MODULE 7
MIXED MOTION WORKED EXAMPLES

Mixed Motion Worked Examples bring together the key ideas of straight-line motion into realistic situations where concepts must work together rather than separately. In real life, motion rarely appears in neat textbook pieces: a vehicle may accelerate, slow down, change direction, or combine several effects at once. These examples are designed to help you connect distance, displacement, speed, velocity, acceleration, and equations of motion into one coherent picture. Do not rush through them; the goal is not just getting answers, but developing the confidence to interpret motion logically and comfortably. Once these ideas connect, straight-line motion becomes less intimidating and much more enjoyable.

MISCELLANEOUS WORKED EXAMPLES ON MOTION IN STRAIGHT LINE

Example 21

(a) A daladala moves east. Kipanga walks inside it. Explain clearly what "Kipanga's velocity relative to the ground" means, and why it depends on direction.

(b) A daladala moves at 15m/s. Kipanga inside the daladala walks forward at 2m/s relative to the daladala.

(i) Find Kipanga's speed relative to the ground.

(ii) If he instead walks toward the back at 2m/s, find his speed relative to the ground.

Solution

(a) "Velocity relative to the ground" means **the speed and direction at which Kipanga's position changes as measured by an observer standing on the ground**. It depends on direction because if Kipanga walks in the **same direction** as the daladala, his ground velocity increases (the velocities add). If he walks in the **opposite direction**, his ground velocity decreases (the velocities subtract). So direction changes the final velocity even if the walking speed is the same.

(b) **For the same direction:**

$$\text{Kipanga's speed relative to ground}$$
$$= \text{bus speed} + \text{Kipanga's speed relative to bus}$$
$$15\text{m/s} + 2\text{m/s} = 17\text{m/s}$$

(i) The Kipanga's speed relative to the ground is 17m/s.

For the opposite direction:

$$\text{Kipanga's speed relative to ground}$$
$$= \text{bus speed} - \text{Kipanga's speed relative to bus}$$
$$15\text{m/s} - 2\text{m/s} = 13\text{m/s}$$

(ii) The Kipanga's speed relative to the ground is 13m/s.

Example 22

(a) Explain why distance can be large even when displacement is zero.

(b) A daladala cruised past a roadside checkpoint at a steady 25m/s. The officer signaled it to stop, but the driver kept going. The officer watched it pull away for 2s, then jumped onto a police bike. Starting from rest, he accelerated uniformly at 4m/s^2, determined to catch the daladala, which continued at a constant 25m/s.

(i) How many seconds after the daladala passed the checkpoint does the police catch it?

(ii) How far from the checkpoint does the catch occur?

Solution

(a) Distance measures the total path covered, while displacement depends only on the starting and final positions. So if you return to the starting point, displacement becomes zero even though distance is not.

(b) Let ***t*** be the time (in seconds) measured from the instant the daladala passes the checkpoint to the moment it is caught by the police.

Solution

Daladala motion

The daladala moves at a constant velocity of 25m/s.

And for the body moving with constant velocity; $\text{displacement} = \text{velocity} \times \text{time}$

So after time t, its distance from the checkpoint is: $s_d = 25t$

Police motion

The police starts chasing 2s later.

So by time t, the police has been moving for: $t_p = t - 2$

The police starts from rest and accelerates uniformly at 4m/s^2.

Distance covered by the police in time t_p is given by: $s = ut + 1/2at^2$

So $s_p = \frac{1}{2}a\left(t_p\right)^2$ $\left(u = 0 \text{ and thus } ut_p = 0\right)$

$s_p = 1/2 \times 4(t - 2)^2$ (Substituting $t_p = t - 2$)

$s_p = 2(t - 2)^2$

Catching condition

The catch occurs when both are at the same position:

$$s_d = s_p$$

So:

$$2(t - 2)^2 = 25t$$

From which;

$$2t^2 - 33t + 8 = 0$$

Solving gives: $t = 12.5s$

(i) The police catches the daladala 12.5s after it passed the checkpoint.

Distance of the catch

$$s_d = 25t = 25m/s \times 12.5s = 312.5m$$

(ii) The catch occurs 312.5m from the checkpoint.

Be careful!

Many students mistakenly write $t_p = t + 2$ instead of $t_p = t - 2$.

That is wrong because t is measured from the moment the daladala passes the checkpoint.

The police starts **2 seconds later**, so by time, t, the police has been moving for **less time**, not more.

Therefore:

$$\text{Police time} = \text{Total time} - \text{Delay} = t - 2$$

Hence, using $t + 2$ would mean the police started **before** the daladala passed the checkpoint, which contradicts the story and leads to a wrong model.

Example 23

(a) A car moves at constant speed in a circle. Explain why it is accelerating even though the speed is constant.

(b) Car A moves east at 18m/s, Car B moves west at 12m/s.

(i) Find their closing speed.

(ii) If they are 450m apart, find time to meet.

Solution

(a) Velocity changes when direction changes. In circular motion the direction changes continuously, so velocity changes and acceleration exists.

(b) The two cars were moving in the opposite directions (one east and another west) and thus:

$$\text{Closing speed} = \textbf{relative speed of approach} = 18\text{m/s} + 12\text{m/s} = 30\text{m/s}$$

(i) The closing speed is 30m/s.

If the distance (in metres) travelled by car A up to a meeting point is x, then the distance travelled by car B to the same point will be $450 - x$.

Then, using: distance $=$ speed $\times$ time;

$$x = 18t \ldots\ldots(i)$$

And;

$$450 - x = 12t \ldots\ldots(ii)$$

Substituting (i) to (ii) gives;

$$450 - 18t = 12t; t = 15\text{s}$$

Alternative solution

The time to the meeting point can be found more directly by using the fact that:

$$t = \frac{\text{Distance}}{\text{Closing speed}} = \frac{450\text{m}}{30\text{m/s}} = 15\text{s}.$$

The time to the meeting point is 15s.

Example 24

(a) Explain why in real life average speed is not always the mean of the starting and final speeds.

(b) A test driver sends a car straight forward along a road for 400m in 30s. He then immediately reverses along the same line and travels 300m back in 50s. Calculate:

- (i) the average **speed** of the car,
- (ii) the average **velocity** of the car.

Solution

(a) Average speed is defined as total distance divided by total time. It depends on how the speed changes during the journey, not only on the starting and final speeds. The mean of the starting and final speeds, $(u + v)/2$, gives the average speed **only when the acceleration is uniform (constant) and the motion is along a straight line with no change of direction**. In most real motions the acceleration is not constant (or the motion involves stopping, changing direction, or different road conditions), so the speed does not increase or decrease evenly with time. In such cases, $(u + v)/2$ is not reliable, and the correct average speed must be found from total distance and total time.

(b) **Total distance travelled**

The car moves 400m forward and 300m backward.

$$\text{Total distance} = 400\text{m} + 300\text{m} = 700\text{m}$$

Total time taken

$$\text{Total time} = 30\text{s} + 50\text{s} = 80\text{s}$$

(i) Average speed

$$\text{Average speed} = \text{total distance} \div \text{total time}$$

$$\text{Average speed} = \frac{700\text{m}}{80\text{s}} = 8.75\text{m/s}$$

The average speed is 8.75m/s.

(ii) Average velocity

Forward displacement $= +400\text{m}$

Backward displacement $= -300\text{m}$

$\text{Net displacement} = 400\text{m} - 300\text{m} = 100\text{m}$ forward

$$\text{Average velocity} = \text{displacement} \div \text{total time}$$

$$\text{Average velocity} = \frac{100\text{m}}{80\text{s}} = 1.25\text{m/s}$$

The average velocity is 1.25m/s forward.

Interesting!

For the same motion, the magnitude of the average velocity is much smaller than the average speed. This is because the car covered a long total distance (700m), so its average speed is fairly large. However, it reversed for most of the journey, so its net displacement is only 100m from the starting point. Since average velocity depends on displacement (not total distance), its magnitude becomes much smaller.

Example 25

(a) Kipute argues: *"At the highest point, the ball stops, so acceleration must be zero."* Explain why this argument is incorrect.

(b) A ball is projected vertically upward from the ground with initial speed u and reaches a maximum height H.

(i) Show that the time to reach the highest point is $t_{up} = \frac{u}{g}$.

(ii) Show that the maximum height is $H = \frac{u^2}{2g}$.

(iii) For $u = 24.5\text{m/s}$, find H. Take $g = 9.8\text{m/s}^2$.

Solution

(a) The argument is incorrect because stopping does not mean acceleration is zero. At the highest point the velocity becomes 0m/s momentarily, but gravity is still acting downward. Since gravity acts continuously, the acceleration remains downward with magnitude g.

(b)

(i) At the highest point, $v = 0\text{m/s}$.

Using**:** $v = u + at$, where: $a = -g$:

$$0 = u + (-g)t_{up}$$

So $gt_{up} = u$

From which; $\mathbf{t_{up} = \frac{u}{g}}$.

(ii) Using: $v^2 = u^2 + 2as$, with $v = 0\text{m/s}, a = -g$, and displacement to the top, $s = H$:

$$0 = u^2 + 2(-g)H$$

$$0 = u^2 - 2gH$$

So $2gH = u^2$

From which; $\mathbf{H} = \frac{\mathbf{u^2}}{\mathbf{2g}}$.

(iii) Substituting $u = 24.5\text{m/s}, g = 9.8\text{m/s}^2$:

$$H = \frac{(24.5\text{m/s})^2}{2 \times 9.8\text{m/s}^2} = 30.625\text{m}$$

Thus, $H = 30.625\text{m}$.

Example 26

(a) Explain why two stones of different masses fall with the same acceleration when air resistance is neglected.

(b) Kipute says: *"A ball thrown vertically upward with initial speed 14.7m/s reaches the highest point after 3s. I used the value of g as* $9.8m/s^2$*."* Without reworking the whole problem, show that Kipute's claim is incorrect.

Solution

(a) Gravity gives the same acceleration, g to all bodies because acceleration due to gravity is independent of mass (when air resistance is ignored).

(b) This can be done by calculating the value of g (constant) by using Kipute's claimed values then comparing it with the given standard value.

The time to reach the highest point is given by $t_{up} = \frac{u}{g}$.

From which;

$$g = \frac{u}{t_{up}}$$

Substituting Kipute's claimed values;

$$g = \frac{14.7\text{m/s}}{3\text{s}} = 4.9\text{m/s}^2 \neq 9.8\text{m/s}^2$$

Since the calculated value of g by using Kipute's claimed values is not equal to the known value of acceleration due to gravity, Kipute's claim is incorrect.

Example 27

(a) Explain why time going up equals time coming down when an object returns to the same height (air resistance neglected).

(b) Ball A is thrown vertically upward from the ground with initial speed of 24.5m/s. At the same instant, Ball B is dropped from rest from a point 40m above the ground. Take $g = 9.8\text{m/s}^2$.

(i) Find the time when they meet.

(ii) Find the height above the ground where they meet.

Solution

(a) The motion is symmetric because the magnitude of acceleration due to gravity is constant. Consequently, the object loses speed at the same rate while rising as it gains speed while falling back to the same level.

(b) Using: $s = ut + 1/2at^2$

For ball A (thrown upward from ground):

$u = 24.5\text{m/s}, s = h_A, a = -g = -9.8\text{m/s}^2$

Substituting $h_A = 24.5t - \frac{1}{2} \times 9.8t^2$

From which:

$$h_A = 24.5t - 4.9t^2 \ldots\ldots \text{(i)}$$

For ball B (dropped from 40m):

If ball A has moved the distance of h_A, then at the meeting point, ball B will move the distance of $40 - h_A$.

So: $u = 0m/s, s = 40 - h_A, a = +g = +9.8m/s^2$

Substituting $40 - h_A = 0 \times t + \frac{1}{2} \times 9.8t^2$

From which:

$$40 - h_A = 4.9t^2 \text{ or } h_A = 40 - 4.9t^2 \dots\dots \text{(ii)}$$

Substituting (ii) to (i) gives:

$$40 - 4.9t^2 = 24.5t - 4.9t^2; t = 1.63s$$

(i) The time of meeting is $1.63s$.

Substituting the value of t in (ii) gives:

$$h_A = 40 - 4.9(1.63)^2 = 26.98m$$

The height above the ground where they meet is $26.98m$.

Alternative solution

The question can be solved by using the **relative motion method** which is shorter.

Since the two balls are moving in opposite directions:

Relative acceleration, $a_R = a_A + a_B = -9.8m/s^2 + 9.8m/s^2 = 0$

And;

Relative initial velocity, $u_R = u_A + u_B = 24.5m/s + 0m/s = 24.5m/s$

Then, the equation $s = ut + 1/2at^2$ becomes:

$$s = u_R t + 1/2a_R t^2$$

Where: $u_R = 24.5m/s, s = 40m, a_R = 0$

Substituting $40 = 24.5t$; $\mathbf{t = 1.63s}$

Also for ball A; $s = ut + 1/2at^2$

Substituting $\mathbf{h_A} = 24.5 \times 1.63 - \frac{1}{2} \times 9.8(1.63)^2 = \mathbf{26.9m}$

Example 28

(a) Two cars move toward each other. Explain why their relative speed is the sum of their speeds.

(b) Ball A is thrown vertically upward from the ground with initial velocity of 24.5m/s. One second later, ball B is thrown upward from the same point with initial velocity of 34.3m/s.

Take $g = 9.8\text{m/s}^2$.

(i) Find the time (measured from A's launch) when Ball B catches Ball A.

(ii) Find the height where they meet.

Solution

(a) If two cars move in opposite directions, the separation between them reduces by both distances each second, so relative speed adds.

(b) Using: $s = ut + 1/2at^2$

For ball A:

$u = 24.5\text{m/s}, s = h_A, t_A = t, a = -g = -9.8\text{m/s}^2$

Substituting $h_A = 24.5t - \frac{1}{2} \times 9.8t^2$

From which:

$$h_A = 24.5t - 4.9t^2 \dots\dots (i)$$

For ball B: thrown 1s later.

$u = 34.3\text{m/s}, s = h_B, t_B = t - 1, a = -g = -9.8\text{m/s}^2$

Substituting $h_B = 34.3(t-1) - \frac{1}{2} \times 9.8(t-1)^2$

From which:

$$h_B = -4.9t^2 + 44.1t - 39.2 \ldots\ldots \text{(ii)}$$

But at the catching point; $h_A = h_B$

It follows that:

$$24.5t - 4.9t^2 = -4.9t^2 + 44.1t - 39.2 \text{ or } 19.6t = 39.2; t = 2s$$

(i) The time from A's launch is 2s *(So B catches A exactly 1 second after B is launched.)*

(ii) Using $h_A = 24.5t - 4.9t^2$ (from (i))

Substituting $h_A = (24.5 \times 2 - 4.9 \times 2^2)\text{m} = 29.4\text{m}$

The height of meeting is 29.4m

Example 29

(a) State one reason why distance is always greater than or equal to the magnitude of displacement.

(b) Ball A is thrown vertically upward from the ground with initial velocity of 19.6m/s. One second later, ball B is thrown upward from the same point with velocity u_B. Find u_B so that both balls land at the same time. Take $g = 9.8\text{m/s}^2$

Solution

Distance counts the full path length, while displacement is the straight-line change from start to end, so distance cannot be smaller than displacement magnitude.

For a vertical throw returning to the same level, time of flight is $T = \frac{2u}{g}$.

So ball A time of flight:

$$T_A = \frac{2(19.6\text{m/s})}{9.8\text{m/s}^2} = 4\text{s}$$

Ball B is thrown **one second later**, so for both to land together, ball B must land at $t = 4\text{s}$ from A's launch. Therefore, ball B's time in the air must be $(4 - 1)\text{s}$ or 3s.

For ball B: $T_B = \frac{2u_B}{g}$

Substituting $\frac{2u_B}{9.8\text{m/s}^2} = 3\text{s}$

From which: $u_B = 14.7\text{m/s}$

Example 30

(a) A ball is thrown vertically upward. At some moment it is still moving upward but slowing down. Explain how this is possible.

(b) A boy wants to throw a ball from a ground so that it just reaches a roof 25m above the ground.

(i) Find the required initial speed.

(ii) Find the time to reach the roof.

Solution

(a) The ball continues moving upward because it has an initial upward velocity. However, gravity acts downward throughout the motion, opposing the upward motion. As a result, the upward velocity decreases each second until it becomes zero at the highest point.

(b) "Just reaches" means it arrives at the top point of its motion (maximum height) at the roof, so $v = 0\text{m/s}$ there.

But the maximum height is given by the following equation:

$$H = \frac{u^2}{2g}$$

From which:

$$u = \sqrt{(2gH)} = \sqrt{(2 \times 9.8 \times 25)} = 22.1\text{m/s}$$

(i) The required initial speed is 22.1m/s.

$$t_{up} = \frac{u}{g} = \frac{22.1\text{m/s}}{9.8\text{m/s}^2} = 2.26\text{s}$$

(ii) The time to reach the roof is 2.26s.

Example 31

(a) In real life, does time going up always equal time coming down for vertical throw? Explain briefly.

(b) A ball is thrown vertically upward with $u = 19.6\text{m/s}$.

(i) Find ideal time of flight.

(ii) State whether the real time of flight with air resistance is likely to be greater, smaller, or the same, and why.

Solution

(a) Not always.

Reason

With **air resistance** in real life, symmetry breaks (time coming down becomes greater than time going up) because resistive force opposes motion both upward and downward, changing the acceleration magnitude differently in each phase.

(b)

(i) Ideal time of flight: $T = \frac{2u}{g} = \frac{2(19.6)}{9.8} = \mathbf{4s}$.

(ii) The real time of flight is more likely to be **greater than 4s** because air resistance reduces downward velocity, increasing the time of descent by a greater amount than it reduces the time of ascent.

Example 32

(a) A mango and a stone are dropped at the same time from the same height. Many students expect the heavier stone to land first. In reality, they hit the ground together. Explain why.

(b) A ball is dropped from 20m and rebounds to 12.8m. Take $g = 9.8\text{m/s}^2$.

(i) Find velocity just before impact.

(ii) Find velocity just after rebound.

(iii) Find total time from release until it reaches the top of the rebound.

Solution

(a) Near the Earth's surface, gravity gives the same acceleration to all objects regardless of their mass (ignoring air resistance). Although the stone is heavier, it does not receive a greater acceleration. Both objects start from rest and gain speed at the same rate, so they reach the ground at the same time.

(b) Using: $v^2 = u^2 + 2as$, with $u = 0, a = +g = 9.8\text{m/s}^2$, and, $s = h = 20\text{m}$:

Substituting $v^2 = 0^2 + 2 \times 9.8\text{m/s}^2 \times 20\text{m}$

From which: $v = \sqrt{(2 \times 9.8\text{m/s}^2 \times 20\text{m})} = 19.8\text{m/s}$

(i) The velocity just before impact is 19.8m/s.

After the rebound, the rebound velocity will be initial velocity for ascending to the maximum height (H) of 12.8m.

Thus using:

$$H = \frac{u^2}{2g}$$

From which:

$$u = \sqrt{(2gH)} = \sqrt{(2 \times 9.8\text{m/s}^2 \times 12.8\text{m})} = 15.8\text{m/s}$$

(ii) The velocity just after rebound is 15.8m/s.

Time to reach the ground can be found by using:

$$s = ut + 1/2at^2$$

Where: $u = 0, a = +g = 9.8\text{m/s}^2, s = h = 20\text{m}$

Substituting $20 = 0 + 1/2 \times 9.8t^2$; $\mathbf{t = 2.02s}$

After rebound, time to reach the maximum height, H (12.8m) can be found by using:

$$\mathbf{t_{up}} = \frac{u}{g} = \frac{15.8\text{m/s}}{9.8\text{m/s}^2} = \mathbf{1.61s}$$

Total time $= 2.02\text{s} + 1.61\text{s} = 3.63\text{s}$

(iii) The total time from release until it reaches the top of the rebound is 3.63s.

Example 33

(a) Explain why the acceleration of a ball thrown upward is still downward even when the ball is moving upward.

(b) **Kipanga** was solving a vertical-motion problem. He calculated the time when a ball is at a height of 5m **on its way downward** and obtained 4.32s. When he presented his work, **Mr. Akilikubwa** marked the answer wrong and explained that Kipanga's mistake was taking the displacement as $s = -5\text{m}$ instead of $s = +5\text{m}$. Mr. Akilikubwa added that Kipanga had **correctly** used the acceleration due to gravity as $a = -9.8\text{m/s}^2$.

(i) Explain clearly why taking $s = -5m$ was inappropriate for a point that is 5m above the point of projection, even if the ball is moving downward at that moment.

(ii) Help Kipanga to calculate the correct time when the ball is at 5m on its way downward.

(iii) What does the time 4.32s really represent?

Solution

(a) Gravity always acts downward, so the acceleration remains downward throughout the motion. The body rises only because it was given an initial upward velocity, not because it has an upward acceleration. As it moves upward, gravity steadily reduces the upward velocity until the velocity becomes zero at the highest point.

(b)

(i) Displacement s depends on the **position of the ball relative to the starting point**, not on the direction the ball is moving at that instant. Since the ball is 5m above the point of projection, its displacement must be s = +5m whether it is rising or falling.

The fact that the ball is moving downward is shown by the **sign of velocity**, not by changing the sign of displacement.

(ii)

Obtaining value of u from the incorrectly determined value of t

Using: $s = ut + 1/2at^2$

Where: $s = -5m$, (As incorrectly used by Kipanga), $t = 4.32s$, $a = -9.8m/s^2$

Substituting Kipanga's values:

$$-5\text{m} = \text{u}(4.32\text{s}) + 1/2(-9.8\text{m/s}^2)(4.32\text{s})^2$$

From which; $\text{u} = 20.01\text{m/s}$

So Kipanga's working implies the ball was thrown with initial velocity about 20.01m/s.

Determining correct value of t

Because the ball is still 5m above the point of projection: $\text{s} = +5\text{m}$

Again using $\text{s} = \text{ut} + 1/2\text{at}^2$

Where: $\text{u} = 20.01\text{m/s}, \text{a} = -9.8\text{m/s}^2, \text{s} = 5\text{m}$

Substituting $5\text{m} = (20.01\text{m/s})\text{t} + 1/2(-9.8\text{m/s}^2)\text{t}^2$

Rearrange to the quadratic equation:

$$4.9\text{t}^2 - 20.01\text{t} + 5 = 0$$

From which; $\text{t} = 0.27\text{s}$ or 3.82s

Since the ball is on its way downward, the suitable time is the larger one which is 3.81s.

The correct time is 3.82s.

(iii) It represents the moment when the ball is **5m below** the point of projection on its way downward.

With these miscellaneous worked examples now complete, the picture should feel nicely connected. It is time to stretch our understanding a little further. Welcome to the Digging Deeper Exercise!