constant; it would increase or decrease.*

Think Like a Physicist: *Constant velocity does not mean no forces; it means zero resultant force.*

REAL Example 10

Kipute rides a bicycle along a straight road. While she pedals, the bicycle gains velocity. When she stops pedalling, the bicycle continues moving but its velocity

gradually decreases until it stops. Use Newton's second law to explain both phases of motion.

Solution

During pedalling, the driving force from the chain and wheel-ground interaction exceeds resistive forces (friction and air resistance), giving a forward resultant force. A forward resultant force produces forward acceleration, so velocity increases. When she stops pedalling, the driving force drops greatly while friction and air resistance remain backward. The resultant force becomes backward, giving backward acceleration (deceleration), so velocity decreases.

Making Sense of the Answer: *This matches everyday riding: you gain velocity when pushing forward, and you lose velocity when the forward push disappears.*

Think Like a Physicist: *Speeding up or slowing down depends on whether the resultant force is in the same direction as, or opposite to, the velocity.*

REAL Example 11

Mr. Akilikubwa pushes a loaded trolley with a nearly steady push, but the trolley's velocity increases slowly at first and then increases more noticeably after it has been moving for some time. Explain why the acceleration may increase even when the applied force is nearly constant.

Solution

Reason:

The resistive force can decrease after motion begins, increasing the resultant force and therefore increasing acceleration.

Explanation:

At the start, frictional effects (especially static friction or higher starting resistance)

can be larger. Once the trolley is moving, the resistive force may reduce slightly (rolling becomes smoother, wheels align, surfaces settle). If the applied force stays nearly constant but the resistive force decreases, the resultant force increases. By Newton's second law, a larger resultant force produces a larger acceleration.

Making Sense of the Answer: *Many objects feel hardest to start moving; once rolling smoothly, the same push produces more noticeable change in motion.*

Think Like a Physicist: *Acceleration changes when the resultant force changes, even if the applied force remains the same.*

HOT Example 12

A body of mass 4kg moves along a straight line with velocity 12m/s. A constant resistive force of 10N acts opposite to its motion. At the same time, a driving force of 26N acts in the direction of motion for 3s, after which the driving force is removed but the resistive force continues.

- (a) Find the acceleration during the first 3s.
- (b) Find the velocity after 3s.
- (c) After the driving force is removed, find the additional time taken for the body to come to rest.

Solution

(a) **First 3s:**

$$\text{Resultant force} = 26\text{N} - 10\text{N} = 16\text{N}$$

Using $F = ma$

$$16\text{N} = 4\text{kg} \times a, a = 4\text{m/s}^2$$

The acceleration during the first 3s is 4m/s^2 .

(b) Using $v = u + at$

$$v = 12\text{m/s} + (4\text{m/s}^2)(3\text{s}), v = 24\text{m/s}$$

The velocity after 3s is 24m/s.

(c) After 3s, only resistive force acts opposite to the motion direction. So resultant force = -10N .

Using $F = ma$

$$-10\text{N} = 4\text{kg} \times a; a = -2.5\text{m/s}^2$$

Now use $v = u + at$ with $v = 0\text{m/s}$, $u = 24\text{m/s}$

$$0\text{m/s} = 24\text{m/s} + (-2.5\text{m/s}^2)t$$

$$2.5t = 24; t = 9.6\text{s}$$

The additional time to come to rest is 9.6s.

Making Sense of the Answer: *While the driving force acts, resultant force is forward so velocity increases. After it is removed, only motion resistance remains so the body must slow down to rest.*

Think Like a Physicist: *When forces change in stages, analyze the motion in stages as well, using the final velocity of one stage as the initial velocity of the next.*

HOT Example 13

A body of mass 2kg moves along a straight line. A constant driving force of 14N acts forward. The resistive force is not constant; it increases with velocity and is given by $R = 2v$, where R is in N and v is in m/s. At a certain instant, the velocity is 4m/s.

(a) Find the resistive force at that instant.

(b) Find the resultant force at that instant.

(c) Find the acceleration at that instant.

Solution

$$(a) R = 2v = 2(4\text{m/s}) = 8\text{N}$$

$$(b) \text{Resultant force} = 14\text{N} - 8\text{N} = 6\text{N}$$

(c) Using $F = ma$

$$6\text{N} = 2\text{kg} \times a; a = 3\text{m/s}^2$$

The acceleration is 3m/s^2 .

Making Sense of the Answer: Higher velocity gives higher motion resistance, so the resultant force becomes smaller than the driving force, giving a moderate acceleration.

Think Like a Physicist: *If resistive force depends on velocity, acceleration is not constant; evaluate forces at the instant asked.*

HOT Example 14

A body moves along a straight line with constant velocity 15m/s . A constant resultant force of 12N then acts opposite to its motion. After 5s , the body starts to reverse its direction of motion. Find the mass of the body.

Solution

Since velocity varies continuously, the instant at which the body reverses direction is the instant at which its velocity becomes zero. Therefore, the body's velocity is momentarily zero at $t = 5\text{s}$.

Using $v = u + at$; where: $v = 0\text{m/s}$, $u = 15\text{m/s}$, $t = 5\text{s}$

$$0\text{m/s} = 15\text{m/s} + a(5\text{s}); a = -3\text{m/s}^2$$

Now use $F = ma$; where: $F = -12\text{N}$, $a = -3\text{m/s}^2$

$$-12\text{N} = m(-3\text{m/s}^2); m = 4\text{kg}$$

The mass of the body is 4kg.

Making Sense of the Answer: *A larger mass would resist the change more and would take longer to stop and reverse under the same force, so 4kg is reasonable.*

Think Like a Physicist: *“Reverses direction” usually means the velocity becomes zero at the turning instant, then changes sign.*

HOT Example 15

A force acts on a body and changes its momentum from 6kgm/s to 18kgm/s in 0.4s along a straight line. Assume the force is constant during this time.

- (a) Find the resultant force.
 (b) If the mass of the body is 3kg, find the change in velocity during this time.

Solution

(a) Using $F = \frac{\Delta p}{\Delta t}$

Change in momentum, $\Delta p = 18\text{kgm/s} - 6\text{kgm/s} = 12\text{kgm/s}$

Time, $\Delta t = 0.4\text{s}$

$$F = \frac{\Delta p}{\Delta t} = \frac{12\text{kgm/s}}{0.4\text{s}}; F = 30\text{N}$$

The resultant force is 30N.

(b) Using $p = mv$, change in momentum $\Delta p = m\Delta v$

$$12\text{kgm/s} = 3\text{kg} \times \Delta v; \Delta v = 4\text{m/s}$$

The change in velocity is 4m/s.

Making Sense of the Answer: *Even when a force acts for a very short time, a sufficiently large force can produce a significant change in momentum.*

Think Like a Physicist: *When a force acts for a short time, it is often more efficient to use the momentum form of Newton's second law rather than $F = ma$.*

HOT Example 16

A body of mass 6kg moves along a straight line with velocity 18m/s. A constant resistive force of 9N acts opposite to the motion until the body comes to rest.

- (a) Find the acceleration of the body.
- (b) Find the time taken to come to rest.
- (c) Find the distance travelled before coming to rest.

Solution

(a) Resultant force = -9N

Using $F = ma$

$$-9\text{N} = 6\text{kg} \times a; a = -1.5\text{m/s}^2$$

The acceleration is -1.5m/s^2 .

(b) Using $v = u + at$; where: $v = 0\text{m/s}$, $u = 18\text{m/s}$, $a = -1.5\text{m/s}^2$

$$0\text{m/s} = 18\text{m/s} + (-1.5\text{m/s}^2)t$$

$$1.5t = 18; t = 12\text{s}$$

The time taken to come to rest is 12s.

(c) Using $s = ut + \frac{1}{2}at^2$

$$s = (18\text{m/s})(12\text{s}) + \frac{1}{2}(-1.5\text{m/s}^2)(12\text{s})^2$$

$$s = 216\text{m} - 0.75(144)\text{m}$$

$$s = 216\text{m} - 108\text{m}$$

$$s = 108\text{m}$$

The stopping distance is 108m.

Making Sense of the Answer: *A small resistive force gives a small deceleration, so the body takes a long time and a long distance to stop.*

Think Like a Physicist: *Once you have acceleration, use kinematics to connect velocity change to time and distance.*

The worked examples have filled the plate nicely; now it is time to enjoy the third law and see what new ideas it brings to the table.